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# Statistics of Some Atmospheric Turbulence Records Relevant to Aircraft Response Calculations

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# Statistics of Some Atmospheric Turbulence Records Relevant to Aircraft Response Calculations

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## SUMMARY

This report illustrates the results of application to turbulence velocity records of several new methods for characterizing atmospheric turbulence that are described in Ref. 1. The methods illustrated include maximum likelihood estimation of the integral scale and intensity of records obeying the von Karman transverse power spectral form, constrained least-squares estimation of the parameters of a parametric representation of autocorrelation functions, estimation of the power spectral density of the instantaneous variance of a record with temporally fluctuating variance, and estimation of the probability density functions of various turbulence components. The report also contains descriptions of the computer programs used in the computations, and a full listing of these programs. The computational methods illustrated herein were developed by the first named author. The computer programs and their explanation contained in the Appendices were written and exercised by the second named author.

## TURBULENCE MODEL

In the work described in this report, we shall assume that the turbulence velocity records under consideration can be modeled as

$$w(t) = w_s(t) + w_f(t)$$

$$= w_s(t) + \sigma_f(t)z(t), \tag{1.1}$$

where

$$w_f(t) = \sigma_f(t)z(t) \tag{1.2}$$

with

$$\sigma_f(t) \geq 0$$

and

$$E\{z(t)\} = 0, \quad E\{z^2(t)\} = 1. \tag{1.3}$$

The three processes $\{w_s(t)\}$, $\{\sigma_f(t)\}$, and $\{z(t)\}$ are assumed to be stationary and mutually statistically independent. Furthermore, we shall assume that $\{z(t)\}$ is a Gaussian process. The "slow" turbulence component $w_s(t)$ is assumed to contain predominately very low frequencies (or large wave-numbers) relative to the "fast" component $w_f(t)$ which may be regarded as ordinary turbulence with a slowly varying standard deviation $\sigma_f(t)$. This model is more completely described in Section 1 of the companion report [1] or Section 2 of its predecessor [5].

# MAXIMUM LIKELIHOOD ESTIMATION OF THE INTEGRAL SCALE AND INTENSITY OF THE VERTICAL RECORD FROM FLIGHT 8 RUN 2 (CONVECTIVE CONDITIONS)

The vertical record shown in Fig. 1 illustrates a turbulence velocity history with negligible low-frequency component $w_s(t)$. The power spectral density of the vertical record in Fig. 1 is shown in Fig. 2. The method used to compute the power spectral density of Fig. 2 is described in Appendix B of Ref. 2, where the value used for M was 6590.5 m which corresponds to 1024 temporal sample points. Before computing the power spectral density of the record, its mean value was computed and removed.

Also shown in Fig. 2 is the von Karman transverse power spectral density

$$\Phi_{KT}(k) = \sigma^2 L \frac{1+188.75L^2k^2}{[1+70.78L^2k^2]^{11/6}} \tag{2.1}$$

The values of L and $\sigma^2$ in Eq. (2.1) – as plotted in Fig. 2 – are

$$L = 309.4 \text{ m}, \quad \sigma^2 = 1.326 \text{ (m/sec)}^2. \tag{2.2}$$

These values were computed using the maximum likelihood method derived in Sec. 3 of Ref. 1. The specific equation used to compute the value of L in Eq. (2.2) was Eq. (3.26) of Ref. 1 with the aid of Eqs. (3.34) and (3.35) of Ref. 1. Details of this computation are described in Appendix F of Ref. 1. Using the value of L obtained by Eq. (3.26) of Ref. 1, Eq. (3.25) of Ref. 1 was then used to compute the values of $\sigma^2$ given in Eq. (2.2) above.

The von Karman transverse spectrum shown in Fig. 2 provides an excellent fit to the spectrum computed from the turbulence record. In particular, note that asymptotic (high wavenumber) slopes of the empirical and von Karman spectra agree very well. We also have computed the value of $\sigma^2$ directly – by squaring and averaging the time history sample points. The value of $\sigma^2$ obtained in this manner was

$$\sigma^2 = 1.331 \text{ (m/sec)}^2. \tag{2.3}$$

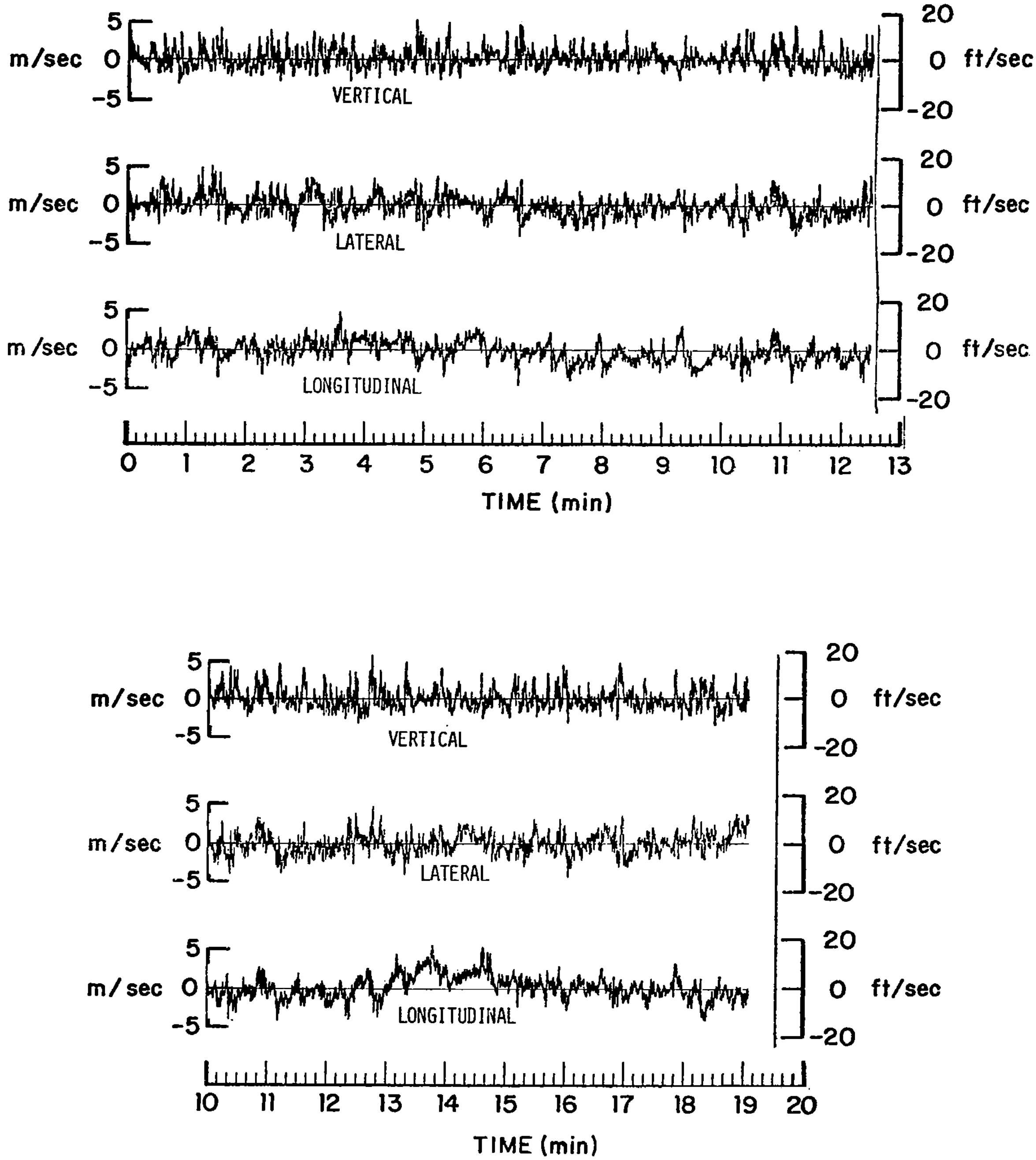


FIG. 1. LOW-ALTITUDE TURBULENCE RECORDS.[CONVECTIVE CONDITIONS. AIRCRAFT SPEED 129 m/sec (422 ft/sec).] (Ref. 3, Fig. 4, p. 282).

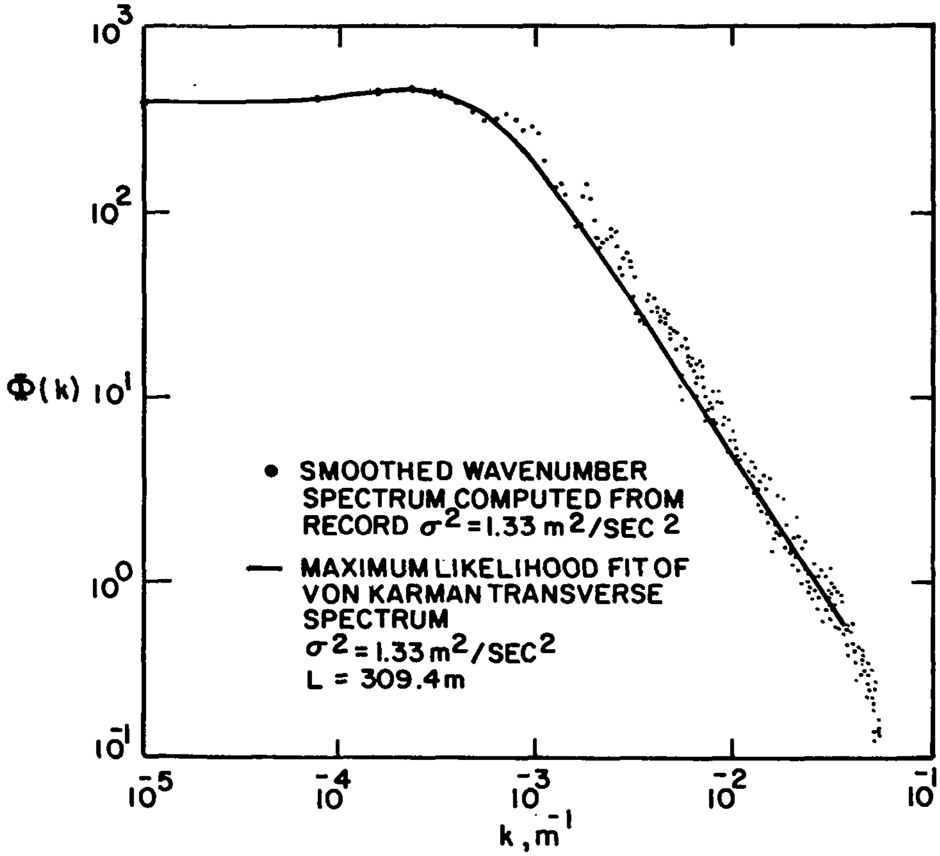


FIG. 2. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM VERTICAL RECORD SHOWN IN FIG. 1 AND MAXIMUM LIKELIHOOD FIT OF von KARMAN TRANSVERSE SPECTRUM.

The maximum likelihood method used to compute the value of $\sigma^2$ given by Eq. (2.2) is *not* the same as the squaring and averaging procedure used to compute the value of Eq. (2.3). However, quite remarkably, the two values agree to the first three significant figures. Since the assumption of a von Karman transverse spectrum was used in the computation of $\sigma^2$ given by Eq. (2.2), the close agreement of the values of $\sigma^2$ given by Eqs. (2.2) and (2.3) provides verification of the excellent representation of the empirical spectrum that is provided by the von Karman transverse spectrum of Eq. (2.1).

The autocorrelation function of the vertical record shown in Fig. 1 is compared in Fig. 3 with the von Karman transverse autocorrelation function:

$$\phi_{KT}(\xi) \triangleq \sigma^2 \frac{2^{2/3}}{\Gamma(1/3)} (\beta\xi/L)^{1/3}[K_{1/3}(\beta\xi/L) - \frac{\beta\xi}{2L} K_{-2/3}(\beta\xi/L)], \tag{2.4}$$

where

$$\beta \triangleq \frac{2\sqrt{\pi}\ \Gamma(11/6)}{5\ \Gamma(4/3)}, \tag{2.5}$$

where the $K_n(\cdot)$ in Eq. (2.4) are modified Bessel functions of the second kind of order n and $\Gamma(\cdot)$ is the gamma function. Values of the Bessel functions in Eq. (2.4) were obtained from the tabulation of p. 228 of Ref. 4, where we note that $K_{-2/3}(x) = K_{2/3}(x)$. The empirical autocorrelation function in Fig. 3 was computed from the vertical record of Fig. 1 by the method described in Appendix B of Ref. 2. Both autocorrelation functions shown in Fig. 3 are normalized to unity at the origin. The value of integral scale L used in the von Karman form of Eq. (2.4) is that given by Eq. (2.2).

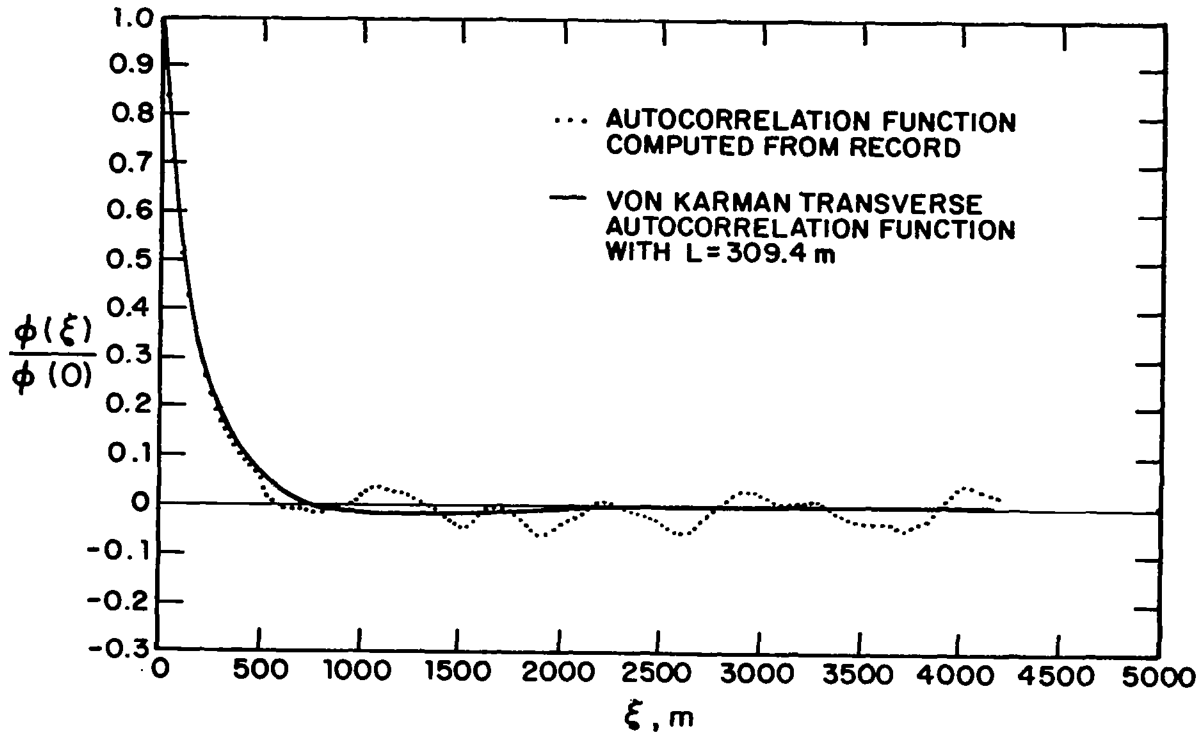


FIG. 3. COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM VERTICAL RECORD SHOWN IN FIG. 1 AND von KARMAN TRANSVERSE AUTOCORRELATION FUNCTION.

## CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS OF LATERAL RECORD FROM FLIGHT 32 RUN 4 (WIND-SHEAR CONDITIONS)

The lateral record shown in Fig. 4 illustrates a turbulence velocity history with a relatively strong low-frequency component $w_S(t)$. The power spectral density of the lateral record in Fig. 4 is shown in Fig. 5 (solid dots), which was computed by the method described in Appendix B of Ref. 2 — the value of M used in the computation was 9613.3 m which corresponds to 1024 temporal sample points. Before computing the power spectral density of the record, its mean value was computed and removed.

Also plotted in Fig. 5 is the von Karman transverse spectrum of Eq. (2.1) evaluated from the parameters

$$L = 265.5 \text{ m}, \quad \sigma^2 = 5.315 \text{ m}^2/\text{sec}^2. \tag{3.1}$$

These parameter values were arrived at using the constrained least-squares estimation method described in Sec. 4 of Ref. 1. This method postulates that within an interval $0 \le \xi \le \xi_H$, the autocorrelation function of a record is of the form

$$\underset{\sim}{\phi}(\xi) \triangleq \sigma_f^2 \phi_K(\xi;L) + \sum_{i=0}^{m} a_i \xi^i \ , \ 0 \le \xi \le \xi_H, \tag{3.2}$$

where $\sigma_f^2 \phi_K(\xi;L)$ is the appropriate (transverse or longitudinal) von Karman autocorrelation function, and the mth degree polynomial in Eq. (3.2) represents the autocorrelation function of the "slow" turbulence component $w_S(t)$ within the interval $0 \le \xi \le \xi_H$. The least-squares estimation procedure constrains the relationship between $\sigma_f^2$ and L using the portion of the wavenumber spectrum of the record (in the "high-frequency" region) between two wavenumbers $k_\ell$ and $k_u$ as described in Sec. 4 of Ref. 1, where in the present case, we used $k_\ell = 10^{-3} m^{-1}$ and $k_u = 4 \times 10^{-2} m^{-1}$. Equation (4.4) of Ref. 1 is the equation of constraint. The resulting relationship between $\sigma_f^2$ and L for the present example is plotted in Fig. 6.

Figure 7 displays the autocorrelation function of the lateral record shown in Fig. 4. To determine the general behavior of the constrained least-squares estimation method

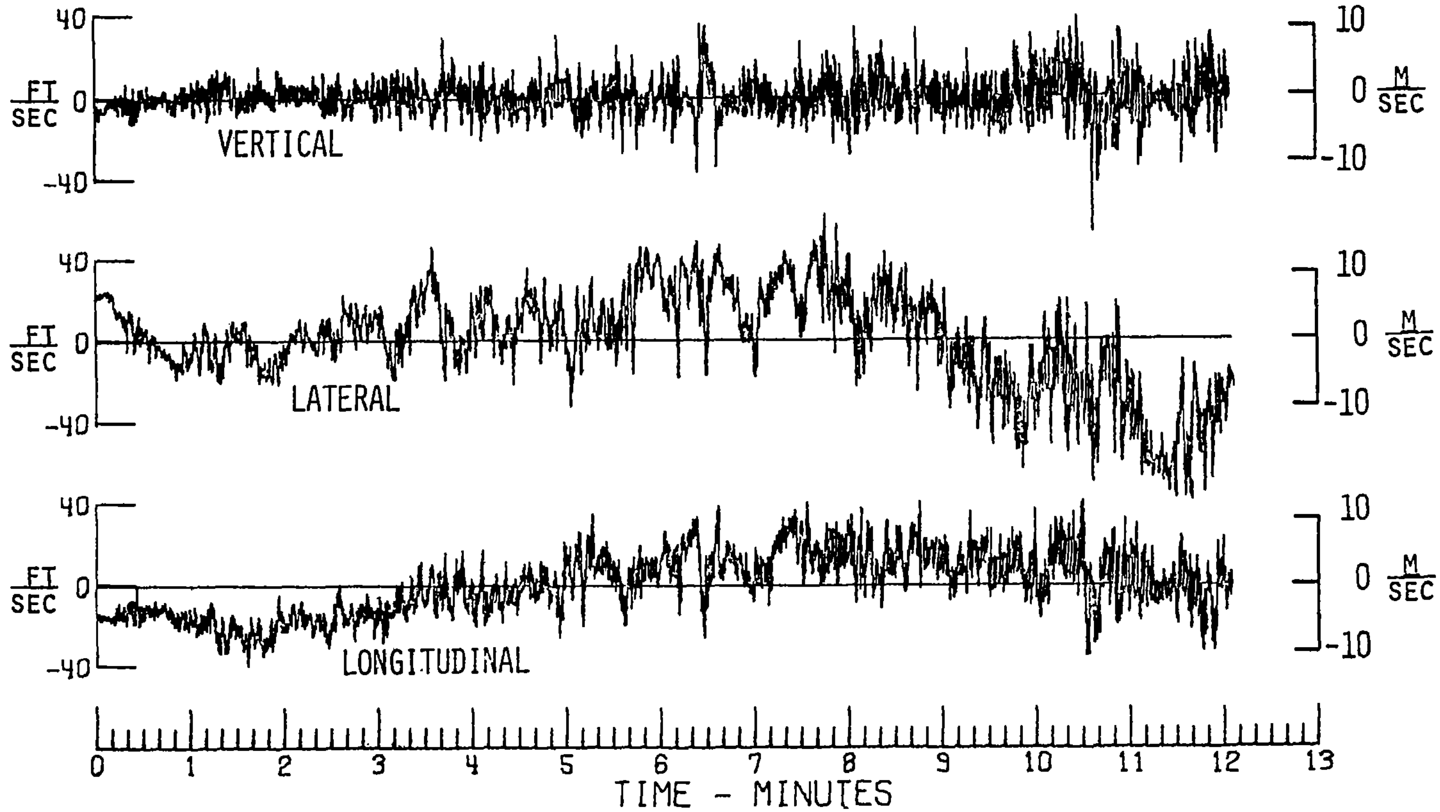


FIG. 4. TURBULENCE RECORDS CONTAINING STRONG "SLOW" COMPONENTS $w_s(t)$. [WIND SHEAR CONDITIONS. AIRCRAFT SPEED 188 m/sec (616 ft/sec).] Ref. 3, Fig. 6, p. 283.)

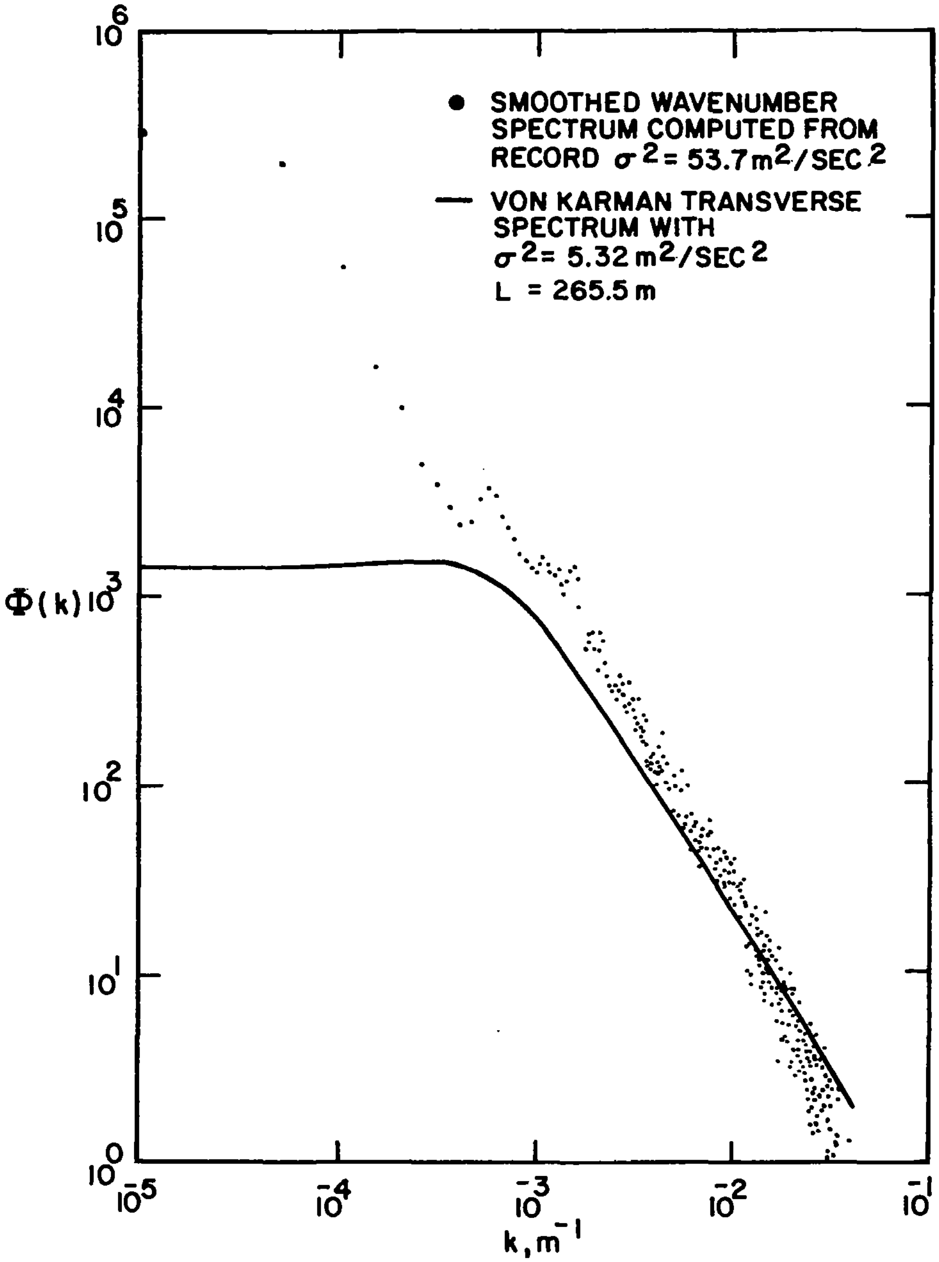


FIG. 5. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM LATERAL RECORD SHOWN IN FIG. 4 AND von KARMAN TRANSVERSE SPECTRUM OBTAINED BY CONSTRAINED LEAST-SQUARES FIT TO THE (EMPIRICAL) AUTOCORRELATION FUNCTION. Von KARMAN SPECTRUM CHARACTERIZES "FAST" TURBULENCE COMPONENT ONLY.

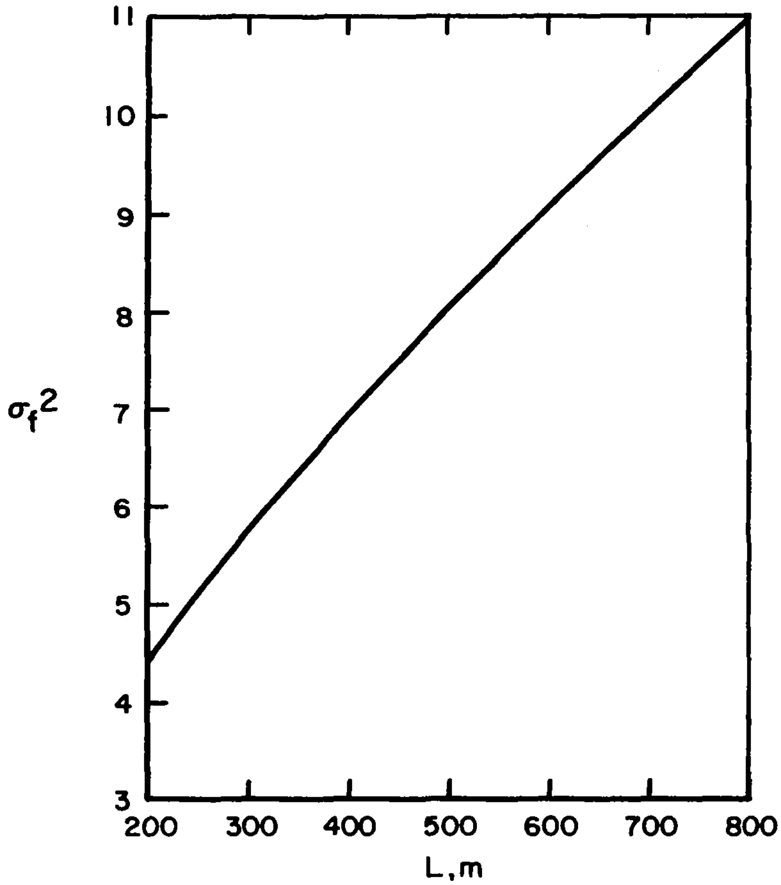


FIG. 6. CONSTRAINT BETWEEN $\sigma_f^2$ AND L FOR CONSTRAINED LEAST-SQUARES ESTIMATION PROCEDURE APPLIED TO LATERAL RECORD SHOWN IN FIG. 4.

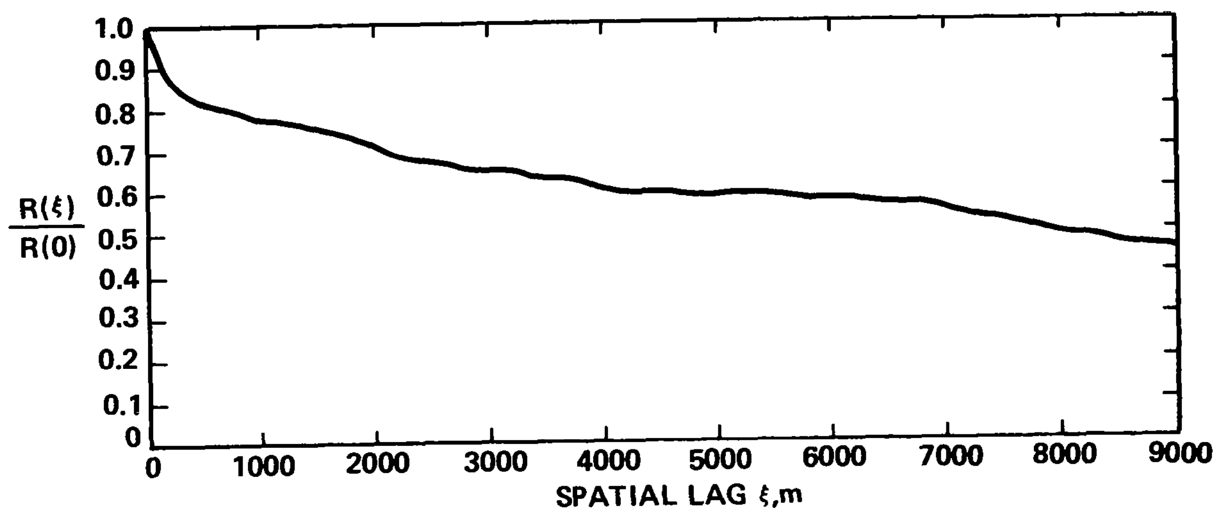


FIG. 7. AUTOCORRELATION FUNCTION OF LATERAL RECORD SHOWN IN FIG. 4.

for the autocorrelation function shown in Fig. 7, the method was exercised a number of times for the values of $\xi_H$ and m listed in Table 1.

Four autocorrelation function representations $\phi(\xi)$ given by Eq. (3.2) for four different sets of values of $\xi_H$ and m are plotted on expanded scales in Fig. 8 along with the empirical autocorrelation function $R(\xi)$ of the lateral record shown in Fig. 4. The von Karman transverse spectrum of Eq. (2.1) for one of these cases ($\xi_H$ = 5998.1 m and m = 3) is plotted in Fig. 5 for comparison with the "high-frequency" portion of the empirical spectrum. The values of L and $\sigma_f^2 = \sigma^2$ used in the evaluation of Eq. (2.1) shown in Fig. 5 are those given by Eq. (3.1), which were taken from Table 1. From Fig. 5, we see that the asymptotic slope of the empirical spectrum is somewhat steeper than the -5/3 slope of the von Karman spectrum. Hence, for this record, one of the basic assumptions in the constrained least-squares fit method is not well satisfied. Because of this discrepancy in slopes, none of four curves $\phi(\xi)$ shown in Fig. 8 fits well the "von Karman" region of the autocorrelation function near $\xi = 0$. Nevertheless, the knee of the von Karman spectrum shown in Fig. 5 would appear to have about the right position.

Figure 9 displays the autocorrelation function of the lateral component of the wind shear record with the von Karman autocorrelation component $\sigma_f^2\phi_K(\xi;L)$ removed. That is, the solid curve in Fig. 8 is the autocorrelation function $R(\xi)$ of the lateral record shown in Fig. 4 after subtraction of the von Karman autocorrelation function component

$$\phi_{KT}(\xi) \equiv \sigma_f^2\phi_K(\xi;L) \qquad (3.3)$$

evaluated from Eq. (2.4) with the values of L and $\sigma^2$ given by Eq. (3.1). The dashed curve is the cubic (3rd degree of polynomial) that best represents the solid curve in an integral least-squares sense over the lag region from 0 to 10,000 meters. We see from Fig. 9 that a third-degree polynomial represents very nicely the autocorrelation function of the slow turbulence component $w_s(t)$ over a 10,000 meter lag interval. Such polynomial representations are the characterizations suggested in Sec. 1 of Ref. 1 for describing the "slow" turbulence component $w_s(t)$ for aircraft response calculations.

TABLE 1. CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS FOR WIND-SHEAR LATERAL RECORD

| $\xi_H$ m | m | $\sigma_f^2$ $m^2/sec^2$ | L m | $\underset{\sim}{\phi}(0)$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|---|
| 1004.5 | 1 | 5.518 | 281.5 | 52.28 | 46.77 | $-.582\times10^{-2}$ | | |
| 1999.6 | 1 | 6.486 | 361.1 | 51.68 | 45.19 | $-.341\times10^{-2}$ | | |
| 3004.2 | 1 | 6.046 | 324.1 | 51.76 | 45.14 | $-.390\times10^{-2}$ | | |
| 3004.2 | 2 | 5.793 | 303.6 | 51.81 | 46.02 | $-.431\times10^{-2}$ | $.115\times10^{-6}$ | |
| 4496.9 | 2 | 5.428 | 274.7 | 51.96 | 46.53 | $-.517\times10^{-2}$ | $.409\times10^{-6}$ | |
| 5998.9 | 2 | 5.265 | 261.8 | 51.99 | 46.73 | $-.542\times10^{-2}$ | $.468\times10^{-6}$ | |
| 5998.9 | 3 | 5.315 | 265.5 | 51.98 | 46.66 | $-.533\times10^{-2}$ | $.434\times10^{-6}$ | $.353\times10^{-11}$ |
| 7998.6 | 3 | 4.459 | 201.0 | 52.29 | 47.83 | $-.723\times10^{-2}$ | $.116\times10^{-5}$ | $-.739\times10^{-10}$ |
| 9998.2 | 3 | 4.742 | 221.8 | 52.20 | 47.46 | $-.674\times10^{-2}$ | $.102\times10^{-5}$ | $-.624\times10^{-10}$ |
| 9998.2 | 4 | 4.530 | 206.3 | 52.28 | 47.75 | $-.721\times10^{-2}$ | $.122\times10^{-5}$ | $-.913\times10^{-10}$ |
| | | | | | | | | $a_4=.141\times10^{-14}$ |

Exact value of R(0) is 53.66 $m^2/sec^2$.

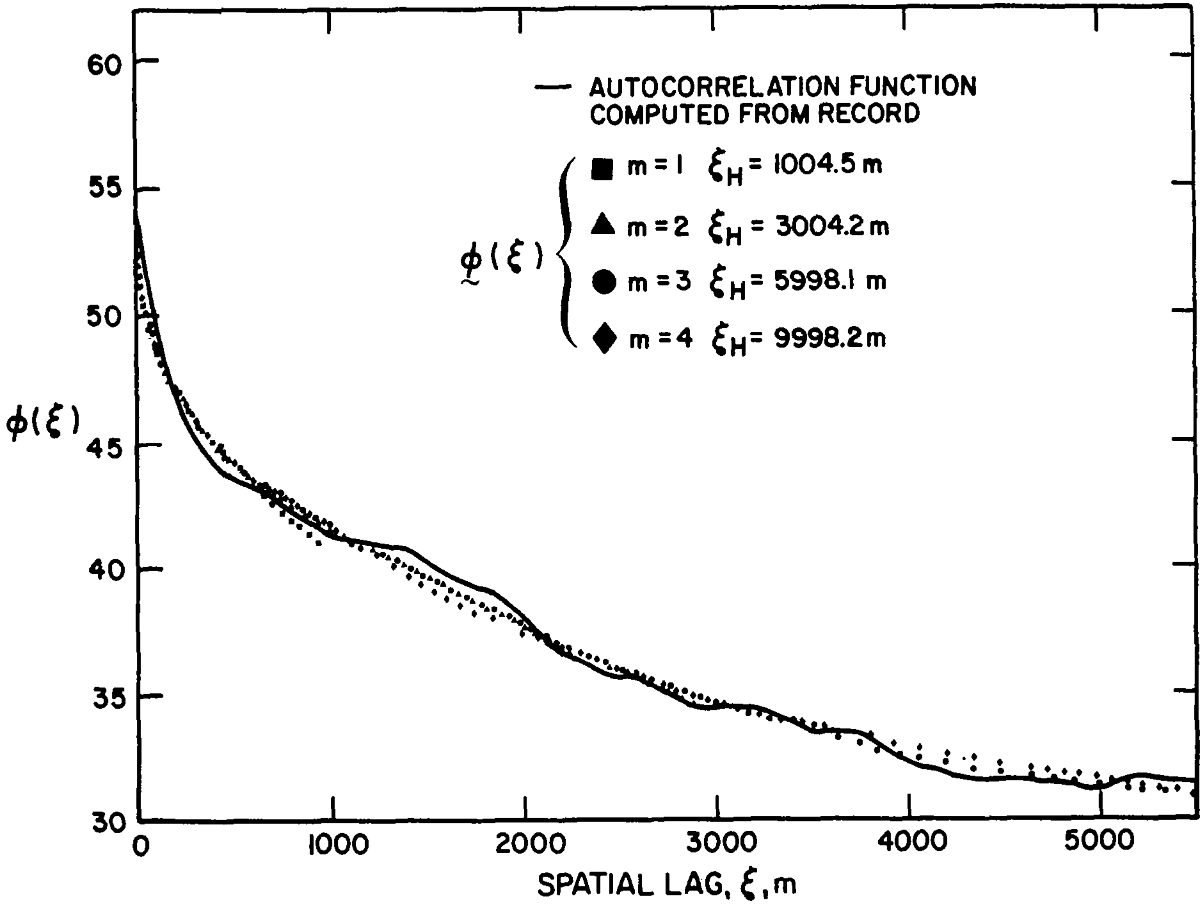


FIG. 8. COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM LATERAL RECORD SHOWN IN FIG. 4 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2).

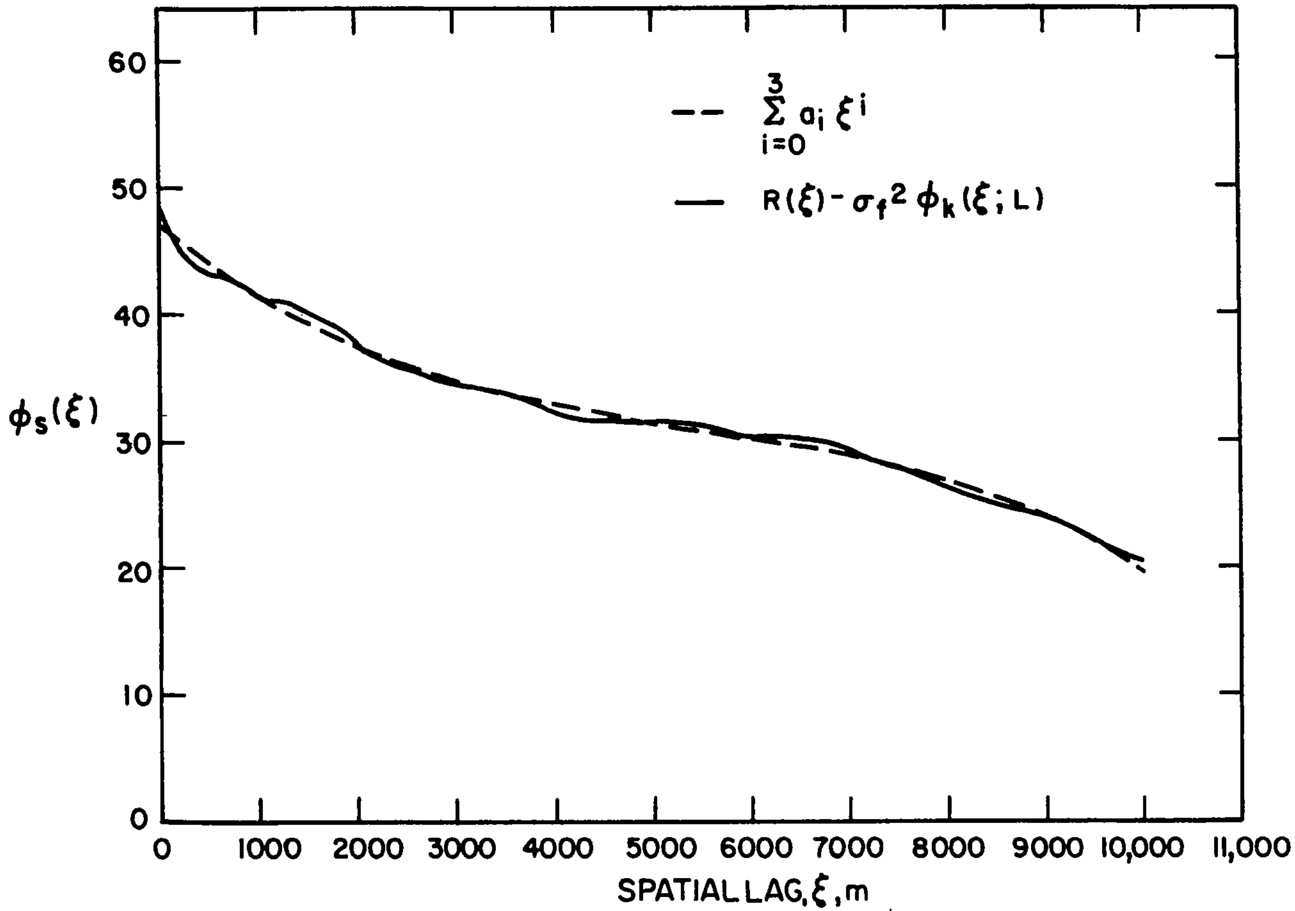


FIG. 9. COMPARISON OF AUTOCORRELATION FUNCTION $R(\xi)$ OF LATERAL RECORD SHOWN IN FIG. 4 MINUS AUTOCORRELATION FUNCTION $\sigma_f^2\ \phi_K(\xi;L)$ OF von KARMAN COMPONENT AND INTEGRAL LEAST-SQUARES THIRD-DEGREE POLYNOMIAL APPROXIMATION.

## CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS OF LATERAL RECORD FROM FLIGHT 30 RUN 8 (MOUNTAIN-WAVE CONDITIONS)

The lateral record shown in Fig. 10 illustrates a record with an exceptionally strong low-frequency component $w_s(t)$ relative to the "fast" component $w_f(t)$. Figures 11 to 15 illustrate, respectively, the same quantities for the lateral record shown in Fig. 10 that Figs. 5 to 9 displayed for the lateral record in Fig. 4. Similarly, Table 2 displays for the lateral record in Fig. 10 quantities comparable to the quantities displayed in Table 1 for the lateral record in Fig. 4. Computations of the material in Table 2 and Figs. 11 to 15 were carried out using the same methods as in the case of Table 1 and Figs. 5 to 9.

The value of M used in computing the empirical spectrum in Fig. 11 was 10,089 m which corresponds to 1024 temporal sample points. The von Karman transverse spectrum plotted in Fig. 11 was computed using the parameter values

$$L = 128.9 \text{ m}, \ \sigma^2 = 0.684 \text{ m}^2/\text{sec}^2 \qquad (4.1)$$

which correspond to the case $\xi_H$ = 2295.6 meters and a 2nd degree polynomial (m=2) in the autocorrelation function representation of Eq. (3.2).

In computing the constraint relationship between $\sigma_f^2$ and L displayed in Fig. 12, the lower and upper wavenumbers used were $k_\ell = 10^{-3}\text{m}^{-1}$ and $k_u = 4 \times 10^{-2}\text{m}^{-1}$.

*Discussion.* The four fits to the empirical autocorrelation function shown in Fig. 14(a) illustrates misleading results that the method described in Sec. 4 of Ref. 1 can yield when it is not used properly. Although each of the four fits provided by Eq. (3.2) to the empirical autocorrelation function appears reasonable to the eye, reference to Table 2 shows that the largest integral scale obtained for these four cases is that corresponding to $\xi_H$ = 699.5 meters and m = 1 which yielded L = 118.6 meters, whereas the next largest value of L for the four cases is L = 69.9 meters for the case $\xi_H$ = 896.6 meters and m = 2. The discrepancy between these two values of L is quite large. The problem here is that the polynomial in the right-band of Eq. (3.2) is actually representing part of the von Karman portion of the empirical

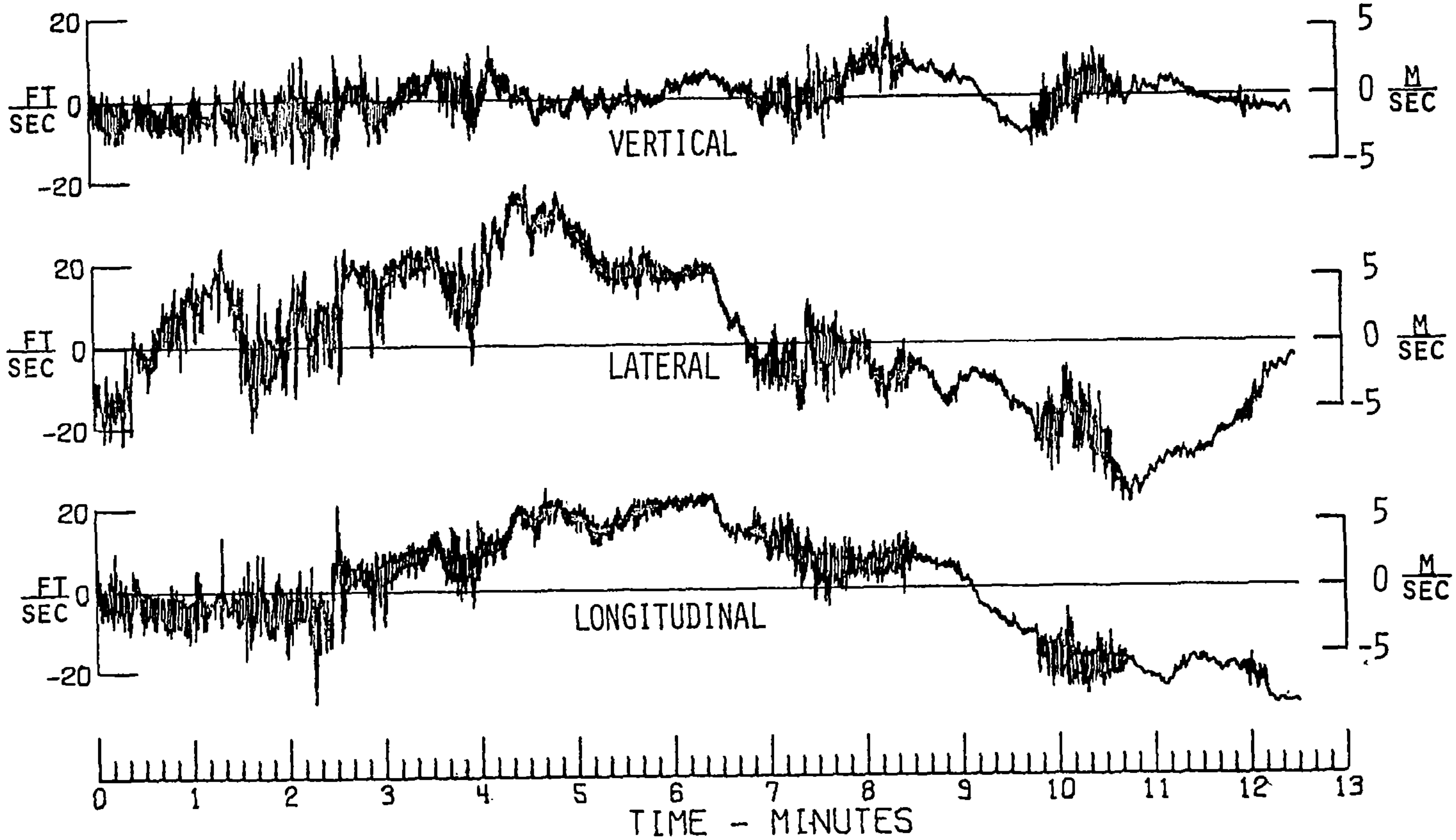


FIG. 10. TURBULENCE RECORDS CONTAINING EXCEPTIONALLY STRONG "SLOW" COMPONENTS $w_s(t)$. [MOUNTAIN WAVE CONDITIONS. AIRCRAFT SPEED 197 m/sec (646 ft/sec).] (Ref. 3, FIG. 10, p. 285.)

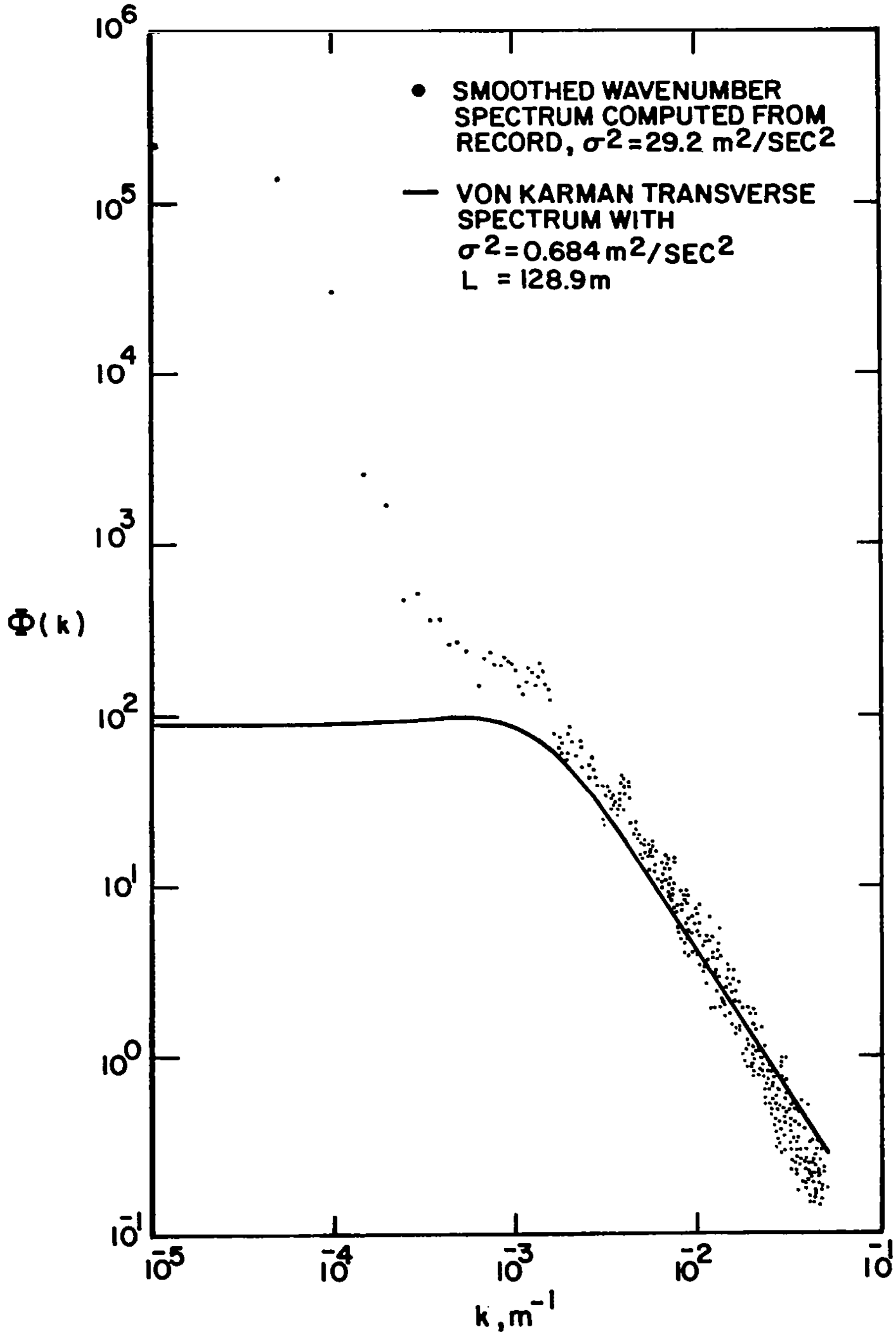


FIG. 11. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM LATERAL RECORD SHOWN IN FIG. 10 AND von KARMAN TRANSVERSE SPECTRUM OBTAINED BY CONSTRAINED LEAST-SQUARES FIT TO THE (EMPIRICAL) AUTOCORRELATION FUNCTION. Von KARMAN SPECTRUM CHARACTERIZES "FAST" TURBULENCE COMPONENT ONLY.

