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DOPPLER-CANCELLED RESPONSE TO VLF GRAVITATIONAL WAVES

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16. ABSTRACT

This report discusses the interaction of long periodic gravitational waves with a three-link microwave system known as the Doppler Cancelling System. This system, which was developed for a gravitational redshift experiment, uses one-way and two-way Doppler information to construct the beat signal of two reference oscillators moving with respect to each other. The geometric optics approximation is used to derive the frequency shift produced on a light signal propagating in a gravitational wave space-time. The signature left on the Doppler-cancelled beat by bursts and continuous gravitational waves is analyzed. A comparison is made between the response to gravitational waves of the Doppler Cancelling System and that of a (NASA) Doppler tracking system which employs two-way, round-trip radio waves. A three-fold repetition of the gravitational wave form is found to be a common feature of the response functions of both systems. These two functions otherwise exhibit interesting differences.

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TECHNICAL MEMORANDUM

DOPPLER-CANCELLED RESPONSE TO VLF GRAVITATIONAL WAVES

I. INTRODUCTION

On several occasions [1,2] it has been pointed out that Doppler tracking of a distant spacecraft can, under certain conditions, provide a valid technique in the search for Very Low Frequency (VLF) gravitational radiation. The basic system consists of the Earth and a distant spacecraft considered as free masses communicating through microwaves: a monochromatic electromagnetic wave is emitted at a station, relayed by the spacecraft, and, finally, received at a station where its frequency is compared to that of a reference oscillator. The resulting frequency shift is primarily caused by the Doppler effect. Gravitational waves have been suggested among the potential causes of Doppler residuals [3]. More precisely, it has been pointed out [4] that those plane gravitational waves, for which the product of their frequency by the round-trip light time is a large integer, would produce preferred peaks in the spectra of the Doppler residuals. This result is based on a "two-way" configuration in which the light signal interacting with the gravitational wave follows a round trip [5]. Similar spectral properties can, in general, be expected for combinations of microwave links other than the two-way type just mentioned, but the response to gravitational waves will change accordingly.

Recently, NASA and the Smithsonian Astrophysical Observatory (SAO) [6] have accomplished a verification of an important quantitative prediction of the principle of equivalence. The experiment compared the frequency of a space-borne atomic clock with that of an identical clock at an Earth station by using one-way (spacecraft-Earth) and two-way (Earth-spacecraft-Earth) Doppler information. This experiment in particular demonstrated that under certain conditions a combination of one- and two-way microwave links provides, in real time, a beat signal which is independent of the otherwise dominating Doppler effect but does depend on other weaker effects.

The system which removes the Doppler effect has been called the Doppler Cancelling System, and the Doppler-cancelled signal is the corresponding observable [7].

Calling $\delta\psi_{1w}$, $\delta\psi_{2w}$ the accumulated cycles in the one-way down link and two-way link, respectively, the phase of the Doppler-cancelled beat signal is defined by

$$\delta\psi = \delta\psi_{1w} - \frac{1}{2}\,\delta\psi_{2w} \tag{1}$$

and its fractional frequency by

$$\frac{\Delta f}{f} = \frac{\Delta f}{f}\bigg)_{1w} - \frac{1}{2}\,\frac{\Delta f}{f}\bigg)_{2w} \quad , \tag{2}$$

where f is the carrier frequency and $\Delta f/f)_{1w}$ and $\Delta f/f)_{2w}$ are the frequency shifts in the one-way and two-way signals.

By definition, the Doppler cancelling technique will cancel every frequency shift which is doubled when the path length of the radio wave is doubled. For plane, long periodic gravitational waves, which may be considered as inducing time-dependent perturbations in the time of propagation of the radio signal, the frequency shift caused by the space-time curvature during the round trip is not twice the frequency shift corresponding to the one-way path, in general.[1] Thus, the Doppler cancelling technique is expected to respond to gravitational waves of sufficiently low frequency for geometric optics to apply but high enough to resonate the Earth-spacecraft system and avoid local measurements.

