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Influence of Friction Forces on the Motion of V/TOL Aircraft During Landing Operations on Ships at Sea

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NOMENCLATURE

A_a	pneumatic area
A_h	hydraulic area
A_n	net orifice area
C	distance from center of gravity of ship to landing platform
C_d	coefficient of discharge
D_{ij}	direction cosines
$\mathcal{D}_{ij}$	direction cosines
d_{ij}	direction cosines
$\bar{F}_a^w$	force acting at axle of wheel w
$\bar{F}_g^w$	force acting at point of wheel contact
$\bar{F}_{N_a}^w$	force acting normal to shock strut at axle of wheel w
$\bar{F}_s^w$	shock-strut force at wheel w
$\bar{f}_{ia}^w$	body axes components of axle force at wheel w
$\bar{f}_{ig}^w$	body axes components of forces generated at point of wheel contact
f_f^w	internal friction forces at wheel w
$\bar{g}$	gravity vector
H^w	hydraulic force parameter
$\hat{I},\hat{J},\hat{K}$	triad of mutually orthogonal unit vectors in directions of platform axes
$\hat{i},\hat{j},\hat{k}$	triad of mutually orthogonal unit vectors in directions of aircraft body axes
$\bar{\bar{I}}$	unit dyadic
K^w	tire stiffness at wheel w
ℓ^w,m^w,n^w	direction cosines of shock strut relative to aircraft axes
M_i^w	components of moment vector

M^w	mass of wheel w
$\hat{n}$	unit vector normal to plane of wheel
P,Q,R	components of ship's angular velocity vector
p,q,r	components of aircraft's angular velocity vector
$\hat{p}$	unit vector in plane of wheel
p	pneumatic pressure
p_a	air pressure in upper chamber
p_h	hydraulic pressure in lower chamber
$\bar{r}^w$	position vector of wheel w relative to origin of aircraft axes
$\bar{R}^w$	position vector of wheel w relative to ship's coordinate system
$\bar{R}^{cg}$	position vector of aircraft center of gravity relative to ship axes
$\hat{s}$	unit vector in direction of shock strut
s	shock-strut axial stroke
U,V,W	components of ship's linear velocity vector
u,v,w	components of aircraft linear velocity
$\bar{V}^w$	velocity of wheel w relative to landing platform
$\bar{V}_{cg}$	velocity of aircraft's center of gravity
X_i	ship axes coordinates
X_i^P	platform axes coordinates
X_i^w	coordinates of wheel w relative to ship's coordinate system
X_i^{cg}	coordinates of aircraft's center of gravity relative to ship's reference frame
X_i^o	initial coordinates
x_i^A	coordinates of point of attachment of shock strut
x_i^w	coordinates of wheel w relative to aircraft axes
x_{io}^w	coordinates of wheel w when shock strut is in fully extended position

ψ, θ, ϕ	aircraft Euler angles
Ψ, Θ, Φ	ship Euler angles
Θ_i	Euler angles corresponding to direction cosines
κ^w	angle in plane of landing platform between X_1^P axis and velocity vector at wheel w
κ_K^w	angle κ when friction is kinetic
κ_s^w	angle κ when friction is static
τ^w	angle between plane of wheel and velocity vector at wheel w
μ^w	coefficient of friction at wheel w
δ^w	tire deflection at wheel w
$\bar{\omega}$	aircraft angular velocity vector
ρ	density of hydraulic fluid

INFLUENCE OF FRICTION FORCES ON THE MOTION OF VTOL AIRCRAFT DURING LANDING OPERATIONS ON SHIPS AT SEA

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SUMMARY

The equations describing the friction forces generated during landing operations on ships at sea have been formulated. These forces depend on the platform reaction and the coefficient of friction. The platform reaction depends on the relative sink rate and the shock-absorbing capability of the landing gear. The coefficient of friction is assumed to vary with the surface condition of the landing platform and the angle of yaw of the aircraft relative to the landing platform. The landings contemplated are by aircraft of the VTOL type equipped with conventional oleo-pneumatic landing gears. Because aircraft of this type land with little or no forward speed, spinup forces have been neglected. Simplifications have been introduced to reduce the complexity of the mathematical description of the tire and shock-strut characteristics. It has been shown elsewhere that, for normal impact without tire bottoming, reasonable variations in the force-deflection characteristics of the tire have only a relatively small effect on the calculated behavior of the landing gear. Approximating the actual complicated force-deflection characteristics of the tire by a linear relationship appears to be adequate for practical purposes. Although the internal friction forces in the shock strut have been included in the landing gear model, experimental data suggest that these forces can often be neglected without impairing the validity of the model equations. By including only those characteristics of the tire and shock strut that contribute significantly to the generation of landing gear forces, a set of relatively simple equations is obtained. Nevertheless, the equations are considered adequate for practical purposes.

INTRODUCTION

Landing VTOL aircraft on ships at sea is complicated by the motion and surface condition of the landing platform, which moves in response to the motions of the sea, and which is exposed to a variety of atmospheric conditions. An understanding of the landing phenomena requires that a mathematical model, which includes platform reactions and friction forces, be formulated and used to simulate the landing maneuver. In assessing the platform reaction forces likely to be encountered, the actual complicated force-deflection characteristics of the tire have been approximated by a linear relationship. Moreover, in the event that tire bottoming occurs, a linear segment approximation which takes into account the increased stiffness of the tire that results from bottoming yields good results (Milwitzky and Cook, 1953). A limited amount of experimental data obtained during drop tests at the Langley Research

Center indicates that the behavior of the landing gear is relatively insensitive to variations in the air compression process, and that variations between isothermal and near adiabatic compression have only a secondary effect on the calculated behavior of the landing gear. Consequently, variations in the polytropic exponent have been neglected. The importance of friction forces derives from the fact that, if friction is not sufficient to prevent sliding, the aircraft may be damaged by colliding with adjacent structures. It is unlikely that VTOL aircraft landings will be attempted when the decks are awash, but during rough weather the landing platform will be exposed to rain and possibly sea spray which could reduce the coefficients of friction below safe levels. Another possibility to be considered is the likelihood that VTOL aircraft may produce a shower of spray as the jets interact with the surface of the sea during the approach. If the resulting spray were to be blown over the landing platform, the friction forces would be reduced significantly. In view of these considerations, it is important to determine the reactions and friction forces likely to be encountered during a variety of sea states and landing conditions. By including these forces in the equations of the mathematical model, the motion of the aircraft during the landing maneuver can be computed and the possibility of sliding determined. In formulating the friction force equations, bending will be neglected and it will be assumed that each landing gear has a fixed orientation relative to aircraft body axes. In many cases, the influence of elasticity may be neglected without serious error, but in some instances, particularly when the landing gear attachment points experience large displacements relative to the nodal points of the flexible system, the interaction between the deformations of the structure and the landing gear may be required to represent the system adequately. Expressions for the hydraulic, air-compression, and internal friction forces generated in the shock strut are derived in appendix B.

KINEMATICS AND DYNAMICS OF AIRCRAFT-SHIP INTERACTIONS

Forces and Moments

Friction forces and platform reactions- Relative to the ship's coordinate system $(X_1X_2X_3)$, which originates at the center of gravity, the landing platform lies in the plane $(X_3 - C) = 0$, where C is negative constant. The X_1X_3 plane of the ship's coordinate system corresponds to the plane to symmetry of the vessel and the X_2 axis is normal to it. A platform coordinate system $(X_1^PX_2^PX_3^P)$ having axes parallel to the axes of the ship's coordinate system is used to specify vector components relative to the platform (see fig. 1).

Subsequent to the instant at which wheel w makes contact with the landing platform, the aircraft is subjected to a reaction force F_3^W which is normal to the landing platform and a friction force which is coplanar with the platform. Relative to platform axes, the friction force has components F_1^W and F_2^W. Hence, subsequent to wheel contact, the force vector $\bar{F}^W$ generated by wheel w has components

$$\bar{F}^W = F_1^W\hat{i} + F_2^W\hat{j} + F_3^W\hat{k} \qquad (1)$$

where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are a triad of mutually orthogonal unit vectors in the directions of the platform axes and the superscript w denotes the wheel being considered.

Therefore, if the dynamic response of the aircraft subsequent to the wheel contact is being computed, the wheel reactions and friction forces of equation (1) must be added to the inertia, thrust, aerodynamic, gravitational, and shock-strut forces to complete the mathematical description of the force system.

The corresponding wheel forces relative to aircraft body axes are f_i^w where

$$f_i^w = d_{ij}F_j^w \quad , \qquad i,j = 1,2,3 \qquad (2)$$

and d_{ij} are direction cosines (appendix A). The force generated by each wheel is obtained by assigning the appropriate wheel number.

In this and subsequent equations, the summation convention is assumed; that is, if in any term an index occurs twice, the term is to be summed with respect to that index for all admissible values of the index.

