NASA TM-83146

NASA Technical Memorandum 83146

NASA-TM-83146 19810025294

APPLICATION OF A LOCAL LINEARIZATION TECHNIQUE FOR THE SOLUTION OF A SYSTEM OF STIFF DIFFERENTIAL EQUATIONS ASSOCIATED WITH THE SIMULATION OF A MAGNETIC BEARING ASSEMBLY

KEMPER S. KIBLER AND GARY A. MCDANIEL

SEPTEMBER 1981

Corrected Copy

NASA
National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

SUMMARY

A local linearization technique has been successfully used to solve a system of stiff differential equations associated with a Magnetic Bearing Assembly. The technique has proven to be accurate, stable and extremely efficient. A variable order Adams method with a stiff option was used as a reference case.

INTRODUCTION

Certain models encountered in simulation studies require the solution of systems of stiff, nonlinear differential equations. Solving a system for the state as a function of time using general purpose flexible integration routines becomes costly because of overhead, satisfying error tolerance requirements and the fine integration step size which may be required to represent properly the system response. The problem is compounded when one or several of these models are required to be interfaced as subsystems of a large dynamic simulation which may have many time consuming derivative evaluations as well as slowly varying states. In such applications, the stiffness of the entire system may be amplified, thus requiring the entire simulation to operate at a very small integration step size. A method for solving this problem is to apply a stiff differential integration technique to only the stiff models of the simulation, thus allowing for stiff and nonstiff integration algorithms to be used, and consequently improving the efficiency of the overall simulation. Therefore, a numerical integration technique, preferably with one derivative evaluation per step, which is stable, accurate, and efficient is needed to solve these stiff simulation models.

An integration technique which exhibits the desired characteristics is the local linearization (LL) technique described in reference 1. The LL algorithm is especially suited for models for which the Jacobian matrix is easily obtained.

An excellent model to demonstrate the application of the LL technique is the Magnetic Bearing Assembly (MBA) described in appendix A. The MBA is a magnetic actuator which is used in critical solar, stellar and earth pointing applications on Space Shuttle missions. This model has eigenvalues ranging from 10 to 2000. However, if the MBA's are interfaced into the Annular Suspension and Pointing System (ASPS) simulation model, eigenvalues ranging from 0.1 to 2000 are obtained, thus considerably amplifying the stiffness of the system. The ASPS is described in reference 2

The purpose of this study is to demonstrate the LL technique to simulate a MBA or other similar models which cannot be solved efficiently using general purpose, flexible integration methods.

The LL performance is documented in this report against a general purpose, variable order Adams method with a stiff option (VOADAM) which assumes a dense Jacobian.

SYMBOLS

B, C, u, v, r, q, δ	submatrices required for matrix inversion
K_1, K_2, K_3, K_4	MBA current loop parameters
K_{LD}, K_p, K_I	

R_{AC}, L_{LE}, R_D, Lg_o	MBA circuit parameters
$A(t)$	time varying Jacobian matrix
b_{ij}	elements of B, where i, j = 1, 2, 3, 4, 5
a_{ij}	elements of A(t), where i, j = 1, 2, 3, 4, 5
C_1, C_2, C_3	MBA scale factors
D	matrix representing special 5x5 inverse
d_{ij}	elements of D, where i, j = 1, 2, 3, 4, 5
$\overline{F}$	n-dimensional vector composed of general nonlinear time-varying functions of the state vector $\overline{Z}(t)$
f_U	force generated by upper magnetic pole, N
f_L	force generated by lower magnetic pole, N
f	MBA force output, N
f_c	force command input to MBA, N
F_{max}	maximum MBA force output, N
g	actual magnetic gap, m
g_o	nominal magnetic gap, m
h	integration step size, seconds
I	identity matrix
I_C	MBA current command, A
I_o	MBA bias current, A
K	MBA force constant, Nm/A^2
m	payload mass, Kg
P, Q	matrix coefficients in general LL algorithm for solution of nonlinear time-varying systems
s	Laplace operator
t	time, seconds
T_1, T_2, T_3, T_4, T_5	common terms used in inverting the 5x5 matrix B

V	MBA voltage command, V
$\overline{Z}(t)$	n-dimensional vector representing system states
Δg	difference between nominal and actual magnetic gap, m
$\hat{\Delta g}$	estimate of Δg,m
$\eta_1, \eta_2, \eta_3, \eta_4$	functions of magnetic gap and gap coefficients
ω_B	MBA bandwith, rad/sec
ζ	actual gap coefficient, /m
$\hat{\zeta}$	estimated gap coefficient, /m

Subscripts:

k	integer constant
L	denotes lower MBA pole
n	denotes steps in construction of submatrices, n = 2, 3, 4, 5
U	denotes upper MBA pole

Dot over symbol denotes time derivative

Abbreviation:

ASPS	Annular Suspension and Pointing System
CPU	Central Processing Unit
LL	Local Linearization
MBA	Magnetic Bearing Assembly
VOADAM	Variable Order Adams with stiff option

PROBLEM DESCRIPTION

Shown in figure 1 is a block diagram of the MBA. The simplified ASPS dynamics shown was used purely for MBA model checkout and is not included in the state equations. The state differential equation of the MBA is given by:

$$\dot{\bar{Z}} = \bar{F}\,(\bar{Z},\ f_c,\ \Delta g,\ \Delta\dot{g},\ \Delta\hat{g},\ t) \tag{1}$$

where $\bar{Z}$, f_c, Δg, $\Delta\dot{g}$, $\Delta\hat{g}$ and t represent the state vector, the command force, gap, gap rate, gap estimate, and time, respectively. The LL solution at t_{k+1} is given by:

$$\bar{Z}_{k+1} = \bar{Z}_k + P_k\,\dot{\bar{Z}}_k + Q_k\,\frac{\partial\bar{F}}{\partial t}\,(\bar{Z}_k,\ f_{c_k},\ \Delta g_k,\ \Delta\dot{g}_k,\ \Delta\hat{g}_k,\ t_k) \tag{2}$$

where

$$P = A^{-1}\,(e^{Ah} - I) \tag{3}$$

$$Q = A^{-1}\,(P - hI) \tag{4}$$

$$A = \partial\dot{\bar{Z}}/\partial\bar{Z} \tag{5}$$

$$\frac{\partial\bar{F}}{\partial t} = \frac{\partial\bar{F}}{\partial f_c}\,\frac{\partial f_c}{\partial t} + \frac{\partial\bar{F}}{\partial\Delta g}\,\frac{\partial\Delta g}{\partial t} + \frac{\partial\bar{F}}{\partial\Delta\dot{g}}\,\frac{\partial\Delta\dot{g}}{\partial t} + \frac{\partial\bar{F}}{\partial\Delta\hat{g}}\,\frac{\partial\Delta\hat{g}}{\partial t} \tag{6}$$

I = nxn identity matrix

and h is the integration step size. The order of the system is n, which for this system is 5 for the upper pole and 5 for the lower pole of the MBA. The n-differential equations of (1), the A matrix (5) and the n-vector (6) are given in appendix B.

The approach taken to efficiently solve equation (2) was to substitute a first order Padé approximation for the matrix exponential and to develop a special matrix inverse to take advantage of the zero elements of the resulting 5x5 matrix. Another approach which was not investigated, but should also be efficient when used in conjunction with the first order Padé approximation is to solve systems of differential equations in lieu of computing inverses. This would probably reduce the number of operations required to obtain a solution as compared to the approach taken.