The purpose of this work is to identify the response of the Doppler-cancelled observable to pulses and continuous gravitational waves. Section II treats the effect of the curvature of a plane gravitational wave space-time on signal propagation. The induced frequency shift on the light signal is computed by means of a new approach which uses the well-known [8,9] analogy between a metric gravitational field and a medium with an equivalent index of refraction. The result is compared with that of a previous calculation [4] based on a different approach, and they are found to be identical. In Section III the response of the Doppler Cancelling System to bursts of gravitational waves and continuous radiation is identified. Response functions and spectra are constructed by using the mathematical methods recently proposed [10] to deal with gravitational wave effects on light propagation.

II. EFFECT OF A GRAVITATIONAL WAVE ON SIGNAL PROPAGATION

A plane gravitational wave is described, in the "transverse, traceless" gauge [11], by a space-time with a Lorentz metric

1. This holds provided that the wavelength of the gravitational wave is not too large in comparison to the Earth-spacecraft distance. Otherwise, the locality of the measurement implies zero response for every Doppler technique.

$$ds^2 = \eta_{ab}\, dx^a\, dx^b - A_+ (dx^2 - dy^2) + 2A_x\, dx\, dy \quad , \tag{3}$$

where the approximately inertial frame of reference has been oriented so that the gravitational wave travels along the z-axis. The amplitudes A_+, A_x of the two independent states of polarization are a retarded solution of the wave equation in Minkowski space-time

$$A_i = A_i\,(ct - z) \quad , \qquad i = +, x \quad . \tag{4}$$

Pointwise they satisfy the weak field condition

$$|A_i| << 1 \quad . \tag{5}$$

Perturbations of the Minkowski metric in general cause time delays in the propagation of light signals. For a wave-like perturbation, as in equation (3), the variations in accumulated phase and the frequency shift induced on a monochromatic electromagnetic wave can be computed using two approaches. One technique [4] consists of computing the frequency shift by integrating the equation of parallel transport of tetrad components of a null vector, the four-dimensional wave vector, defined by the metric (3). A different technique is used here. It consists of treating relativistic effects on a null path by geometrical optics in a three-dimensional space with an equivalent index of refraction. This procedure gives directly the time delay due to propagation and the corresponding accumulated cycles. Time differentiation gives the number of cycles counted in the unit interval of time, and the frequency shift is obtained.

Work to linear order in A and 1/c. Consider that a light signal is exchanged by an Earth station and a distant spacecraft and define the cosine directions of the geocentric position vector of the spacecraft

$$\mu = \frac{dz}{dr} \quad , \quad \beta = \frac{dx}{dr} \quad , \quad \gamma = \frac{dy}{dr} \quad ; \quad dr = (dx^2 + dy^2 + dz^2)^{\frac{1}{2}} \quad . \tag{6}$$

Note that (Fig. 1)

$$\beta^2 - \gamma^2 = (1 - \mu^2) \cos 2\phi \quad ; \quad 2\beta\gamma = (1 - \mu^2) \sin 2\phi \quad , \tag{7}$$

ϕ being the longitude of the spacecraft.

