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GLOBAL FUNCTION APPROACH IN STRUCTURAL ANALYSIS: BASIC APPROACH, NUMERICAL RESULTS

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FOREWORD

The present report documents work performed under Contract NAS1-10843 during the period October 1, 1979 to September 30, 1980.

The basic formulation and software for execution of the global function approach were developed under contract funding. In a separate but related effort, automatic procedures were developed for the selection of some of the parameters governing the analysis strategy used in the program. This work was funded under the LMSC independent research program but is included in this report for completeness.

INTRODUCTION

The structural response to a given environment is described by the differential equations of motion of deformable bodies. Analytic solutions of such problems for a reasonably large class of structural configurations are not within the realm of the possible. Consequently, the mathematical problem is recast into a numerical problem for solution on the computer.

The output from the computer consists of a sequence of numbers, in some way representing the functions satisfying equilibrium equations and boundary conditions. If the solution is represented by a linear combination of a set of "basis functions" then the components of the output vector consist of the coefficients in this linear combination. This is the case if we use the Galerkin or Rayleigh-Ritz procedures. If we use the finite difference or finite element procedures, the solution function is represented by its values at a number of discrete locations within the structure. Because these discretized methods are readily applied in a computer program for a general type of structure, they have been gaining popularity. This applies in particular to the finite element method. The finite element method may be considered as a Rayleigh-Ritz analysis in which the basis functions are localized. Confusion is avoided if the classical form of the Rayleigh-Ritz analysis is referred to as the "Global Function Approach".

New technology in the space and energy fields has led to a growing demand for accurate analysis which at times cannot be met due to the limits set by available budget for computer time. In response to this need for more efficient numerical analysis, the possibilities have been explored of reducing the number of freedoms in the system through a revival of the global function approach. Nagy (Reference 1) analyzed trusses using buckling modes as Ritz functions.

This approach is straightforward if it can be assumed the deformation is inextensional, but is not directly applicable if the strain energy due to stretching of the neutral axis (middle surface for shells) must be included. A more general way to automatically select a suitable set of basis vectors and a method to control the accuracy of the solution was first presented in Reference 2. In that case, the global function approach is used in connection with a finite element model. The basis functions are represented by a set of basis vectors and the interpolating shape functions. Approximate solutions to the initial system are sought in the reduced space defined by all linear combinations of the basis vectors. We refer to the reduced space as the (infinite) set of trial vectors. Basis vectors can be obtained through solution of the initial (discrete) system and the accuracy of solutions in the reduced system can be assessed in the discrete system. Such procedures were further developed by Noor, References 3 and 4. In Reference 2, orthonormalized nonlinear solutions at different load levels are used as basis vectors while Noor proposed to use the so-called path derivatives. In both cases, the procedure involves a return to the discrete system (the finite element model) for evaluation of the error and automatic generation of new vectors when the size of the error suggests such action.

In a finite-element formulation of the structural problem, we have

$$M\ddot{X} + SX - F = 0 \quad (1)$$

where M is the mass matrix, S a nonlinear algebraic stiffness operator, and F the vector of external forces. The vector X represents the freedoms in the finite-element formulation; that is displacement and rotation components at the structural nodes.

A global function formulation may be obtained by introduction into the finite element formulation of the substitution

$$X = Tq \quad (2)$$

where each column in the matrix T represents one of the basis vectors. A basis function is defined by the finite-element discretization, i.e., by the displacement and rotation components at structural nodes and by the local shape functions peculiar to the element. The components of the vector q are coefficients in a linear combination of basis vectors. These coefficients are the degrees of freedom in a reduced nonlinear algebraic equation system obtained through substitution of Eq. (2) into Eq. (1) and summation over the elements. The ith equation is of the form

$$\bar{M}_i\ddot{q}_i + \Sigma A_{ij}q_j + \Sigma\Sigma B_{ijk}q_jq_k + \Sigma\Sigma\Sigma C_{ijkl}q_j q_k q_l = \bar{F}_i \quad (i,j,k,l = 1,I) \tag{3}$$

where $\bar{M}$ and $\bar{F}$ are generalized masses and forces corresponding to the ith basis function. The nonlinear terms derive from the stiffness operator.

Solution Method

An assemblage of computer programs was developed which is based on the use of global functions together with a finite element model. This assemblage includes the STAGSC-1 program (Reference 5). The structural model is always defined by STAGSC-1 input data. In non-linear elastic analysis, the program user has options to define global functions (as input) or to obtain such functions through solution of the discrete system. Eigenmodes, buckling or vibration, or nonlinear solutions to the static equilibrium equations can be included (users choice). A special feature of the solution method is a problem-adaptive solution strategy in which automatic choice and continuous modification of certain strategy parameters allows for efficient analysis.

The automatic feature is based on the ideas first proposed in Reference 2. In that case, the use of global functions represents a powerful way to determine initial estimates for iterative solution of the discrete system. Initial estimates are obtained through integration of a reduced displacement space spanned by the selected basis vectors. Successful operation of such software requires the availability of adequate methods for specification of the basis vectors and a satisfactory step size selector sensing when a return to the discrete system for updating is desirable. In References 3 and 4, Noor uses the path derivatives as basis vectors. These are defined in terms of the coefficients in Eq. (3). Unfortunately, the number of distinct coefficients is very large and if many different elements are included in the structural model, severe disc storage problems will result. Therefore, the automatically selected basis vectors are defined in terms of nonlinear solutions to the discrete system at a sequence of different load levels. This is equivalent to use of numerically determined path derivatives. The disadvantage in this case is that solution accuracy may limit the number of solutions that profitably can be included as basis vectors.

