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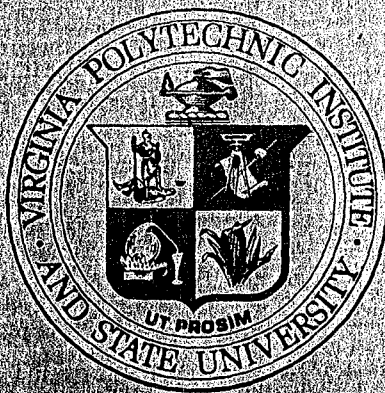
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IN DISTRIBUTED PARAMETER SYSTEMS ADAPTIVE  
IDENTIFICATION AND CONTROL

FINAL REPORT  
for NASA Grant NAG-I-7

by

C. Richard Johnson, Jr.



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THE REDUCED ORDER MODEL PROBLEM  
IN DISTRIBUTED PARAMETER SYSTEMS ADAPTIVE  
IDENTIFICATION AND CONTROL

FINAL REPORT

for NASA Grant NAG-I-7

by

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## 1. INTRODUCTION

This document is the final report on NASA Grant NAG-I-7 entitled "The Reduced Order Model Problem in Distributed Parameter Systems Adaptive Identification and Control" conducted at Virginia Polytechnic Institute and State University under the direction of C. Richard Johnson, Jr. This grant was to end September 30, 1981 but due to the recent relocation of the principal investigator and NASA policy prohibiting grant transfer, this report is being filed prior to the original termination date.

A summary of the last year's efforts (funded by the supplemental grant) follows this introduction. This technical summary is followed by a section detailing examples (some unstable) of adaptive modal control applied to a pinned-pinned beam with (slightly) inaccurate mode shape prespecification. A list of the journal and conference papers supported by NASA Grant NAG-I-7 concludes this final report.

## 2. TECHNICAL SUMMARY

The final 12 months of study summarized by this section began with the initial objective of the investigation of reduced order adaptive modal control with the intent to develop software for on-line control of the free-free beam test fixture at LaRC. Toward this objective, the effort was initially spent investigating reduced order adaptive control of the simpler, low-order, lumped parameter models for rigid structures. In this study [1], adaptive controllers having fixed orders (direct adaptive control approach) and fixed plant model orders (indirect adaptive control approach), both orders being smaller than that required to satisfy current theoretical guarantees of desired asymptotic performance and closed loop stability, were applied to several test examples. The resulting performance in each case was simulated digitally and various performance indices were tabulated. In many cases, the results reinforced the trends predicted by generally accepted "rules of thumb". However, since no general theory was available to accurately predict the results of the reduced order adaptive control, cases were found that violated the expected results. It was obvious at this point that a great deal of work remained in this area before reduced order adaptive control could be confidently applied even to the simpler, low-order lumped parameter plants studied.

During the course of the above study and in connection with discussions at the Workshop on Structural Dynamics and Control of Large Space Structures conducted at LaRC in October, and further work at VPI&SU, a second fundamental difficulty with the adaptive modal control [2] objective became more apparent. The problem was the basic assumption that an LSS could indeed be decoupled preceding the application of reduced order adaptive control. Such decoupling required that the spatial

eigenshapes characteristic of the structure be known a priori. Given the complexity and possible time varying character of likely LSS candidates, it appeared that satisfactory decoupling may require exceedingly accurate a priori mode shape approximations such as were unlikely to be available.

Consideration of this eigenshape prespecification problem, the in-house pursuit of the strategy in [2] by the LaRC Staff, and the excessive damping difficulties encountered with the LaRC test facility caused a shift, reported in [3], in emphasis away from the proposed reduced-order adaptive modal control focus. This shift was to the exploration of alternative solutions to adaptive control of LSS, in contrast to the strict modal control most often considered by the LSS community. Since a flexible structure may be "decoupled" to varying degrees by slightly inaccurate mode shapes, but very little is known about the required degree of decoupling needed to retain adequate performance (including stability), some other modeling method was required that could accurately represent the structure in all cases of decoupling error. In such a case the transfer function matrix from the actuators to the sensors is a reasonable candidate. It was therefore believed that considerable benefit could be derived from a study of more refined models from multivariable systems theory in the context of application to LSS control.

The further work for this grant period thus proceeded in two separate, yet self supporting directions. The first sought to lend support to the belief on the part of the authors that modal control was of limited use without highly accurate mode shape foreknowledge and was therefore not appropriate in all cases. The second direction explored in detail the

possibilities of the control of LSS through multivariable adaptive methods. Here, recent results in the multivariable control field were compared with respect to theoretical deficiencies and likely problems in application to LSS. These results appeared in an interim report [3] and in a conference publication [4]. As noted in [3] much work remains in this direction. In the first direction, examples were found that clearly ruled out adaptive modal control of plants whose prespecified decoupling mode shapes were in slight error. These results were cited in [4] and are presented in more detail in section 3 of this report.

Currently three topics seem of further interest:

1. Detailed relationships and transformations between modal separated-variable, nondiagonal matrix fraction and state variable descriptions. This study is required for evaluation of the effects of sensor and actuator locations and dynamics and inaccurate mode prespecification, e.g. the translation of inexact mode shape prespecification from finite element modeling to the relocation of matrix fraction singularities and to the loss of diagonal dominance in the state matrix.
2. Interpretation for flexible structures of the a priori plant structural knowledge currently required for stable multivariable adaptive control. These conditions have been recently stated as foreknowledge of the interactor matrix  $\sigma_r$ , less restrictively, of the controllability (or observability) indices. Such foreknowledge should be compared to that for adaptive modal control, i.e. adequately accurate mode shape prespecification. It appears that the two may occasionally be equivalent. Such a conjecture should be evaluated.

3. Acceptable modeling bounds for stable reduced-order, time-separated identification and control. Recently developed error bounds for reduced-order adaptive identification quantify the amount of inaccuracy in submodel identification. This region of attraction could be mapped into a "stability margin" concept for spillover due to use of the identified submodel in designing a controller for the full system. A "measurable" quantity that could be monitored indicating a satisfactory degree of identification for controller design should be sought.

We plan to pursue these objectives in subsequent research at Cornell University.

### 3. An Example of Adaptive Modal Control with Inaccurate Mode Shape Prespecification

In the literature, control of large space structures (LSS) is often based upon modal decomposition of the distributed parameter model of the structure [5, 6]. This method relies on the ability to decouple the multiple-input multiple-output (MIMO) relation between actuators and sensors into a set of independent single-input single-output (SISO) subsystems relating modal forces to modal deflections. Such a decomposition for a distributed parameter system requires an infinity of modal subsystems to exactly describe the behavior. In practice, noise levels and finite bandwidth actuators and sensors limit the number of modal subsystems required for accurate description to some large, but finite value. If the plant could be so decoupled into modal form (requiring as many sensors and actuators as modes), the parameters of this set of subsystems obtained, and sufficient processing power were available to solve the large set of SISO control problems on-line, then the desired closed loop character of the system could be assured through classical SISO design procedures.

To decouple the plant, it is required that the exact spatial mode shapes are known [7]. In light of the complex and possibly time varying character of LSS, satisfaction of this requirement is a virtual impossibility. However, if the mode shapes are not exact but very nearly so, then the coupling between approximate modal subsystems will be small and a robust modal control objective may provide acceptable closed loop performance. To provide such robust control using currently available design procedures, it is necessary that some a priori information about the structure be available. The nature of the required information, of



course, depends on the particular design procedure to be applied, but it remains an open question whether all the required foreknowledge will indeed be available for any control method. This is especially in doubt in view of the severely reduced order nature of implementable, real time controllers applied to LSS.

To reduce the amount of a priori information necessary to design a controller for LSS, adaptive control seems a promising approach. The success of adaptive control in many cases where the plant characteristics are incompletely known or where modeling inaccuracies are present (e.g. linearization of a non-linear system [8]) suggest that inaccuracies in spatial mode shapes and initial dynamic (temporal mode) parameters may be accommodated by adaptive modal control. According to the current theory, however, the application of adaptive control can provide desirable closed loop characteristics (including stability) only when very restrictive assumptions about the plant can be made. The required assumption that the correct dynamic order of the plant be known is perhaps the most obvious obstacle to the successful adaptive control of distributed parameter systems, and LSS in particular. This is a particularly difficult assumption to satisfy when the spatial mode shapes used for modal control are inaccurate since the plant cannot be decoupled into subsystems of known order.

Contrary to more commonly encountered descriptions of the advantages of particular algorithms or methods, the purpose of this section of this section of this report is to present an example of adaptive modal control of LSS that does not result in acceptable closed loop performance. It is hoped that the example will serve to provide a clearer picture of the true applicability of current adaptive algorithms for modal control of LSS.

In particular, it points out the need for a more complete understanding of the effects of spatial mode shape errors on the decoupling of a flexible structure for control purposes. Also, it presents a case for the further development of adaptive control theory when the plant order is unknown.

