NASA Contractor Report 165764

AEROELASTIC ANALYSIS OF A TROPOSKIEN-TYPE WIND TURBINE BLADE

F. Nitzsche

Department of Aeronautics and Astronautics
Stanford University
Stanford, CA 94305

Grant NGL 05-020-243
July 1981

NASA
National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

AEROELASTIC ANALYSIS OF A TROPOSKIEN-TYPE WIND TURBINE BLADE

F.Nitzsche
Stanford University
Department of Aeronautics & Astronautics
Stanford, CA 94305 USA

SUMMARY

The linear aeroelastic equations for one curved blade of a vertical axis wind turbine are presented in state vector form. An attractive method, based on a simple integrating matrix scheme together with the transfer matrix idea, is proposed as a convenient way of solving the associated eigenvalue problem for general support conditions.

INTRODUCTION

The troposkien shape has been frequently invoked in structural modeling of vertical axis wind turbine (VAWT) blades [1]. Although it is free of bending stress in the equilibrium position determined by a constant angular velocity, tests have indicated that under certain conditions serious vibrations about the original shape may occur. They involve both chordwise and flatwise bending as well as torsion of the blade.

Few theoretical works have been published on the determination of natural frequencies and vibration mode shapes of such special but nevertheless important kinds of blade geometry [2-4]. The tendency among investigators has been to deal with a more general geometry, which brings perhaps unnecessary complexity into the problem and requires the use of strictly numerical procedures for obtaining reliable results. Futhermore, the shape of actual VAWT blades has been very close to the troposkien; good approximations like those suggested by Reis & Blackwell [5] are commonly used in practice. One can recall that the complicated flow field generated by VAWT already adds an important source of inaccuracy to the aerodynamic load estimation and consequently to problems such as flutter speed determination. Therefore it is important for preliminary design purposes, to develop semi-analytical methods based on the troposkien idealization, which will enable quick analyses and general understanding of the conditions that may affect blade stability. The intention of this paper is to present an approach with the aforementioned characteristics.

"OPTIMUM" LINEARIZED EQUATIONS OF MOTION

The general second-degree nonlinear aeroelastic equations of motion for a slender, nonuniform and extensible blade were presented by Kaza & Kvartenik [6]. For physical understanding of the aeroelastic phenomena, however, such degree of complexity does not seem necessary. It would be more interesting to seek a set of linear equations that allows the study of the relative importance of various parameters on the solution. These parameters include CG offset, shear deformations, rotary inertia and different kinds of blade support, along with the possibility of extending the model to a more realistic one containing the influence of tower and guy-cable stiffness as well as the other blades.

When gravity effects are neglected, the troposkien can be seen as the plane curve described by a light rope rotating at constant angular velocity. Its shape is known in closed analytic form [1]. Working in the same fashion as Ashley [4], small perturbations from this equilibrium shape are assumed. After adding torsion to chordwise and flatwise bending and by assuming simple harmonic motion in time, the final order of the governing system of differential equations is found to be 12. However, the choice of the dependent variables remains open. If, as in reference 4, the three cartesian components of the linear displacement related to the bending, the rotation related to the torsion and the incremental tension are selected, a set of coupled linear partial differential equations subject to one geometric constraint equation is obtained. Asymptotic solutions may be useful, but only simplified versions of the problem could be carried out without a great deal of difficulty. An integrating matrix approach, like the one suggested by Hunter [7], could be employed in order to solve the eigenvalue problem for the free vibration analysis. The choice of the spatial independent variable as the distance along the blade introduces technical difficulties however, and the method fails. A more "natural" spatial independent variable is discovered to be the angle formed by the tangent to the undeformed shape and the axis of rotation. Even then, however, an unnecessary degree of algebraic difficulty is introduced. Without discarding the possibility of the solutions by the aforementioned means, the Reissner's variational principle of the elasticity suggests that the best dependent variables are likely to be the 12x1 state vector of "natural" quantities connected with both equilibrium and deformation of the blade: bending moments, torque, shears, bending slopes and displacements. The integrating matrix [7] together with the transfer matrix method [8], is expected to constitute an efficient and powerful tool for dealing with the vibration of such beams subjected to general boundary conditions.