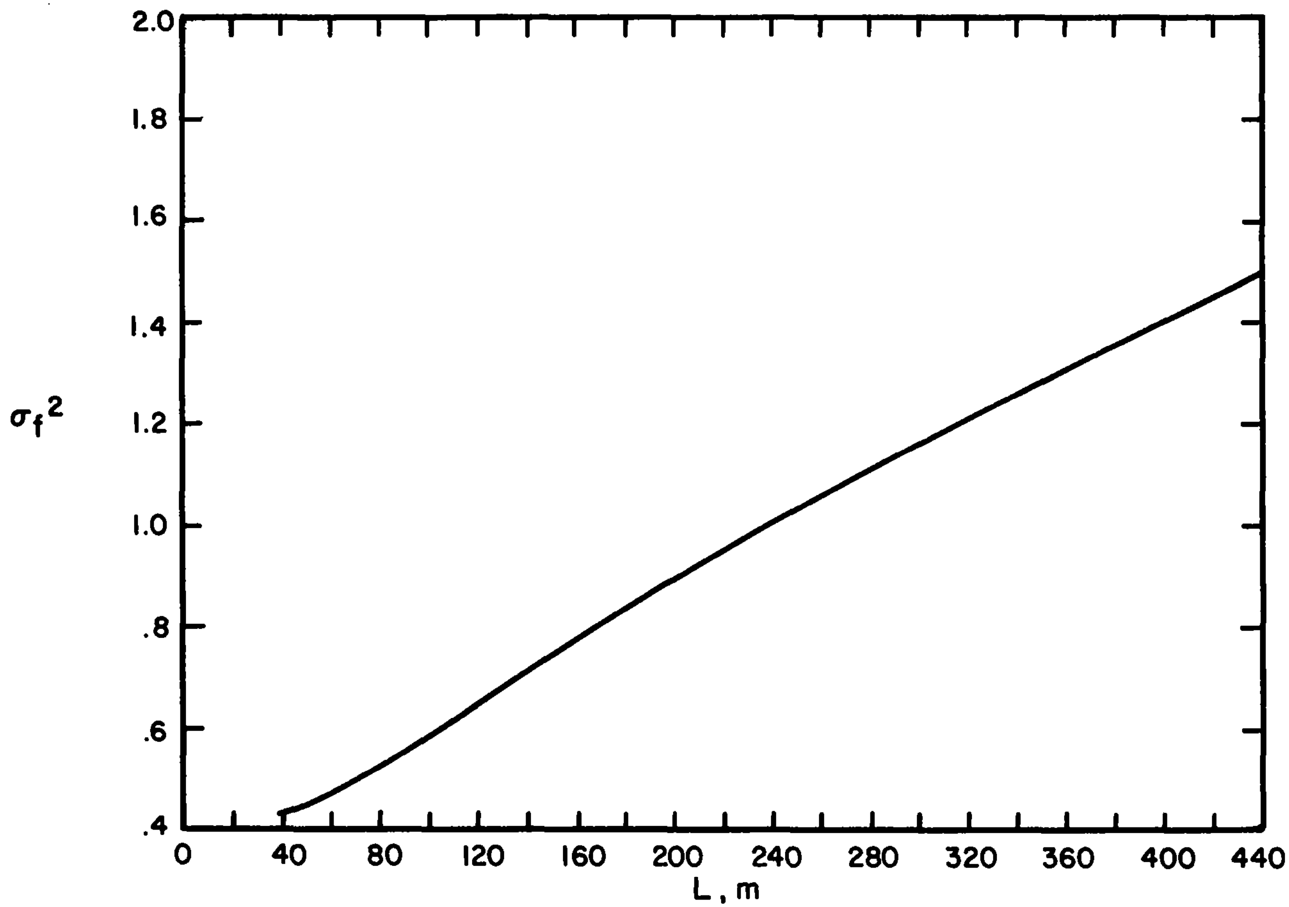


FIG. 12. CONSTRAINT BETWEEN $\sigma_f^2$ AND L FOR CONSTRAINED LEAST-SQUARES ESTIMATION PROCEDURE APPLIED TO LATERAL RECORD SHOWN IN FIG. 10.

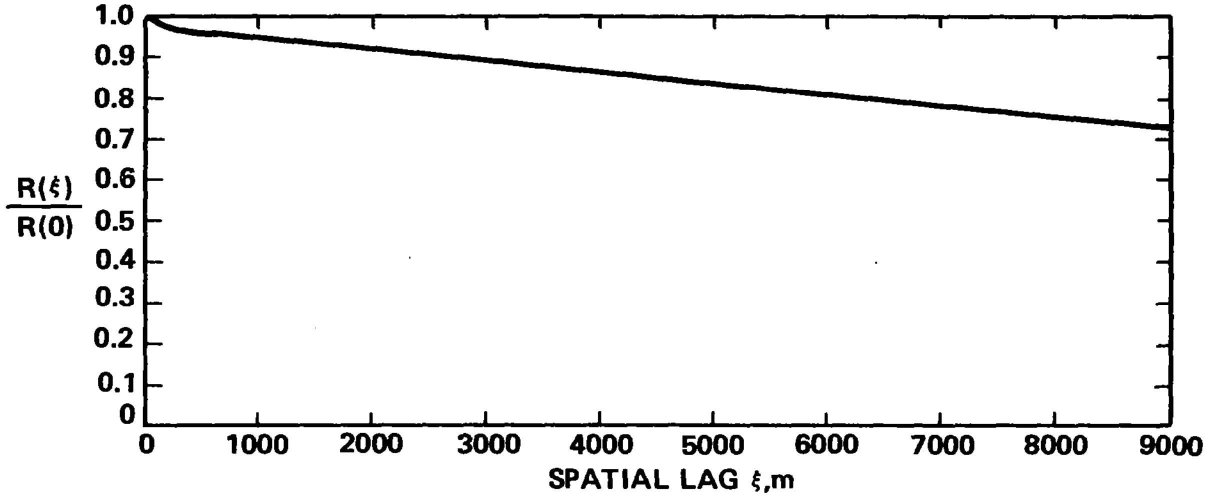


FIG. 13. AUTOCORRELATION FUNCTION OF LATERAL RECORD SHOW IN FIG. 10. [FROM MAT PROJECT, NASA LANGLEY RESEARCH CENTER.]

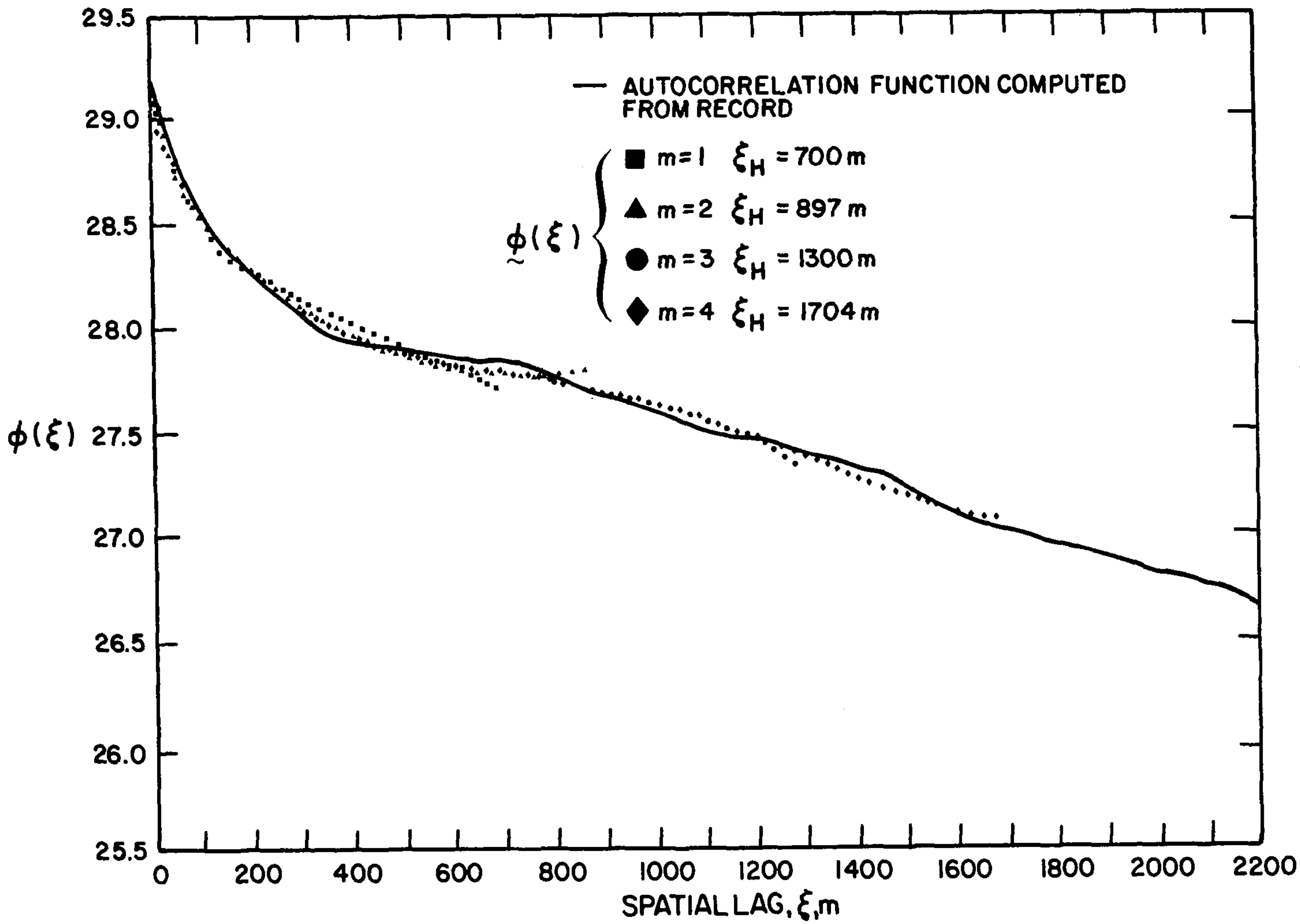


FIG. 14(a). COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM LATERAL RECORD SHOWN IN FIG. 10 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2). VALUES OF $\xi_H$ USED IN OBTAINING THE ABOVE RESULTS WERE TOO SMALL.

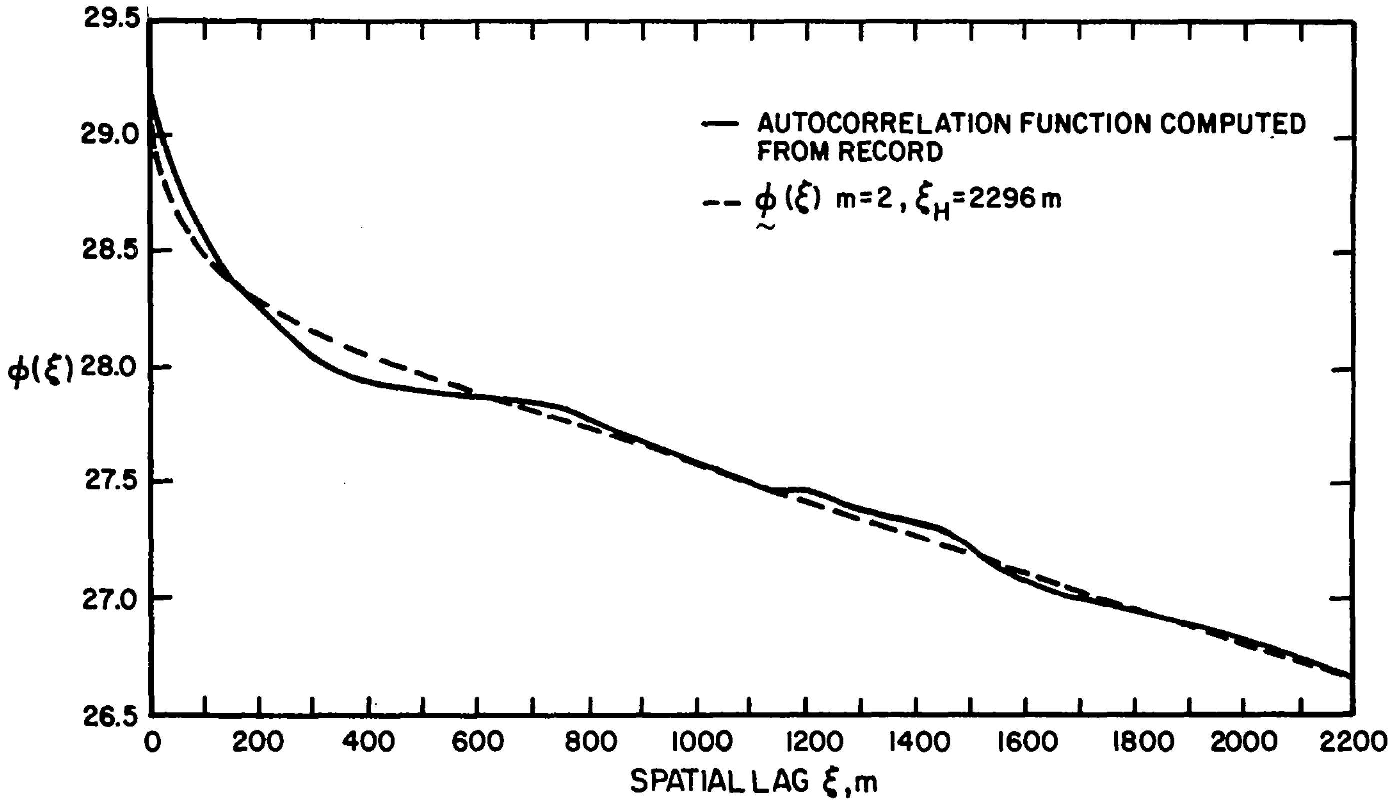


FIG. 14(b). COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM LATERAL RECORD SHOWN IN FIG. 10 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2).

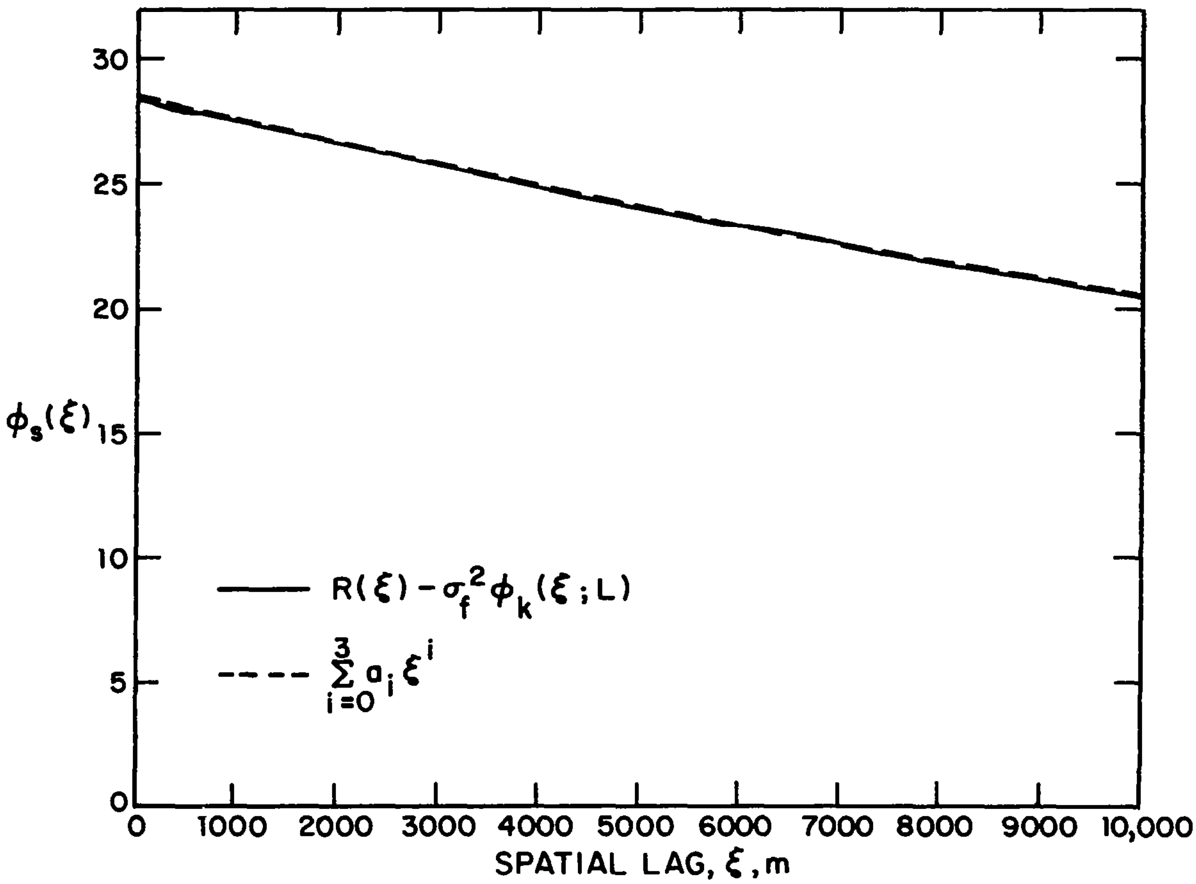


FIG. 15. COMPARISON OF AUTOCORRELATION FUNCTION $R(\xi)$ OF LATERAL RECORD SHOWN IN FIG. 10 MINUS AUTOCORRELATION FUNCTION $\sigma_f^2(\xi;L)$ OF von KARMAN COMPONENT AND INTEGRAL LEAST-SQUARES THIRD-DEGREE POLYNOMIAL APPROXIMATION.

TABLE 2. CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS FOR MOUNTAIN-WAVE LATERAL RECORD

| $\xi_H$ m | m | $\sigma_f^2$ $m^2/sec^2$ | L m | $\underset{\sim}{\phi}(0)$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|---|
| 502.5 | 0 | .944 | 217.2 | 28.97 | 28.03 | | | |
| 699.5 | 1 | .651 | 118.6 | 29.12 | 28.47 | $-.109\times10^{-2}$ | | |
| 699.5 | 2 | .467 | 57.1 | 29.35 | 28.89 | $-.376\times10^{-2}$ | $.340\times10^{-5}$ | |
| 896.6 | 2 | .495 | 69.9 | 29.26 | 28.76 | $-.272\times10^{-2}$ | $.187\times10^{-5}$ | |
| 1103.5 | 3 | .455 | 50.8 | 29.38 | 28.92 | $-.439\times10^{-2}$ | $.612\times10^{-5}$ | $-.305\ 10^{-8}$ |
| 1300.5 | 3 | .490 | 67.4 | 29.30 | 28.81 | $-.331\times10^{-2}$ | $.373\times10^{-5}$ | $-.159\ 10^{-8}$ |
| 1704.5 | 4 | .474 | 60.2 | 29.37 | 28.89 | $-.431\times10^{-2}$ | $.670\times10^{-5}$ | $-.485\ 10^{-8}$ |
| | | | | | | | | $a_4=.119\ 10^{-11}$ |
| 2295.6 | 2 | .684 | 128.9 | 29.07 | 28.39 | $-.821\times10^{-3}$ | $.171\times10^{-7}$ | |
| 2502.5 | 3 | .653 | 119.1 | 29.12 | 28.47 | $-.117\times10^{-2}$ | $.360\times10^{-6}$ | $-.933\times10^{-10}$ |

Exact value of R(0) is 29.22 $m^2/sec^2$

autocorrelation function that occurs near the origin $\xi = 0$. When this misleading behavior takes place, the value of $\sigma_f^2$ in Eq. (3.2) obtained by the least integral-squared fit is somewhat smaller than it should be; hence, the value obtained for the integral scale L also is too small as can be seen from Fig. 12. *This misleading behavior can largely be avoided by choosing $\xi_H$ as large as possible and m as small as possible consistent with achieving a "reasonable" representation of the autocorrelation function of the "slow" component $w_s(t)$ by the polynomial in Eq. (3.2).** Following this rule, we would never expect to have to choose m larger than 3. Mathematically, this misleading behavior is a consequence of the fact that the polynomial in the right-hand side of Eq. (3.2) is not orthogonal to the von Karman autocorrelation function which is the first term in the right-hand side.

In the present example, this problem is aggravated by the fact that the asymptotic slope of the empirical spectrum shown in Fig. 11 is somewhat steeper than the -5/3 asymptotic slope of the von Karman spectrum also shown in Fig. 11. (Thus, in this case also, one of the basic assumptions used in developing the method is not satisfied by the turbulence data.)

If we follow the above italicized rule, we see from the empirical autocorrelation function shown in Fig. 14(a) that for $\xi$ larger than about 750 meters, we cannot reasonably use m = 1 [which is a linear approximation to the autocorrelation function of the "slow" component $w_s(t)$]; however, if we let m increase to a value of 2 (quadratic approximation) then we can reasonably choose $\xi_H$ to be 2200 meters. Using $\xi_H = 2295.6$ meters and m = 2, we obtained the fit shown in Fig. 14(b) which yielded the values of L and $\sigma_f^2$ given by Eq. (4.1).

Upon first inspection of Fig. 11, the value of L = 128.9 m given by Eq. (4.1) appears too small. However, closer inspection shows several (about 4) weak resonances between $k = 7.5 \times 10^{-4} m^{-1}$ and $k = 1.43 \times 10^{-3} m^{-1}$, with the spectrum dropping off fairly abruptly beyond the latter value until it almost reaches the von Karman curve. Hence, we should expect several weak oscillations in the autocorrelation function with periods equal to the reciprocal values of the above frequencies — i.e., periods ranging from $0.70 \times 10^3 = 700$ m to $0.13 \times 10^4 = 1300$ m. Referring to Fig. 14(b), we see that our least-squares fit underestimates the empirical autocorrelation function at the origin, overestimates at

*Trade-off criteria between choices of $\xi_H$ and m are discussed in detail in Appendix G of Ref. 1.

$\xi$ = 350 m and again underestimates at $\xi$ = 700 m, which according to the above is one period of the oscillation with the smallest period. At $\xi$ = 750 m we see another peak in the empirical autocorrelation function, which is the period of the resonance located in the spectrum slightly to the left of the resonance at $k = 1.43 \times 10^{-3} m^{-1}$. Looking back in the region of $\xi \approx$ 350 to 400 m in Fig. 14(b), we see that these two oscillations have added in phase in that region to produce a relatively large discrepancy between the empirical autocorrelation function and our least squares fit. Finally, in the region of Fig. 14(b) between $\xi$ = 1200 m and $\xi$ = 1450 m we observe all 4 of the above oscillations adding almost in phase in this region — as expected from the appearance of the spectrum. Hence, some destructive interference must have occurred in the region near $\xi$ = 750 m which further explains why the discrepancy between the empirical and least-squares curves is less near $\xi$ = 750 m than near $\xi$ = 350 m. Hence, the main discrepancies between the empirical and least-squares fit of the autocorrelation function shown in Fig. 14(b) can be explained by the approximately 4 weak resonances between $k = 7.5 \times 10^{-4} m^{-1}$ and $1.43 \times 10^{-3} m^{-1}$ in Fig. 11. If these resonances were removed from the empirical spectrum shown in Fig. 11, the knee of the von Karman curve would appear to be in about the right position. Hence, the values of L and $\sigma^2$ given by Eq. (4.1) — which characterize the von Karman component of the turbulence — appear to be about right.

Finally, we note that the presence of spectral peaks or "resonances" such as those discussed above will tend to bias the values of L and $\sigma^2$ obtained using the maximum likelihood method developed in Sec. 3 of Ref. 1 since this method assumes that the turbulence obeys the von Karman spectral form. In contrast, such spectral peaks produce oscillations in the autocorrelation function, and the constrained least-squares estimation procedure developed in Sec. 4 of Ref. 1 tends to average out — i.e., ignore — these oscillations. The constrained least-squares procedure therefore should produce better estimates of the integral scale and intensity of the von Karman component of turbulence records in these situations.

## CONSTRAINED LEAST-SQUARES AND MAXIMUM LIKELIHOOD ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS OF VERTICAL RECORD FROM FLIGHT 30 RUN 8 (MOUNTAIN-WAVE CONDITIONS)

Figures 16 to 20 illustrate, respectively, the same quantities for the vertical record shown in Fig. 10 that Figs. 11 to 15 displayed for the lateral record in Fig. 10. Similarly, Table 3 displays for the vertical record in Fig. 10 quantities comparable to the quantities displayed in Table 2 for the lateral record in Fig. 10. The compulations for the material in Table 3 and Figs. 16 to 20 were carried out using the same methods as used in the cases of Tables 1 and 2 and Figs. 5 to 15.

The value of M used in computing the empirical spectrum in Fig. 16 was 10,089 m which corresponds to 1024 temporal sample points. This is the same value of M as used in computation of the spectrum in Fig. 11 for the lateral component.

The von Karman transverse spectrum plotted in Fig. 16 was computed using the parameter values

$$L = 68.4 \text{ m}, \quad \sigma^2 = 0.470 \text{ m}^2/\text{sec}^2 \qquad (5.1)$$

which correspond to the case $\xi_H = 1202$ meters and a second degree polynomial (m=2) in the autocorrelation function representation of Eq. (3.2).

In computing the constraint relationship between $\sigma_f^2$ and L displayed in Fig. 17, the lower and upper wavenumbers used were the same values as those used in the previous two examples – namely, $k_\ell = 10^{-3}\text{m}^{-1}$ and $k_u = 4 \times 10^{-2}\text{m}^{-1}$.

*Discussion.* The vertical record displayed in Fig. 10 is much better behaved than either of the previous two records studied, and this improved behavior is reflected in our results. The record is better behaved for four reasons: (1) the asymptotic slope of the empirical spectrum shown in Fig. 16 agrees very well with the asymptotic slope of -5/3 of the von Karman spectrum; (2) the empirical spectrum in Fig. 16 has a better developed "knee" than the empirical spectra shown in Figs. 5 and 11; (3) the fractional energy in the "resonances" in the spectrum in Fig. 16 is less than in the previous two cases as may be seen by comparing the

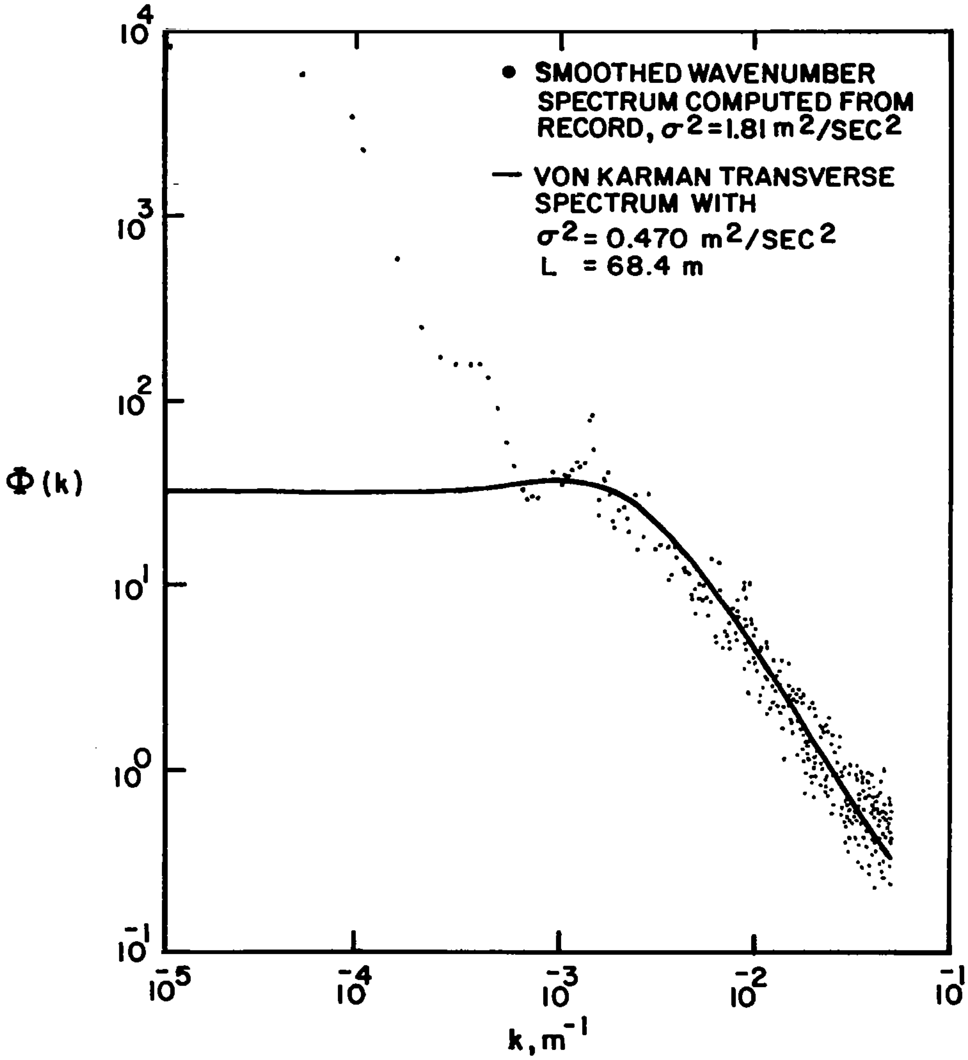


FIG. 16. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM VERTICAL RECORD SHOWN IN FIG. 10 AND von KARMAN TRANSVERSE SPECTRUM OBTAINED BY CONSTRAINED LEAST-SQUARES FIT TO THE (EMPIRICAL) AUTOCORRELATION FUNCTION. Von KARMAN SPECTRUM CHARACTERIZES "FAST" TURBULENCE COMPONENT ONLY.

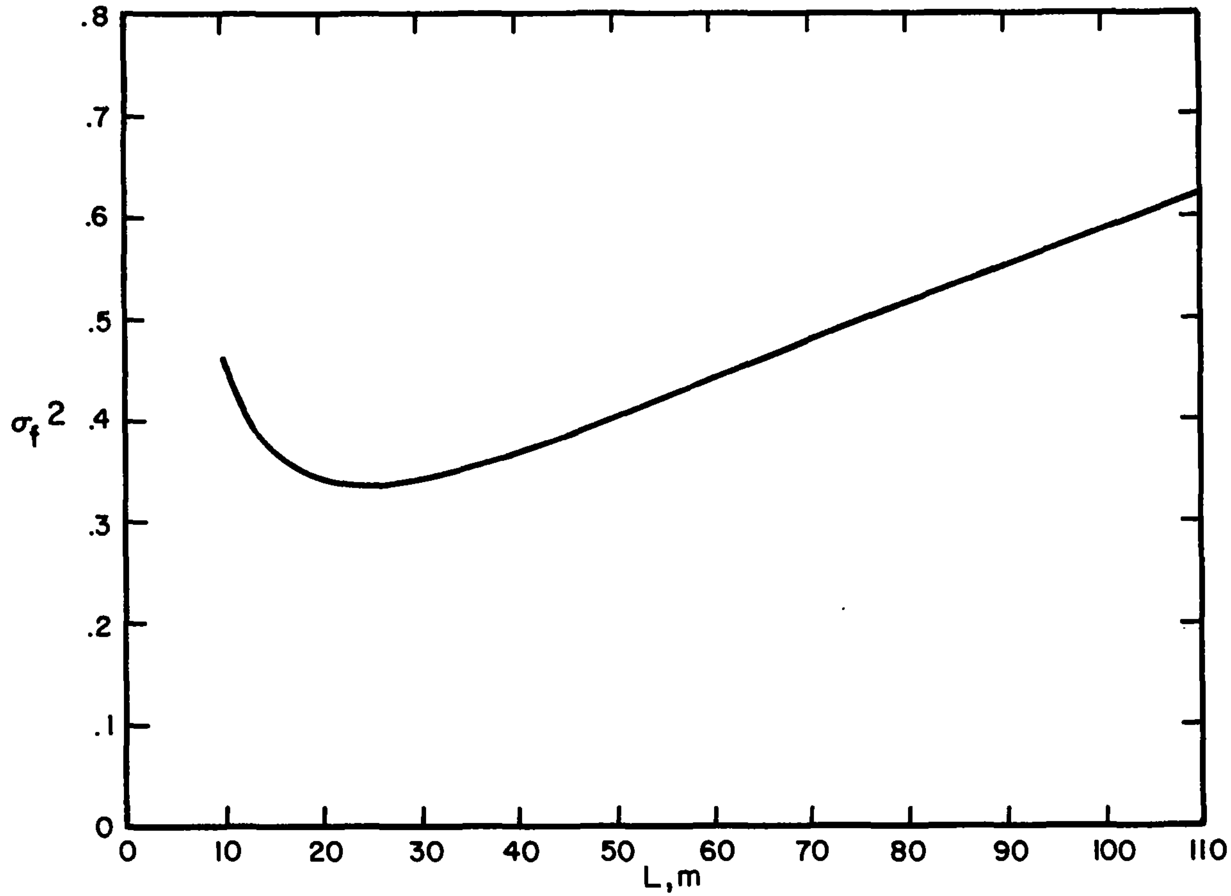


FIG. 17. CONSTRAINT BETWEEN $\sigma_f^2$ AND L FOR CONSTRAINED LEAST-SQUARES ESTIMATION PROCEDURE APPLIED TO VERTICAL RECORD SHOWN IN FIG. 10.

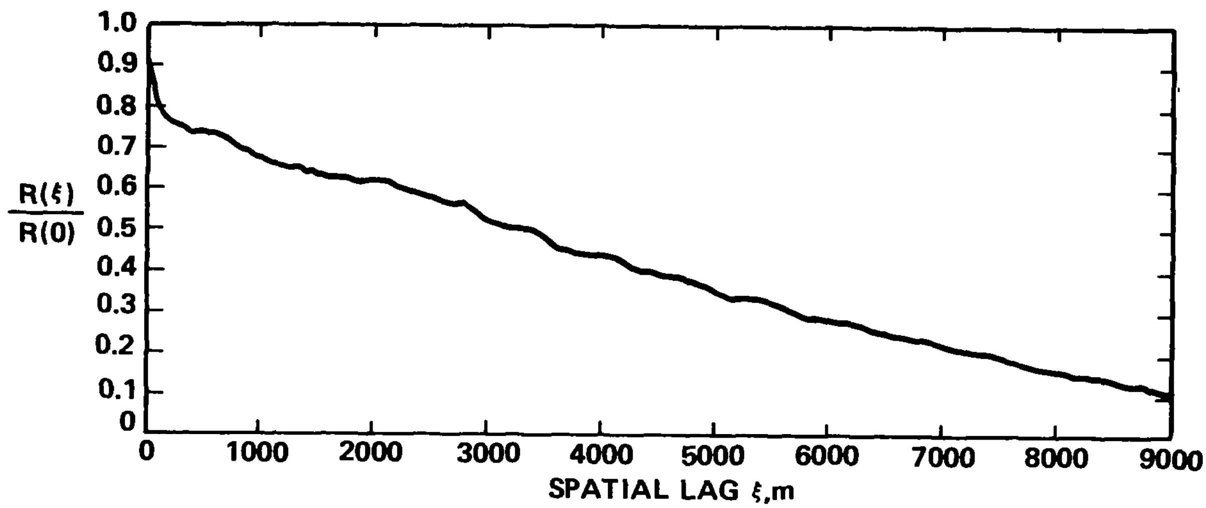


FIG. 18. AUTOCORRELATION FUNCTION OF VERTICAL RECORD SHOWN IN FIG. 10. [FROM MAT PROJECT, NASA LANGLEY RESEARCH CENTER.]

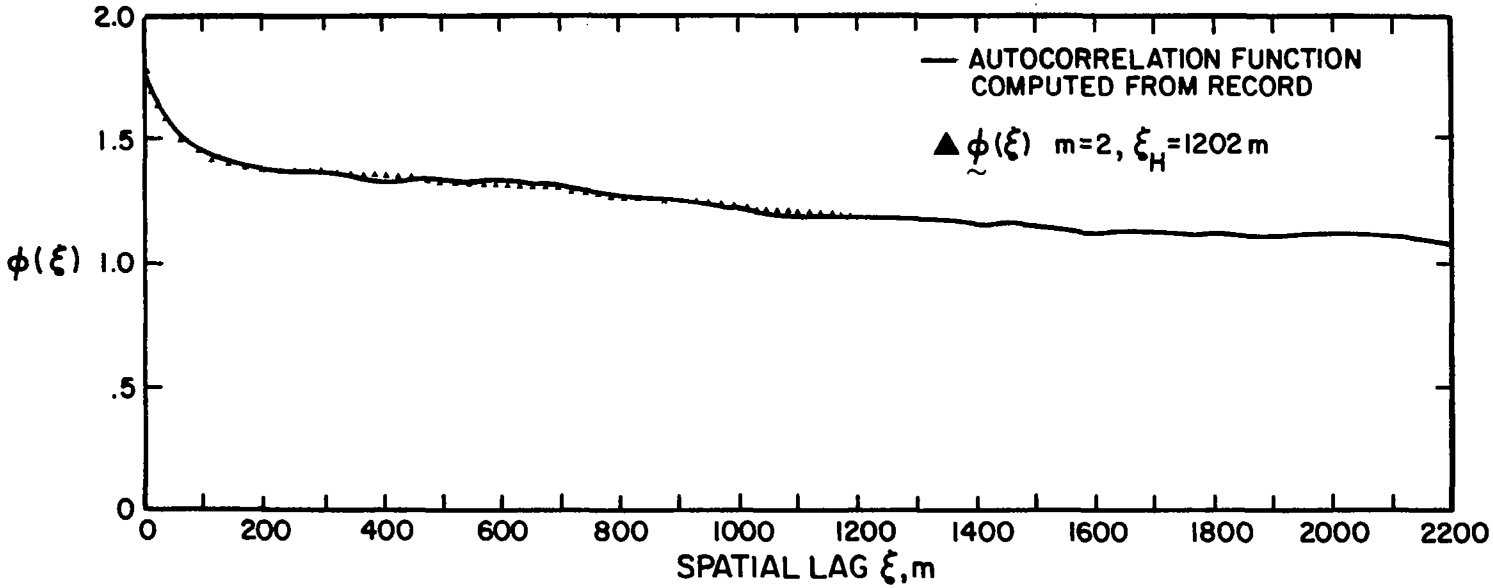


FIG. 19. COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM VERTICAL RECORD SHOWN IN FIG. 10 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2).

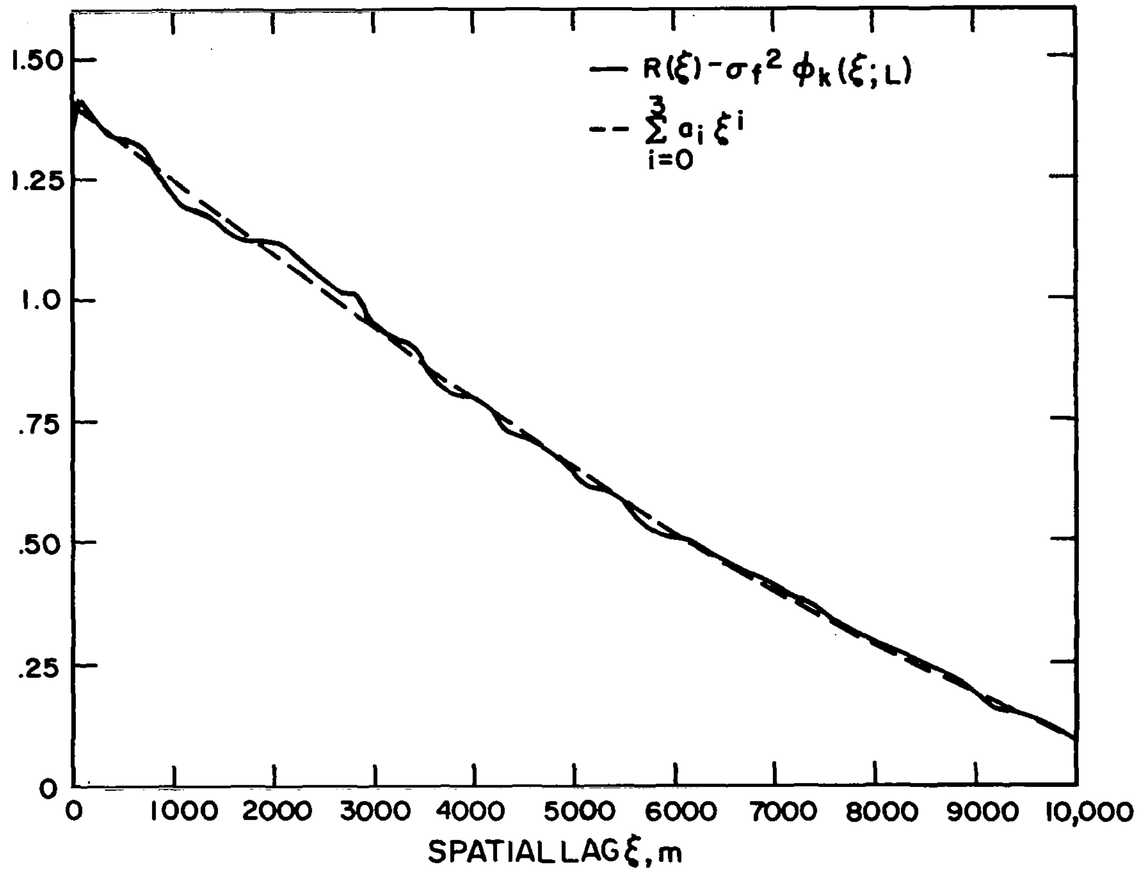


FIG. 20. COMPARISON OF AUTOCORRELATION FUNCTION $R(\xi)$ OF VERTICAL RECORD SHOWN IN FIG. 10 MINUS AUTOCORRELATION FUNCTION $\sigma_f^2\phi_K(\xi;L)$ of von KARMAN COMPONENT AND INTEGRAL LEAST-SQUARES THIRD-DEGREE POLYNOMIAL APPROXIMATION.

TABLE 3. CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS FOR MOUNTAIN-WAVE VERTICAL RECORD

| $\xi_H$ m | m | $\sigma_f^2$ $m^2/sec^2$ | L m | $\underset{\sim}{\phi}(0)$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|---|
| 295.6 | 0 | .486 | 72.7 | 1.872 | 1.39 | | | |
| 798.1 | 1 | .462 | 66.4 | 1.877 | 1.42 | $-.157\times10^{-3}$ | | |
| 798.1 | 2 | .423 | 56.0 | 1.877 | 1.45 | $-.350\times10^{-3}$ | $.202\times10^{-5}$ | |
| 995.1 | 1 | .446 | 62.2 | 1.874 | 1.43 | $-.192\times10^{-3}$ | | |
| 995.1 | 2 | .474 | 69.8 | 1.876 | 1.40 | $-.745\times10^{-4}$ | $-.101\times10^{-6}$ | |
| 1202.0 | 1 | .437 | 59.8 | 1.874 | 1.44 | $-.210\times10^{-3}$ | | |
| 1202.0 | 2 | .470 | 68.4 | 1.876 | 1.41 | $-.949\times10^{-4}$ | $-.855\times10^{-7}$ | |
| 1399.0 | 1 | .443 | 61.1 | 1.875 | 1.43 | $-.201\times10^{-3}$ | | |
| 1399.0 | 2 | .443 | 61.4 | 1.875 | 1.43 | $-.198\times10^{-3}$ | $-.194\times10^{-5}$ | |
| 2502.5 | 1 | .488 | 73.2 | 1.881 | 1.39 | $-.148\times10^{-3}$ | | |
| 2502.5 | 2 | .429 | 57.6 | 1.874 | 1.45 | $-.255\times10^{-3}$ | $.409\times10^{-7}$ | |
| 2502.5 | 3 | .422 | 55.9 | 1.874 | 1.45 | $-.280\times10^{-3}$ | $.632\times10^{-7}$ | $-.565\times10^{-11}$ |
| 2502.5 | 4 | .492 | 74.4 | 1.870 | 1.38 | $.110\times10^{-3}$ | $-.522\times10^{-6}$ | $.323\times10^{-9}$ |
| | | | | | | | | $a_4=-.616\times10^{-13}$ |

Exact value of R(0) is 1.812 $m^2/sec^2$.

"oscillations" in Fig. 19 with those in Figs. 8 and 14(b), and finally (4) the fraction of the "power" of the record in the von Karman component is larger than in the previous two cases as is most easily seen by comparing Fig. 18 with Figs. 7 and 13.

In view of the above considerations, it is not surprising to see relatively less spread in the values of L shown in Table 3 than found in either of the previous two cases. Examination of the empirical autocorrelation function in Fig. 19 shows that the largest value of $\xi_H$ for which we can expect a 2nd degree polynomial to represent well the autocorrelation function component from the "slow" turbulence component $w_s(t)$ is about $\xi_H = 1200$ m. Hence, the values of L and $\sigma_f^2$ from Table 3 that we choose to use for our von Karman curve in Fig. 16 were those given by Eq. (5.1) which were obtained using $\xi_H = 1202$ meters and m = 2. Values of L and $\sigma_f^2$ from the case $\xi_H = 798$ meters and m = 1 also would have provided reasonable choice (L = 66.4 meters and $\sigma_f^2 = 0.462\ m^2/sec^2$ as would have been the values obtained from the case $\xi_H = 995$ meters and m = 2 (L = 69.8 meters and $\sigma_f^2 = 0.474\ m^2/sec^2$). Very little discrepancy is found in the values of L from these three choices.

*Maximum Likelihood Estimation of Integral Scale and Intensity of von Karman Component.* The knee in the empirical spectrum shown in Fig. 16 is sufficiently well developed to apply the maximum likelihood method described in Sec. 3 of Ref. 1 to estimate the values of L and $\sigma^2$ of the von Karman component of the record. The likelihood equations were solved using spectrum sample points in the range from $k = 6.5 \times 10^{-4} m^{-1}$ to $k = 3 \times 10^{-2} m^{-1}$, which the reader may verify from Fig. 16 as the range dominated by the von Karman part of the spectrum. Values of L and $\sigma^2$ obtained using the maximum likelihood method were

$$L = 70.0\ m,\ \sigma^2 = 0.456\ m^2/sec^2. \qquad (5.2)$$

Very little difference is observed among the various values of the integral scale cited above.

## WAVENUMBER SPECTRAL DENSITY OF INSTANTANEOUS VARIANCE OF "FAST" COMPONENT OF VERTICAL RECORD FROM FLIGHT 30 RUN 8 (MOUNTAIN-WAVE CONDITIONS)

In Sec. 6.2 of Ref. 5, a method was developed for estimating the wavenumber spectrum of the instantaneous variance $\sigma_f^2(t)$ of the fast component $w_f(t)$ of a turbulence record, where time is translated to the spatial variable x using the relationship x = Vt where V is the aircraft speed. The procedure used to compute the wavenumber spectrum of $\sigma_f^2(t)$ is as follows:

(1) High-pass filter the record to eliminate the low frequency component $w_s(t)$. In our computations with the vertical record from Flight 30 Run 8, we used first and second order digital Butterworth filters as described in Chapter 12 and Appendix C of Ref. 6.

(2) Find the wavenumber spectral density and autocorrelation function of the high-pass filtered record using the method described in Appendix B of Ref. 2.

(3) Square the high-pass filtered record and find the wavenumber spectral density and autocorrelation function of the squared high-pass filtered record using the method described in Appendix B of Ref. 2.

(4) Compute the wavenumber spectrum of $\sigma_f^2(t)$ using the formula:

$$\Phi_{\sigma_f^2}(k) = \{E[\sigma_f^2]\}^2 \left\{ \delta(k) + \int_{-M}^{M} p_0(\xi) \left( \frac{R_{w_h^2}(\xi) - [R_{w_h}(0)]^2 - 2[R_{w_h}(\xi)]^2}{[R_{w_h}(0)]^2 + 2[R_{w_h}(\xi)]^2} \right) e^{-i2\pi k\xi} d\xi \right\}, \qquad (6.1)$$

where $p_0(\xi)$ is the Papoulis window function [7]

$$p_0(\xi) = \begin{cases} \frac{1}{\pi} \left|\sin \frac{\pi\xi}{M}\right| + \left(1 - \frac{|\xi|}{M}\right) \cos \frac{\pi\xi}{M} , & |\xi| \leq M \\ 0 & , |\xi| > M - \end{cases} \quad (6.2)$$

see Appendix B of Ref. 2. Equation (6.1) is essentially the same as Eq. (6.49) of Ref. 5 except for translation of time t to distance x using x = Vt where V is aircraft speed, and inclusion of the Papoulis window $p_0(\xi)$ to smooth the fluctuations normally associated with power spectral estimates of empirical waveforms. The aircraft speed for Flight 30 Run 8 was 197.05 m/sec and the value of M used was 10,089 m which corresponded to 1024 sample points.

The wavenumber spectra obtained by the above procedure are shown on log-log coordinates in Fig. 21 for three different high-pass filter cutoff frequencies (wavenumbers), where NS designates the number of filter sections used [6], and $k_c$ designates the (3 dB) cutoff frequency (wavenumber) of the filters.

Except for statistical fluctuations associated with the finite length of the record, the wavenumber spectra $\Phi_{\sigma_f^2}(k)$ should be independent of $k_c$ if our basic turbulence model

$$w_f(t) = \sigma_f^2(t)\ z(t) \quad (6.3)$$

is valid. From Fig. 21, we see that good agreement among the three spectra is obtained for values of $k < 10^{-3}m^{-1}$.

For values of $k > 10^{-3}m^{-1}$, the three spectra are shown vertically separated to avoid confusion among the three spectra. The agreement in the wavenumber range $k > 10^{-3}m^{-1}$ is poorer. However, in this wavenumber region, the locally stationary assumption leading to Eq. (6.34) of Ref. 5 is not valid — that is, a basic assumption in the derivation of Eq. (6.1) above is that the spectral content of $\sigma_f^2$ is negligible in the region $k > 10^{-3}m^{-1}$. Hence, the spectra in Fig. 21 in the region $k > 10^{-3}m^{-1}$ cannot be believed.

The same three spectra of Fig. 21 are plotted on linear coordinates in Fig. 22 for the wavenumber region $k < 10^{-3}m^{-1}$ where the spectra are believed to be valid.