In Reference 3, Noor bases the return key on the change in a structural stiffness parameter. A change by ten percent in the value of this parameter prompts return to the discrete system. In Reference 4 he uses, as in Reference 2, the norm of the error in the equation system for the discrete model. In both cases, the analysis is very efficient. In Reference 4, the collapse analysis of an axially compressed prismatic shell (the "pear shaped cylinder") is presented. Return to the discrete system is dictated by an error norm (normalized with respect to the norm of the load vector) exceeding 0.05. The load steps i.e. the load increments between returns to the discrete system in that case are very large; indicating a potential for substantial savings in computer cost in non-linear elastic analysis.

Extensive experimentation with automatic selection has indicated that both a stiffness parameter and the error norm may be useful for step size control. However, suitable values of the parameters governing return to the discrete system are not only case dependent but in a given case may may also vary considerably with the load level. It appears that the potential for savings in computer time by use of global functions can only be realized if a problem-adaptive computational strategy is available. Then, the return key is adjusted in response to the characteristics of the problem so that an efficient analysis can be obtained in a variety of cases without preceding experimentation with the step size selection. The procedure involves a number of strategy parameters. A set of default values for these parameters has been selected to be used when the analyst lacks special knowledge of the behavior of his system and therefore declines to make a different choice.

Default Strategy

The initial basis vectors are obtained through solution of STAGSC-1. This program does not contain procedures for automatic choice of the initial load step. The user must define the ititial load and the initial step in STAGSC-1. The user also defines the initial and the maximum numbers of basis vectors, N_i and N_m. Default values are $N_i = 4$, $N_m = 6$.

On each return to the discrete system, a nonlinear solution of this discrete system is obtained and included in the data base. If the number of basis vectors in the data base exceeds N_m, the program gives preference to those corresponding to higher load levels when the basis vectors are selected.

A check on linear dependence among the basis vectors is performed and vectors that are not sufficiently distinct are discarded. The number of basis vectors therefore can be less than N_i and remain less than N_m. During the computations, the program attempts to set the return key so that solution of the discrete system will require approximately five iterations. The stiffness parameter included in the strategy is represented by the diagonal elements in the factored matrix corresponding to the reduced system. The following notations are used:

ε = error norm = $||\delta U||/||f||$ where δU is the first variation of the total potential energy (i.e., the residuals) and f is the vector of applied forces (including reactions).

A = vector of diagonal elements in the factored matrix associated with the reduced system.

$\bar{A}_i$ = ratio between present value of the elements of A and corresponding values at last return to the discrete system (initial values for the first step).

N = number of iterations for convergence at last return to the discrete system.

$f = 10^{-k(N-5)}$ where k is an input constant.

Solution of the discrete system and updating of the set of basis vectors by inclusion of the current solution is dictated by any of the following events:

$$
\begin{array}{ll}
(1) & \varepsilon > \varepsilon_q \\
(2) & \bar{A}_i > \delta_U \text{ for some } i \\
(3) & \bar{A}_i < \delta_L \text{ for some } i \\
(4) & \Delta P \text{ is reached (max load step)}
\end{array}
\tag{4}
$$

where ε_q, δ_U, δ_L are input parameters.

Whenever convergence occurs on the return to the discrete system, the adjustment depends on which criterion prompted the return.

$$
\begin{array}{l}
\text{If } \varepsilon > \varepsilon_q, \text{ then } \varepsilon_q \rightarrow f\ \varepsilon_q \\
\bar{A}_i > \delta_U, \text{ then } \delta_U \rightarrow 1 + f\ (\delta_U - 1) \\
\bar{A}_i < \delta_L, \text{ then } \delta_L \rightarrow 1/[1 + f\ (1/\delta_L - 1)] \\
\Delta P \text{ reached, then } \Delta P \rightarrow f\ \Delta P
\end{array}
\tag{5}
$$

If divergence occurs on return to the discrete system, f is set to 0.5 and all four return keys are accordingly adjusted.

Initial efforts established that efficient analysis would be achieved in a variety of cases with the choice:

$$
\varepsilon_q = 0.2,\ \delta_U = 4,\ \delta_L = 0.3, \text{ and } k = 0.08 \tag{6}
$$

This strategy should be considered as a first cut only. Additional improvements are certainly possible. For example, a good initial value of ε_q can probably be surmised from the relation between error norm and convergence rate in the first series of solutions in STAGSC-1. Also it may be better to adjust all the strategy parameters on any return to the discrete system. while a more efficient automated strategy may be forthcoming, the present version was evaluated through comparison to solution with STAGSC-1 in a study of five structural configurations with significant nonlinearity. The automated strategy used was in all cases based on the default values for the parameters.

Benchmark Cases

The automated global function procedure is primarily intended for use in the analysis of cases requiring large amounts of computer time. In such cases, the time spent on integration in the reduced system would be insignificant. Further reductions of the computer time in auxiliary programs is possible, that is, in the routine used to define and to integrate the reduced system. In terms of required computer time, the five benchmark cases range somewhat below the class of problems for which computer cost is a serious issue. A straight comparison of computer run times required by the global function analysis and the standards STAGSC-1 may not be conclusive. Therefore, the total number of iterations and the number of reformulations and factorings of the second variation are also recorded in each case. In all the cases, the analysis was carried well into the non-linear range but never beyond possible limit points since STAGSC-1 presently does not include efficient methods to handle such cases.

Case 1: "Pear-Shaped Cyliner" in Compression

The first test case is the Pear-Shaped Cylinder considered in Reference 7. Dimensions and material data are shown in Figure 1. The cylinder is subjected to uniform axial shortening. The maximum load factor was set to 8.27 corresponding to an axial shortening of 00.00004201 m. which is within a couple of percent of the critical value. The corresponding axial load is 12793 N. This value is well above the critical load reported in Reference 6, possibly caused by a tendency of the finite element configuration used to "lock" as the rotations become relatively large. Whatever the reason, this discrepancy has no impact on the solution procedures.