AEROELASTIC ANALYSIS OF THE BLADE

Assuming small perturbations from the equilibrium shape is equivalent to constructing a linear field of deformations on a prestressed, untwisted, plane troposkien-curved rod. The governing equations for a three-dimensionally curved rod under such a general initial stress configuration were obtained by different authors. Taking Nair & Hegemier's derivation [9], based on the principle of virtual work, neglecting cross section warping and introducing the conditions defined by the troposkien hypothesis, one obtains a set of 12 linear first-order ordinary differential equations with nonconstant coefficients. These govern the 12 perturbation quantities chosen to be the dependent variables:

$$
\begin{aligned}
&Q_1' + \kappa_o T + f_1 = 0 && M_1' + \kappa_o M_3 - Q_2 - T_o\chi_1 + m_1 = 0\\
&Q_2' + f_2 = 0 && M_2' + Q_1 - T_o\chi_2 + m_2 = 0\\
&T' - \kappa_o Q_1 + f_3 = 0 && M_3' - \kappa_o M_1 + m_3 = 0\\
&u' + \kappa_o w - \chi_2 - Q_1/G\bar{A} = 0 && \chi_2' - M_2/EI_{\xi\xi} = 0\\
&v' + \chi_1 - Q_2/G\bar{A} = 0 && \alpha' - \kappa_o\chi_1 - M_3/GJ = 0
\end{aligned}
$$

$$w' - \kappa_o u - T/EA[1-e^2(EA/EI_{\eta\eta})]^{-1} + eM_1/EI_{\eta\eta}[1-e^2(EA/EI_{\eta\eta})]^{-1} = 0$$

$$\chi_1' + \kappa_o\alpha - M_1/EI_{\eta\eta}[1-e^2(EA/EI_{\eta\eta})]^{-1} + eT/EI_{\eta\eta}[1-e^2(EA/EI_{\eta\eta})]^{-1} = 0$$

where Figure-1 together with the list below, define the nomenclature used: $(\ )'$= differentiation with respect to s; s= spatial independent variable, taken as distance along the locus of the section shear centers (elastic axis); Q_{1-2}= internal shear components; T= tension along the blade; M_{1-2-3}= internal moment components; u,v,w= elastic axis displacements; χ_{1-2},α= rotations about the elastic axis (α= angle of attack); κ_o= local initial curvature, given by the troposkien; T_o= local initial tension, given by the troposkien; e= CG offset, constant along the blade; $EI_{\xi\xi}$,$EI_{\eta\eta}$ = bending rigidities; EA= longitudinal rigidity; GJ= torsional rigidity; $G\bar{A}$= effective shear rigidity; f_{1-2-3},m_{1-2-3}= external loads (forces and moments) of both inertial and aerodynamic origin.

It became evident during the development that some extensibility should be allowed in this formulation in order to get the most rational set of equations. If inextensibility is assumed by simply letting EA become infinitely large, important information is lost. Shear deformation is also included for completeness.

The analysis proceeds by grouping the dependent variables in a 12x1 state vector y, which contains generalized internal forces y_F and generalized displacements y_D. The external loads are also collected in a 12x1 vector p:

$$y = \{y_F \mid y_D\}^T = \{Q_1\ M_1\ Q_2\ M_2\ T\ M_3 \mid u\ \chi_1\ v\ \chi_2\ w\ \alpha\}^T$$

$$p = \{f \mid 0\}^T = \{f_1\ m_1\ f_2\ m_2\ f_3\ m_3 \mid 0\ 0\ 0\ 0\ 0\ 0\}^T$$

where $f = f^{(A)} + f^{(J)}$, respectively its aerodynamic and inertial components.

Hamilton's principle is involked to evaluate f as function of the generalized displacements. By taking both the translational and rotational parts of the kinetic energy, coriolis and rotary inertia effects are automatically included. After the rigid blade inertial load is subtracted from the equations, the following approximations appear to be significant to bring all inertial terms into a convenient form:

$$\iint \rho\xi^2 d\xi d\eta \ll \iint \rho\eta^2 d\xi d\eta \approx \iint \rho(\xi^2+\eta^2) d\xi d\eta$$

Here coordinates ξ and η are defined in Figure-1, and ρ is the blade mass per unit of volume.