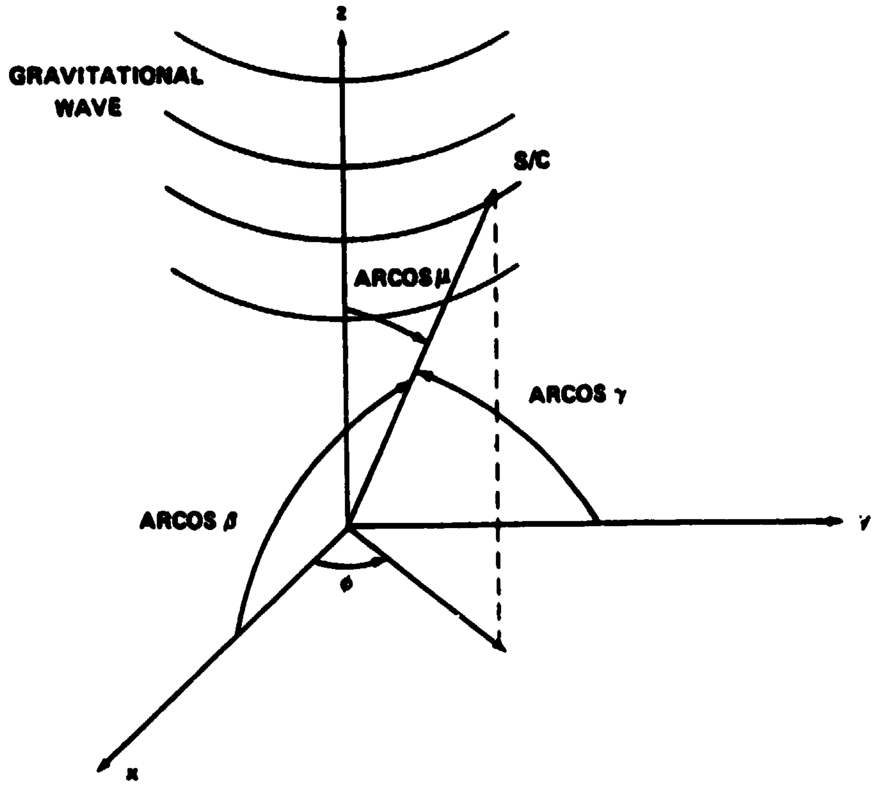

Figure 1. Gravitational wave and Earth-spacecraft geometry.

The path of a light signal in space-time satisfies

$$ds^2 = 0 \quad . \tag{8}$$

From equation (3) the time delay between emission and reception of the light signal is formally given by

$$c(t_r - t_e) = \int_{x_e}^{x_r} \left[1 + \frac{1 - \mu^2}{2} (A_+ \cos 2\phi - A_x \sin 2\phi)\right] dr \quad . \tag{9}$$

The integrand in equation (9) may be considered as an equivalent index of refraction. The integral must be extended from the Earth station, with coordinates ($\underline{r}_1$, t_1), to the spacecraft, with coordinates ($\underline{r}_2$, t_2), and back to the receiving station, with coordinates ($\underline{r}_3$, t_3). We adapt the origin of coordinates so that $\underline{r}_1 = \underline{r}_3 = 0$. Then, the parametric equations of the forward path are, in first approximation,

$$x = \beta r \quad ; \quad y = \gamma r \quad ; \quad z = \mu r \tag{10}$$

$$0 \leq x \leq x_2 \quad , \quad 0 \leq y \leq y_2 \quad , \quad 0 \leq z \leq z_2 \quad ,$$

which imply

$$A_i(ct - z) = A_i[r(1 - \mu)] \tag{11}$$

along the forward path. For simplicity, we set

$$\ell = r_2 \quad ; \quad \tilde{h} = \cos 2\phi A_+ - \sin 2\phi \, A_x \quad . \tag{12}$$

Evaluation of equation (9) along the approximately null trajectory (10) gives

$$c(t_2 - t_1) = \ell + \frac{1 + \mu}{2} \int_0^\ell \tilde{h}(u) \, du \quad , \tag{13}$$

where $u \equiv r(1 - \mu)$ is an outgoing null coordinate. The parametric equations of the backwards path are, in the same approximation,

$$x - x_2 = -\beta(r - \ell) \quad , \quad y - y_2 = -\gamma(r - \ell) \quad , \quad z - z_2 = -\mu(r - \ell) \quad , \tag{14}$$

which imply

$$A_i \, (ct - z) = A_i \, [r \, (1 + \mu)] \quad , \quad i = +, x \tag{15}$$

along the backwards path. Thus,

$$c(t_3 - t_2) = \ell - \frac{1 - \mu}{2} \int_0^\ell \tilde{h}(v) \, dv \quad , \tag{16}$$

where $v \equiv r(1 + \mu)$ is an ingoing, null coordinate. From equations (13) and (16) it follows that the total time delay for a round trip is, to linear order in the A's and 1/c,

$$c(t_3 - t_1) = 2\ell + \frac{1}{2}\left[(1 + \mu) \int_0^\ell \tilde{h}(u)\, du + (1 - \mu) \int_0^\ell \tilde{h}(v)\, dv\right] . \tag{17}$$