If the Padé approximation

$$e^{Ah} \simeq (I - Ah/2)^{-1} (I + Ah/2) \tag{7}$$

is substituted into equations (3) and (4) P becomes:

$$P = h (I - Ah/2)^{-1} \tag{8}$$

and Q becomes:

$$Q = Ph/2 \tag{9}$$

The matrix $\left(I - \frac{Ah}{2}\right)$ can be inverted using standard system matrix routines. However, to take advantage of the many zero elements, a special inverse was obtained using the partioning and bordering technique discussed in reference 3 and summarized in appendix C. This final code resulted in approximately a 76 percent reduction in execution time required for the inverse problem.

RESULTS

Several test cases were examined comparing the LL and VOADAM results. Forcing functions used were a step, triangular wave and sine wave. The triangular wave had a frequency of 1 Hz and a slope of 1 Newton/second. The sine wave had an amplitude of 4 Newtons and frequencies of 2.5 and 10 Hz. The LL test cases were run with a integration step size of 0.001 seconds. The error criteria used for the VOADAM solutions was selected such that the solution was to have no more than 125 units of error per 1,000,000 units of magnitude. Two test cases were made with each forcing function except for the step input where only one case was run. The two cases run were with and without the $\frac{\partial \bar{F}}{\partial t}$ vector zeroed. The step, which was input at t = 0.005 seconds with a magnitude of 1.5 newtons, was only run with $\frac{\partial \bar{F}}{\partial t}$ zeroed since by definition the derivative of a step has infinite slope. The purpose of examining the effect of zeroing the $\frac{\partial \bar{F}}{\partial t}$ vector is that in practice it is seldom available. In some applications a backward difference calculation of $\frac{\partial \bar{F}}{\partial t}$ can be used, however, caution should be exercised when using this procedure.

The LL and VOADAM solutions to a 2.5 Hz sine wave is shown in figure 2. The variables plotted are the MBA output, f, the error between the VOADAM and LL solutions, and the magnetic gap change, Δg, resulting from applying f to a 90 Kg mass. Figures 3 to 5 show the LL and VOADAM solutions to a 10 Hz sine wave, a triangular wave and a step input, respectively. Presented in tables I to IV is a digital representation of portions of the data shown in the time history plots. The maximum error which occurred during the run is shown enclosed in the box.

The data presented shows LL to be extremely accurate even with the $\frac{\partial \bar{F}}{\partial t}$ vector zeroed. As mentioned previously, the LL data presented was run at only one integration step size, 0.001 seconds. No attempt was made at the writing of this report to vary the step size or to investigate methods of approximating the $\frac{\partial \bar{F}}{\partial t}$ vector with backward differences or other numerical methods of computing derivatives. It is reasonable to assume, however, that the LL results for $\frac{\partial \bar{F}}{\partial t}$ equal to zero could be improved by one or both of the above methods.

Computer execution times comparing LL to VOADAM for an input sine wave frequency of 1, 5, and 10 Hz are shown in the following table:

Frequency (Hz)	1	5	10
LL	2.7	2.7	2.7
VOADAM	24.2	31.1	28.0
%CPU saved	88.8	91.3	90.3

The execution times are expressed as CPU-second/second of run time and the percent CPU saved is the amount of CPU time saved by LL. For timing purposes, a step size of 0.001 seconds was used for LL while VOADAM adjusts the step size to satisfy a maximum error criteria of 125 units of error per 1,000,000 units of magnitude. VOADAM was required to return to the calling program at 0.01 second intervals. Although this is not the most efficient use of VOADAM, it was required to simulate the operating environment of the MBA. The MBA receives inputs at 0.01 second intervals from a digital controller.

CONCLUDING REMARKS

A local linearization technique (LL) has been successfully used to solve a system of stiff differential equations associated with a Magnetic Bearing Assembly. The LL technique proved to be accurate, stable and extremely efficient when compared against a general purpose flexible Adams integration method with a stiff option. In large dynamic simulations, which require the simulation of models such as the Magnetic Bearing Assembly, the LL technique, when applicable, appears to be one of the most efficient methods to employ. When considering budget constraints, the LL technique may very well determine whether or not models such as the MBA can be included, without simplication, in large dynamic simulations.

The LL technique presented in this report is not restricted to the application discussed herein. For example, in the linear time-varying case, the Jacobian and LL coefficients (P and Q) are constant, which makes the LL technique especially suitable for real time simulation applications since only one initial computation of the Jacobian is required.

REFERENCES

1. Barker, L. E.; Bowles, R. L.; and Williams, L. H.: Development and Application of a Local Linearization Algorithm for the Integration of Quaternion Rate Equations in Real-Time Flight Simulation Problems. NASA TN D-7347, 1973.

2. Keckler, C. R.; Kibler, K. S.; and Rowell, L. F.: Determination of ASPS Performance for Large Payloads in the Shuttle Orbiter Disturbance Environment. NASA TM 80136, 1979.

3. Westlake, J. R.: A Handbook of Numerical Matrix Inversion and Solution of Linear Equations, John Wiley and Sons, Inc., New York, 1968.

APPENDIX A

DESCRIPTION OF THE MAGNETIC BEARING ASSEMBLY

The Magnetic Bearing Assembly (MBA) is a magnetic actuator used for fine pointing control in the Annular Suspension and Pointing System (ASPS) which is described in reference 2. Shown in figure 1 is a block diagram of the hardware model and the associated current loop electronics. Actuators of this type are inherently nonlinear. This particular model uses bias current linearization to remove the current squared nonlinearity and a signal proportional to the gap is used to multiply the coil currents to compensate for the inverse-gap-squared relationship. A current control loop is used to obtain the desired actuator response. The MBA parameters are shown in table V. The current control loop parameters are tabulated in table VI. The following functional relationships are used to calculate the bias current and current control loop parameters.

$$I_o = \frac{g_o}{2}\sqrt{\frac{F_{max}}{K}} \qquad \text{(A-1)}$$

$$K_{LD} = L_{LE}\, L_{g_o}\, \omega_B^2 \qquad \text{(A-2)}$$

$$K_p = (Lg_o\, R_{AC} + Lg_o\, R_D + L_{LE}\, R_{AC})\, \omega_B^2 \qquad \text{(A-3)}$$

$$K_I = R_{AC}\, R_D\, \omega_B^2 \qquad \text{(A-4)}$$

$$K_3 = Lg_o \qquad \text{(A-5)}$$

$$K_4 = R_{AC} \tag{A-6}$$

where ω_B is the bandwidth and F_{max} is the maximum force output. The parameters K_1 and K_2 were selected to give desired damping characteristics. The value of the remaining parameters are the result of laboratory measurements.