As a first approximation, any reasonable aerodynamic theory is adequate to complete the aeroelastic problem formulation. The quasisteady strip theory can be employed without a great deal of difficulty. The author cites Kaza & Kvaternik's derivation as appropriate [6].

Simple harmonic motion is assumed in order to eliminate the time dependence: this approach is known to be suitable for analyzing both free vibration and flutter. It follows that the inertial (J) and aerodynamic (A) loads can be written as:

$$p = [\ J + A\ ]\ y = \left(\begin{bmatrix} 0 & J_{FD} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & A_{FD} \\ 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} y_F \\ y_D \end{Bmatrix}$$

where J_{FD} and A_{FD} are 6x6 submatrices dependent on the harmonic frequency. In Appendix A the nondimensional forms of the equations are presented for the free vibration problem in particular.

SOLUTION OF THE AEROELASTIC PROBLEM

The main objective of this paper is to propose a convenient method for solving the foregoing aeroelastic problem. Numerical results are expected to be published in a later work. Therefore, the author will now describe how to obtain the eigenvalue problem associated with either free vibration or flutter analysis.

Lehman [10] presents a modified version of an integrating matrix scheme apparently first used by Hunter [7] which can be adapted for obtaining transfer matrices relating state vectors at different stations of a one-dimensional elastic structure. Appendix B gives a detailed derivation of the global transfer matrix U_o^n relating state vectors at the two ends of the blade:

$$y_n = U_o^n\, y_o$$

The enforcement of any physically possible set of boundary conditions at y_o and y_n will lead to the determination of the corresponding eigenfrequencies (as well as airspeed in the case of flutter). Such eigenfrequencies are related to the complex roots of the so called frequency determinant associated with the particular set of selected boundary conditions. The accuracy of the method relies on the number of discretization points taken during the integration process, as well as on the basic polynomial from which the integrating matrix is constructed (cf.ref.10). The transfer matrix capability for dealing with inhomogeneous boundary conditions, through the introduction of a new transfer matrix of very simple form expressing the local inhomogeneity, is what really makes this approach attractive. The construction of more elaborate models of VAWT, including for example guy wires and tower stiffness, seems possible without difficulty. Spring type boundary conditions are easily handled by a single matrix multiplication of the form:

$$U_o^{n+1} = K\, U_o^n$$

Here U_o^{n+1} is the new global transfer matrix and K is the spring transfer matrix.

CONCLUDING REMARKS

A semi-analytical method based on an integrating matrix scheme was proposed for obtaining the transfer matrix of a troposkien-curved blade. Such an approach is expected to be very efficient to study both free vibrations and flutter of VAWT.

APPENDIX A

NONDIMENSIONAL EQUATIONS FOR FREE VIBRATION IN STATE VECTOR FORM

$$\bar{y}' = [\,\bar{S} + \bar{J}\,]\,\bar{y}(\bar{s})$$

$$\bar{y} = \begin{Bmatrix} \bar{Q}_1 \\ \bar{M}_1 \\ \bar{Q}_2 \\ \bar{M}_2 \\ \bar{T} \\ \bar{M}_3 \\ \bar{u} \\ \chi_1 \\ \bar{v} \\ \chi_2 \\ \bar{w} \\ \alpha \end{Bmatrix}_{12\text{x}1} \qquad \bar{S} = \begin{bmatrix} \bar{S}_{FF} & \bar{S}_{FD} \\ \bar{S}_{DF} & \bar{S}_{DD} \end{bmatrix}_{12\text{x}12} \qquad \bar{J} = \begin{bmatrix} 0 & \bar{J}_{FD} \\ 0 & 0 \end{bmatrix}_{12\text{x}12}$$

$$\bar{S}_{FF} = -\bar{S}_{DD}^{T} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & -\bar{\kappa}_o & \cdot \\ \cdot & \cdot & \ell/c & \cdot & \cdot & -\bar{\kappa}_o \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\ell/c & \cdot & \cdot & \cdot & \cdot & \cdot \\ \bar{\kappa}_o & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \bar{\kappa}_o & \cdot & \cdot & \cdot & \cdot \end{bmatrix}_{6\text{x}6};$$

