A COMPARATIVE STUDY OF THE HARRIS-PRIESTER, JACCHIA-ROBERTS, AND MSIS ATMOSPHERIC DENSITY MODELS IN THE CONTEXT OF SATELLITE ORBIT DETERMINATION*

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ABSTRACT

Extensive comparisons of the Harris-Priester, Jacchia-Roberts, and MSIS (Mass Spectrometer/Incoherent Scatter) atmospheric density models as used in satellite orbit determination are summarized. The quantities compared include Bayesian weighted least squares differential correction statistics and orbit solution consistency and accuracy.

*This work was supported by the Operations Analysis Section, Operational Orbit Support Branch, Goddard Space Flight Center, National Aeronautics and Space Administration, under Contract NAS 5-24300.

SECTION 1 - INTRODUCTION

Atmospheric drag is a significant perturbation of Earth satellite orbits with perigee heights of less than 1000 kilometers. The acceleration of a spherical satellite due to atmospheric drag is given by the equation

$$\vec{A}_{drag} = -\frac{1}{2}\frac{C_D}{m} A \rho |\vec{V}| \vec{V}$$

where ρ = atmospheric density at the position of the satellite

$\vec{V}$ = satellite velocity relative to the atmosphere

A = satellite reference cross-sectional area

C_D = satellite drag coefficient

m = satellite mass

Therefore, calculation of the drag acceleration requires knowledge of the atmospheric density as a function of position and time.

This paper presents the results of a comparative study of three different global atmospheric density models in the context of orbit determination. The three models compared are the Harris-Priester (H-P) model, the Jacchia-Roberts (J-R) model, and the Mass Spectrometer/Incoherent Scatter (MSIS) model.

The Harris-Priester model is based on theoretical temperature profile solutions of the heat conduction equation under hydrostatic equilibrium conditions. The model assumes two heat sources: solar extreme ultraviolet (EUV) heating and an artificial heat source that produces the diurnal variation deduced from satellite drag calculations. In the modified Harris-Priester model used for this study, the EUV heating level is selected by choosing among 10 different altitude-density profile tables representing 10 different levels of solar flux, and the diurnal variation is modeled by a correction calculated using a power of a cosine (References 1 and 2).

The Jacchia-Roberts model is based on empirical temperature profiles scaled by an upper boundary exospheric temperature (T_∞). Analytic density calculation is accomplished through integration of thermodynamic equations. The modeling includes corrections for EUV heating, solar particle flux (so-called geomagnetic) heating, semiannual variations, seasonal variations, and the diurnal variation (References 2 and 3).

The MSIS model is based on fitting spherical surface harmonic expansions to match the angular dependence exhibited by mass spectrometer and incoherent scatter measurements. The MSIS formulation includes sections that model EUV heating, solar particle flux heating, annual variations, semiannual variations, diurnal variations, semidiurnal variations, terdiurnal variations, and departures from diffusive equilibrium. MSIS modeling has been implemented in a special GTDS load module. Dr. Hedin and his associates at the Goddard Space Flight Center, who developed the model (Reference 4), contributed advice and some of their program subroutines during the GTDS implementation.

Table 1 shows sample density profiles for the three atmospheric models with two different solar EUV levels and one geomagnetic activity level. Figure 1 shows the Jacchia-Roberts and MSIS densities, relative to the Harris-Priester density, as a function of altitude. The figure shows maximum ratios as high as 2.0 but, as is apparent from the table, the three profiles are quite similar in overall shape.

TABLE 1. ATMOSPHERIC DENSITIES COMPUTED USING HARRIS-PRIESTER, JACCHIA-ROBERTS, AND MSIS MODELS

ALTITUDE (km)	DENSITY (kg/km^3)					
	HARRIS-PRIESTER		JACCHIA-ROBERTS		MSIS	
	$F_{10.7} = 125.0$	$F_{10.7} = 150.0$	$F_{10.7} = 116.2$, $\bar{F}_{10.7} = 135.1$	$F_{10.7} = 140.0$, $\bar{F}_{10.7} = 165.3$	$F_{10.7} = 116.2$, $\bar{F}_{10.7} = 135.1$	$F_{10.7} = 140.0$, $\bar{F}_{10.7} = 165.3$
150	.205 E + 1	.206 E + 1	.193 E + 1	.210 E + 1	.203 E + 1	.204 E + 1
200	.224 E 0	.255 E 0	.228 E 0	.270 E 0	.274 E 0	.313 E 0
250	.459 E – 1	.583 E – 1	.559 E – 1	.721 E – 1	.636 E – 1	.802 E – 1
300	.129 E – 1	.178 E – 1	.177 E – 1	.249 E – 1	.187 E – 1	.255 E – 1
350	.425 E – 2	.631 E – 2	.637 E – 2	.977 E – 2	.633 E – 2	.926 E – 2
400	.155 E –2	.247 E – 2	.246 E – 2	.413 E – 2	.236 E – 2	.368 E – 2
450	.521 E – 3	.879 E – 3	.835 E – 3	.157 E – 2	.780 E – 3	.131 E – 2
500	.218 E – 3	.392 E – 3	.353 E – 3	.724 E – 3	.324 E – 3	.582 E – 3
550	.963 E – 4	.182 E – 3	.155 E – 3	.344 E – 3	.139 E – 3	.266 E – 3
600	.451 E – 4	.851 E – 4	.706 E – 4	.169 E – 3	.619 E – 4	.125 E – 3
650	.227 E – 4	.451 E – 4	.339 E – 4	.851 E – 4	.285 E – 4	.600 E – 4
700	.112 E – 4	.217 E – 4	.154 E – 4	.394 E – 4	.120 E – 4	.259 E – 4
750	.691 E – 5	.127 E – 4	.878 E – 5	.219 E – 4	.623 E – 5	.134 E – 4
800	.464 E – 5	.804 E – 5	.548 E – 5	.128 E – 4	.352 E – 5	.728 E – 5
850	.316 E – 5	.462 E – 5	.348 E – 5	.737 E – 5	.200 E – 5	.378 E – 5
900	.245 E – 5	.301 E – 5	.258 E – 5	.500 E – 5	.137 E – 5	.236 E – 5
950	.198 E – 5	.201 E – 5	.201 E – 5	.361 E – 5	.102 E – 5	.158 E – 5
1000	.163 E – 5	.141 E – 5	.155 E – 5	.262 E – 5	.761 E – 6	.107 E – 5