APPENDIX B

STATE DIFFERENTIAL EQUATIONS, "A" MATRICES AND THE $\frac{\partial \overline{F}}{\partial t}$ VECTORS

The differential equations are

Upper Pole

$$\dot{Z}_{1U} = K_2 \ (I_{CU} - Z_{4U}) \tag{B-1}$$

$$\dot{Z}_{2U} = (Z_{1U} + K_1 \ (I_{CU} - Z_{4U}) - K_4 \ Z_{2U})/K_3 \tag{B-2}$$

$$\dot{Z}_{3U} = K_I \ Z_{2U} \tag{B-3}$$

$$\dot{Z}_{4U} = \frac{1}{L_{LE}} \ (V_U + Z_{5U} \ R_{AC} - Z_{4U} \ R_{AC} - R_D \ Z_{4U}) \tag{B-4}$$

$$\dot{Z}_{5U} = \frac{n_3}{Lg_o} \ (+ R_{AC} \ Z_{4U} - \frac{C_2 \ \Delta\dot{g} \ Z_{5U}}{n_3^2} - R_{AC} \ Z_{5U}) \tag{B-5}$$

Lower Pole

$$\dot{Z}_{1L} = K_2 \ (I_{CL} - Z_{4L}) \tag{B-6}$$

$$\dot{Z}_{2L} = (Z_{1L} + K_1 (I_{CL} - Z_{4L}) - K_4 Z_{2L}) / K_3 \quad \text{(B-7)}$$

$$\dot{Z}_{3L} = K_I Z_{2L} \quad \text{(B-8)}$$

$$\dot{Z}_{4L} = \frac{1}{L_{LE}} (V_L + Z_{5L} R_{AC} - Z_{4L} R_{AC} - R_D Z_{4L}) \quad \text{(B-9)}$$

$$\dot{Z}_{5L} = \frac{\eta_4}{Lg_o} (+ R_{AC} Z_{4L} + \frac{C_2 \Delta \dot{g} Z_{5L}}{\eta_4^2} - R_{AC} Z_{5L}) \quad \text{(B-10)}$$

where

$$I_{CU} = (I_o + f_c C_1) \eta_1 \quad \text{(B-11)}$$

$$I_{CL} = (I_o - f_c C_1) \eta_2 \quad \text{(B-12)}$$

$$\eta_1 = (1 - \hat{\zeta} \Delta \hat{g}) \quad \text{(B-13)}$$

$$\eta_2 = (1 + \hat{\zeta} \Delta \hat{g}) \quad \text{(B-14)}$$

$$\eta_3 = (1 - \zeta \Delta g) \quad \text{(B-15)}$$

$$\eta_4 = (1 + \zeta \Delta g) \tag{B-16}$$

$$V_U = K_{LD} \dot{Z}_{2U} + K_p Z_{2U} + Z_{3U} \quad -V_{LIM} \lesssim V_U \lesssim V_{LIM} \tag{B-17}$$

$$V_L = K_{LD} \dot{Z}_{2L} + K_p Z_{2L} + Z_{3L} \quad -V_{LIM} \lesssim V_L \lesssim V_{LIM} \tag{B-18}$$

$$C_1 = g_o^2 /(4KI_o) \tag{B-19}$$

$$C_2 = \zeta L g_o \tag{B-20}$$

$$F_U = (C_3/\eta_3^2) Z_{5U}^2 \tag{B-21}$$

$$F_L = (C_3/\eta_4^2) Z_{5L}^2 \tag{B-22}$$

$$C_3 = K/g_o^2 \tag{B-23}$$

The $\partial \overline{F}/\partial t$ vector for the upper pole is given by:

$$\frac{\partial \dot{Z}_{1U}}{\partial t} = K_2 \, [\eta_1 \, C_1 \frac{\partial fc}{\partial t} - (I_o + C_1 \, f_c) \, \hat{\zeta} \, \frac{\partial \Delta \hat{g}}{\partial t} \,] = K_2 \, . \, \text{VAR1} \qquad \text{(B-24)}$$

$$\frac{\partial \dot{Z}_{2U}}{\partial t} = \frac{K_1}{K_3} . \, \text{VAR1} \qquad \text{(B-25)}$$

$$\frac{\partial \dot{Z}_{3U}}{\partial t} = 0 \qquad \text{(B-26)}$$

$$\frac{\partial \dot{Z}_{4U}}{\partial t} = \frac{K_{LD} \, K_1}{L_{LE} \, K_3} \, . \, \text{VAR1} \qquad \text{(B-27)}$$

$$\frac{\partial \dot{Z}_{4U}}{\partial t} = 0 \qquad -V_{LIM} > V_U > V_{LIM} \qquad \text{(B-28)}$$

$$\frac{\partial \dot{Z}_{5U}}{\partial t} = \frac{1}{Lg_o} \left[\left(\zeta \, R_{AC}[Z_{5U} - Z_{4U}] - \frac{C_2 \, \Delta \dot{g} \, Z_{5U} \, \zeta}{\eta_3^2} \right) \frac{\partial \Delta g}{\partial t} - \frac{C_2 \, Z_{5U}}{\eta_3} \, \frac{\partial \Delta \dot{g}}{\partial t} \right] \qquad \text{(B-29)}$$

The $\partial \overline{F} / \partial t$ vector for the lower pole is given by:

$$\frac{\partial \dot{Z}_{1L}}{\partial t} = K_2 \left[-C_1 \, \eta_2 \frac{\partial fc}{\partial t} + (I_o - C_1 \, f_c) \, \hat{\zeta} \, \frac{\partial \Delta \hat{g}}{\partial t} \right] = K_2 \, . \, \text{VAR2} \qquad \text{(B-30)}$$

$$\frac{\partial \dot{Z}_{2L}}{\partial t} = \frac{K_1}{K_3} \, . \, \text{VAR2} \qquad \text{(B-31)}$$

$$\frac{\partial \dot{Z}_{3L}}{\partial t} = 0 \qquad \text{(B-32)}$$

$$\frac{\partial \dot{Z}_{4L}}{\partial t} = \frac{K_{LD}\ K_1}{L_{LE}\ K_3} \cdot \text{VAR2} \qquad \text{(B-33)}$$

$$\frac{\partial \dot{Z}_{4L}}{\partial t} = 0 \qquad -V_{LIM} > V_L > V_{LIM} \qquad \text{(B-34)}$$

$$\frac{\partial \dot{Z}_{5L}}{\partial t} = \frac{1}{Lg_o}\left[\left(\zeta\ R_{AC}\ [Z_{4L} - Z_{5L}] - \frac{C_2\ \Delta\dot{g}\ Z_{5L}\ \zeta}{\eta_4^2}\right)\frac{\partial \Delta g}{\partial t} + \frac{C_2\ Z_{5L}}{\eta_4}\ \frac{\partial \Delta \dot{g}}{\partial t}\right] \qquad \text{(B-35)}$$

The "A" matrix, $\frac{\partial \dot{Z}_i}{\partial Z_j}$ (i = 1, . . ., 5, j = 1, . . ., 5) for the upper pole is given by:

$$\begin{bmatrix} 0 & 0 & 0 & a_{14} & 0 \\ a_{21} & a_{22} & 0 & a_{24} & 0 \\ 0 & a_{32} & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix} \qquad \text{(B-36)}$$

where

$$a_{14} = -K_2 \qquad \text{(B-37)}$$

$$a_{21} = 1/K_3 \tag{B-38}$$

$$a_{22} = -K_4/K_3 \tag{B-39}$$

$$a_{24} = -K_1/K_3 \tag{B-40}$$

$$a_{32} = K_I \tag{B-41}$$

$$a_{41} = K_{LD}/K_3 L_{LE} \tag{B-42}$$

$$a_{42} = \frac{1}{L_{LE}}\left(\frac{-K_4 \; K_{LD}}{K_3} + K_p\right) \tag{B-43}$$

$$a_{43} = 1/L_{LE} \tag{B-44}$$

$$a_{44} = -\frac{1}{L_{LE}}\left(\frac{K_1 \; K_{LD}}{K_3} + R_{AC} + R_D\right) \tag{B-45}$$

$$a_{45} = R_{AC}/L_{LE} \tag{B-46}$$

$$a_{54} = \frac{\eta_3 R_{AC}}{Lg_o} \tag{B-47}$$

$$a_{55} = \frac{-\eta_3}{Lg_o}\left(\frac{C_2 \Delta\dot{g}}{\eta_3^2} + R_{AC}\right) \tag{B-48}$$

The following elements are zero:

a_{11}, a_{12}, a_{13}, a_{15}, a_{23}, a_{25}, a_{31}, a_{33}, a_{34}, a_{35}, a_{51}, a_{52}, and a_{53}.