$$\bar{S}_{DF} = \left(\frac{V\ell c}{EI_{\eta\eta}}\right) \begin{bmatrix} \gamma a_1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ & a_2 & \cdot & \cdot & -x_\alpha a_2 & \cdot \\ & & \gamma a_1 & \cdot & \cdot & \cdot \\ & & & a_3 & \cdot & \cdot \\ & \text{sym} & & & r_\alpha^2 a_2 & \cdot \\ & & & & & a_4 \end{bmatrix}_{6\text{x}6} \qquad {}^{\backprime}\bar{S}_{FD} = \left(\frac{\ell}{c}\right) \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & s_\phi^{-1} & & & & \cdot \\ & & \cdot & & & \cdot \\ & & & s_\phi^{-1} & & \cdot \\ & & & & \cdot & \cdot \\ & & & & & \cdot \end{bmatrix}_{6\text{x}6}$$

$$\bar{J}_{FD} = \left(\frac{mc\Omega^2}{V}\right)\left[-(\bar{\omega}^2 + \frac{1}{2})\,{}^{\backprime}M\,B\,{}^{\backprime}M - \frac{1}{2}\,{}^{\backprime}M\,\Phi_1\,{}^{\backprime}M + 2i\bar{\omega}\,{}^{\backprime}M\,\Phi_2\,{}^{\backprime}M + r_\alpha^2[-\bar{\omega}^2\,{}^{\backprime}C + i\bar{\omega}\,\Phi_3\,]\right]_{6\text{x}6}$$

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & 0 & \Phi_{12} \\ & I & 0 \\ \text{sym} & & -\Phi_{11} \end{bmatrix}_{6\text{x}6} \qquad \Phi_2 = \begin{bmatrix} 0 & \Phi_{21} & 0 \\ -\Phi_{21}^T & 0 & \Phi_{22} \\ 0 & -\Phi_{22}^T & 0 \end{bmatrix}_{6\text{x}6} \qquad \Phi_3 = \begin{bmatrix} 0 & \Phi_{31} & 0 \\ -\Phi_{31}^T & 0 & \Phi_{32} \\ 0 & -\Phi_{32}^T & 0 \end{bmatrix}_{6\text{x}6}$$

$$\Phi_{11} = \begin{bmatrix} -c_{2\phi} & -s_{2\phi} \\ -s_{2\phi} & c_{2\phi} \end{bmatrix} \qquad \Phi_{21} = \begin{bmatrix} s_\phi & \cdot \\ -c_\phi & \cdot \end{bmatrix} \qquad {}^{\backprime}\Phi_{31} = \begin{bmatrix} \cdot & \cdot \\ \cdot & s_\phi \end{bmatrix}$$

$$\Phi_{12} = \begin{bmatrix} -s_{2\phi} & c_{2\phi} \\ c_{2\phi} & s_{2\phi} \end{bmatrix} \qquad \Phi_{22} = \begin{bmatrix} c_\phi & s_\phi \\ \cdot & \cdot \end{bmatrix} \qquad {}^{\backprime}\Phi_{32} = \begin{bmatrix} \cdot & \cdot \\ \cdot & c_\phi \end{bmatrix}, \text{ all 2x2 matrices}$$

$${}^{\backprime}M = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ & x_\alpha & & & & \cdot \\ & & 1 & & & \cdot \\ & & & r_\alpha & & \cdot \\ & & & & 1 & \cdot \\ & & & & & x_\alpha \end{bmatrix}_{6\text{x}6} \qquad B = \begin{bmatrix} 1 & \cdot & \cdot & \cdot & \cdot & -1 \\ & 1 & \cdot & \cdot & 1 & \cdot \\ & & 1 & \cdot & \cdot & \cdot \\ & & & 1 & \cdot & \cdot \\ & \text{sym} & & & 1 & \cdot \\ & & & & & 1 \end{bmatrix}_{6\text{x}6} \qquad {}^{\backprime}C = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & 1 & & & & \cdot \\ & & \cdot & & & \cdot \\ & & & -1 & & \cdot \\ & & & & \cdot & \cdot \\ & & & & & 1 \end{bmatrix}_{6\text{x}6}$$