We define the emitted, transmitted, and received frequency as the number of cycles dN per unit of time,[2]

$$f_1 = \frac{dN}{dt_1} \quad , \quad f_2 = \frac{dN}{dt_2} \quad , \quad f_3 = \frac{dN}{dt_3} \quad , \tag{18}$$

respectively. Equations (13) and (16) give the frequency shift in the upward and downward paths, respectively.

$$\left.\frac{\Delta f}{f}\right)_{up} \equiv \frac{dt_1}{dt_2} - 1 = -\frac{\dot{\ell}}{c} + \frac{1 + \mu}{2} (\tilde{h}_1 - \tilde{h}_2)$$

$$\left.\frac{\Delta f}{f}\right)_{down} \equiv \frac{dt_2}{dt_3} - 1 = -\frac{\dot{\ell}}{c} + \frac{1 - \mu}{2} (\tilde{h}_2 - \tilde{h}_3) \quad , \tag{19}$$

where $\tilde{h}_i \equiv \tilde{h}\,(ct_i - z_i)$. The Doppler-cancelled frequency shift is

$$\frac{\Delta f}{f} = \frac{1}{2}\left(\left.\frac{\Delta f}{f}\right|_{down} - \left.\frac{\Delta f}{f}\right|_{up}\right) = -\frac{1 + \mu}{4}\tilde{h}_1 + \frac{1}{2}\tilde{h}_2 - \frac{1 - \mu}{4}\tilde{h}_3 \quad , \tag{20}$$

while the two-way frequency shift corresponding to the round trip is

$$\left.\frac{\Delta f}{f}\right)_{2w} = \left.\frac{\Delta f}{f}\right)_{up} + \left.\frac{\Delta f}{f}\right)_{down} = \frac{1 + \mu}{2}\tilde{h}_1 - \mu\tilde{h}_2 - \frac{1 - \mu}{2}\tilde{h}_3 - \frac{2\dot{\ell}}{c} \quad . \tag{21}$$

2. These definitions do not account for the time scales of the Earth and on-board clocks being offset by a nonzero relative gravitational field (redshift). This timing effect must be considered separately.

Equation (19) coincides with the one derived by integrating the equation of parallel transport of the electromagnetic wave vector, i.e., the null geodesic equation [4]. The present approach does not make explicit use of the geodesic equation. This equation is, however, automatically satisfied by virtue of equation (8) and the fact that, in geometric optics, light rays are normal to surfaces of constant phase.

III. RESPONSE TO BURSTS AND CONTINUOUS GRAVITATIONAL WAVES

From equation (20) it follows that the response of the Doppler-cancelled observable to a sufficiently short ($< \ell/c$) burst of gravitational waves consists — as for equation (21) — of a threefold repetition of the waveform, though with somewhat different weighting coefficients. In particular, for orthogonally incident gravitational waves ($\mu = 0$), the three-pulse structure is preserved.

Hellings [10] has pointed out that an optimum technique for the detection of a signature of known form in data corrupted by white noise is the matched filter. He has further shown how to construct one appropriate for the two-way Doppler observable. The same method can be used for the Doppler-cancelled observable as follows: A matched filter [12] is operationally defined by its transfer function

$$F(\omega) = \alpha\, S^*(\omega)\, e^{-i\omega\tau_m} \quad , \tag{22}$$

where α is a constant which can be precisely defined in terms of the noise power, $S^*(\omega)$ is the complex conjugate of the Fourier transform of the theoretical signature, and τ_m is the time of maximal signal-to-noise ratio. We define

$$\tau_1 = 2\ell/c \quad , \quad \tau_2 = \ell(1+\mu)/c \quad , \quad \tau_3 = 0 \tag{23}$$

$$a_1 = -\frac{1+\mu}{4} \quad , \quad a_2 = \frac{1}{2} \quad , \quad a_3 = -\frac{1-\mu}{4} \tag{24}$$

and

$$\tilde{y}(\tau) = \sum_{i=1}^{3} a_i\, \tilde{h}(\tau - \tau_i) \quad . \tag{25}$$