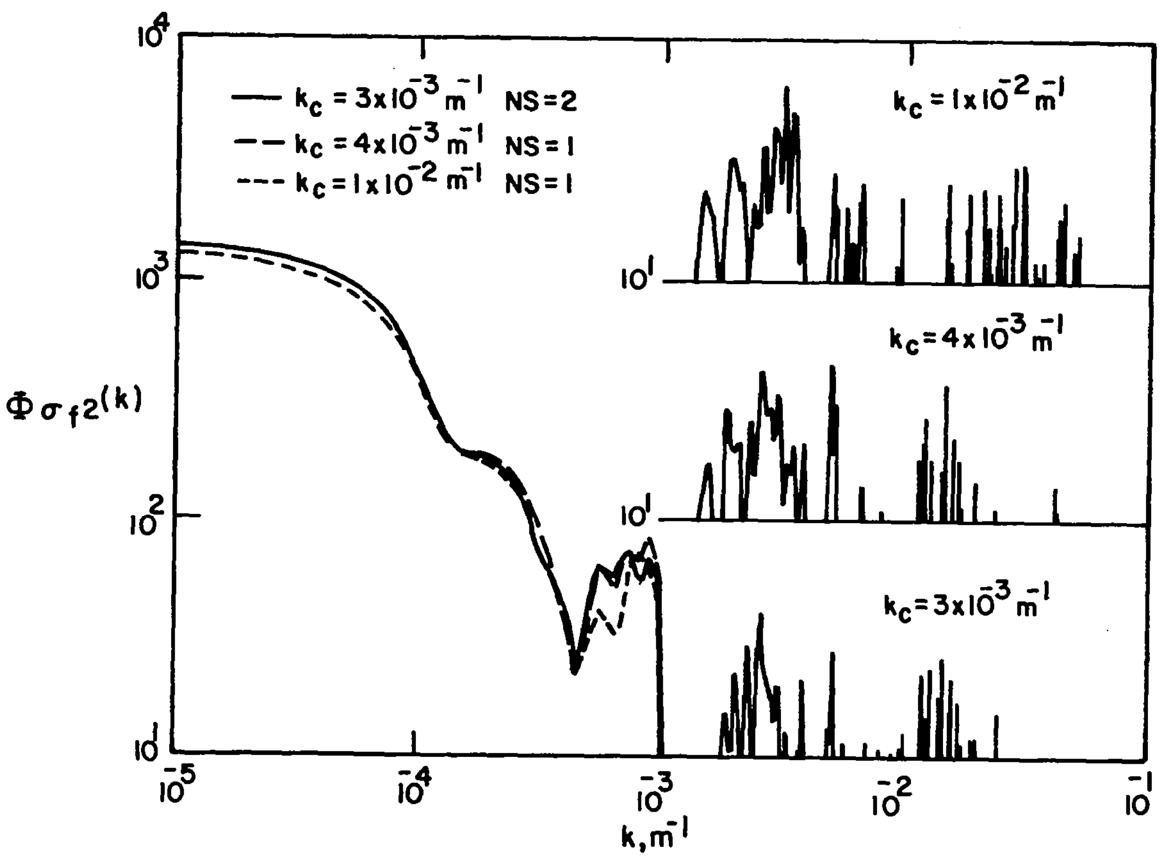


FIG. 21. WAVENUMBER SPECTRA OF INSTANTANEOUS VARIANCE $\sigma_f^2(t)$ OF "FAST" COMPONENT $w_f(t)$ OF VERTICAL RECORD SHOWN IN FIG. 10. THE THREE SPECTRA SHOWN WERE COMPUTED FROM THE SAME RECORD USING THREE DIFFERENT HIGH-PASS CUTOFF WAVENUMBERS.

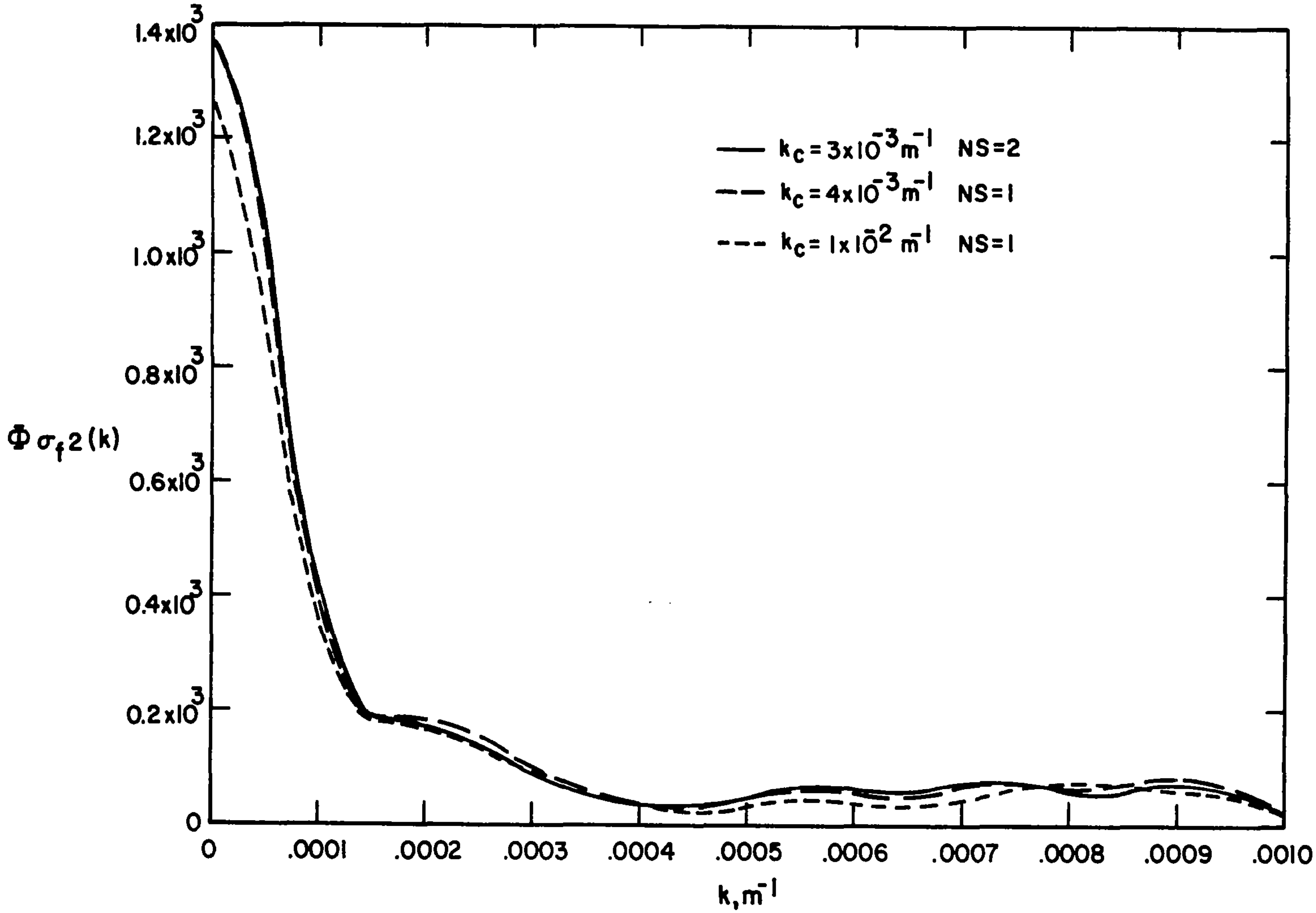


FIG. 22. WAVENUMBER SPECTRA OF INSTANTANEOUS VARIANCE $\sigma_f^2(t)$ OF "FAST" COMPONENT $w_f(t)$ OF VERTICAL RECORD SHOWN IN FIG. 10. THE THREE SPECTRA SHOWN WERE COMPUTED FROM THE SAME RECORD USING THREE DIFFERENT HIGH-PASS CUTOFF WAVENUMBERS.

The three high-pass filter cutoff wavenumbers used in obtaining the spectra of Figs. 21 and 22 may be compared with the spectrum of the vertical record w(t) for Flight 30 Run 8 shown in Fig. 16.

# PROBABILITY DENSITY FUNCTIONS OF INSTANTANEOUS VARIANCE $\sigma_f^2(t)$ AND SLOW TURBULENCE COMPONENT $w_s(t)$ OF VERTICAL RECORD FROM FLIGHT 30 RUN 8 (MOUNTAIN-WAVE CONDITIONS)

*Probability density of $\sigma_f^2(t)$.* In Sec. 6.3 of Ref. 5, a method is developed for estimating the probability density of the instantaneous variance $\sigma_f^2(t)$ of the "fast component" $w_f(t)$ of our turbulence model. Using that method, the probability density of $\sigma_f^2(t)$ for the vertical record shown in Fig. 10 was estimated using moments of $\sigma_f^2(t)$ through orders 3, 4, 5, and 6. A high-pass two stage digital Butterworth filter [6] with cut-off wavenumber $k_c = 3 \times 10^{-3} m^{-1}$ was used in this procedure. The resulting probability densities obtained using Eq. (6.77) of Ref. 5 are plotted in Fig. 23 for the cases where moments through the third and sixth were used. The cases using moments through the fourth and fifth fell between the two curves shown in Fig. 23. All four approximations are sufficiently close to one another so that little practical significance can be ascribed to their differences.

Each of the four computed density functions had an integrable singularity at the origin. Thus, neither of the two curves shown in Fig. 23 has a finite value at $\sigma_f^2 = 0$. The reason for the integrable singularity at the origin is apparent when we examine the vertical record shown in Fig. 10. In the region between about 9 min 0 sec and 9 min 45 sec, the fast turbulence component $w_f(t)$ is virtually absent; hence, $\sigma_f^2(t)$ is very nearly zero during this time interval. This behavior is undoubtedly responsible for the singularity in $p(\sigma_f^2)$ at $\sigma_f^2 = 0$.

The probability density function of $\sigma_f^2(t)$ is useful for calculation of aircraft-response mean exceedance rates. See Eq. (4.8) of Ref. 5.

*Probability density of $w_s(t)$.* In Sec. 6.4 of Ref. 5, a method is developed for estimating the probability density of the "slow" turbulence component $w_s(t)$ from a turbulence time history. Using that method, the probability density of $w_s(t)$ for the vertical record shown in Fig. 10 was estimated using moments of $w_s(t)$ through the fourth. This technique also requires the moments through the fourth of the fast turbulence component $w_f(t)$ which were obtained using a high-pass two stage digital Butterworth filter [6] with cut-off wavenumber $k_c = 3 \times 10^{-3} m^{-1}$ as before. The probability

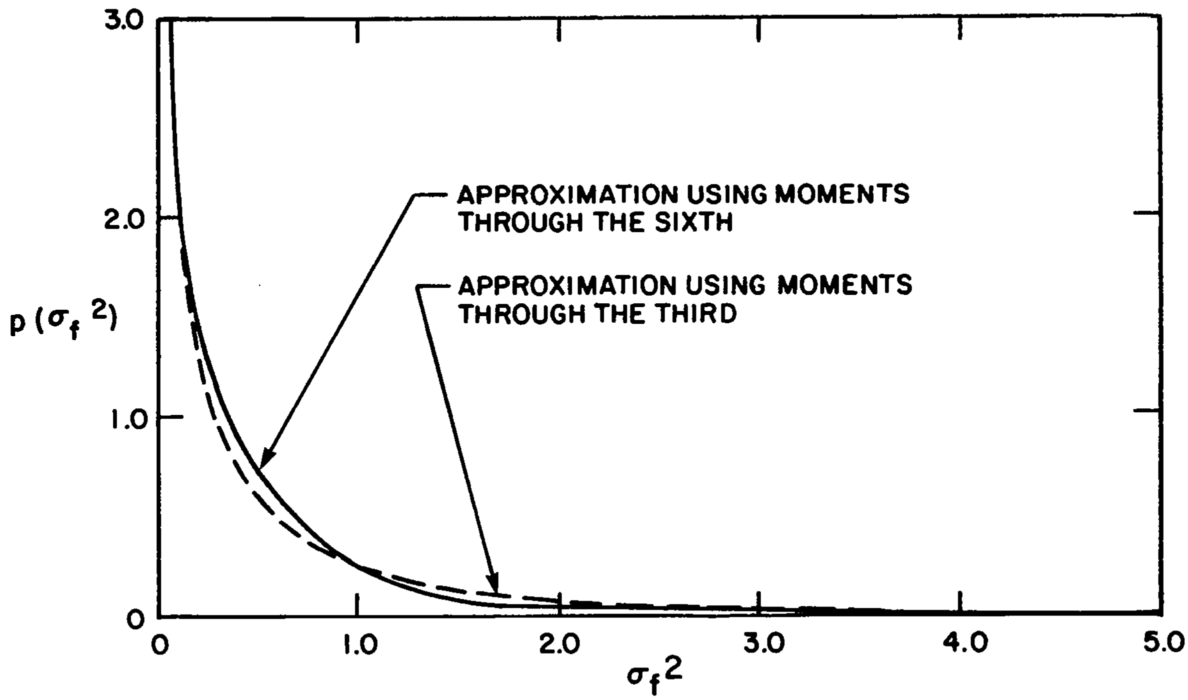


FIG. 23. PROBABILITY DENSITY FUNCTIONS OF INSTANTANEOUS VARIANCE $\sigma_f^2(t)$ OF THE "FAST" COMPONENT $w_f(t)$ OF VERTICAL RECORD SHOWN IN FIG. 10.

density of $w_s(t)$ is actually computed using Eq. (6.93) of Ref. 5, and is displayed in Fig. 24 for the vertical record shown in Fig. 10. Also shown there is the Gaussian density function with the same mean and variance.

The main purpose in estimating the probability density of $w_s(t)$ is to make a first check on the assumption that $w_s(t)$ is a stationary Gaussian process. Considering the obviously small number of statistical degrees-of-freedom in the component $w_s(t)$ of the vertical record shown in Fig. 10, we conclude from Fig. 24 that the deviation of the probability density of $w_s(t)$ from the Gaussian curve is not statistically significant.

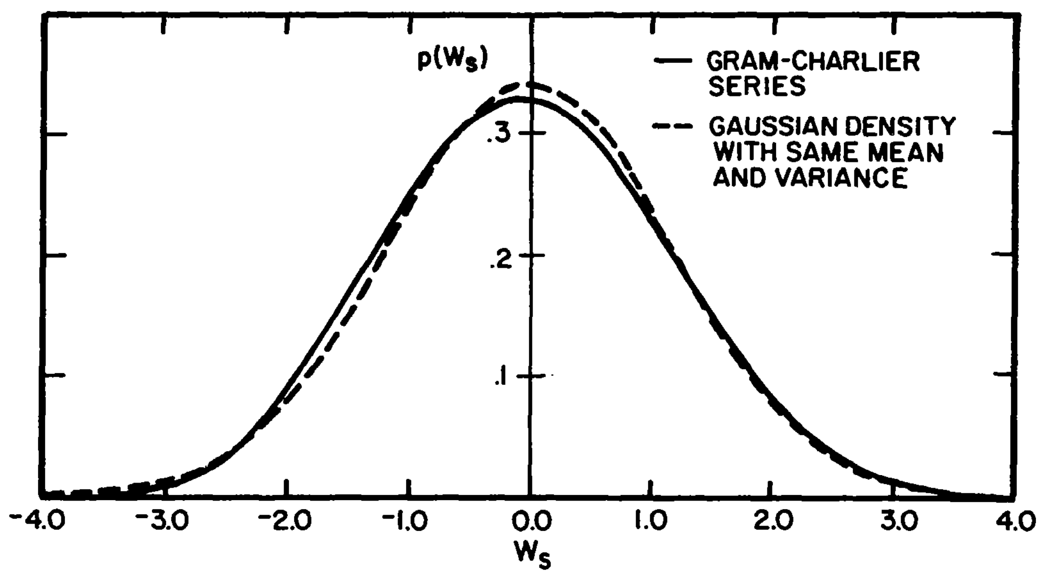


FIG. 24. ESTIMATE OF THE PROBABILITY DENSITY OF THE "SLOW" COMPONENT $w_S(t)$ OF THE VERTICAL RECORD SHOWN IN FIG. 10 USING THE GRAM-CHARLIER EXPANSION AND MOMENTS THROUGH THE FOURTH.

# CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS OF LONGITUDINAL RECORD FROM FLIGHT 30 RUN 8 (MOUNTAIN-WAVE CONDITIONS) AND VERTICAL AND LONGITUDINAL RECORDS FROM FLIGHT 32 RUN 4 (WIND-SHEAR CONDITIONS)

Here, we discuss collectively the results from three records, the longitudinal record from Flight 30 Run 8 (mountain-wave conditions) and the vertical and longitudinal records from Flight 32 Run 4 (wind-shear conditions). This completes all three records from Flight 30 Run 8 and Flight 32 Run 4.

The power spectral density computed from the longitudinal record of Fig. 10 is shown by the solid dots in Fig. 25, and the power spectral densities computed from the vertical and longitudinal records of Fig. 4 are shown by the solid dots in Figs. 26 and 27, respectively. All three of these spectra were computed by the method described in Appendix B of Ref. 2. The value of M used in the computation involving the mountain-wave record was 10,089 m as before, whereas the value of M used in the compulations involving the wind-shear records was 9613.3 m. These values of M correspond in each case to 1024 sample points. Before computing these power spectra, the mean values of the records were computed and removed.

Von Karman longitudinal spectra are also plotted in Figs. 25 and 27 using solid lines, and the von Karman transverse spectrum of Eq. (2.1) is shown by the solid line in Fig. 26. The von Karman longitudinal power spectrum is described by

$$\Phi_{KL}(k) = 2\sigma^2 L \frac{1}{[1+70.78L^2k^2]^{5/6}} \; . \qquad (8.1)$$

The values of $\sigma^2$ and L for the von Karman spectra are given in each of the three figures. These values of $\sigma^2$ and L were arrived at using the constrained least-squares estimation method described in Sec. 4 of Ref. 1, which postulates that the autocorrelation functions are of the form of Eq. (3.2), where $\sigma_f^2\phi_K(\xi;L)$ is the appropriate form (transverse or longitudinal) of the von Karman autocorrelation function. In obtaining the von Karman spectra shown in Figs. 25 and 27, we used a value of m = 3, whereas m = 4 was used in

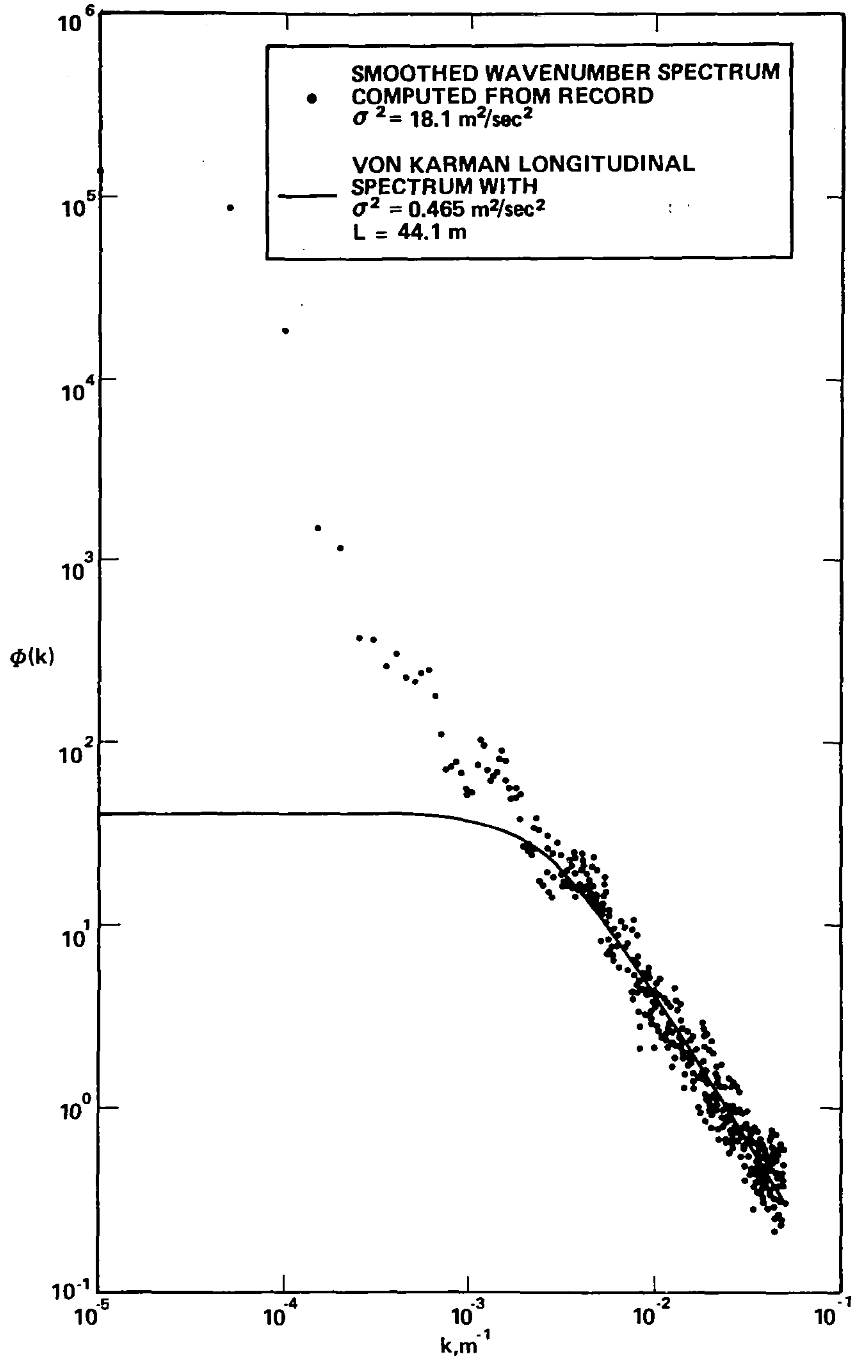


FIG. 25. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM LONGITUDINAL RECORD SHOWN IN FIG. 10 AND VON KARMAN LONGITUDINAL SPECTRUM OBTAINED BY CONSTRAINED LEAST-SQUARES FIT TO THE (EMPIRICAL) AUTOCORRELATION FUNCTION. VON KARMAN SPECTRUM CHARACTERIZES "FAST" TURBULENCE COMPONENT ONLY.

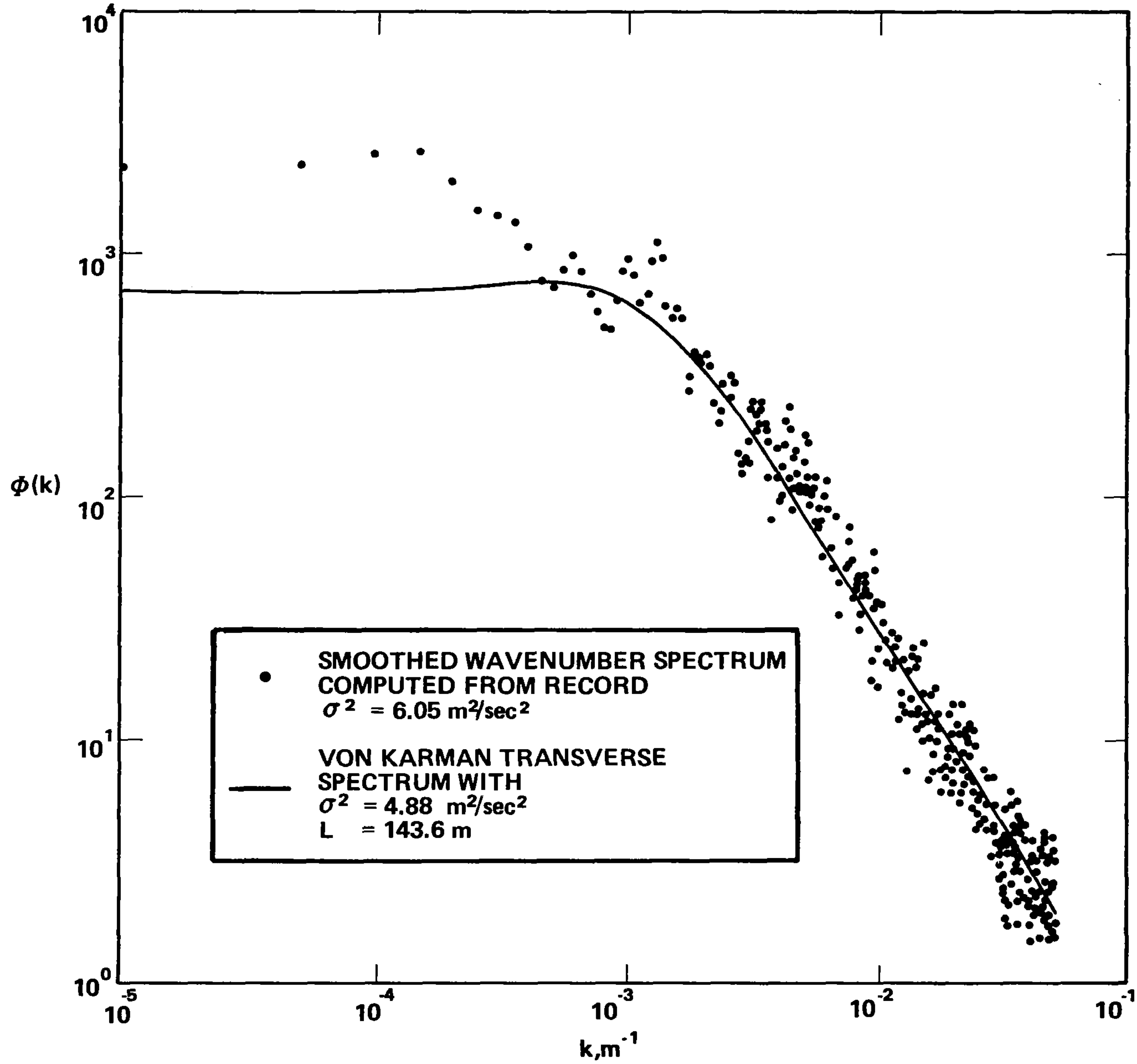


FIG. 26. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM VERTICAL RECORD SHOWN IN FIG. 4 AND VON KARMAN TRANSVERSE SPECTRUM OBTAINED BY CONSTRAINED LEAST-SQUARES FIT TO THE (EMPIRICAL) AUTOCORRELATION FUNCTION. VON KARMAN SPECTRUM CHARACTERIZES "FAST" TURBULENCE COMPONENT ONLY.

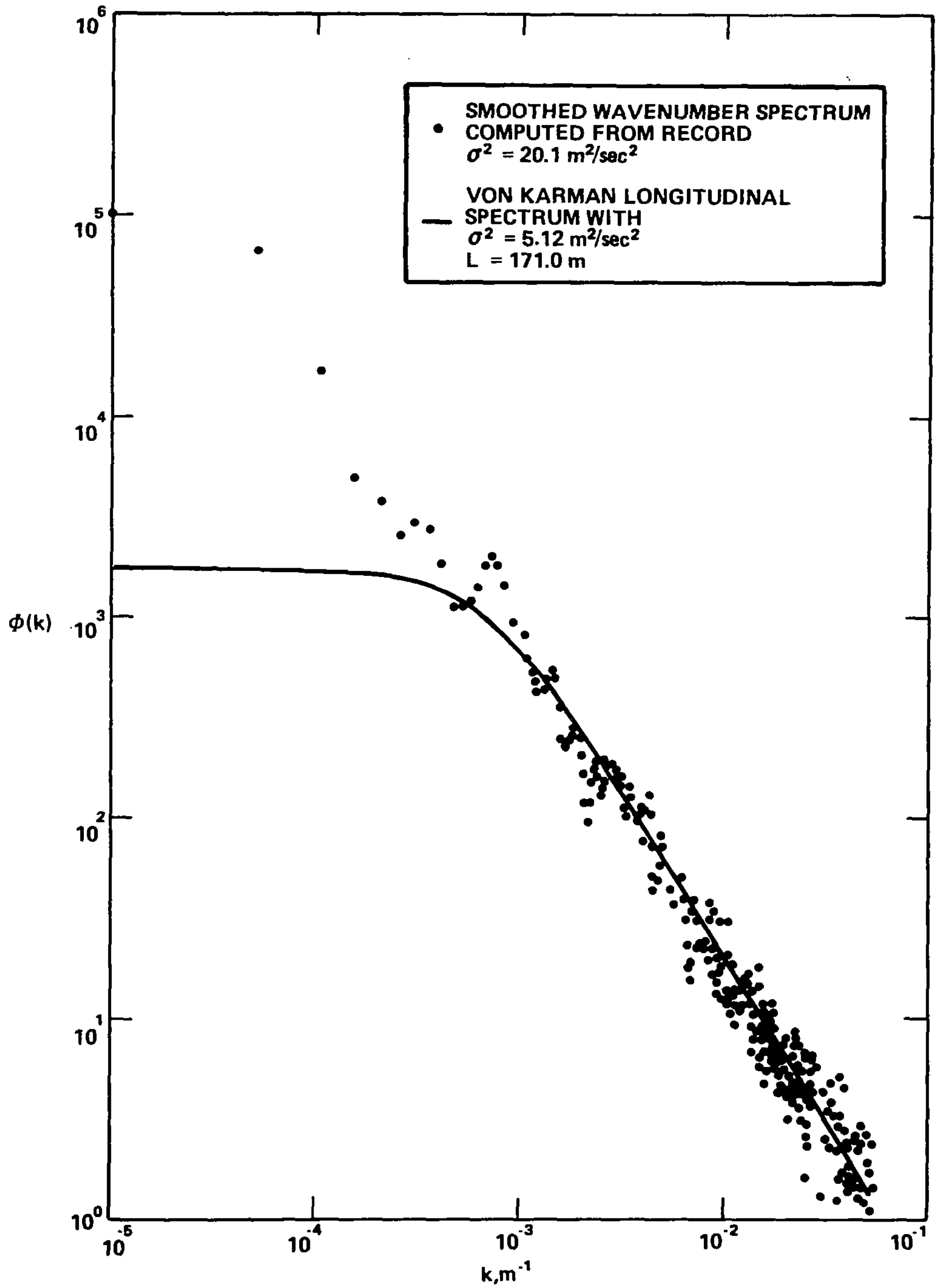


FIG. 27. COMPARISON OF SMOOTHED WAVENUMBER SPECTRUM COMPUTED FROM LONGITUDINAL RECORD SHOWN IN FIG. 4 AND VON KARMAN LONGITUDINAL SPECTRUM OBTAINED BY CONSTRAINED LEAST-SQUARES FIT TO THE (EMPIRICAL) AUTOCORRELATION FUNCTION. VON KARMAN SPECTRUM CHARACTERIZES "FAST" TURBULENCE COMPONENT ONLY.

obtaining the solid curve in Fig. 26. The values of $\xi_H$ used in Figs. 25 through 27 were, respectively, $\xi_H = 1300.5$ m, $\xi_H = 7501.0$ m, and $\xi_H = 2497.0$ m. Values of $k_\ell$ and $k_u$ used in every case were $k_\ell = 10^{-3} m^{-1}$ and $k_u = 4 \times 10^{-2} m^{-1}$. The equation of constraint used in the least-squares method for the mountain-wave longitudinal component is shown in Fig. 28. The equations of constraint for the other two components were not plotted.

Figures 29 through 31 show the autocorrelation functions of the three records under consideration. The autocorrelation function shown in Fig. 29 is dominated by the "slow" component, whereas the "slow" component contributes relatively little to the autocorrelation function shown in Fig. 30. The relative contribution of the "slow" component to the autocorrelation function shown in Fig. 31 is midway between the other two cases.

The autocorrelation function representation, Eq. (3.2), provided by the constrained least-squares estimation procedure of Ref. 1 is shown plotted in Figs. 32 through 34 for the three records under consideration. Each figure contains the results of several pairs of the parameters $\xi_H$ and m. We may observe from these figures that the constrained least-squares fits $\phi(\xi)$ of Eq. (3.2) to the autocorrelation functions computed directly from the records are, in general, quite good.

Figures 35 through 37 show the autocorrelation function representations of the "slow" turbulence component

$$\phi_s(\xi) = \sum_{i=0}^{m} a_i \xi^i \qquad (8.2)$$

obtained after removal of the von Karman component $\sigma_f^2 \phi_K(\xi;L)$ obtained in the constrained least-squares procedure. The value m = 3 was used in all three fits. Figures 35 through 37 were computed by first subtracting from the empirical autocorrelation functions $R(\xi)$ the von Karman autocorrelation functions with parameters $\sigma^2$ and L as given in Figs. 25 through 27 respectively. The resulting differences, $R(\xi) - \sigma_f^2 \phi_K(\xi,L)$ are shown by the solid lines in Figs. 35 through 37. We then computed an integral least-squares fit

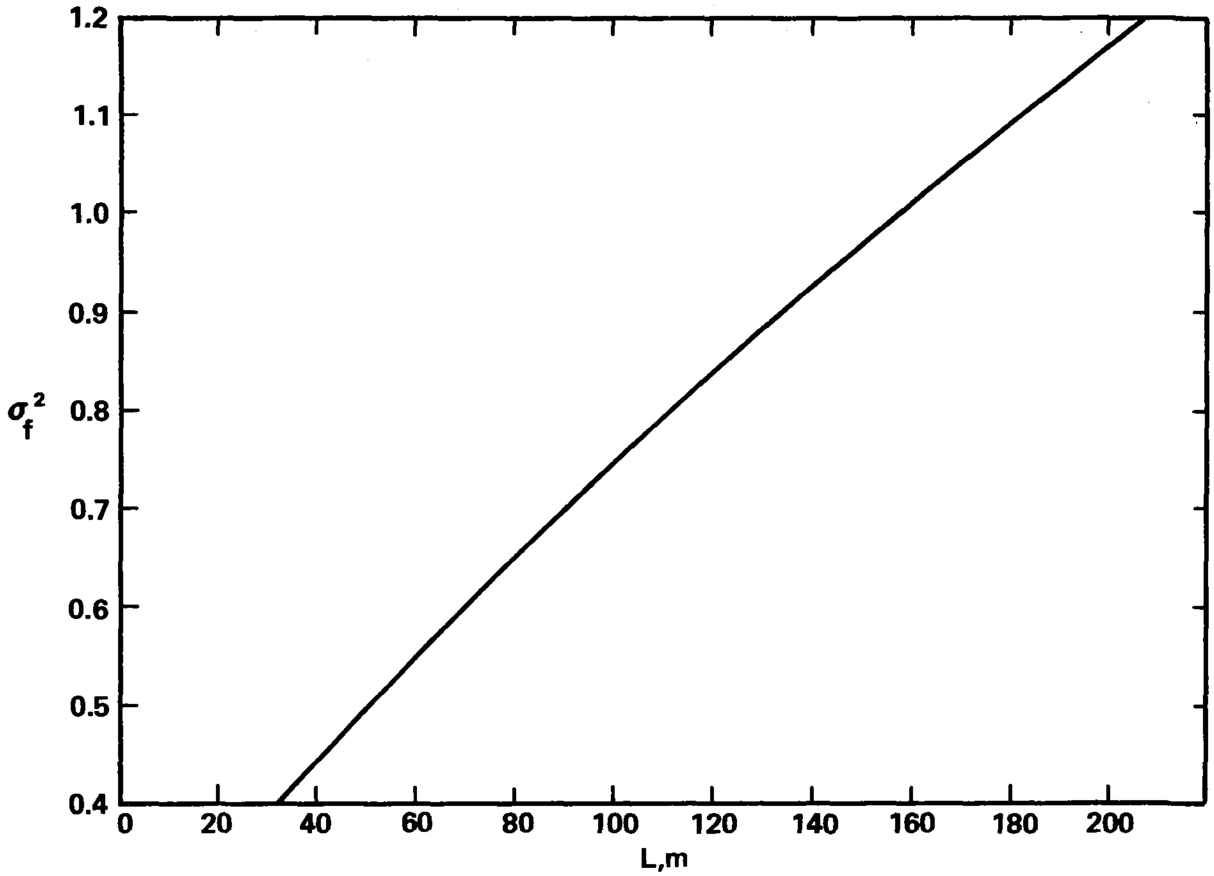


FIG. 28. CONSTRAINT BETWEEN $\sigma_f^2$ AND L FOR CONSTRAINED LEAST-SQUARES ESTIMATION PROCEDURE APPLIED TO LONGITUDINAL RECORD SHOWN IN FIG. 10.

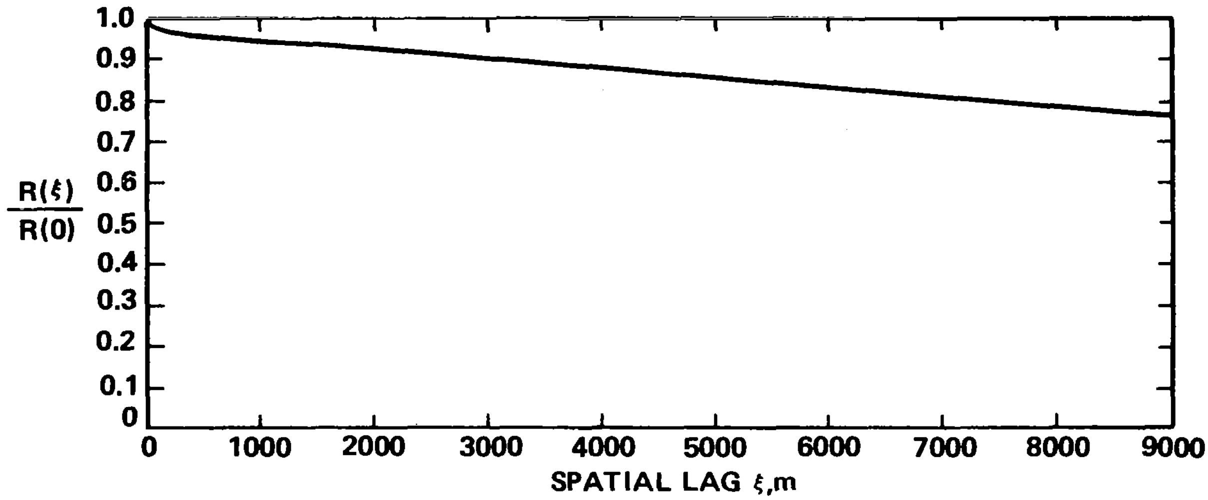


FIG. 29. AUTOCORRELATION FUNCTION OF LONGITUDINAL RECORD SHOWN IN FIG. 10 (MOUNTAIN-WAVE CONDITIONS). [FROM MAT PROJECT, NASA LANGLEY RESEARCH CENTER.]

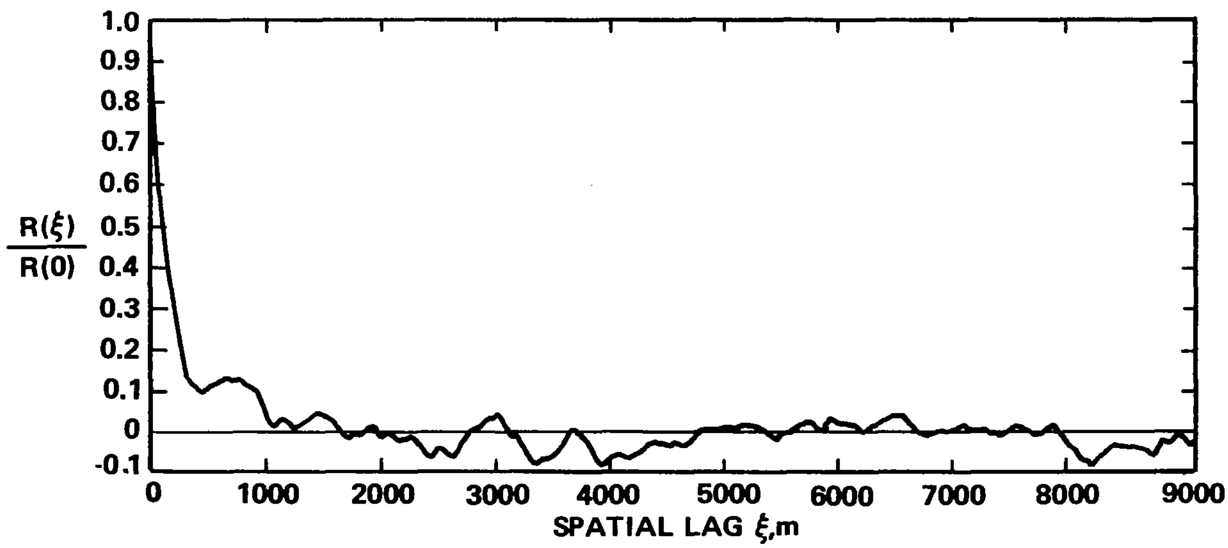


FIG. 30. AUTOCORRELATION FUNCTION OF VERTICAL RECORD SHOWN IN FIG. 4 (WIND-SHEAR CONDITIONS). [FROM MAT PROJECT, NASA LANGLEY RESEARCH CENTER.]

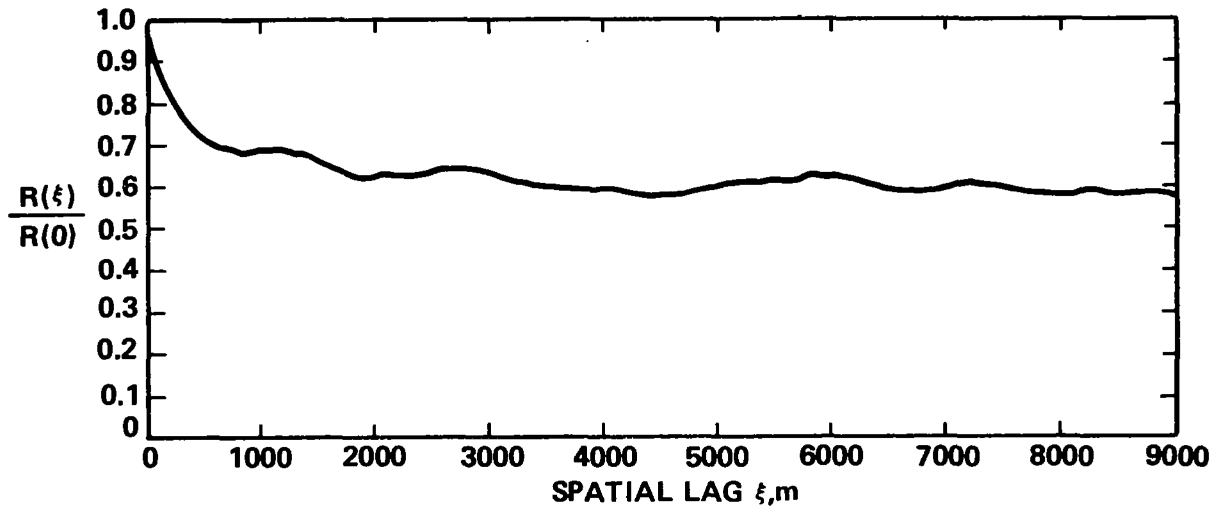


FIG. 31. AUTOCORRELATION FUNCTION OF LONGITUDINAL RECORD SHOWN IN FIG. 4 (WIND-SHEAR CONDITIONS). [FROM MAT PROJECT, NASA LANGLEY RESEARCH CENTER.]

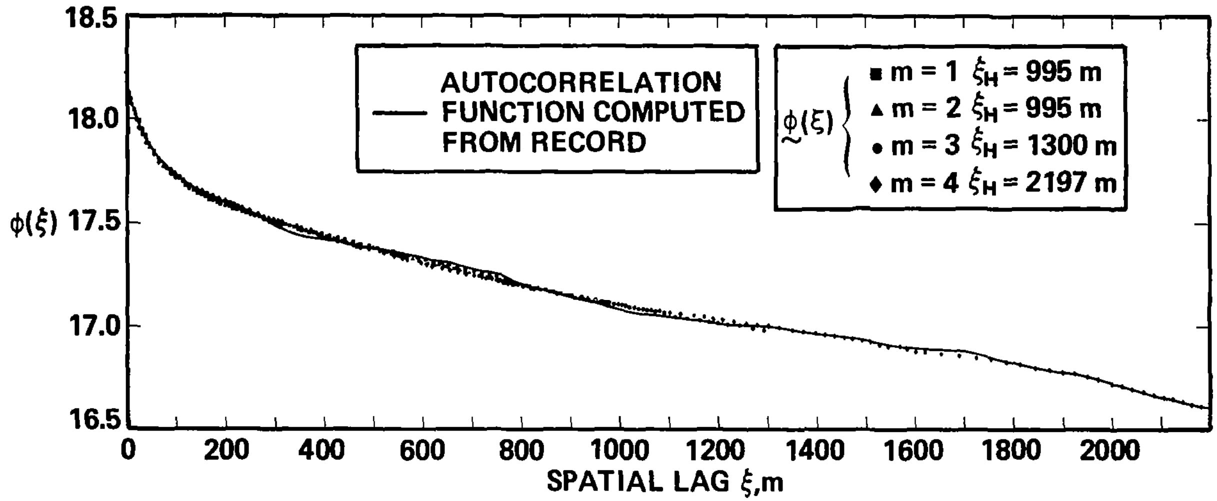


FIG. 32. COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM LONGITUDINAL RECORD SHOWN IN FIG. 10 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2).

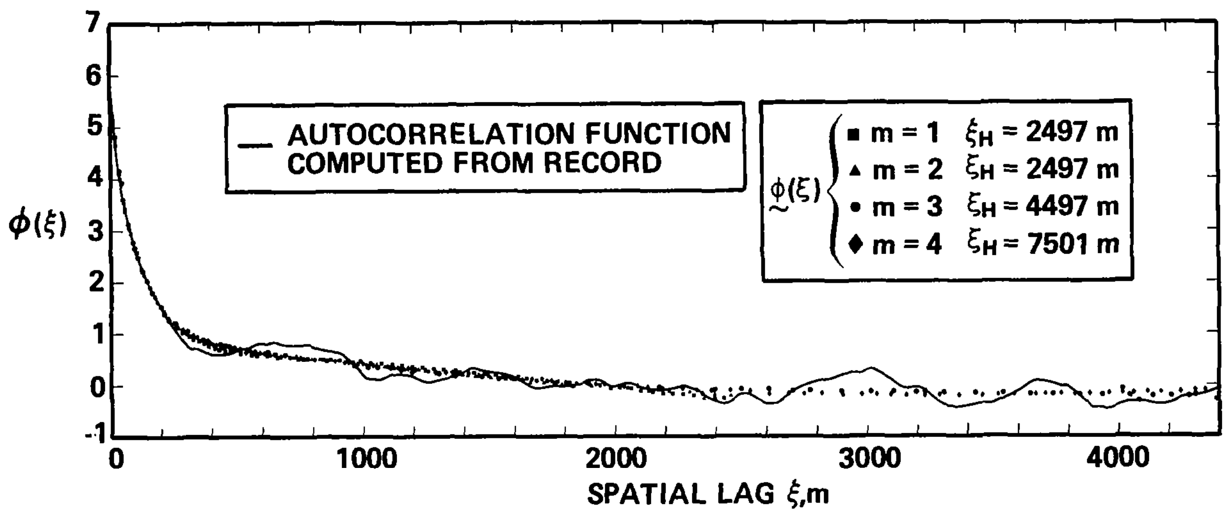


FIG. 33. COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM VERTICAL RECORD SHOWN IN IN FIG. 4 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2).

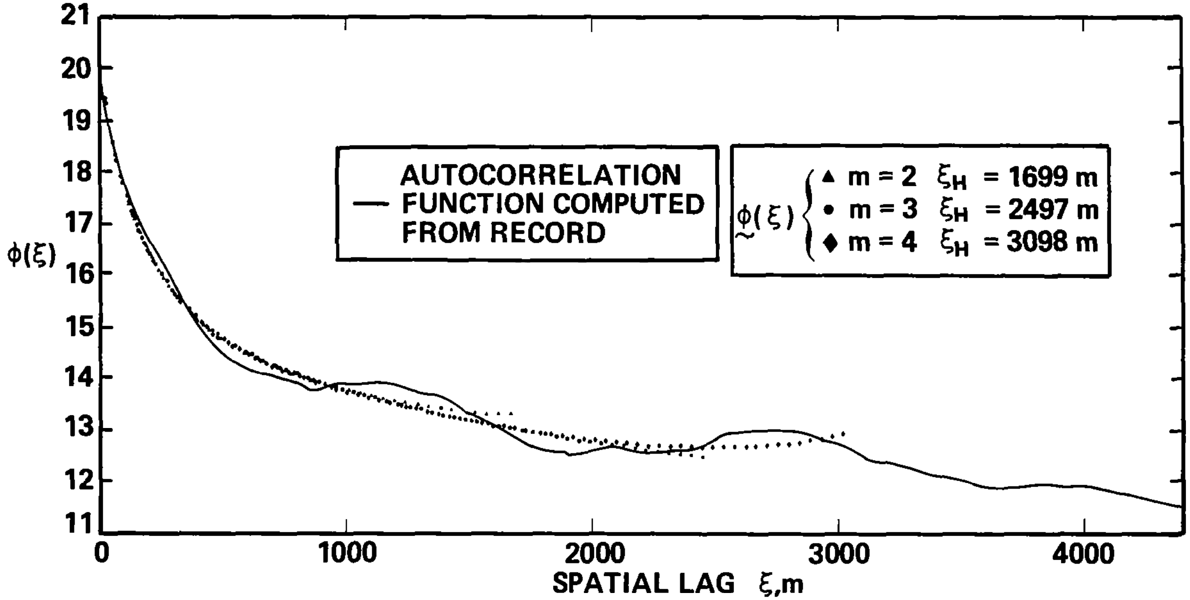


FIG. 34. COMPARISON OF AUTOCORRELATION FUNCTION COMPUTED FROM LONGITUDINAL RECORD SHOWN IN FIG. 4 AND CONSTRAINED LEAST-SQUARES FIT OF AUTOCORRELATION MODEL OF EQ. (3.2).

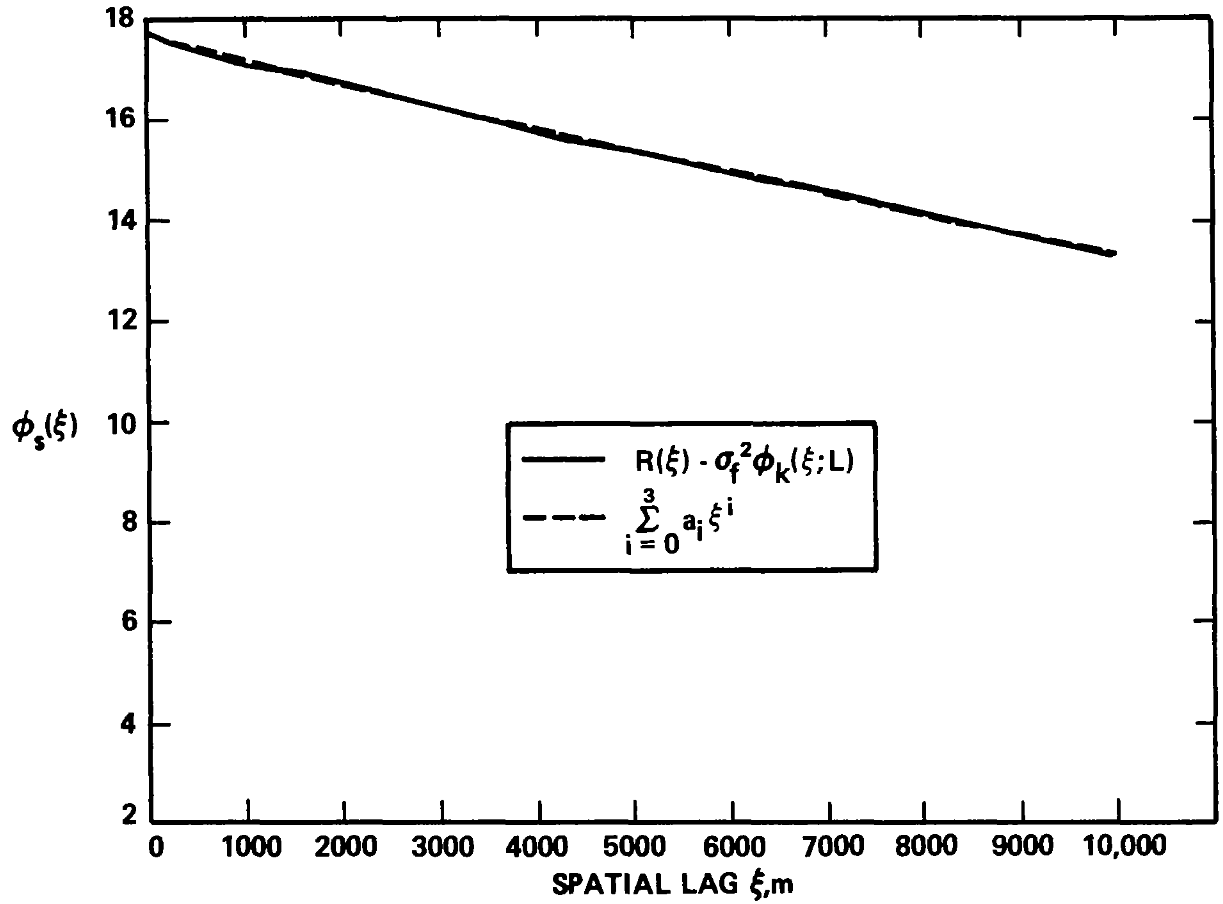


FIG. 35. COMPARISON OF AUTOCORRELATION FUNCTION $R(\xi)$ OF LONGITUDINAL RECORD SHOWN IN FIG. 10 MINUS AUTOCORRELATION FUNCTION $\sigma_f^2 \phi_k(\xi;L)$ OF VON KARMAN COMPONENT AND INTEGRAL LEAST-SQUARES THIRD-DEGREE POLYNOMIAL APPROXIMATION.