where the new symbols are: $a_1 = EI_{\eta\eta}/c^2GA$; $a_2 = [1-(x_\alpha/r_\alpha)^2]^{-1}$; $a_3 = EI_{\eta\eta}/EI_{\xi\xi}$ $a_4 = EI_{\eta\eta}/GJ$; GA= shear rigidity; $(\bar{\ })$= dimensionless quantities: (force)/V, (moment)/Vc, (displacement)/c; $\grave{(\ )}$= diagonal matrix; I= identity matrix; 0= null matrix; $(\ )^T$= transpose matrix; (.)=zero element; x_α= dimensionless cross section static unbalance (in units of chord); r_α= dimensionless cross section radius of gyration (in units of chord); ℓ= blade length; c= blade chord; m= blade total mass; V= vertical component of the initial tension (constant for a gravity-free troposkien); Ω= angular velocity of the VAWT; ω= harmonic frequency; ϕ= angle defining the troposkien shape (Figure-2); γ= shear coefficient; $\bar{s} = s/\ell$; $\bar{\kappa}_o = \ell\kappa_o$; $\bar{\omega} = \omega/\Omega$; s_x=sin x; c_x = cos x $i = \sqrt{-1}$; sym= symmetric matrix.

APPENDIX B

TRANSFER MATRIX DERIVATION WITH THE USE OF THE INTEGRATING MATRIX

In state vector form, the nondimensional aeroelastic equations can be written as:

$$\bar{y}' = [\ \bar{S} + \bar{J} + \bar{A}\ ]\ \bar{y} = Z\ \bar{y} \tag{B-1}$$

where $\bar{S}$, $\bar{J}$ and $\bar{A}$ are 12x12 previously defined matrices and $\bar{y}$ is the 12x1 dimensionless state vector of the dependent variables. When the integral operator L (cf.ref.11) is applied, one gets:

$$\bar{y} = L\ Z\ \bar{y} + k \tag{B-2}$$

where k is a constant vector, dependent upon the boundary conditions.Premultiplying equation B-2 by a boundary condition matrix D_o (cf.ref.7),the state vector at one end is isolated since, by their intrinsic structure, $D_o L = 0$. Therefore $\bar{y}_o = k$ and B-2 can be rewritten as:

$$\bar{y}_o = (\ I - L\ Z\ )\ \bar{y} \tag{B-3}$$

One notices that the term in parentheses is, by definition the inverse of the transfer matrix U. Hence,

$$\bar{y} = U\ \bar{y}_o = (\ I - L\ Z\ )^{-1}\ \bar{y}_o \tag{B-4}$$

where I stands for the unit matrix. In particular, premultiplying U by another boundary condition matrix at the opposite end D_n, yields:

$$\bar{y}_n = D_n\ U\ \bar{y}_o = U_o^n\ \bar{y}_o \tag{B-5}$$

that is, the relation between state vectors at the two ends of the blade. The transfer matrix U_o^n can be seen as a product of intermediate transfer matrices relating conditions at adjacent points along the integration path. The manipulation of most of these matrices is facilitated by their block structure and sparseness.

REFERENCES

1. Blackwell, B.F. and Reis, G.E.,"Blade Shape for a Troposkien Type of Vertical Axis Wind Turbine,"Sandia Laboratories Report SAND 74-0154,Apr.1974.
2. Ham, N.D.,"Aeroelastic Analysis of a Troposkien-Type Wind Turbine," Sandia Laboratories Report SAND 77-0026, Apr.1977.
3. Ham, N.D.,"Flutter of Darrieus Wind Turbine Blades,"Proceedings of Wind Turbine Structural Dynamics Workshop, NASA CP-2034/CONF-771148,1977pp.77-91.
4. Ashley, H.,"Use of Asymptotic Methods in Vibration Analysis,"Proceedings of Wind Turbine Struct.Dynam.Workshop,NASA CP-2034/CONF-771148,1977pp.39-52.
5. Reis, G.E. and Blackwell, B.F.,"Practical Approximations to a Troposkien by Straight-Line and Circular-Arc Segments,"Sandia Laboratories Report SAND 74-0100, March 1975.
6. Kaza, K.R.V. and Kvaternik, R.G.,"Aeroelastic Equations of Motion of a Darrieus Vertical-Axis Wind-Turbine Blade,"NASA TM-79295, Dec.1979.
7. Hunter, W.F.,"Integrating-Matrix Method for Determining the Natural Vibration Characteristics of Propeller Blades,"NASA TN D-6064, Dec.1970.
8. Pestel, E.C. and Leckie, F.A., Matrix Methods in Elastomechanics, Mc. Graw-Hill, 1963.
9. Nair, S. and Hegemier, G.,"Effect of Initial Stresses on the Small Deformations of a Composite Rod,"AIAA Journal,V.16,No.3,Mar.1978,pp.212-217.
10. Lehman, L.,"A Hybrid State Vector Approach to Aeroelastic Analysis with Application to Composite Lifting Surfaces,"in AIAA/ASME/ASCE/AHS 22nd. Structures, Struct.Dynamics and Materials Conf.,Part 2, Apr.1981,pp.821-31.