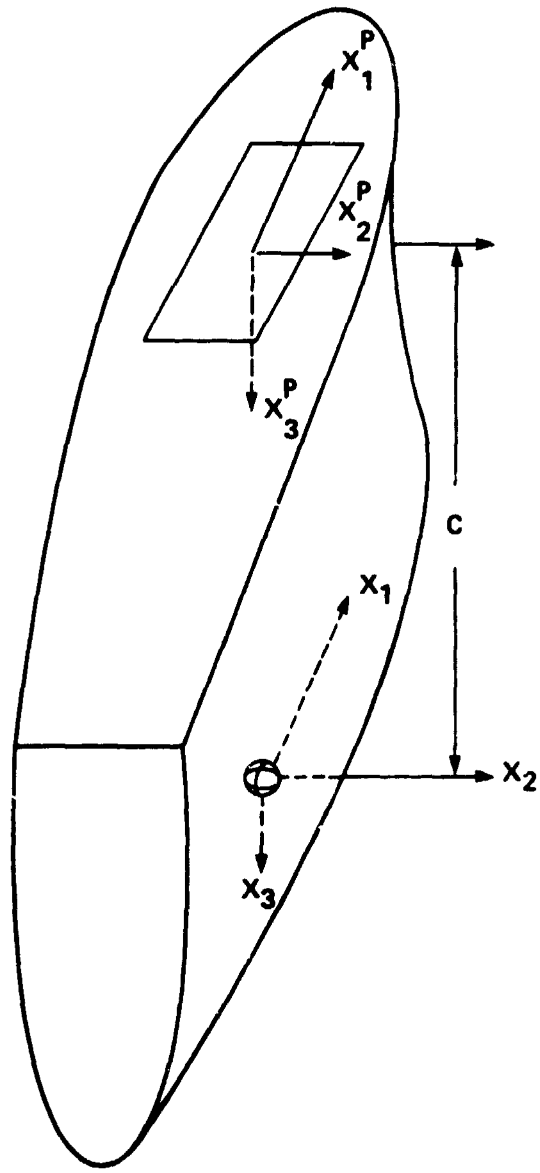


Figure 1.- Systems of reference axes, including ship and platform.

<u>Moments produced by friction forces and platform reactions</u>- The ith moment component produced by the platform reaction and the friction force at wheel w is the vector cross-product of the position vector of wheel w and the friction force vector:

$$M_i^w = (x_j^w f_k^w - x_k^w f_j^w) \qquad (3)$$

where x_i^w are the aircraft body axes positional components of wheel w. It should be noted that i, j, and k must be in cyclic order in equation (3).

From equation (2),

$$f_k^w = d_{kn}F_n^w \; ; \quad f_j^w = d_{jn}F_n^w$$

Substitution of these values in equation (3) yields the following moment equation:

$$M_i^w = (x_j^w d_{kn} - x_k^w d_{jn})F_n^w \tag{4}$$

Therefore, the individual components are

$$M_1^w = (x_2^w d_{3n} - x_3^w d_{2n})F_n^w$$

$$M_2^w = (x_3^w d_{1n} - x_1^w d_{3n})F_n^w$$

$$M_3^w = (x_1^w d_{2n} - x_2^w d_{1n})F_n^w$$

$$n = 1,2,3$$

When these equations are summed on n, the moment components assume the form

$$M_1^w = [x_2^w(d_{31}F_1^w + d_{32}F_2^w + d_{33}F_3^w) - x_3^w(d_{21}F_1^w + d_{22}F_2^w + d_{23}F_3^w)] \tag{5}$$

$$M_2^w = [x_3^w(d_{11}F_1^w + d_{12}F_2^w + d_{13}F_3^w) - x_1^w(d_{31}F_1^w + d_{32}F_2^w + d_{33}F_3^w)] \tag{6}$$

$$M_3^w = [x_1^w(d_{21}F_1^w + d_{22}F_2^w + d_{23}F_3^w) - x_2^w(d_{11}F_1^w + d_{12}F_2^w + d_{13}F_3^w)] \tag{7}$$

Orientation of Friction Force Vector

Relative velocities- To determine the orientation of the friction force vector in the plane of the landing platform, it is necessary to know the relative velocity of each landing wheel at the instant of touchdown. The friction force at each wheel will be in a direction opposite to the direction of the aircraft velocity relative to the landing platform at the point of wheel contact (Ijff, 1972). The point at which wheel w makes contact with the landing platform has a position vector $\bar{R}^w$ relative to the origin of the ship's coordinate system. The angle in the plane of the landing platform between the direction of the X_1^P axis and the velocity vector at wheel w is denoted by κ^w (see fig. 2). When the magnitude of the wheel velocity relative to the landing platform is known, the angle κ^w is obtained as follows. The velocity of wheel w relative to the landing platform is $\bar{V}^w$, where

$$\bar{V}^w = V_1^w\hat{I} + V_2^w\hat{J} + V_3^w\hat{K} \tag{8}$$

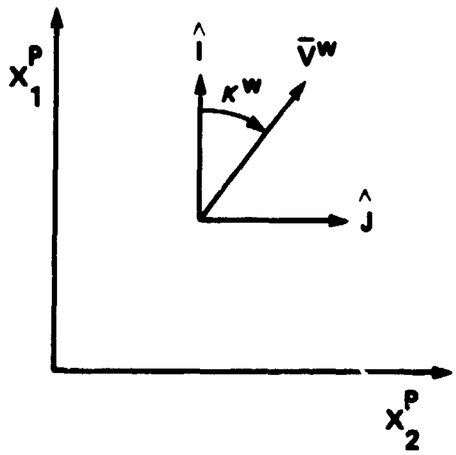


Figure 2.- Systems of platform reference axes.

This velocity is obtained by subtracting the velocity of the point of wheel contact on the platform from the wheel's velocity relative to the platform:

$$\bar{V}^W = D(\bar{V}_{cg} + \bar{\omega}x\bar{r}^W) - (\bar{V}_{ship} + \bar{\omega}_{ship}x\bar{R}^W)$$

In matrix notation these equations assume the form

$$\begin{pmatrix} V_1^W \\ V_2^W \\ V_3^W \end{pmatrix} = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{pmatrix} \begin{pmatrix} v_1^W \\ v_2^W \\ v_3^W \end{pmatrix} - \begin{pmatrix} U + QX_3^W - RX_2^W \\ V + RX_1^W - PX_3^W \\ W + PX_2^W - QX_1^W \end{pmatrix} \tag{9}$$

where U,V,W and P,Q,R are the components of the ship's linear and angular velocity vectors, respectively.

For experiments of this type, the most important components of ship motion are heaving, pitching, and rolling. Hence, by assuming that

$$U = V = R = 0$$

the amount of computation is reduced, and equation (9) assumes the simpler form

$$\begin{pmatrix} V_1^W \\ V_2^W \\ V_3^W \end{pmatrix} = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{pmatrix} \begin{pmatrix} v_1^W \\ v_2^W \\ v_3^W \end{pmatrix} - \begin{pmatrix} QX_3^W \\ -PX_3^W \\ W + PX_2^W - QX_1^W \end{pmatrix} \tag{10}$$

Moreover,

$$\begin{pmatrix} \upsilon_1^w \\ \upsilon_2^w \\ \upsilon_3^w \end{pmatrix} = (\bar{V}_{cg} + \bar{\omega}x\bar{r}^w) = \begin{pmatrix} u + qx_3^w - rx_2^w \\ v + rx_1^w - px_3^w \\ w + px_2^w - qx_1^w \end{pmatrix} \tag{11}$$

where $\bar{V}_{cg}$ is the velocity of the aircraft's center of gravity and $\bar{r}^w$ is the position vector of wheel w relative to the origin of the aircraft body axes system (Etkin, 1972). Hence, in terms of the velocity components V_1^w and V_2^w, the angle κ^w is (see fig. 2)

$$\kappa^w = \tan^{-1}(V_2^w/V_1^w) \tag{12}$$

Equation (10) can be solved for the velocity components V_i^w when the X_i^w coordinates of the point of wheel contact on the platform are known. To determine these coordinates, it is first necessary to compute the position vector $\bar{R}^{cg}$ where

$$\bar{R}^{cg} = X_1^{cg}\hat{i} + X_2^{cg}\hat{j} + X_3^{cg}\hat{k} \tag{13}$$

and X_i^{cg} constitutes the locus of points on the trajectory of the aircraft center of gravity projected onto the plane $(X_3 - C) = 0$, which is the plane of the landing platform. Hence the distance of any point on the projected trajectory from the origin of the ship's coordinate system is

$$\bar{R}^{cg} = X_1^{cg}\hat{i} + X_2^{cg}\hat{j} + C\hat{k} \tag{14}$$

where C is a negative constant. By substituting $X_i^w = X_i^{cg}$ and $x_i^w = 0$ in equations (10) and (11), respectively, the velocity components of the aircraft's center of gravity relative to the landing platform are obtained:

$$\begin{pmatrix} V_1^{cg} \\ V_2^{cg} \\ V_3^{cg} \end{pmatrix} = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} QC \\ -PC \\ W + PX_2^{cg} - QX_1^{cg} \end{pmatrix} \tag{15}$$

Therefore, the velocity components V_1^{cg} and V_2^{cg} are

$$V_1^{cg} = (d_{11}u + d_{21}v + d_{31}w - QC) \tag{16}$$

$$V_2^{cg} = (d_{12}u + d_{22}v + d_{32}w + PC) \tag{17}$$

The velocities V_1^{cg} and V_2^{cg} can be integrated to yield the coordinates X_1^{cg} and X_2^{cg}. Therefore, the required coordinates are

$$X_1^{cg} = X_{10}^{cg} + \int V_1^{cg}\, dt \tag{18}$$

$$X_2^{cg} = X_{20}^{cg} + \int V_2^{cg}\, dt \tag{19}$$

where X_{10}^{cg} and X_{20}^{cg} are initial coordinates.