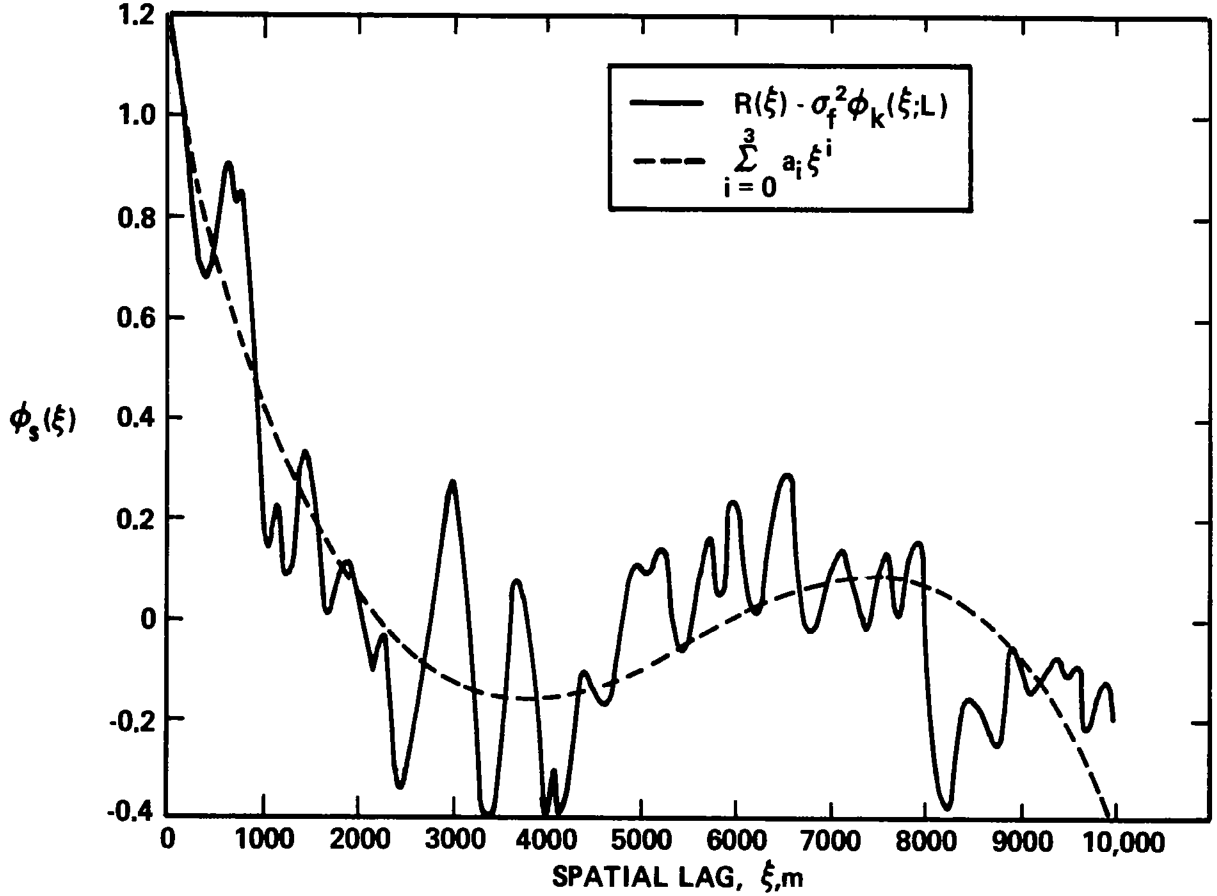


FIG. 36. COMPARISON OF AUTOCORRELATION FUNCTION $R(\xi)$ OF VERTICAL RECORD SHOWN IN FIG. 4 MINUS AUTOCORRELATION FUNCTION $\sigma_f^2\phi_k(\xi;L)$ OF VON KARMAN COMPONENT AND INTEGRAL LEAST-SQUARES THIRD-DEGREE POLYNOMIAL APPROXIMATION.

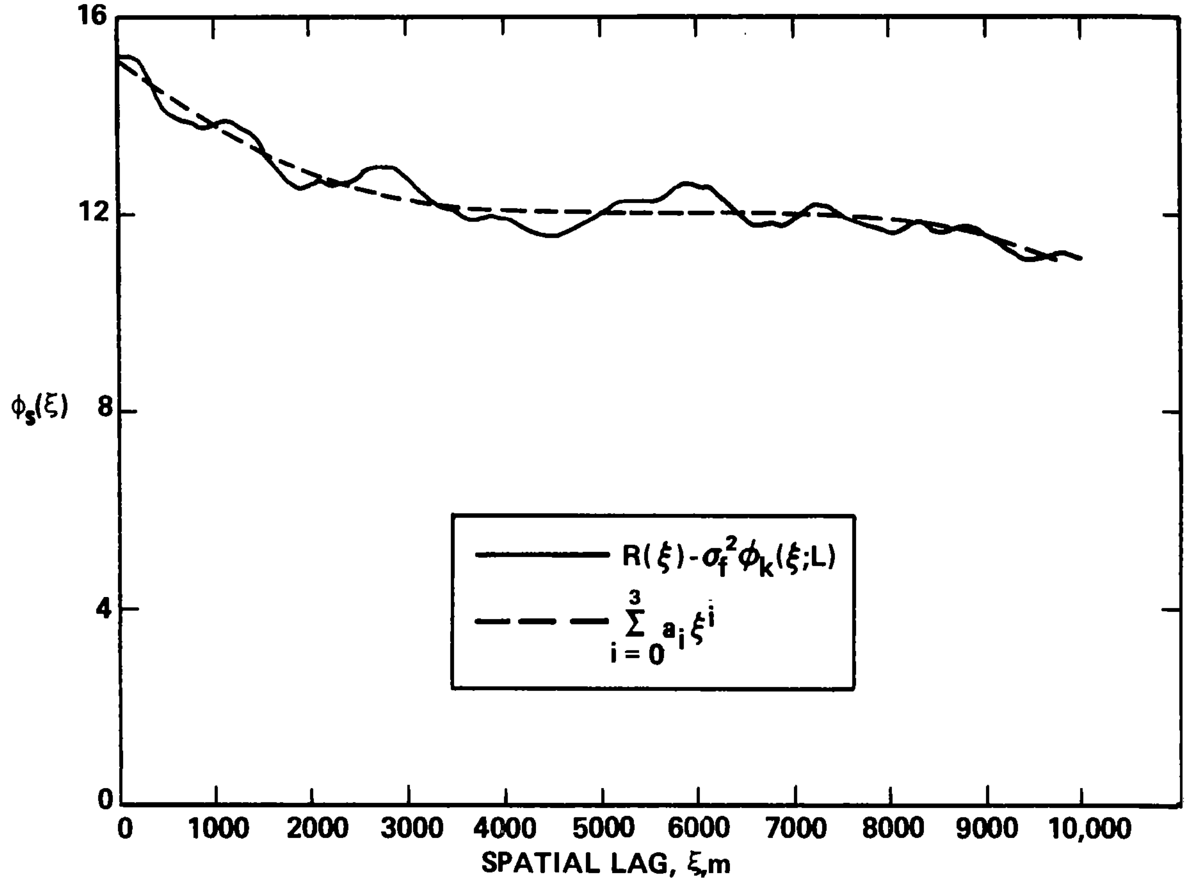


FIG. 37. COMPARISON OF AUTOCORRELATION FUNCTION $R(\xi)$ OF LONGITUDINAL RECORD SHOWN IN FIG. 4 MINUS AUTOCORRELATION FUNCTION $\sigma_f^2\phi_k(\xi;L)$ OF VON KARMAN COMPONENT AND INTEGRAL LEAST-SQUARES THIRD-DEGREE POLYNOMIAL APPROXIMATION.

of Eq. (8.2) to the functions $R(\xi) - \sigma_f^2\phi_K(\xi,L)$ over the lag intervals $0 \leq \xi \leq 10{,}000$ m using the third-degree polynomial ($m = 3$) of Eq. (8.2). These third-degree polynomial representations are shown by the dashed lines in Figs. 35 through 37.

Tables 4 through 6 show the values of $\sigma_f^2$, L, $\phi(0)$, and $a_0$ through $a_m$ for each combination of values of $\xi_H$ and m used in the constrained least-squares fit of Eq. (3.2) to the autocorrelation functions of each of the three records under consideration. With the exception of the cases where $m = 1$, we observe relatively little spread in the values of the integral scale L of the von Karman component in each of Tables 4 through 6.

TABLE 4. CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS FOR MOUNTAIN-WAVE LONGITUDINAL RECORD

| $\xi_H$ m | m | $\sigma_f^2$ $m^2/sec^2$ | L m | $\tilde{\phi}(0)$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|---|---|---|---|---|
| 995.1 | 1 | .550 | 60.4 | 18.221 | 17.671 | $-.582\times10^{-3}$ | | |
| 995.1 | 2 | .481 | 47.2 | 18.221 | 17.740 | $-.829\times10^{-3}$ | $.198\times10^{-6}$ | |
| 1300.5 | 2 | .475 | 46.1 | 18.220 | 17.744 | $-.837\times10^{-3}$ | $.193\times10^{-6}$ | |
| 1300.5 | 3 | .465 | 44.1 | 18.220 | 17.755 | $-.889\times10^{-3}$ | $.266\times10^{-6}$ | $-.310\times10^{-10}$ |
| 2197.1 | 3 | .414 | 34.5 | 18.223 | 17.809 | $-1.146\times10^{-3}$ | $.589\times10^{-6}$ | $-.143\times10^{-9}$ |
| 2197.1 | 4 | .443 | 39.8 | 18.221 | 17.778 | $-.977\times10^{-3}$ | $.310\times10^{-6}$ | $.303\times10^{-10}$ |
| | | | | | | | | $a_4=-.365\times10^{-13}$ |

Exact value of R(0) is 18.147 $m^2/sec^2$

TABLE 5. CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS FOR WIND-SHEAR VERTICAL RECORD

| $\xi_H$ m | m | $\sigma_f^2$ m²/sec² | L m | $\underset{\sim}{\phi}(0)$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|---|---|---|---|---|
| 2497.2 | 1 | 5.102 | 154.3 | 6.090 | .989 | $-.518\times10^{-3}$ | | | |
| 2497.2 | 2 | 4.858 | 142.6 | 6.066 | 1.208 | $-.909\times10^{-3}$ | $.139\times10^{-5}$ | | |
| 4496.9 | 2 | 5.036 | 151.1 | 6.095 | 1.058 | $-.712\times10^{-3}$ | $.986\times10^{-5}$ | | |
| 4496.9 | 3 | 4.710 | 135.6 | 6.070 | 1.360 | $-1.303\times10^{-3}$ | $.388\times10^{-6}$ | $-.40\times10^{-10}$ | |
| 7501.0 | 3 | 4.775 | 138.6 | 6.066 | 1.291 | $-1.106\times10^{-3}$ | $.258\times10^{-6}$ | $-.18\times10^{-10}$ | |
| 7501.0 | 4 | 4.881 | 143.6 | 6.073 | 1.192 | $-.909\times10^{-3}$ | $.154\times10^{-6}$ | $.25\times10^{-11}$ | $-.13\times10^{-14}$ |
| 9003.1 | 4 | 4.709 | 135.5 | 6.065 | 1.356 | $-1.249\ 10^{-3}$ | $.340\times10^{-6}$ | $-.34\times10^{-10}$ | $.11\times10^{-14}$ |

Exact value of R(0) is 6.053 m²/sec².

TABLE 6. CONSTRAINED LEAST-SQUARES ESTIMATION OF AUTOCORRELATION FUNCTION PARAMETERS FOR WIND-SHEAR LONGITUDINAL RECORD.

| $\xi_H$ m | m | $\sigma_f^2$ $m^2/sec^2$ | L m | $\underset{\sim}{\phi}(0)$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|---|---|---|---|---|
| 1699.2 | 2 | 5.200 | 175.3 | 20.430 | 15.230 | $-.197\times10^{-2}$ | $.486\times10^{-6}$ | | |
| 2102.9 | 3 | 4.262 | 128.7 | 20.705 | 16.443 | $-.536\times10^{-2}$ | $.370\times10^{-5}$ | $-.10\times10^{-8}$ | |
| 2497.0 | 3 | 5.120 | 171.0 | 20.406 | 15.286 | $-.190\times10^{-2}$ | $.384\times10^{-6}$ | $-.29\times10^{-10}$ | |
| 3098.0 | 4 | 4.965 | 163.2 | 20.482 | 15.517 | $-.258\times10^{-2}$ | $.108\times10^{-5}$ | $-.35\times10^{-9}$ | $.6\times10^{-13}$ |
| 3605.0 | 4 | 5.016 | 166.0 | 20.347 | 15.330 | $-.149\times10^{-2}$ | $-.599\times10^{-6}$ | $.56\times10^{-9}$ | $-1.0\times10^{-13}$ |

Exact value of R(0) is 20.072 $m^2/sec^2$.

## METHODS FOR COMPUTATION OF THE INTEGRAL SCALE AND INTENSITY OF THE "SLOW" TURBULENCE COMPONENT

If $\phi(\xi)$ is the autocorrelation function of a turbulence record which is a function of the spatial lag variable $\xi$, and

$$\sigma^2 \equiv \phi(0) \tag{9.1}$$

is the mean-square value of the record, then the integral scale L is defined as [p. 43 of Ref. 8]

$$L \triangleq \frac{1}{\sigma^2} \int_0^{\infty} \phi(\xi) d\xi . \tag{9.2}$$

Let us define the wavenumber spectrum of the record as

$$\Phi(k) \triangleq \int_{-\infty}^{\infty} \phi(\xi) e^{-i2\pi k\xi} d\xi . \tag{9.3}$$

Hence,

$$\Phi(0) = \int_{-\infty}^{\infty} \phi(\xi) d\xi = 2 \int_0^{\infty} \phi(\xi) d\xi , \tag{9.4}$$

since $\phi(\xi)$ is an even function of $\xi$. From Eqs. (9.2) and (9.4), we may express the integral scale in terms of $\Phi(0)$ as

$$L = \frac{1}{2\sigma^2} \Phi(0) . \tag{9.5}$$

In the case of the von Karman transverse wavenumber spectrum which we have chosen to characterize the "fast" turbulence component, we have instead of Eq. (9.5),

$$\Phi_{KT}(0) = \sigma_f^2 L_f \ , \tag{9.6}$$

according to Eq. (3.18) on p. 80 of Ref. 1, where in Eq. (9.6) we have used subscripts f to denote characteristics of the fast turbulence component. From Eqs. (9.5) and (9.6), we see that there is a factor of two between the definition $L_f$ for the von Karman transverse integral scale and the definition given by Eq. (9.5). This is customary — e.g., see Ref. 9.

Let us denote the wavenumber spectra of w(t), $w_s(t)$ and $w_f(t)$ in our turbulence model of Eq. (1.1) by $\Phi_w(k)$, $\Phi_s(k)$, and $\Phi_f(k)$. Then, from the assumed statistical independence of $\{w_s(t)\}$ and $\{w_f(t)\}$, it follows that

$$\Phi_w(k) = \Phi_s(k) + \Phi_f(k) \ ; \tag{9.7}$$

hence,

$$\Phi_s(0) = \Phi_w(0) - \Phi_f(0) \ . \tag{9.8}$$

Applying Eq. (9.5) to the slow turbulence component, and using subscripts s to denote "slow," we have

$$L_s = \frac{1}{2\sigma_s^2} \Phi_s(0) \ . \tag{9.9}$$

If we now recognize that the fast turbulence component whose spectrum is $\Phi_f(k)$ has been modeled by the von Karman transverse spectrum; hence, $\Phi_f(k) = \Phi_{KT}(k)$, we may combine Eqs. (9.6), (9.8), and (9.9) to yield the desired expression for the integral scale of the slow turbulence component:

$$L_s = \frac{1}{2\sigma_s^2} [\Phi_w(0) - \sigma_f^2 L_f] \ , \tag{9.10}$$

where all quantities on the right-hand side are easily estimated from methods discussed earlier. The mean-square value $\sigma_s^2$ of the slow component is obtained from the mean-square value of the original record and the mean-square value of the fast component by subtraction:

$$\sigma_s^2 = \sigma_w^2 - \sigma_f^2 \qquad (9.11)$$

as may be seen from Eq. (9.7).

*Initial evaluations of $L_s$ for the first three records discussed in this report.* The parameters required to evaluate $L_s$ for the first three records with a "slow" component discussed earlier in this report are listed below in metric system units.

| | Flt. 32 Run 4 (Wind Shear Lat.) | | Flt. 30 Run 8 (Mt. Wave Lat.) | | Flt. 30 Run 8 (Mt. Wave Vert.) | |
|---|---|---|---|---|---|---|
| $\Phi_w(0)$ | $3.001\times10^5 m^3/sec^2$ | | $2.187\times10^5 m^3/sec^2$ | | $8.502\times10^3 m^3/sec^2$ | |
| $\sigma_w^2$ | 53.66 | $m^2/sec^2$ | 29.23 | $m^2/sec^2$ | 1.812 | $m^2/sec^2$ |
| $\sigma_f^2$ | 5.315 | $m^2/sec^2$ | 0.684 | $m^2/sec^2$ | 0.470 | $m^2/sec^2$ |
| $L_f$ | 265.5 | m | 128.9 | m | 68.4 | m |

Substitution of the above parameters into Eqs. (9.10) and (9.11) yields the following values for the integral scale $L_s$ of the slow turbulence components of the three records:

wind-shear lateral: $L_s$ = 3090 m (9.12a)

mountain-wave lateral: $L_s$ = 3829 m (9.12b)

mountain-wave vertical: $L_s$ = 3156 m. (9.12c)

*Discussion.* Rough checks may be made for each of the above values of $L_s$ by comparing them with plots of the auto-correlation functions of the slow turbulence components shown in Figs. 9, 15, and 20 respectively. From the definition of L given by Eq. (9.2), it is evident from the above mentioned figures that each of the values of $L_s$ given by Eq. (9.12) is too small. The reason that these computed values of $L_s$ are too small may be seen from Eq. (9.10). To compute them, we used the values of the smoothed spectra $\Phi_w(0)$ evaluated at k = 0. The process of smoothing the wavenumber spectra to get improved statistical reliability reduced the values of the wavenumber spectra evaluated at k = 0. Hence, we require an alternative method to estimate the integral scales $L_s$. Such an improved method has been developed in Appendix H of Ref. 1. This method is based directly on integration of the autocorrelation function representation of Eq. (8.2) after extrapolating its "tail" using a decaying exponential with continuous slope at $\xi = \xi_H$. The resulting formula for $L_s$ derived in Appendix H of Ref. 1 [Eq. (H.14)] is

$$L_s = \frac{1}{a_0}\left[\sum_{j=0}^{m} \frac{a_j}{j+1} \xi_H^{j+1} - \frac{\left(\sum_{j=0}^{m} a_j \xi_H^j\right)^2}{\sum_{j=1}^{m} j a_j \xi_H^{j-1}}\right] \tag{9.13}$$

where $a_0, \cdots, a_m$ are the coefficients of the autocorrelation representation of Eq. (8.2), and $\xi_H$ = 10,000 m is the upper limit of the lag parameter used in the integral least-squares fit.

*Improved evaluations of $L_s$ for the three records from Flight 30 Run 8 and the three records from Flight 32 Run 4.* Using the autocorrelation function representation of Eq. (8.2) with m = 3 over the interval $0 \leq \xi \leq 10{,}000$ m that is displayed for the six records of interest by the dashed lines in Figs. 9, 15, 20, 35, 36, and 37, we have computed $L_s$ using Eq. (9.13). The resulting values of integral scale are shown in Table 7.

TABLE 7. VALUES OF INTEGRAL SCALE $L_s$ OF THE SLOW TURBULENCE COMPONENT COMPUTED USING EQ. (9.13).

| | | |
|---|---|---|
| wind-shear lateral | Fig. 9 | $L_s$ = 8,511 m |
| mountain-wave lateral | Fig. 15 | $L_s$ = 37,094 m |
| mountain-wave vertical | Fig. 20 | $L_s$ = 4,997 m |
| mountain-wave longitudinal | Fig. 35 | $L_s$ = 32,384 m |
| wind-shear vertical | Fig. 36 | $L_s$ = 825 m* |
| wind-shear longitudinal | Fig. 37 | $L_s$ = 17,052 m |

*The value of $L_s$ = 825 m for the wind-shear vertical case is completely unreliable because of the behavior of the autocorrelation function representation of Eq. (8.2) shown in Fig. 36.

From examination of the dashed autocorrelation function representations in each of the above cited figures, we can conclude that the method of Eq. (9.13) should provide reasonable values of $L_s$ for all records except the wind-shear vertical record whose autocorrelation function is shown in Fig. 36. This lack of reliability is noted in Table 7.

# APPENDIX A
## INTRODUCTION TO COMPUTER PROGRAMS

The Appendices to this report document the following computer programs developed to characterize nonGaussian atmospheric turbulence records relevant to aircraft response calculations:

a. maximum likelihood estimation of the integral scale and variance of von Karman turbulence,

b. constrained least-squares estimation of turbulence autocorrelation function parameters,

c. power spectral density of the instantaneous variance $\sigma_f^2(t)$,

d. probability density estimation of the instantaneous variance $\sigma_f^2(t)$ and the "slow" turbulence component $w_s(t)$.

The turbulence models used to develop these programs are described in BBN Report 4319, Characterization, Parameter Estimation, and Aircraft Response Statistics of Atmospheric Turbulence (Ref. 1).

These Appendices explain the program usage, program inputs and outputs, and give typical teletype printouts. Included are source program listings and printouts of typical output data files. The programs were written in FORTRAN IV. The notation used in this report is consistent with that of the above cited reference.

The source program and subroutine listings are contained in Appendix F in alphabetic order by program name. The source programs and subroutines contain comment statements to aid the user in following the flow of the computations.

Table 8 contains a summary of the purpose of each program, subroutines required, and the form of the input/output of each program.

## TABLE 8. ATMOSPHERIC TURBULENCE MODELING PROGRAMS.

| Appendix | Main Program | Subroutines | Purpose | Inputs | Outputs* |
|---|---|---|---|---|---|
| B.1.1 | ATURB2 | CFFT<br>SIMP | compute $\Phi_\ell(k)$, $R_\ell(\xi)$, $\Phi_\rho(k)$ | turbulence samples, TTY inputs | (PHILK): $\Phi_\ell(k)$<br>(AUTO): $R_\ell(\xi)$<br>(DSPS): $\Phi_\rho(k)$ |
| B.1.2 | PART2 | AK<br>AKDAT<br>GAM<br>PARAB<br>SIMQ | computes $E(L)$, $LG(k_i,L)$, $\phi_K(\xi)$ and $\Phi_K(k)$ | data file (PHILK) and TTY inputs | (LG):$LG(k_i)$<br>(PHIxI): $\phi_K(\xi)$<br>(PHIK): $\Phi_K(k)$ |
| C.1.1 | ATURB3 | CFFT<br>SIMP | computes proven spectrum autocorrelation for nonGaussian turbulence samples — similar to ATURB2 | turbulence | (PHILK): $\Phi_\ell(k)$<br>(AUTO): $R(\xi)$<br>(DSPS): $\Phi_\rho(k)$ |
| C.1.2 | PART5 | -- | computes $\sigma^2(L_j)$, $j=1,\ldots 15$ | (PHILK)<br>TTY inputs | TTY:<br>$\sigma^2(L_j)$ |
| C.1.3 | FINAL | AK1<br>AKDAT<br>ANRP1<br>FnDECT<br>DGELG<br>GAM<br>PAR1&2<br>SET<br>SIMP2<br>SIMQ<br>TRAP3&6 | computes $\underset{\sim}{\phi}(\xi)$ | TTY inputs | (ITM2): $\underset{\sim}{\phi}(\xi)$<br>(ITM2L):<br>$\sigma_f^2\, L\Phi_K(kL)$ |
| D.1.1 | ATURB4 | CFFT<br>HPDES<br>SIMP | Filter data, produce power spectrum & autocorrelation fn | turbulence samples & TTY inputs | (PHILK):$\Phi_\ell(k)$<br>(AUTO):$R_{w_h}(\xi)$<br>TTY inputs |
| D.1.1 (cont.) | ATUR4A | CFFT1<br>HPDES<br>SIMP | Filter data, square, produce power spectrum and autocorrelation | same as above | (FPSD2):$\Phi_\ell(k)$<br>(AUTF2):$R_{w_h^2}(\xi)$ |
| D.1.2 | ITEM3 | CFFT1 | compute $R_{\sigma_f^2}(\xi)$, $\Phi_{\sigma_f^2}(k)$ | (AUTO)<br>(AUTF2) & TTY inputs | (RSIGF):$R_{\sigma_f^2}(\xi)$<br>(PHIF):$\Phi_{\sigma_f^2}(k)$ |
| E.1.1 | MOMENT | BIN | computes $\alpha_w^{[k]}$, $\alpha_{w_h}^{[k]}$, $\alpha_{w_h^2}^{[k]}$, and $\alpha_{\sigma_f^2}^{[k]}$ $k=1,\ldots 8$ | turbulence samples & TTY input | TTY: moments listed to left |
| E.1.2 | GDIST6 | GAM | probability density $\rho_{\sigma_f^2}$ | TTY input | TTY output |
| E.1.3 | ITEM4 | FAC1 | probability density $\rho_{w_s}$ | TTY input | (PROB):$\rho_{w_s}$ & TTY output |

*Data files in parentheses.

## APPENDIX B

## MAXIMUM LIKELIHOOD ESTIMATION OF THE INTEGRAL SCALE AND VARIANCE OF von KARMAN TURBULENCE

Two main programs, ATURB2.F4 and PART2.F4, are used in computing the integral scale and variance of von Karman turbulence. The development of the maximum likelihood estimation technique is discussed in Sec. 3 of Ref. 1 and Appendix F of Ref. 1. The first program, ATURB2, computes the two sided power spectrum, $S_j$, the autocorrelation function, $R_{\rho}(\xi)$, and the two sided smoothed power spectrum, $\Phi_{\rho}(k)$, of the turbulence record, w(t), being processed. The second program, PART2, calculates the integral scale of the stationary turbulence sample and its variance.

A fast Fourier transform subroutine is utilized in computing the power spectra and autocorrelation function. The positive frequency domain values of the spectra and the values of the autocorrelation function are stored in three separate output data files. The data file PHILK, containing $S_j$ values, is used by the program PART2. In addition to computing the integral scale and variance, PART2 computes the von Karman autocorrelation function $\phi_K(\xi)$ and von Karman spectrum $\Phi_{KT}(k)$ using values of $\sigma^2$ and length scale L determined by the program.

### Program Outlines and Usage

Program ATURB2.F4: Computes the two sided power spectrum, $S_j$, of turbulence data, the autocorrelation function $\phi$, and smoothed power spectrum $\Phi$.

a. Subroutines

   i. CFFT – fast Fourier transform routine
   ii. SIMP – integration by Simpson's rule.

b. Inputs* [from the teletype (TTY) unless otherwise noted]

   i. speed of craft in m/sec – (V)†
   ii. number of points in Fourier transform (NPTS) and power of two of that number (MPWRN)
   iii. number of points in turbulence record (NOPTS) and sampling rate of data (SRATE)

---

*Unless otherwise noted a "G" format is used for numerical inputs from the teletype.

†Alphanumerics in parenthesis represents equivalent variable name in source program.

iv. number of points for smoothed power spectrum (MPTS) and power of two of that number (MPWRM)

v. name of data file (A5 format) containing turbulence values w(t) and number of points in the array (NXRAY; input file has 0 to NXRAY-1 points divided into 4 columns with a format of 4 (E15.7); input values in units of ft/sec – program converts values to m/sec); this data file is read by the program.

vi. answer yes (Y) if program check on integration is desired (value printed on TTY should equal value printed in data file) or no (N) if program check is not desired.

c. Outputs (to various data files or to the TTY)

i. data file, PHILK, containing positive frequency (k) domain values of power spectrum $S_j$, equation 3.27, Ref. 3.

ii. data file, AUTO, containing autocorrelation values

iii. data file, DSPS, containing values of the smoothed power spectrum

d. Example

A typical teletype printout of the execution of this program is shown in Fig. B.1. The user supplied information discussed in b is underlined in the example. The first page of each output file mentioned is shown in Figs. B.2 through B.4.

**Program PART2.F4:** Computes the integral scale, L, of a stationary turbulence sample and its variance (Eq. 3.25, Ref. 1).

a. Subroutines:

i. GAM – computes gamma function Γ

ii. AK and AKDAT – compute modified Bessel functions of fractional orders 1/3 and 2/3, AK uses data stored in AKDAT

iii. SIMQ – simultaneous linear equation routine from the IBM Scientific Subroutine Package

iv. PARAB – curve fitting routine

```
@LOADER
*ATURB2,CFFT,SIMP$

ATURB2 36K CORE, 212 WORDS FREE
LOADER USED 39+5K CORE

EXIT.
^C
@SAVE (CORE FROM) 20 (TO) 777777 (ON)     ATURB2.SAV [NEW FILE]
@ST


INPUT SPEED OF CRAFT (M/SEC)
187.76


INPUT TOTAL NO. OF POINTS TO BE USED IN 2L M. OF DATA
AND POWER OF TWO OF THAT NO.
16384
14


INPUT NO. OF POINTS OF W(X) TO  BE READ
AND SAMPLING RATE OF DATA 14592.,.05


INPUT VALUE OF MPTS
AND POWER OF TWO OF THAT NO.
1024
10


INPUT DATA FILE NAME THAT CONTAINS SAMPLES
OF W(X) AND NO. POINTS NXRAY
NASA1
14592


PERFORM INTEGRATION CHECK (Y OR N)    Y

                                        program output should match
                                        value of σ² of data files
INTEGRAL OF PHI OF L (K) =  0.4802E+02
 0.3594628E-02  0.0000000      1478369.       0.0000000   intermediate
  848732.1      0.0000000      244530.9       0.0000000   output, useful
  118152.3       26228.80                                 for debugging
 0.4658256                                                only
 0.7216021      0.7061144
  848688.2       244518.3      118146.2

CPU TIME: 4:0.23        ELAPSED TIME: 10:5.85
NO EXECUTION ERRORS DETECTED

EXIT.
^C
```

FIG. B.1. TTY PRINTOUT FOR RUNNING PROGRAM ATURB2.

; <RFISHER>PHILK.DAT;5    Mon 7-Nov-77 9:47AM    PAGE 1

DATA FILE CREATED BY PROGRAM ATURB2

POWER SPECTRUM OF PHI OF L(K)
DATA TAKEN FROM FILE VERT

32768 DATA POINTS WERE USED IN 2L = 210894.8460 METER
1024 DATA POINTS WERE USED IN M = 6590.4639 METER

9968 ZEROS WERE ADDED TO DATA

MEAN VALUE OF W(X) = 0.83463E-01 M/SEC
MEAN SQ. VALUE = 0.13305E+01 (M/SEC)**2

<W OF L(X)**2> = 1.3305

PRINTOUT OF THE VALUES OF THE POWER SPECTRUM

| K | PS VALUE | K CONTD | PS VALUE CONTD |
|---|---|---|---|
| 0.000000 | 0.1419E-09 | 0.038844 | 0.5791E+00 |
| 0.000005 | 0.1622E+04 | 0.038849 | 0.1750E+01 |
| 0.000009 | 0.6052E+03 | 0.038853 | 0.4770E+00 |
| 0.000014 | 0.2032E+02 | 0.038858 | 0.2895E+00 |
| 0.000019 | 0.4472E+02 | 0.038863 | 0.1043E-01 |
| 0.000024 | 0.5508E+03 | 0.038868 | 0.7540E+00 |
| 0.000028 | 0.1108E+03 | 0.038872 | 0.4492E-01 |
| 0.000033 | 0.7429E+02 | 0.038877 | 0.1940E+00 |
| 0.000038 | 0.5229E+03 | 0.038882 | 0.4078E+00 |
| 0.000043 | 0.2987E+03 | 0.038887 | 0.6840E-01 |
| 0.000047 | 0.4791E+03 | 0.038891 | 0.8488E+00 |
| 0.000052 | 0.3334E+03 | 0.038896 | 0.1444E+00 |
| 0.000057 | 0.1399E+03 | 0.038901 | 0.2073E-01 |
| 0.000062 | 0.1206E+03 | 0.038906 | 0.1883E+00 |
| 0.000066 | 0.1233E+03 | 0.038910 | 0.9916E+00 |
| 0.000071 | 0.1368E+03 | 0.038915 | 0.8517E+00 |
| 0.000076 | 0.5743E+03 | 0.038920 | 0.1654E-02 |
| 0.000081 | 0.9769E+02 | 0.038925 | 0.2768E+00 |
| 0.000085 | 0.3835E+03 | 0.038929 | 0.8511E+00 |
| 0.000090 | 0.8038E+02 | 0.038934 | 0.6148E+00 |
| 0.000095 | 0.5548E+03 | 0.038939 | 0.7542E+00 |
| 0.000100 | 0.7102E+03 | 0.038944 | 0.1934E+01 |
| 0.000104 | 0.3008E+03 | 0.038948 | 0.8529E-01 |
| 0.000109 | 0.3210E+03 | 0.038953 | 0.5254E+00 |
| 0.000114 | 0.7278E+03 | 0.038958 | 0.1431E+00 |
| 0.000119 | 0.1500E+03 | 0.038963 | 0.4173E+00 |
| 0.000123 | 0.1513E+04 | 0.038967 | 0.2629E+01 |

FIG. B.2. OUTPUT DATA FILE PHILK.

: <RFISHER>AUTWSV.DAT;1 Fri 28-Apr-78 4:51PM PAGE 1

↑ AUTO renamed to AUTWSV

DATA FILE CREATED BY PROGRAM ATURB2

AUTOCORRELATION OF STATIONARY SAMPLE
DATA TAKEN FROM FILE NASV

16384 DATA POINTS WERE USED IN 2L = 153812.9920 METER
1024 DATA POINTS WERE USED IN M = 9613.3120 METER

1796 ZEROS WERE ADDED TO DATA

MEAN VALUE OF W(X) = -0.43757E-01 M/SEC
MEAN SQ. VALUE = 0.60534E+01 (M/SEC)**2

<W OF L(X)**2> = 6.0533

TRUCATION POINT WAS 13000.00 METERS
WHICH CONTAINS 1384 POINTS

PRINTOUT OF THE VALUES OF THE AUTOCORRELATION

| X | RL | RL/R0 |
|---|---|---|
| 0.0000000 | 6.053274 | 1.000000 |
| 9.388000 | 5.531511 | 0.9138248 |
| 18.77600 | 5.046111 | 0.8336167 |
| 28.16400 | 4.635600 | 0.7658003 |
| 37.55200 | 4.277285 | 0.7066268 |
| 46.94000 | 3.941774 | 0.6511804 |
| 56.32800 | 3.612217 | 0.5967377 |
| 65.71600 | 3.323236 | 0.5489980 |
| 75.10400 | 3.083437 | 0.5093834 |
| 84.49200 | 2.885883 | 0.4767475 |
| 93.88000 | 2.686956 | 0.4438848 |
| 103.2680 | 2.529167 | 0.4178180 |
| 112.6560 | 2.397351 | 0.3960421 |
| 122.0440 | 2.274019 | 0.3756676 |
| 131.4320 | 2.177932 | 0.3597940 |
| 140.8200 | 2.081459 | 0.3438567 |
| 150.2080 | 1.979035 | 0.3269362 |
| 159.5960 | 1.874740 | 0.3097068 |
| 168.9840 | 1.796849 | 0.2968393 |
| 178.3720 | 1.730489 | 0.2858766 |
| 187.7600 | 1.661303 | 0.2744470 |
| 197.1480 | 1.577359 | 0.2605794 |
| 206.5360 | 1.491435 | 0.2463848 |
| 215.9240 | 1.401096 | 0.2314609 |
| 225.3120 | 1.312728 | 0.2168625 |

FIG. B.3. OUTPUT DATA FILE AUTO.

<RFISHER>DSPS6.DAT;1 Fri 28-Apr-78 4:52PM PAGE 1

↑ Renamed from DSPS

DATA FILE CREATED BY PROGRAM ATURB2

SMOOTHED POWER SPECTRUM PHI OF P(K)
DATA TAKEN FROM DATA FILE NASV

16384 DATA POINTS WERE USED IN 2L = 153812.992Ø METER
1Ø24 DATA POINTS WERE USED IN M = 9613.312Ø METER

MEAN VALUE OF W(X) = -Ø.43757E-Ø1 M/SEC
MEAN SQ. VALUE = Ø.6Ø534E+Ø1 (M/SEC)**2

<W OF L(X)**2> = 6.Ø533

1796 ZEROS WERE ADDED TO DATA

RL(Ø) = Ø.6Ø53E+Ø1

PRINTOUT OF VALUES OF THE SMOOTHED POWER SPECTRUM

| K | SPS VALUE | K CONTD | SPS VALUE CONTD |
|---|---|---|---|
| Ø.ØØØØØØ | Ø.2211E+Ø4 | Ø.Ø2663Ø | Ø.5582E+Ø1 |
| Ø.ØØØØ52 | Ø.2324E+Ø4 | Ø.Ø26682 | Ø.5629E+Ø1 |
| Ø.ØØØ1Ø4 | Ø.26Ø1E+Ø4 | Ø.Ø26734 | Ø.5344E+Ø1 |
| Ø.ØØØ156 | Ø.26Ø1E+Ø4 | Ø.Ø26786 | Ø.5Ø22E+Ø1 |
| Ø.ØØØ2Ø8 | Ø.2Ø3ØE+Ø4 | Ø.Ø26838 | Ø.574ØE+Ø1 |
| Ø.ØØØ26Ø | Ø.1521E+Ø4 | Ø.Ø2689Ø | Ø.6989E+Ø1 |
| Ø.ØØØ312 | Ø.1446E+Ø4 | Ø.Ø26942 | Ø.6953E+Ø1 |
| Ø.ØØØ364 | Ø.1371E+Ø4 | Ø.Ø26994 | Ø.5436E+Ø1 |
| Ø.ØØØ416 | Ø.1Ø86E+Ø4 | Ø.Ø27Ø46 | Ø.43ØØE+Ø1 |
| Ø.ØØØ468 | Ø.7951E+Ø3 | Ø.Ø27Ø98 | Ø.4341E+Ø1 |
| Ø.ØØØ52Ø | Ø.7422E+Ø3 | Ø.Ø2715Ø | Ø.4698E+Ø1 |
| Ø.ØØØ572 | Ø.8975E+Ø3 | Ø.Ø272Ø2 | Ø.5Ø71E+Ø1 |
| Ø.ØØØ624 | Ø.9883E+Ø3 | Ø.Ø27254 | Ø.5294E+Ø1 |
| Ø.ØØØ676 | Ø.8745E+Ø3 | Ø.Ø273Ø6 | Ø.5114E+Ø1 |
| Ø.ØØØ728 | Ø.6968E+Ø3 | Ø.Ø27358 | Ø.4857E+Ø1 |
| Ø.ØØØ78Ø | Ø.5919E+Ø3 | Ø.Ø2741Ø | Ø.5411E+Ø1 |
| Ø.ØØØ832 | Ø.4981E+Ø3 | Ø.Ø27462 | Ø.6Ø69E+Ø1 |
| Ø.ØØØ884 | Ø.493ØE+Ø3 | Ø.Ø27514 | Ø.529ØE+Ø1 |
| Ø.ØØØ936 | Ø.6537E+Ø3 | Ø.Ø27566 | Ø.44Ø2E+Ø1 |
| Ø.ØØØ988 | Ø.8731E+Ø3 | Ø.Ø27618 | Ø.4795E+Ø1 |
| Ø.ØØ1Ø4Ø | Ø.967ØE+Ø3 | Ø.Ø2767Ø | Ø.5412E+Ø1 |
| Ø.ØØ1Ø92 | Ø.8431E+Ø3 | Ø.Ø27722 | Ø.5693E+Ø1 |
| Ø.ØØ1144 | Ø.6466E+Ø3 | Ø.Ø27774 | Ø.6767E+Ø1 |

FIG. B.4. OUTPUT DATA FILE DSPS.

b. Inputs and teletype outputs

i. number of data points (NPTS); step size of k (DELK); and step size of $\xi$ (DELX)

ii. program automatically calls in data file "PHILK" containing $S_j$ values

iii. summation index N(N); initial estimate of L (ALV(1)); and step size of L ($\Delta L \cong$ STPL)

iv. program prints computed values of E(L), E(L+ΔL) and questions whether the values are positive and negative (opposite signs) – answer yes (Y) or no (N); the program then outputs the interpolated value of L; this value, L', is used to compute the new values of E(L') and E(L'+ΔL/10)

v. again the program questions whether the values of E(L) and E(L+ΔL/10) are of opposite sign; if the answer is yes (Y) the program computes the final value of L, if the answer is no (N) the program has to be restated with a new value of L [abort (A) program]

vi. after program computes and prints out value of $\sigma^2$ the program can be aborted (A) or continued (C) for the computation of LG($k_j$,L) Eq. 3.35, Ref. 1, and the von Karman spectrum and autocorrelation.

c. Outputs

i. values of L and $\sigma^2$ (Eq. 3.25, Ref. 1) are printed out on TTY

ii. data file LG containing values of LG($k_j$,L) (Eq. 3.35, Ref. 1)

iii. data file PHIXI containing values of the von Karman autocorrelation function:

$$\phi_K(\xi) = \sigma^2 \frac{2^{2/3}}{\Gamma(\frac{1}{3})} (\beta\xi/L)^{1/3} K_{1/3}(\beta\xi/L)$$

$$- \frac{\beta\xi}{2L} K_{-2/3}(\beta\xi/L)$$

which is similar to Eq. 4.48, Ref. 1.

iv. data file PHIK containing values of the von Karman spectrum $\Phi_{KT}(k)$ or $F_j(L)$, Eq. 3.20, Ref. 1.

d. Example

A typical teletype printout of this interactive program is shown in Fig. B.5. Again, the user supplied information is underscored. Examples of the first page of the output data files are shown in Figs. B.6 through B.8.

```
@LOADER
*PART2,GAM,AK,AKDAT,PARAB,SIMQ$

PART2 24K CORE, 899 WORDS FREE
LOADER USED 26+5K CORE

EXIT.
^C
@SAVE (CORE FROM) 20 (TO) 777777 (ON) PART2.SAV [NEW FILE]
@ST

INPUT NO. OF POINTS TO BE READ, DELK,DELX
6450,4.7416995E-06,6.436 ,
INPUT N,L, & STEP SIZE OF L
6326,305.,30.5 <-------------- ΔL ≅ .1*L

ON    1 PASS E = -0.8844976E-03
ON    2 PASS E =  0.4644040E-02
ARE THERE POS. AND NEG. E'S (Y OR N) Y <- If answer is no restart
                                          program with another
FOR  2 PASS INTERPOLATTED L =   309.8796  estimate of L
ON    3 PASS E =  0.9391967E-04
ON    4 PASS E = -0.5123543E-03
ARE THERE POS. AND NEG. E'S (Y OR N) Y

FOR  4 PASS INTERPOLATTED L =   309.4071
CONTINUE OR ABORT? (C OR A) C

SIG2 =    1.326393
CONTINUE OR ABORT? (C OR A) C
EXIT
```

FIG. B.5. TTY PRINTOUT FOR RUNNING PROGRAM PART2.

OUTPUT OF ITEM 1, PART 4, PHASE II ATMO. TURB.
CREATED BY PROGRAM PART2
FOR L = 309.4071 METERS

| K | LG | K CONTD | LG CONTD |
|---|---|---|---|
| 0.4741700E-05 | -0.2538761E-03 | 0.1500274E-01 | -1.665575 |
| 0.9483399E-05 | -0.1015040E-02 | 0.1500748E-01 | -1.665575 |
| 0.1422510E-04 | -0.2282104E-02 | 0.1501222E-01 | -1.665576 |
| 0.1896680E-04 | -0.4052757E-02 | 0.1501696E-01 | -1.665577 |
| 0.2370850E-04 | -0.6323783E-02 | 0.1502170E-01 | -1.665577 |
| 0.2845020E-04 | -0.9091070E-02 | 0.1502645E-01 | -1.665578 |
| 0.3319190E-04 | -0.1234963E-01 | 0.1503119E-01 | -1.665579 |
| 0.3793360E-04 | -0.1609363E-01 | 0.1503593E-01 | -1.665579 |
| 0.4267530E-04 | -0.2031638E-01 | 0.1504067E-01 | -1.665580 |
| 0.4741700E-04 | -0.2501044E-01 | 0.1504541E-01 | -1.665581 |
| 0.5215869E-04 | -0.3016757E-01 | 0.1505015E-01 | -1.665581 |
| 0.5690039E-04 | -0.3577880E-01 | 0.1505490E-01 | -1.665582 |
| 0.6164209E-04 | -0.4183448E-01 | 0.1505964E-01 | -1.665583 |
| 0.6638379E-04 | -0.4832431E-01 | 0.1506438E-01 | -1.665584 |
| 0.7112549E-04 | -0.5523736E-01 | 0.1506912E-01 | -1.665584 |
| 0.7586719E-04 | -0.6256217E-01 | 0.1507386E-01 | -1.665585 |
| 0.8060889E-04 | -0.7028672E-01 | 0.1507860E-01 | -1.665586 |
| 0.8535059E-04 | -0.7839854E-01 | 0.1508335E-01 | -1.665586 |
| 0.9009229E-04 | -0.8688474E-01 | 0.1508809E-01 | -1.665587 |
| 0.9483399E-04 | -0.9573203E-01 | 0.1509283E-01 | -1.665588 |
| 0.9957569E-04 | -0.1049268 | 0.1509757E-01 | -1.665588 |
| 0.1043174E-03 | -0.1144552 | 0.1510231E-01 | -1.665589 |
| 0.1090591E-03 | -0.1243030 | 0.1510705E-01 | -1.665590 |
| 0.1138008E-03 | -0.1344560 | 0.1511180E-01 | -1.665590 |
| 0.1185425E-03 | -0.1448997 | 0.1511654E-01 | -1.665591 |
| 0.1232842E-03 | -0.1556195 | 0.1512128E-01 | -1.665592 |
| 0.1280259E-03 | -0.1666008 | 0.1512602E-01 | -1.665592 |
| 0.1327676E-03 | -0.1778290 | 0.1513076E-01 | -1.665593 |
| 0.1375093E-03 | -0.1892895 | 0.1513550E-01 | -1.665594 |
| 0.1422510E-03 | -0.2009678 | 0.1514025E-01 | -1.665594 |
| 0.1469927E-03 | -0.2128494 | 0.1514499E-01 | -1.665595 |
| 0.1517344E-03 | -0.2249200 | 0.1514973E-01 | -1.665596 |
| 0.1564761E-03 | -0.2371655 | 0.1515447E-01 | -1.665596 |
| 0.1612178E-03 | -0.2495720 | 0.1515921E-01 | -1.665597 |
| 0.1659595E-03 | -0.2621257 | 0.1516396E-01 | -1.665598 |
| 0.1707012E-03 | -0.2748132 | 0.1516870E-01 | -1.665598 |
| 0.1754429E-03 | -0.2876214 | 0.1517344E-01 | -1.665599 |
| 0.1801846E-03 | -0.3005372 | 0.1517818E-01 | -1.665600 |
| 0.1849263E-03 | -0.3135481 | 0.1518292E-01 | -1.665600 |
| 0.1896680E-03 | -0.3266419 | 0.1518766E-01 | -1.665601 |
| 0.1944097E-03 | -0.3398067 | 0.1519241E-01 | -1.665602 |
| 0.1991514E-03 | -0.3530309 | 0.1519715E-01 | -1.665602 |
| 0.2038931E-03 | -0.3663033 | 0.1520189E-01 | -1.665603 |
| 0.2086348E-03 | -0.3796131 | 0.1520663E-01 | -1.665604 |
| 0.2133765E-03 | -0.3929498 | 0.1521137E-01 | -1.665604 |
| 0.2181182E-03 | -0.4063035 | 0.1521611E-01 | -1.665605 |
| 0.2228599E-03 | -0.4196644 | 0.1522086E-01 | -1.665606 |

FIG. B.6. OUTPUT DATA FILE LG.

DATA FILE CREATED BY PROGRAM PART2

OUTPUT FOR PART 7 OF ITEM I,PHASE II
CALCULATIONS OF VON-KARMEN AUTOCORRELATION FN
SIGMA SORD = 1.326393

Comment: headings not correct in main program format

| ξ | βξ/L | $\phi_K(\xi)$ |
|---|---|---|
| 41.42915 | Ø.1ØØØØØØ | Ø.9711831 |
| 82.8583Ø | Ø.2ØØØØØØ | Ø.78Ø9889 |
| 124.2874 | Ø.3ØØØØØØ | Ø.636343Ø |
| 165.7166 | Ø.4ØØØØØØ | Ø.5221149 |
| 2Ø7.1457 | Ø.5ØØØØØØ | Ø.4288443 |
| 248.5749 | Ø.6ØØØØØØ | Ø.3515248 |
| 29Ø.ØØ4Ø | Ø.7ØØØØØØ | Ø.287Ø4Ø3 |
| 331.4332 | Ø.8ØØØØØØ | Ø.2331573 |
| 372.8623 | Ø.9ØØØØØØ | Ø.1881397 |
| 414.2915 | 1.ØØØØØØ | Ø.15Ø2596 |
| 455.72Ø6 | 1.1ØØØØØ | Ø.1185698 |
| 497.1498 | 1.2ØØØØØ | Ø.92Ø1595E-Ø1 |
| 538.5789 | 1.3ØØØØØ | Ø.6982Ø34E-Ø1 |
| 58Ø.ØØ81 | 1.4ØØØØØ | Ø.513369ØE-Ø1 |
| 621.4372 | 1.5ØØØØØ | Ø.3597379E-Ø1 |
| 662.8664 | 1.6ØØØØØ | Ø.2325935E-Ø1 |
| 7Ø4.2955 | 1.7ØØØØØ | Ø.12813Ø3E-Ø1 |
| 745.7247 | 1.8ØØØØØ | Ø.4265Ø6ØE-Ø2 |
| 787.1538 | 1.9ØØØØØ | -Ø.2655269E-Ø2 |
| 828.583Ø | 2.ØØØØØØ | -Ø.8212838E-Ø2 |
| 87Ø.Ø121 | 2.1ØØØØØ | -Ø.1262164E-Ø1 |
| 911.4412 | 2.2ØØØØØ | -Ø.16Ø3264E-Ø1 |
| 952.87Ø4 | 2.3ØØØØØ | -Ø.1864146E-Ø1 |
| 994.2995 | 2.4ØØØØØ | -Ø.2Ø54586E-Ø1 |
| 1Ø35.729 | 2.5ØØØØØ | -Ø.219Ø8Ø8E-Ø1 |
| 1Ø77.158 | 2.6ØØØØØ | -Ø.2277357E-Ø1 |
| 1118.587 | 2.7ØØØØØ | -Ø.232441ØE-Ø1 |
| 116Ø.Ø16 | 2.8ØØØØØ | -Ø.234ØØ22E-Ø1 |
| 12Ø1.445 | 2.9ØØØØØ | -Ø.2329913E-Ø1 |
| 1242.874 | 3.ØØØØØØ | -Ø.2298791E-Ø1 |
| 1284.3Ø4 | 3.1ØØØØØ | -Ø.225Ø897E-Ø1 |
| 1325.733 | 3.2ØØØØØ | -Ø.21899Ø3E-Ø1 |
| 1367.162 | 3.3ØØØØØ | -Ø.2119117E-Ø1 |
| 14Ø8.591 | 3.4ØØØØØ | -Ø.2Ø4Ø945E-Ø1 |
| 145Ø.Ø2Ø | 3.5ØØØØØ | -Ø.1958Ø42E-Ø1 |
| 1491.449 | 3.6ØØØØØ | -Ø.18715Ø2E-Ø1 |
| 1532.878 | 3.7ØØØØØ | -Ø.17831Ø7E-Ø1 |
| 1574.3Ø8 | 3.8ØØØØØ | -Ø.1694Ø23E-Ø1 |
| 1615.737 | 3.9ØØØØØ | -Ø.16Ø4555E-Ø1 |
| 1657.166 | 4.ØØØØØØ | -Ø.1517245E-Ø1 |
| 1698.595 | 4.1ØØØØØ | -Ø.1431294E-Ø1 |
| 174Ø.Ø24 | 4.2ØØØØØ | -Ø.1347734E-Ø1 |
| 1781.453 | 4.3ØØØØØ | -Ø.1267253E-Ø1 |

FIG. B.7. OUTPUT DATA FILE PHIXI.