Equation (25) shows that if a feature is identified in the Doppler residuals at the time τ, similar features should be recognizable at previous instants τ_1 and τ_2. We assume that $\tilde{y}(\tau)$ and $\tilde{h}(\tau)$ are of integrable square over the real line and denote by $y(\omega)$, $h(\omega)$ the respective Fourier transforms. Thus, the gravitational wave signature is, in the frequency domain,

$$y(\omega) = h(\omega) \sum_{j=1}^{3} a_j e^{-i\omega\tau_j} \quad . \tag{26}$$

The corresponding matched filter is, using equation (22),

$$F(\omega) = \alpha h^*(\omega) e^{-i\omega\tau_m} \sum_{j=1}^{3} a_j e^{i\omega\tau j} \tag{27}$$

in the frequency domain and

$$\tilde{F}(\tau) = \alpha \int_{-\infty}^{\infty} \tilde{h}(\tau') g(\tau - \tau') d\tau' \tag{28}$$

in the time domain, where

$$g(\tau) = \sum_{i=1}^{3} a_i \delta[\tau - (\tau_m - \tau_i)] \quad . \tag{29}$$

We put $\tau_m = \tau_3$. From equations (28) and (29), it follows that the filtering procedure consists in matching the input signal to the theoretical signature run backwards from $t = \tau_m$ to $t = \tau_1$.

According to some astrophysical models [13], pulses coming from different events may combine stochastically, producing a signal with a higher amplitude than in the case of a pulse from a single event. It is, therefore, useful to average the filter — or, equivalently, $g(\tau)$ — over the angle of incidence. The result is

$$g_{ave}(\tau) = \frac{1}{2} \int_{-1}^{1} g(\tau)\, d\mu$$

$$= -\frac{1}{4}\left[\delta(\tau) + \delta(\tau - 2\ell/c)\right] + \begin{cases} c/4\ell & 0 < t < 2\ell/c \\ 0 & \text{elsewhere} \end{cases} \quad . \tag{30}$$

The corresponding filter is the convolution of equation (30) with the gravitational waveform $\tilde{h}(t)$. The second pulse repetition, whose position in time depends on μ, has been averaged out. The same happens for the two-way observable, which has, in fact, the same time sequence [equation (23)] as the Doppler-cancelled observable. What distinguishes equation (30) from the corresponding filter of the two-way system [equation (37) in Reference 10]

$$g_{ave,2w}(t) = \delta(t) - \delta(t - 2\ell/c) + \begin{cases} \dfrac{t - \ell/c}{(\ell/c)^2} & 0 < t < 2\ell/c \\ 0 & \text{elsewhere} \end{cases} \quad , \tag{31}$$

is the sign of the correlation between the spikes at $t = 0$ and $t = 2\ell/c$: this is positive in equation (30), negative in equation (31).

The Doppler-cancelled response to continuous gravitational waves is now considered. The autocorrelation function

$$R(\tau) = \int_{-\infty}^{\infty} \tilde{y}(\tau')\, \tilde{y}(\tau - \tau')\, d\tau' = \frac{1}{4\pi} \int_{-\infty}^{\infty} |y(\omega)|^2 e^{i\omega\tau}\, d\omega \tag{32}$$

is, for the process [equation (25)],

$$R(\tau) = \frac{1}{4\pi} \int_{-\infty}^{\infty} |h(\omega)|^2 \sum_{j,\ell=1}^{3} a_j\, a_\ell \cos \omega(\tau_j - \tau_\ell)\, e^{i\omega\tau}\, d\omega$$

$$= \frac{1}{16\pi} \int_{-\infty}^{\infty} |h(\omega)|^2 \big(1 + \cos^2 \omega\ell + \mu^2 \sin^2 \omega\ell$$
$$- 2 \cos \omega\ell \cos \omega\ell\mu$$
$$- 2\mu \sin \omega\ell \sin \omega\ell\mu\big)\, e^{i\omega\tau}\, d\omega \ . \tag{33}$$