The velocity of the aircraft center of gravity normal to the landing platform is given by

$$V_3^{cg} = [(d_{13}u + d_{23}v + d_{33}w) - (W + PX_2^{cg} - QX_1^{cg})] \tag{20}$$

Since X_1^{cg} and X_2^{cg} are known from equations (18) and (19), respectively, equation (20) can be integrated to yield

$$X_3^{cg} = X_{30}^{cg} + \int V_3^{cg}\, dt \tag{21}$$

where X_{30}^{cg} is the initial distance from the X_1X_2 plane of the ship's coordinate system. The initial values X_{10}^{cg}, X_{20}^{cg}, and X_{30}^{cg} are ship axes components.

When initial conditions are given in Earth-fixed axes, a transformation from these axes to ship axes is required. Given that $\bar{X}_{10}^{cg}$ are initial values in Earth-fixed axes, the corresponding ship axes components are (from appendix A)

$$\begin{pmatrix} X_{10}^{cg} \\ X_{20}^{cg} \\ X_{30}^{cg} \end{pmatrix} = [T]_{ES} \begin{pmatrix} \bar{X}_{10}^{cg} \\ \bar{X}_{20}^{cg} \\ \bar{X}_{30}^{cg} \end{pmatrix} \tag{22}$$

When equations (18) through (21) are modified in accordance with this transformation, the coordinates of the aircraft trajectory assume the form:

$$\begin{pmatrix} X_1^{cg} \\ X_2^{cg} \\ X_3^{cg} \end{pmatrix} = [T]_{ES} \begin{pmatrix} \bar{X}_{10}^{cg} \\ \bar{X}_{20}^{cg} \\ \bar{X}_{30}^{cg} \end{pmatrix} + \int \begin{pmatrix} V_1^{cg} \\ V_2^{cg} \\ V_3^{cg} \end{pmatrix} dt \tag{23}$$

Let the origin location of the landing platform with respect to the ship's coordinate system be $X^P = (X^P_{10}, X^P_{20}, C)$; then the height of the aircraft center of gravity above the landing platform is X^P_3, where

$$X^P_3 = (X^{cg}_3 - C)$$

Moreover,

$$X^P_1 = (X^{cg}_1 - X^P_{10})$$

$$X^P_2 = (X^{cg}_2 - X^P_{20})$$

Wheel w will make contact with the landing platform when (Howard, 1977)

$$X^P_3 = -(d_{13}x^w_1 + d_{23}x^w_2 + d_{33}x^w_3) \tag{24}$$

(see fig. 3).

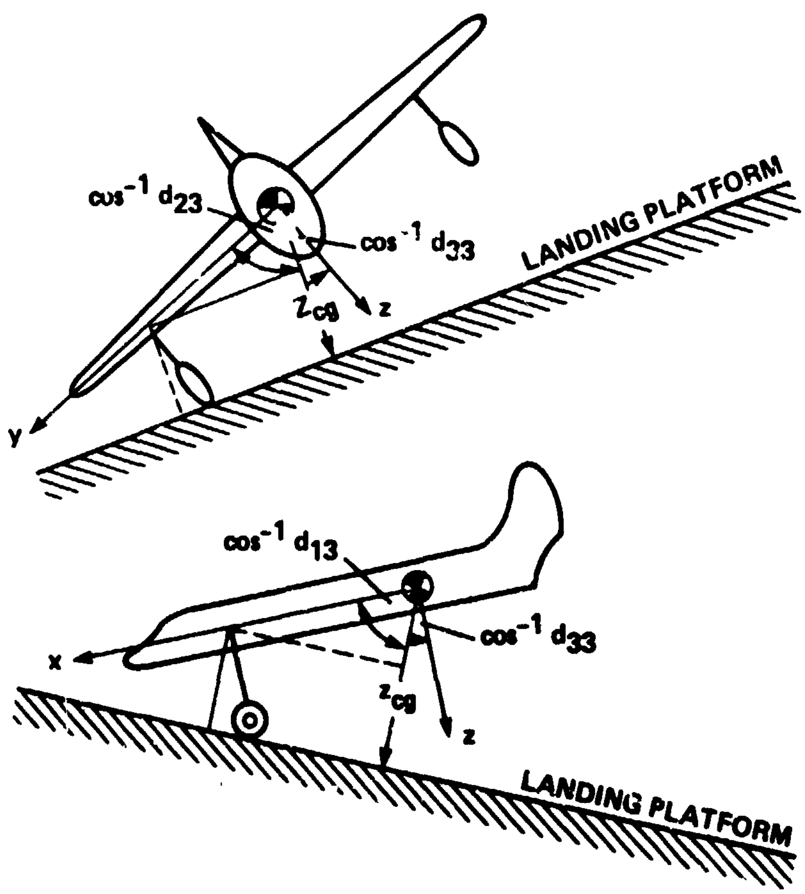


Figure 3.- Aircraft attitude relative to landing platform.

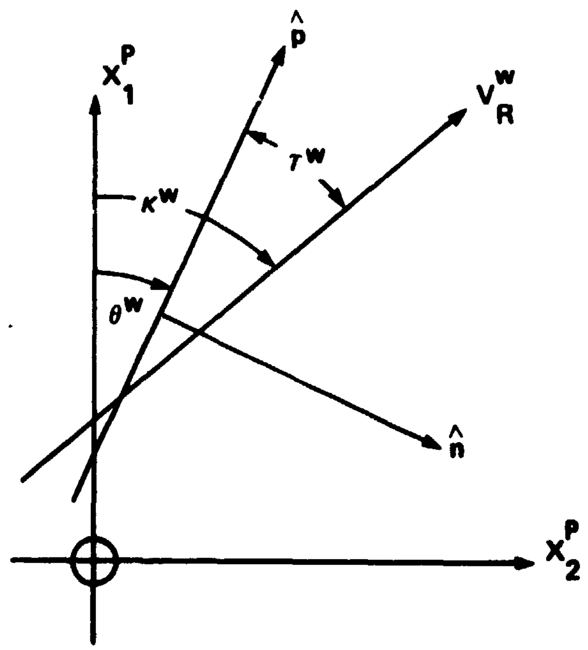
When this condition is satisfied, the computed values of X_1^{cg} and X_2^{cg} may be used to determine the point of contact of wheel w from

$$X_1^w = X_1^{cg} + d_{11}x_1^w + d_{21}x_2^w + d_{31}x_3^w \tag{25}$$

$$X_2^w = X_2^{cg} + d_{12}x_1^w + d_{22}x_2^w + d_{32}x_3^w \tag{26}$$

The relative velocity components can now be evaluated by substituting from equations (25) and (26) in equation (10), and the angle κ^w determined from equation (12).

Kinetic friction force- The friction force vector $\bar{F}^w$ at wheel w is opposite the resultant velocity vector and is equal to the product of the coefficient of friction and the platform reaction. It is assumed that the coefficient of friction is a function of the angle τ^w, which is the angle between the resultant velocity vector at wheel w and the line of intersection of the rolling plane of the wheel with the landing platform. Denoting the direction of this line by the unit vector $\hat{p}$ and the perpendicular direction by the unit vector $\hat{n}$, the velocity components relative to these directions are (see sketch (a))



Sketch (a)

$$V_p^w = V_R^w \cos(\kappa^w - \theta_3^w) = V_R^w \cos \tau^w \tag{27}$$

$$V_n^w = V_R^w \sin(\kappa^w - \theta_3^w) = V_R^w \sin \tau^w \tag{28}$$

where κ^w is defined by equation (12) and θ_3^w is the aircraft angle of yaw with respect to platform axes. The velocity components at wheel w relative to platform axes are V_1^w and V_2^w and the resultant velocity in the plane of the platform is V_R^w.