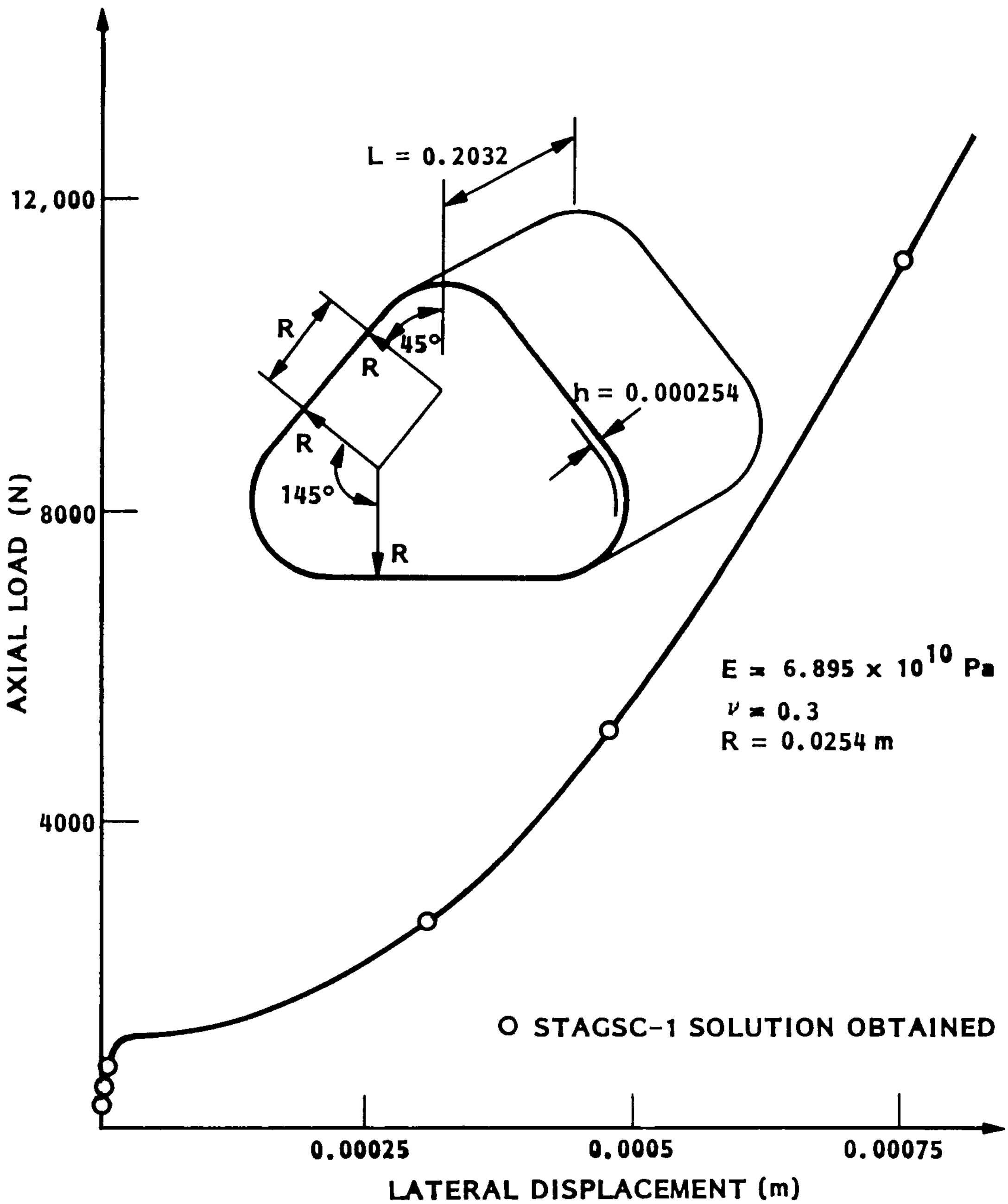

Fig. 1 Deformation of Pear-Shaped Cylinder

Due to the presence of two symmetry planes, only one-quarter of the shell need be considered. A finite difference grid is defined over this model with 5 uniformly spaced gridlines axially and 43 approximately uniformly spaced gridlines circumferentially.

While the analysis with STAGSC-1 required a total of 226 iterations and 13 refactorings, the corresponding numbers with the automated global function analysis are 25 iterations and 4 refactorings. The total run time (CP-time CDC 175,NOS-BE) is 605 seconds with STAGSC-1 and 240 seconds with the global analysis. However, with 1300 degrees of freedom and an average bandwith of 40 this is a relatively inexpensive case. The time spent on formulation and solution of the reduced system is still significant, approximately half the total run time. The indication then is that for a larger case the use of global functions for extrapolation may reduce the run time by a factor between 2.5 and 5.0.

The load level at which STAGSC-1 solutions were obtained are indicated in Figure 1.

The first three solutions represent results from the initial STAGSC-1 analysis. The first return was dictated by excessive change in the stiffness parameter, the second by the size of the error norm. In each of these instances, the number of iterations in the discrete systems is five and no parameter adjustment was required. On the last return (maximum step size) the number of iterations is six.

Case 2: Bending of a Long Cylinder

As a second test case, a long cylinder subject to a constant bending moment was selected. The cylinder has a radius of 0.127 m. and a length of 3.048 m., as shown in Figure 2. Due to the symmetry conditions, only one-quarter of the cylinder needs to be included in the structural model. Over this model a uniform 11 x 11 finite element grid is applied. This corresponds to a total of 939 degrees of freedom and an average bandwidth of 82. The bending moment is applied at one end and symmetry conditions are enforced at the other.