The "A" matrix for the lower pole is given by:

Elements a_{14} through a_{45} are identical to the upper pole "A" matrix elements. The remaining two elements are:

$$a_{54} = \frac{\eta_4 R_{AC}}{Lg_o} \tag{B-49}$$

$$a_{55} = \frac{\eta_4}{Lg_o}\left(\frac{C_2 \Delta\dot{g}}{\eta_4^2} - R_{AC}\right) \tag{B-50}$$

If $V_U < -V_{LIM}$ or $V_U > V_{LIM}$ the following upper pole elements are set to:

$$a_{41} = a_{42} = a_{43} = 0 \tag{B-51}$$

$$a_{44} = -(R_{AC} + R_D)/L_{LE} \tag{B-52}$$

If $V_L < -V_{LIM}$ or $V_L > V_{LIM}$ the following lower pole elements are set to:

$$a_{41} = a_{42} = a_{43} = 0 \tag{B-53}$$

$$a_{4,4} = -(R_{AC} + R_D)/L_{LE} \tag{B-54}$$

For steady state initialization:

$$Z_{1U} = Z_{2U} = Z_{1L} = Z_{2L} = 0 \tag{B-55}$$

$$Z_{3U} = Z_{3L} = I_o R_D \tag{B-56}$$

$$Z_{4U} = Z_{5U} = Z_{4L} = Z_{5L} = I_o \tag{B-57}$$

APPENDIX C

MATRIX INVERSION TECHNIQUE

Given the matrix B

$$B = \left[\begin{array}{c|c} B_{n-1} & u_n \\ \hline v_n & b_{nn} \end{array}\right] \tag{C-1}$$

where B_{n-1} is a (n-1) x (n-1) square matrix, v_n is a 1 x (n-1) row, matrix, u_n is a (n-1) x 1 column matrix, b_{nn} is a scalar. Assume the inverse to be of the form

$$B^{-1} = \left[\begin{array}{c|c} C_{n-1} & r_n \\ \hline q_n & \delta^{-1}_n \end{array}\right] \tag{C-2}$$

where C_{n-1} is a (n-1) x (n-1) square matrix, q_n is a 1 x (n-1) row matrix, r_n is a (n-1) x 1 column matrix, δ^{-1}_n is a scalar. Using the relationship $BB^{-1} = I$ as in reference 2, it follows that:

$$\delta_n = b_{nn} - v_n B^{-1}_{n-1} u_n \tag{C-3}$$

$$q_n = -\delta^{-1}_n v_n B^{-1}_{n-1} \tag{C-4}$$

$$r_n = -B^{-1}_{n-1} \, u_n \, \delta^{-1}_n \tag{C-5}$$

$$C_{n-1} = B^{-1}_{n-1} - B^{-1}_{n-1} \, u_n \, q_n \tag{C-6}$$

The 5x5 matrix inverse was calculated by starting with n = 2 to get the 2x2 inverse, with n = 3 to get the 3x3 inverse, etc., until n = 5.

If B is of the form

$$B = \begin{bmatrix} b_{11} & 0 & 0 & b_{14} & 0 \\ b_{21} & b_{22} & 0 & b_{24} & 0 \\ 0 & b_{32} & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ 0 & 0 & 0 & b_{54} & b_{55} \end{bmatrix} \tag{C-7}$$

the final 5x5 inverse, D, is given by the following equations. Equations (C-8) through (C-23) give the 4x4 inverse. Equations (C-24) through (C-33) give the 5x5 inverse extended from the 4x4 inverse.

$$d_{44} = \frac{1}{b_{44} - b_{14} \, T_1 - b_{24} \, T_2} \tag{C-8}$$

$$d_{41} = - d_{44} \, T_1 \tag{C-9}$$

$$d_{42} = - d_{44} \, T_2 \tag{C-10}$$

$$d_{43} = - \frac{d_{44}\ b_{43}}{b_{33}} \tag{C-11}$$

$$d_{14} = - \frac{d_{44}\ b_{14}}{b_{11}} \tag{C-12}$$

$$d_{24} = -d_{44} \left(\frac{b_{24}}{b_{22}} - \frac{b_{14}\ b_{21}}{b_{11}\ b_{22}} \right) \tag{C-13}$$

$$d_{34} = -d_{44} \left(\frac{b_{14}\ b_{21}\ b_{32}}{b_{11}\ b_{22}\ b_{33}} - \frac{b_{24}\ b_{32}}{b_{22}\ b_{33}} \right) \tag{C-14}$$

$$d_{11} = \frac{1}{b_{11}} - d_{41}\ T_3 \tag{C-15}$$

$$d_{21} = - \frac{b_{21}}{b_{11}\ b_{22}} - d_{41}\ T_4 \tag{C-16}$$

$$d_{31} = \frac{b_{21}\ b_{32}}{b_{11}\ b_{22}\ b_{33}} - d_{41}\ T_5 \tag{C-17}$$

$$d_{12} = - d_{42}\ T_3 \tag{C-18}$$

$$d_{22} = \frac{1}{b_{22}} - d_{42}\ T_4 \tag{C-19}$$

$$d_{32} = - \frac{b_{32}}{b_{22}\ b_{33}} - d_{42}\ T_5 \tag{C-20}$$

$$d_{13} = - d_{43}\ T_3 \tag{C-21}$$

$$d_{23} = - d_{43}\ T_4 \tag{C-22}$$

$$d_{33} = \frac{1}{b_{33}} - d_{43}\ T_5 \tag{C-23}$$

$$d_{55} = \frac{1}{b_{55} - b_{54}\ d_{44}\ b_{45}} \tag{C-24}$$

$$d_{51} = - d_{55}\ b_{54}\ d_{41} \tag{C-25}$$

$$d_{52} = - d_{55}\ b_{54}\ d_{42} \tag{C-26}$$

$$d_{53} = - d_{55}\ b_{54}\ d_{43} \tag{C-27}$$

$$d_{54} = - d_{55}\ b_{54}\ d_{44} \tag{C-28}$$

$$d_{15} = - d_{55}\, b_{45}\, d_{14} \tag{C-29}$$

$$d_{25} = - d_{55}\, b_{45}\, d_{24} \tag{C-30}$$

$$d_{35} = - d_{55}\, b_{45}\, d_{34} \tag{C-31}$$

$$d_{45} = - d_{55}\, b_{45}\, d_{44} \tag{C-32}$$

$$d_{ij} = d_{ij} + d_{55}\, d_{i4}\, b_{45}\, b_{54}\, d_{4j} \quad i, j = 1, 2, 3, 4 \tag{C-33}$$

where

$$T_1 = \frac{b_{41}}{b_{11}} - \frac{b_{42}\, b_{21}}{b_{11}\, b_{22}} + \frac{b_{43}\, b_{21}\, b_{32}}{b_{11}\, b_{22}\, b_{33}} \tag{C-34}$$

$$T_2 = \frac{b_{42}}{b_{22}} - \frac{b_{43}\, b_{32}}{b_{22}\, b_{33}} \tag{C-35}$$

$$T_3 = \frac{b_{14}}{b_{11}} \tag{C-36}$$

$$T_4 = - \frac{b_{21}\, b_{14}}{b_{11}\, b_{22}} + \frac{b_{24}}{b_{22}} \tag{C-37}$$

and

$$T_5 = \frac{b_{21}\ b_{32}\ b_{14}}{b_{11}\ b_{22}\ b_{33}} - \frac{b_{32}\ b_{24}}{b_{22}\ b_{33}} \qquad \text{(C-38)}$$