: <RFISHER>PHIK.DAT;1 Mon 7-Nov-77 4:37PM PAGE 1

DATA FILE CREATED BY PROGRAM PART2

OUTPUT FOR PART 8 OF ITEM I, PHASE II
CALCULATIONS OF PHI K WITH L = 309.4071 AND SIG SQRD = 1.326393
**

Comment: 1) headings should line up with columns
2) PHIK $\doteq F_j(L)$

| K | PHIK | K CONTD | PHIK CONTD |
|---|---|---|---|
| 0.4741699E-05 | 410.4475 | 0.1500274E-01 | 2.432225 |
| 0.9483399E-05 | 410.6035 | 0.1500748E-01 | 2.430946 |
| 0.1422510E-04 | 410.8626 | 0.1501222E-01 | 2.429668 |
| 0.1896680E-04 | 411.2235 | 0.1501696E-01 | 2.428391 |
| 0.2370850E-04 | 411.6845 | 0.1502170E-01 | 2.427115 |
| 0.2845020E-04 | 412.2432 | 0.1502645E-01 | 2.425840 |
| 0.3319190E-04 | 412.8968 | 0.1503119E-01 | 2.424566 |
| 0.3793360E-04 | 413.6421 | 0.1503593E-01 | 2.423293 |
| 0.4267529E-04 | 414.4755 | 0.1504067E-01 | 2.422021 |
| 0.4741699E-04 | 415.3928 | 0.1504541E-01 | 2.420751 |
| 0.5215869E-04 | 416.3897 | 0.1505015E-01 | 2.419481 |
| 0.5690039E-04 | 417.4612 | 0.1505490E-01 | 2.418213 |
| 0.6164209E-04 | 418.6022 | 0.1505964E-01 | 2.416945 |
| 0.6638379E-04 | 419.8074 | 0.1506438E-01 | 2.415679 |
| 0.7112549E-04 | 421.0711 | 0.1506912E-01 | 2.414414 |
| 0.7586719E-04 | 422.3873 | 0.1507386E-01 | 2.413150 |
| 0.8060889E-04 | 423.7500 | 0.1507860E-01 | 2.411886 |
| 0.8535059E-04 | 425.1530 | 0.1508335E-01 | 2.410624 |
| 0.9009229E-04 | 426.5900 | 0.1508809E-01 | 2.409363 |
| 0.9483399E-04 | 428.0545 | 0.1509283E-01 | 2.408103 |
| 0.9957569E-04 | 429.5402 | 0.1509757E-01 | 2.406844 |
| 0.1043174E-03 | 431.0406 | 0.1510231E-01 | 2.405586 |
| 0.1090591E-03 | 432.5493 | 0.1510705E-01 | 2.404329 |
| 0.1138008E-03 | 434.0599 | 0.1511180E-01 | 2.403073 |
| 0.1185425E-03 | 435.5663 | 0.1511654E-01 | 2.401819 |
| 0.1232842E-03 | 437.0623 | 0.1512128E-01 | 2.400565 |
| 0.1280259E-03 | 438.5420 | 0.1512602E-01 | 2.399313 |
| 0.1327676E-03 | 439.9994 | 0.1513076E-01 | 2.398061 |
| 0.1375093E-03 | 441.4290 | 0.1513550E-01 | 2.396810 |
| 0.1422510E-03 | 442.8254 | 0.1514025E-01 | 2.395561 |
| 0.1469927E-03 | 444.1835 | 0.1514499E-01 | 2.394312 |
| 0.1517344E-03 | 445.4982 | 0.1514973E-01 | 2.393065 |
| 0.1564761E-03 | 446.7650 | 0.1515447E-01 | 2.391818 |
| 0.1612178E-03 | 447.9793 | 0.1515921E-01 | 2.390573 |
| 0.1659595E-03 | 449.1372 | 0.1516395E-01 | 2.389329 |
| 0.1707012E-03 | 450.2347 | 0.1516870E-01 | 2.388085 |
| 0.1754429E-03 | 451.2684 | 0.1517344E-01 | 2.386843 |
| 0.1801846E-03 | 452.2349 | 0.1517818E-01 | 2.385602 |
| 0.1849263E-03 | 453.1312 | 0.1518292E-01 | 2.384361 |
| 0.1896680E-03 | 453.9548 | 0.1518766E-01 | 2.383122 |
| 0.1944097E-03 | 454.7033 | 0.1519241E-01 | 2.381884 |
| 0.1991514E-03 | 455.3744 | 0.1519715E-01 | 2.380647 |
| 0.2038931E-03 | 455.9665 | 0.1520189E-01 | 2.379411 |

FIG. B.8. OUTPUT DATA FILE PHIK.

# APPENDIX C

## CONSTRAINED LEAST-SQUARES ESTIMATION OF TURBULENCE AUTOCORRELATION FUNCTION PARAMETERS

This series of programs computes the maximum likelihood estimate of $\sigma_f^2$ and length scale L for nonGaussian atmospheric turbulence. The method used is generally described in Sec. 4 of Ref. 1. The computer method used to compute $\sigma^2$ vs L is similar to that already discussed in Sec. B.1. The program, PART5, is used to compute up to 15 pairs of $\sigma^2(L)$;L values.

These values are used by program FINAL according to the method outlined in Sec. 4 of Ref. 1 to determine $\sigma_f$ and L. There are several versions of FINAL tailored to particular turbulence cases. Only one example is covered by this report.

### Program Outlines and Usage

**Program ATURB3.F4:** Computes the two sided power spectrum, $S_i$, the autocorrelation function, and smoothed power spectrum of the nonGaussian turbulence samples. Usage of this program is similar to that outlined in Sec. B.1.1 of this report.

**Program PART5.F4:** Computes $\sigma^2$ for up to 15 different values of length scale L (Eq. 4.46, Ref. 1).

a. Subroutines – none

b. Inputs

- i. number of data points (M41); step size of k(DELK); and step size of $\xi$ (DELX)
- ii. whether turbulence record is transverse (T) or longitudinal (L)
- iii. program reads in data file (PHILK) containing values of $S_i$ computed by ATURB3.
- iv. lower and upper bounds of k (KL,KU) and number of data points between the two limits (N)
- v. index counter (J) [from 1 to a limit of 15 successively] and value of integral scale L (AL)
- vi. option to continue or abort program after each computation of $\sigma^2(L)$.

c. Outputs

- i. index counter (J) and calculated value of $\sigma^2$ for the L chosen (Eq. 4.14, Ref. 1), printed out on TTY.

d. Example

A typical TTY interaction is shown in Fig. C.1.

Program FINAL.F4: Computes constrained least-squares estimation of turbulence autocorrelation function $\underset{\sim}{\phi}(\xi)$ (Eq. 4.1, Ref. 1)

a. Subroutines

i. AK1 and AKDAT — similar to AK and AKDAT described in Sec. B.1.1 except more data points are contained in AKDAT
ii. GAM and SIMQ — described in Sec. B.1.1
iii. PAR1 and PAR2 — curve fitting routines
iv. DGELG — simultaneous linear equation routine using double precision — from the IBM Scientific Subroutine Package
v. TRAP3 and TRAP6 — subroutines for integration by trapezoidal rule
vi. ANRP1 — interpolation routine
vii. SIMP2 — integration by Simpson's rule
viii. Function DECT and subroutine SET are incorporated in the main program body of FINAL.

b. Inputs and TTY printout

i. if the program has been executed once and the contents of all registers saved answer yes (Y) and the program will skip the input data phase (skip to step vi), otherwise answer no (N)
ii. input step size of $\xi$ (DELX) and number of combinations of $\sigma^2$ and L pairs that have been computed previously and *edited* into the main program FINAL (maximum of 11, indexed from 0 to 10). [Since these values are placed in the main body of the program there are different versions of FINAL for each different turbulence record]
iii. is turbulence record transverse (T) or longitudinal (L)?
iv. integer value of $\xi_H/L$-(NXIH), $\xi_H$(EH), and value of m (MM) used in the summation of Eq. 4.1, Ref. 1.
v. program automatically reads in data from file AUTO which contains values of the autocorrelation function computed by program ATURB3

[program prints out value of $\Phi(\xi_H)$]

```
@LOADER
*PART5$

PART5 16K CORE, 1022 WORDS FREE
LOADER USED 18+5K CORE

EXIT.
^C
@ST

INPUT NO. OF POINTS TO BE READ, DELK,DELX
12916,3.09744513E-06,9   .8525

IS RECORD TRANSVERSE OR LONGITUDIONAL (T OR L) T

INPUT KL,KU,N
324,12916,12592

INPUT J,L 1,20.

FOR J =   1 SIG2 =   0.3392710
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 1,25.

FOR J =   1 SIG2 =   0.3340475
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 3,35.

FOR J =   3 SIG2 =   0.3517353
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 4,45.

FOR J =   4 SIG2 =   0.3832800
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 5,55.

FOR J =   5 SIG2 =   0.4192905
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 6,65.

FOR J =   6 SIG2 =   0.4566356
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 7,75.

FOR J =   7 SIG2 =   0.4940900
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 8,85.

FOR J =   8 SIG2 =   0.5311373
PICK ANOTHER J,L (Y OR N) Y

INPUT J,L 9,95.

FOR J =   9 SIG2 =   0.5675591
PICK ANOTHER J,L (Y OR N) N

CPU TIME: 4:18.85      ELAPSED TIME: 14:46.23
NO EXECUTION ERRORS DETECTED

EXIT.
^C
@
```

FIG. C.1. TELETYPE PRINTOUT FOR RUNNING PART5.

vi. choose an initial value of $\sigma^2$ and its associated array index i; i.e., which index i in the array, $L_i(\sigma^2)$, $\sigma^2$ is closest to; the array $L_i(\sigma^2)$ was discussed in Sec. C.1.3b ii.

[program then prints out interpolated value of L for $\sigma^2$ value chosen; the value of $dL/d\sigma^2$ (Eq. 4.47, Ref. 1); and IER should equal $\phi$ or there is an error in the simultaneous linear equation routine]

[program outputs solution of $\sigma^2$ as explained in Sec. 4, Ref. 1, pages 106-109. When $\sigma^2_{INPUT} \cong \sigma^2_{OUTPUT}$ a solution has been reached]

vii. stop loop if $\sigma^2_{INPUT} = \sigma^2_{OUTPUT}$; if equality is not reached program reverts to step vi above, if equality is reached latest value of $\sigma^2$ becomes $\sigma^2_f$ [if equality is reached the program outputs the interpolated value of the length scale L associated with $\sigma^2_f$ and the coefficients of the polynomial approximation to the autocorrelation function of the slow component as discussed in Eq. 4.1 of Ref. 1]

viii. input value of step size of k (DELK) save image of core if program is to be rerun

c. Output data files

i. data file ITM2 contains values of the autocorrelation function $\phi(\xi)$

ii. data file ITM2L contains values of the normalized spectra $\sigma^2_f L\Phi_K(kL)$, Eq. 4.16, Ref. 1.

d. Examples

Figure C.2 is an example of the execution of program FINAL while the first pages from data files ITM2 and ITM2L are shown in Figs. C.3 and C.4.

```
@LOADER
*FINAL,AK1,AKDAT,ANRP1,GAM,PAR1,PAR2,SIMP2,SIMQ,TRAP3,TRAP6,DGELG$

FINAL 17K CORE, 245 WORDS FREE
LOADER USED 20+5K CORE

EXIT.
^C
@SAVE (CORE FROM) 20 (TO) 777777 (ON) FINAL.SAV;6 [NEW VERSION]
@ST

HAS DATA BEEN COMPUTED ?
N


INPUT DELX,LMAX
9.8525,10

IS RECORD TRANSVERSE OR LONGITUDINAL (T OR L) T

INPUT HIGHEST INDEX OF XI, EH, & M,  122,1202.005,1

RL(NXIH) =    1.186804
SIMPSONS RULE USED IN I5& I6 UP TO L INDEX =  5
PICK A SIGMA SQRD AND ITS ASSOCIATED L INDEX
.443,4

FOR SIGMA SQRD =  0.4430000      L INT =   61.37044
DLDS =     268.2701
IER =                0
FOR NCOUNT = 0 SIGMA SQRD =  0.4287123156550342D+00

STOP LOOP ? (Y OR N) N

PICK A SIGMA SQRD AND ITS ASSOCIATED L INDEX
.44,4

FOR SIGMA SQRD =  0.4400000      L INT =   60.56512
DLDS =     268.6132
IER =                0
FOR NCOUNT = 1 SIGMA SQRD =  0.4327416339782993D+00

STOP LOOP ? (Y OR N) N

PICK A SIGMA SQRD AND ITS ASSOCIATED L INDEX
.435,3

FOR SIGMA SQRD =  0.4350000      L INT =   59.22062
DLDS =     269.1852
IER =                0
FOR NCOUNT = 2 SIGMA SQRD =  0.4396278931497085D+00

STOP LOOP ? (Y OR N) N

PICK A SIGMA SQRD AND ITS ASSOCIATED L INDEX
.437,3

FOR SIGMA SQRD =  0.4370000      L INT =   59.75876
DLDS =     268.9564
IER =                0
FOR NCOUNT = 3 SIGMA SQRD =  0.4368476183747002D+00

STOP LOOP ? (Y OR N) Y

ON LAST PASS NCOUNT =  3 L =    59.75876
AND SIGMA SQRD =  0.4368476183747002D+00
A( 0) =    1.436821
A( 1) = -0.2101162E-03
INPUT DELK   4.9559E-05


CPU TIME: 21.79 ELAPSED TIME: 3:1.07
NO EXECUTION ERRORS DETECTED

EXIT.
^C
```

FIG. C.2. TELETYPE PRINTOUT FOR RUNNING PROGRAM FINAL.

DATA FILE CREATED BY PROGRAM FINAL
WITH M = 1 AND LENGTH = 1399.055 AUTOCOR. OF FILE VERT2

OUTPUT FOR PART 9.K
WITH L = 61.10212 AND SIGMA SQRD = 0.44262162495807450D+00
COEFFICIENT A(0) = 1.432016
COEFFICIENT A(1) = -0.2007736E-03

| XI | R(XI) | XI CONTD | R(XI) CONTD |
|---|---|---|---|
| 0.0000000 | 1.874637 | 695.4260 | 1.292224 |
| 8.181482 | 1.754460 | 703.6075 | 1.290595 |
| 16.36296 | 1.689349 | 711.7889 | 1.288966 |
| 24.54445 | 1.639438 | 719.9704 | 1.287335 |
| 32.72593 | 1.599677 | 728.1519 | 1.285703 |
| 40.90741 | 1.566909 | 736.3334 | 1.284070 |
| 49.08889 | 1.539465 | 744.5149 | 1.282437 |
| 57.27037 | 1.516304 | 752.6964 | 1.280803 |
| 65.45186 | 1.496680 | 760.8778 | 1.279168 |
| 73.63334 | 1.480015 | 769.0593 | 1.277532 |
| 81.81482 | 1.465732 | 777.2408 | 1.275896 |
| 89.99630 | 1.453514 | 785.4223 | 1.274259 |
| 98.17778 | 1.443010 | 793.6038 | 1.272622 |
| 106.3593 | 1.433951 | 801.7852 | 1.270985 |
| 114.5407 | 1.426150 | 809.9667 | 1.269347 |
| 122.7222 | 1.419331 | 818.1482 | 1.267708 |
| 130.9037 | 1.413496 | 826.3297 | 1.266111 |
| 139.0852 | 1.408357 | 834.5112 | 1.264468 |
| 147.2667 | 1.403872 | 842.6927 | 1.262825 |
| 155.4482 | 1.399920 | 850.8741 | 1.261183 |
| 163.6296 | 1.396423 | 859.0556 | 1.259540 |
| 171.8111 | 1.393309 | 867.2371 | 1.257897 |
| 179.9926 | 1.390528 | 875.4186 | 1.256255 |
| 188.1741 | 1.388015 | 883.6001 | 1.254612 |
| 196.3556 | 1.385737 | 891.7815 | 1.252970 |
| 204.5371 | 1.383639 | 899.9630 | 1.251327 |
| 212.7185 | 1.381708 | 908.1445 | 1.249684 |
| 220.9000 | 1.379908 | 916.3260 | 1.248042 |
| 229.0815 | 1.378214 | 924.5075 | 1.246399 |
| 237.2630 | 1.376605 | 932.6890 | 1.244756 |
| 245.4445 | 1.375066 | 940.8704 | 1.243114 |
| 253.6259 | 1.373583 | 949.0519 | 1.241471 |
| 261.8074 | 1.372144 | 957.2334 | 1.239829 |
| 269.9889 | 1.370738 | 965.4149 | 1.238186 |
| 278.1704 | 1.369356 | 973.5964 | 1.236543 |
| 286.3519 | 1.367990 | 981.7778 | 1.234901 |
| 294.5334 | 1.366636 | 989.9593 | 1.233258 |
| 302.7148 | 1.365238 | 998.1408 | 1.231615 |
| 310.8963 | 1.363943 | 1006.322 | 1.229973 |

FIG. C.3. OUTPUT DATA FILE ITM2.

: <RFISHER>ITM2L.DAT;7 Thu 13-Apr-78 3:33PM PAGE 1

DATA FILE CREATED BY PROGRAM FINAL
WITH M = 1 AND LENGTH = 1399.055 AUTOCOR. OF FILE VERT2

OUTPUT FOR PART 9.L
WITH L = 61.10212 AND SIGMA SQRD = 0.44262162495807450+00

| K | L*SIGMA SQRD*PHIK | K CONTD | L*SIGMA SQRD*PHIK CONTD |
|---|---|---|---|
| 0.0000000 | 27.04512 | 0.2537421E-01 | 0.9893473 |
| 0.4955900E-04 | 27.05972 | 0.2542377E-01 | 0.9861678 |
| 0.9911800E-04 | 27.10322 | 0.2547333E-01 | 0.9830046 |
| 0.1486770E-03 | 27.17468 | 0.2552289E-01 | 0.9798576 |
| 0.1982360E-03 | 27.27250 | 0.2557244E-01 | 0.9767265 |
| 0.2477950E-03 | 27.39492 | 0.2562200E-01 | 0.9736113 |
| 0.2973540E-03 | 27.53912 | 0.2567156E-01 | 0.9705120 |
| 0.3469130E-03 | 27.70224 | 0.2572112E-01 | 0.9674285 |
| 0.3964720E-03 | 27.88132 | 0.2577068E-01 | 0.9643604 |
| 0.4460310E-03 | 28.07196 | 0.2582024E-01 | 0.9613077 |
| 0.4955900E-03 | 28.27141 | 0.2586980E-01 | 0.9582707 |
| 0.5451490E-03 | 28.47563 | 0.2591936E-01 | 0.9552487 |
| 0.5947080E-03 | 28.68093 | 0.2596892E-01 | 0.9522419 |
| 0.6442670E-03 | 28.88372 | 0.2601848E-01 | 0.9492503 |
| 0.6938260E-03 | 29.08057 | 0.2606803E-01 | 0.9462735 |
| 0.7433850E-03 | 29.26826 | 0.2611759E-01 | 0.9433116 |
| 0.7929440E-03 | 29.44337 | 0.2616715E-01 | 0.9403645 |
| 0.8425030E-03 | 29.60477 | 0.2621671E-01 | 0.9374319 |
| 0.8920620E-03 | 29.74855 | 0.2626627E-01 | 0.9345141 |
| 0.9416210E-03 | 29.87358 | 0.2631583E-01 | 0.9315105 |
| 0.9911800E-03 | 29.97794 | 0.2636539E-01 | 0.9287214 |
| 0.1040739E-02 | 30.06047 | 0.2641495E-01 | 0.9258465 |
| 0.1090298E-02 | 30.12026 | 0.2646451E-01 | 0.9229858 |
| 0.1139857E-02 | 30.15670 | 0.2651407E-01 | 0.9201392 |
| 0.1189416E-02 | 30.16946 | 0.2656362E-01 | 0.9173066 |
| 0.1238975E-02 | 30.15851 | 0.2661318E-01 | 0.9144877 |
| 0.1288534E-02 | 30.12433 | 0.2666274E-01 | 0.9116827 |
| 0.1338093E-02 | 30.06643 | 0.2671230E-01 | 0.9088914 |
| 0.1387652E-02 | 29.98631 | 0.2676186E-01 | 0.9061136 |
| 0.1437211E-02 | 29.88441 | 0.2681142E-01 | 0.9033495 |
| 0.1486770E-02 | 29.76152 | 0.2686098E-01 | 0.9005986 |
| 0.1536329E-02 | 29.61890 | 0.2691054E-01 | 0.8978612 |
| 0.1585888E-02 | 29.45732 | 0.2696010E-01 | 0.8951369 |
| 0.1635447E-02 | 29.27801 | 0.2700966E-01 | 0.8924260 |
| 0.1685006E-02 | 29.08211 | 0.2705921E-01 | 0.8897280 |
| 0.1734565E-02 | 28.87081 | 0.2710877E-01 | 0.8870430 |
| 0.1784124E-02 | 28.64530 | 0.2715833E-01 | 0.8843709 |
| 0.1833683E-02 | 28.40675 | 0.2720789E-01 | 0.8817116 |
| 0.1883242E-02 | 28.15633 | 0.2725745E-01 | 0.8790650 |
| 0.1932801E-02 | 27.89519 | 0.2730701E-01 | 0.8764312 |
| 0.1982360E-02 | 27.62442 | 0.2735657E-01 | 0.8738097 |
| 0.2031919E-02 | 27.34508 | 0.2740613E-01 | 0.8712008 |

FIG. C.4. OUTPUT DATA FILE ITM2L.

## APPENDIX D

## POWER SPECTRAL DENSITY OF THE INSTANTANEOUS VARIANCE $\sigma_f^2(t)$

The estimation procedure for calculating the power spectrum of $\sigma_f^2$ is discussed in Sec. 6.2 of Ref. 5. There are two main programs used to compute $\sigma_f^2$–ATURB4.F4 and ITEM3.F4. The first program, ATURB4, computes the two sided power spectrum of high pass filtered atmospheric turbulence data, $w_h(t)$. The unsmoothed power spectral density, $\Phi_\ell(k)$, and its autocorrelation, $R_{w_h}(\xi)$, of the high pass filtered data are all computed by this one program. From the square of the high pass filtered samples, $w_h^2$, the sample spectrum and autocorrelation function, $R_{w_h^2}(\xi)$ are formed by a slight variation of ATURB4 called ATUR4A.F4.

The second program, ITEM3, computes the autocorrelation function, $R_{\sigma_f^2}(\tau)$, Eq. 6.40, Ref. 5, and the two sided smoothed spectrum of $\sigma_f^2(x)$:

$$\Phi_{\sigma_f^2}(k) = \{E[\sigma_f^2]\}\left\{\delta(f) + \int_{-M}^{M} p_0(\xi)\left(\frac{R_{w_h^2}(\xi)-[R_{w_h^2}(0)]^2-2[R_{w_h}(\xi)]^2}{[R_{w_h}(0)]^2+2\,[R_{w_h}(\xi)]^2}\right) \times e^{-i2\pi k\xi}d\xi\right\}$$

$$\text{where } p_0(\xi) = \left\{\frac{1}{\pi}\left|\sin\frac{\pi\xi}{M}\right| + \left(1-\frac{|\xi|}{M}\right)\cos\frac{\pi\xi}{M}\right\} |\xi| \leq M$$

This equation is similar to Eq. 6.49, Ref. 5 except the Papoulis window, $p_0(\xi)$, has been added to smooth the power spectrum.

### Program Outlines and Usage

**Program ATURB4:** Computes the two sided power spectra and autocorrelation function of high pass filtered turbulence samples. (ATUR4A parallels ATURB4 except it operates on the square of high pass filtered turbulence samples).

a. Subroutines

i. CFFT and CFFT1 — fast Fourier transform routines for ATURB4 and ATUR4A respectively
ii. SIMP — integration by Simpson's rule
iii. HPDES — digital high pass filter (routine C-2 of Appendix C of Ref. 6)

b. Inputs

i. through v are the same as Sec. B.1.1b i to v
vi. input the cut-off frequency $k_c$(FC) for the filter routine; sampling interval (TS) in seconds; and number of filter sections (NS), see Ref. 6
vii. yes (Y) or no (N) to performing integration check as per B.1.1b vi

c. Outputs

i. data files PHILK and FPSD2 contain positive frequency domain (k) values of the power spectrum of the high pass filtered data and the high pass filtered squared samples respectively
ii. data files AUTO and AUTF2 contain the autocorrelation functions of the filtered and filtered squared data

d. Example

The teletype printout involved with program ATURB4 is shown in Fig. D.1. ATUR4A would involve an identical interaction (except CFFT1 would be loaded instead of CFFT with the main program). Figures D.2 through D.5 show examples of outputs of ATURB4 — PHILK and AUTO; and ATUR4A — FPSD2 and AUTF2.

**Program ITEM3.F4:** Computes the spectrum $\Phi_{\sigma_f^2}(k)$ and autocorrelation function $R_{\sigma_f^2}(\xi)$.

a. Subroutines

i. CFFT1 — fast Fourier transform program

b. Inputs

i. step size of $\xi$ (DELX)
ii. program automatically reads data from files AUTO and AUTF2 containing $R_{w_h}$ and $R_{w_h^2}$

```
LOADER U^C
^C
@LOADER
*ATURB4,CFFT,SIMP,HPDES$

ATURB4 69K CORE, 762 WORDS FREE
LOADER USED 72+5K CORE

EXIT.
^C
@SAVE (CORE FROM) 20 (TO) 777777 (ON) ATURB4.SAV;2 [NEW VERSION]
@ST


INPUT SPEED OF CRAFT (M/SEC)
197.05


INPUT TOTAL NO. OF POINTS TO BE USED IN 2L M. OF DATA
AND POWER OF TWO OF THAT NO.
32768
15


INPUT NO. OF POINTS OF W(X) TO  BE READ
AND SAMPLING RATE OF DATA 15000,.05


INPUT VALUE OF MPTS
AND POWER OF TWO OF THAT NO.
1024
10


INPUT DATA FILE NAME THAT CONTAINS SAMPLES
OF W(X) AND NO. POINTS NXRAY
VERT2
15000

INPUT CUT-OFF FREQ,SAMPLING INTERVAL,NO. OF FILTER SECTIONS5.9115E-01,.0
**5,2
PERFORM INTEGRATION CHECK (Y OR N)   Y


INTEGRAL OF PHI OF L (K) =  0.2211E+00

CPU TIME: 6:50.24        ELAPSED TIME: 13:19.00
NO EXECUTION ERRORS DETECTED

EXIT.
^C
```

FIG. D.1. TELETYPE PRINTOUT FOR RUNNING ATURB4.

DATA FILE CREATED BY PROGRAM ATURB4

HIGH PASS FILTERED DATA
CUT-OFF FREQ = 0.5911500 (K = 0.3000000E-02)

POWER SPECTRUM OF PHI OF L(K)
DATA TAKEN FROM FILE VERT2

32768 DATA POINTS WERE USED IN 2L = 322846.7190 METER
1024 DATA POINTS WERE USED IN M = 10088.9600 METER

17760 ZEROS WERE ADDED TO DATA

MEAN VALUE OF W(X) = -0.74214E-05 M/SEC
MEAN SQ. VALUE = 0.22110E+00 (M/SEC)**2

<W OF L(X)**2> = 0.2210

PRINTOUT OF THE VALUES OF THE POWER SPECTRUM

| K | PS VALUE | K CONTD | PS VALUE CONTD |
|---|---|---|---|
| 0.000000 | 0.1348E-13 | 0.025374 | 0.8273E+00 |
| 0.000003 | 0.1048E-04 | 0.025377 | 0.1553E+00 |
| 0.000006 | 0.9635E-05 | 0.025380 | 0.9468E-01 |
| 0.000009 | 0.7124E-05 | 0.025384 | 0.4748E-01 |
| 0.000012 | 0.9423E-05 | 0.025387 | 0.1841E-01 |
| 0.000015 | 0.7117E-05 | 0.025390 | 0.1567E+00 |
| 0.000019 | 0.9116E-05 | 0.025393 | 0.5845E+00 |
| 0.000022 | 0.7376E-05 | 0.025396 | 0.9412E+00 |
| 0.000025 | 0.8759E-05 | 0.025399 | 0.8532E+00 |
| 0.000028 | 0.7711E-05 | 0.025402 | 0.4245E+00 |
| 0.000031 | 0.8414E-05 | 0.025405 | 0.1030E+01 |
| 0.000034 | 0.8044E-05 | 0.025408 | 0.1135E+01 |
| 0.000037 | 0.8110E-05 | 0.025411 | 0.3692E+00 |
| 0.000040 | 0.8260E-05 | 0.025415 | 0.7724E+00 |
| 0.000043 | 0.7895E-05 | 0.025418 | 0.6522E+00 |
| 0.000046 | 0.8417E-05 | 0.025421 | 0.2323E+00 |
| 0.000050 | 0.7823E-05 | 0.025424 | 0.2026E-01 |
| 0.000053 | 0.8465E-05 | 0.025427 | 0.4337E+00 |
| 0.000056 | 0.7831E-05 | 0.025430 | 0.8168E+00 |
| 0.000059 | 0.8410E-05 | 0.025433 | 0.5790E+00 |
| 0.000062 | 0.7951E-05 | 0.025436 | 0.1324E+01 |
| 0.000065 | 0.8351E-05 | 0.025439 | 0.1206E+01 |
| 0.000068 | 0.8036E-05 | 0.025442 | 0.4376E+00 |
| 0.000071 | 0.8143E-05 | 0.025446 | 0.1843E+01 |

FIG. D.2. OUTPUT DATA FILE PHILK.

```
DATA FILE CREATED BY PROGRAM ATURB4

AUTOCORRELATION OF
HIGH PASS FILTERED NON-HOMOGENEOUS SAMPLE
DATA TAKEN FROM FILE VERT2

 32768 DATA POINTS  WERE USED IN 2L =      322846.7190 METER
 1024 DATA POINTS WERE USED IN M =        10088.9600 METER

 17768 ZEROS WERE ADDED TO DATA

MEAN VALUE OF W(X) =     -0.74214E-05 M/SEC
MEAN SQ. VALUE =      0.22110E+00 (M/SEC)**2

<W OF L(X)**2> =        0.2210

 TRUCATION POINT WAS   13000.00     METERS
WHICH CONTAINS    1319 POINTS

PRINTOUT OF THE VALUES OF THE AUTOCORRELATION
```

| X | RL | RL/R0 |
|---|---|---|
| 0.0000000 | 0.2210828 | 1.0000000 |
| 9.852500 | 0.1366631 | 0.6181535 |
| 19.70500 | 0.7223809E-01 | 0.3267467 |
| 29.55750 | 0.2718233E-01 | 0.1229509 |
| 39.41000 | -0.5175197E-02 | -0.2340841E-01 |
| 49.26250 | -0.2545530E-01 | -0.1151392 |
| 59.11500 | -0.3989603E-01 | -0.1804574 |
| 68.96750 | -0.4644818E-01 | -0.2100940 |
| 78.82000 | -0.4698692E-01 | -0.2125308 |
| 88.57250 | -0.4595287E-01 | -0.2078536 |
| 98.52500 | -0.4678660E-01 | -0.2116248 |
| 108.3775 | -0.4089274E-01 | -0.1849657 |
| 118.2300 | -0.3525967E-01 | -0.1594862 |
| 128.0825 | -0.3052934E-01 | -0.1380901 |
| 137.9350 | -0.2747492E-01 | -0.1242743 |
| 147.7875 | -0.2431214E-01 | -0.1099685 |
| 157.6400 | -0.2179220E-01 | -0.9857031E-01 |
| 167.4925 | -0.1709930E-01 | -0.7734341E-01 |
| 177.3450 | -0.1068496E-01 | -0.4833010E-01 |
| 187.1975 | -0.3274173E-02 | -0.1480971E-01 |
| 197.0500 | 0.1146689E-02 | 0.5186693E-02 |
| 206.9025 | 0.6017922E-02 | 0.2722021E-01 |
| 216.7550 | 0.9448106E-02 | 0.4273559E-01 |
| 226.6075 | 0.1239062E-01 | 0.5604516E-01 |

FIG. D.3. OUTPUT DATA FILE AUTO.

DATA FILE CREATED BY PROGRAM ATURB4

HIGH PASS FILTERED DATA
CUT-OFF FREQ = 0.5911500 (K = 0.3000000E-02)

FILTERED VERSION SQUARED BEFORE TRANSFORMING
POWER SPECTRUM OF PHI OF L(K)
DATA TAKEN FROM FILE VERT2

32768 DATA POINTS WERE USED IN 2L = 322846.7190 METER
1024 DATA POINTS WERE USED IN M = 10088.9600 METER

17768 ZEROS WERE ADDED TO DATA

MEAN VALUE OF W(X) = -0.74214E-05 M/SEC
MEAN SQ. VALUE = 0.22110E+00 (M/SEC)**2

<W OF L(X)**2> = 0.4627

PRINTOUT OF THE VALUES OF THE POWER SPECTRUM

| K | PS VALUE | K CONTD | PS VALUE CONTD |
|---|---|---|---|
| 0.000000 | 0.7224E+04 | 0.025374 | 0.2112E+01 |
| 0.000003 | 0.3957E+04 | 0.025377 | 0.3822E+01 |
| 0.000006 | 0.6459E+03 | 0.025380 | 0.2336E+01 |
| 0.000009 | 0.1359E+04 | 0.025384 | 0.2389E+01 |
| 0.000012 | 0.1438E+04 | 0.025387 | 0.5410E+01 |
| 0.000015 | 0.5900E+03 | 0.025390 | 0.3918E+01 |
| 0.000019 | 0.3731E+03 | 0.025393 | 0.7773E+00 |
| 0.000022 | 0.1381E+03 | 0.025396 | 0.2330E+01 |
| 0.000025 | 0.7375E+02 | 0.025399 | 0.3844E+01 |
| 0.000028 | 0.3612E+03 | 0.025402 | 0.2807E+01 |
| 0.000031 | 0.5751E+03 | 0.025405 | 0.2275E+01 |
| 0.000034 | 0.3753E+03 | 0.025408 | 0.1228E+01 |
| 0.000037 | 0.2580E+02 | 0.025411 | 0.1186E+00 |
| 0.000040 | 0.2465E+03 | 0.025415 | 0.1594E+01 |
| 0.000043 | 0.4334E+03 | 0.025418 | 0.2782E+01 |
| 0.000046 | 0.1599E+03 | 0.025421 | 0.1292E+01 |
| 0.000050 | 0.2256E+03 | 0.025424 | 0.7048E+00 |
| 0.000053 | 0.3186E+03 | 0.025427 | 0.3248E+01 |
| 0.000056 | 0.1056E+03 | 0.025430 | 0.4490E+01 |
| 0.000059 | 0.2861E+02 | 0.025433 | 0.1749E+01 |
| 0.000062 | 0.2717E+02 | 0.025436 | 0.4573E+00 |
| 0.000065 | 0.6028E+02 | 0.025439 | 0.3822E+01 |
| 0.000068 | 0.1258E+03 | 0.025442 | 0.7261E+01 |

FIG. D.4. OUTPUT DATA FILE FPSD2.

DATA FILE CREATED BY PROGRAM ATURB4

AUTOCORRELATION OF
HIGH PASS FILTERED AND SQUARED NON-HOMOGENEOUS SAMPLE
DATA TAKEN FROM FILE VERT2

32768 DATA POINTS WERE USED IN 2L = 322846.7190 METER
1024 DATA POINTS WERE USED IN M = 10088.9600 METER

17768 ZEROS WERE ADDED TO DATA

MEAN VALUE OF W(X) = -0.74214E-05 M/SEC
MEAN SQ. VALUE = 0.22110E+00 (M/SEC)**2

<W OF L(X)**2> = 0.4627

TRUCATION POINT WAS 13000.00 METERS
WHICH CONTAINS 1319 POINTS

PRINTOUT OF THE VALUES OF THE AUTOCORRELATION

| X | RL | RL/R0 |
|---|---|---|
| 0.0000000 | 0.4629917 | 1.0000000 |
| 9.852500 | 0.2634734 | 0.5690672 |
| 19.70500 | 0.1654293 | 0.3573051 |
| 29.55750 | 0.1367036 | 0.2952614 |
| 39.41000 | 0.1272111 | 0.2747590 |
| 49.26250 | 0.1450926 | 0.3133805 |
| 59.11500 | 0.1577147 | 0.3406427 |
| 68.96750 | 0.1564650 | 0.3379434 |
| 78.82000 | 0.1460383 | 0.3154231 |
| 88.57250 | 0.1341455 | 0.2897362 |
| 98.52500 | 0.1194356 | 0.2579648 |
| 108.3775 | 0.1148353 | 0.2480288 |
| 118.2300 | 0.1207423 | 0.2607872 |
| 128.0825 | 0.1143026 | 0.2468782 |
| 137.9350 | 0.1129238 | 0.2439003 |
| 147.7875 | 0.1129470 | 0.2439503 |
| 157.6400 | 0.1232595 | 0.2662241 |
| 167.4925 | 0.1278594 | 0.2761592 |
| 177.3450 | 0.1114554 | 0.2407286 |
| 187.1975 | 0.1117146 | 0.2412886 |
| 197.0500 | 0.1127996 | 0.2436319 |
| 206.9025 | 0.1065844 | 0.2302079 |
| 216.7550 | 0.1123870 | 0.2427407 |
| 226.6075 | 0.1149910 | 0.2483651 |

FIG. D.5. OUTPUT DATA FILE AUTF2.

iii. input $\sigma_f^2$ (SIGF) and L determined in Sec. C.1.3

iv. number of points in Fourier transform (MPTS) and power of two of that number (MPWRM)

c. Outputs

i. data file RSIGF containing values of $R_{\sigma_f^2}(\xi)$

ii. data file PHIF containing the smoothed power spectrum values of $\Phi_{\sigma_f^2}(k)$.

d. Example

An example of the input through the teletype is given in Fig. D.6 while Figs. D.7 and D.8 show one page from each of the data files RSIGF and PHIF.

```
@LOADER
*ITEM3,CFFT1$

ITEM3 11K CORE, 353 WORDS FREE
LOADER USED 14+5K CORE

EXIT.
^C
@ST


INPUT DELX
9.8525

RWH(MAOFF) =   0.2880105E-02
RWH2(MAOFF) =   0.4643580E-01
INPUT SIGMA SQRD, L
.4609,65.89

INPUT M AND POWER OF 2
1024,10


CPU TIME: 52.13 ELAPSED TIME: 1:53.21
NO EXECUTION ERRORS DETECTED

EXIT.
^C
```

FIG. D.6. TELETYPE OUTPUT FOR RUNNING ITEM3.

```
!  <RFISHER>RSIGF.DAT;3   Thu 1-Dec-77 4:30PM          PAGE 1

DATA FILE CREATED BY PROGRAM ITEM3
FOR SIGMA SQUARED =  0.4609000      AND AL =    65.89000


  XI          RSIGF
 0.0000000     0.6707419
  9.852500     0.6490613
  19.70500     0.5924702
  29.55750     0.5766969
  39.41000     0.5522717
  49.26250     0.6143047
  59.11500     0.6435365
  68.96750     0.6248567
  78.82000     0.5821151
  88.67250     0.5366453
  98.52500     0.4764114
  108.3775     0.4671271
  118.2300     0.4993594
  128.0825     0.4785250
  137.9350     0.4760772
  147.7875     0.4792910
  157.6400     0.5254913
  167.4925     0.5491249
  177.3450     0.4821481
  187.1975     0.4853142
  197.0500     0.4902162
  206.9025     0.4625451
  216.7550     0.4866718
  226.6075     0.4966468
  236.4600     0.5135362
  246.3125     0.5233681
  256.1650     0.5119246
  266.0175     0.4612299
  275.8700     0.4421263
  285.7225     0.4679804
  295.5750     0.4891256
  305.4275     0.5169329
  315.2800     0.5298064
  325.1325     0.5117880
  334.9850     0.5550355
  344.8375     0.5224204
  354.6900     0.4742931
  364.5425     0.4484167
  374.3950     0.4570057
  384.2475     0.4884470
  394.1000     0.5384777
  403.9525     0.5555076
  413.8050     0.5482229
  423.6575     0.4959692
  433.5100     0.4385263
  443.3625     0.4048591
```

FIG. D.7.  OUTPUT DATA FILE RSIGF.

; <RFISHER>PHIF.DAT;3    Thu 1-Dec-77 4:30PM    PAGE 1

DATA FILE CREATED BY PROGRAM ITEM3

SMOOTHED POWER SPECTRUM PHI OF SIGMA SQRD F(K)

1024 DATA POINTS WERE USED IN M =    10080.9600 FT.
RWH(0) =   0.2210828

PRINTOUT OF VALUES OF THE SMOOTHED POWER SPECTRUM

| K | SPS VALUE | K CONTD | SPS VALUE CONTD |
|---|---|---|---|
| 0.000000 | 0.1388E+04 | 0.025374 | 0.1005E+01 |
| 0.000050 | 0.1007E+04 | 0.025424 | 0.3967E+00 |
| 0.000099 | 0.4127E+03 | 0.025473 | -0.1887E+01 |
| 0.000149 | 0.1884E+03 | 0.025523 | -0.3033E+01 |
| 0.000198 | 0.1740E+03 | 0.025573 | -0.7258E+00 |
| 0.000248 | 0.1394E+03 | 0.025622 | 0.3225E+01 |
| 0.000297 | 0.8768E+02 | 0.025672 | 0.3959E+01 |
| 0.000347 | 0.5885E+02 | 0.025721 | 0.1401E+01 |
| 0.000396 | 0.3664E+02 | 0.025771 | 0.1112E+01 |
| 0.000446 | 0.2377E+02 | 0.025820 | 0.3751E+01 |
| 0.000496 | 0.4061E+02 | 0.025870 | 0.3992E+01 |
| 0.000545 | 0.6361E+02 | 0.025919 | 0.1706E+01 |
| 0.000595 | 0.6199E+02 | 0.025969 | 0.6458E+00 |
| 0.000644 | 0.5725E+02 | 0.026019 | -0.1602E+00 |
| 0.000694 | 0.7100E+02 | 0.026068 | -0.1048E+01 |
| 0.000743 | 0.7442E+02 | 0.026118 | 0.3609E+00 |
| 0.000793 | 0.5749E+02 | 0.026167 | 0.1975E+01 |
| 0.000843 | 0.5533E+02 | 0.026217 | 0.2240E+01 |
| 0.000892 | 0.7045E+02 | 0.026266 | 0.2586E+01 |
| 0.000942 | 0.6358E+02 | 0.026316 | 0.1789E+01 |
| 0.000991 | 0.8858E+02 | 0.026365 | 0.8118E+00 |
| 0.001041 | -0.1601E+01 | 0.026415 | 0.1144E+01 |
| 0.001090 | -0.9461E+01 | 0.026465 | 0.1232E+01 |
| 0.001140 | -0.6385E+01 | 0.026514 | 0.8538E+00 |
| 0.001189 | -0.5456E+01 | 0.026564 | -0.5051E+00 |
| 0.001239 | -0.7733E+01 | 0.026613 | -0.2148E+01 |
| 0.001289 | -0.9891E+01 | 0.026663 | -0.2239E+01 |
| 0.001338 | -0.6303E+01 | 0.026712 | -0.1680E+01 |
| 0.001388 | 0.3264E+01 | 0.026762 | -0.1419E+01 |
| 0.001437 | 0.8319E+01 | 0.026811 | -0.1143E+01 |
| 0.001487 | 0.8763E+01 | 0.026861 | -0.1863E+01 |
| 0.001536 | 0.9509E+01 | 0.026911 | -0.4201E+01 |
| 0.001586 | 0.7367E+01 | 0.026960 | -0.4662E+01 |
| 0.001635 | 0.8612E+00 | 0.027010 | -0.1828E+01 |
| 0.001685 | -0.7582E+01 | 0.027059 | 0.4374E-02 |
| 0.001735 | -0.9394E+01 | 0.027109 | -0.2595E+00 |
| 0.001784 | 0.2060E+01 | 0.027158 | 0.1573E+00 |
| 0.001834 | 0.1579E+02 | 0.027208 | 0.1628E+01 |
| 0.001883 | 0.1587E+02 | 0.027258 | 0.1666E+01 |

FIG. D.8. OUTPUT DATA FILE PHIF.

# APPENDIX E

## PROBABILITY DENSITY ESTIMATION OF THE INSTANTANEOUS VARIANCE $\sigma_f^2(t)$ AND THE "SLOW" TURBULENCE COMPONENT $w_s(t)$

The methods used to develop the probability density function of $\sigma_f^2(t)$ and $w_s(t)$ are given in Sec. 6.3 and 6.4 of Ref. 5. Three programs are necessary to carry out the computations — MOMENT.F4, ITEM.F4, and GDIST6.F4. MOMENT calculates the moments of $\sigma_f^2$ as per Eq. 6.71, Ref. 5. GDIST computes the probability density function of $\sigma_f^2$ as described in Ref. 5 — BBN Report 3476, pages 88-91. ITEM4 calculates the moments of $w_s$, $M_{w_s}$, Eq. 6.87, Ref. 5 and the probability density function $P_{w_s}$, Eq. 6.93, Ref. 5.

### Program Outlines and Usage

**Program MOMENT.F4:** Computes first 8 moments of $\sigma_f^2$; $M_{\sigma_f^2}$, Eq. 6.71, Ref. 5.

a. Subroutines

- i. BIN — tabulates the number of samples in a particular bin
- ii. BINSQ — counts the number of filtered squared samples in a certain bin
- iii. HPDES — high pass digital filter routine from Ref. 6

b. Inputs

- i. number of data points to be used (NPTS)
- ii. name to be used for output data file [FLE, A5 format]
- iii. bin width (BINW)
- iv. number of bins (NBIN)
- v. cut-off frequency $k_c$ (FC); sampling interval (TS), and number of filter section (NS)
- vi. bin width and number of bins for filtered data (BINW and NBIN)
- vii. bin width and number of bins for filtered squared data (BINW1 and NBIN1)
- viii. input $\sigma_f^2$ (computed in Sec. C.1.3)

c. Outputs (at TTY)

- i. moments of w(t); $M^n_w$ n = 1 to 8
- ii. moments of $w_h$; $M^n_{w_h}$ n = 1 to 8

iii. moments of $w_h^2$; $M^n_{w_h}$ $n = 1$ to 8

iv. moments of $\sigma_f^2$; $M^n_{\sigma_f^2}$ $n = 1$ to 8

d. Example

Figure E.1 contains an example of the teletype inputs and outputs.

**Program GDIST6.F4:** Computes the probability density distribution $P_{\sigma_f^2}$

a. Subroutine

i. GAM — computes the gamma function $\Gamma$

b. Inputs

i. moments of $\sigma_f^2$; $M_{\sigma_f^2}$

c. Outputs (at the TTY)

i. $\gamma$ — Eq. 6.75, Ref. 5
ii. $P_{\sigma_f^2}$ — the probability density function of $\sigma_f^2$

d. Example

Figure E.2 contains a *partial* listing of the terminal printout for program GDIST6.