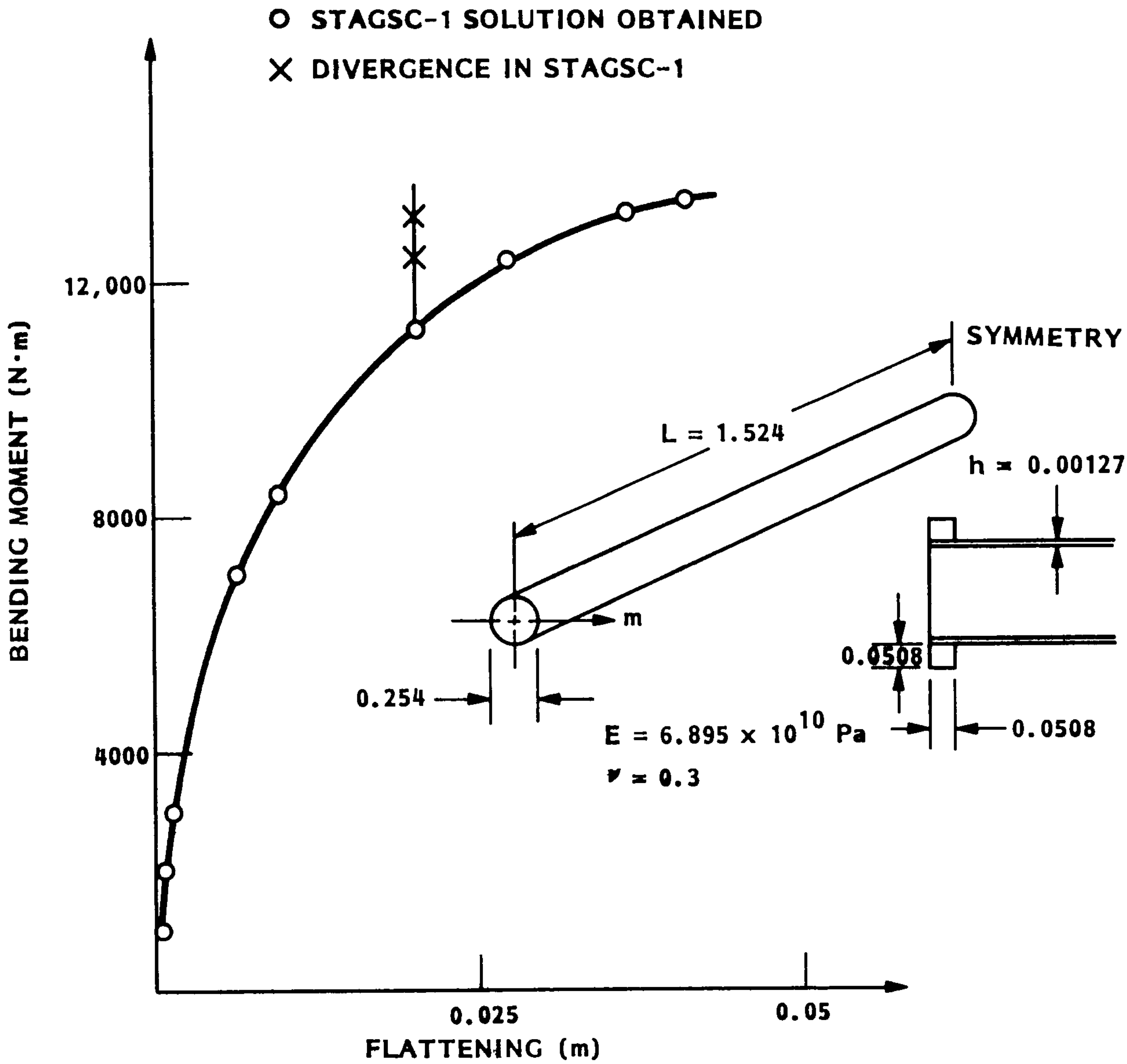

Fig. 2 Bending of Long Cylinder

A hefty end ring is applied at the loaded end, preventing significant cross-section warping.

Since convergence of the buckling load is from above, the shortwave local buckling observed in Reference 7 is suppressed by use of a coarse net and a pure Brazier effect is displayed. For an infinitely long cylinder, Brazier, Reference 8, predicts collapse under a bending moment of

$$M = \frac{2\sqrt{2}}{9} \frac{E \pi a h^2}{\sqrt{1 - \nu^2}} = 14{,}620 \text{ N·m} \qquad (7)$$

The cross-section at the point of collapse is then flattened by 0.05588 m. (0.22 times the diameter).

The results of the STAGSC-1 shown in Figure 2 appear to be in relatively good agreement with those by Brazier. The analysis was interrupted when the bending moment reached 13445 N·m. At that point, convergence is very difficult and the load step must be very small. With more accurate extrapolation, the global function analysis was carried somewhat further M_{max} = 13705 N·m. The points at which STAGSC-1 solutions were obtained (including the initial solutions and subsequent returns for updating) are indicated in Figure 2 together with two points at which convergence failed. The total run time with the global function analysis was 335 sec CP as compared to 1030 with STAGSC-1. With the global analysis, the total number of iterations is 50 and the number of refactorings 20. Corresponding numbers with STAGSC-1 are 450 and 34. In this case, about 75 percent of the global analysis run time is spent in STAGSC-1. The indication for a large case with this general behavior is a reduction in computer cost by use of global function extrapolation of 3 to 4.