TABLE V.- MBA PARAMETERS

Maximum force output, F_{max}, N	34.25
Bandwidth, ω_B, rad/sec	628
Bias current, I_o, A	0.55522
Nominal gap, g_o, m	0.00762
Actuator force constant, K, $\frac{N.m^2}{A^2}$	0.00161284
Gap coefficients, ζ and $\hat{\zeta}$,/m	127.795
Mass inductance measured at nominal gap, Lg_o, H	0.4347
Leakage inductance, L_{LE}, H	0.243
AC resistance, R_{AC},ohms	238
DC resistance, R_D, ohms	7.4

TABLE VI.- CURRENT LOOP PARAMETERS

Parameter	Value
K_{LD}	4.16×10^4
K_p	6.48×10^7
K_I	6.94×10^8
K_1	0.00136
K_2	0.2845
K_3	0.4347
K_4	238

TABLE I.- 2.5 HZ SINE WAVE COMPARISON (BOX DENOTES MAXIMUM ERROR)

t	VOADAM	LL, $\frac{\partial \overline{F}}{\partial t} = 0$	LL, $\frac{\partial \overline{F}}{\partial t} \neq 0$	error, $\frac{\partial \overline{F}}{\partial t} = 0$	error, $\frac{\partial \overline{F}}{\partial t} \neq 0$
.210000000	.117449790	.148293576	.116824035	.030843786	-.000625755
.211000000	.054503260	.085357574	.053876353	.030854314	-.000626907
.212000000	-.008456853	.022400377	-.009084756	.030857230	-.000627904
.213000000	-.071415009	-.040562478	-.072043755	.030852532	-.000628745
.214000000	-.134355673	-.103515452	-.134985104	.030840221	-.000629431
.215000000	-.197263310	-.166443010	-.197893272	.030820301	-.000629961
.216000000	-.260122395	-.229329620	-.260752731	.030792775	-.000630336
.217000000	-.322917413	-.292159762	-.323547967	.030757650	-.000630555
.218000000	-.385632865	-.354917929	-.386263483	.030714936	-.000630618
.219000000	-.448253272	-.417588630	-.448883798	.030664641	-.000630526
.220000000	-.510763177	-.480156398	-.511393456	.030606779	-.000630279
.221000000	-.573147152	-.542605789	-.573777028	.030541363	-.000629876
.222000000	-.635389798	-.604921389	-.636019116	.030468409	-.000629318
.223000000	-.697475750	-.667087615	-.698104356	.030387935	-.000628605
.224000000	-.759389684	-.729089723	-.760017422	.030299961	-.000627738
.225000000	-.821116316	-.790911807	-.821743032	.030204509	-.000626716
.226000000	-.882640407	-.852538807	-.883265948	.030101600	-.000625540
.227000000	-.943946772	-.913955510	-.944570983	.029991262	-.000624211
.228000000	-1.005020275	-.975146754	-1.005643003	.029873520	-.000622728
.229000000	-1.065845839	-1.036097435	-1.066466932	.029748405	-.000621093

TABLE II.- 10 HZ SINE WAVE COMPARISON (BOX DENOTES MAXIMUM ERROR)

t	VOADAM	LL, $\frac{\partial \overline{F}}{\partial t} = 0$	LL, $\frac{\partial \overline{F}}{\partial t} \neq 0$	error, $\frac{\partial \overline{F}}{\partial t} = 0$	error, $\frac{\partial \overline{F}}{\partial t} \neq 0$
.460000000	.513067197	.638874723	.511264777	.125787525	-.001822420
.461000000	.256069556	.382617966	.254010125	.126528409	-.002079432
.462000000	-.001919442	.124848986	-.004248421	.126768427	-.002328980
.463000000	-.259912634	-.133413296	-.262489983	.126499338	-.002577349
.464000000	-.516860116	-.391147979	-.519693729	.125732137	-.002813613
.465000000	-.771808873	-.647336238	-.774842931	.124472635	-.003034058
.466000000	-1.023688522	-.900965369	-1.026928999	.122723153	-.003240477
.467000000	-1.271522438	-1.151032816	-1.274955495	.120489622	-.003433056
.468000000	-1.514330397	-1.396550151	-1.517942083	.117780246	-.003611686
.469000000	-1.751152373	-1.636546999	-1.754928434	.114605374	-.003776061
.470000000	-1.981052266	-1.870074899	-1.984978028	.110977368	-.003925762
.471000000	-2.203121571	-2.096211060	-2.207181683	.106910511	-.004060312
.472000000	-2.416482944	-2.314062036	-2.420662156	.102420908	-.004179212
.473000000	-2.620293642	-2.522767259	-2.624575626	.097526382	-.004281985
.474000000	-2.813748851	-2.721502461	-2.818117040	.092246390	-.004368189
.475000000	-2.996084862	-2.909482934	-3.000522296	.086601927	-.004437435
.476000000	-3.166582075	-3.085966639	-3.171071474	.080615435	-.004489400
.477000000	-3.324567843	-3.250257140	-3.329091675	.074310703	-.004523832
.478000000	-3.469419126	-3.401706355	-3.473959679	.067712771	-.004540553
.479000000	-3.600564942	-3.539717111	-3.605104402	.060847831	-.004539460

TABLE III.- TRIANGULAR WAVE COMPARISON (BOX DENOTES MAXIMUM ERROR)

t	VOADAM	LL, $\frac{\partial \overline{F}}{\partial t} = 0$	LL, $\frac{\partial \overline{F}}{\partial t} \neq 0$	error, $\frac{\partial \overline{F}}{\partial t} = 0$	error, $\frac{\partial \overline{F}}{\partial t} \neq 0$
.541000000	.499271067	.498859295	.499337367	-.000411772	.000066300
.542000000	.499841323	.499663338	.500043179	-.000177985	.000201856
.543000000	.499947785	.500023750	.500181731	.000075964	.000233946
.544000000	.499615176	.499908124	.499828048	.000292949	.000212873
.545000000	.498926654	.499380925	.499095039	.000454271	.000168384
.546000000	.497978622	.498539647	.498096720	.000561025	.000118098
.547000000	.496860664	.497482348	.496931430	.000621683	.000070765
.548000000	.495645870	.496293016	.495676123	.000647145	.000030253
.549000000	.494388219	.495036759	.494386464	.000648540	-.000001755
.550000000	.493124903	.493760065	.493099624	.000635162	-.000025279
.551000000	.491879290	.492493394	.491838045	.000614105	-.000041245
.552000000	.490664179	.491254554	.490613202	.000590375	-.000050977
.553000000	.489484820	.490052042	.489428937	.000567222	-.000055883
.554000000	.488341439	.488887967	.488284159	.000546528	-.000057281
.555000000	.487231212	.487760394	.487174910	.000529182	-.000056302
.556000000	.486149720	.486665122	.486095856	.000515402	-.000053865
.557000000	.485091961	.485596948	.485041292	.000504986	-.000050669
.558000000	.484053002	.484550510	.484005779	.000497508	-.000047223
.559000000	.483028362	.483520807	.482984495	.000492445	-.000043867
.560000000	.482014203	.482503472	.481973390	.000489269	-.000040813