The corresponding spectral density (one-sided)

$$S(\omega) = |y(\omega)|^2 \qquad 0 \leq \omega < \infty \tag{34}$$

is

$$S(\omega) = \frac{1}{4} |h(\omega)|^2 (1 + \cos^2 \omega\ell + \mu^2 \sin^2 \omega\ell - 2 \cos \omega\ell \cos \omega\ell\mu - 2\mu \sin \omega\ell \sin \omega\ell\mu) \quad . \tag{35}$$

For gravitational waves orthogonal to the line of sight,

$$S(\omega) = |h(\omega)|^2 \sin^4 \frac{\omega\ell}{2} \qquad (\mu = 0) \quad . \tag{36}$$

This spectrum has exactly twice the periodicity of the corresponding two-way Doppler spectrum [equation (24) of Reference 4]

$$S_{2w}(\omega) = |h(\omega)|^2 \sin^2 \omega\ell \qquad (\mu = 0) \quad . \tag{37}$$

Averaging equation (35) over μ gives

$$S_{ave}(\omega) = \frac{5}{12} |h(\omega)|^2 \left(1 + \frac{1}{5} \cos 2\omega\ell - \frac{3}{5(\omega\ell)^2} + \frac{3}{5} \frac{\cos 2\omega\ell}{(\omega\ell)^2} \right) . \tag{38}$$

Equation (38) is the spectrum of the Doppler-cancelled response function to an isotropic background of random gravitational waves.

In Figure 2 the autocorrelations of the Doppler-cancelled response (left) and of the two-way Doppler (right) are compared in the case of a flat spectrum [$h(\omega)$ = const.] at angles of incidence θ = 30°, 60°, 90°, and isotropic. The analytic expression of the two-way spectrum is [10]

$$S_{2w}(\omega) = |h(\omega)|^2 (\mu^2 + \sin^2 \omega\ell + \mu^2 \cos^2 \omega\ell - 2\mu \sin \omega\ell \sin \omega\ell\mu - 2\mu^2 \cos \omega\ell \cos \omega\ell\mu) \tag{39}$$