This segment is organized as follows. Section I presents the details of the simplified flexible plant used in the example. The plant was simplified in that the reduced order problems [7] were not present; all the component modes of the plant entered into the control design. Even so, it was found that the regulation performance was highly sensitive to errors in the mode shapes used for decoupling. These reduced order problems in the control of an actual LSS would have to be addressed, in addition to the problems encountered here, in a realistic application. The adaptive modal control is described in Section II. The question of what initial assumptions must be met is discussed there. Section III discusses the results of the digital simulation of the plant and adaptive controller in closed loop. For clarity, the simulations are partitioned into five groups. The characteristics of each group are:

- Case 1: Open loop plant simulation
- Case 2: Closed loop control - no mode shape errors/ no initial parameter errors.
- Case 3: Closed loop control - no mode shape errors/ - 5% initial parameter errors.
- Case 4: Closed loop control - shape errors per Table 1/no initial parameter errors.
- Case 5: Closed loop control - shape errors per Table 1/ - 5% initial parameter errors.
- Case 6: Closed loop control - shape errors per Table 2/ - 5% initial parameter errors.

The rather large amount of simulation data was included to allow conclusions to be drawn concerning the source of performance degradation by isolating the introduction of errors. Following Section III, a summary of results and a discussion of the outlook of the method will be given in the conclusion of part 3 of this final report.

### Section I: Plant Example

The plant used in the simulation was based on a flexible, uniform, simply supported (pinned-pinned) beam of normalized length  $\ell = 1.0$ . The behavior of such a structure in one spatial dimension can be described by the following partial differential equation (PDE) [5]

$$M \frac{\partial^2}{\partial t^2} u(x,t) + EI \frac{\partial^4}{\partial x^4} u(x,t) = F(x,t), \quad (1)$$

Where  $M$ ,  $E$ , and  $I$  are mass per unit length, Young's modulus, and section area moment of inertia, respectively, and the  $u(x,t)$  and  $f(x,t)$  are the output deflection and input force, respectively. The associated boundary conditions are

$$\begin{aligned} u(0,t) &= u(\ell,t) = 0 \\ \frac{\partial^2}{\partial x^2} u(0,t) &= \frac{\partial^2}{\partial x^2} u(\ell,t). \end{aligned} \quad (2)$$

The characteristic solutions of this PDE in the spatial coordinate  $x$  are of the form [5]

$$\phi_k(x) = \sqrt{2} \sin \frac{k\pi x}{\ell} \quad (3)$$

where the index  $k$  is the mode number.

For this example, the rigid body modes were not used, and only the first 5 flexible modes were retained to represent this simplified

structure. These mode shapes are plotted in figure 1, each having a normalized amplitude of 1.0. The modal deflection measurements at the collocated sensor/actuator locations on the beam are also shown in figure 1, indicated by asterisks on each mode shape. Note that all five modes were controllable and observable with these locations. Five sensor/actuator locations were used to assure that the modal transformations used in the control process, to be described in section II, would be invertible matrices.

The characteristic temporal solutions of the PDE in (1) each satisfy a second order ordinary differential equation of the form

$$M_i \ddot{u}_{m_i}(t) + B_u \dot{u}_{m_i}(t) + k_i u_{m_i}(t) = f_{m_i} \quad (4)$$

where the  $M_i$ ,  $B_i$ , and  $k_i$  are modal mass, modal damping, and modal stiffness respectively for the  $i^{\text{th}}$  mode. The  $u_{m_i}$  and  $f_{m_i}$  are the modal deflections and forces, respectively. To be consistent with the very lightly damped character of LSS, the  $B_i$  for each mode were chosen such that the eigenvalues for all modes were uniformly damped at 0.1% of critical damping. The mode frequencies were given the convenient values

$$\omega_k = k, \quad k = 1, \dots, 5. \quad (5)$$

The input-output relation can be represented in state space form as

$$\begin{aligned} \dot{\underline{x}}(t) &= A\underline{x}(t) + B\underline{f}(t); & \underline{x}(t): 10 \times 1, & A: 10 \times 10 \\ & & B: 10 \times 5, & \underline{f}(t): 5 \times 1 \\ \underline{u}(t) &= C\underline{x}(t) & \underline{u}(t): 5 \times 1, & C: 5 \times 10 \end{aligned}$$

In the plant simulation, the state variables represented modal states, so the spatial operators in B and C were the exact modal transformations derived from the shape measurements in figure 1. The A matrix was diagonal, having the plant eigenvalues as diagonal elements.

The open loop response of the beam to an impulse force at all actuators (simulation case 1) was as shown in figure 2. Here the output, input, and feedback are displayed in modal form, one plot for each modal signal. The digital sample interval for all simulations was  $DT = 0.1$  sec, so the plots represent 100 sec. of plant behavior. Only 100 samples were taken of the modal response for each plot due to storage limitations in the simulation software. This clearly violates the Nyquist Sampling Theorem for accurate preservation of the sampled data. As a result, the plotted data does not exactly represent the corresponding signals, at times misrepresenting the oscillation frequency and the signal envelope. However the general trends in the response envelopes are preserved and clearly visible. In this respect, figure 2 shows the very lightly damped character of each modal deflection, along with the envelope modulations and smoothing which are due solely to the undersampled data presentation.

The next section describes the adaptive modal control applied to this lightly damped plant.

## Section II: Adaptive Controller

This example utilized an adaptive controller providing simultaneous plant identification and control. The adaptive control [2] was modal in form, requiring the plant to be decoupled using a set of modal transformations to generate modal signals from actual plant input-output measurements:

$$\begin{aligned} \underline{u}_m(t) &= (M^0)^{-1} \underline{u}(t) \\ \underline{f}_m(t) &= (M^0)^{-1} \underline{f}(t) \end{aligned} \quad (7)$$

where  $M^0$  is the collection of modal deflections at the sensor measurement points (modal transformation matrix) and  $\underline{u}_m$  and  $\underline{f}_m$  are the modal

outputs and inputs, respectively. Each modal input-output pair  $(f_{m_i}, u_{m_i})$  would be related by a second order ODE of the form (4), independent of all other input-output pairs, only if the modal transformation was the unique one for this particular structure.

To identify the dynamic parameters of each mode, a recursive least squares formulation minimizing the equation error was employed [9]. In discrete time, the modal systems could be described by a second order difference equation of the following form

$$\begin{aligned} u_{m_i}(k) &= a_i u_{m_i}(k-1) + b_i u_{m_i}(k-2) \\ &+ c_i f_{m_i}(k-1) + d_i f_{m_i}(k-2) \\ &= \phi_i^T(k-1) \theta_i \end{aligned} \quad (8)$$

where

$$\begin{aligned} \phi_i^T(k-1) &= [u_{m_i}(k-1), u_{m_i}(k-2), f_{m_i}(k-1), f_{m_i}(k-2)] \\ \theta_i^T &= [a_i, b_i, c_i, d_i]. \end{aligned} \quad (9)$$

It was desired that an estimated output

$$\hat{u}_{m_i}(k) = \phi_i^T(k-1) \hat{\theta}_i(k) \quad (10)$$

approach  $u_{m_i}(k)$  as the parameter estimates  $\hat{\theta}_i(k)$  approached the true values  $\theta_i$ . The parameter estimates were adjusted according to the well-known least squares algorithm [9]:

$$\hat{\theta}_i(k) = \hat{\theta}_i(k-1) + \frac{P_i(k-2) \phi_i(k-1) [u_{m_i}(k) - \phi_i^T(k-1) \hat{\theta}_i(k-1)]}{1 + \phi_i^T(k-1) P_i(k-2) \phi_i(k-1)} \quad (11)$$

where  $P_i(k)$  was a positive definite weighting matrix given by

$$P_i(k-1) = P_i(k-2) - \frac{P_i(k-2) \phi_i(k-1) \phi_i^T(k-1) P_i(k-2)}{1 + \phi_i^T(k-1) P_i(k-2) \phi_i(k-1)} \quad (12)$$

With  $P_i(0)$  any positive definite matrix. If sufficient excitation can be assured, the parameter estimates  $\hat{\theta}_i(k)$  are guaranteed to converge exponentially to the true values  $\theta_i$  as  $k \rightarrow \infty$  [10]. When the order of the identifier is different from that of the modal block being identified, or when sufficient excitation cannot be assumed, the parameter estimate convergence as well as the location of convergence points is in question. These difficulties will be discussed further in connection with the simulation results in section III.