The equation for angle τ^w may be written in the following alternative forms (see eq. (A19)):

$$\tau^w = \left[\tan^{-1}\frac{V_2^w}{V_1^w} - \theta_3^w\right]$$

$$\tau^w = \tan^{-1}\left(\frac{V_2^w - V_1^w \tan\theta_3^w}{V_1^w + V_2^w \tan\theta_3^w}\right) = (\kappa^w - \theta_3^w)$$

Relative to these directions, the kinetics friction force vector $\bar{F}_f^w$ assumes the form

$$\bar{F}_f^w = \mu^w(\cos\tau^w\hat{p} + \sin\tau^w\hat{n})F_3^w \qquad (29)$$

Relative to aircraft body axes, the kinetic friction force and the platform reaction have components f_{ig}^w, where

$$\begin{pmatrix} f_{1g}^w \\ f_{2g}^w \\ f_{3g}^w \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} \begin{pmatrix} F_3^w\mu^w \cos\tau^w \\ F_3^w\mu^w \sin\tau^w \\ F_3^w \end{pmatrix} \qquad (30)$$

and

$$\begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ \sin\theta_1 \sin\theta_2 & \cos\theta_1 & \sin\theta_1 \cos\theta_2 \\ \cos\theta_1 \sin\theta_2 & -\sin\theta_1 & \cos\theta_1 \cos\theta_2 \end{pmatrix}$$

Kinetic friction moments- When the components of the force vector from equation (30) are substituted in equations (5) through (7), and the coefficients D_{ij} substituted for d_{ij}, the moment components are obtained as functions of the platform reactions. These are

$$M_1^w = [x_2^w(D_{31}\mu^w \cos \tau^w + D_{32}\mu^w \sin \tau^w + D_{33})$$

$$- x_3^w(D_{21}\mu^w \cos \tau^w + D_{22}\mu^w \sin \tau^w + D_{23})]F_3^w \quad (31)$$

$$M_2^w = [x_3^w(D_{11}\mu^w \cos \tau^w + D_{12}\mu^w \sin \tau^w + D_{13})$$

$$- x_1^w(D_{31}\mu^w \cos \tau^w + D_{32}\mu^w \sin \tau^w + D_{33})]F_3^w \quad (32)$$

$$M_3^w = [x_1^w(D_{21}\mu^w \cos \tau^w + D_{22}\mu^w \sin \tau^w + D_{33})$$

$$- x_2^w(D_{11}\mu^w \cos \tau^w + D_{12}\mu^w \sin \tau^w + D_{13})]F_3^w \quad (33)$$

Experimental evidence indicates that, in the case of normal impact without tire bottoming, the assumption of a linear force-deflection relationship for the tire is adequate for practical purposes (Milwitzky and Cook, 1953). Hence, by assuming that each tire is a linear spring with an effective stiffness K_ε^w, the force resulting from a deflection δ^w can be expressed in the simple form

$$F_3^w = -K_\varepsilon^w \delta^w \quad (34)$$

Moreover, in the event that tire bottoming occurs, a linear segment approximation which takes into account the increased stiffness of the tire that results from bottoming yields good results.

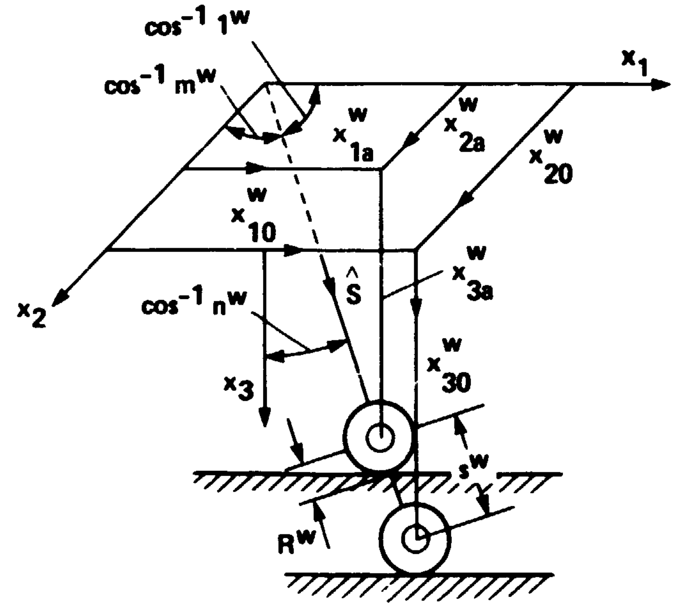
Subsequent to tire and shock-strut deflection, the moment arms become functions of the tire and shock-strut deflections; that is

$$\left.\begin{aligned} x_1^w &= (x_{10}^w - \ell^w s^w) + (\ell^w R^w - D_{13}\delta^w) \\ x_2^w &= (x_{20}^w - m^w s^w) + (m^w R^w - D_{23}\delta^w) \\ x_3^w &= (x_{30}^w - n^w s^w) + (n^w R^w - D_{33}\delta^w) \end{aligned}\right\} \quad (35)$$

where x_{io}^w are the coordinates of the axle at wheel w when the shock strut is in the fully extended position; ℓ^w, m^w, and n^w are direction cosines; s^w is the shock-strut deflection, and R^w is the radius of wheel w (see fig. 4). Hence, in terms of these moment arms and tire forces, the moment equations are

$$M_1^w = [x_3^w(D_{21}\mu^w \cos \tau^w + D_{22}\mu^w \sin \tau^w + D_{23})$$

$$- x_2^w(D_{31}\mu^w \cos \tau^w + D_{32}\mu^w \sin \tau^w + D_{33})]K_\varepsilon^w \delta^w \quad (36)$$



x^w_{10} = COORDINATES OF AXLE AT WHEEL w WHEN SHOCK STRUT IS IN FULLY EXTENDED POSITION

R^w = RADIUS OF WHEEL w

Figure 4.- Orientation of landing gear relative to aircraft body axes.

$$M_2^w = [x_1^w(D_{31}\mu^w \cos \tau^w + D_{32}\mu^w \sin \tau^w + D_{33}) - x_3^w(D_{11}\mu^w \cos \tau^w + D_{12}\mu^w \sin \tau^w + D_{13})]K_\varepsilon^w\delta^w \quad (37)$$

$$M_3^w = [x_2^w(D_{11}\mu^w \cos \tau^w + D_{12}\mu^w \sin \tau^w + D_{13}) - x_1^w(D_{21}\mu^w \cos \tau^w + D_{22}\mu^w \sin \tau^w + D_{23})]K_\varepsilon^w\delta^w \quad (38)$$

The forces relative to aircraft body axes are

$$\begin{pmatrix} f_{1g}^w \\ f_{2g}^w \\ f_{3g}^w \end{pmatrix} = -K_e^w\delta^w \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} \begin{pmatrix} \mu^w \cos \tau^w \\ \mu^w \sin \tau^w \\ 1 \end{pmatrix} \quad (39)$$

It should be noticed that, when the aircraft rebounds, $\delta^w = 0$ and the friction forces and moments vanish.

Static friction forces- Subsequent to wheel contact, wheel w is exposed to a static friction force. If this force is not sufficient to prevent sliding, a kinetic friction force will be generated. The kinetic friction force will persist until the velocity of wheel w in the plane of the landing platform is dissipated, at which point a static friction force is again encountered. During static friction conditions, the velocity of the point of wheel contact in the plane of the landing platform is zero. In this case, the wheel axle will move relative to the point of wheel contact by stretching the tire spring and the aircraft will oscillate under the restoring influence of the tire springs.

If torsional stiffness is neglected, each tire may be treated as a three-spring system: a lateral spring that acts to restore deformations of the tire normal to the plane of the wheel, a spring that resists displacements in the direction of the strut axis, and a spring normal to these two directions. That is, if $\hat{n}$ is a unit vector normal to the plane of the wheel and $\hat{s}$ a unit vector coaxial with the shock strut, then the tire resists motions in the directions of $\hat{n}$, $\hat{s}$, and $\hat{n} \times \hat{s}$.

In order to determine the spring forces, it is necessary to know the velocity components of the wheel axle in the directions of $\hat{n}$, $\hat{s}$, and $\hat{n} \times \hat{s}$,

$$\begin{aligned} \hat{n} &= L\hat{i} + M\hat{j} + M\hat{k} \\ \hat{s} &= \ell\hat{i} + m\hat{j} + n\hat{k} \qquad \text{(39a)} \\ \hat{n} \times \hat{s} &= (nM - mN)\hat{i} + (\ell N - nL)\hat{j} + (mL - \ell M)\hat{k} \end{aligned}$$

and the direction cosines $\ell^w, m^w, n^w, L^w, M^w, N^w$ must be ascertained from the geometry of a particular aircraft. The tire displacements are δ_n^w normal to the plane of the wheel, δ_v^w in the direction of the strut axis, and δ_ρ^w normal to these two directions. Before touch-down, the displacement of the wheel axle relative to the point of wheel contact is zero. After touch-down, the velocity components of the wheel axle are

$$\begin{pmatrix} \dot{\delta}_\rho^w \\ \dot{\delta}_\eta^w \\ \dot{\delta}_v^w \end{pmatrix} = \begin{pmatrix} n^wM^w - m^wN^w & \ell^wN^w - n^wL^w & m^wL^w - \ell^wM^w \\ L^w & M^w & N^w \\ \ell^w & m^w & n^w \end{pmatrix} \left[\begin{pmatrix} u_a^w + qx_{3a}^w - rx_{2a}^w \\ v_a^w + rx_{1a}^w - px_{3a}^w \\ w_a^w + px_{2a}^w - qx_{1a}^w \end{pmatrix} - \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix} \begin{pmatrix} QX_3^W \\ -PX_3^W \\ W + PX_2^W - QX_1^W \end{pmatrix} \right]$$

and

$$u_a^w = (u - \ell^w \dot{s}^w) \quad ; \quad v_a^w = (v - m^w \dot{s}^w) \quad ; \quad w_a^w = (w - n^w \dot{s}^w)$$

$$x_{1a}^w = (x_{1o}^w - \ell^w s^w) \quad ; \quad x_{2a}^w = (x_{2o}^w - m^w s^w) \quad ; \quad x_{3a}^w = (x_{3o}^w - n^w s^w)$$

The x_{1o}^w are the coordinates of the axle when the shock strut is in the fully extended position and s^w is the shock-strut deflection at wheel w.