**Program ITEM4.F4:** Calculates the probability density function of $w_s(t)$: $P_{w_s}$

a. Subroutine

i. FAC1 — factorial routine

b. Inputs

i. moments of $w_h$ and $w$; $M^n_{w_h}$ and $M^n_w$, $n = 1$ to 8

ii. $\sigma_f^2$

```
:LOADER
:MOMENT,BIN,BINSQ,HPDESG

MOMENT 20K CORE, 366 WORDS FREE
LOADER USED 23+5K CORE

EXIT.
^C
:ST

INPUT NO. OF DATA POINTS15000

INPUT DATA RECORD NAME  VERT2


 INPUT BIN WIDTH
 NO. OF BINS TOTAL
.014
1000


TOTAL NO. OF POINTS USED FOR 1ST PASS =15000.00
MOMENTS OF W(X)
 K =  1  ALPHW =   0.4662677E-05
 K =  2  ALPHW =    1.811600
 K =  3  ALPHW =   0.2195678
 K =  4  ALPHW =    10.63055
 K =  5  ALPHW =    5.433067
 K =  6  ALPHW =    116.9995
 K =  7  ALPHW =    162.7640
 K =  8  ALPHW =    1994.063
INPUT CUT-OFF FREQ,SAMPLING INTERVAL,NO. OF FILTER SECTIONS
5.9115E-01,.05,2

MOMENTS OF FILTERED W

 INPUT BIN WIDTH
 NO. OF BINS TOTAL
.014
1000


TOTAL NO. OF POINTS USED FOR 1ST PASS =15000.00
 K =  1  ALPHWH = -0.2426659E-04
 K =  2  ALPHWH =  0.2211058
 K =  3  ALPHWH =  0.8224017E-02
 K =  4  ALPHWH =  0.4627320
 K =  5  ALPHWH =  0.1394177
 K =  6  ALPHWH =   2.671960
 K =  7  ALPHWH =   2.300871
 K =  8  ALPHWH =   25.76930
COMPUTE MOMENTTS FOR W*W

 INPUT BIN WIDTH
 NO. OF BINS
.03571
700


TOTAL NO. OF POINTS USED FOR 1ST PASS =15000.00
 K =  1  ALPHW2 =  0.2261919
 K =  2  ALPHW2 =  0.4630958
 K =  3  ALPHW2 =   2.676421
 K =  4  ALPHW2 =   25.05040
 K =  5  ALPHW2 =   318.6409
 K =  6  ALPHW2 =   4431.121
 K =  7  ALPHW2 =   65037.35
 K =  8  ALPHW2 =   1017433.
COMPUTE MOMENTS OF SIGMA SQRD F
INPUT SIG SQRD F .4609

N=  1  ALSIG =  0.4609000
N=  2  ALSIG =  0.6409275
N=  3  ALSIG =   1.309566
N=  4  ALSIG =   4.245520
N=  5  ALSIG =   11.84454
N=  6  ALSIG =   30.51175
N=  7  ALSIG =   71.05787
N=  8  ALSIG =   149.1700

CPU TIME: 1:22.33       ELAPSED TIME: 9:59.24
NO EXECUTION ERRORS DETECTED

EXIT.
```

FIG. E.1. TELETYPE PRINTOUT OF INPUTS AND OUTPUTS OF PROGRAM MOMENT.

```
F40
*GAM=GAM
GAM      ERRORS DETECTED: 0

 9K CORE USED

*^C
@LOADER *
*GDIST6,GAM$

GDIST6 3K CORE, 197 WORDS FREE
LOADER USED 6+5K CORE

EXIT.
^C
@ST

INPUT ALPHF :
.4609
.6409275
1.509566
4.24552
11.84454
30.51175

GAMMA =  0.4957514
```

| X | F1PRM | F2PRM | F3PRM | F4PRM |
|---|---|---|---|---|
| 0.07 | 2.629587 | 2.568655 | 2.490394 | 2.440631 |
| 0.13 | 1.730845 | 1.729292 | 1.753583 | 1.798062 |
| 0.20 | 1.316895 | 1.340347 | 1.403987 | 1.480339 |
| 0.26 | 1.063073 | 1.098345 | 1.177042 | 1.261538 |
| 0.33 | 0.8864120 | 0.9266947 | 1.008244 | 1.089494 |
| 0.39 | 0.7543590 | 0.7957138 | 0.8734558 | 0.9457618 |
| 0.46 | 0.6510620 | 0.6911143 | 0.7614079 | 0.8221444 |
| 0.52 | 0.5676976 | 0.6050170 | 0.6660199 | 0.7143386 |
| 0.59 | 0.4988822 | 0.5326439 | 0.5836338 | 0.6197396 |
| 0.65 | 0.4411022 | 0.4708831 | 0.5118475 | 0.5365624 |
| 0.72 | 0.3919436 | 0.4175922 | 0.4489650 | 0.4634521 |
| 0.79 | 0.3496791 | 0.3712275 | 0.3937150 | 0.3993006 |
| 0.85 | 0.3130320 | 0.3306351 | 0.3450979 | 0.3431546 |
| 0.92 | 0.2810325 | 0.2949255 | 0.3022969 | 0.2941690 |
| 0.98 | 0.2529274 | 0.2633948 | 0.2646258 | 0.2515816 |
| 1.05 | 0.2281204 | 0.2354738 | 0.2314955 | 0.2146990 |
| 1.11 | 0.2061320 | 0.2106943 | 0.2023926 | 0.1828883 |
| 1.18 | 0.1865708 | 0.1886646 | 0.1768647 | 0.1555721 |
| 1.24 | 0.1691139 | 0.1690534 | 0.1545104 | 0.1322240 |
| 1.31 | 0.1534916 | 0.1515770 | 0.1349719 | 0.1123657 |
| 1.37 | 0.1394771 | 0.1359909 | 0.1179288 | 0.9556370E-01 |
| 1.44 | 0.1268777 | 0.1220822 | 0.1030944 | 0.8142707E-01 |
| 1.51 | 0.1155286 | 0.1096651 | 0.9021136E-01 | 0.6960408E-01 |

FIG. E.2. TELETYPE PRINTOUT FOR RUNNING GDIST6.

c. Outputs

i. data file PROB containing moments of $w_s$, $M_{w_s}$, and the probability density function $P_{w_s}$.

d. Example

Figure E.3 is an example of the teletype printout created by program ITEM4 and Fig. E.4 contains the first page of data file PROB.

```
ITEM4 3K CORE, 364 WORDS FREE
LOADER USED 6+5K CORE

EXIT.
^C
@ST

INPUT ALPHWH,ALPHW (8 VALUES)
-.2426659E-04,.4662677E-05
.2211058,1.8116
.8224017E-02,.2195678
.462732,10.63055
.1394177,5.433067
2.67196,116.9995
2.380871,162.7648
25.7693,1994.063

INPUT SIGMA SQRD F  .4609

PROB DENSITY FN.

CPU TIME: 1.87  ELAPSED TIME: 1:47.82
NO EXECUTION ERRORS DETECTED

EXIT.
^C
@LIST PROB.DAT;3
@
```

FIG. E.3. TELETYPE PRINTOUT FOR RUNNING ITEM4.

```
DATA FILE CREATED BY PROGRAM ITEM4
 ATMOSPHERIC TURBULENCE TASK, PHASE II
FOR SIGMA SQRD F =  0.4609000

THE VALUES OF ALPHA OF WS ARE:
N =   1ALPHA WS =  0.3969848E-04
N =   2ALPHA WS =   1.350700
N =   3ALPHA WS =  0.1949038
N =   4ALPHA WS =   4.884669
N =   5ALPHA WS =   3.326249
N =   6ALPHA WS =   18.19409
N =   7ALPHA WS =   56.67557
N =   8ALPHA WS =  -344.2492


THE PROBABILITY DENSITY FN. IS:
        WS                  P(WS)
  0.0000000             0.3294248
 -0.4702189E-01         0.3300615
 -0.9404378E-01         0.3302435
 -0.1410657             0.3299658
 -0.1880876             0.3292255
 -0.2351094             0.3280209
 -0.2821313             0.3263524
 -0.3291532             0.3242220
 -0.3761751             0.3216337
 -0.4231970             0.3185931
 -0.4702189             0.3151080
 -0.5172408             0.3111880
 -0.5642627             0.3068443
 -0.6112845             0.3020904
 -0.6583064             0.2969412
 -0.7053283             0.2914138
 -0.7523502             0.2855266
 -0.7993721             0.2792998
 -0.8463940             0.2727550
 -0.8934159             0.2659153
 -0.9404378             0.2588050
 -0.9874597             0.2514495
  -1.034482             0.2438750
  -1.081503             0.2361088
  -1.128525             0.2281785
  -1.175547             0.2201123
  -1.222569             0.2119386
  -1.269591             0.2036858
  -1.316613             0.1953822
  -1.363635             0.1870557
  -1.410657             0.1787340
  -1.457679             0.1704437
  -1.504700             0.1622108
  -1.551722             0.1540602
```

FIG. E.4. OUTPUT DATA FILE PROB.

# APPENDIX F: COMPUTER PROGRAM LISTING

Subroutine AK1

```
      SUBROUTINE AK(I,XP1,AKZ)
C
      COMMON/AA/AK1(100),AK2(100)
C
C
      IF (XP1.GT.10..OR.XP1.LT..1) GO TO 10
      PI = 3.14159265
      IXMID = IFIX(XP1/.1 + .5)
      IF (I.EQ.1) AKZ = AK1(IXMID)*PI/2.
      IF (I.EQ.2) AKZ = AK2(IXMID)*PI/2.
C
      GO TO 30
C
10    IF (XP1.LT..1) TYPE 40,XP1
      IF (XP1.GT.10.) TYPE 20,XP1
20    FORMAT(' X GT 10.0, CANNOT COMPUTE, X =',G)
40    FORMAT(' X LT 0.1, CANNOT COMPUTE, X =',G)
30    RETURN
C
      END
```

Subroutine AK

```
C <RFISHER>AK.F4;10      1-Nov-77 11:55:59     EDIT BY RFISHER
        SUBROUTINE AK(I,XP1,AKZ)
C
C       COMPUTES MODIFIED BESSEL FNS OF FRACTIONAL ORDER 1/3
C       AND 2/3. TABLE OF K(1/3) AND K(2/3) FROM MATH HANDBOOK
C       VALUES OF AK1 ARE FOR MODIFIED BESSEL FNS: (2/PI)*K(1/3)
C       FOR VALUES OF X FROM .1 TO 5.1 IN .1 STEPS
C       VALUES OF AK2 ARE FOR MODIFIED BESSEL FNS: (2/PI)*K(2/3)
C       FOR VALUES OF X AS ABOVE
C       USES SUBROUTINE PARAB TO FIT 3 DATA PTS TO INTERPOLATE
C       BETWEEN STEP OF .1
C
        COMMON/AA/AK1(51),AK2(51)
        COMMON/BB/X(3),AKI(3),XPL
C
C
        XPL = XP1
C
C       CHECK IF WITHIN RANGE OF .1 TO 5.1
C
        IF (XPL.GT.5..OR.XPL.LT..1) GO TO 10
        IXP1 = IFIX(10.*XP1)
        PI = 3.14159265
        IXMID = IFIX(XPL/.1 + .5)
C
C       CHECK IF INTERPOLATION NECESSARY
C
        IF ((10.*XP1).EQ.IXP1.AND.I.EQ.1) AKZ = AK1(IXMID)*PI/2.
        IF ((10.*XP1).EQ.IXP1.AND.I.EQ.2) AKZ = AK2(IXMID)*PI/2.
        IF ((10.*XP1).EQ.IXP1) GO TO 30
C
        IXLOW = IXMID - 1
        IXHIG = IXMID + 1
C
C       CHECK LOWER  BOUND OF ARRAY
C
        IF (IXMID.EQ.1) IXHIG = 3
        IF (IXMID.EQ.1) IXLOW = 1
        IF (IXMID.EQ.1) IXMID = 2
        AKI(1) = AK2(IXLOW)
        AKI(2) = AK2(IXMID)
        AKI(3) = AK2(IXHIG)
        IF (I.EQ.1) AKI(1) = AK1(IXLOW)
        IF (I.EQ.1) AKI(2) = AK1(IXMID)
        IF (I.EQ.1) AKI(3) = AK1(IXHIG)
        X(1) = .1*IXLOW
        X(2) = .1*IXMID
        X(3) = .1*IXHIG
        CALL PARAB(AKZ)
        AKZ = AKZ*PI/2.
        GO TO 30
```

```
C
10        IF (XPL.LT..1) TYPE 40,XPL
          IF (XPL.GT.5.) TYPE 20,XPL
20        FORMAT(' X GT 5.0, CANNOT COMPUTE, X =',G)
40        FORMAT(' X LT 0.1, CANNOT COMPUTE, X =',G)
30        RETURN
C
          END
```

Subroutine AKDAT (for use with AK)

```
C        USED BY SUBROUTINE AK, ATMOSPHERIC TURBULENCE
C        VALUES OF AK1 ARE FOR MODIFIED BESSEL FNS: (2/PI)*K(1/3)
C        FOR VALUES OF X FROM .1 TO 5.1 IN .1 STEPS
C        VALUES OF AK2 ARE FOR MODIFIED BESSEL FNS: (2/PI)*K(2/3)
C        FOR VALUES AS ABOVE
         BLOCK DATA
C
         COMMON/AA/AK1(51),AK2(51)
C
         DATA AK1/1.8461,1.2601,.9607,.7676,.6296,.5253,.4434,
     1   .3776,.3238,.27911,.24167,.21001,.18306,.160,.14016,.12302,
     2   .10818,.09527,.08402,.07419,.06559,.05805,.05142,.04559,
     3   .04045,.03592,.03192,.02838,.025249,.022476,.020018,.017838,
     4   .015902,.014183,.012654,.011295,.010085,.009008,.008049,
     5   .007194,.006432,.005752,.005145,.004604,.004120,.003688,
     6   .003303,.0029578,.0026495,.0023739,.0021273/
C
         DATA AK2/3.026,1.7837,1.2716,.9681,.7678,.6257,.5187,.4354,
     1   .3688,.3148,.27024,.23312,.20191,.17547,.15294,.13364,.11704,
     2   .10270,.09027,.07947,.07007,.06185,.05466,.04835,.04282,.03795,
     3   .03366,.029877,.02654,.023591,.020982,.018672,.016625,.01481,
     4   .0132,.01177,.010499,.009369,.008362,.007468,.006671,.005961,
     5   .005329,.004764,.004261,.003812,.003411,.003053,.0027332,
     6   .0024474,.002192/
C
         END
```

Subroutine AKDAT (for use with AK1)

```
C <RFISHER>AKDAT.F4;2       1-NOV-77 10:00:30     EDIT BY RFISHER
      BLOCK DATA
C
C     USED BY SUBROUTINE AK, ATMOSPHERIC TURBULENCE
C     VALUES OF AK1 ARE FOR MODIFIED BESSEL FNS; (2/PI)*K(1/3)
C     FOR VALUES OF X FROM .1 TO 10.0 IN .1 STEPS
C     VALUES OF AK2 ARE FOR MODIFIED BESSEL FNS; (2/PI)*K(2/3)
C     FOR VALUES OF X AS ABOVE
C
C
      COMMON/AA/AK1(100),AK2(100)
C
      DATA AK1/1.8461,1.2601,.9607,.7676,.6296,.5253,.4434,
     1 .3776,.3238,.27911,.24167,.21001,.18306,.160,.14016,.12302,
     2 .10818,.09527,.08402,.07419,.06559,.05805,.05142,.04559,
     3 .04045,.03592,.03192,.02838,.025249,.022476,.020018,.017838,
     4 .015902,.014183,.012654,.011295,.010085,.009008,.008049,
     5 .007194,.006432,.005752,.005145,.004604,.004120,.003688,
     6 .003303,.0029578,.0026495,.0023739,.0021273,.0019067,.0017093,
     7 .0015325,.0013743,.0012326,.0011057,.000992,.0008901,
     8 .0007988,.0007169,.0006469,.0005778,.0005188,.0004658,
     9 .0004184,.0003758,.0003375,.0003032,.00027245,.00024481,
     A .00022,.00019772,.00017779,.00015974,.0001436,.0001291,
     B .00011608,.00010438,.00009386,.8441E-04,.7592E-04,.6828E-04,
     C .6141E-4,.5525E-4,.4971E-4,.4472E-4,.4042E-4,.3621E-4,
     D .3258E-4,.29322E-4,.26389E-4,.23751E-4,.21377E-4,.19242E-4,
     E .17321E-4,.15593E-4,.14038E-4,.12639E-4,.11379E-4/
C
      DATA AK2/3.026,1.7837,1.2716,.9681,.7678,.6257,.5187,.4354,
     1 .3688,.3148,.27024,.23312,.20191,.17547,.15294,.13364,.11704,
     2 .1027,.09027,.07947,.07007,.06185,.05466,.04835,.04282,.03795,
     3 .03366,.029877,.02654,.023591,.020982,.018672,.016625,.01481,
     4 .0132,.01177,.010499,.009369,.008362,.007468,.006671,.005961,
     5 .005329,.004764,.004261,.003812,.003411,.003053,.0027332,
     6 .0024474,.002192,.0019637,.0017594,.0015767,.0014132,
     7 .0012669,.001136,.0010187,.0009137,.0008196,.0007354,.0006599,
     8 .0005922,.0005315,.0004771,.0004283,.0003846,.0003454,
     9 .0003102,.0002786,.00025026,.00022483,.000202,.00018151,
     A .00016312,.0001466,.00013176,.00011844,.00010647,.00009572,
     B .8607E-4,.7739E-4,.6959E-4,.6259E-4,.5629E-4,.5063E-4,.4554E-4,
     C .4097E-4,.3686E-4,.3316E-4,.29837E-4,.26847E-4,.24159E-4,
     D .21741E-4,.195666E-4,.1761E-4,.15851E-4,.14268E-4,.12843E-4,
     E .11562E-4/
C
      END
```

Subroutine ANRP1

```
C         INTERPOLATION ROUTINE
C
          SUBROUTINE ANTRP(JMAX,AKNWN,AKJ,NDX,AL,ALINT)
C
          DIMENSION AKJ(0/11),AL(0/11)
C
          IF (NDX.EQ.JMAX) GO TO 10
C
          ALINT = (AL(NDX+1)*(AKNWN-AKJ(NDX))-AL(NDX)*(AKNWN-AKJ(NDX+1)))
     1           /(AKJ(NDX+1)-AKJ(NDX))
          GO TO 20
10        TYPE 5,JSIG
5         FORMAT(' JSIG = JMAX, INTERPOLATION OUT OF RANGE')
20        RETURN
C
          END
```

Program ATUR4A

```
C         PROGRAM ATURB4.  ATMOSPHERIC TURBULENCE TASK
C         ITEM 3, PARTS 1,1.A. PHASE II OF ATMOSPHERIC TURBULENCE
C         MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C         SPECTRUM CALCULATIONS (AS IN PHASE I)
C
C         REQUIRES SUBROUTINES:
C                   CFFT1: TO PERFORM FFT
C                   SIMP: TO INTEGRATE BY SIMPSIONS RULE
C                   HPDES: HIGH PASS FILTER
C
C         READS NASA DATA STORED IN DATA FILE TO BE NAMED
C         PRODUCES DATA FILES:
C                   PHILK: VALUES OF PSD PHI OF L(K)
C                   AUTO: AUTOCORRELATION OF PSD
C
          COMMON/S3/W(0/65537)
          COMMON/CC/A(3),B(3),C(3),GR(2,10),F(4,3)
C
C         INPUT INITIAL PARAMETERS
C
          TYPE 800
800       FORMAT(/1X,'INPUT SPEED OF CRAFT (M/SEC)',/$)
          ACCEPT 801,V
801       FORMAT(G)
          TYPE 802
802       FORMAT(/1X,'INPUT TOTAL NO. OF POINTS TO BE USED IN 2L M.',
     1    ' OF DATA',/1X,'AND POWER OF TWO OF THAT NO.',/$)
          ACCEPT 803,NPTS
          ACCEPT 803,MPWRN
803       FORMAT(I6)
          TYPE 702
          ACCEPT 825,NOPTS,SRATE
825       FORMAT(2G)
702       FORMAT(/1X,'INPUT NO. OF POINTS OF W(X) TO  BE READ',/1X,
     1    'AND SAMPLING RATE OF DATA ',$)
          TYPE 804
804       FORMAT(/1X,'INPUT VALUE OF MPTS',
     1    /,' AND POWER OF TWO OF THAT NO.',/$)
          ACCEPT 803,MPTS
          ACCEPT 803,MPWRM
C
          MZERO = NPTS-NOPTS
          NLAST = NPTS-1
          TIME = SRATE*NPTS
          TWOL = V*TIME
          PTS = FLOAT(NPTS)
          DELX = TWOL/PTS
          FTM = MZERO*DELX
          FTDAT = TWOL-FTM
          DELK = 1./TWOL
          FTM1 = MPTS*DELX
```

```
C
C       READ IN THE DATA W(X) AND CONVERT TO M/SEC
C
        TYPE 805
806     FORMAT(/1X,'INPUT DATA FILE NAME THAT CONTAINS SAMPLES',
     1  /1X,'OF W(X) AND NO. POINTS NXRAY',/$)
        ACCEPT 807,FLE
        ACCEPT 825,NXRAY
807     FORMAT(A5)

        CALL IFILE(20,FLE)
        NX1 = NXRAY/4
        DO 550 I=0,NX1-1
        READ(20,551)W(I),W(I+NX1),W(I+2*NX1),W(I+3*NX1)
        W(I+NX1) = W(I+NX1)*.3048
        W(I+2*NX1) = W(I+2*NX1)*.3048
        W(I+3*NX1) = W(I+3*NX1)*.3048
550     W(I) = W(I)*.3048
551     FORMAT(4(E15.7))
        END FILE 20
C
C       HIGH PASS FILTER THE DATA
C
        TYPE 9
9       FORMAT(' INPUT CUT-OFF FREQ,SAMPLING INTERVAL,'
     1  'NO. OF FILTER SECTIONS',/$)
        ACCEPT 2,FC,TS,NS
2       FORMAT(3G)
        FK = FC/V
        CALL HPDES(FC,TS,NS)
        DO 140 N=1,NS+1
        DO 140 M=1,2
140     F(N,M) = 0.0
        DO 150 M=0,NXRAY-1
        F(1,3) = W(M)
        DO 160 N=1,NS
        TEMP = A(N)*(F(N,3)-2.*F(N,2)+F(N,1))
160     F(N+1,3) = TEMP-B(N)*F(N+1,2)-C(N)*F(N+1,1)
        DO 170 N=1,NS+1
        DO 170 MM=1,2
170     F(N,MM) = F(N,MM+1)
150     W(M) = F(NS+1,3)
C
C       COMPUTE AND SUBTRACT OUT MEAN VALUE
C       COMPUTE THE SAMPLE VARIANCE
C
        WBAR = 0.0
        VAR = 0.0
        DO 610 JJ = 0,NOPTS-1
610     WBAR = WBAR + W(JJ)
        WBAR = WBAR/FLOAT(NOPTS)
        DO 600 I=0,NOPTS-1
        W(I) = W(I) - WBAR
```

```
600       VAR = VAR + W(I)*W(I)
          VAR = VAR/FLOAT(NOPTS)
C
C         SQUARE W(I)
C
          DO 1111 I=0,NXRAY-1
1111      W(I) = W(I)*W(I)
C
C         COMPUTE INTEGRAL OF W(K)**2 USING TRAPIZOIDAL RULE
C
          WSUM = 0.0
          DO 370 I=1,NOPTS-2
370       WSUM = WSUM+W(I)*W(I)
          WSUM = WSUM + .5*W(0)*W(0)+.5*W(NOPTS-1)*W(NOPTS-1)
          WSUM = DELX*WSUM/FTDAT
C
C         ADD ZEROS TO DATA
C
          DO 10 I=NOPTS,NLAST
10        W(I) = 0.0
C
C         MAKE ARRAY COMPLEX
C
          DO 512 J=NPTS-1,0,-1
512       W(2*J) = W(J)
          DO 513 J=1,2*NPTS-1,2
513       W(J) = 0.0
C
C         COMPUTE PHI L(K)
C
          CALL CFFT(MPWRN,NPTS,W,1)

          NHALF = NPTS/2
          NMIN1 = NHALF-1
          TMS = TWOL*TWOL/FTDAT
          DO 11 I=0,2*NLAST,2
          W(I+1) = 0.0
          W(I) = W(I)*TMS
11        CONTINUE
          W(2*NPTS-1) = 0.0
C
C         OUTPUT VALUES OF PHI OF L(K) TO DATA FILE (FIRST HALF ONLY)
C
          CALL OFILE (20,'FPSD2')
          WRITE(20,900)
          WRITE(20,950) FC,FK,FLE
950       FORMAT(/1X,'HIGH PASS FILTERED DATA',/1X,'CUT-OFF FREQ =',G,
     1    ' (K =',G,')',//1X,
     2    'FILTERED VERSION SQUARED BEFORE TRANSFORMING',/1X,
     3    'POWER SPECTRUM OF PHI OF L(K)',/1X,
     3    'DATA TAKEN FROM FILE ',A5)
          WRITE(20,906) NPTS,TWOL,MPTS,FTM1
          WRITE(20,907) MZERO
          WRITE(20,910) WBAR,VAR
          WRITE(20,951) WSUM
```

```
951     FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
        WRITE(20,952)
952     FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE POWER SPECTRUM',
      1 /1X,5X,'K',10X,'PS VALUE',8X,'K CONTD',4X,'PS VALUE CONTD',/)
        MHALF = NPTS/4
        M41 = MHALF-1
        DO 823 I=0,M41
        DEL = DELK*I
        XX = W(2*I)
        K = MHALF+I
        DEL2 = DELK*K
        YY= W(2*K)
823     WRITE(20,903) DEL,XX,DEL2,YY
        DEL3 = DELK*NHALF
        ZZ = W(2*NHALF)
        WRITE(20,908) DEL3,ZZ
        END FILE 20
C
C       PERFORM INTEGRATION CHECK
C
        TYPE 810
810     FORMAT(/1X,'PERFORM INTEGRATION CHECK (Y OR N)    ',$)
        ACCEPT 807,CHK
        IF (CHK.EQ.'N') GO TO 23
        EDR = NPTS*DELK
        CALL SIMP(0.0,EDR,DELK,NPTS,ANS)
        TYPE 811,ANS
811     FORMAT(//1X,'INTEGRAL OF PHI OF L (K) =',E12.4)
C
C       OBTAIN AUTOCORRELATION FUNCTION
C
23      CALL CFFT(MPWRN,NPTS,W,2)
C
C       TYPE OUT AUTOCORRELATION TO DATA FILE
C
        XAOFF = 13000.
        RFCON = PTS/TWOL
        DR0 = W(0)*RFCON
        MAOFF = 13000./DELX
C
        CALL OFILE (20,'AUTF2')
        WRITE(20,900)
        WRITE(20,1950) FLE
1950    FORMAT(/1X,'AUTOCORRELATION OF',/1X,
      1 'HIGH PASS FILTERED AND SQUARED NON-HOMOGENEOUS SAMPLE',/1X,
      2 'DATA TAKEN FROM FILE ',A5)
        WRITE(20,906) NPTS,TWOL,MPTS,FTM1
        WRITE(20,907) MZERO
        WRITE(20,910) WBAR,VAR
        WRITE(20,951) WSUM
        WRITE(20,381) XAOFF,MAOFF
```

```
381       FORMAT(/1X,' TRUCATION POINT WAS',G,' METERS',/1X,
     1    'WHICH CONTAINS ',I7,' POINTS')
          WRITE(20,1952)
1952      FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE AUTOCORRELATION',
     1    /1X,8X,'X',12X,'    RL    ',9X,'RL/R0')
          DO 1823 I=0,MAOFF
          DEL = DELX*I
          XX = W(2*I)*RFCON
          YY = XX/DR0
1823      WRITE(20,915)DEL,XX,YY
915       FORMAT(1X,3(2X,G))
          END FILE 20
C
900       FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB4')
903       FORMAT(1X,F10.6,3X,F12.4,3X,F10.6,3X,E12.4)
905       FORMAT(//1X,'RL(0) = ',F12.4)
906       FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1    ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1    ' METER')
907       FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
908       FORMAT(29X,F10.4,3X,E12.4)
910       FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1    'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
C
C
4999      END
```

Program ATURB2

```
C <RFISHER>ATURB2.F4;5      1-NOV-77 10:14:27    EDIT BY RFISHER
C        PROGRAM ATP2.  ATMOSPHERIC TURBULENCE TASK
C        ITEM 1, PARTS 1,1.A. PHASE II OF ATMOSPHERIC TURBULENCE
C        MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C        SPECTRUM CALCULATIONS (AS IN PHASE I)
C
C        REQUIRES SUBROUTINES:
C
C                CFFT: TO PERFORM FFT
C                SIMP: TO INTEGRATE BY SIMPSIONS RULE
C
C        READS NASA DATA STORED IN DATA FILE TO BE NAMED
C        PRODUCES DATA FILES:
C                PHILK: VALUES OF PSD PHI OF L(K)
C                AUTO: AUTOCORRELATION OF PSD
C                DSPS: SMOOTHED POWER SPECTRUM
C
         COMMON/SG/W(0/65537)
C
C        INPUT INITIAL PARAMETERS
C
         TYPE 800
800      FORMAT(/1X,'INPUT SPEED OF CRAFT (M/SEC)',/$)
         ACCEPT 801,V
801      FORMAT(G)
         TYPE 802
802      FORMAT(/1X,'INPUT TOTAL NO. OF POINTS TO BE USED IN 2L M.',
     1   ' OF DATA',/1X,'AND POWER OF TWO OF THAT NO.',/$)
         ACCEPT 803,NPTS
         ACCEPT 803,MPWRN
803      FORMAT(I6)
         TYPE 702
         ACCEPT 825,NOPTS,SRATE
825      FORMAT(2G)
702      FORMAT(/1X,'INPUT NO. OF POINTS OF W(X) TO  BE READ',/1X,
     1   'AND SAMPLING RATE OF DATA ',$)
         TYPE 804
804      FORMAT(/1X,'INPUT VALUE OF MPTS',
     1   /,' AND POWER OF TWO OF THAT NO.',/$)
         ACCEPT 803,MPTS
         ACCEPT 803,MPWRM
C
         MZERO = NPTS-NOPTS
         NLAST = NPTS-1
         TIME = SRATE*NPTS
         TWOL = V*TIME
         PTS = FLOAT(NPTS)
         DELX = TWOL/PTS
         FTM = MZERO*DELX
         FTDAT = TWOL-FTM
         DELK = 1./TWOL
         FTM1 = MPTS*DELX
```

```
C
C       READ IN THE DATA W(X) AND CONVERT TO M/SEC
C
        TYPE 806
806     FORMAT(/1X,'INPUT DATA FILE NAME THAT CONTAINS SAMPLES',
     1  /1X,'OF W(X) AND NO. POINTS NXRAY',/$)
        ACCEPT 807,FLE
        ACCEPT 825,NXRAY
807     FORMAT(A5)

        CALL IFILE(20,FLE)
        NX1 = NXRAY/4
        DO 550 I=0,NX1-1
        READ(20,551)W(I),W(I+5700),W(I+11400),W(I+17100)
        W(I+5700) = W(I+5700)*.3048
        W(I+11400) = W(I+11400)*.3048
        W(I+17100) = W(I+17100)*.3048
550     W(I) = W(I)*.3048
551     FORMAT(4(E15.7))
        END FILE 20

C
C       COMPUTE AND SUBTRACT OUT MEAN VALUE
C       COMPUTE THE SAMPLE VARIANCE
C
        WBAR = 0.0
        VAR = 0.0
        DO 610 JJ = 0,NOPTS-1
610     WBAR = WBAR + W(JJ)
        WBAR = WBAR/FLOAT(NOPTS)
        DO 600 I=0,NOPTS-1
        W(I) = W(I) - WBAR
600     VAR = VAR + W(I)*W(I)
        VAR = VAR/FLOAT(NOPTS)
C
C       COMPUTE INTEGRAL OF W(K)**2 USING TRAPIZOIDAL RULE
C
        WSUM = 0.0
        DO 370 I=1,NOPTS-2
370     WSUM = WSUM+W(I)*W(I)
        WSUM = WSUM + .5*W(0)*W(0)+.5*W(NOPTS-1)*W(NOPTS-1)
        WSUM = DELX*WSUM/FTDAT
C
C       ADD ZEROS TO DATA
C
        DO 10 I=NOPTS,NLAST
10      W(I) = 0.0
C
C       MAKE ARRAY COMPLEX
C
        DO 512 J=NPTS-1,0,-1
```

```
512     W(2*J) = W(J)
        DO 513 J=1,2*NPTS-1,2
513     W(J) = 0.0
C
C       COMPUTE PHI L(K)
C
        CALL CFFT(MPWRN,NPTS,W,1)

        NHALF = NPTS/2
        NMIN1 = NHALF-1
        TMS = TWOL*TWOL/FTDAT
        DO 11 I=0,2*NLAST,2
        W(I+1) = 0.0
        W(I) = W(I)*TMS
11      CONTINUE
        W(2*NPTS-1) = 0.0
C
C       OUTPUT VALUES OF PHI OF L(K) TO DATA FILE (FIRST HALF ONLY)
C
        CALL OFILE (20,'PHILK')
        WRITE(20,900)
        WRITE(20,950)FLE
950     FORMAT(/1X,'POWER SPECTRUM OF PHI OF L(K)',/1X,
     1  'DATA TAKEN FROM FILE ',A5)
        WRITE(20,906)NPTS,TWOL,MPTS,FTM1
        WRITE(20,907)MZERO
        WRITE(20,910)WBAR,VAR
        WRITE(20,951)WSUM
951     FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
        WRITE(20,952)
952     FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE POWER SPECTRUM',
     1  /1X,5X,'K',10X,'PS VALUE',8X,'K CONTD',4X,'PS VALUE CONTD',/)
        MHALF = NPTS/4
        M41 = MHALF-1
        DO 823 I=0,M41
        DEL = DELK*I
        XX = W(2*I)
        K = MHALF+I
        DEL2 = DELK*K
        YY= W(2*K)
823     WRITE(20,903)DEL,XX,DEL2,YY
        DEL3 = DELK*NHALF
        ZZ = W(2*NHALF)
        WRITE(20,908)DEL3,ZZ
        END FILE 20
C
C       PERFORM INTEGRATION CHECK
C
        TYPE 810
```

```
810     FORMAT(/1X,'PERFORM INTEGRATION CHECK (Y OR N)    ',$)
        ACCEPT 807,CHK
        IF (CHK.EQ.'N') GO TO 23
        EDR = NPTS*DELK
        CALL SIMP(0.0,EDR,DELK,NPTS,ANS)
        TYPE 811,ANS
811     FORMAT(//1X,'INTEGRAL OF PHI OF L (K) =',E12.4)
C
C       OBTAIN AUTOCORRELATION FUNCTION
        TYPE 4444,W(0),W(1),W(2),W(3)
        TYPE 4444,W(4),W(5),W(6),W(7)
        TYPE 4444,W(8),W(10)
        TYPE  4444,W(NPTS)
        TYPE 4444,W(NPTS-2),W(NPTS+2)
        TYPE 4444,W(2*NLAST-2),W(2*NLAST-4),W(2*NLAST-6)
4444    FORMAT(4G)
C
23      CALL CFFT(MPWRN,NPTS,W,2)
C
C       TYPE OUT AUTOCORRELATION TO DATA FILE
C
        XAOFF = 13000.
        RFCON = PTS/TWOL
        DR0 = W(0)*RFCON
        MAOFF = 13000./DELX
C
        CALL OFILE (20,'AUTO')
        WRITE(20,900)
        WRITE(20,1950)FLE
1950    FORMAT(/1X,'AUTOCORRELATION OF STATIONARY SAMPLE',/1X,
     1  'DATA TAKEN FROM FILE ',A5)
        WRITE(20,906)NPTS,TWOL,MPTS,FTM1
        WRITE(20,907)MZERO
        WRITE(20,910)WBAR,VAR
        WRITE(20,951)WSUM
        WRITE(20,381)XAOFF,MAOFF
381     FORMAT(/1X,' TRUCATION POINT WAS',G,' METERS',/1X,
     1  'WHICH CONTAINS ',I7,' POINTS')
        WRITE(20,1952)
1952    FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE AUTOCORRELATION',
     1  /1X,8X,'X',12X,'    RL     ',9X,'RL/R0')
        DO 1823 I=0,MAOFF
        DEL = DELX*I
        XX = W(2*I)*RFCON
        YY = XX/DR0
1823    WRITE(20,915)DEL,XX,YY
915     FORMAT(1X,3(2X,G))
        END FILE 20
C
C       COMPUTE WINDOW FN. AND RL(Z)/(1-/Z//(2L-M))
```

```
C
      DRØ = W(Ø)/DELX
      PI = 3.141592654
      FTM1 = MPTS*DELX
      DO 13 J=Ø,MPTS
      ARG = DELX*J
      CC = PI*ARG/FTM1
      DD = 1.-ARG/FTM1
      EE = ABS(SIN(CC))
      WINDO = EE/PI+DD*COS(CC)
      DVSOR = 1.-ARG/FTDAT
13    W(2*J) = W(2*J)*WINDO/(DELX*DVSOR)
      M2PWR = MPWRM+1
      M2PTS = 2*MPTS
      FT2M = 2.*FTM1
      MMIN1 = MPTS-1
      DO 14 JK=1,MMIN1
      KK = M2PTS-JK
14    W(2*KK) = W(2*JK)
      DO 521 K=1,2*M2PTS-1,2
521   W(K) = Ø.Ø
C
C     COMPUTE SMOOTHED POWER SPECTRUM
C
      CALL CFFT(M2PWR,M2PTS,W,2)
C
C     PRINT DATA FILE 'DSPS' , FIRST HALF OF DATA POINTS
C
      CALL OFILE(2Ø,'DSPS')
      WRITE(2Ø,9ØØ)
      WRITE(2Ø,9Ø1)FLE
      WRITE(2Ø,9Ø6)NPTS,TWOL,MPTS,FTM1
      WRITE(2Ø,91Ø)WBAR,VAR
      WRITE(2Ø,951)WSUM
      WRITE(2Ø,9Ø7)MZERO
      WRITE(2Ø,9Ø5)DRØ
      WRITE(2Ø,9Ø2)
      MHALF = MPTS/2
      M41 = MHALF-1
      DELK = 1./FT2M
      DO 56 I=Ø,M41
      DEL = DELK*I
      XX = W(2*I)*FT2M
      K = MHALF+I
      DEL2 = DELK*K
      YY = W(2*K)*FT2M
56    WRITE(2Ø,9Ø3)DEL,XX,DEL2.YY
      DEL3 = DELK*MPTS
      ZZ = W(2*MPTS)*FT2M
      WRITE(2Ø,9Ø8)DEL3,ZZ
      END FILE 2Ø
```

```
900       FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB2')
901       FORMAT(//1X,'SMOOTHED POWER SPECTRUM PHI OF P(K)',
     1    /1X,'DATA TAKEN FROM DATA FILE ',A5)
902       FORMAT(//1X,'PRINTOUT OF VALUES OF THE SMOOTHED',
     1    ' POWER SPECTRUM',/5X,'K',10X,'SPS VALUE',8X,'K CONTD',
     2    4X,'SPS VALUE CONTD',/)
903       FORMAT(1X,F10.6,3X,E12.4.3X,F10.6,3X,E12.4)
905       FORMAT(//1X,'RL(0) = ',E12.4)
906       FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1    ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1    ' METER')
907       FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
908       FORMAT(29X,F10.4,3X,E12.4)
910       FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1    'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
C
C
4999      END
```

Program ATURB3

```
C       PROGRAM ATURB2.  ATMOSPHERIC TURBULENCE TASK
C       ITEM 2, PARTS 1,1.A'. PHASE II OF ATMOSPHERIC TURBULENCE
C       MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C       SPECTRUM CALCULATIONS (AS IN PHASE I)
C
C       REQUIRES SUBROUTINES:
C
C               CFFT: TO PERFORM FFT
C               SIMP: TO INTEGRATE BY SIMPSIONS RULE
C
C       READS NASA DATA STORED IN DATA FILE TO BE NAMED
C       PRODUCES DATA FILES:
C               PHILK: VALUES OF PSD PHI OF L(K)
C               AUTO: AUTOCORRELATION OF PSD
C               DSPS: SMOOTHED POWER SPECTRUM
C
        COMMON/SG/W(0/65537)
C
C       INPUT INITIAL PARAMETERS
C
        TYPE 800
800     FORMAT(/1X,'INPUT SPEED OF CRAFT (M/SEC)',/$)
        ACCEPT 801,V
801     FORMAT(G)
        TYPE 802
802     FORMAT(/1X,'INPUT TOTAL NO. OF POINTS TO BE USED IN 2L M.',
     1  ' OF DATA',/1X,'AND POWER OF TWO OF THAT NO.',/$)
        ACCEPT 803,NPTS
        ACCEPT 803,MPWRN
803     FORMAT(I6)
        TYPE 702
        ACCEPT 825,NOPTS,SRATE
825     FORMAT(2G)
702     FORMAT(/1X,'INPUT NO. OF POINTS OF W(X) TO  BE READ',/1X,
     1  'AND SAMPLING RATE OF DATA ',$)
        TYPE 804
804     FORMAT(/1X,'INPUT VALUE OF MPTS',
     1  /,' AND POWER OF TWO OF THAT NO.',/$)
        ACCEPT 803,MPTS
        ACCEPT 803,MPWRM
C
        MZERO = NPTS-NOPTS
        NLAST = NPTS-1
        TIME = SRATE*NPTS
        TWOL = V*TIME
        PTS = FLOAT(NPTS)
        DELX = TWOL/PTS
        FTM = MZERO*DELX
        FTDAT = TWOL-FTM
        DELK = 1./TWOL
        FTM1 = MPTS*DELX
```

```
C
C       READ IN THE DATA W(X) AND CONVERT TO M/SEC
C
        TYPE 806
806     FORMAT(/1X,'INPUT DATA FILE NAME THAT CONTAINS SAMPLES',
     1  /1X,'OF W(X) AND NO. POINTS NXRAY',/$)
        ACCEPT 807,FLE
        ACCEPT 825,NXRAY
807     FORMAT(A5)

        CALL IFILE(20,FLE)
        NX1 = NXRAY/4
        DO 550 I=0,NX1-1
        READ(20,551)W(I),W(I+NX1),W(I+2*NX1),W(I+3*NX1)
        W(I+NX1) = W(I+NX1)*.3048
        W(I+2*NX1) = W(I+2*NX1)*.3048
        W(I+3*NX1) = W(I+3*NX1)*.3048
550     W(I) = W(I)*.3048
551     FORMAT(4(E15.7))
        END FILE 20

C
C       COMPUTE AND SUBTRACT OUT MEAN VALUE
C       COMPUTE THE SAMPLE VARIANCE
C
        WBAR = 0.0
        VAR = 0.0
        DO 610 JJ = 0,NOPTS-1
610     WBAR = WBAR + W(JJ)
        WBAR = WBAR/FLOAT(NOPTS)
        DO 600 I=0,NOPTS-1
        W(I) = W(I) - WBAR
600     VAR = VAR + W(I)*W(I)
        VAR = VAR/FLOAT(NOPTS)
C
C       COMPUTE INTEGRAL OF W(K)**2 USING TRAPIZOIDAL RULE
C
        WSUM = 0.0
        DO 370 I=1,NOPTS-2
370     WSUM = WSUM+W(I)*W(I)
        WSUM = WSUM + .5*W(0)*W(0)+.5*W(NOPTS-1)*W(NOPTS-1)
        WSUM = DELX*WSUM/FTDAT
C
C       ADD ZEROS TO DATA
C
        DO 10 I=NOPTS,NLAST
10      W(I) = 0.0
C
C       MAKE ARRAY COMPLEX
C
        DO 512 J=NPTS-1,0,-1
```

```
512     W(2*J) = W(J)
        DO 513 J=1,2*NPTS-1,2
513     W(J) = 0.0
C
C       COMPUTE PHI L(K)
C
        CALL CFFT(MPWRN,NPTS,W,1)

        NHALF = NPTS/2
        NMIN1 = NHALF-1
        TMS = TWOL*TWOL/FTDAT
        DO 11 I=0,2*NLAST,2
        W(I+1) = 0.0
        W(I) = W(I)*TMS
11      CONTINUE
        W(2*NPTS-1) = 0.0
C
C       OUTPUT VALUES OF PHI OF L(K) TO DATA FILE (FIRST HALF ONLY)
C
        CALL OFILE (20,'PHILK')
        WRITE(20,900)
        WRITE(20,950)FLE
950     FORMAT(/1X,'POWER SPECTRUM OF PHI OF L(K)',/1X,
     1  'DATA TAKEN FROM FILE ',A5)
        WRITE(20,906)NPTS,TWOL,MPTS,FTM1
        WRITE(20,907)MZERO
        WRITE(20,910)WBAR,VAR
        WRITE(20,951)WSUM
951     FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
        WRITE(20,952)
952     FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE POWER SPECTRUM',
     1  /1X,5X,'K',10X,'PS VALUE',8X,'K CONTD',4X,'PS VALUE CONTD',/)
        MHALF = NPTS/4
        M41 = MHALF-1
        DO 823 I=0,M41
        DEL = DELK*I
        XX = W(2*I)
        K = MHALF+I
        DEL2 = DELK*K
        YY= W(2*K)
823     WRITE(20,903)DEL,XX,DEL2,YY
        DEL3 = DELK*NHALF
        ZZ = W(2*NHALF)
        WRITE(20,908)DEL3,ZZ
        END FILE 20
C
C       ABOVE WAS DESCRIPTIVE DATA FILE NOW MAKE DATA FILE THAT
C       IS READ BY OTHER PROGRAMS
C
        CALL OFILE(21,'PSD')
        DO 1010 JK=1,32
        K = 500*(JK-1)
```

```
1010    WRITE(21,9606)(W(2*I),I=0+K,499+K)
9606    FORMAT(500G)
        WRITE(21,9606)(W(2*I),I=16000,16384)
        END FILE 21
C
C       PERFORM INTEGRATION CHECK
C
        TYPE 810
810     FORMAT(/1X,'PERFORM INTEGRATION CHECK (Y OR N)    ',$)
        ACCEPT 807,CHK
        IF (CHK.EQ.'N') GO TO 23
        EDR = NPTS*DELK
        CALL SIMP(0.0,EDR,DELK,NPTS,ANS)
        TYPE 811,ANS
811     FORMAT(//1X,'INTEGRAL OF PHI OF L (K) =',E12.4)
C
C       OBTAIN AUTOCORRELATION FUNCTION
        TYPE 4444,W(0),W(1),W(2),W(3)
        TYPE 4444,W(4),W(5),W(6),W(7)
        TYPE 4444,W(8),W(10)
        TYPE  4444,W(NPTS)
        TYPE 4444,W(NPTS-2),W(NPTS+2)
        TYPE 4444,W(2*NLAST-2),W(2*NLAST-4),W(2*NLAST-6)
4444    FORMAT(4G)
C
23      CALL CFFT(MPWRN,NPTS,W,2)
C
C       TYPE OUT AUTOCORRELATION TO DATA FILE
C
        XAOFF = 13000.
        RFCON = PTS/TWOL
        DR0 = W(0)*RFCON
        MAOFF = 13000./DELX
C
        CALL OFILE (20,'AUTO')
        WRITE(20,900)
        WRITE(20,1950)FLE
1950    FORMAT(/1X,'AUTOCORRELATION OF STATIONARY SAMPLE',/1X,
     1  'DATA TAKEN FROM FILE ',A5)
        WRITE(20,906)NPTS,TWOL,MPTS,FTM1
        WRITE(20,907)MZERO
        WRITE(20,910)WBAR,VAR
        WRITE(20,951)WSUM
        WRITE(20,381)XAOFF,MAOFF
381     FORMAT(/1X,' TRUCATION POINT WAS',G,' METERS',/1X,
     1  'WHICH CONTAINS ',I7,' POINTS')
        WRITE(20,1952)
1952    FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE AUTOCORRELATION',
     1  /1X,8X,'X',12X,'    RL    ',9X,'RL/R0')
        DO 1823 I=0,MAOFF
        DEL = DELX*I
        XX = W(2*I)*RFCON
        YY = XX/DR0
```

```
1823    WRITE(20,915)DEL,XX,YY
915     FORMAT(1X,3(2X,G))
        END FILE 20
C
C       ABOVE WAS DES CRIPTIVE DATA FILE NOW MAKE DATA FILE
C       TO BE READ BY FOLLOWING PROGRAMS
C
C
C*****DONT FORGET THESE NUMBERS MUST BE MULTIPLIED BY RFCON ******
C
        IEND = MAOFF/500
        CALL OFILE(21,'RLNH')
        DO 1011 JK = 1,IEND
        K = 500*(JK-1)
1011    WRITE(21,9606)(W(2*I),I=0+K,499+K)
        KEND = 500*(IEND-1)
        WRITE(21,9606)(W(2*I),I=KEND,MAOFF)
C
C       COMPUTE WINDOW FN, AND RL(Z)/(1-/Z//(2L-M))
C
        DR0 = W(0)/DELX
        PI = 3.141592654
        FTM1 = MPTS*DELX
        DO 13 J=0,MPTS
        ARG = DELX*J
        CC = PI*ARG/FTM1
        DD = 1.-ARG/FTM1
        EE = ABS(SIN(CC))
        WINDO = EE/PI+DD*COS(CC)
        DVSOR = 1.-ARG/FTDAT
13      W(2*J) = W(2*J)*WINDO/(DELX*DVSOR)
        M2PWR = MPWRM+1
        M2PTS = 2*MPTS
        FT2M = 2.*FTM1
        MMIN1 = MPTS-1
        DO 14 JK=1,MMIN1
        KK = M2PTS-JK
14      W(2*KK) = W(2*JK)
        DO 521 K=1,2*M2PTS-1,2
521     W(K) = 0.0
C
C       COMPUTE SMOOTHED POWER SPECTRUM
C
        CALL CFFT(M2PWR,M2PTS,W,2)
C
C       PRINT DATA FILE 'DSPS' , FIRST HALF OF DATA POINTS
```

```
C
         CALL OFILE(20,'DSPS')
         WRITE(20,900)
         WRITE(20,901)FLE
         WRITE(20,906)NPTS,TNOL,MPTS,FTM1
         WRITE(20,910)MEAR,VAR
         WRITE(20,911)WSUM
         WRITE(20,907)MZERO
         WRITE(20,905)DR0
         WRITE(22,902)
         MHALF = MPTS/2
         M41 = MHALF+1
         DELK = 1./FT2M
         DO 56 I=0,M41
         DEL = DELK*I
         XX = W(2*I)*FT2M
         K = MHALF+I
         DEL2 = DELK*K
         YY = W(2*K)*FT2M
56       WRITE(20,903)DEL,XX,DEL2,YY
         DEL3 = DELK*MPTS
         ZZ = W(2*MPTS)*FT2M
         WRITE(20,908)DEL3,ZZ
         END FILE 22

900      FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB3')
901      FORMAT(//1X,'SMOOTHED POWER SPECTRUM PHI OF R(K)',
       1 /1X,'DATA TAKEN FROM DATA FILE ',A5)
902      FORMAT(//1X,'PRINTOUT OF VALUES OF THE SMOOTHED',
       1 ' POWER SPECTRUM',/5X,'K',10X,'SPS VALUE',8X,'K CONTD',
       2 4X,'SPS VALUE CONTD',/)
903      FORMAT(1X,F10.6,3X,E12.4,3X,F10.6,3X,E12.4)
905      FORMAT(//1X,'RL(0) = ',E12.4)
906      FORMAT(//1X,I6,' DATA POINTS WERE USED IN 2L = ',F15.4,
       1 ' METER',/1X,I6,' DATA POINTS WERE USED IN M = ',F16.4,
       1 ' METER')
907      FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
908      FORMAT(29X,F12.4,3X,E12.4)
910      FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
       1 'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
C
C
4999     END
```