By considering the parameter estimates to be close to the true ones, classical design techniques could be used to determine a suitable control law to obtain the desired behavior of each modal system in closed loop. For this example, the difference equation equivalent of a state observer in conjunction with state variable feedback was used to place the closed loop eigenvalues at the desired locations [11]. The resulting modal feedback signal  $g_{m_i}(k)$ , for the  $i^{\text{th}}$  mode, was of the form

$$g_{m_i}(k) = \hat{\delta}_{1_i} g_{m_i}(k-1) + \hat{\delta}_{2_i} g_{m_i}(k-2) + \hat{\delta}_{3_i} f_{m_i}(k-1) \\ + \hat{\delta}_{4_i} f_{m_i}(k-2) + \hat{\delta}_{5_i} u_{m_i}(k-1) + \hat{\delta}_{6_i} u_{m_i}(k-2) \quad (13)$$

where the  $\hat{\delta}_{j_i}$  were determined by the pole placement objective, the estimated plant coefficients, and the observer pole locations. The observer poles were all placed at  $z = 0.1$  to provide fast convergence, yet some measure of noise immunity for the state estimates. The closed loop pole placement objective for this example was to augment the very slight natural damping of the plant. Specifically, it was desired to move the plant open loop z-plane eigenvalues away from the unit circle toward the origin along radial lines, to place the closed loop eigenvalues along a circle of radius

$$r_p = e^{-\alpha DT}$$

where the response time constant was  $1/\alpha$  sec. For all the simulations, the desired closed loop time constant was set at 20 sec. Such a small amount of closed loop damping was selected based on two practical constraints. For one, it is well known that large increases in damping, and hence large increases in bandwidth, require large control efforts which would tend to drive the structure out of the linear region of operation. Secondly, pole placement design based on only 5 modes of a flexible LSS would almost surely be a reduced-order problem. Here, large control efforts on the modeled portion of the plant tend to destabilize the modes in the unmodeled portion [12]. Therefore, a realistic control objective on an actual LSS would be restricted to rather small improvements in the modal damping of the modeled modes.

At each iteration, the vector of modal feedback signals  $\underline{g}_m(k) = [g_{m_i}(k)]$  was inverse modal transformed to obtain the actual feedback applied to the actuators:

$$\underline{g}(k) = M^o \underline{g}_m(k) \quad (14)$$

where in the absence of a reference input after the initial disturbance pulse (regulation objective) the feedback was the sole input and

$$\underline{g}(k) = \underline{f}(k), \quad k > 1. \quad (15)$$

In the case where the spatial mode shapes were not exact, the decoupling would not be complete and a modal control scheme was expected to suffer performance degradation depending on the extent of the shape errors. To test the effect of shape errors, the modal transformation used to decouple the plant signals as in (7) was perturbed from the exact value  $M^o$ . The errors were introduced to the actual



mode shapes at each sensor/actuator location by an additive random number uniformly distributed in magnitude to a bound that increased with mode frequency. The resulting decoupling mode shapes are indicated at the sensor locations by asterisks in figures 3 and 4, the errors being the differences between the asterisk locations and the actual shapes indicated by solid lines. The actual errors at each measurement point are indicated in table 1 for figure 3, and in table 2 for figure 4. The introduction of random errors was not meant to imply that errors are likely to be generated in this way, but to remove any bias on the part of the authors in picking shape errors. Note that a smooth shape drawn through the asterisks would differ very little from the actual mode shapes. It will be shown later that the slight difference between the errors in figures 3 (table 1) and 4 (table 2) can cause startling changes in the performance of the closed loop control system.

The next section will discuss the regulation performance when these particular errors in the decoupling mode shapes are introduced into the adaptive controller.

### Section III: Closed Loop Simulation Results

Case 2: To establish a base for performance comparison when mode shape and parameter errors are introduced, the first closed loop simulation contained no such errors. It therefore represented the best possible situation for control design since the plant characteristics were known exactly. In this case, it was expected that the adaptive controller would behave as a fixed controller and that the closed loop response in a regulation task would have the familiar damped exponential envelope in each modal deflection, characteristic of linear time-invariant systems. The desired closed loop eigenvalues for all modes

were located along a uniform damping locus in the z-plane corresponding to a time constant of 20 sec. The results of simulating this case are shown in figure 5. The plots are arranged by modes in columns, with each row representing a particular signal or parameter variation in time. The first row displays the magnitude of the estimated plant eigenvalue for each mode. As expected, the magnitudes remained at their correct values, all very slightly less than 1.0, making the closed loop system linear and time invariant. The second row plots the trace of the positive definite weighting matrix P for each mode. If the least squares identification algorithm were sufficiently excited, then the trace would be driven to zero as t became large. The rate at which the trace of P decreased was determined only by the magnitude of the signals  $u_m(k)$  and  $f_u(k)$  in (12) and hence did not depend on the prediction error (shown in row three). In this case, the plant input-output signals shown in the last three rows were small and exponentially decreasing in amplitude and, as a result, the trace of P did not become small. This effect was seen to be significant when errors were introduced into the other simulations to be subsequently discussed. In all simulations, the first iteration represented the system under the influence of a unit pulse in force applied to all actuators. For this reason, the last row of plots, i.e. the input forces applied to the plant in modal form, had much larger scales than the feedback and output signal plots due to the presence of the large initial pulse.

Case 3: If the initial estimates of the plant dynamic parameters were in error, yet the decoupling spatial shapes were exact, then correct identification of the unknown parameters in each mode would imply correct closed loop performance of the entire system. This was due to the fact that the plant would be exactly decoupled for all dynamic parameter

estimates. Referring to figure 6, note that the estimated eigenvalue magnitudes approached the correct values of 0.9990 from their initial values of 0.95000 as the trace of P becomes small in each modal identifier. The prediction error in modal outputs (row three), weighted by the trace of P, drove the parameter estimates toward their correct values. Notice the significantly larger feedback and output signals as compared to figure 5 (Case 2), and also the non-exponential envelopes surrounding the signals. For this particular initial parameterization and pole placement objective, the closed loop response was initially unstable. As the signal magnitudes became large, the modal identifiers became sufficiently excited and the estimated parameters converged to the actual ones. Thereafter, each closed loop modal system behaved as in figure 5, showing the desired exponentially damped response. This is an example where a fixed controller, based on some initial parameter estimates, would be unstable yet an adaptive controller could provide a stable closed loop system.

Case 4: Figure 7 represents the response to the opposite situation as in figure 6. For the simulation in figure 7, the "correct" initial parameters were used in each modal identifier, but the decoupling mode shapes were in error according to table 1 and figure 3. In the plots of the prediction error in modal outputs, row three, the errors due to cross-coupling in the (assumed) decoupled controller can be seen. Compare with figure 5 (Case 2) which was exactly decoupled. The feedback signals in figure 7 are larger in magnitude and of longer duration than those in figure 5, indicating the dissatisfaction of the controller with a decoupled parameterization of the non-decoupled plant. This effect was large enough in mode 4 to cause the identifier to alter than mode's parameter estimates from the "correct" initial values to some other set, as is evident from

the plots in row one of figure 7. This is possible because a vector of parameters  $\theta_j$  may come quite close to solving (8) for all  $k$ , and the "correct" initial value  $\theta_j$  may not be altered significantly by the adaptive algorithm. This is the case for mode 1 in figure 7, for example. The small amount of shape distortion used for figure 7 does not cause severe problems in the regulation performance, as stability of the closed loop system is retained.

Case 5: When both initial parameter uncertainty and decoupling shape distortion are present, however, the resulting performance of the adaptive regulator is quite unpredictable. This unpredictability is the main warning of this section of this report. In figure 8, the same initial parameter error as for figure 6 and the mode shape distortion given by table 1 were both present in the adaptive modal controller. Comparing figure 8 with figures 5, 6 and 7, note first the much larger prediction errors in all modes for figure 8. Also, due to the large magnitudes of the plant input and output signals, the trace of  $P$  decreased rapidly to zero for each mode. This caused the parameter update to become less sensitive to the large prediction errors and the parameters appeared to converge to some fairly steady values as  $t$  approached 100 sec. This "steady state" parameterization did not, however, provide acceptable closed loop performance, as shown in rows 3-6. Indeed, had the trace of  $P$  not become small due, e.g., to the action of a "forgetting factor" [13] in the least squares algorithm, the parameters would likely have undergone larger variations, making performance even more unpredictable. Such highly non-linear and time-varying behavior is extremely difficult to describe, and appears a major deterrent to the successful application of adaptive modal control to LSS when plant decoupling cannot be assured.

Case 6: As testimony to the extreme sensitivity of adaptive modal control to shape errors in this example, consider the regulation performance shown in figure 9. This simulation was identical to that of figure 8 except that the shape errors of table 2 and figure 4 were used. Note the erratic parameter variation and obvious instability of the closed loop regulator for this moderate increase in shape distortion over that used in figure 7 (Case 5).