The forces generated by the tire displacements are

$$\begin{pmatrix} f_p^w \\ f_n^w \\ f_v^w \end{pmatrix} = - \begin{pmatrix} k_p^w & 0 & 0 \\ 0 & k_n^w & 0 \\ 0 & 0 & k_v^w \end{pmatrix} \begin{pmatrix} \delta_p^w \\ \delta_n^w \\ \delta_v^w \end{pmatrix}$$

where

$$k_n^w = k_n^w(p^w, \delta^w) \quad ; \quad k_v^w = k_v^w(p^w, \delta^w) \quad ; \quad k_p^w = k_p^w(p^w, \delta^w)$$

and p^w and δ^w are the pressure and tire deflection, respectively, at wheel w.

If after touch-down wheel w has a velocity component V_p^w in the direction of unit vector $\hat{p}$, then a kinetic friction force will be generated. The magnitude of the force will be

$$F_p^w = \mu^w F_3^w \cos \tau^w \quad ; \quad \mu^w = \mu^w(\tau^w)$$

Likewise, a velocity component V_n^w perpendicular to V_p^w will be accompanied by a kinetic friction force F_n^w, where

$$F_n^w = \mu^w F_3^w \sin \tau^w \quad ; \quad \mu^w = \mu^w(\tau^w)$$

Generally, these two components will exist simultaneously and give rise to the resultant kinetic friction force:

$$F_f^w = \mu^w F_3^w$$

The tire force components (F_{iT}^w) relative to the landing platform are obtained by transforming the tire forces from strut axes to platform axes. These are:

$$\begin{pmatrix} F^W_{1T} \\ F^W_{2T} \\ F^W_{3T} \end{pmatrix} = -\begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \end{pmatrix} \begin{pmatrix} L^W & \ell^W & n^W M^W - m^W N^W \\ M^W & m^W & \ell^W N^W - n^W L^W \\ N^W & n^W & m^W L^W - \ell^W M^W \end{pmatrix} \begin{pmatrix} k^W_n \delta^W_n \\ k^W_v \delta^W_v \\ k^W_p \delta^W_p \end{pmatrix}$$

A static friction force will persist until the friction force exceeds the spring force, that is, until

$$(F^W_f)^2 > [(F^W_{1T})^2 + (F^W_{2T})^2]$$

When this condition is satisfied, the tire starts to slide and kinetic friction forces begin to operate. Relative to aircraft axis, the tire force components are

$$\begin{pmatrix} f^W_{1T} \\ f^W_{2T} \\ f^W_{3T} \end{pmatrix} = -\begin{pmatrix} L^W & \ell^W & n^W M^W - m^W N^W \\ M^W & m^W & \ell^W N^W - n^W L^W \\ N^W & n^W & m^W L^W - \ell^W M^W \end{pmatrix} \begin{pmatrix} k^W_n \delta^W_n \\ k^W_v \delta^W_v \\ k^W_p \delta^W_p \end{pmatrix}$$

When the aircraft touches down, that is, when the wheel contact condition (eq. (24)) is satisfied, the tire deflection rates are calculated and integrated and the corresponding spring forces determined. These are then compared with the forces required to produce sliding. When the kinetic friction forces exceed the spring forces, sliding ensues and the forces are modified accordingly.

Effective tire stiffness- The effective tire stiffness at any orientation is a function of the component stiffnesses of the tire, the direction cosines defining the orientation of the shock strut and wheel relative to aircraft body axes, and the spatial orientation of the aircraft relative to the landing platform. The tire force produced by a tire deflection δ normal to the landing platform is $K_e \delta \hat{K}$, where K_e is the effective tire stiffness and $\hat{K}$ is a unit vector normal to the landing platform. The corresponding tire forces in the directions of $\hat{n}$, $\hat{s}$, and $\hat{n} \times \hat{s}$ are, respectively:

$$(LD_{13} + MD_{23} + ND_{33})\delta^W K^W_n(p^W, \delta^W)$$

$$(\ell D_{13} + mD_{23} + nD_{33})\delta^W K^W_v(p^W, \delta^W)$$

$$[(nM - mN)D_{13} + (\ell N - nL)D_{23} + (mL - \ell M)D_{33}]\delta^W K^W_p(p^W \delta^W)$$

Therefore,

$$K_e^w = \{(LD_{13} + MD_{23} + ND_{33})^2K_n^2 + (\ell D_{13} + mD_{23} + nD_{33})^2K_v^2$$
$$+ [(nM - mN)D_{13} + (\ell N - nL)D_{23} + (mL - \ell M)D_{33}]^2K_p^2\}^{1/2}$$

KINEMATICS AND DYNAMICS OF LANDING GEAR

Landing Gear Forces

Resultant force- The friction force and the platform reaction force are reacted at the wheel axle by a resultant force, which can be resolved into a component coaxial with the shock strut and a component normal to the shock strut.

Axle force- The force $\bar{F}_a^w$ acting at the wheel axle and the force $\bar{F}_g^w$ acting at the point of wheel contact with the landing platform combine with the gravity force $M^w\bar{g}$ to impart an acceleration to the wheel, that is,

$$\bar{F}_a^w + \bar{F}_g^w + M^w\bar{g} = M^w\ddot{\bar{r}}^w$$

where $\bar{g}$ is the gravity acceleration vector and $\ddot{\bar{r}}$ is the acceleration of wheel w (Milwitzky and Cook, 1953). Therefore,

$$\bar{F}_a^w = M^w(\ddot{\bar{r}}^w - \bar{g}) - \bar{F}_g^w \tag{40}$$

The resultant force acting at the axle of wheel w has components f_{ia}^w relative to aircraft body axes. The force components normal to the shock strut may be expressed in terms of these components as follows.

The scalar magnitude of the body axes components f_{ia}^w normal to the shock strut are

$$f_{1a}^w \sin(\cos^{-1} \ell^w) \;; \quad f_{2a}^w \sin(\cos^{-1} m^w) \;; \quad f_{3a}^w \sin(\cos^{-1} n^w) \tag{41}$$

In terms of the aircraft body axes components f_{ia}^w, the normal force $\bar{F}_{N_a}^w$ has vector components as follows (Wills, 1958)

$$\bar{F}_{N_a}^w = f_{1a}^w(\hat{s} \times \hat{i}) \times \hat{s} + f_{2a}^w(\hat{s} \times \hat{j}) \times \hat{s} + f_{3a}^w(\hat{s} \times \hat{k}) \times \hat{s} \tag{42}$$

Using vector identities, this equation can be rewritten as follows:

$$\bar{F}_{N_a}^w = \{f_{1a}^w[\hat{i} - (\hat{i} \cdot \hat{S})\hat{S}] + f_{2a}^w[\hat{j} - (\hat{j} \cdot \hat{S})\hat{S}] + f_{3a}^w[\hat{k} - (\hat{k} \cdot \hat{S})\hat{S}]\} \tag{43}$$

By substitution for $\hat{S}$ from equation (39a), equation (43) assumes the following form:

$$\bar{F}^w_{N_a} = \{[(1 - \ell\ell)f^w_{1a} - \ell m f^w_{2a} - \ell n f^w_{3a}]\hat{i} + [-m\ell f^w_{1a} + (1 - mm)f^w_{2a} - mnf^w_{3a}]\hat{j}$$
$$+ [-n\ell f^w_{1a} - nmf^w_{2a} + (1 - nn)f^w_{3a}]\hat{k}\} \qquad (44)$$

<u>Shock strut and platform forces</u>- In addition to the force $\bar{F}^w_{N_a}$ normal to the shock strut and the axial strut force $\bar{F}^w_s$ applied at the axle, the wheel is subjected to the platform reaction and friction force $\bar{F}^w_g$. This force is applied at the point of wheel contact and has components f^w_{ig} relative to aircraft body axes. The resultant of these forces is

$$F^w = (\bar{F}^w_{N_a} + \bar{F}^w_s + F^w_g)$$
$$= \{[(1 - \ell\ell)f^w_{1a} - \ell m f^w_{2a} - \ell n f^w_{3a} + \ell F^w_s + f^w_{1g}]\hat{i} + [-m\ell f^w_{1a} + (1 - mm)f^w_{2a}$$
$$- mnf^w_{3a} + mF^w_s + f^w_{2g}]\hat{j} + [-n\ell f^w_{1a} - nmf^w_{2a} + (1 - nn)f^w_{3a} + nF^w_s + f^w_g]\hat{k}\}$$
$$= M^w(\ddot{\bar{r}}^w - \bar{g}) \qquad (45)$$