TABLE IV.- STEP COMPARISON (BOX DENOTES MAXIMUM ERROR)

t	VOADAM	LL, $\frac{\partial \overline{F}}{\partial t} = 0$	error, $\frac{\partial \overline{F}}{\partial t} = 0$
.005000000	0.000000000	0.000000000	0.000000000
.006000000	.162252170	.147347430	-.014904740
.007000000	.493552465	.480234865	-.013317600
.008000000	.844168912	.837443361	-.006725552
.009000000	1.146302109	1.146273165	-.000028943
.010000000	1.376957376	1.381929791	.004972415
.011000000	1.536057579	1.543985964	.007928385
.012000000	1.633890697	1.642993687	.009102990
.013000000	1.684160618	1.693122399	.008961780
.014000000	1.700371363	1.708349390	.007978028
.015000000	1.694159157	1.700710234	.006551077
.016000000	1.674726022	1.679707480	.004981459
.017000000	1.648861594	1.652335073	.003473479
.018000000	1.621246513	1.623396321	.002149808
.019000000	1.594859358	1.595929238	.001069880
.020000000	1.571388877	1.571636786	.000247909
.021000000	1.551602082	1.551270672	-.000331410
.022000000	1.535648199	1.534947799	-.000700399
.023000000	1.523294906	1.522395924	-.000898982
.024000000	1.514102440	1.513134309	-.000968131