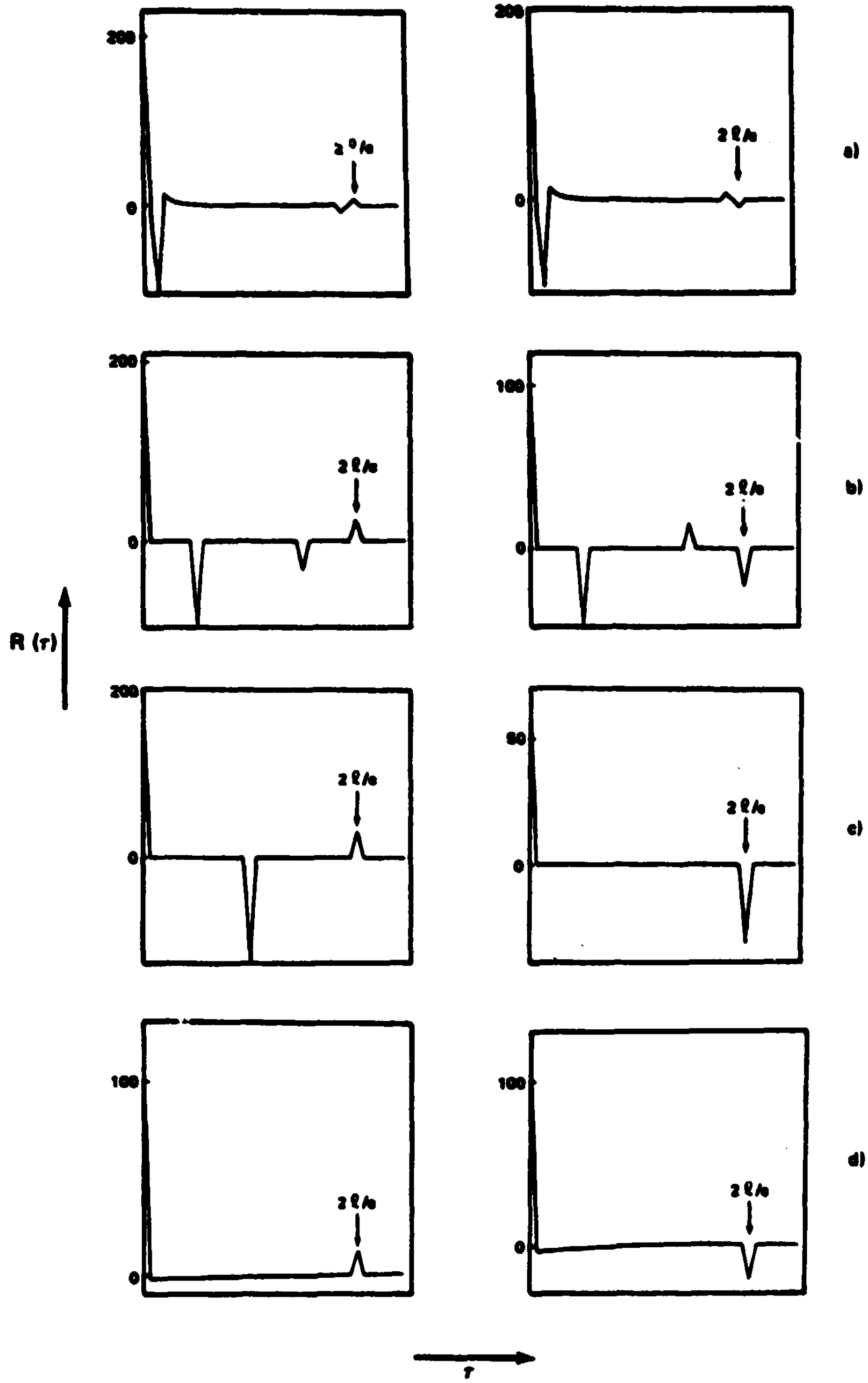

Figure 2. Three-way Doppler-cancelled (left) and two-way Doppler (right) autocorrelation function (one-sided). A constant spectrum is assumed in both cases. The angles of incidence are (a) 30°, (b) 60°, (c) 90°, (d) isotropic. The numerical values on the y-axes are defined by the number of points (128) used in the calculation.

for an angle of incidence $\theta = \cos^{-1} \mu$ and

$$S_{ave,2w}(\omega) = |h(\omega)|^2 \left(1 - \frac{1}{3} \cos 2\omega\ell - \frac{3}{(\omega\ell)^2} - \frac{\cos 2\omega\ell}{(\omega\ell)^2} + \frac{2 \sin 2\omega\ell}{(\omega\ell)^3}\right) \tag{40}$$

for averaged angle of incidence. In Figure 2 it is interesting to note that at $\theta = 90°$ the Doppler-cancelled autocorrelation has a peak at $t = \ell/c$. All the intermediate features disappear by averaging over θ. The peak at $t = 2\ell/c$ is 1/10, in the Doppler-cancelled case, and -1/6, in the two-way case, of that at $t = 0$, as is evident from the form of equations (38) and (40) in the limit of large values for $\omega\ell$.

The numerical calculation evaluated equations (35) and (39) at selected values of μ, and equations (38) and (40), each at 128 equally spaced points, with eight points in each of the intervals $2n\pi \leq \omega\ell \leq 2(n+1)\pi$, $n = 0, 1, 2, \ldots 15$. A Fast Fourier Transform routine was then used to obtain the corresponding autocorrelation function [equation (32)].