Program ATURB4

```
C       PROGRAM ATURB4.  ATMOSPHERIC TURBULENCE TASK
C       ITEM 3, PARTS 1,1,A. PHASE II OF ATMOSPHERIC TURBULENCE
C       MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C       SPECTRUM CALCULATIONS (AS IN PHASE I)
C
C       REQUIRES SUBROUTINES:
C               CFFT: TO PERFORM FFT
C               SIMP: TO INTEGRATE BY SIMPSIONS RULE
C               HPDES: HIGH PASS FILTER
C
C       READS NASA DATA STORED IN DATA FILE TO BE NAMED
C       PRODUCES DATA FILES:
C               PHILK: VALUES OF PSD PHI OF L(K)
C               AUTO: AUTOCORRELATION OF PSD
C
        COMMON/SG/W(0/65537)
        COMMON/CC/A(3),B(3),C(3),GR(2,10),F(4,3)
C
C       INPUT INITIAL PARAMETERS
C
        TYPE 800
800     FORMAT(/1X,'INPUT SPEED OF CRAFT (M/SEC)',/$)
        ACCEPT 801,V
801     FORMAT(G)
        TYPE 802
802     FORMAT(/1X,'INPUT TOTAL NO. OF POINTS TO BE USED IN 2L M.',
     1  ' OF DATA',/1X,'AND POWER OF TWO OF THAT NO.',/$)
        ACCEPT 803,NPTS
        ACCEPT 803,MPWRN
803     FORMAT(I6)
        TYPE 702
        ACCEPT 825,NOPTS,SRATE
825     FORMAT(2G)
702     FORMAT(/1X,'INPUT NO. OF POINTS OF W(X) TO  BE READ',/1X,
     1  'AND SAMPLING RATE OF DATA ',$)
        TYPE 804
804     FORMAT(/1X,'INPUT VALUE OF MPTS',
     1  /,' AND POWER OF TWO OF THAT NO.',/$)
        ACCEPT 803,MPTS
        ACCEPT 803,MPWRM
C
        MZERO = NPTS-NOPTS
        NLAST = NPTS-1
        TIME = SRATE*NPTS
        TWOL = V*TIME
        PTS = FLOAT(NPTS)
        DELX = TWOL/PTS
        FTM = MZERO*DELX
        FTDAT = TWOL-FTM
        DELK = 1./TWOL
        FTM1 = MPTS*DELX
```

```
C
C       READ IN THE DATA W(X) AND CONVERT TO M/SEC
C
        TYPE 806
806     FORMAT(/1X,'INPUT DATA FILE NAME THAT CONTAINS SAMPLES',
     1  /1X,'OF W(X) AND NO. POINTS NXRAY',/$)
        ACCEPT 807,FLE
        ACCEPT 825,NXRAY
807     FORMAT(A5)

        CALL IFILE(20,FLE)
        NX1 = NXRAY/4
        DO 550 I=0,NX1-1
        READ(20,551)W(I),W(I+NX1),W(I+2*NX1),W(I+3*NX1)
        W(I+NX1) = W(I+NX1)*.3048
        W(I+2*NX1) = W(I+2*NX1)*.3048
        W(I+3*NX1) = W(I+3*NX1)*.3048
550     W(I) = W(I)*.3048
551     FORMAT(4(E15.7))
        END FILE 20
C
C       HIGH PASS FILTER THE DATA
C
        TYPE 9
9       FORMAT(' INPUT CUT-OFF FREQ,SAMPLING INTERVAL,'
     1  'NO. OF FILTER SECTIONS',$)
        ACCEPT 2,FC,TS,NS
2       FORMAT(3G)
        FK = FC/V
        CALL HPDES(FC,TS,NS)
        DO 140 N=1,NS+1
        DO 140 M=1,2
140     F(N,M) = 0.0
        DO 150 M=0,NXRAY-1
        F(1,3) = W(M)
        DO 160 N=1,NS
        TEMP = A(N)*(F(N,3)-2.*F(N,2)+F(N,1))
160     F(N+1,3) = TEMP-B(N)*F(N+1,2)-C(N)*F(N+1,1)
        DO 170 N=1,NS+1
        DO 170 MM=1,2
170     F(N,MM) =  F(N,MM+1)
150     W(M) = F(NS+1,3)
C
C       COMPUTE AND SUBTRACT OUT MEAN VALUE
C       COMPUTE THE SAMPLE VARIANCE
C
        WBAR = 0.0
        VAR = 0.0
        DO 610 JJ = 0,NOPTS-1
610     WBAR = WBAR + W(JJ)
        WBAR = WBAR/FLOAT(NOPTS)
        DO 600 I=0,NOPTS-1
        W(I) = W(I) - WBAR
```

```
600       VAR = VAR + W(I)*W(I)
          VAR = VAR/FLOAT(NOPTS)
C
C         COMPUTE INTEGRAL OF W(K)**2 USING TRAPIZOIDAL RULE
C
          WSUM = Ø.Ø
          DO 37Ø I=1,NOPTS-2
37Ø       WSUM = WSUM+W(I)*W(I)
          WSUM = WSUM + .5*W(Ø)*W(Ø)+.5*W(NOPTS-1)*W(NOPTS-1)
          WSUM = DELX*WSUM/FTDAT
C
C         ADD ZEROS TO DATA
C
          DO 1Ø I=NOPTS,NLAST
1Ø        W(I) = Ø.Ø
C
C         MAKE ARRAY COMPLEX
C
          DO 512 J=NPTS-1,Ø,-1
512       W(2*J) = W(J)
          DO 513 J=1,2*NPTS-1,2
513       W(J) = Ø.Ø
C
C         COMPUTE PHI L(K)
C
          CALL CFFT(MPWRN,NPTS,W,1)

          NHALF = NPTS/2
          NMIN1 = NHALF-1
          TMS = TWOL*TWOL/FTDAT
          DO 11 I=Ø,2*NLAST,2
          W(I+1) = Ø.Ø
          W(I) = W(I)*TMS
11        CONTINUE
          W(2*NPTS-1) = Ø.Ø
C
C         OUTPUT VALUES OF PHI OF L(K) TO DATA FILE (FIRST HALF ONLY)
C
          CALL OFILE (2Ø,'PHILK')
          WRITE(2Ø,9ØØ)
          WRITE(2Ø,95Ø)FC,FK,FLE
95Ø       FORMAT(/1X,'HIGH PASS FILTERED DATA',/1X,'CUT-OFF FREQ =',G,
     1    ' (K =',G,')',//1X,
     2    'POWER SPECTRUM OF PHI OF L(K)',/1X,
     3    'DATA TAKEN FROM FILE ',A5)
          WRITE(2Ø,9Ø6)NPTS,TWOL,MPTS,FTM1
          WRITE(2Ø,9Ø7)MZERO
          WRITE(2Ø,91Ø)WBAR,VAR
          WRITE(2Ø,951)WSUM
951       FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
          WRITE(2Ø,952)
```

```
952     FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE POWER SPECTRUM',
     1  /1X,5X,'K',10X,'PS VALUE',8X,'K CONTD',4X,'PS VALUE CONTD',/)
        MHALF = NPTS/4
        M41 = MHALF-1
        DO 823 I=0,M41
        DEL = DELK*I
        XX = W(2*I)
        K = MHALF+I
        DEL2 = DELK*K
        YY= W(2*K)
823     WRITE(20,903)DEL,XX,DEL2,YY
        DEL3 = DELK*NHALF
        ZZ = W(2*NHALF)
        WRITE(20,908)DEL3,ZZ
        END FILE 20
C
C       PERFORM INTEGRATION CHECK
C
        TYPE 810
810     FORMAT(/1X,'PERFORM INTEGRATION CHECK (Y OR N)    ',S)
        ACCEPT 807,CHK
        IF (CHK.EQ.'N') GO TO 23
        EDR = NPTS*DELK
        CALL SIMP(0.0,EDR,DELK,NPTS,ANS)
        TYPE 811,ANS
811     FORMAT(//1X,'INTEGRAL OF PHI OF L (K) =',E12.4)
C
C       OBTAIN AUTOCORRELATION FUNCTION
C
23      CALL CFFT(MPWRN,NPTS,W,2)
C
C       TYPE OUT AUTOCORRELATION TO DATA FILE
C
        XAOFF = 13000.
        RFCON = PTS/TWOL
        DR0 = W(0)*RFCON
        MAOFF = 13000./DELX
C
        CALL OFILE (20,'AUTO')
        WRITE(20,900)
        WRITE(20,1950)FLE
1950    FORMAT(/1X,'AUTOCORRELATION OF',/1X,
     1  'HIGH PASS FILTERED NON-HOMOGENEOUS SAMPLE',/1X,
     2  'DATA TAKEN FROM FILE ',A5)
        WRITE(20,906)NPTS,TWOL,MPTS,FTM1
        WRITE(20,907)MZERO
        WRITE(20,910)WBAR,VAR
        WRITE(20,951)WSUM
        WRITE(20,381)XAOFF,MAOFF
381     FORMAT(/1X,' TRUCATION POINT WAS',G,' METERS',/1X,
     1  'WHICH CONTAINS ',I7,' POINTS')
        WRITE(20,1952)
```

```
1952    FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE AUTOCORRELATION',
     1  /1X,8X,'X',12X,'    RL    ',9X,'RL/R0')
        DO 1823 I=0,MAOFF
        DEL = DELX*I
        XX = W(2*I)*RFCON
        YY = XX/DR0
1823    WRITE(20,915)DEL,XX,YY
915     FORMAT(1X,3(2X,G))
        END FILE 20
C

900     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB4')
903     FORMAT(1X,F10.6,3X,E12.4,3X,F10.6,3X,E12.4)
905     FORMAT(//1X,'RL(0) = ',E12.4)
906     FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1  ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1  ' METER')
907     FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
908     FORMAT(29X,F10.4,3X,E12.4)
910     FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1  'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
C
C
4999    END
```

Subroutine BIN

```
      SUBROUTINE BIN(S)
C     PROGRAM BIN,  ATMOSPHERIC TURBULENCE TASK
C     ITEM 4
C
      COMMON/AA/BIN1(501),BIN2(501),TOTAL,NBIN,BINW,NPTS
      DIMENSION S(0/15000)

      TYPE 798
798   FORMAT(/1X,' INPUT BIN WIDTH',/1X,' NO. OF BINS TOTAL',/$)
      ACCEPT 744,BINW
      ACCEPT 744,NBIN
744   FORMAT(G)

      NBIN2 = NBIN/2
      DO 10 J=1,NBIN2
      BIN1(J) = 0.0
10    BIN2(J) = 0.0
      TOTAL = 0.0
      TBIN = 0.0

      DO 11 J= 0,NPTS-1
      IF (S(J).LT.0.0) GO TO 50
      IF (S(J).EQ.0.0) BIN1(1) = BIN1(1)+1.
      IF (S(J).EQ.0.0) GO TO 11
      RSLT1 = S(J)/BINW
      IR = IFIX(RSLT1) + 1
      BIN1(IR) = BIN1(IR) + 1.
      GO TO 11
50    RSLT2 = ABS(S(J))/BINW
      IR = IFIX(RSLT2) + 1
      BIN2(IR) = BIN2(IR) + 1.
11    CONTINUE

      TOTAL = 0.0
      DO 13 J=1,NBIN2
13    TOTAL = TOTAL + BIN1(J) + BIN2(J)
      TYPE 802,TOTAL
802   FORMAT(/1X,'TOTAL NO. OF POINTS USED FOR IST PASS =',F8.2)

C
      RETURN
C
      END
```

Subroutine BINSQ

```
      SUBROUTINE BINSQ(S)
C     PROGRAM BINSQ.  ATMOSPHERIC TURBULENCE TASK
C     ITEM 4
C
      COMMON/BB/BIN3(700),TOT1,NBIN1,BINW1,NPTS1
      DIMENSION S(0/15000)

      TYPE 798
798   FORMAT(' INPUT BIN WIDTH ',/1X,' NO. OF BINS',/$)
      ACCEPT 744,BINW1
      ACCEPT 744,NBIN1
744   FORMAT(G)

      DO 10 J=1,NBIN1
10    BIN3(J) = 0.0
      TOT1 = 0.0
      TBIN = 0.0

      DO 11 J= 0,NPTS1-1
      IF (S(J).EQ.0.0) BIN3(1) = BIN3(1)+1.
      IF (S(J).EQ.0.0) GO TO 11
      RSLT = S(J)/BINW1
      IR = IFIX(RSLT) + 1
      BIN3(IR) = BIN3(IR) + 1.0
11    CONTINUE
      TOT1 = 0.0
      DO 13 J=1,NBIN1
13    TOT1 = TOT1 +  BIN3(J)
      TYPE 802,TOT1
802   FORMAT(/1X,'TOTAL NO. OF POINTS USED FOR IST PASS =',F8.2)

C
      RETURN
C
      END
```

Subroutine CFFT1

```
C       SUBROUTINE CFFT.  CALCULATES FFT OF ANY DATA ARRAY
C       NUMBER OF DATA POINTS IS POWER OF 2(M)
C
C
        SUBROUTINE CFFT(MPOWR,NPTS,S,NWAY)

        DIMENSION B(2)
        COMPLEX S(0/5000),U,T,W1
        EQUIVALENCE (W1,B)
        DO 301 I=NPTS,1,-1
        NRAY = I-1
301     S(I) = S(NRAY)
D454    FORMAT(1X,4(E12.4,3X))
D       TYPE 454,S(1),S(2)
D       TYPE 454,S(3),S(4)
D       TYPE 454,S(5),S(6)
D       TYPE 454,S(NPTS/2+1)
D       TYPE 454,S(NPTS/2),S(NPTS/2+2)
D       TYPE 454,S(NPTS),S(NPTS-1),S(NPTS-2)
D       TYPE 454,S(NPTS/2-1),S(NPTS/2-2)
D       TYPE 454,S(NPTS/2+3),S(NPTS/2+4)
C
C
        NV2 =NPTS/2
        NM1 = NPTS-1
        J = 1
        DS = 1./FLOAT(NPTS)
        DO 7 I=1,NM1
        IF (I.GE.J) GO TO 5
        T = S(J)
        S(J) = S(I)
        S(I) = T
5       K = NV2
6       IF (K.GE.J) GO TO 7
        J= J-K
        K=K/2
        GO TO 6
7       J = J+K
        PI = 3.141592654
        DO 30 L=1,MPOWR
        LE =2**L
        LE1 = LE/2
        FLE1= FLOAT(LE1)
        U = (1.,0.0)
        PL=PI/FLE1
        PL =-PL
        B(1) = COS(PL)
        B(2) = SIN(PL)
        DO 20 J=1,LE1
        DO 11 I=J,NPTS,LE
        IP =I+LE1
        T=S(IP)*U
        S(IP)=S(I)-T
```

```
11      S(I) =S(I)+T
20      U=U*W1
30      CONTINUE
        NOP = NPTS-1
        IF(NWAY.EQ.1) GO TO 200
        DO 40 I=0,NOP
        IDX = I+1
        XX2 = DS*REAL(S(IDX))
40      S(I) = CMPLX(XX2,0.0)
        GO TO 210
200     DO 300 I=0,NOP
        IDX1 = I+1
        XYZ = (CABS(S(IDX1)))*DS
        XX3 = XYZ*XYZ
300     S(I) = CMPLX(XX3,0.0)
210     CONTINUE

D       TYPE 454,S(0),S(1)
D       TYPE 454,S(2),S(3)
D       TYPE 454,S(4),S(5)
D       TYPE 454,S(6),S(7)
D       TYPE 454,S(NPTS-4),S(NPTS-3)
D       TYPE 454,S(NPTS-2),S(NPTS-1)
        RETURN
        END
```

Subroutine CFFT

```
C       SUBROUTINE CFFT.  CALCULATES FFT OF ANY DATA ARRAY
C       NUMBER OF DATA POINTS IS POWER OF 2(M)
C
C
        SUBROUTINE CFFT(MPOWR,NPTS,S,NWAY)

        DIMENSION B(2)
        COMPLEX S(Ø/32768),U,T,W1
        EQUIVALENCE (W1,B)
        DO 3Ø1 I=NPTS,1,-1
        NRAY = I-1
3Ø1     S(I) = S(NRAY)
D454    FORMAT(1X,4(E12.4,3X))
D       TYPE 454,S(1),S(2)
D       TYPE 454,S(3),S(4)
D       TYPE 454,S(5),S(6)
D       TYPE 454,S(NPTS/2+1)
D       TYPE 454,S(NPTS/2),S(NPTS/2+2)
D       TYPE 454,S(NPTS),S(NPTS-1),S(NPTS-2)
D       TYPE 454,S(NPTS/2-1),S(NPTS/2-2)
D       TYPE 454,S(NPTS/2+3),S(NPTS/2+4)
C
C
        NV2 =NPTS/2
        NM1 = NPTS-1
        J = 1
        DS = 1./FLOAT(NPTS)
        DO 7 I=1,NM1
        IF (I.GE.J) GO TO 5
        T = S(J)
        S(J) = S(I)
        S(I) = T
5       K = NV2
6       IF (K.GE.J) GO TO 7
        J= J-K
        K=K/2
        GO TO 6
7       J = J+K
        PI = 3.141592654
        DO 3Ø L=1,MPOWR
        LE =2**L
        LE1 = LE/2
        FLE1= FLOAT(LE1)
        U = (1.,Ø.Ø)
        PL=PI/FLE1
        PL =-PL
        B(1) = COS(PL)
        B(2) = SIN(PL)
        DO 2Ø J=1,LE1
        DO 11 I=J,NPTS,LE
        IP =I+LE1
        T=S(IP)*U
        S(IP)=S(I)-T
```

```
11      S(I) =S(I)+T
20      U=U*W1
30      CONTINUE
        NOP = NPTS-1
        IF(NWAY.EQ.1) GO TO 200
        DO 40 I=0,NOP
        IDX = I+1
        XX2 = DS*REAL(S(IDX))
40      S(I) = CMPLX(XX2,0.0)
        GO TO 210
200     DO 300 I=0,NOP
        IDX1 = I+1
        XYZ = (CABS(S(IDX1)))*DS
        XX3 = XYZ*XYZ
300     S(I) = CMPLX(XX3,0.0)
210     CONTINUE

D       TYPE 454,S(0),S(1)
D       TYPE 454,S(2),S(3)
D       TYPE 454,S(4),S(5)
D       TYPE 454,S(6),S(7)
D       TYPE 454,S(NPTS-4),S(NPTS-3)
D       TYPE 454,S(NPTS-2),S(NPTS-1)
        RETURN
        END
```

Subroutine DGELG

```
C
C     ..................................................................
C
C        SUBROUTINE DGELG
C
C        PURPOSE
C           TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.
C
C        USAGE
C           CALL DGELG(R,A,M,N,EPS,IER)
C
C        DESCRIPTION OF PARAMETERS
C           R      - DOUBLE PRECISION M BY N RIGHT HAND SIDE MATRIX
C                    (DESTROYED). ON RETURN R CONTAINS THE SOLUTIONS
C                    OF THE EQUATIONS.
C           A      - DOUBLE PRECISION M BY M COEFFICIENT MATRIX
C                    (DESTROYED).
C           M      - THE NUMBER OF EQUATIONS IN THE SYSTEM.
C           N      - THE NUMBER OF RIGHT HAND SIDE VECTORS.
C           EPS    - SINGLE PRECISION INPUT CONSTANT WHICH IS USED AS
C                    RELATIVE TOLERANCE FOR TEST ON LOSS OF
C                    SIGNIFICANCE.
C           IER    - RESULTING ERROR PARAMETER CODED AS FOLLOWS
C                    IER=0  - NO ERROR,
C                    IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
C                             PIVOT ELEMENT AT ANY ELIMINATION STEP
C                             EQUAL TO 0,
C                    IER=K  - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
C                             CANCE INDICATED AT ELIMINATION STEP K+1,
C                             WHERE PIVOT ELEMENT WAS LESS THAN OR
C                             EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
C                             ABSOLUTELY GREATEST ELEMENT OF MATRIX A.
C
C        REMARKS
C           INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE
C           IN M*N RESP. M*M SUCCESSIVE STORAGE LOCATIONS. ON RETURN
C           SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
C           THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
C           GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
C           ARE DIFFERENT FROM 0. HOWEVER WARNING IER=K - IF GIVEN -
C           INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL
C           SCALED MATRIX A AND APPROPRIATE TOLERANCE EPS, IER=K MAY BE
C           INTERPRETED THAT MATRIX A HAS THE RANK K. NO WARNING IS
C           GIVEN IN CASE M=1.
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION WITH
C           COMPLETE PIVOTING.
```

```
C
C     ..................................................................
C
      SUBROUTINE DGELG(R,A,M,N,EPS,IER)
C
C
      DIMENSION A(1),R(1)
      DOUBLE PRECISION R,A,PIV,TB,TOL,PIVI
      IF(M)23,23,1
C
C     SEARCH FOR GREATEST ELEMENT IN MATRIX A
    1 IER=0
      PIV=0.D0
      MM=M*M
      NM=N*M
      DO 3 L=1,MM
      TB=DABS(A(L))
      IF(TB-PIV)3,3,2
    2 PIV=TB
      I=L
    3 CONTINUE
      TOL=EPS*PIV
C     A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).
C
C
C     START ELIMINATION LOOP
      LST=1
      DO 17 K=1,M
C
C     TEST ON SINGULARITY
      IF(PIV)23,23,4
    4 IF(IER)7,5,7
    5 IF(PIV-TOL)6,6,7
    6 IER=K-1
    7 PIVI=1.D0/A(I)
      J=(I-1)/M
      I=I-J*M-K
      J=J+1-K
C     I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
C
C     PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
      DO 8 L=K,NM,M
      LL=L+I
      TB=PIVI*R(LL)
      R(LL)=R(L)
    8 R(L)=TB
C
C     IS ELIMINATION TERMINATED
      IF(K-M)9,18,18
C
```

```
C       COLUMN INTERCHANGE IN MATRIX A
      9 LEND=LST+M-K
        IF(J)12,12,10
     10 II=J*M
        DO 11 L=LST,LEND
        TB=A(L)
        LL=L+II
        A(L)=A(LL)
     11 A(LL)=TB
C
C       ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
     12 DO 13 L=LST,MM,M
        LL=L+I
        TB=PIVI*A(LL)
        A(LL)=A(L)
     13 A(L)=TB
C
C       SAVE COLUMN INTERCHANGE INFORMATION
        A(LST)=J
C
C       ELEMENT REDUCTION AND NEXT PIVOT SEARCH
        PIV=0.D0
        LST=LST+1
        J=0
        DO 16 II=LST,LEND
        PIVI=-A(II)
        IST=II+M
        J=J+1
        DO 15 L=IST,MM,M
        LL=L-J
        A(L)=A(L)+PIVI*A(LL)
        TB=DABS(A(L))
        IF(TB-PIV)15,15,14
     14 PIV=TB
        I=L
     15 CONTINUE
        DO 16 L=K,NM,M
        LL=L+J
     16 R(LL)=R(LL)+PIVI*R(L)
     17 LST=LST+M
C       END OF ELIMINATION LOOP
C
C
C       BACK SUBSTITUTION AND BACK INTERCHANGE
     18 IF(M-1)23,22,19
     19 IST=MM+M
        LST=M+1
        DO 21 I=2,M
        II=LST-I
        IST=IST-LST
        L=IST-M
```

```
      L=A(L)+.5D0
      DO 21 J=II,NM,M
      TB=R(J)
      LL=J
      DO 20 K=IST,MM,M
      LL=LL+1
   20 TB=TB-A(K)*R(LL)
      K=J+L
      R(J)=R(K)
   21 R(K)=TB
   22 RETURN
C
C
C     ERROR RETURN
   23 IER=-1
      RETURN
      END
```

Subroutine FAC1

```
C         SUBROUTINE FAC1: COMPUTES K!/((K-M)!M!)
C
          SUBROUTINE FAC1(K,M,AKM)
C
          IF (K.EQ.Ø.OR.K.EQ.1) GO TO 35
          FK =FLOAT(K)
          FKC = FK
          DO 3Ø I=1,K-1
          AI =FLOAT(I)
3Ø        FK = FK*(FKC-AI)
35        IF (K.EQ.1.OR.K.EQ.Ø) FK =1.
          IF (M.EQ.1.OR.M.EQ.Ø) GO TO 45
          FM = FLOAT(M)
          FMC = FM
          DO 4Ø I=1,M-1
          AI = FLOAT(I)
4Ø        FM = FM*(FMC-AI)
45        IF (M.EQ.1.OR.M.EQ.Ø) FM = 1.
          FKM = FLOAT(K)-FLOAT(M)
          IF (FKM.EQ.1..OR.FKM.EQ.Ø.Ø) GO TO 55
          KM = K-M-1
          FKMC = FKM
          DO 5Ø I=1,KM
          AI = FLOAT(I)
5Ø        FKM = FKM*(FKMC-AI)
55        IF (FKM.EQ.1.Ø.OR.FKM.EQ.Ø.Ø)FKM=1.Ø
          AKM = FK/(FKM*FM)
          RETURN
          END
```

Program FINAL

```
C       PROGRAM FINAL.  ATMOSPHERIC TURBULENCE TASK
C       ITEM 2(REVISION 2&3), PARTS 6,7,8, AND 9. PHASE II OF ATMOS-
C       PHERIC TURBULENCE
C       MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C       COMPONENT WITH LOW FREQUENCY CONTAMINATION
C
C       REQUIRES SUBROUTINES GAM,PAR1  & INTERPOLATION
C        ROUTINE:   ANRP1; AK1 REQUIRES PAR,AKDAT,SIMQ,DGELG;
C       USES SUBROUTINES TRAP(3&6) AND SIMP2 FOR INTEGRATIONS
C       ALSO USES SUBROUTINE SET AND FUNCTION DECT INCLUDED IN PROGRAM
C
C       LMAX MAX = 10; NCOUNT MAX = 10; NXIH MAX = 290; M MAX = 4
C
C       DATA FILES READ BY PROGRAM ARE:
C               AUTO: AUTOCORRELATION -FROM ATURB2
C       DATA FILE CREATED BY THIS PROGRAM ARE:
C               ITM2: VALUES OF CAP R (XI), PART 9.K, ITEM 2
C               ITM2L: VALUES OF SIGMA SQRD*L*PHI OF K(KL), PART 9.L
C
        DOUBLE PRECISION AMTRX,XC,DIJ,A,SUM,AM,S1,S2,RI,CJ,ALG2
        DOUBLE PRECISION S3,S4
C
        COMMON/BB/X(3),AKI(3),XPL
        COMMON/TR5/DELX1,NXIH1,AI5I(0/290)
        COMMON/TR2/DELX,NXIH,AI2T(0/290)
C
        DIMENSION DIJ(7,7),RI(7),CJ(7)
        DIMENSION AL(0/11),SIG2(0/11),PH1(0/100),PH2(0/100)
        DIMENSION PHI(0/11,0/290),PHI1(0/11,0/290),AI(0/6)
        DIMENSION AI3(0/4,0/11),AI4(0/4,0/11),AI5(0/11),AI6(0/11)
        DIMENSION AI7(0/9),AI8(0/4),RL(0/290)
        DIMENSION SIG2J(0/11),XC(6),A(6,6),AMTRX(36)
        DIMENSION AI1(0/11),AI2(0/11),AI4T(0/4),AI3T(0/4)
        DIMENSION NDO(0/11),NUPR(0/11),ELEFT(0/11)
        DIMENSION EWAY(0/11),AI5X(0/11),AI6X(0/11)
C
C
C
        TYPE 450
450     FORMAT(' HAS DATA BEEN COMPUTED ?',/$)
        ACCEPT 807,AND11
        IF (AND11.EQ.'Y') GO TO 451
C
C
        TYPE 1001
1001    FORMAT(/1X,'INPUT DELX,LMAX',/$)
        ACCEPT 1750,DELX,LMAX
1750    FORMAT(2G)
        TYPE 1751
1751    FORMAT(' IS RECORD TRANSVERSE OR LONGITUDIONAL (T OR L) ',$)
        ACCEPT 807,WHICH
```

```
807     FORMAT(A5)
        DELX1= DELX
C
C
C       SIGMA SQRD & L COMPUTED FROM PROGRAM PART5
C
        AL(0) = 20.
        AL(1) = 30.
        AL(2) = 40.
        AL(3) = 50.
        AL(4) = 60.
        AL(5) = 70.
        AL(6) = 80.
        AL(7) = 90.
        AL(8) = 100.
        AL(9) = 110.
        AL(10) = 120.
        AL(11) = 130.
        SIG2(0) = .339271
        SIG2(1) = .3400205
        SIG2(2) = .3666069
        SIG2(3) = .4009919
        SIG2(4) = .4378971
        SIG2(5) = .4753911
        SIG2(6) = .5126825
        SIG2(7) = .5494336
        SIG2(8) = .5855068
        SIG2(9) = .6208566
        SIG2(10) = .6554832
        SIG2(11) = .73219
C
C       PART 7. REQUIRES SUBROUTINES GAM, TRAP(3&6),AK
C       COMPUTES VON-KARMEN AUTOCORRELATION
C
        TYPE 1511
1511    FORMAT(' INPUT HIGHEST INDEX OF XI, EH, & M,  '$)
        ACCEPT 1752,NXIH,EH,MM
1752    FORMAT(3G)
        NXIH1=NXIH
        PI = 3.14159265
        C116 = 11./6.
        C56 = 5./6.
        C43 = 4./3.
        CALL GAM(C116,G1)
        CALL GAM(C43,G2)
        GAM13 = 2.67893853
        BETA = 2.*G1*(PI**.5)/(5.*G2)
        C23 = 2./3.
        C13 = 1./3.
        CONST = -BETA*(2.**C23)/GAM13
        CONS1 = BETA/(GAM13*2.**C13)
```

```
      CONS2 = ((2.**C23)/GAM13)
      PH2(0) = 0.0
      PH1(0) = 1.0
      DO 1070 I=1,100
      E1 = .1*I
      CALL AK(1,E1,AK1)
      CALL AK(2,E1,AK2)
      PH1(I) = CONS2*(E1**C13)*AK1-CONS2*(E1**C13)*AK2*E1/2.
      IF (WHICH.EQ.'L')PH1(I) = CONS2*
     1                        (E1**C13)*AK1
      PH2(I)=CONS1*((E1**C43)*AK1 -
     1                        8.*(E1**C13)*AK2/3.)
      IF (WHICH.EQ.'L')PH2(I) = CONST*(E1**C13)*AK2
1070  CONTINUE
D     TYPE 6100,PH1(97),PH1(98),PH1(99),PH1(100)
D6100 FORMAT(' PH1: ',4G)
D     TYPE 6101,PH2(97),PH2(98),PH2(99),PH2(100)
D6101 FORMAT(' PH2 :',4G)
C
C     DO INTEGRALS 1 TO 4
C     PH1 AND PH2 = 0 IF MORE THAN 100 PTS USED (NUPR IS ARRAY INDEX)
C     IF LESS THAN 100 PTS USED IN INTEGRAL, SIMPSON'S RULE
C     IS USED UP TO LAST "EVEN" PT IN INTEGRAL. THE END OF THE
C     INTEGRAL IS DONE AS PER HILDEBRAND (P.111) FOR THE REMAINDING
C     PART OF THE INTEGRAL (USING TERMS UP TO THE FOURTH POWER)
C
      EBAR = .1/BETA
      DO 6068 L = 0,LMAX+1
      EHMAX = EH/AL(L)
      NUPR(L) = IFIX(EHMAX/EBAR)
      NDO(L) = 0
      IF (NUPR(L).GT.100) NUPR(L) = 100
      IF (NUPR(L).EQ.100) GO TO 444
      NDO(L) = 1
      AODD = FLOAT(NUPR(L))/2.
      NODD = NUPR(L)/2
      REM = AODD - FLOAT(NODD)
      IF (REM.GE..5) NUPR(L) = NUPR(L) -1
      ELEFT(L) = EHMAX - FLOAT(NUPR(L))*EBAR
D     TYPE 447,AODD,NODD,REM,ELEFT(L)
D447  FORMAT(' AODD,NODD,REM,ELEFT(L) :',G,2X,I3,2X,2G)
444   CONTINUE
D     TYPE 6003,NUPR(L),L
D6003 FORMAT(' NUPR(L) = ',I4,' L INDEX =',I3)
      DO 3002 I=0,NUPR(L)
      E = EBAR*I
      AI2I(I) = PH1(I)*PH2(I)*E
      AI5I(I) = PH1(I)*PH1(I)
      DO 3003 J=0,MM
      J1 = J+1
      PHI(J,I) = (E**J1)*PH2(I)
      PHI1(J,I) = (E**J)*PH1(I)
```

```
3ØØ3      CONTINUE
3ØØ2      CONTINUE
          CALL SIMP(EBAR,NUPR(L),AI2I,A1)
          CALL SIMP(EBAR,NUPR(L),AI5I,A2)
          AI1(L) = A1
          AI2(L) = A2
          IF (NDO(L).EQ.Ø) GO TO 445
D         TYPE 449,L,AI1(L),AI2(L)
D449      FORMAT(' FOR L =',I3,' AI1 =',G,' AI2 =',G)
          AI1(L) = AI1(L) + DECT(ELEFT(L),AI2I,NUPR(L))
D         TYPE 448,AI1(L)
D448      FORMAT(' AI1 =',G)
          AI2(L) = AI2(L) + DECT(ELEFT(L),AI5I,NUPR(L))
445       CONTINUE
D         TYPE 66Ø9,L,AI1(L),AI2(L)
D66Ø9     FORMAT(1X,'L,AI1,AI2 :',1X,I4,2G)
          DO 3ØØ4 JJ=Ø,MM
          DO 31Ø4 I=Ø,NUPR(L)
          AI2I(I) = PHI(JJ,I)
31Ø4      AI5I(I) = PHI1(JJ,I)
          CALL SIMP(EBAR,NUPR(L),AI2I,A1)
          AI3(JJ,L) = A1
          CALL SIMP(EBAR,NUPR(L),AI5I,A2)
          AI4(JJ,L) = A2
          IF (NDO(L).EQ.Ø) GO TO 446
D         TYPE 453,JJ,L,AI3(JJ,L),AI4(JJ,L)
D453      FORMAT('JJ,L,AI3,AI4:',I3,1X,I2,1X,2G)
          AI3(JJ,L) = AI3(JJ,L) + DECT(ELEFT(L),AI2I,NUPR(L))
          AI4(JJ,L) = AI4(JJ,L) + DECT(ELEFT(L),AI5I,NUPR(L))
446       CONTINUE
D         TYPE 66Ø8,JJ,L,AI3(JJ,L),AI4(JJ,L)
D66Ø8     FORMAT(1X,'J,L,AI3,AI4 :',2X,2I4,2G)
3ØØ4      CONTINUE
6Ø68      CONTINUE
C
C         READ IN AUTOCORRELATION FN.
C
          CALL IFILE (2Ø,'AUTO')
          READ(2Ø,9ØØ)
          READ(2Ø,195Ø)FLE
195Ø      FORMAT(/1X,'AUTOCORRELATION OF STATIONARY SAMPLE',/1X,
     1    'DATA TAKEN FROM FILE ',A5)
          READ(2Ø,9Ø6)NPTS,TWOL,MPTS,FTM1
          READ(2Ø,9Ø7)MZERO
          READ(2Ø,91Ø)WBAR,VAR
          READ(2Ø,951)WSUM
951       FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
          READ(2Ø,381)XAOFF,MAOFF
381       FORMAT(/1X,' TRUCATION POINT WAS',G,' METERS',/1X,
     1    'WHICH CONTAINS ',I7,' POINTS')
          READ(2Ø,1952)
```

```
1952    FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE AUTOCORRELATION',
     1  /1X,8X,'X',12X,'    RL    ',9X,'RL/RØ')
        DO 1823 I=Ø,NXIH
1823    READ(2Ø,1915)DEL,RL(I),YY
1915    FORMAT(1X,3(2X,G))
        END FILE 2Ø
        TYPE 4324,RL(NXIH)
4324    FORMAT( ' RL(NXIH) =',G)
C
C       DETERMINE WHETHER SIMPSON'S RULE OR TRAP. RULE IS TO BE
C       USED IN INTEGRAL I5 & I6
C
        DO 7Ø23 L=Ø,LMAX+1
        EB = DELX/AL(L)
        EWAY(L) = 'TRP'
7Ø23    IF (EB.GT.EBAR) EWAY(L) = 'SMP'
        LCHNG = -2
        TP = 1
        DO 7Ø24 L=Ø,LMAX+1
        IF (EWAY(L).EQ.'SMP') TP = Ø
7Ø24    IF (EWAY(L).EQ.'SMP') LCHNG = L
        IF (TP.EQ.1) TYPE 7ØØ6
7ØØ6    FORMAT(' SIMPSONS RULE NOT USED FOR I5 & I6')
        IF (TP.EQ.Ø) TYPE 7ØØ7,LCHNG
7ØØ7    FORMAT(' SIMPSONS RULE USED IN I5& I6 UP TO L INDEX =',I3)
C
C       DO INTEGRAL I5 AND I6 WHEN E/L > EBAR
C
        IF (TP.EQ.1) GO TO 7ØØ8
        DO 7Ø5Ø L=Ø,LMAX+1
        IF (EWAY(L).EQ.'TRP') GO TO 7Ø5Ø
        ALBAR = AL(L)*EBAR
        DO 7ØØ2 I=Ø,NUPR(L)
        ALDT = I*ALBAR
        NPLCE = IFIX(ALDT/DELX+.5)
        IXMID = NPLCE
        IXLOW = IXMID - 1
        IXHIG = IXMID+ 1
        IF (IXMID.EQ.Ø) IXLOW = Ø
        IF (IXMID.EQ.Ø) IXHIG = 2
        IF (IXMID.EQ.Ø) IXMID = 1
        IF (IXMID.GE.NXIH) IXLOW = NXIH-2
        IF (IXMID.GE.NXIH) IXHIG = NXIH
        IF (IXMID.GE.NXIH) IXMID = NXIH-1
        AKI(1) = RL(IXLOW)
        AKI(2) = RL(IXMID)
        AKI(3) = RL(IXHIG)
        X(1) = IXLOW*DELX
        X(2) = IXMID*DELX
        X(3) = IXHIG*DELX
        XPL = ALDT
        CALL PARAB(AKZ)
        RLI = AKZ
        E=EBAR*I
        AI2I(I) = E*PH2(I)*RLI
```

```
7002    AI5I(I) = PH1(I)*RLI
        CALL SIMP(EBAR,NUPR(L),AI2I,A3)
        CALL SIMP(EBAR,NUPR(L),AI5I,A4)
        AI5(L) = A3/RL(0)
        AI6(L) = A4/RL(0)
D       TYPE 7449,L,AI5(L),AI6(L)
D7449   FORMAT(' L,AI5,AI6:',3G)
        IF (NDO(L).EQ.0) GO TO 7050
        AI5(L) = AI5(L) + (DECT(ELEFT(L),AI2I,NUPR(L)))/RL(0)
        AI6(L) = AI6(L) + (DECT(ELEFT(L),AI5I,NUPR(L)))/RL(0)
D       TYPE 7450,AI5(L),AI6(L)
D7450   FORMAT(' AI5,AI6:',2G)
7050    CONTINUE
C
C
C
C       INTERPOLATE PH1 AND PH2 WHEN E/L < EBAR
C
7008    DO 1050 L=0,LMAX+1
        IF (L.LE.LCHNG) GO TO 1050
        PHI(L,0) = 1.0
        PHI1(L,0) = 0.0
        EB = DELX/AL(L)
D       TYPE 1071,EB,EBAR
D1071   FORMAT(' EB =',G,' EBAR =',G)
        XMAX2 = DELX*NXIH
        XMAX1 = 100.*EBAR
        DO 2071 JX = 1,NXIH
        IXMID = IFIX(EB*JX/EBAR+.5)
        IXLOW = IXMID-1
        IXHIG = IXMID+1
        IF (IXMID.GE.100) IXHIG = 100
        IF (IXMID.GE.100) IXLOW = 98
        IF (IXMID.GE.100) IXMID = 99
        IF (IXMID.EQ.0) IXHIG = 2
        IF (IXMID.EQ.0) IXLOW = 0
        IF (IXMID.EQ.0) IXMID = 1
        AKI(1) = PH1(IXLOW)
        AKI(2) = PH1(IXMID)
        AKI(3) = PH1(IXHIG)
        X(1) = IXLOW*EBAR
        X(2) = IXMID*EBAR
        X(3) = IXHIG*EBAR
        XPL = EB*JX
        IF (XPL.GT.XMAX1) GO TO 2072
        CALL PARAB(AKZ)
        PHI(L,JX) = AKZ
        AKI(1) = PH2(IXLOW)
        AKI(2) = PH2(IXMID)
        AKI(3) = PH2(IXHIG)
        X(1) = IXLOW*EBAR
        X(2) = IXMID*EBAR
        X(3) = IXHIG*EBAR
        XPL = EB*JX
        CALL PARAB(AKZ)
        PHI1(L,JX) = AKZ
```

```
2072    IF (XPL.GT.XMAX1) PHI(L,JX) = 0.0
        IF (XPL.GT.XMAX1) PHI1(L,JX) = 0.0
2071    CONTINUE
1050    CONTINUE
C
C       INTEGRALS 5 AND 6 WHEN E/L < EBAR
C
        DO 3010 L=0,LMAX+1
        IF (L.LE.LCHNG) GO TO 3010
        DO 3011 I=0,NXIH
        AI5I(I) = DELX*I*PHI1(L,I)*RL(I)
3011    AI2I(I) = PHI(L,I)*RL(I)
        CALL TRAP2(AIXY)
        AI6(L) = AIXY/(RL(0)*AL(L))
        CALL TRAP5(AIXY)
        AI5(L) = AIXY/(RL(0)*AL(L)*AL(L))
3010    CONTINUE
D       DO 3019 L=0,LMAX+1
D       IF (EWAY(L).EQ.'SMP') GO TO 3019
D       TYPE 6007,L,AI5(L)
D       TYPE 6008,L,AI6(L)
D3019   CONTINUE
D6007   FORMAT(' FOR L=',I3,' AI5 =',G)
D6008   FORMAT(' FOR L=',I3,' AI6 =',G)
C
C       INTEGRAL I8(J)
C
        DO 1080 J2 = 0,MM
        DO 1081 I=0,NXIH
1081    AI2I(I) = ((DELX*I)**J2)*RL(I)
        CALL TRAP2(AIXY)
1080    AI8(J2) = AIXY
C
C       INTEGRAL I7(J)
C
        MMTWO = 2*MM
        DO 1084  J2 =0,MMTWO
        IEX = J2+1
1084    AI7(J2) = (EH**IEX)/(FLOAT(J2) + 1.)
D       TYPE 5219,AI7(MMTWO)
D5219   FORMAT(' FINAL VALUE OF I7 IS:',/1X,G)
C
C       START ITERATION
C
451     NCOUNT = -1
19      NCOUNT = NCOUNT +1
        TYPE 4
4       FORMAT(' PICK A SIGMA SQRD AND ITS ASSOCIATED L INDEX',/$)
        ACCEPT 1753,SIG2J(NCOUNT),LSIG
1753    FORMAT(2G)
C
```

```
C         DETERMINE CLOSEST MIDPT FOR INTERPOLATION ROUTINES
C
          DIY1 = ABS(SIG2J(NCOUNT)-SIG2(LSIG))
          DIY2 = ABS(SIG2J(NCOUNT)-SIG2(LSIG+1))
          LCTR = LSIG
          IF (DIY2.LT.DIY1) LCTR = LSIG + 1
D         TYPE 7028,LSIG,LCTR
D7028     FORMAT(' LSIG =',I3,' LCTR =',I3)
C
C         CALL INTERPOLATION ROUTINES
C
          CALL SET(AL,SIG2,SIG2J(NCOUNT),LCTR,BK1)
          ALINT = BK1
          TYPE 6056,SIG2J(NCOUNT),ALINT
6056      FORMAT(' FOR SIGMA SQRD =',G,' L INT =',G)
C
          IF (LCTR.LT.(LCHNG+2)) GO TO 7012
          DO 7010 L=0,LMAX+1
          AI5X(L) = AI5(L)*AL(L)*AL(L)
7010      AI6X(L) = AI6(L)*AL(L)
          CALL SET(AI5X,AL,ALINT,LCTR,BK1)
          AI5T = BK1/(ALINT*ALINT)
          CALL SET(AI6X,AL,ALINT,LCTR,BK1)
          AI6I = BK1/ALINT
          GO TO 7011
7012      CALL SET(AI5,AL,ALINT,LCTR,BK1)
          AI5T = BK1
          CALL SET(AI6,AL,ALINT,LCTR,BK1)
          AI6I = BK1
7011      CALL SET(AI1,AL,ALINT,LCTR,BK1)
          AI1T = BK1
          CALL ANTRP(LMAX,ALINT,AL,LSIG,AI2,AI2T)
C
D         TYPE 9084,AI5T,AI6I,AI1T,AI2T
D9084     FORMAT(' FOR INT AI5I =',G,'AI6I =',G,'AI1T =',G,'AI2T =',G)
          DO 6069 IJK =0,MM
          AKI(1) = AI3(IJK,LCTR-1)
          AKI(2) = AI3(IJK,LCTR)
          AKI(3) = AI3(IJK,LCTR+1)
          X(1) = AL(LCTR-1)
          X(2) = AL(LCTR)
          X(3) = AL(LCTR+1)
          XPL = ALINT
          CALL PARAB(AKZ)
          AI3T(IJK) = AKZ
          AKI(1) = AI4(IJK,LCTR-1)
          AKI(2) = AI4(IJK,LCTR)
          AKI(3) = AI4(IJK,LCTR+1)
          X(1) = AL(LCTR-1)
          X(2) = AL(LCTR)
          X(3) = AL(LCTR+1)
          XPL = ALINT
          CALL PARAB(AKZ)
          AI4T(IJK) = AKZ
```

```
D       TYPE 6010,IJK,AI3T(IJK),AI4T(IJK)
D6010   FORMAT(' J =',I3,'AI3T =',G,'AI4T =',G)
6069    CONTINUE
C
C       COMPUTE DEL(L)/DEL(SIGMA SQRD)
C
        AKI(1) = SIG2(LCTR-1)
        AKI(2) = SIG2(LCTR)
        AKI(3) = SIG2(LCTR+1)
        X(1) = AL(LCTR-1)
        X(2) = AL(LCTR)
        X(3) = AL(LCTR+1)
        XPL = SIG2J(NCOUNT)
        CALL PAR1(DLDS)
        TYPE 7004,DLDS
7004    FORMAT(' DLDS = ',G)
C
C
C       COMPUTE COEFFICIENTS XC = R.H.S., ARRAY A = L.H.S.
C
        XC(1) = RL(0)*(-SIG2J(NCOUNT)*DLDS*AI5T+ALINT*AI6T)
        A(1,1) = -SIG2J(NCOUNT)*DLDS*AI1T+ALINT*AI2T
        DO 3013 K=2,MM+2
        K2 = K-2
        L=K
        L2 = K2
        XC(K) = AI0(K2)/ALINT**K2
        A(K,1) = ALINT*AI4T(K2)
        A(1,L) = -SIG2J(NCOUNT)*DLDS*(ALINT**L2)*AI3T(L2)
     1           + (ALINT**(L-1))*AI4T(L2)
        DO 3014 LA = 2,MM+2
        LAK = K+LA-4
3014    A(K,LA) = (ALINT**(LA-LAK-2))*AI7(LAK)
3013    CONTINUE
D       TYPE 7003,XC(1),A(1,1),A(1,2)
D       TYPE 7005,XC(2),A(2,1),A(2,2)
D7003   FORMAT(' XC(1),A(1,1),A(1,2):',3G)
D7005   FORMAT(' XC(2),A(2,1),A(2,2):',3G)
C
C       DO SIM. LINEAR EQN USING DOUBLE PRECISION PROGRAM DGELG
C
        MM2 = MM+2
        ALG2 = DLOG10(2.D0)
        MM3 = MM+3
        DO 10 I=1,MM2
        DO 10 J=1,MM2
10      DIJ(I,J) = DLOG10(DABS(A(I,J)))/ALG2
        DO 20 I=1,MM2
20      DIJ(I,MM3) = DLOG10(DABS(XC(I)))/ALG2
C       IF MM =4 USE DOUBLE PRECISION CONSTANTS BELOW
```

```
C         IF NOT USE FOLLOWING LOGIC
          DMM23 = 42.D0
          DMM2 = 6.D0
          DMM3 = 7.D0
          IF (MM.NE.0) GO TO 6683
          DMM23 = 6.D0
          DMM2 = 2.D0
          DMM3 = 3.D0
          GO TO 6681
6683      IF (MM.NE.1) GO TO 6680
          DMM23 = 12.D0
          DMM2 = 3.D0
          DMM3 = 4.D0
          GO TO 6681
6680      IF (MM.NE.2) GO TO 6682
          DMM23 = 20.D0
          DMM2 = 4.D0
          DMM3 = 5.D0
          GO TO 6681
6682      IF (MM.NE.3) GO TO 6681
          DMM23 = 30.D0
          DMM2 = 5.D0
          DMM3 = 6.D0
6681      SUM = 0.0
          DO 30 J=1,MM3
          DO 30 I=1,MM2
30        SUM = SUM +DIJ(I,J)
          AM = -SUM/DMM23
          DO 40 I=1,MM2
          S1 = 0.0
          DO 50 J=1,MM3
50        S1 = S1 + (DIJ(I,J)+AM)
40        RI(I) = -S1/DMM3
          DO 60 J=1,MM3
          S2 = 0.0
          DO 70 I=1,MM2
70        S2 = S2 + (DIJ(I,J)+AM)
60        CJ(J) = -S2/DMM2
          DO 80 J=1,MM2
          IX = (J-1)*MM2
          DO 80 I=1,MM2
80        AMTRX(I+IX) = A(I,J)*2.D0**(RI(I)+CJ(J)+AM)
          DO 90 I=1,MM2
90        XC(I) = XC(I)*2.D0**(RI(I)+CJ(MM3)+AM)
          S3 = 0.0
          DO 110 I=1,MM2
110       S3 = S3 + RI(I)
          S4 = 0.0
          DO 120 J=1,MM3
120       S4 = S4 + CJ(J)
          CALL DGELG(XC,AMTRX,MM2,1,1.E-10,IER)
          TYPE 3016,IER
```

```
3016    FORMAT(' IER = ',G)
        DO 100 I=1,MM2
100     XC(I) = XC(I)*2.**(CJ(I)-CJ(MM3))
        SIG2J(NCOUNT+1) = XC(1)
        TYPE 1620,NCOUNT,XC(1)
1620    FORMAT(' FOR NCOUNT =',I2,' SIGMA SQRD =',G,/1X)
C
C
28      TYPE 1627
1627    FORMAT(' STOP LOOP ? (Y OR N) ',$)
        ACCEPT 807,STOP
        IF (STOP.EQ.'N') GO TO 19
C
C       COMPUTE FINAL A'S
C
        DO 3017 L=2,MM2
3017    AI(L-2) = XC(L)
C
C       TYPE OUT FINAL RESULTS
C
31      TYPE 1628,NCOUNT,ALINT,XC(1)
1628    FORMAT(' ON LAST PASS NCOUNT =',I3,' L =',G,/1X,
     1  'AND SIGMA SQRD =',G)
        DO 32 I2 = 0,MM
32      TYPE 1629,I2,AI(I2)
1629    FORMAT(' A(',I2,') =',G)
C
C       COMPUTE R(XI), PART 9.K
C
        CALL OFILE(20,'ITM2')
        WRITE(20,915),MM,EH,FLE
        WRITE(20,1632)ALINT,XC(1)
1632    FORMAT(/1X,'OUTPUT FOR PART 9.K',/1X,'WITH L =',G,
     1  ' AND SIGMA SQRD =',G)
        DO 7016 I=0,MM
7016    WRITE(20,7015),I,AI(I)
7015    FORMAT(' COEFFICIENT A(',I1,') = ',G)
        WRITE(20,1631)
1631    FORMAT(/3X/,'  XI',12X,'R(XI)',12X,'XI CONTD',8X,'R(XI) CONTD')
        ARG = ALINT*.1/BETA
        NX2 = IFIX(EH/ARG)
        NX12 = (NX2/2) - 1
        DO 41 I=0,NX12
        E1 = ARG*I
        E2 = ARG*(I+NX12+1)
        SUM1 = 0.
        SUM2= 0.
        DO 42 I0 = 0,MM
        SUM2 = SUM2+AI(I0)*E2**I0
42      SUM1 = SUM1+AI(I0)*E1**I0
        IF (I.GT.100) R = SUM1
        IF (I.GT.100) GO TO 43
        R = XC(1)*PH1(I)+SUM1
```

```
43      IF ((I+NX12+1).GT.100) R2 = SUM2
        IF ((I+NX12+1).GT.100) GO TO 41
        R2 = XC(1)*PH1(I+NX12)+SUM2
41      WRITE(20,1630)E1,R,E2,R2
1630    FORMAT(4G)
        END FILE 20
C
C       DO PART 9.L, COMPUTE SIGMA SQRD*L*PHIK
C
        CALL OFILE(21,'ITM2L')
        WRITE(21,915),MM,EH,FLE
        WRITE(21,1633)ALINT,XC(1)
1633    FORMAT(/' OUTPUT FOR PART 9.L',/1X,'WITH L =',G,' AND SIGMA',
     1  ' SQRD =',G,//1X,'   K',8X,'L*SIGMA SQRD*PHIK',5X,'K CONTD',4X,
     2  'L*SIGMA SQRD*PHIK CONTD')
        SIG2L = XC(1)*ALINT
        TYPE 7013
7013    FORMAT(' INPUT DELK ',2X.$)
        ACCEPT 7014,DELK2
7014    FORMAT(G)
        DO 44 I = 0,511
        EXC1 = DELK2*I
        EXC2 = DELK2*(I+512)
        AKLR = ALINT*EXC1
        AKLR1 = ALINT*EXC2
        ALKI2 = AKLR*AKLR
        ALKI3 = AKLR1*AKLR1
        PHIK = (2./(1.+70.78*ALKI2)**C56)*SIG2L
        PHIK1 = (2./(1.+70.78*ALKI3)**C56)*SIG2L
        IF (WHICH.EQ.'T') PHIK = ((1.+188.75*ALKI2)/(1.+70.78*ALKI2)
     1  **C116)*SIG2L
        IF (WHICH.EQ.'T') PHIK1 = ((1.+188.75*ALKI3)/(1.+70.78*ALKI3)
     1  **C116)*SIG2L
44      WRITE(21,1630)EXC1,PHIK,EXC2,PHIK1
        END FILE 21
C
915     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM FINAL',/1X,
     1  'WITH M =',I3,' AND LENGTH =',G,' AUTOCOR. OF FILE ',A5)

900     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB3')
903     FORMAT(1X,F10.6,3X,E12.4.3X,F10.6,3X,E12.4)
906     FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1  ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1  ' METER')
907     FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
910     FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1  'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
C
9999    END
C
```

```
C
      FUNCTION DECT(ULEFT,AIXZ,NAU1)
      DIMENSION AIXZ(0/290)
      DECT = ULEFT*(2.6402778*AIXZ(NAU1)-3.852778*AIXZ(NAU1-1)
     1         +3.6333333*AIXZ(NAU1-2)-1.7694444*AIXZ(NAU1-3)
     2          +.34861111*AIXZ(NAU1-4))
      RETURN
C
      END
C
      SUBROUTINE SET(A1,B1,C1,LSIG,BK1)
C
      DIMENSION A1(0/11),B1(0/11)
      COMMON/BB/X(3),AKI(3),XPL
C
      AKI(1) = A1(LSIG-1)
      AKI(2) = A1(LSIG)
      AKI(3) = A1(LSIG+1)
      X(1) = B1(LSIG-1)
      X(2) = B1(LSIG)
      X(3) = B1(LSIG+1)
      XPL = C1
      CALL PARAB(AKZ)
      BK1 = AKZ
C
      RETURN
C
      END
```