Each return to the discrete system was caused by excessive error in the first variation of the discrete system. Figure 3 shows how this error varies with the applied load. At each point of update, the error returns to zero. The default value ε_q = 0.2, is for this case somewhat too small; convergence is obtained in two iterations and the value of ε_q is eventually increased

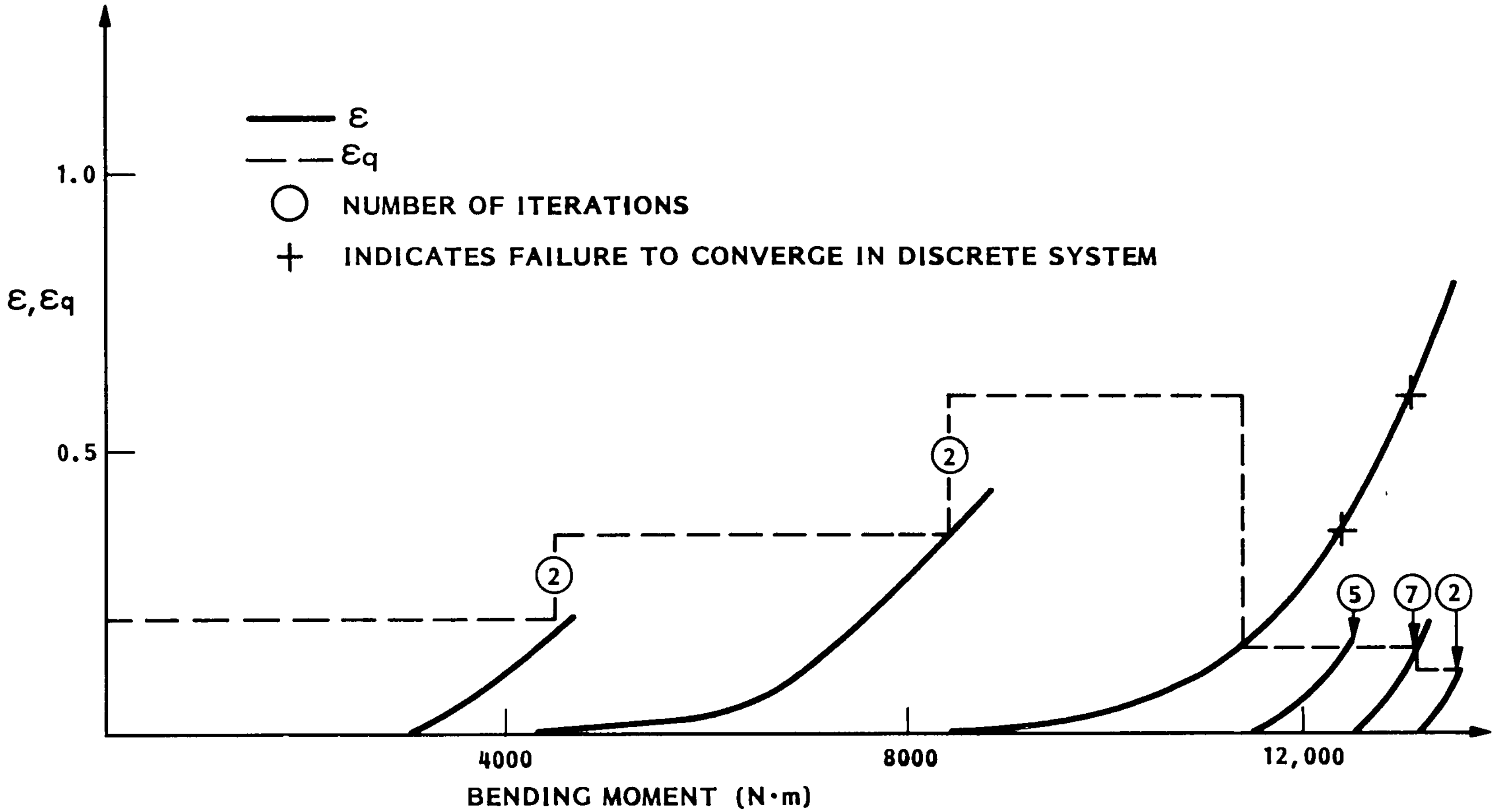

Fig. 3 Error Norm as Function of Load, Long Cylinder

to 0.6. Convergence becomes more difficult as the critical load is approached and ε_q is reduced by a factor of four after divergence in two consecutive efforts.

Case 3: Spherical Shell Subjected to Point Force

The case of a spherical cap clamped at the edge and subjected to an inward directed central point force at the apex is considered as a third case. The shell geometry shown in Figure 4 is identical to that considered by Fitch in Reference 10. The bifurcation buckling analysis with nonlinear prestress in Reference 10 indicated buckling into a mode with four circumferential waves when the displacement under the point load reaches 0.046736 m.

The case considered here is the nonlinear behavior of the cap when in addition to the central point load two small forces are applied such that deformation in a four-wave pattern is triggered. These forces are held constant and the midpoint displacement is gradually increased. A 90-degree sector of the cap is analyzed with symmetry conditions applied along the meridians. The finite difference grid is uniform in the circumferential direction and varied in the meridional direction with the grid spacing tightened around the apex and at the clamped edge. The grid includes 16 grid lines in each direction. This system contains 1966 degrees of freedom and the average bandwith is 127.

The STAGSC-1 works quite well up to a midpoint displacement of about 0.0254 m. After that convergence is more difficult. The computations were interrupted when the displacement reached 0.040132 m. This does not correspond to a maximum in the load displacement curve but even with a step size of 0.000254 m. (deformation) it is at that point necessary to refactor on each load step. The results including the growth of a four-wave buckling pattern are shown in Figure 4.