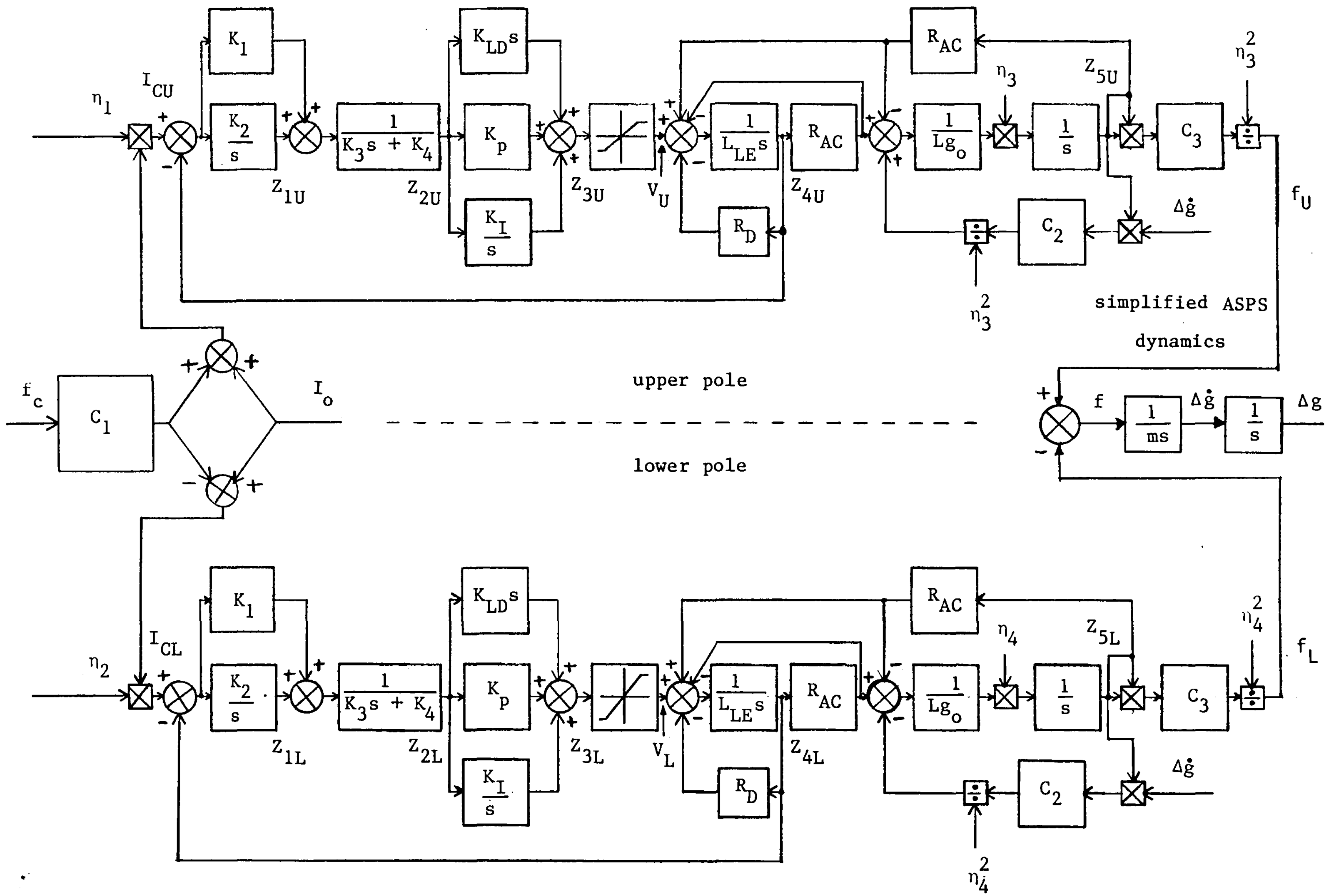

Figure 1.- Magnetic Bearing Assembly Model

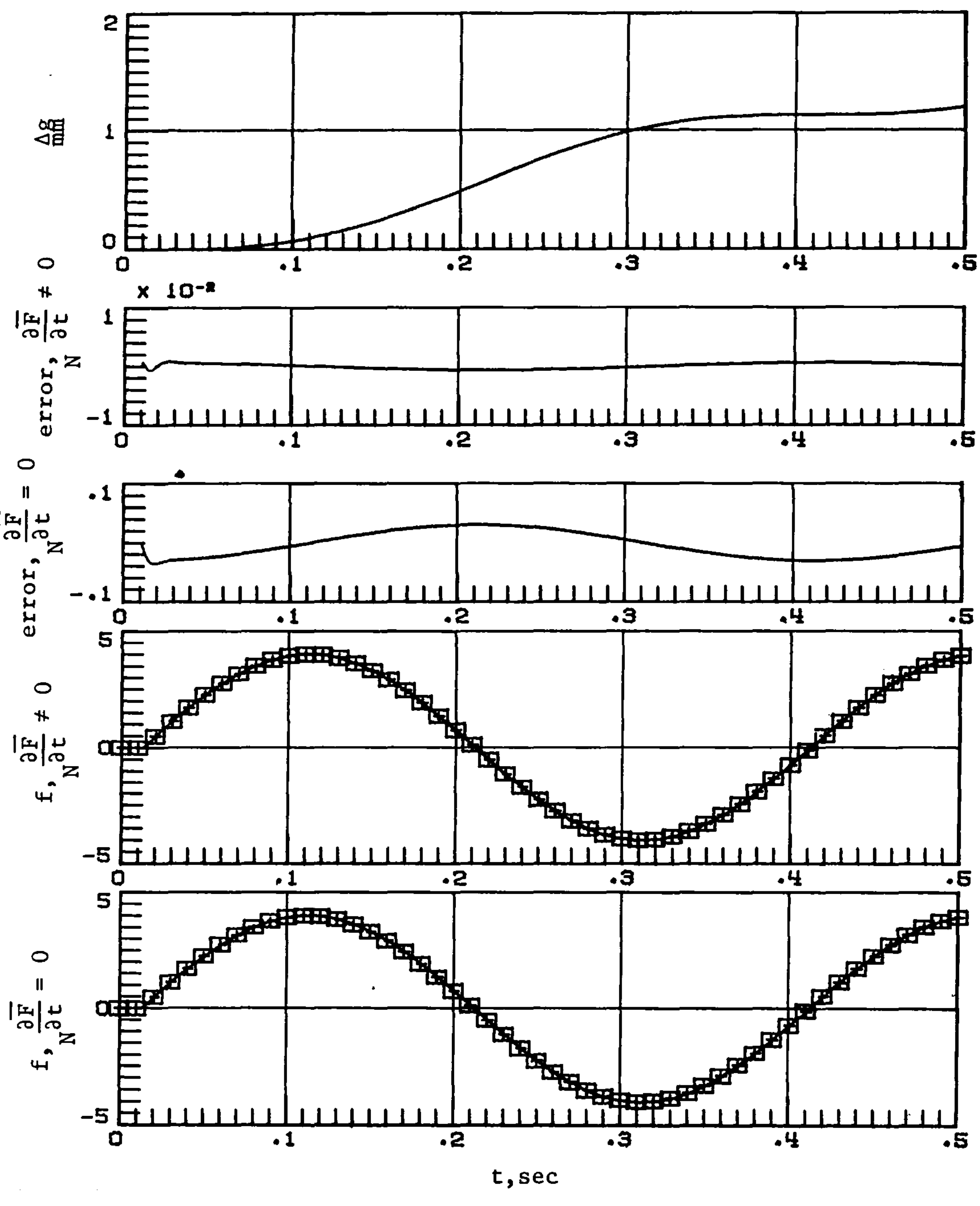

Figure 2.- 2.5 Hz sine wave comparison (+ symbol denotes VOADAM solution)

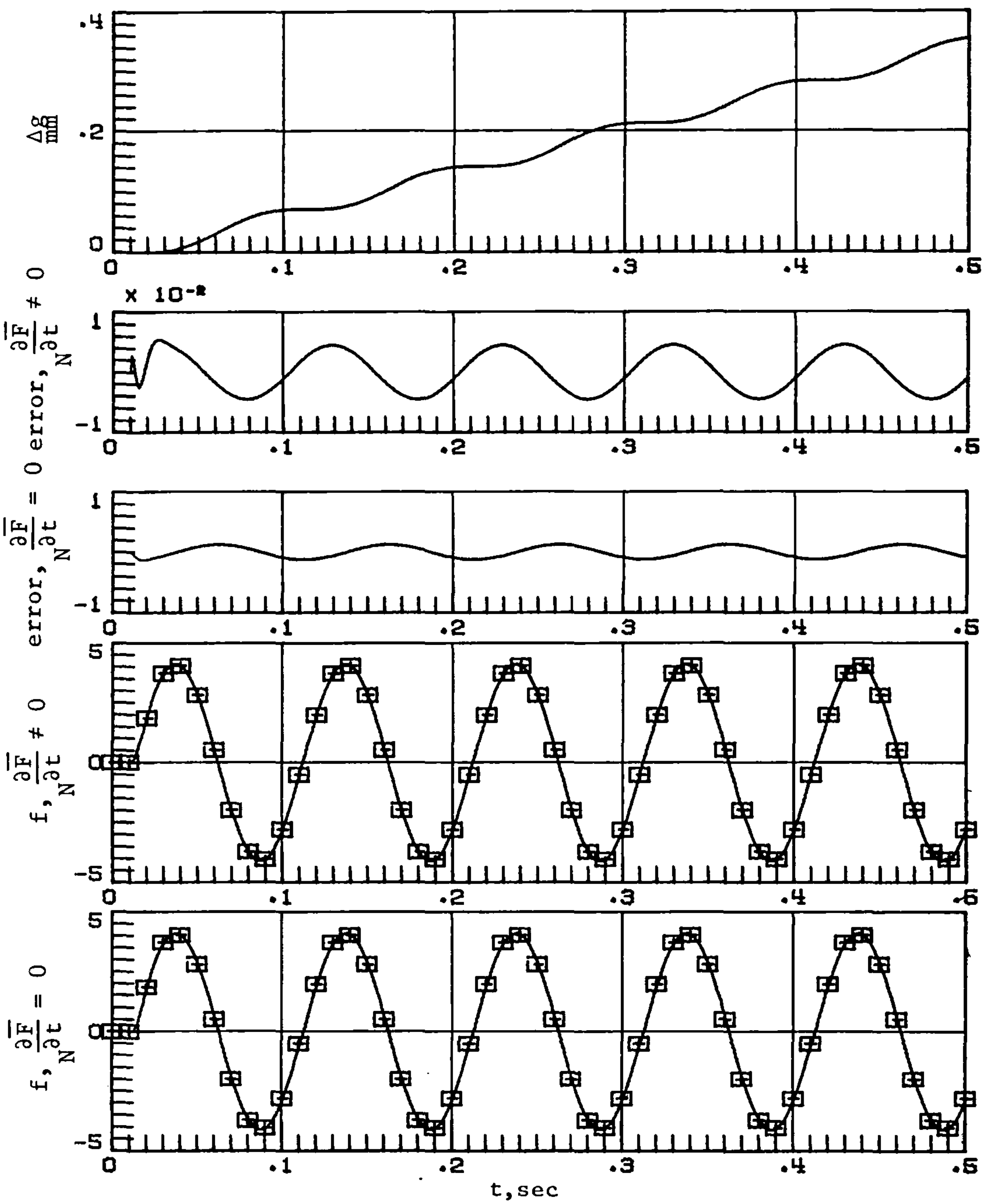

Figure 3.- 10 Hz sine wave comparison (+ symbol denotes VOADAM solution)