This equation may be written more concisely as

$$[F^w] = [L^w][F^w_a] + F^w_s\begin{pmatrix} \ell^w \\ m^w \\ n^w \end{pmatrix} + [F^w_g] = M^w([\ddot{x}^w] - [g]) \qquad (46)$$

where

$$[F^w] = \begin{pmatrix} f^w_1 \\ f^w_2 \\ f^w_3 \end{pmatrix}$$

$$[L^w] = \begin{pmatrix} 1 - \ell^w\ell^w & -\ell^w m^w & -\ell^w n^w \\ -m^w\ell^w & 1 - m^w m^w & -m^w n^w \\ -n^w\ell^w & -n^w m^w & 1 - n^w n^w \end{pmatrix}$$

$$[F_a^w] = \begin{pmatrix} f_{1a}^w \\ f_{2a}^w \\ f_{3a}^w \end{pmatrix}$$

$$[F_g^w] = \begin{pmatrix} f_{1g}^w \\ f_{2g}^w \\ f_{3g}^w \end{pmatrix}$$

From equation (40),

$$[F_a^w] = -[F_g^w] + M^w([\ddot{X}^w] - [g]) \tag{47}$$

Substituting from equation (39) yields

$$[F_a^w] = K_\varepsilon^w \delta^w [D][F_f^w] + M^w([\ddot{X}^w] - [g]) \tag{48}$$

$$[D] = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}$$

$$[F_f^w] = \begin{pmatrix} \mu^w \cos \tau^w \\ \mu^w \sin \tau^w \\ 1 \end{pmatrix}$$

Substituting from equations (48) and (39) in equation (46) gives the equation of motion of the wheel mass M^w:

$$[L^w]\{K_\varepsilon^w \delta^w [D][F_f^w] + M^w([\ddot{X}^w] - [g])\} + F_s^w \begin{pmatrix} \ell^w \\ m^w \\ n^w \end{pmatrix} - K_\varepsilon^w \delta^w [D][F_f^w] = M^w([\ddot{X}^w] - [g]) \tag{49}$$

Equation (49) gives the forces acting at wheel w and establishes a relationship between the forces due to tire deflection and the shock-strut forces.

The corresponding force acting on the aircraft is $[F_A^W]$, where

$$[F_A^W] = -[L^W]\{K_\epsilon^W \delta^W [D][F_f^W] + M^W([\ddot{X}^W] - [g])\} - F_s^W \begin{pmatrix} \ell^W \\ m^W \\ n^W \end{pmatrix} \qquad (50)$$

and

$$(F_A^W) = \begin{pmatrix} f_1^W \\ f_2^W \\ f_3^W \end{pmatrix}_A$$

that is, the axial strut force F_s^W due to hydraulic resistance, air compression, and internal bearing friction combines with $F_{N_s}^W$ and the prevailing aerodynamic, thrust, inertia, and gravity forces to modify the aircraft motion subsequent to wheel contact.

Influence of wheel mass- The fact that the mass of the wheel is a relatively small fraction of the total mass of the aircraft suggests that the wheel mass can be neglected without impairing the validity of the calculated results (Milwitzky and Cook, 1953). With this modification, the equations relating the tire forces to the strut forces assume the simpler form:

$$K_\epsilon^W \delta^W [L^W][D][F_f^W] + F_s^W \begin{pmatrix} \ell^W \\ m^W \\ n^W \end{pmatrix} = K_\epsilon^W \delta^W [D][F_f^W] \qquad (51)$$

By combining like terms, this equation can be rearranged as

$$K_\epsilon^W \delta^W ([I] - [L^W])[D][F_f^W] = F_s^W \begin{pmatrix} \ell^W \\ m^W \\ n^W \end{pmatrix}$$

where

$$[I] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K_{\varepsilon}^{w}\delta^{w}\begin{pmatrix} \ell\ell & \ell m & \ell n \\ m\ell & mm & mn \\ n\ell & nm & nn \end{pmatrix}\begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}\begin{pmatrix} \mu^{w}\cos\tau^{w} \\ \mu^{w}\sin\tau^{w} \\ 1 \end{pmatrix} = F_{s}^{w}\begin{pmatrix} \ell \\ m \\ n \end{pmatrix}$$

Matrix multiplication yields

$$K_{\varepsilon}^{w}\delta^{w}\begin{pmatrix} \ell\mathcal{D}_{11} & \ell\mathcal{D}_{12} & \ell\mathcal{D}_{13} \\ m\mathcal{D}_{11} & m\mathcal{D}_{12} & m\mathcal{D}_{13} \\ n\mathcal{D}_{11} & n\mathcal{D}_{12} & n\mathcal{D}_{13} \end{pmatrix}\begin{pmatrix} \mu^{w}\cos\tau^{w} \\ \mu^{w}\sin\tau^{w} \\ 1 \end{pmatrix} = F_{s}^{w}\begin{matrix} \ell \\ m \\ n \end{matrix}$$

where

$$\mathcal{D}_{11} = \ell D_{11} + mD_{21} + nD_{31}$$

$$\mathcal{D}_{12} = \ell D_{12} + mD_{22} + nD_{32}$$

$$\mathcal{D}_{13} = \ell D_{13} + mD_{23} + nD_{33}$$

Therefore,

$$K_{\varepsilon}^{w}\delta^{w}(\mathcal{D}_{11}\mu^{w}\cos\tau^{w} + \mathcal{D}_{12}\mu^{w}\sin\tau^{w} + \mathcal{D}_{13}) = F_{s}^{w} = F_{s}^{w}(S,\dot{S}) \tag{52}$$

For zero wheel mass, the landing gear forces acting on the aircraft are

$$-\left\{K_{\varepsilon}^{w}\delta^{w}[L^{w}][D][F_{f}^{w}] + F_{s}^{w} \cdot \begin{pmatrix} \ell^{w} \\ m^{w} \\ n^{w} \end{pmatrix}\right\} \tag{53}$$

Equation (51) permits this equation to be written in the simpler form:

$$-K_{\varepsilon}^{w}\delta^{w}[D][F_{f}^{w}] = [F_{A}^{w}] \tag{54}$$

<u>Shock-strut closure and tire deflection rates</u>- Subsequent to the time when

$$X_{3}^{p} = -(d_{13}x_{1g}^{w} + d_{23}x_{2g}^{w} + d_{33}x_{3g}^{w})$$

platform reaction forces, friction forces, and shock-strut forces are generated in accordance with the equations formulated. Equation (52) gives the relationship between the tire deflection forces and the shock-strut forces. The rate of tire deflection is $\dot{\delta}^W$, where

$$\dot{\delta}^W = d_{13}(u_a^W + qx_{3a}^W - rx_{2a}^W) + d_{23}(v_a^W + rx_{1a}^W - px_{3a}^W)$$

$$+ d_{33}(w_a^W + px_{2a}^W - qx_{1a}^W) - (W + PX_2^W \cdot QX_1^W) \tag{55}$$

where

$$u_a^W = (u - \ell^W \dot{s}^W) \; ; \quad v_a^W = (v - m^W \dot{s}^W) \; ; \quad w_a^W = (w - n^W \dot{s}^W)$$

and

$$x^W(x_{10}^W - \ell^W s^W) \; ; \quad x_{2a}^W = (x_{20}^W - m^W s^W) \; ; \quad x_{3a}^W = (x_{30}^W - n^W s^W)$$

Equations (52) and (55) can be solved to obtain δ^W and s^W, which can then be used to compute the platform reactions and friction forces and the forces generated in the shock strut. When these forces are added to the aerodynamic, inertia, thrust, and gravity forces, the response of the aircraft subsequent to wheel contact can be determined.

CONCLUSIONS

The equations describing the friction forces generated during landing operations on ships at sea have been formulated. To simplify the formulation, it has been assumed that the force-deflection characteristics of the tire are linear, and that the behavior of the shock strut is relatively insensitive to variations in the air compression process. By ignoring variations in the polytropic exponent and by including only those characteristics of the tire and shock strut that contribute significantly to the generation of landing gear forces, a set of relatively simple equations is obtained. Nevertheless, these equations are considered adequate for practical purposes.

APPENDIX A

TRANSFORMATIONS

Transformation of Motion Vector Components

A set of vector components in a coordinate system that is rotationally fixed is related to the components in the aircraft body axes by a transformation equation of the form

$$[A] = [T]_{EA}[E] \tag{A1}$$

where

$[A]$ column vector of motion components in aircraft reference system

$[T]_{EA}$ matrix that effects a transformation from fixed axes to aircraft body axes

$[E]$ column vector of motion components in fixed reference system

Likewise, the components of a vector in the fixed reference system are related to the components in the moving ship reference system by a transformation of the same form. That is,

$$[S] = [T]_{ES}[E] \tag{A2}$$

where

$[S]$ column vector of motion vector components relative to ship axes

$[T]_{ES}$ matrix that effects a transformation from fixed axes to moving ship axes

Similarly, a triad of ship axes components can be transformed to aircraft body axes by the transformation equation

$$[A] = [T]_{SA}[S] \tag{A3}$$

where

$[T]_{SA}$ matrix that effects a transformation from ship axes to aircraft body axes

Substituting from equation (A2) in equation (A3) gives a transformation from fixed axes to ship axes, followed by a transformation from ship axes to aircraft axes:

$$[A] = [T]_{SA}[T]_{ES}[E] \tag{A4}$$

Finally, substituting from equation (A1) in equation (A4) yields the following matrix equation:

$$[A] = [T]_{SA}[T]_{ES}[T]_{EA}^{-1} A$$

Therefore, $[T]_{SA}[T]_{ES}[T]_{EA}^{-1} = [I]$, where $[I]$ is the unit matrix. Solving this matrix equation for $[T]_{SA}$ yields

$$[T]_{SA} = [T]_{EA}[T]_{ES}^{-1} \tag{A5}$$

Since only orthogonal transformations are being considered, the inverse of a transformation matrix equals the transpose of the matrix and equation (A5) simplifies accordingly. That is,

$$[T]_{ES}^{-1} = [T]_{ES}^{T} \tag{A6}$$

where superscript T denotes transposition.