Before leaving the discussion of the adaptive modal regulator performance for this example, it should be pointed out that the ill-behavior demonstrated should not be taken to be representative of all cases of pole placement objectives, decoupling shape errors, and initial dynamic parameter choices. In this sense, the example presented here was somewhat contrived. And yet, the errors introduced were not outside the range of physical possibility but seemed quite small indeed. The point to be made is that a much more extensive characterization of the effects of decoupling mode shape errors on the performance of adaptive modal control must be available before such regulation techniques may be safely applied to physical LSS, even if the reduced-order effects can be avoided.

#### CONCLUSIONS

Although the example simulated was without the reduced order effects normally associated with control of DPS, it demonstrated the sensitivity and unpredictability of adaptive modal control when the spatial mode shapes used for decoupling were inaccurate. In particular, this example provided the following observations:

- As expected, when the mode shapes and initial dynamic parameters are known exactly, the regulation performance was as predicted by the classical pole placement design procedure.

This was an unrealistic situation in that the reduced order problems were not included, and that the representation of the structure was completely known a priori. Recognizing the current difficulties with reduced order control theory, further simulations were instead presented that treated the problems of inaccurate a priori information about the structure.

- When the spatial decoupling shapes were accurately pre-specified but the dynamic parameters of each mode were not, adaptive modal control performed as expected, in this case outperforming a fixed controller with the same initial modal parameter errors. The regulation performance in this case was worsened from the previous case containing no initial errors due to the larger transient signals during the "learning phase" of the adaptive algorithm.
- When the initial dynamic parameters are set to the values obtained above after the identification algorithm had converged, but the mode shapes were inaccurate within certain bounds, moderate performance degradation under closed loop regulation was observed. This degradation was due to the coupling between modal subsystems that could not be accounted for by a modal identification and control algorithm.
- When both initial parameter errors and shape errors were introduced, the regulation performance was very much worse than that obtained for only one class of error. Instability of the closed loop system was observed for surprisingly small mode shape errors. This result was not indicated by the previous simulation performance, although it seemed principally due, again, to the inability of the decoupled identification

algorithm to find a parameterization of the non-decoupled plant suitable for closed loop control. The choice of initial parameter error and pole placement objective has a significant, but loosely predictable effect on the behavior of closed loop control for these shape errors.

In the light of the many publications proclaiming the usefulness of particular adaptive control methods in specific instances, this paper has sought to present evidence to the contrary. It was meant to point out the severe deficiencies and immaturity of the current theory of adaptive modal control in application to LSS.

Rather than discouraging the application of adaptive modal control, since the prospect may suffer merely from the poor understanding presently available, it seems that further development of the theory may prove quite profitable. Such development can be divided into two logical directions[3]. One is the retention of the modal form of modeling and control. This may involve some sort of adaption on the mode shapes themselves to provide increasingly accurate structure decoupling. Also, a more complete understanding of the relation between initial dynamic parameter errors, pole placement objectives, and mode shape errors is indicated. The second direction abandons the modal ideas for a more general multi-input multi-output approach to modeling and control. As one might expect, additional complexity is introduced in this way, however, advantages may be gained to justify the effort. In addition, the exploration of the equivalences of modal and non-modal representations should prove invaluable in comparing results and generating new insights. In short, a great deal of work remains to adequately expose the underlying behavioral mechanisms of the adaptive modal control of LSS.

Figure 1

Mode Shapes with Measurement Locations

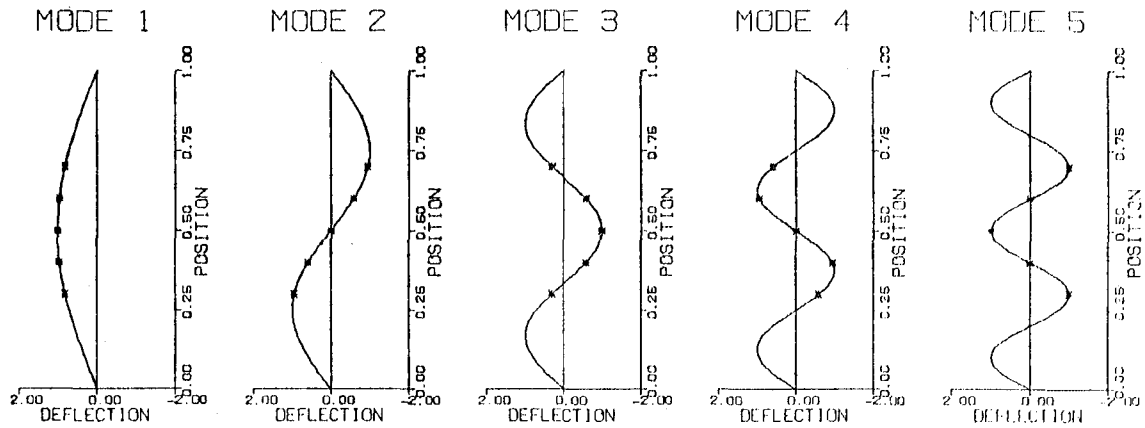


Figure 2

Open Loop Modal Response

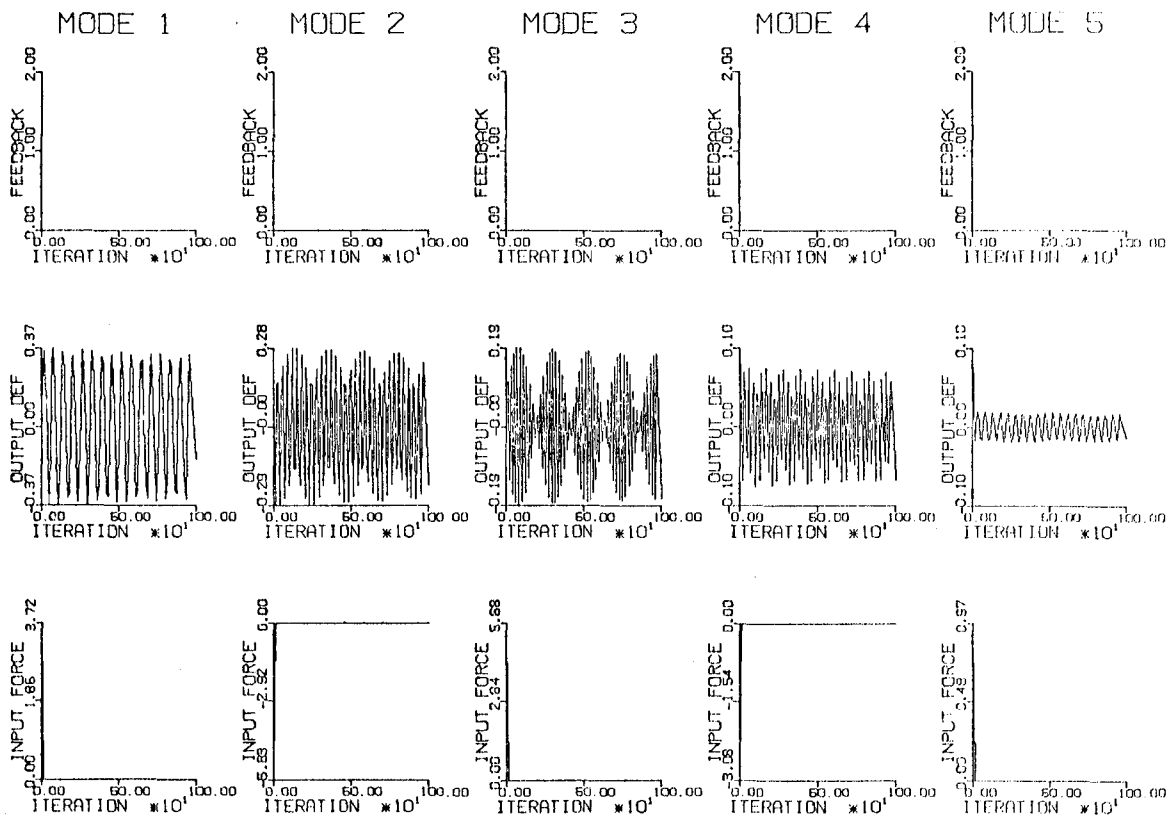




Figure 3  
Mode Shapes with Additive Random Errors

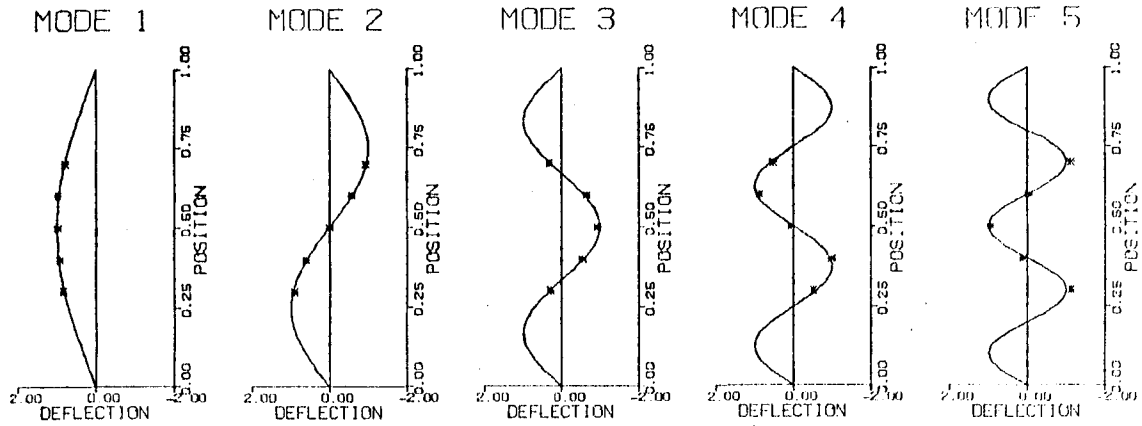


Table 1  
Shape Errors for Figure 3

SENSOR	MODE 1	MODE 2	MODE 3	MODE 4	MODE 5
5	-.014	0.012	0.027	-.063	-.107
4	0.022	0.012	-.061	-.073	-.042
3	-.009	-.012	0.057	0.078	-.046
2	-.024	0.020	0.039	-.032	0.116
1	0.023	-.035	-.010	0.070	-.122