With the global function analysis the maximum load factor, 2.0, was easily reached. Load levels at which STAGSC-1 solutions were obtained are shown in Figure 4. The error bound, $\varepsilon_q = 0.2$, is much too severe in this case

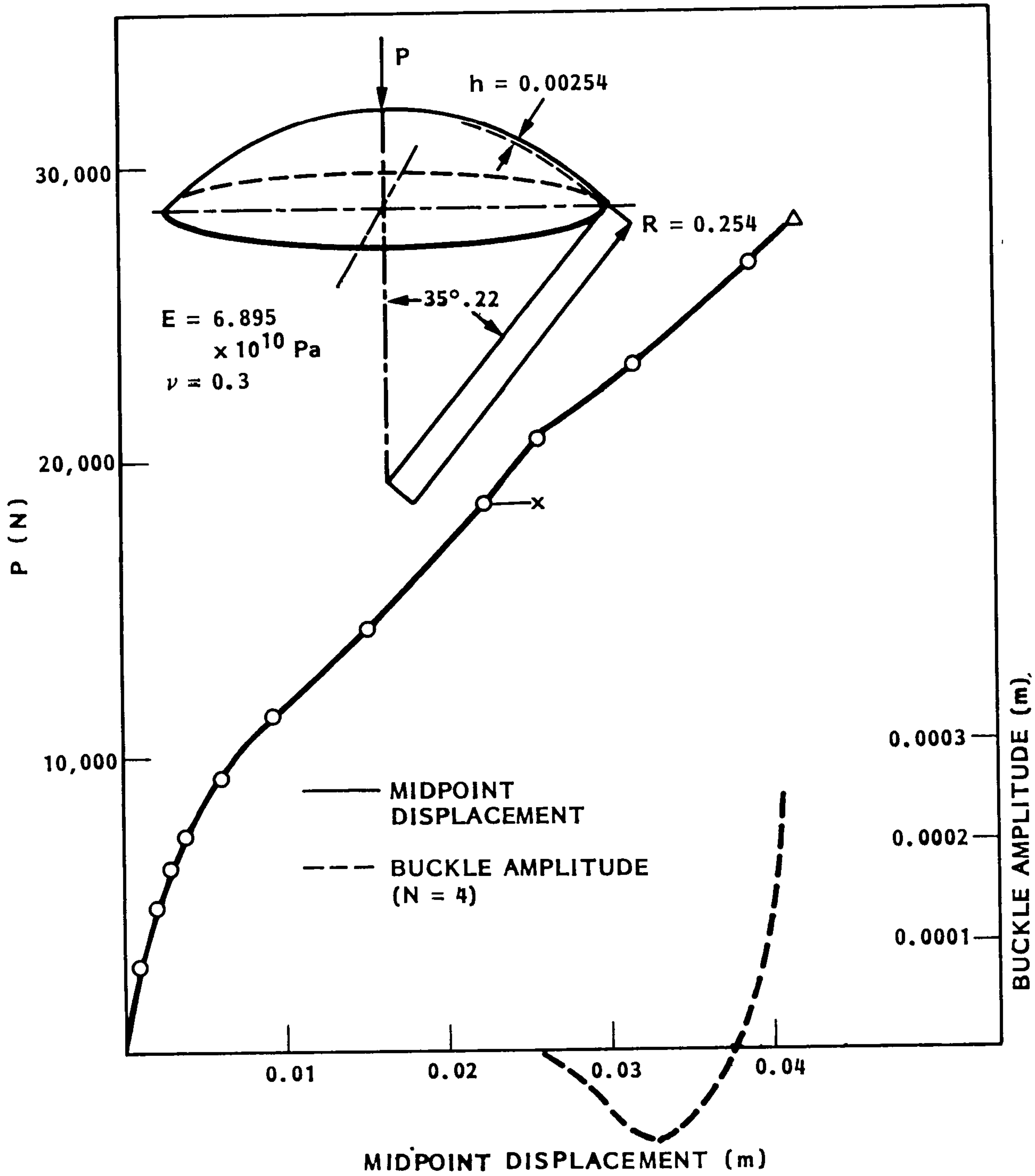

Fig. 4 Deformation of Spherical Cap

the AUTORITZ analysis is not efficient. However, the return key is adjusted (to $\varepsilon_q = 10.5$) and a suitable step size is eventually chosen. A displacement of 0.040132 m. was reached with a total of 52 iterations and 20 reformulations with factoring of the second variation. The corresponding values with STAGSC-1 are 196 iterations and 20 refactorings. The run time is 1898 sec with STAGSC-1 and 971 sec with AUTORITZ of which about 70 percent is spent in the STAGSC-1 program. The saving in computer time for a case of this type then is 2.0 to 2.7 and it will be considerably more in favor of AUTORITZ if the analysis is carried further.

Case 4: Panel with Initial Imperfections

The fourth case is a cylindrical panel subjected to axial compression in the form of uniform end-shortening. The properties of the panel are shown in Figure 5. All the four edges are simply supported. The panel is free from initial stresses but deviates from the true geometric shape. The initial imperfection is represented by a lateral displacement (in meters) of the form

$$\begin{aligned} W_o &= 0.00127 \sin \frac{x\pi}{L} \sin (12\ y) + 0.000508 \sin \frac{x\pi}{L} \sin (24\ y) \\ &+ 0.000508 \sin \frac{2x\pi}{L} \sin (12\ y) + 0.000508 \sin \frac{3x\pi}{L} \sin (24\ y) \\ &+ 0.000508 \sin \frac{2x\pi}{L} \sin (12\ y) + 0.000254 \sin \frac{3x\pi}{L} \sin (24\ y) \\ &+ 0.0000508 \sin \frac{3x\pi}{L} \sin (36\ y) \end{aligned} \tag{8}$$

A uniform 19 x 19 finite element grid was used resulting in a system with 3044 degrees of freedom and an average bandwidth of 151. The lateral displacement at the midpoint and the axial shortening are shown in Figure 5 as functions of the total axial load. With STAGSC-1 the maximum load, 3,247,200 N., corresponding to an axial shortening of 0.01143 m., is reached after a total of 106 iterations and 9 refactorings. The total run time is 1096 sec. The corresponding values with AUTORITZ are 23 iterations, 6 refactorings, and a total run time of 786 sec. The three initial load steps