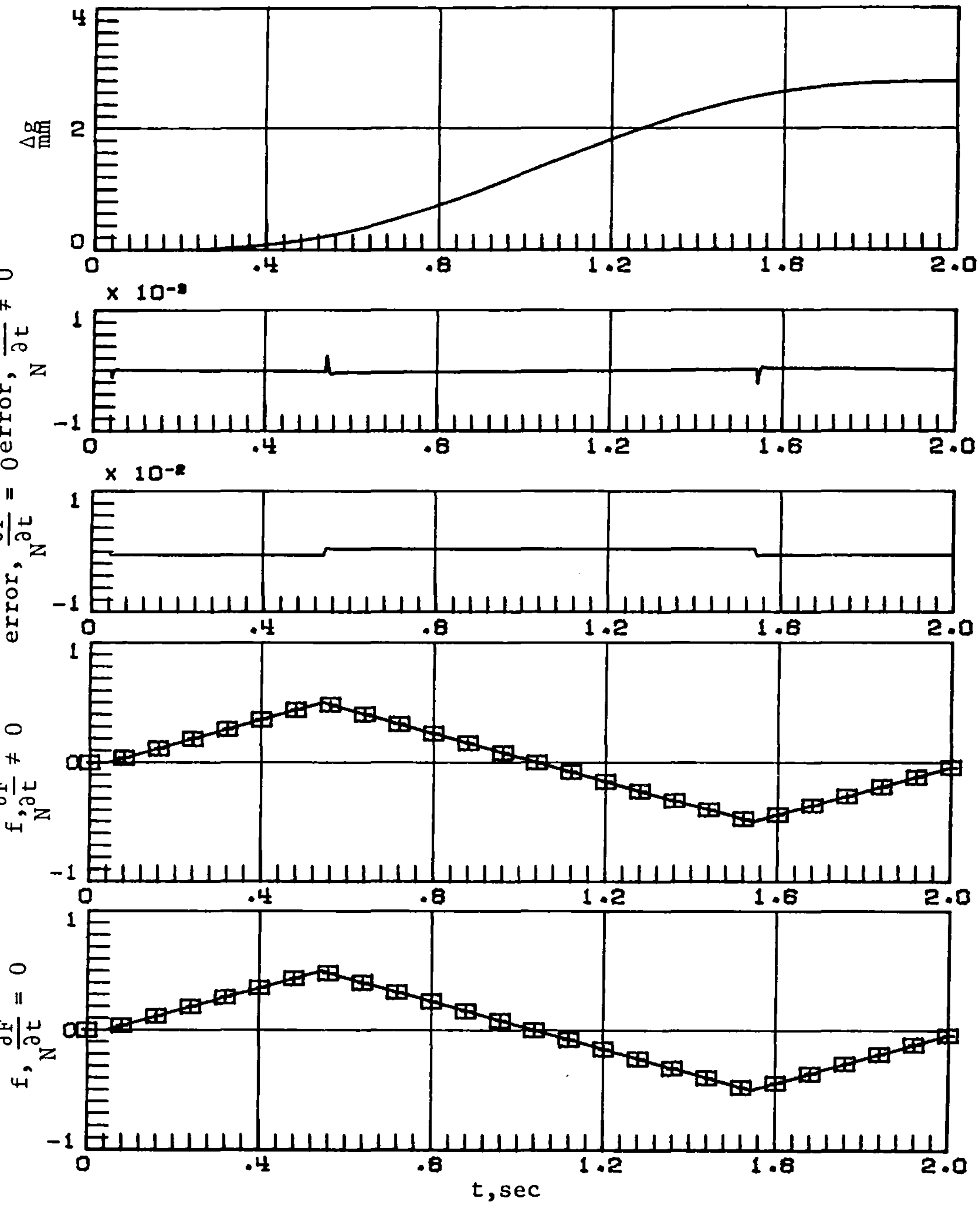

Figure 4.- Triangular wave comparison (+ symbol denotes VOADAM solution)

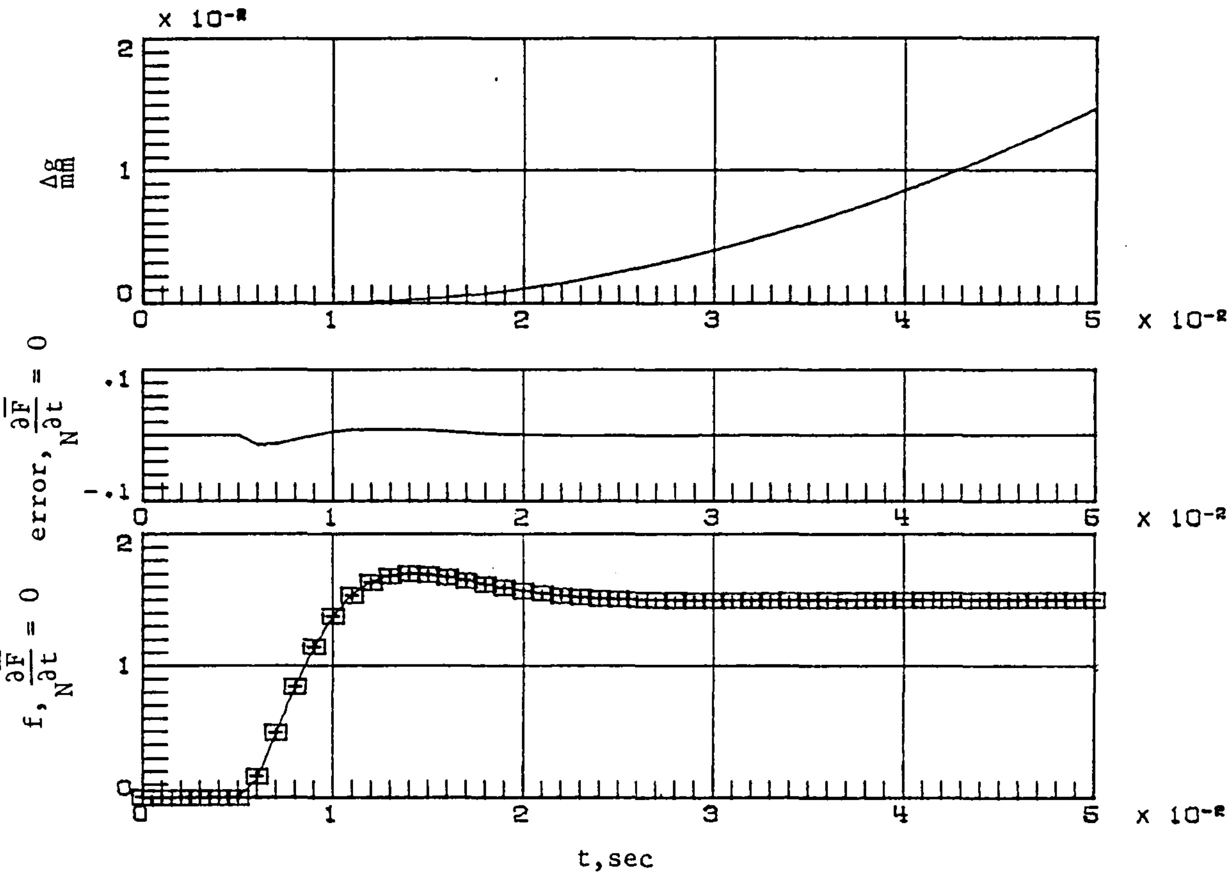

Figure 5.- Step comparison (+ symbol denotes VOADAM solution)

1. Report No. NASA TM-83146	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Application of a Local Linearization Technique for the Solution of a System of Stiff Differential Equations Associated with the Simulation of a Magnetic Bearing Assembly		5. Report Date September 1981
		6. Performing Organization Code 506-61-43-06
7. Author(s) Kemper S. Kibler and Gary A. McDaniel *		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		14. Sponsoring Agency Code

15. Supplementary Notes

* Gary McDaniel, Sperry System Management

16. Abstract

A digital local linearization technique has been successfully used to solve a system of stiff differential equations which simulate a Magnetic Bearing Assembly. The technique has proven to be accurate, stable, and efficient when compared to a general purpose variable order Adams method with a stiff option.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
local linearization technique stiff differential equations Magnetic Bearing Assembly simulation	Unclassified - Unlimited Subject Category 61

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	37	A03

End of Document