Figure 4  
Mode Shapes with Larger Random Errors

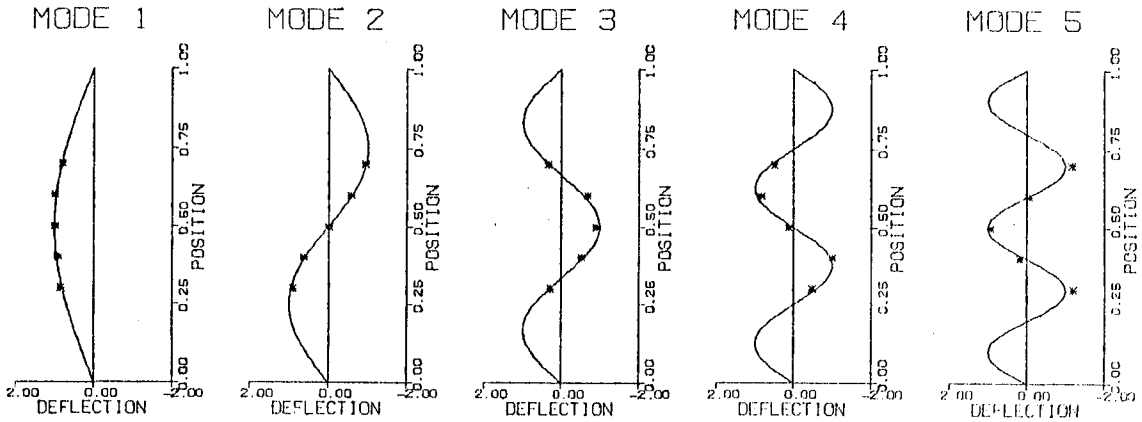
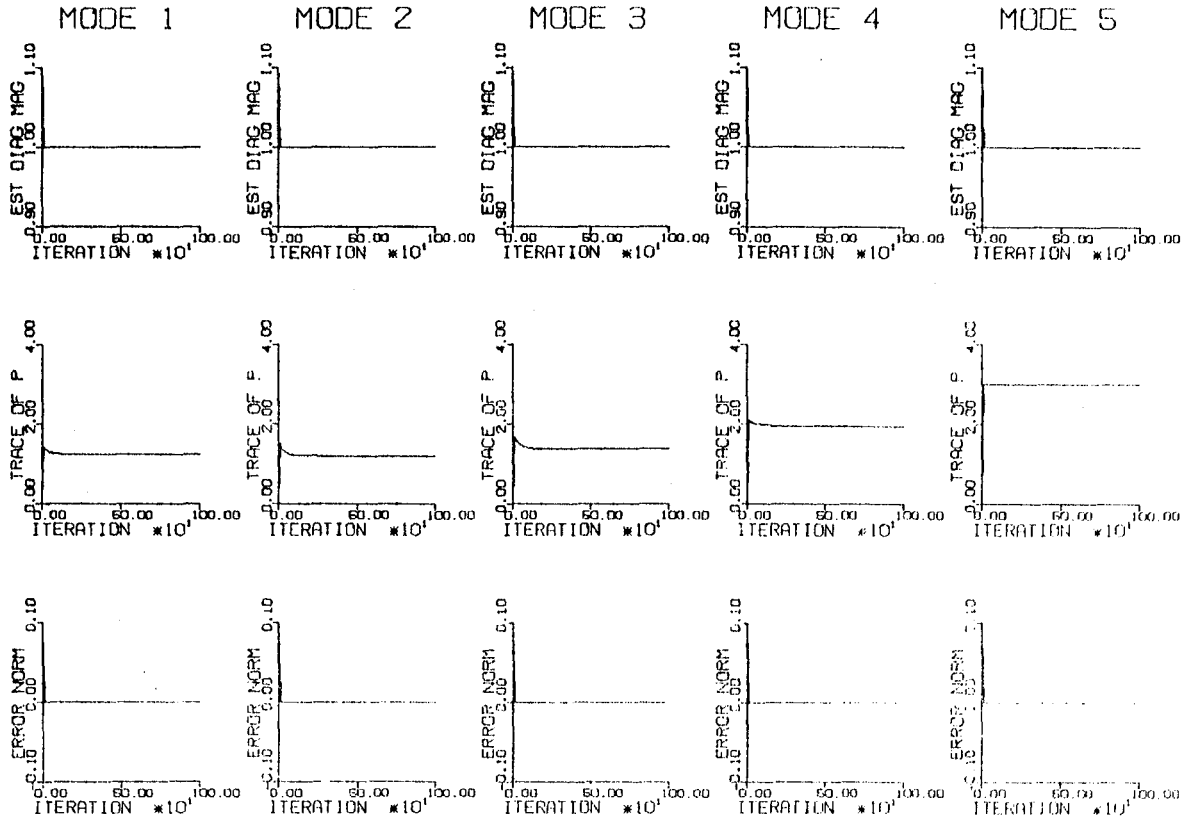


Table 2  
Shape Errors for Figure 4

SENSOR	MODE 1	MODE 2	MODE 3	MODE 4	MODE 5
5	-.021	0.017	0.041	-.095	-.161
4	0.033	0.018	-.091	-.109	+.062
3	-.014	-.018	0.085	0.117	-.068
2	-.037	0.030	0.059	-.048	0.174
1	0.035	-.053	-.015	0.106	-.183