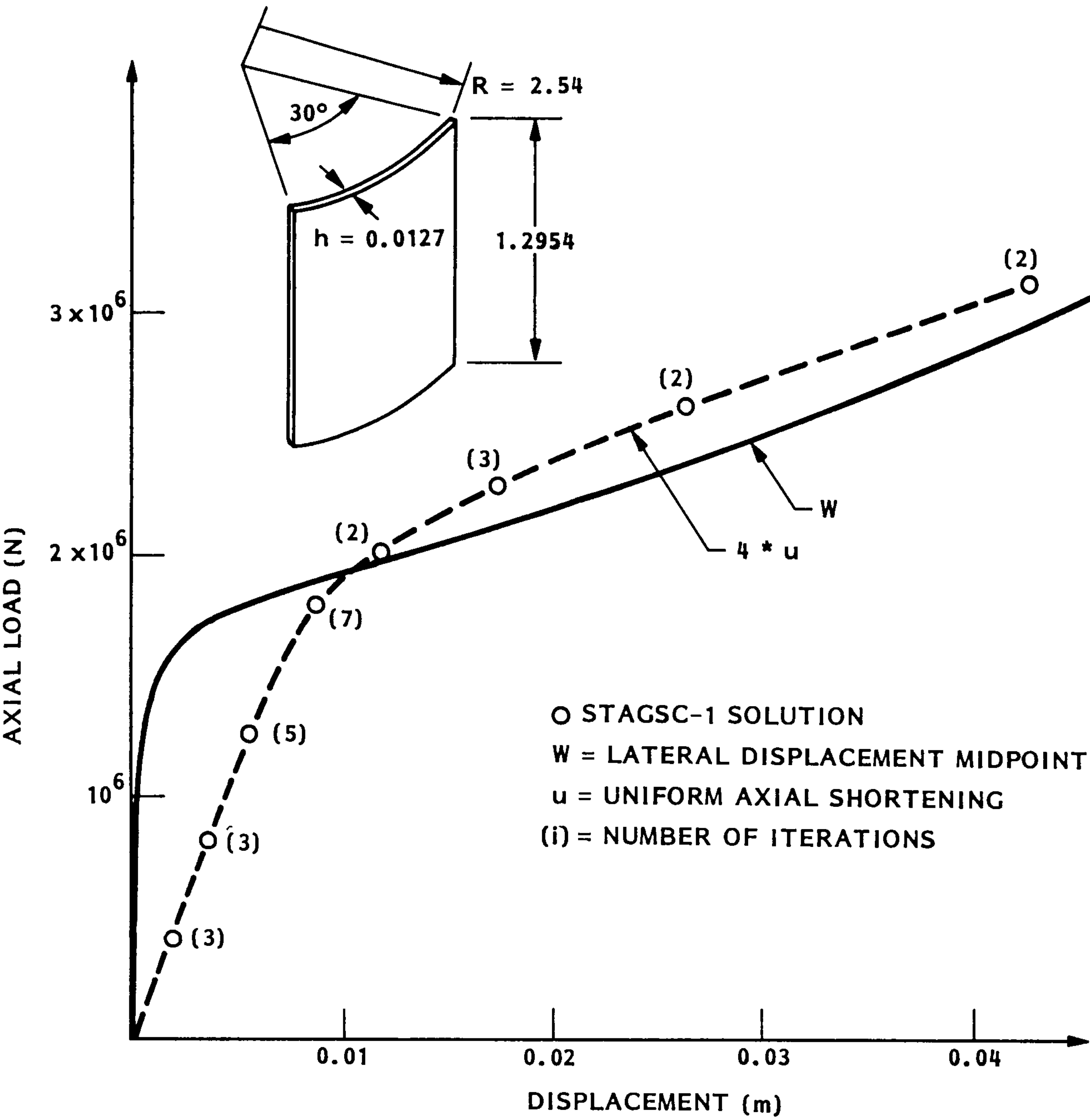

Fig. 5 Deformation of Imperfect Panel

and the returns to the discrete system for updating are indicated in the figure. The first three returns are governed by the size of the maximum load step. This is the case in which the global analysis compares least favorably with a straight STAGSC-1 analysis. The reason seems to be that the STAGSC-1 strategy works unusually well in this case. The saving in computer time by use of global functions in this case is only by a factor of 1.4. With further improvement of the efficiency of the automated strategy, a saving by a factor of two seems possible.

Case 5: Cylinder with Cutouts

The last case considered was a cylinder with two diametrically opposite rectangular cutouts. The geometry of the shell together with some results of the analysis are shown in Figure 6. Here, u_o represents the uniform end shortening and W the lateral displacement midways on the edge of the cut-out. As in Case 1 (pear shaped cylinder), the maximum load reached exceeds the previously established in analysis as well as in experiments (Reference 12). This tendency of the element to lock with relatively large rotations occurs despite the fact that a very fine grid has been used as indicated in Figure 7. The model has 7055 degrees of freedom and the average bandwidth is 205.

In the analysis, the uniform end shortening was gradually increased to 0.00009144 m. which is just below the limit point for the model and corresponds to a total force of 12900 N. With STAGSC-1, this load level was reached with a total of 145 iterations and 12 refactorings. By use of the automated strategy, it was only necessary to formulate and factor the second variation 6 times and the total number of iterations was 16. The total run time with STAGSC-1 was 5267 sec and the global analysis 1856 sec CP time; hence, the use of global functions for extrapolations leads to a considerable saving - a factor 2.8 between the run times.