Substituting from equation (A6) in equation (A5) yields the required transformation from ship axes to aircraft body axes. That is,

$$[T]_{SA} = [T]_{EA}[T]_{ES}^{T} \tag{A7}$$

In terms of the Euler angles ψ, θ, and ϕ and with the conventional aeronautical rotation sequence, the required transformation matrices are (McRuer et al., 1973)

$$[T]_{EA} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[T]_{EA} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \sin\psi\cos\phi & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\phi\cos\theta \\ \cos\psi\cos\phi\sin\theta + \sin\psi\sin\phi & \sin\psi\cos\phi\sin\theta - \cos\psi\sin\phi & \cos\phi\cos\theta \end{bmatrix} \tag{A8}$$

For the transformation from fixed axes to ship axes, the Euler angles will be denoted by the capital Greek letters Ψ, Θ, and Φ. In terms of this notation, the transformation matrix $[T]_{ES}$ assumes a form identical to equation (A8)

$$
[T]_{ES} = \begin{bmatrix} \cos\Theta \cos\Psi & \cos\Theta \sin\Psi & -\sin\Theta \\ \sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi & \sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi & \sin\Phi \cos\Theta \\ \cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Phi & \sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Theta & \cos\Phi \cos\Theta \end{bmatrix} \qquad \text{(A9)}
$$

The following equation gives the transposed form of this matrix:

$$
[T]_{ES}^{T} = \begin{bmatrix} \cos\Theta \cos\Psi & \sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi & \cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Phi \\ \cos\Theta \sin\Psi & \sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi & \sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Phi \\ -\sin\Theta & \sin\Phi \cos\Theta & \cos\Phi \cos\Theta \end{bmatrix} \qquad \text{(A10)}
$$

A computer program, which solves the equations of the mathematical model of the aircraft, evaluates angles ψ, θ, and ϕ and hence determines the attitude of the aircraft as a function of time. The Euler angles are then used to compute the elements of the transformation matrix $[T]_{EA}$.

To determine the attitude of the ship, the components of the ship's angular velocity vector are measured and used to formulate the equations (McRuer et al., 1973)

$$
\left.\begin{aligned} P &= \dot{\Phi} - \dot{\Psi} \sin\Theta \\ Q &= \dot{\Theta} \cos\Phi + \dot{\Psi} \cos\Theta \sin\Phi \\ R &= \dot{\Psi} \cos\Theta \cos\Phi - \dot{\Theta} \sin\Phi \end{aligned}\right\} \qquad \text{(A11)}
$$

Solving these equations for $\dot{\Phi}$, $\dot{\Theta}$, and $\dot{\Psi}$, we obtain

$$
\left.\begin{aligned} \dot{\Phi} &= P + (Q \sin\Phi + R \cos\Phi)\tan\Theta \\ \dot{\Theta} &= Q \cos\Phi - R \sin\Phi \\ \dot{\Psi} &= (Q \sin\Phi + R \cos\Phi)\sec\Theta \end{aligned}\right\} \qquad \text{(A12)}
$$

The solution of these equations yields the required Euler angles Φ, θ, and Ψ, which are then used to determine the elements of the transformation matrix $[T]_{ES}$. After the formulation and transposition of this matrix, the product of $[T]_{EA}$ and $[T]_{ES}^{T}$ is formed.

The transformation of vector components from ship axes to aircraft body axes is given by equation (A7):

$$
[T]_{SA} = [T]_{EA}[T]_{ES}^{T}
$$

In terms of the direction cosines d_{ij}, relating aircraft body axes to ship axes, this matrix equation assumes the form

$$[T]_{SA} = \begin{bmatrix} d_{11} d_{12} d_{13} \\ d_{21} d_{22} d_{23} \\ d_{31} d_{32} d_{33} \end{bmatrix} = [D]$$

The direction cosines d_{ij} have the following values:

$$d_{11} = (\cos\theta \cos\psi \cos\Theta \cos\Psi + \cos\theta \sin\psi \cos\Theta \sin\Psi + \sin\theta \sin\Theta)$$

$$d_{12} = [\cos\theta \cos\psi(\sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi)$$
$$+ \cos\theta \sin\psi(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi) - \sin\theta \sin\Phi \cos\Theta]$$

$$d_{13} = [\cos\theta \cos\psi(\cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Theta)$$
$$+ \cos\theta \sin\psi(\sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Phi) - \sin\theta \cos\Phi \cos\Theta]$$

$$d_{21} = [(\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi)\cos\Theta \cos\Psi$$
$$+ (\sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi)\cos\Theta \sin\Psi - \sin\phi \cos\theta \sin\Theta]$$

$$d_{22} = [(\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi)(\sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi)$$
$$+ (\sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi)(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi)$$
$$+ \sin\phi \cos\theta \sin\Phi \cos\Theta]$$

$$d_{23} = [(\sin\phi \sin\theta \cos\psi - \sin\psi \cos\phi)(\cos\Psi \cos\Phi \sin\Theta + \sin\Psi \sin\Phi)$$
$$+ (\sin\psi \sin\theta \sin\phi + \cos\psi \cos\phi)(\sin\Psi \cos\Phi \sin\Theta - \cos\Psi \sin\Phi)$$
$$+ \sin\phi \cos\theta \cos\Phi \cos\Theta]$$

$$d_{31} = [(\cos\psi \cos\phi \sin\theta + \sin\psi \sin\phi)(\cos\Theta \cos\Psi)$$
$$+ (\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi)\cos\Theta \sin\Psi - \cos\phi \cos\theta \sin\Theta]$$

$$d_{32} = [(\cos\psi \cos\phi \sin\theta + \sin\psi \sin\phi)(\sin\Phi \sin\Theta \cos\Psi - \sin\Psi \cos\Phi)$$
$$+ (\sin\psi \cos\phi \sin\theta - \cos\psi \sin\phi)(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi)$$
$$+ \cos\phi \cos\theta \sin\Phi \cos\Theta]$$

$$d_{33} = [(\cos\psi\cos\phi\sin\theta + \sin\psi\sin\phi)(\cos\Psi\cos\Phi\sin\Theta + \sin\Psi\sin\Phi)$$
$$+ (\sin\psi\cos\phi\sin\theta - \cos\psi\sin\phi)(\sin\Psi\cos\Phi\sin\Theta - \cos\Psi\sin\Phi)$$
$$+ \cos\phi\cos\theta\cos\Phi\cos\Theta]$$

Although the direction cosines are useful for transformation purposes, they are not convenient measures of aircraft attitude. A conversion from direction cosines to a set of Euler angles that represents the attitude of the aircraft relative to the ship can be effected by the method described by Meyer et al. (1967).

For the conventional aeronautical rotation sequence, a rotation matrix can be generated as the product of three rotation matrices as follows:

$$[D] = [T_1(\theta_1)][T_2(\theta_2)][T_3(\theta_3)] \qquad (A13)$$

where

$$[T_1(\theta_1)] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

$$[T_2(\theta_2)] = \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}$$

$$[T_3(\theta_3)] = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The product matrix, equation (A13), yields the following direction cosine:

$$d_{11} = \cos\theta_2\cos\theta_3$$
$$d_{12} = \cos\theta_2\sin\theta_3$$
$$d_{13} = -\sin\theta_2$$
$$d_{21} = \sin\theta_1\sin\theta_2\cos\theta_3 - \sin\theta_3\cos\theta_1$$
$$d_{22} = \sin\theta_3\sin\theta_2\sin\theta_1 + \cos\theta_3\cos\theta_1$$

$$d_{23} = \sin \theta_1 \cos \theta_2$$

$$d_{31} = \cos \theta_3 \cos \theta_1 \sin \theta_2 + \sin \theta_3 \sin \theta_1$$

$$d_{32} = \sin \theta_3 \cos \theta_1 \sin \theta_2 - \cos \theta_3 \sin \theta_1$$

$$d_{33} = \cos \theta_1 \cos \theta_2$$

The following combinations of these equations are required to convert the direction cosines d_{ij} to the Euler angles θ_1, θ_2, and θ_3:

$$d_{11} \sin \theta_3 - d_{12} \cos \theta_3 = 0 \qquad \text{(A14)}$$

$$d_{31} \sin \theta_3 - d_{32} \cos \theta_3 = \sin \theta_1 \qquad \text{(A15)}$$

$$d_{21} \sin \theta_3 - d_{22} \cos \theta_3 = -\cos \theta_1 \qquad \text{(A16)}$$

$$d_{13} = -\sin \theta_2 \qquad \text{(A17)}$$

$$d_{23} \sin \theta_1 + d_{33} \cos \theta_1 = \cos \theta_2 \qquad \text{(A18)}$$

From equation (A14),

$$\tan \theta_3 = \frac{d_{12}}{d_{11}} \qquad \text{(A19)}$$

Equations (A15) and (A16) give

$$\tan \theta_1 = \left[\frac{d_{31} \sin \theta_3 - d_{32} \cos \theta_3}{d_{22} \cos \theta_2 - d_{21} \sin \theta_3}\right]$$

or

$$\tan \theta_1 = \left[\frac{d_{31} \tan \theta_3 - d_{32}}{d_{22} - d_{21} \tan \theta_3}\right]$$

From equations (A17) and (A18)

$$\tan \theta_2 = -\left[\frac{d_{13}}{d_{23} \sin \theta_1 + d_{33} \cos \theta_1}\right]$$

Given the nine direction cosines and the rotational sequence, the three Euler angles θ_1, θ_2, and θ_3 can be computed. For the present application, θ_3 corresponds to the aircraft yaw angle relative to the landing platform. Angles θ_2 and θ_3 are pitch and roll angles, respectively, relative to the landing platform. It should be noted that the computed values of θ_1 are not unique since $\tan \theta$ is a many valued function, that is,

$$\tan \Theta = \tan(n\pi + \Theta)$$

where n is a positive or negative integer. However, for the case being considered and for the angles anticipated, only these solutions corresponding to $n = 0$ will be required.