Figure 5  
Simulation for Case 2: Parameters



Simulation for Case 2: Signals

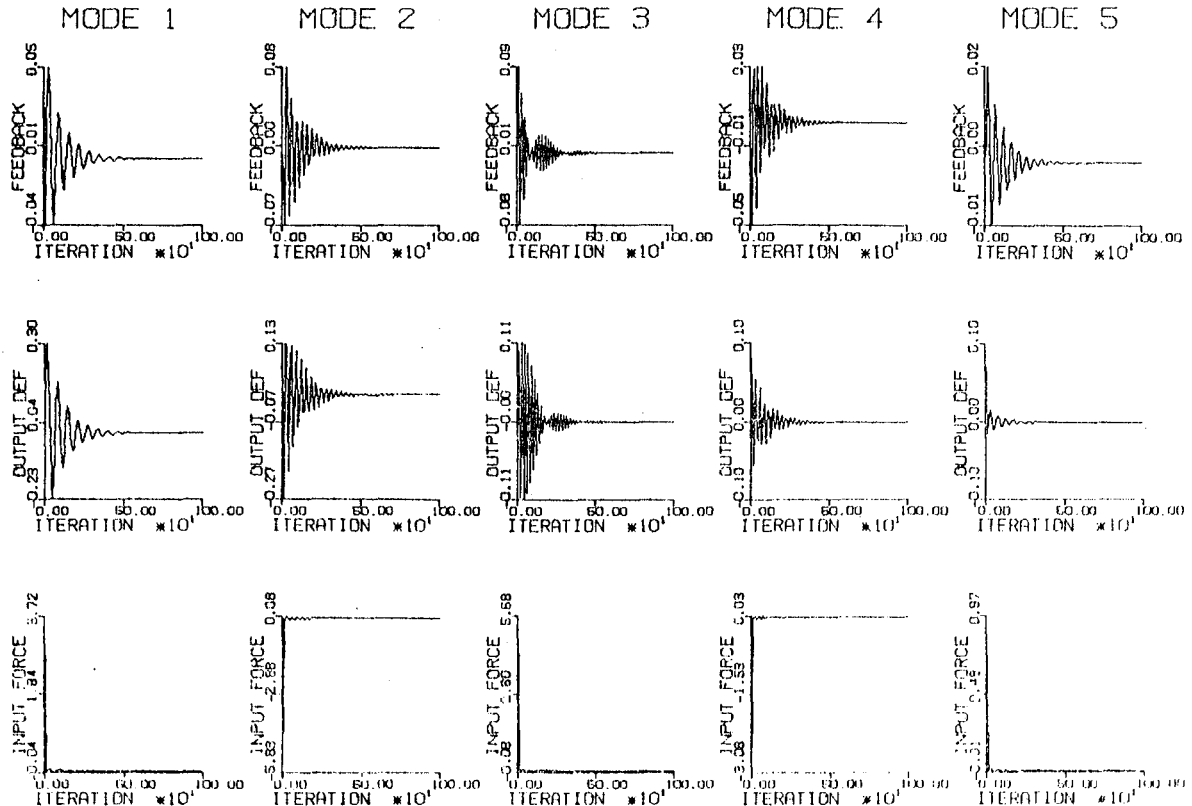
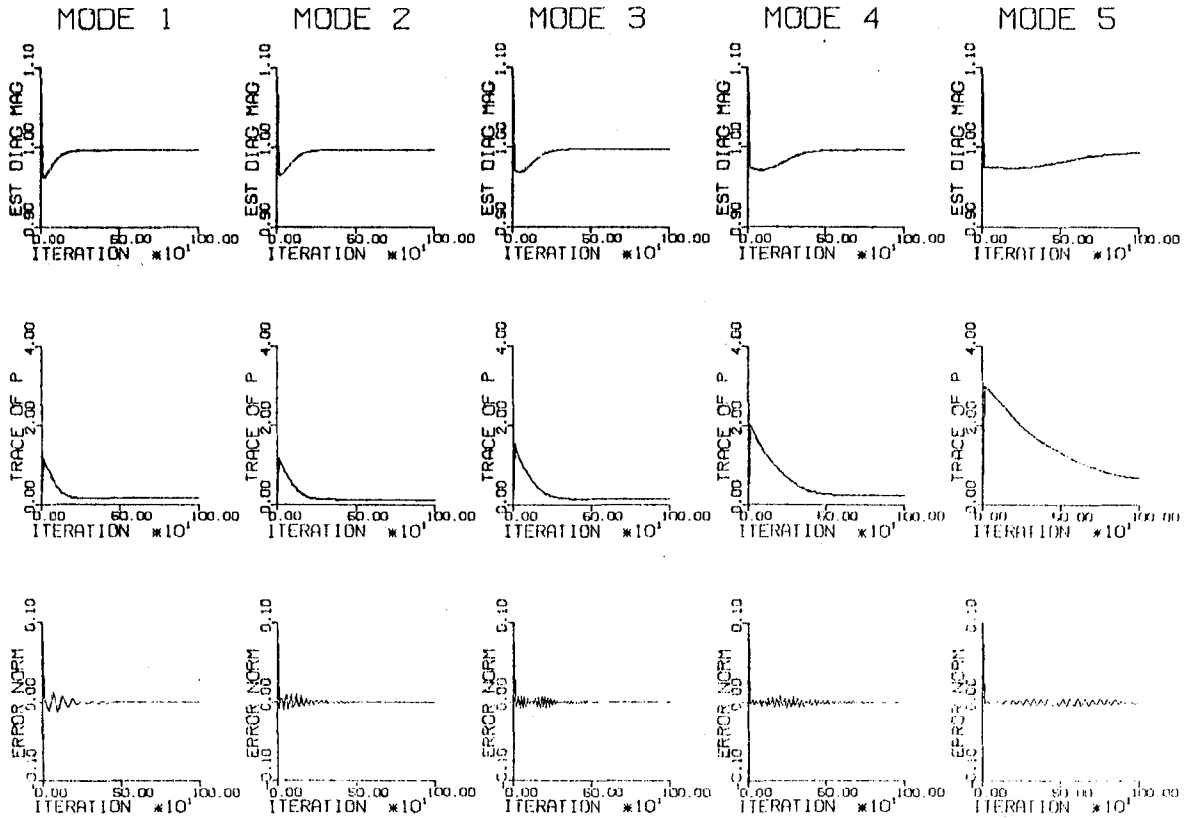


Figure 6  
Simulation for Case 3: Parameters



Simulation for Case 3: Signals

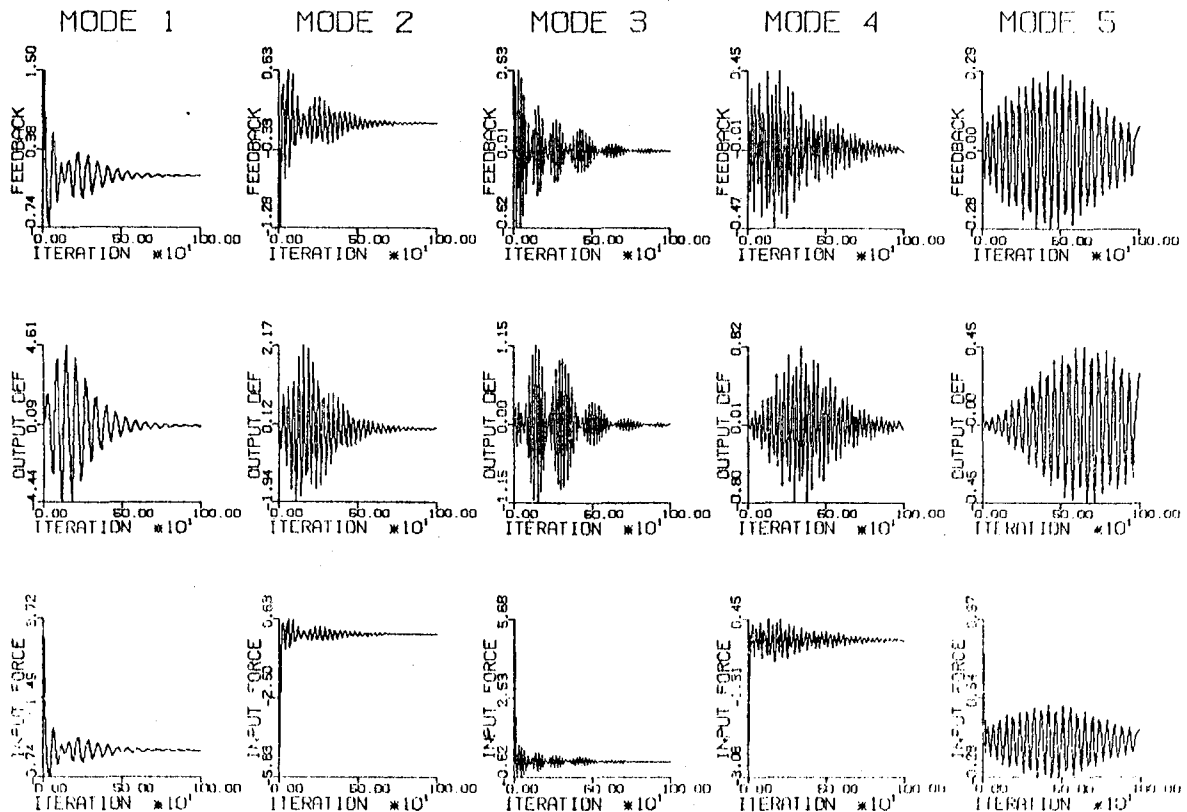
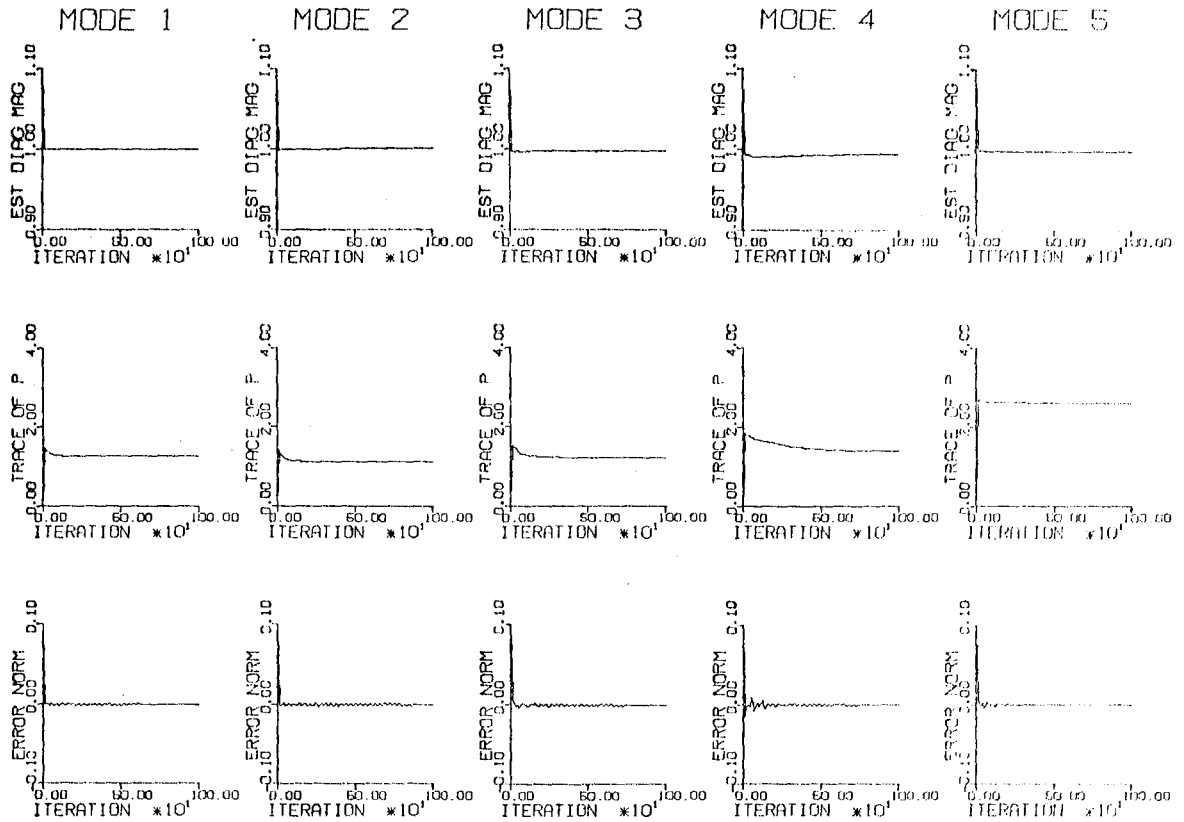