IV. CONCLUSION

As has been pointed out [14,15], the use of multiple time-correlated radio signals offers interesting possibilities not only for redshift experiments but also in the search for VLF gravitational waves. In particular, the same Doppler Cancelling System developed for the NASA-SAO redshift experiment responds to VLF gravitational waves in a precise manner. The form of the response function is identical to that of the two-way Doppler configuration as far as the timing is concerned but is otherwise different. For example, there is always a positive correlation in the Doppler-cancelled response function at two events separated by a round-trip light-time. Further, a three-peak feature is maintained in the case of gravitational waves propagating at a right angle with the line of sight to the spacecraft. This is a most interesting case, because at this angle of incidence the transverse nature of general relativistic gravitational waves would maximize the strength of the coupling with the Earth-spacecraft system.

An analysis of the influence, in the Doppler Cancelling System, of other timing and propagation effects may indicate whether the features of the Doppler-cancelled response function to VLF gravitational waves could be decoupled from those of at least some of the remaining noise sources.

REFERENCES

1. Estabrook, F. B., Hellings, R. W., Wahlquist, H. D., and Wolff, R. S.: Sources of Gravitational Radiation. Proc. of the Battelle Seattle Workshop, ed. by L. L. Smarr, Cambridge University Press, 1979, pp. 37-48.

2. Douglass, D. H. and Braginsky, V. B.: Einstein Centenary Survey. Ed. by S. W. Hawking and W. Israel, Cambridge University Press, 1980, pp. 90-137.

3. Anderson, A. J.: JPL Memorandum 391-560, NASA Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, 1974.

4. Estabrook, F. B. and Wahlquist, H. D.: Gen. Rel. and Gravit., vol. 6, 1975, pp. 439-447.

5. Anderson, J. D. and Estabrook, F. B.: J. of Spacecraft and Rockets, vol. 16, 1979, pp. 120-125; presented as Paper 78-132 at 16th Aerospace Sciences Meeting, Huntsville, Alabama, January 16-18, 1978.

6. Vessot, R. F. C., Levine, M. W., Mattison, E. M., Blomberg, E. L., Hoffman, T. E., Nystrom, G. U., Farrel, B. F., Decher, R., Eby, P. B., Baugher, C. R., Watts, J. W., Teuber, D. L., and Wills, F. D.: Phys. Rev. Letters, vol. 45, 1980, pp. 2081-2084.

7. Vessot, R. F. C. and Levine, M. W.: Gravitational Redshift Space-Probe Experiment, GP-A Project Final Report, Contract NAS8-27969, prepared for National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Huntsville, Alabama 35812. Smithsonian Institution Astrophysical Observatory, Cambridge, Massachusetts 02138, Sect. 5, 1979, pp. 21-28.

8. Weinberg, S. W.: Gravitation and Cosmology. John Wiley and Sons, Inc., Sect. 3.2, 1972.

9. Brill, D. R.: Relativity, Astrophysics and Cosmology. Proc. of the Banff Summer School, Alberta, ed. by W. Israel, D. Reidel Publishing Company, Dordrecht, Holland, 1973, pp. 127-152.

10. Hellings, R. W.: Phys. Rev., vol. D23, 1981, pp. 832-843.

11. Misner, C. W., Thorne, K. S., and Wheeler, J. A.: Gravitation. Freeman, San Francisco, California, Sect. 35.4, 1973.

12. Davenport, W. B., and Root, W. L.: An Introduction to the Theory of Random Signals and Noise. Lincoln Laboratory, McGraw-Hill Book Company, Inc., Sect. 11.7, 1958.

13. Wahlquist, H. D., Anderson, J. D., Estabrook, F. B., and Thorne, K. S.: International Symposium on Experimental Gravitation. Edited by B. Bertotti, Atti dei Covegni Lincei 34, 1977, pp. 335-350.

14. Vessot, R. F. C., and Levine, M. W.: A Close-Up of the Sun. JPL Publication 78-80, NASA Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, September 1, 1978, pp. 457-497.

15. Smarr, L. L., Vessot, R. F. C., Lundquist, C. A., and Decher, R.: Center for Astrophysics Preprint Series No. 1293, Cambridge, Massachusetts 02138. Submitted to General Relativity and Gravitation, 1980.

APPROVAL

DOPPLER-CANCELLED RESPONSE TO VLF GRAVITATIONAL WAVES

By Alessandro Caporali

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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