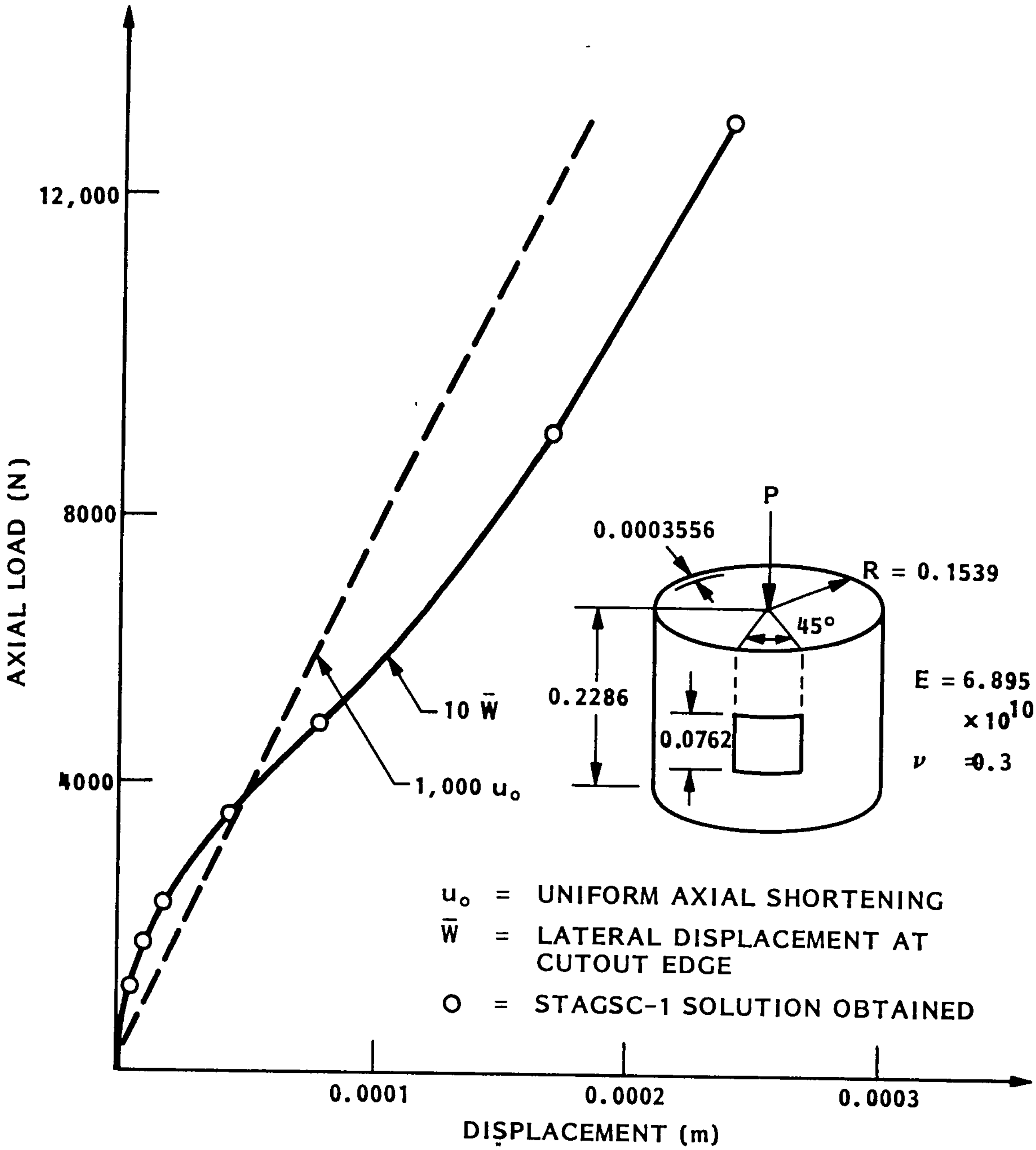

Fig. 6 Deformation of Cylinder with Cutout

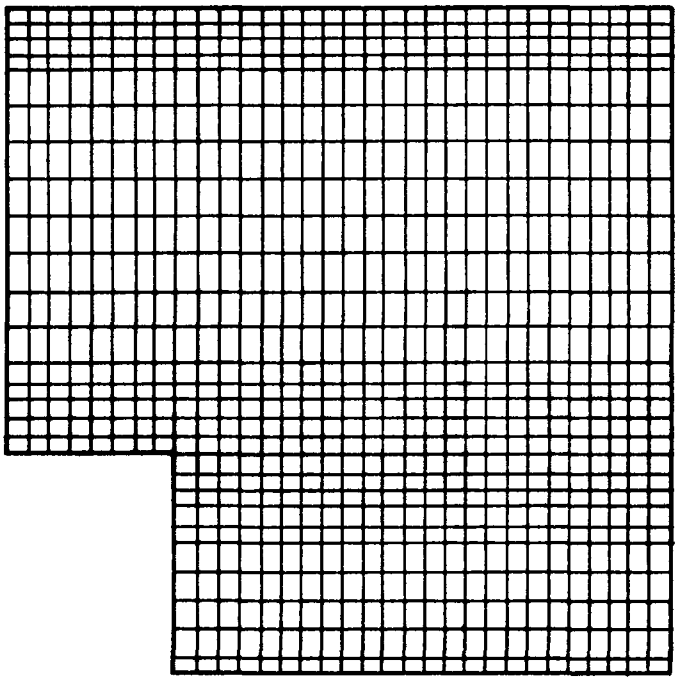

Fig. 7 Finite Element Grid for Cylinder With Cutout

CONCLUSIONS

The present paper discusses the possibilities to improve the efficiency in nonlinear structural analysis by use of global displacement functions. A procedure is presented for accuracy control and automatic generation of the global functions. In essence, the procedure is a way to predict initial estimates for solution of large nonlinear systems with a continuation method. The indication from a few benchmark cases is that an improvement by a factor of three to five is possible in most cases. The sample cases indicate the usefulness of the procedure in solution of nonlinear structural shell problems by the finite element method. However, the basic idea of extrapolation through integration in a reduced solution space may be useful in other applications leading to large and strongly nonlinear equation systems.

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