Figure 7  
Simulation for Case 4: Parameters



Simulation for Case 4: Signals

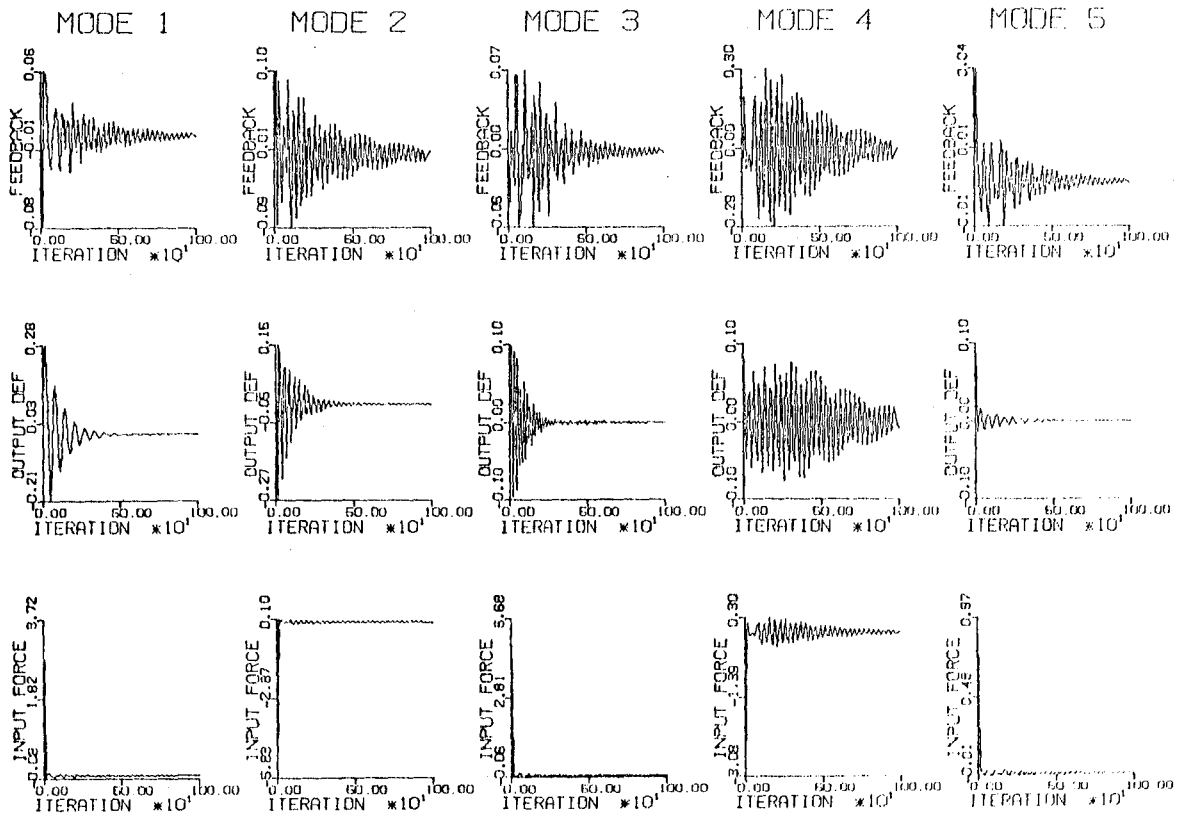
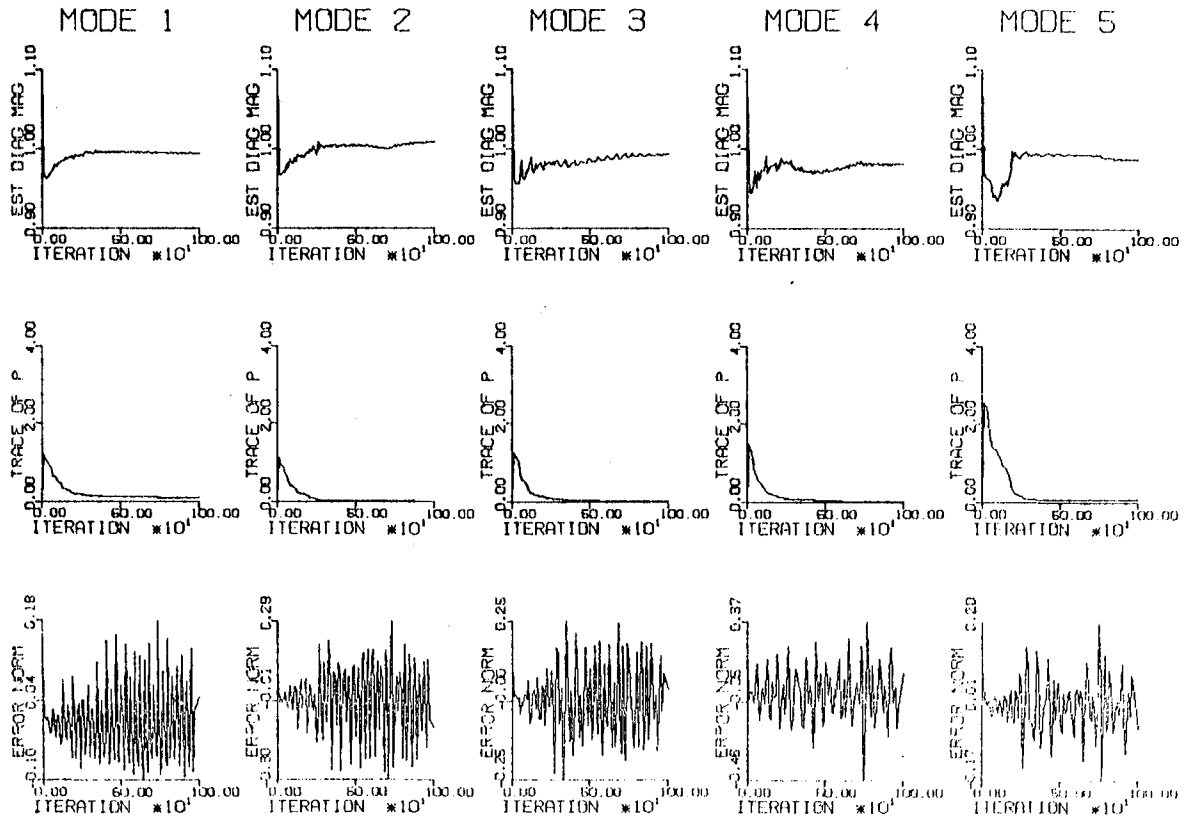