Subroutine GAM

```
C <RFISHER>GAM.F4;1    28-Sep-77 13:47:22    EDIT BY RFISHER
        SUBROUTINE GAM(GAMMA,GG)
C       PROGRAM TO COMPUTE GAMMA FUNCTIONS
C       PROGRAM DERIVED FROM "HANDBOOK OF MATH. FUNCTIONS, NAT.
C       BUREAU OF STANDARDS, APPLIED MATH SERIES 55", 1964
C       PROGRAM USES EQN 6.1.15 (P 256) AND APPROX. 6.1.36 (P.257)
C
C       ANS RETURNED AS GG, INPUT IS GAMMA
C
        IF (GAMMA.GE.2.) GO TO 1
        X = GAMMA -1.
        RES =1.
        GO TO 10
1       N = IFIX(GAMMA)-1
        RES = 1.
        DO 20 I=1,N
20      RES = RES*(GAMMA-FLOAT(I))
        X = GAMMA - FLOAT(N+1)
10      GG = 1. - .577191652*X +.988205891*X**2 - .897056937*X**3
     1       + .918206857*X**4 - .756704078*X**5 + .482199394*X**6
     2       - .193527818*X**7 + .035868343*X**8
C
        GG = GG*RES
C
        RETURN
C
        END
```

Program GDIST6

```
C         PROGRAM TO COMPUTE DIST. SEVERAL WAYS. NOTES FROM  BILL
C         MARK OF SEPT 25, 1977. "APPROXIMATIONS USING DERIVATIVE
C         FORMS OF GENERALIZED GRAM-CHARLIS
C         UPDATED VERSION PAGES 6,7
C         USED WITH ITEM4, AT. TURBULENCE  Part 5
C         REQUIRES SUBROUTINE GAM
C
C
          PI = 3.141592654
          PI2 = PI/2.
          TPI = (2.*PI)**-.5
C
C
          TYPE 7001
7001      FORMAT(' INPUT ALPHF :',/$)
          ACCEPT 702,ALPH1F
          ACCEPT 702,ALPH2F
          ACCEPT 702,ALPH3F
          ACCEPT 702,ALPH4F
          ACCEPT 702,ALPH5F
          ACCEPT 702,ALPH6F
702       FORMAT(G)
C
          ALPB = ALPH2F - ALPH1F*ALPH1F
          ALPB2 = SQRT(ALPB)
          GAMMA = ALPH1F*ALPH1F/ALPB
          TYPE 1010,GAMMA
1010      FOMAT(' GAMMA =',G)
          ALAM = GAMMA/ALPH1F
          GAM2 = GAMMA/2.
          GAM12 = (GAMMA+1.)/2.
          TGAM = 2.**(GAMMA-.5)
          DELX = .1*ALPB2
C
C
          CALL GAM(GAM2,GG1)
          CALL GAM(GAM12,GG2)
C
          BE1 = (GAMMA-1.)/2.
          BETA = GAMMA - 1.
          B2 = BETA*BETA
          B3 = BETA*B2
          B4 = BETA*B3
          AL3 = ALAM*ALAM*ALAM
          AL4 = AL3*ALAM
          G12 = GAMMA*(GAMMA+1.)*(GAMMA+2.)/6.
          G123 = G12*(GAMMA+3.)/4.
          G4 = G123*(GAMMA+4.)/5.
          G5 = G4*(GAMMA+5.)/6.
          CO1 = (ALAM/GAMMA)
          CO2 = (CO1*ALAM/(GAMMA+1.))
```

```
      CO3 = (CO2*ALAM/(GAMMA+2.))
      CO4 = (CO3*ALAM/(GAMMA+3.))
      CO5 = (CO4*ALAM/(GAMMA+4.))
      CO6 = (CO5*ALAM/(GAMMA+5.))
      BB3 = G12*(1.-3.*CO1*ALPH1F+3.*CO2*ALPH2F-CO3*ALPH3F)
      BB4 = G123*(1.-4.*CO1*ALPH1F+6.*CO2*ALPH2F-4.*CO3*ALPH3F+
     1    CO4*ALPH4F)
      BB5 = G4*(1.-5.*CO1*ALPH1F+10.*CO2*ALPH2F-10.*CO3*ALPH3F+5.*
     1 CO4*ALPH4F-CO5*ALPH5F)
      BB6 = G5*(1.-6.*CO1*ALPH1F+15.*CO2*ALPH2F-20.*CO3*ALPH3F+15.*
     1 CO4*ALPH4F-6.*CO5*ALPH5F+CO6*ALPH6F)
C
      FØNØ = Ø.
      FØN1 = Ø.
      FØN2 = Ø.
      F1NØ = Ø.
      F1N1 = Ø.
      F1N2 = Ø.
      F1N3 = Ø.
      F2NØ = Ø.
      F2N1 = Ø.
      F2N2 = Ø.
      F2N3 = Ø.
      F2N4 = Ø.
      F3NØ = Ø.
      F3N1 = Ø.
      F3N2 = Ø.
      F3N3 = Ø.
      F3N4 = Ø.
      F3N5 = Ø.
      F4NØ = Ø.
      F4N1 = Ø.
      F4N2 = Ø.
      F4N3 = Ø.
      F4N4 = Ø.
      F4N5 = Ø.
      F4N6 = Ø.
C
      TYPE 6ØØ
6ØØ   FORMAT(/,'      X',11X,'F1PRM',9X,'F2PRM',9X,'F3PRM',9X,'F4PRM')
C
      DO 2Ø I=1,1ØØ
      X = DELX*I
      ALX = ALAM*X
      PHIØ = (ALAM*EXP(-ALX)*((ALX**BE1)/GG1)*((ALX**BE1)/GG2))/
     1        (TPI*TGAM)
C
      P3 = 1. - 3.*CO1*X + 3.*CO2*X*X - CO3*X*X*X
      P4 = 1. - 4.*CO1*X + 6.*CO2*X*X - 4.*CO3*X*X*X + CO4*X**4
      P5 = 1.-5.*CO1*X+1Ø.*CO2*X*X-1Ø.*CO3*X**3+5.*CO4*X**4-CO5*X**5
      P6 = 1.-6.*CO1*X+15.*CO2*X*X-2Ø.*CO3*X**3+15.*CO4*X**4-
     1    6.*CO5*X**5+CO6*X**6
      F1PRM = PHIØ + BB3*PHIØ*P3
      F2PRM = F1PRM + BB4*PHIØ*P4
      F3PRM = F2PRM + BB5*PHIØ*P5
      F4PRM = F3PRM + BB6*PHIØ*P6
```

```
C
        F0N0 = F0N0 + PHI0
        F0N1 = F0N1 + PHI0*X
        F0N2 = F0N2 + PHI0*X*X
        F1N0 = F1N0 + F1PRM
        F1N1 = F1N1 + FIPRM*X
        F1N2 = F1N2 + F1PRM*X*X
        F1N3 = F1N3 + F1PRM*X**3
        F2N0 = F2N0 + F2PRM
        F2N1 = F2N1 + F2PRM*X
        F2N2 = F2N2 + F2PRM*X*X
        F2N3 = F2N3 + F2PRM*X**3
        F2N4 = F2N4 + F2PRM*X**4
        F3N0 = F3N0 + F3PRM
        F3N1 = F3N1 + F3PRM*X
        F3N2 = F3N2 + F3PRM*X*X
        F3N3 = F3N3 + F3PRM*X**3
        F3N4 = F3N4 + F3PRM*X**4
        F3N5 = F3N5 + F3PRM*X**5
        F4N0 = F4N0 + F4PRM
        F4N1 = F4N1 + F4PRM*X
        F4N2 = F4N2 + F4PRM*X*X
        F4N3 = F4N3 + F4PRM*X**3
        F4N4 = F4N4 + F4PRM*X**4
        F4N5 = F4N5 + F4PRM*X**5
        F4N6 = F4N6 + F4PRM*X**6
        TYPE 700,X,F1PRM,F2PRM,F3PRM,F4PRM
20      CONTINUE
700     FORMAT(F6.2,2X,4G)
C
        TYPE 7004,F0N0,F0N1,F0N2
        TYPE 7004,F1N0,F1N1,F1N2,F1N3
        TYPE 7004,F2N0,F2N1,F2N2,F2N3,F2N4
7004    FORMAT(5G)
        TYPE 7004,F3N0,F3N1,F3N2,F3N3,F3N4
        TYPE 7004,F3N5
        TYPE 7004,F4N0,F4N1,F4N2,F4N3,F4N4
        TYPE 7004,F4N5,F4N6
C
C
        END
```

Subroutine HPDES

```
C       HPDES
C       HIPASS BUTTERWORTH FILTER DESIGN SUBROUTINE
C       INPUTS ARE CUTOFF (3-DB) FREQUENCY FC IN HERTZ,
C               SAMPLING INTERVAL T IN SECONDS, AND
C               NUMBERNS OF FILTER SECTIONS.
C       OUTPUTS ARE NS SETS OF FILTERED COEFFICIENTS, I.E.,
C               A(K) THRU C(K) FOR K=1 THRU NS, AND
C               10 PAIRS OF FREQUENCY AND POWER GAIN, I.E.,
C               GR(1,K) AND GR(2,K) FOR K=1 THRU 10
C       NOTE THAT A(K),B(K),C(K) AND GR(2,10) MUST BE DIMENSIONED
C       IN CALLING PROGM.
C
C       THE DIGITAL FILTER HAS NS SECTIONS IN CASCADE, THE KTH
C       SECTION HAS THE TRANSFER FUNCTION
C
C               A(K)*Z(Z**2-2*Z+1)
C       H(Z) =  ----------------------
C               Z**2+B(K)*Z+C(K)
C
C       THUS IF F(M) ANDG(M) ARE THE INPUT AND OUTPUT OF THE
C       KTH SECTION AT TIME M*T, THEN
C
C       G(M) = A(K)*(F(M)-2*F(M-1)+F(M-2))-B(K)*G(M-1)
C               -C(K)*G)M-2)
C
C
        SUBROUTINE HPDES(FC,T,NS)
        COMMON/CC/A(3),B(3),C(3),GR(2,10),F(4,3)
        PI = 3.1415926536
        WCP = SIN(FC*PI*T)/COS(FC*PI*T)
        DO 120 K=1,NS
        CS = COS(FLOAT(2*(K+NS)-1)*PI/FLOAT(4*NS))
        A(K) = 1./(1.+WCP*WCP-2.*WCP*CS)
        B(K) = 2.*(WCP*WCP-1.)*A(K)
120     C(K) = (1.+WCP*WCP+2.*WCP*CS)*A(K)
        DO 130 K=1,10
        GR(2,K) = .01+.98*FLOAT(K-1)/9.
        X = ATAN(WCP*(1./GR(2,K)-1.)**(-1./FLOAT(4*NS)))
130     GR(1,K) = X/(PI*T)
        RETURN
        END
```

Program ITEM3

```
C       PROGRAM ITEM3.  ATMOSPHERIC TURBULENCE TASK
C       ITEM 3, PARTS 5 AND 6. PHASE II OF ATMOSPHERIC TURB.
C
C       PROGRAM READS DATA FILE :
C               AUTOF= AUTOCORRELATION FN FROM ATURB4
C               AUTF2= AUTOCORRELATION PRODUCES WITH W**2= FROM
C                       MODIFIED VERSION OF ATURB4
C       PROGRAM PRODUCES DATA FILES:
C               RSIGF= VALUES OF R (SIGMA SQRD F), PART 5 ITEM 3
C               PHIF= VALUES OF PHI OF F, PART 6, ITEM 3
C
C       REQUIRES SUBROUTINE CFFT1
C
C       RUN PROGRAM ATURB4 TO DETERMINE PHI L, R L,AND PHI L ,
C       R L FOR SQUARED W(T) VALUES (SEE MARK'S NOTES ITEM3)
C       RUN ITEM2 TO DETERMINE SIGMA SQRD AND L
C
        DIMENSION RWH(0/1300),RWH2(0/1300)
        DIMENSION AUT(0/5000)
C
C
C       INPUT INITIAL PARAMETERS
C
        TYPE 1                          FC (filter cut-off frequency)
1       FORMAT(/1X,'INPUT DELX',/$)
        ACCEPT 1750,DELX ,
1750    FORMAT(3G)
C
C       READ IN AUTOCORRELATION FN.
C
        CALL IFILE (20,'AUTOF')
        READ(20,900)
        READ(20,1950)FLE
1950    FORMAT(/1X,'AUTOCORRELATION OF',/1X,
     1  'HIGH PASS FILTERED NON-HOMOGENEOUS SAMPLE',/1X,
     2  'DATA TAKEN FROM FILE ',A5)
        READ(20,906)NPTS,TWOL,MPTS,FTM1
        READ(20,907)MZERO
        READ(20,910)WBAR,VAR
        READ(20,951)WSUM
951     FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
        READ(20,381)XAOFF,MAOFF
381     FORMAT(/1X,' TRUCATION POINT WAS',G,' METERS',/1X,
     1  'WHICH CONTAINS ',I7,' POINTS')
        READ(20,1952)
1952    FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE AUTOCORRELATION',
     1  /1X,8X,'X',12X,'    RL    ',9X,'RL/R0')
        DO 1823 I=0,MAOFF
1823    READ(20,1915)DEL,RWH(I),YY
1915    FORMAT(1X,3(2X,G))
        END FILE 20
        TYPE 4324,RWH(MAOFF)
```

```
4324    FORMAT( ' RWH(MAOFF) =',G)
C
C       READ IN AUTOCORRELATION FN. OF SQUARED W(T)
C
        CALL IFILE (20,'AUTF2')
        READ(20,900)
1965    FORMAT(/1X,'AUTOCORRELATION OF',/1X,
     1  'HIGH PASS FILTERED AND SQUARED NON-HOMOGENEOUS SAMPLE',/1X,
     2  'DATA TAKEN FROM FILE ',A5)
        READ(20,1965)FLE
        READ(20,906)NPTS,TWOL,MPTS,FTM1
        READ(20,907)MZERO
        READ(20,910)WBAR,VAR
        READ(20,951)WSUM
        READ(20,381)XAOFF,MAOFF
        READ(20,1952)
        DO 1824 I=0,MAOFF
1824    READ(20,1915)DEL,RWH2(I),YY
        END FILE 20
        TYPE 4325,RWH2(MAOFF)
4325    FORMAT( ' RWH2(MAOFF) =',G)
C
C       READ IN SIGMA SQRD, L (AFTER RUNNING ITEM2)
C
        TYPE 111
111     FORMAT(' INPUT SIGMA SQRD, L',/$)
        ACCEPT 2,SIGF,AL
2       FORMAT(2G)
C
C       DO PART 5, R (SIGMA SQRD F)
C
        CALL OFILE(20,'RSIGF')
        WRITE (20,911)
        WRITE(20,1953)SIGF,AL
1953    FORMAT(' FOR SIGMA SQUARED =',G,' AND AL =',G)
        WRITE(20,1954)
1954    FORMAT(//'   XI',7X,'RSIGF')
        SIGF2 = SIGF*SIGF
        DO 500 I=0,MAOFF
        XI = DELX*I
        RSIGF = SIGF2*RWH2(I)/(RWH(0)**2+2.*RWH(I)*RWH(I))
        IF (RSIGF.LT..0001) GO TO 501
500     WRITE(20,2)XI,RSIGF
501     END FILE 20
C
C       DO PART 6, PHI(SIGF)
C
        TYPE 3
3       FORMAT(' INPUT M AND POWER OF 2 ',/$)
        ACCEPT 1750,MPTS,MPWRM
```

```
C
      PI = 3.141592654
      RWHØ2 = RWH(Ø)*RWH(Ø)
      FTM1 = MPTS*DELX
      DO 13 J=Ø,MPTS
      ARG = DELX*J
      CC = PI*ARG/FTM1
      DD = 1.-ARG/FTM1
      EE = ABS(SIN(CC))
      WINDO = EE/PI+DD*COS(CC)
      RTP = RWHØ2+2.*RWH(J)*RWH(J)
13    AUT(2*J) = ((RWH2(J)-RTP)/RTP)*WINDO
      M2PWR = MPWRM+1
      M2PTS = 2*MPTS
      FT2M = 2.*FTM1
      MMIN1 = MPTS-1
      DO 14 JK=1,MMIN1
      KK = M2PTS-JK
14    AUT(2*KK) = AUT(2*JK)
      DO 521 K=1,2*M2PTS-1,2
521   AUT(K) = Ø.Ø
C
C     COMPUTE SMOOTHED POWER SPECTRUM
C
      CALL CFFT(M2PWR,M2PTS,AUT,2)
C
C     PRINT DATA FILE 'PHIF'
C
      CALL OFILE(20,'PHIF')
      WRITE(20,911)
      WRITE(20,9Ø1) FC
9Ø1   FORMAT(//1X,'SMOOTHED POWER SPECTRUM PHI OF SIGMA SQRD F(K)')
      WRITE(20,19Ø6)MPTS,FTM1
19Ø6  FORMAT(/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,' FT.')
      WRITE(2Ø,19Ø5)RWH(Ø)              ADD TO FORMAT STATEMENT 901
19Ø5  FORMAT(' RWH(Ø) = ',G)
      WRITE(2Ø,9Ø2)                 [/(X,' WITH FILTER CUT-OFF=',
      MHALF = MPTS/2                    G,'M**-1')]
      M41 = MHALF-1
      DELK = 1./FT2M
      DO 56 I=Ø,M41
      DEL = DELK*I
      XX = AUT(2*I)*FT2M*SIGF*SIGF
      K = MHALF+I
      DEL2 = DELK*K
      YY = AUT(2*K)*FT2M*SIGF*SIGF
56    WRITE(2Ø,9Ø3)DEL,XX,DEL2,YY
      DEL3 = DELK*MPTS
      ZZ = AUT(2*MPTS)*FT2M*SIGF*SIGF
      WRITE(2Ø,9Ø8)DEL3,ZZ
      END FILE 2Ø
```

```
C
902     FORMAT(//1X,'PRINTOUT OF VALUES OF THE SMOOTHED',
     1  ' POWER SPECTRUM',/5X,'K',10X,'SPS VALUE',8X,'K CONTD',
     2  4X,'SPS VALUE CONTD',/)
908     FORMAT(29X,F10.4,3X,E12.4)
900     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB4')
903     FORMAT(1X,F10.6,3X,E12.4,3X,F10.6,3X,E12.4)
906     FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1  ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1  ' METER')
907     FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
910     FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1  'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
911     FORMAT(' DATA FILE CREATED BY PROGRAM ITEM3')
C
        END
```

Program ITEM4

```
C         PROGRAM ITEM4 ,PART 6    ATMOS. TURBULENCE PHASE II
C
          DIMENSION ALWF(8),ALWS(0/8),ALPHWH(8),PROB(0/100)
          DIMENSION ALPHW(8)
C
          TYPE 1
1         FORMAT(' INPUT ALPHWH,ALPHW (8 VALUES) ',/$)
          ACCEPT 2,ALPHWH(1),ALPHW(1)
          ACCEPT 2,ALPHWH(2),ALPHW(2)
          ACCEPT 2, ALPHWH(3),ALPHW(3)
          ACCEPT 2,ALPHWH(4),ALPHW(4)
          ACCEPT 2, ALPHWH(5),ALPHW(5)
          ACCEPT 2,ALPHWH(6),ALPHW(6)
          ACCEPT 2, ALPHWH(7),ALPHW(7)
          ACCEPT 2,ALPHWH(8),ALPHW(8)
C
2         FORMAT(2G)
          TYPE 3
3         FORMAT(' INPUT SIGMA SQRD F   ',$)
          ACCEPT 2,SIG
C
          AK = (SIG/ALPHWH(2))**.5
          DO 100 K = 1,8
100       ALWF(K) = (AK**K)*ALPHWH(K)
          ALWS(0) = 1.0
          DO 110 N = 1,8
          SUM = 0.0
          DO 120 K=0,N-1
          CALL FAC1(N,K,ANK)
120       SUM = SUM + ANK*ALWF(N-K)*ALWS(K)
110       ALWS(N) = ALPHW(N) - SUM
C
C         COMPUTE PROB DENSITY FN
C
          TYPE 8
8         FORMAT(' PROB DENSITY FN.')
          DELWS = .1*(SQRT(ALPHWH(2)-ALPHWH(1)*ALPHWH(1)))
          SIGWS = ALWS(2)**.5
          TWOSG = 2.*SIGWS*SIGWS
          C1 = 1./((2.*3.14159265)**.5*SIGWS)
          SIG3 = SIGWS**3
          SIG4 = SIGWS**4
          SIG2 = SIGWS**2
C
          DO 130 I=0,100
          WS = DELWS*I
          WS2 = WS*WS
          WS3 = WS2*WS
          WS4 = WS3*WS
          EX = -(WS2/TWOSG)
```

```
130     PROB(I) = C1*EXP(EX)*((1.+(ALWS(3)*(WS3/SIG3-
     1            3.*WS/SIGWS)/(6.*SIG3))+(((ALWS(4)/SIG4)-3.)*
     2            (WS4/SIG4-6.*WS2/SIG2+3.))/24.))
        GAM1 = ALWS(3)/(ALWS(2)**(3./2.))
        GAM2 = (ALWS(4)/(ALWS(2)**2))-3.
C
C
C       PRINT OUT VALUES OF PROBABILITYE
C
        CALL OFILE(20,'PROB')
        WRITE(20,900)
900     FORMAT(' DATA FILE CREATED BY PROGRAM ITEM4',/1X,
     1  ' ATMOSPHERIC TURBULENCE TASK, PHASE II')
        WRITE(20,901)SIG
901     FORMAT(' FOR SIGMA SQRD F =',G,//1X,'THE VALUES OF ALPHA ',
     1  'OF WS ARE:')
        DO 170 N=1,8
170     WRITE(20,902)N,ALWS(N)
902     FORMAT(' N =',I3,'ALPHA WS =',G)
        WRITE(20,903)
903     FORMAT(//1X,'THE PROBABILITY DENSITY FN. IS:'
     1  /7X,'WS',13X,'P(WS)')
        DO 180 I=0,100
        WS = DELWS*I
180     WRITE(20,904)WS,PROB(I)
904     FORMAT(1X,G,2X,G)
        WRITE(20,905)GAM1,GAM2
905     FORMAT(//1X,'COEFFICIENT OF SKEWNESS =',G,/1X,
     1  'COEFFICIENT OF EXCESS =',G)
```

Program MOMENT

```
C         PROGRAM MOMENT   ATMOSPHERIC TURBULENCE PHASE II
C         ITEM 4   Parts 1-4
C
C         CALL SUBROUTINES BIN(SQ),HPDES
C
          COMMON/AA/BIN1(501),BIN2(501),TOTAL,NBIN,BINW,NPTS
          COMMON/BB/BIN3(700),TOT1,NBIN1,BINW1,NPTS1
          COMMON/CC/A(3),B(3),C(3),GR(2,10),F(4,3)
C
          DIMENSION D(8),ALSIG(8)
          DIMENSION ALWS(0/8)
          DIMENSION W(0/15000),ALPHW(8),ALPHWH(8),ALPWH2(8)
C
C         READ IN W(T) DATA
C
          TYPE 1
1         FORMAT(' INPUT NO. OF DATA POINTS',$)
          ACCEPT 2,NPTS
2         FORMAT(3G)
          TYPE 3
3         FORMAT(' INPUT DATA RECORD NAME  ',$)
          ACCEPT 4,FLE
4         FORMAT(A5)
C
C         READ IN FILE AND CONVERT TO METERS
C
          CALL IFILE(20,FLE)
          NX1 = NPTS/4
          DO 550 I=0,NX1-1
          READ(20,551)W(I),W(I+NX1),W(I+2*NX1),W(I+3*NX1)
          W(I+NX1) = W(I+NX1)*.3048
          W(I+2*NX1) = W(I+2*NX1)*.3048
          W(I+3*NX1) = W(I+3*NX1)*.3048
550       W(I) = W(I)*.3048
551       FORMAT(4(E15.7))
          END FILE 20
C
C         SUBTRACT OUT MEAN
C
          WBAR = 0.0
          DO 610 JJ = 0,NPTS-1
610       WBAR = WBAR+W(JJ)
          WBAR = WBAR/FLOAT(NPTS)
          DO 611 I=0,NPTS-1
611       W(I) = W(I)-WBAR
C
C         COMPUTE BINS
C
          CALL BIN(W)
C
C         COMPUTE MOMENTS OF W(T)
```

```
      BINHF = BINW/2.
      NBIN2 = NBIN/2
C
      TYPE 900
900   FORMAT(' MOMENTS OF W(X) ')
      DO 10 K=1,8
      SUM = 0.0
      DO 20 J=1,NBIN2
      WMID1 = (BINW*J-BINHF)**K
      WMID2 = (BINW*(-J)+BINHF)**K
20    SUM = SUM + WMID1*BIN1(J) + WMID2*BIN2(J)
      ALPHW(K) = SUM/TOTAL
10    TYPE 901,K,ALPHW(K)
901   FORMAT('  K =',I3,'  ALPHW = ',G)
C
C     FILTER W(T) AND COMPUTE MOMENTS
C
C     FILTER W(T)
C
      TYPE 9
9     FORMAT(' INPUT CUT-OFF FREQ,SAMPLING INTERVAL,'
     1 'NO. OF FILTER SECTIONS',/$)
      ACCEPT 2,FC,TS,NS
      CALL HPDES(FC,TS,NS)
      DO 140 N=1,NS+1
      DO 140 M=1,2
140   F(N,M) = 0.0
      DO 150 M=0,NPTS-1
      F(1,3) = W(M)
      DO 160 N=1,NS
      TEMP = A(N)*(F(N,3)-2.*F(N,2)+F(N,1))
160   F(N+1,3) = TEMP-B(N)*F(N+1,2)-C(N)*F(N+1,1)
      DO 170 N=1,NS+1
      DO 170 MM=1,2
170   F(N,MM) =  F(N,MM+1)
150   W(M) = F(NS+1,3)
C
C     MOMENTS OF WH
C
      TYPE 5
5     FORMAT(' MOMENTS OF FILTERED W')
C
      CALL BIN(W)
      BINHF = BINW/2.
      NBIN2 = NBIN/2
C
      DO 30 K=1,8
      SUM = 0.0
      DO 40 J = 1,NBIN2
      WMID1 = (BINW*J-BINHF)**K
      WMID2 = (BINW*(-J)+BINHF)**K
```

```
40      SUM = SUM + WMID1*BIN1(J) + WMID2*BIN2(J)
        ALPHWH(K) = SUM/TOTAL
30      TYPE 9Ø2,K,ALPHWH(K)
9Ø2     FORMAT('  K =',I3,'  ALPHWH =',G)
C
C       COMPUTE WH**2, PART 3
C
        DO 5Ø I=Ø,NPTS-1
5Ø      W(I) = W(I)*W(I)
C
C       COMPUTE BINS
C
        TYPE 6
6       FORMAT(' COMPUTE MOMENTS FOR W*W')
C
        NPTS1 = NPTS
        CALL BINSQ(W)
C
        BINHF = BINW1/2.
C
        DO 6Ø K=1,8
        SUM = Ø.Ø
        DO 7Ø J=1,NBIN1
        WMID = (BINW1*J-BINHF)**K
7Ø      SUM = SUM + WMID*BIN3(J)
        ALPWH2(K) = SUM/TOT1
6Ø      TYPE 9Ø3,K,ALPWH2(K)
9Ø3     FORMAT('  K =',I3,'  ALPHW2 =',G)
C
C       PART4, MOMENTS OF SIGMA SQRD F
C
        D(1) = 1.
        D(2) = 3.
        D(3) = 15.
        D(4) = 1Ø5.
        D(5) = 945.
        D(6) = 1Ø395.
        D(7) = 135135.
        D(8) = 2Ø27Ø25.
        TYPE 9Ø5
9Ø5     FORMAT(' COMPUTE MOMENTS OF SIGMA SQRD F')
C
        TYPE 7
7       FORMAT(' INPUT SIG SQRD F ',$)
        ACCEPT 2,SIG
C
        DO 8Ø N=1,8
        ALSIG(N) = ((SIG/ALPWH2(1))**N)*ALPWH2(N)/D(N)
8Ø      TYPE 9Ø4,N,ALSIG(N)
9Ø4     FORMAT(' N =',I3,'  ALSIG =',G)
C
        END
```

Subroutine PAR1

```
      SUBROUTINE PAR1(AKZ)
C
C     CALL SIMQ, FORMS EQN Y**2+DX+EY+F = 0.0
C
      COMMON/BB/X(3),AKI(3),XPL
      DIMENSION Y(3),A(9),B(3)
      EQUIVALENCE(Y,AKI)
C
      YMID = AKI(2)
      DO 20 J=1,3
      B(J) = -Y(J)**2
20    A(J) = X(J)
C
      DO 25 K=4,6
25    A(K) = Y(K-3)
      A(7) = 1.
      A(8) = 1.
      A(9) = 1.
C
      KK = 0
C
      CALL SIMQ(A,B,3,KK)
C
      IF (KK.EQ.1) TYPE 518
518   FORMAT(' K = 1 IN SIMQ; SINGULAR SOLUTION = BAD')
C
      D = B(1)
      E = B(2)
      F = B(3)
      AKZ = (-2.*XPL-E)/D
C
      RETURN
C
      END
```

Subroutine PAR2

```
      SUBROUTINE PARAB(AKZ)
C
C     CALLS SIMQ, FORMS EQN Y**2+DX+EY+F = 0.0
C
      COMMON/BB/X(3),AKI(3),XPL
      DIMENSION Y(3),A(9),B(3)
      EQUIVALENCE(Y,X)
C
      IF (AKI(2).EQ.AKI(3)) AKZ = AKI(2)
      IF (AKI(2).EQ.AKI(3)) GO TO 40
      DO 20 J=1,3
      B(J) = -Y(J)**2
20    A(J) = AKI(J)
C
      DO 25 K=4,6
25    A(K) = Y(K-3)
      A(7) = 1.
      A(8) = 1.
      A(9) = 1.
C
      KK = 0
C
      CALL SIMQ(A,B,3,KK)
C
      IF (KK.EQ.1) TYPE 518,X(1),AKI(1),XPL
518   FORMAT(' K = 1 IN SIMQ; SINGULAR SOLUTION = BAD',/1X,
     1 'X(1)=',G,'AKI(1)=',G,'XPL =',G)
C
      D = B(1)
      E = B(2)
      F = B(3)
      AKZ = (-XPL*XPL-E*XPL-F)/D
C
C
C
40    RETURN
      END
```

Subroutine PARAB

```
C <RFISHER>PARAB.F4;6      1-Nov-77 10:06:33     EDIT BY RFISHER
        SUBROUTINE PARAB(AKZ)
C
C       CALL SIMQ, FORMS EQN Y**2+DX+EY+F = 0.0
C
        COMMON/BB/X(3),AKI(3),XPL
        DIMENSION Y(3),A(9),B(3)
        EQUIVALENCE(Y,AKI)
C
        YMID = AKI(2)
        DO 20 J=1,3
        B(J) = -Y(J)**2
20      A(J) = X(J)
C
        DO 25 K=4,6
25      A(K) = Y(K-3)
        A(7) = 1.
        A(8) = 1.
        A(9) = 1.
C
        KK = 0
C
        CALL SIMQ(A,B,3,KK)
C
        IF (KK.EQ.1) TYPE 518
518     FORMAT(' K = 1 IN SIMQ: SINGULAR SOLUTION = BAD')
C
        D = B(1)
        E = B(2)
        F = B(3)
        CONST = F + D*XPL
        AKZ1 = (-E - (E*E-4.*CONST)**.5)/2.
        AKZ2 = (-E + (E*E-4.*CONST)**.5)/2.
        DIF1 = ABS(AKZ1-YMID)
        DIF2 = ABS(AKZ2-YMID)
        AKZ = AKZ2
        IF (DIF2.GT.DIF1) AKZ = AKZ1
C
        RETURN
C
        END
```

Program PART2

```
C       PROGRAM PART2.  ATMOSPHERIC TURBULENCE TASK
C       ITEM 1, PARTS 2,3,4,5,6,7, AND 8. PHASE II OF ATMOS-
C       PHERIC TURBULENCE
C       MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C       MAXIMUM LIKELIHOOD ESTIMATION OF INTEGRAL SCALE OF VERTICAL
C       COMPONENT OF STATIONARY SAMPLE
C
C       READS IN PSD FROM FILE PHILK (PRODUCED BY ATURB2.F4)
C       PRODUCES DATA FILES;
C               LG; VALUES OF LG(K;L) PART4,ITEM1
C               PHIXI: VALUES OF PHI OF K(XI)- VON-KARMEN AUTO-
C                    CORRELATION FN., PART 7, ITEM 1
C               PHIK: VALUES OF PHI OF K(K)- PART 8, ITEM 1
C
C       REQUIRES SUBROUTINES GAM AND AK
C       SUBR. AK REQUIRES PARAB, AKDAT, SIMQ
C
        DIMENSION PHIKT(6500),DVSR(4),E(20),ALV(20),ALKI2(6500)
        DIMENSION W(0/6500)
C
C       INPUT INITIAL PARAMETERS
C
        TYPE 1
1       FORMAT(/1X,'INPUT NO. OF POINTS TO BE READ, DELK,DELX',/$)
        ACCEPT 1750,NPTS,DELK,DELX
1750    FORMAT(3G)
C
C       INPUT VALUES OF PHI OF L(K)
C
        CALL IFILE (20,'PHILK')
        READ(20,900)
        READ(20,950)FLE
950     FORMAT(/1X,'POWER SPECTRUM OF PHI OF L(K)',/1X,
     1  'DATA TAKEN FROM FILE ',A5)
        READ(20,906)NXZJ,TWOL,MPTS,FTM1
        READ(20,907)MZERO
        READ(20,910)WBAR,VAR
        READ(20,951)WSUM
951     FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
        READ(20,952)
952     FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE POWER SPECTRUM',
     1  /1X,5X,'K',10X,'PS VALUE',8X,'K CONTD',4X,'PS VALUE CONTD',/)
        DO 823 I=0,NPTS
        READ(20,903)DEL,W(I),DEL2,YY
823     CONTINUE
        END FILE 20
C
C       START PART2; COMPUTE INTEGRAL SCALE OF L USING EQN FOR E(L)
C
1040    TYPE 1500
1500    FORMAT(' INPUT N,L, & STEP SIZE OF L ',/$)
        ACCEPT 1750,N,ALV(1),STPL
```

```
C
      AN = FLOAT(N)
      C116 = 11./6.
      DVSR(1) = 1.
      DVSR(2) = 1.
      DVSR(3) = 10.
      DVSR(4) = 10.
C
      NDX = 0
C
1030  NDX = NDX + 1
      E(NDX) = 0.0
C
      SUM1 = 0.0
      DO 1020 JJ=1,N
      ALKI2(JJ) = (ALV(NDX)*DELK*JJ)**2
      PHIKT(JJ) = (1.+188.75*ALKI2(JJ))/(1.+70.78*ALKI2(JJ))**C116
1020  SUM1 = SUM1 + W(JJ)/PHIKT(JJ)
      DO 1010 I=1,N
      GKIL = 117.97*ALKI2(I)*(1.-188.75*ALKI2(I))/
     1       (ALV(NDX)*(1.+70.78*ALKI2(I))*(1.+188.75*ALKI2(I)))
1010  E(NDX) = E(NDX) + GKIL*((SUM1/AN)-(W(I)/PHIKT(I)))
      E(NDX) = E(NDX)/AN
      TYPE 1525,NDX,E(NDX)
1525  FORMAT(' ON ',I3,' PASS E =',G)
C
C     PICK NEW L FOR SECOND AND 4TH PASS
C
      IF (E(NDX).LT.0.0) ALV(NDX+1) = ALV(NDX) + STPL/DVSR(NDX)
      IF (E(NDX).GT.0.0) ALV(NDX+1) = ALV(NDX) - STPL/DVSR(NDX)
C
C     USE LINEAR INTERPOLATION FOR NDX = 2 OR 4
C
      IF (NDX.EQ.1) GO TO 1030
      IF (NDX.EQ.3) GO TO 1030
      TYPE 1503
1503  FORMAT(' ARE THERE POS. AND NEG. E"S (Y OR N) ',$)
      ACCEPT 807,ANS
807   FORMAT(A5)
      IF (ANS.EQ.'N') GO TO 1040
C
C     DO LINEAR INTERPOLATION
C
      ALV(NDX+1) = (ALV(NDX)*(-E(NDX-1))+ALV(NDX-1)*E(NDX))/
     1             (E(NDX)-E(NDX-1))
      TYPE 1502,NDX,ALV(NDX+1)
1502  FORMAT(' FOR ',I2,' PASS INTERPOLATTED L =',G)
      IF (NDX.EQ.2) GO TO 1030
C
C     FINAL L HAS BEEN SELECTED AFTER 4 PASSES
C
      ALF = ALV(NDX+1)
```

```
C
C       PART 4 , COMPUTE LG(KI;L)
C
        CALL OFILE(20,'LG')
        WRITE(20,1507)ALF
1507    FORMAT(' OUTPUT OF ITEM 1, PART 4, PHASE II ATMO. TURB.',
     1  /1X,'CREATED BY PROGRAM PART2',/1X,
     2  ' FOR L =',G,' METERS',/1X,'    K',7X,'LG',7X,'K CONTD',5X,
     3  'LG CONTD')
        NH2 = N/2
        DO 1050 I1 = 1,NH2
        AKZ = DELK*I1
        AKIL = ALF*AKZ
        AK1 = DELK*(NH2+I1)
        AKIL3 = ALF*AK1
        AKIL2 = AKIL*AKIL
        AKIL4 = AKIL3*AKIL3
        ALG = (117.97*AKIL2*(1.-188.75*AKIL2))/
     1        ((1.+70.78*AKIL2)*(1.+188.75*AKIL2))
        ALG2 = (117.97*AKIL4*(1.-188.75*AKIL4))/
     1        ((1.+70.78*AKIL4)*(1.+188.75*AKIL4))
1050    WRITE(20,1505)AKZ,ALG,AK1,ALG2
1505    FORMAT(4G)
        END FILE 20
C
C
        TYPE 1506
1506    FORMAT(' CONTINUE OR ABORT? (C OR A) ',$)
        ACCEPT 807,GOON
        IF (GOON.EQ.'A') GO TO 9999
C
C       DO PART 5
C
        SIG2 = 0.0
        DO 1060 I=1,N
1060    SIG2 = SIG2 + W(I)/(ALF*PHIKT(I))
        SIG2 =SIG2/AN
        TYPE 1508,SIG2
1508    FORMAT(' SIG2 =',G)
C
C       CONTINUE ?
C
        TYPE 1506
        ACCEPT 807,GOON
        IF (GOON.EQ.'A') GO TO 9999
C
C       PART 7, REQUIRES SUBROUTINES GAM AND AK
C
        CALL OFILE(21,'PHIXI')
        WRITE (20,915)
        WRITE(20,1512)SIG2
```

```
1512    FORMAT(/1X,'OUTPUT FOR PART 7 OF ITEM I,PHASE II',/1X,
     1  'CALCULATIONS OF VON-KARMEN AUTOCORRELATION FN',/1X,
     2  'SIGMA SQRD =',G)
        WRITE(20,1511)
1511    FORMAT('    XI',5X,'PHI(XI)',5X,'XI CONTD',5X,'PHI(XI) CONTD')
        PI = 3.14159265
        C43 = 4./3.
        CALL GAM(C116,G1)
        CALL GAM(C43,G2)
        GAM13 = 2.67893853
        BETA = 2.*G1*(PI**.5)/(5.*G2)
        ARG = BETA/ALF
        C13 = 1./3.
        C23 = 2./3.
        C13 = 1./3.
        CONST = SIG2*(2.**C23)/GAM13
        DO 1070 I=1,NH2
        E1 = DELX*I
        E2 = DELX*(I+NH2)
        ETA = E1*ARG
        ETA2 = E2*ARG
        IF (ETA.GT.5..OR.ETA.LT..1)TYPE 1515,ETA
        CALL AK(1,ETA,AK1)
        CALL AK(2,ETA,AK2)
        CALL AK(1,ETA2,AK3)
        CALL AK(2,ETA2,AK4)
        PHIK = CONST*(ETA**C13)*(AK1-AK2*ETA/2.)
        PHIK2 = CONST*(ETA2**C13)*(AK3-AK4*ETA2/2.)
1070    WRITE(20,1505) E1,PHIK,E2,PHIK2
1515    FORMAT(' ETA =',G,' WHICH IS OUT OF RANGE FOR AK SUBR.')
        END FILE 20
C
C       PART 8
C
        CALL OFILE(20,'PHIK')
        WRITE(20,915)
        WRITE(20,1595)ALF,SIG2
1595    FORMAT(/1X,'OUTPUT FOT PART 8 OF ITEM I, PHASE II',/1X,
     1  'CALCULATIONS OF PHI K WITH L =',G,'AND SIG SQRD =',G)
        WRITE(20,1514)
1514    FORMAT('  K',5X,'PHIK',7X,'K CONTD',5X,'PHIK CONTD')
        CONS1 = SIG2*ALF
        STEP = DELK*ALF
        NPTS2 = NPTS/2
        DO 1080 I=1,NPTS2
        AK0 = DELK*I
        AK1 = DELK*(I+NPTS2)
        AK2 = (STEP*I)*(STEP*I)
        AK4 = (STEP*(I+NPTS2))**2
        PHIK = CONS1*(1.+188.75*AK2)/((1.+70.78*AK2)**C116)
        PHIK2 = CONS1*(1.+188.75*AK4)/((1.+70.78*AK4)**C116)
```

```
1080    WRITE(20,1505)AK0,PHIK,AK1,PHIK2
        END FILE 20
C
C
900     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB2')
915     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM PART2')
903     FORMAT(1X,F10.6,3X,E12.4,3X,F10.6,3X,E12.4)
906     FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1  ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1  ' METER')
907     FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
908     FORMAT(29X,F10.4,3X,E12.4)
910     FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1  'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
C
C
9999    END
```

Program PART5

```
C         PROGRAM ITEM2.  ATMOSPHERIC TURBULENCE TASK
C         ITEM 2, PART 5. PHASE II OF ATMOS-
C         PHERIC TURBULENCE
C         MEAN VALUE SUBTRACTED FROM DATA BEFORE COMPUTING SPECTRUM
C         COMPONENT WITH LOW FREQUENCY CONTAMINATION
C
C
C         DATA FILES READ BY PROGRAM ARE:
C                 PHILK: INPUT PSD FROM ATURB2.F4
C
C
          DIMENSION W(0/13000),AL(15),SIG2(15)
C
C         INPUT INITIAL PARAMETERS
C
          TYPE 1001
1001      FORMAT(/1X,'INPUT NO. OF POINTS TO BE READ, DELK,DELX',/$)
          ACCEPT 1750,M41,DELK,DELX
1750      FORMAT(3G)
          TYPE 1751
1751      FORMAT(' IS RECORD TRANSVERSE OR LONGITUDIONAL (T OR L) ',$)
          ACCEPT 807,WHICH
807       FORMAT(A5)
C
C         INPUT VALUES OF PHI OF L(K)
C
          CALL IFILE (20,'PHILK')
          READ(20,900)
          READ(20,950)FLE
950       FORMAT(/1X,'POWER SPECTRUM OF PHI OF L(K)',/1X,
     1    'DATA TAKEN FROM FILE ',A5)
          READ(20,906)NPTS,TWOL,MPTS,FTM1
          READ(20,907)MZERO
          READ(20,910)WBAR,VAR
          READ(20,951)WSUM
951       FORMAT(/1X,'<W OF L(X)**2> = ',F12.4)
          READ(20,952)
952       FORMAT(//1X,'PRINTOUT OF THE VALUES OF THE POWER SPECTRUM',
     1    /1X,5X,'K',10X,'PS VALUE',8X,'K CONTD',4X,'PS VALUE CONTD',/)
          MHALF = NPTS/4
          M51 = MHALF-1
          DO 823 I=0,M51
           K= MHALF+I
          IF (K.GE.M41) K=M41+1
823       READ(20,903)DEL,W(I),DEL2,W(K)
          END FILE 20
C
C
C
          TYPE 1500
1500      FORMAT(' INPUT KL,KU,N ',/$)
          ACCEPT 1752,KL,KU,N
```

```
1752    FORMAT(3G)
C
1010    TYPE 1503
1503    FORMAT(' INPUT J,L ',$)
        ACCEPT 1752,J,AL(J)
        JMAX = J
C
C       DO PART 5
C
        AN = FLOAT(N)
        C116 = 11./6.
        C56 = 5./6.
        SIG2(J) = 0.0
        DO 1060 I=KL,KU
        ALKI2 = (AL(J)*DELK*I)**2
        PHIK = 2./(1.+70.78*ALKI2)**C56
        IF (WHICH.EQ.'T')PHIK=(1.+188.75*ALKI2)/
     1           (1.+70.78*ALKI2)**C116
1060    SIG2(J) = SIG2(J) + W(I)/(AL(J)*PHIK)
        SIG2(J) =SIG2(J)/AN
        TYPE 1508,J,SIG2(J)
1508    FORMAT(' FOR J =',I3,' SIG2 =',G)
C
        TYPE 1502
1502    FORMAT(' PICK ANOTHER J,L (Y OR N) ',$)
        ACCEPT 807,AJL
        IF (AJL.EQ.'Y') GO TO 1010
900     FORMAT(//1X,'DATA FILE CREATED BY PROGRAM ATURB3')
903     FORMAT(1X,F10.6,3X,E12.4,3X,F10.6,3X,E12.4)
906     FORMAT(//1X,I6,' DATA POINTS  WERE USED IN 2L = ',F15.4,
     1  ' METER',/1X,I5,' DATA POINTS WERE USED IN M = ',F16.4,
     1  ' METER')
907     FORMAT(//1X,I6,' ZEROS WERE ADDED TO DATA')
910     FORMAT(//1X,'MEAN VALUE OF W(X) = ',E15.5,' M/SEC',/1X,
     1  'MEAN SQ. VALUE = ',E15.5,' (M/SEC)**2')
        END
```

Subroutine SIMP2

```
C         SUBROUTINE SIMP. USES SIMPSON'S RULE TO DO INTEGRALS.
C         H = SPACING BETWEEN VALUES
C         NPTS = NO. OF POINTS TO BE COVERED. W=ARRAY CONT-
C         AINING VALUES TO BE INTEGRATED. ANS = INTEGRAL OF W(I)
C
          SUBROUTINE SIMP(H,NPTS,W,ANS)
C
          DIMENSION W(0/500)
C
          ARG = H/3.
          ANS1 = 0.0
          DO 10 I=1,NPTS-3,2
10        ANS1 = ANS1 + 4.*W(I)
          ANS2 = 0.0
          DO 11 I=2,NPTS-2,2
11        ANS2 = ANS2 + 2.*W(I)
          ANS = (ANS2+ANS1+W(0)+W(NPTS-1))*ARG
C
          RETURN
C
          END
```

Subroutine SIMP

```
C       SUBROUTINE SIMP. USES SIMPSON'S RULE TO DO INTEGRALS.
C       A = STARTING VALUE, B= END VALUE, H = SPACING BETWEEN
C       VALUES, NPTS = NO. OF POINTS TO BE COVERED, W=ARRAY CONT-
C       AINING VALUES TO BE INTEGRATED, ANS = INTEGRAL OF W(I)
C
        SUBROUTINE SIMP(A,B,H,NPTS,ANS)
C
        COMMON/SG/W(Ø/65537)
C
        ARG = H/3.
        ANS1 = Ø.Ø
        DO 1Ø I=1,NPTS-3,2
1Ø      ANS1 = ANS1 + 4.*W(2*I)
        ANS2 = Ø.Ø
        DO 11 I=2,NPTS-2,2
11      ANS2 = ANS2 + 2.*W(2*I)
        ANS = (ANS2+ANS1+W(Ø)+W(2*NPTS-2))*ARG
C
        RETURN
C
        END
```

Subroutine SIMQ

```
C
C     ..................................................................
C
C        SUBROUTINE SIMQ
C
C        PURPOSE
C           OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS,
C           AX=B
C
C        USAGE
C           CALL SIMQ(A,B,N,KS)
C
C        DESCRIPTION OF PARAMETERS
C           A - MATRIX OF COEFFICIENTS STORED COLUMNWISE.  THESE ARE
C               DESTROYED IN THE COMPUTATION.  THE SIZE OF MATRIX A IS
C               N BY N.
C           B - VECTOR OF ORIGINAL CONSTANTS (LENGTH N). THESE ARE
C               REPLACED BY FINAL SOLUTION VALUES, VECTOR X.
C           N - NUMBER OF EQUATIONS AND VARIABLES. N MUST BE .GT. ONE.
C           KS - OUTPUT DIGIT
C                0 FOR A NORMAL SOLUTION
C                1 FOR A SINGULAR SET OF EQUATIONS
C
C        REMARKS
C           MATRIX A MUST BE GENERAL.
C           IF MATRIX IS SINGULAR , SOLUTION VALUES ARE MEANINGLESS.
C           AN ALTERNATIVE SOLUTION MAY BE OBTAINED BY USING MATRIX
C           INVERSION (MINV) AND MATRIX PRODUCT (GMPRD).
C
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C           NONE
C
C        METHOD
C           METHOD OF SOLUTION IS BY ELIMINATION USING LARGEST PIVOTAL
C           DIVISOR. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING
C           ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL
C           ELEMENTS.
C           THE FORWARD SOLUTION TO OBTAIN VARIABLE N IS DONE IN
C           N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS
C           CALCULATED BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION
C           VALUES ARE DEVELOPED IN VECTOR B, WITH VARIABLE 1 IN B(1),
C           VARIABLE 2 IN B(2),........, VARIABLE N IN B(N).
C           IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.0,
C           THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS
C           TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.
C
C     ..................................................................
C
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
C
```

```
C        FORWARD SOLUTION
C
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DO 30 I=J,N
C
C        SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN
C
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
C
C        TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)
C
      IF(ABS(BIGA)-TOL) 35,35,40
   35 KS=1
      RETURN
C
C        INTERCHANGE ROWS IF NECESSARY
C
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
C
C        DIVIDE EQUATION BY LEADING COEFFICIENT
C
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
C
C        ELIMINATE NEXT VARIABLE
C
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
```

```
C
C          BACK SOLUTION
C
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
```

Subroutine TRAP3

```
C <RFISHER>TRAP2.F4;1      1-Nov-77 10:09:04    EDIT BY RFISHER
        SUBROUTINE TRAP2(AI)
C
C       INTEGRALS BY TRAP. RULE
C
        COMMON/TR2/DELX,NXIH,AI2I(0/290)
C
        AI = 0.0
        DO 10 I=1,NXIH-1
10      AI = AI + AI2I(I)
        AI = .5*AI2I(0) + .5*AI2I(NXIH) + AI
        AI = AI*DELX
C
        RETURN
C
        END
```

Subroutine TRAP6

```
C <RFISHER>TRAP5.F4;1      1-Nov-77 10:09:04     EDIT BY RFISHER
        SUBROUTINE TRAP5(AI)
C
C       INTEGRALS BY TRAP. RULE
C
        COMMON/TR5/DELX1,NXIH1,AI5I(0/290)
C
        AI = 0.0
        DO 10 I=1,NXIH1-1
10      AI = AI + AI5I(I)
        AI = .5*AI5I(0) + .5*AI5I(NXIH1) + AI
        AI = AI*DELX1
C
        RETURN
C
        END
```

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15. Supplementary Notes

Langley technical monitors: Richard H. Rhyne and Harold N. Murrow
Topical Report

16. Abstract

The results of application to turbulence velocity records of several new methods for characterizing atmospheric turbulence for aircraft response calculations are described. The methods illustrated include maximum likelihood estimation of the integral scale and intensity of records obeying the von Karman transverse power spectral form, constrained least-squares estimation of the parameters of a parametric representation of autocorrelation functions, estimation of the power spectral density of the instantaneous variance of a record with temporally fluctuating variance, and estimation of the probability density functions of various turbulence components. Descriptions of the computer programs used in the computations are given, and a full listing of these programs is included.

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