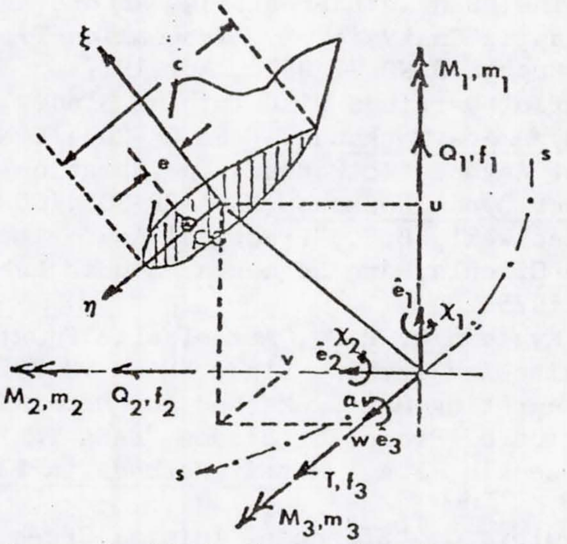

Figure-1: Blade Cross Section

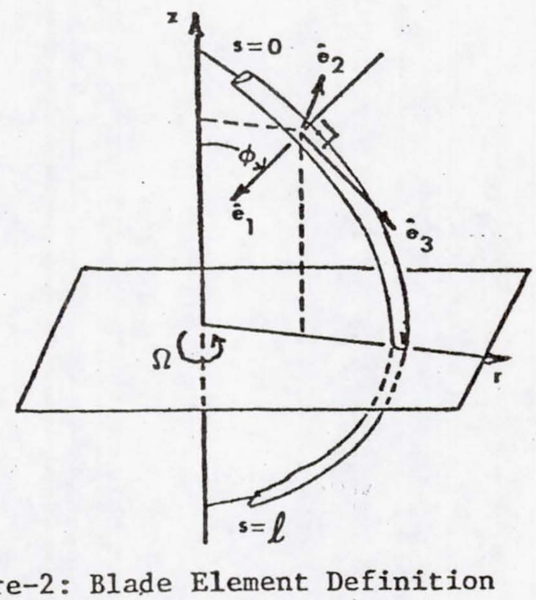

Figure-2: Blade Element Definition

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-165764		

4. Title and Subtitle	5. Report Date
Aeroelastic Analysis of a Troposkien-Type Wind Turbine Blade	July 1981
	6. Performing Organization Code
7. Author(s) F. Nitzsche	8. Performing Organization Report No.
	10. Work Unit No.
9. Performing Organization Name and Address Stanford University Stanford, CA 94305	11. Contract or Grant No. NGL 05-020-243
	13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546	14. Sponsoring Agency Code

15. Supplementary Notes Langley Technical Monitor: R. V. Doggett, Jr.

Paper presented at International Colloquium on Wind Energy, Brighton, England, August 1981.

16. Abstract

The linear aeroelastic equations for one curved blade of a vertical axis wind turbine are presented in state vector form. An attractive method, based on a simple integrating matrix scheme together with the transfer matrix idea, is proposed as a convenient way of solving the associated eigenvalue problem for general support conditions.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Aeroelasticity Wind Turbine Troposkien	Unclassified-Unlimited Subject Category 44

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	11	A02