APPENDIX B

INTERNAL SHOCK-STRUT FORCES

Hydraulic Forces

The hydraulic force F_h is obtained by making use of the equation for the discharge through an orifice (Milwitzky and Cook, 1953)

$$Q = C_d A_n \sqrt{\frac{2}{\rho} (p_h - p_a)} \tag{B1}$$

Q volumetric rate of discharge

C_d coefficient of discharge

A_n net orifice area

p_h hydraulic pressure in lower chamber

p_a air pressure in upper chamber

ρ mass density of hydraulic fluid

Using continuity considerations, the volumetric rate of discharge can be expressed in the alternative form:

$$Q = A_h \dot{S} \tag{B2}$$

where A_h is the hydraulic area and $\dot{S}$ is the telescoping velocity of the shock strut. Equating these two expressions for the volumetric rate of discharge yields the pressure differential

$$p_h - p_a = \frac{\rho A_h^2 \dot{S}^2}{2(C_d A_n)^2} \tag{B3}$$

The hydraulic resistance F_h due to the telescoping of the strut is given by the product

$$(p_h - p_a)A_h = \frac{\rho A_h^3}{2(C_d A_n)^2} \dot{S}^2 = F_h \tag{B4}$$

This equation can be used for both the compression and elongation strokes by introducing the factor $\dot{S}/|\dot{S}|$ to indicate the sign of the hydraulic force as follows:

$$F_h = \frac{\dot{S}}{|\dot{S}|} \frac{PA_h^3}{2(C_d A_n)^2} \dot{S}^2 \qquad \text{(B5)}$$

Pneumatic Forces

The pneumatic force F_a in the upper chamber depends on the inflation pressure and the exposed area. It is assumed that the air pressure obeys the generalized gas law (Milwitzky and Cook, 1953)

$$pv^n = \text{constant} = p_o v_o^n$$

or

$$p = p_o \left(\frac{v_o}{v}\right)^n \qquad \text{(B6)}$$

where

p air pressure in upper chamber of shock strut

p_o air pressure in upper chamber for fully extended strut

v air volume of shock strut

v_o air volume for fully extended strut

The instantaneous volume v is equal to the initial volume minus the swept volume, where the swept volume is the product of the pneumatic area A_a and the stroke of the strut S, that is,

$$v = (v_o - A_a S)$$

Substituting this value in equation (B6) gives

$$p = p_o \left(\frac{v_o}{v_o - A_a S}\right)^n \qquad \text{(B7)}$$

It follows that the pneumatic force is

$$F_a = pA_a = p_o A_a \left(\frac{v_o}{v_o - A_a S}\right)^n \qquad \text{(B8)}$$

The exponent n depends on the rate of compression and rate of heat transfer from the air to the surrounding environment. Low rates of compression correspond approximately to the isothermal case and a value of $n = 1$. Higher rates of compression approach the adiabatic condition and a limiting value of n = 1.4. A limited amount of experimental data obtained in drop tests indicates that, in most practical cases, a value of $n = 1.1$ may be used (Milwitzky and Cook, 1953).

Internal Friction Forces

According to the law of friction, the coefficient of friction μ is defined as the ratio of the friction force to the normal force. This coefficient is somewhat greater under conditions of rest (static friction) than under conditions of sliding (kinetic friction). Machine designers usually classify frictional resistance as friction between dry surface, friction between imperfectly lubricated surfaces, and friction between perfectly lubricated surfaces. The internal friction in landing gear shock struts usually involves relatively high normal pressures and small sliding velocities. Moreover, the usual types of hydraulic fluid used in shock struts have imperfect lubricating properties. Therefore, for the sake of completeness, it will be assumed that the internal friction between the bearings and cylinder walls approaches the dry friction condition. Hence, the internal friction forces which depend on the magnitude of the bearing forces, the orientation of the gear, the spacing of the bearings, and the appropriate coefficients of friction are obtained as follows (fig. 5).

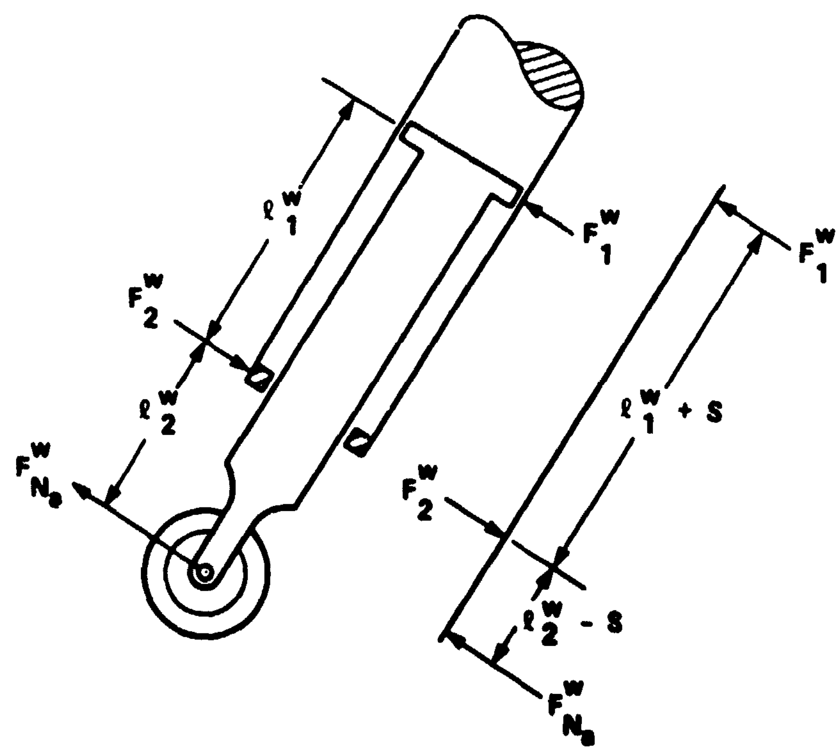


Figure 5.- Shock-strut dimensions.

The bearing forces are related to the force $F^w_{N_a}$ normal to the axis of the shock strut. This force is required to determine the internal friction forces f^w_f, where

$$f^w_f = \frac{\dot{S}}{|\dot{S}|} \left(\mu^w_1 |F^w_1| + \mu^w_2 |F^w_2|\right) \tag{B9}$$

where

f^w_f axial friction force for strut at wheel w

μ^w_1 coefficient of friction for upper bearing (attached to inner cylinder)

F^w_1 normal force on upper bearing (attached to inner cylinder)

μ^w_2 coefficient of friction for lower bearing (attached to outer cylinder)

F^w_2 normal force on lower bearing (attached to outer cylinder)

$\frac{\dot{S}}{|\dot{S}|}$ sign of friction force

$$F^w_1 = f^w_{N_a} \frac{(\ell^w_2 - S)}{(\ell^w_1 + S)}$$

$$F^w_2 = (f^w_{N_a} + F^w_1) = f^w_{N_a}\left(\frac{\ell^w_2 - S}{\ell^w_1 + S} + 1\right)$$

$$f^w_f = \frac{\dot{S}}{|\dot{S}|} \left|F^w_{N_a}\right| \left[\frac{(\ell^w_2 - S)}{(\ell^w_1 + S)} (\mu^w_1 + \mu^w_2) + \mu^w_2\right] \tag{B10}$$

(see fig. 5).

Total Strut Force

The sum of the pneumatic, hydraulic, and internal friction forces gives the total shock-strut force, which is

$$F_s = \left[\frac{\dot{S}}{|\dot{S}|} \frac{PA^3_h}{2(C_d A_n)^2} \dot{S}^2 + p_o A_a \left(\frac{v_o}{v_o - A_a S}\right)^n + \frac{\dot{S}}{|\dot{S}|} \left|F^w_{N_a}\right| \left\{\left(\frac{\ell^w_2 - S}{\ell^w_1 + S}\right) (\mu^w_1 + \mu^w_2) + \mu^w_2\right\}\right] \tag{B11}$$

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