Figure 8  
Simulation for Case 5: Parameters



Simulation for Case 5: Signals

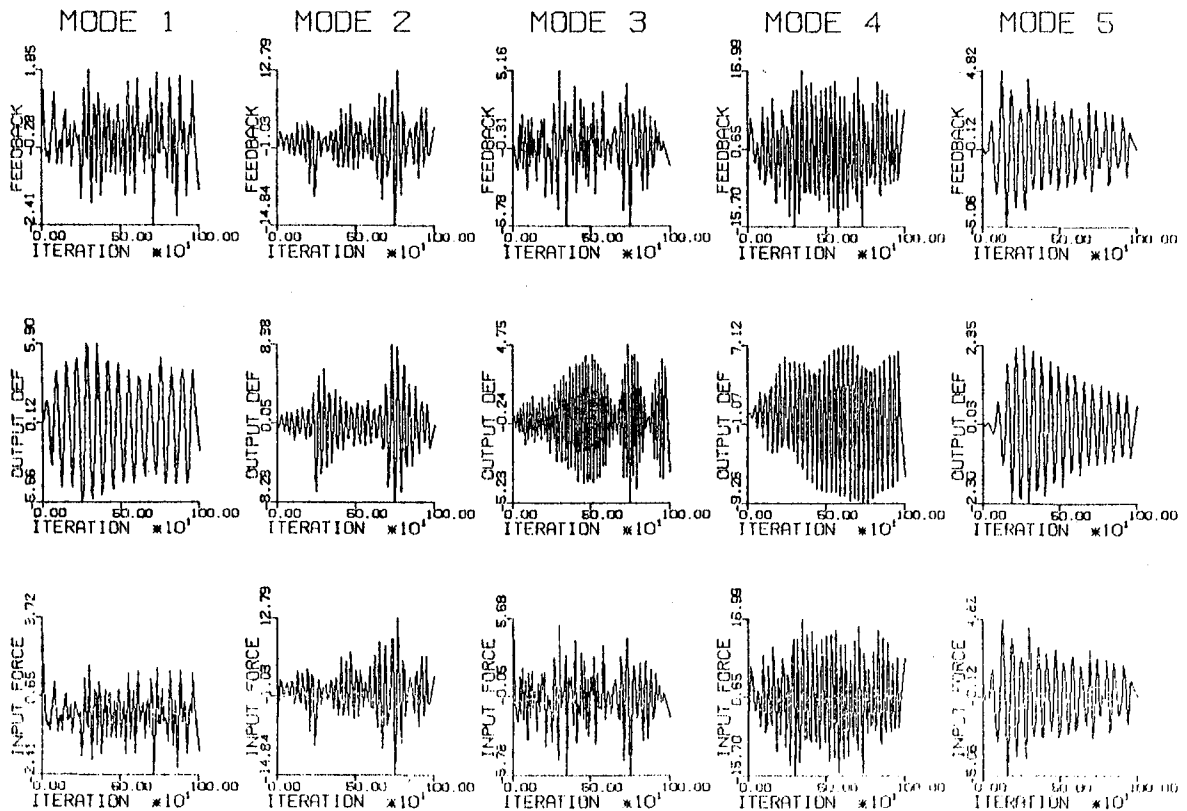
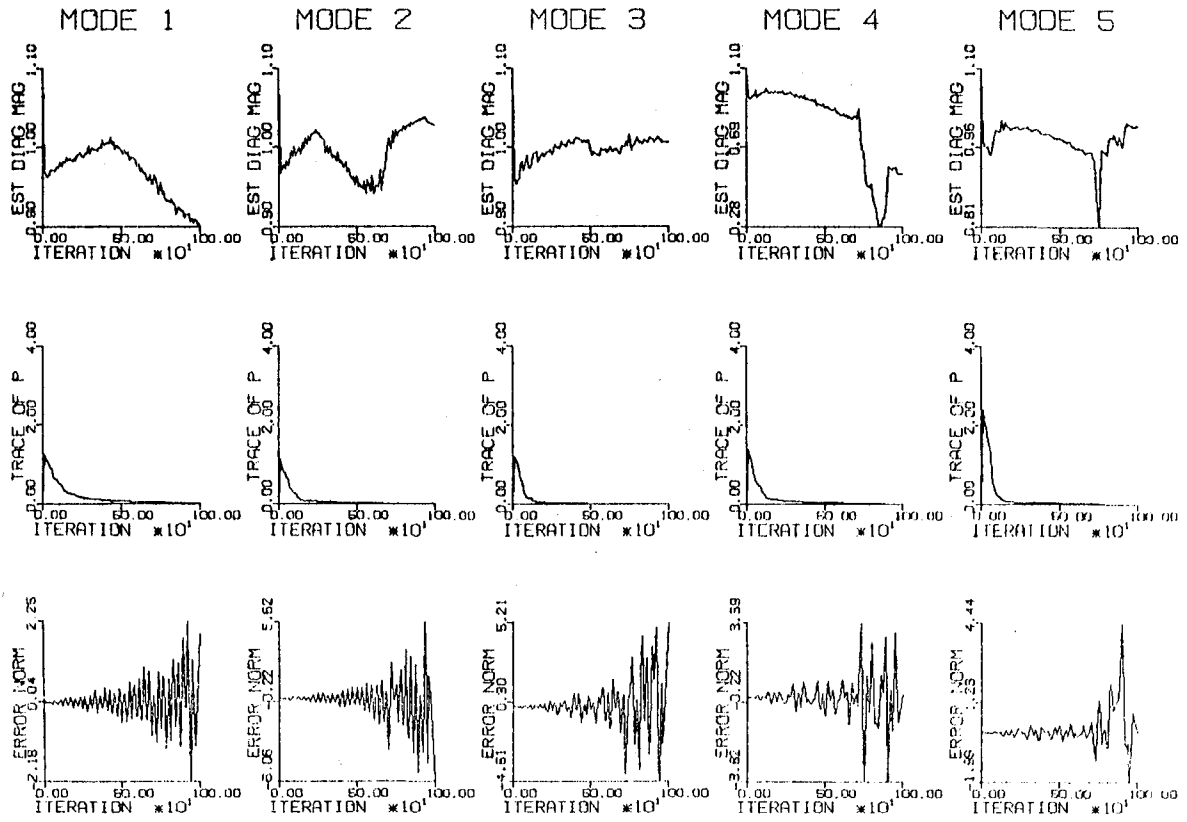
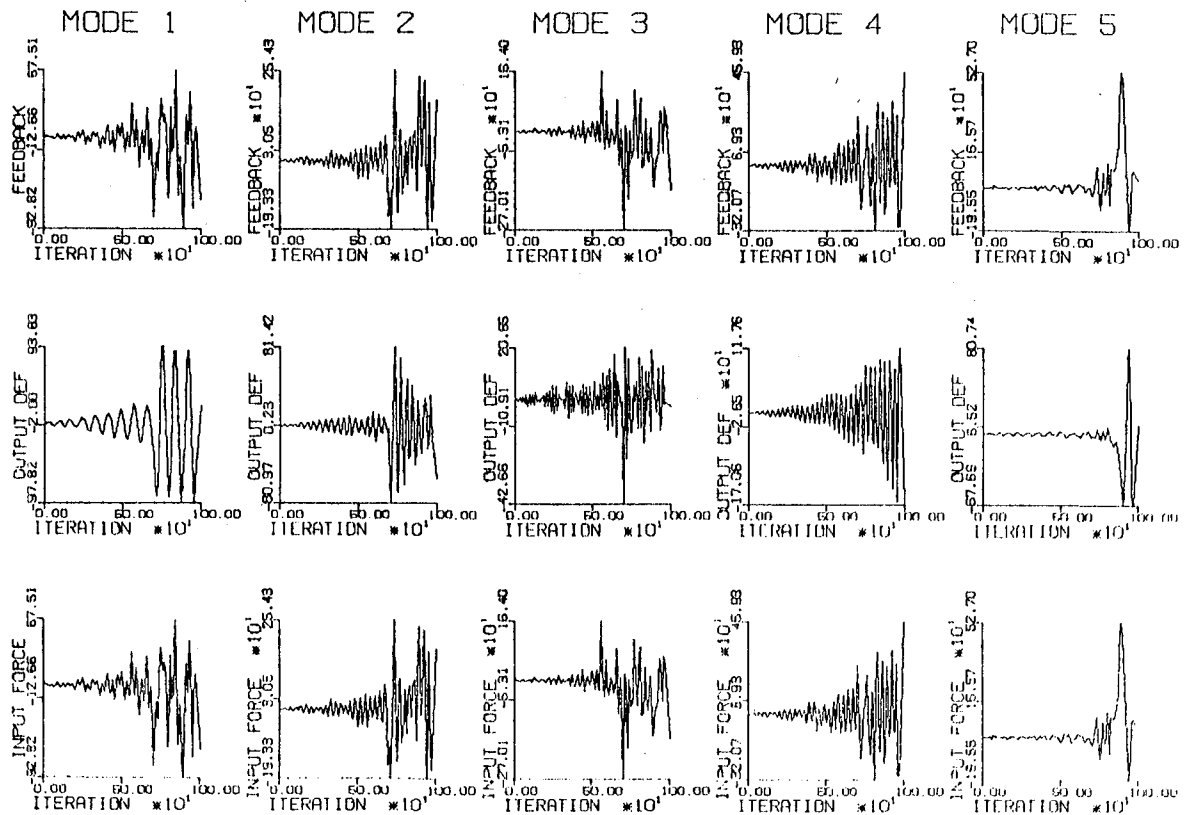


Figure 9  
Simulation for Case 6: Parameters



Simulation for Case 6: Signals



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