8226/81

NOTES: 1. Kp = 3.3 FOR JACCHIA-ROBERTS DENSITY AND Ap = 33 FOR MSIS DENSITY ARE USED.

2. THESE PROFILES ARE FOR AUGUST 30, 1978, AT A LATITUDE OF 46° N, AN EAST LONGITUDE OF 205°, AND A LOCAL SOLAR TIME OF 1:40 P.M.

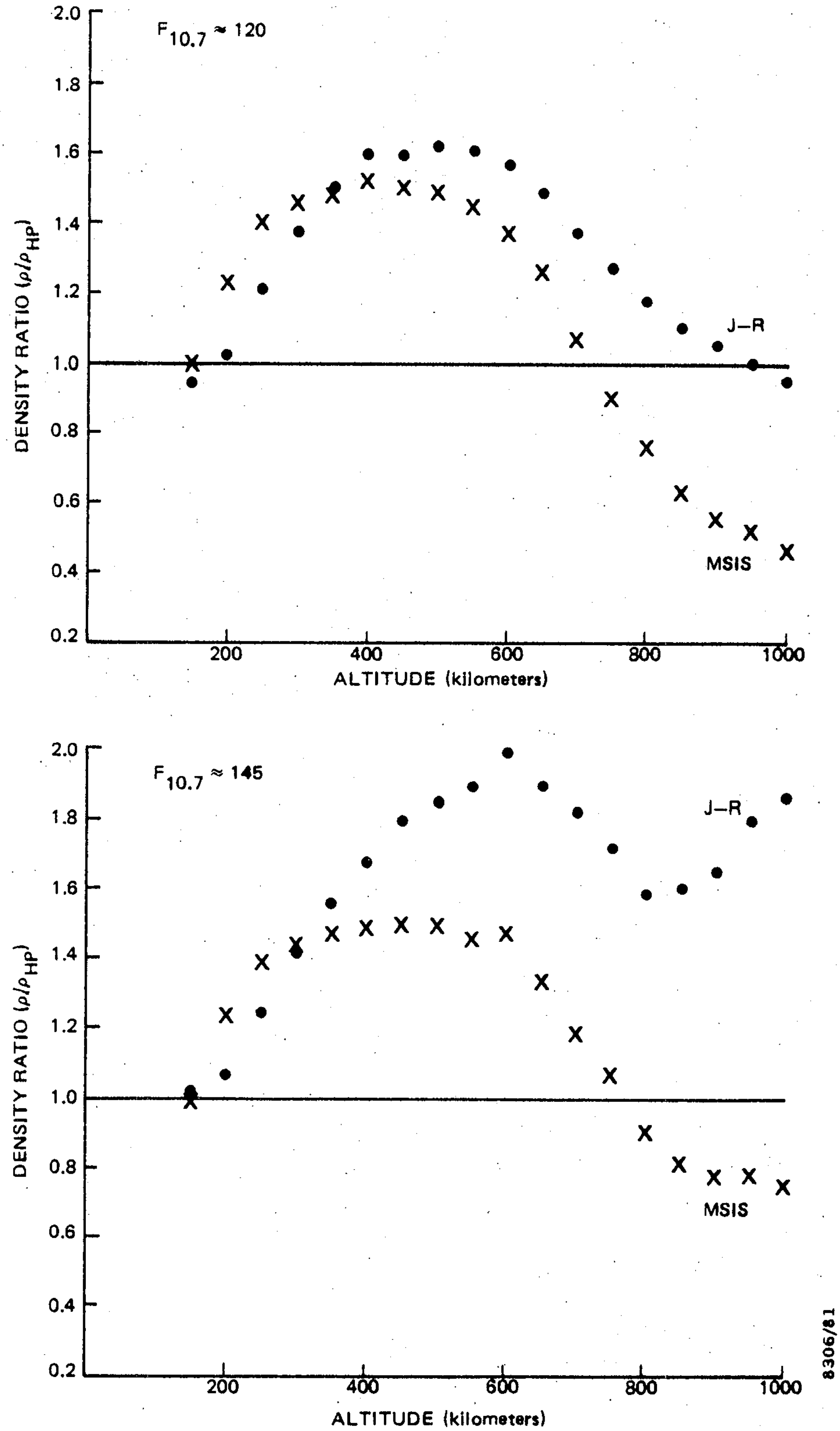

FIGURE 1. ATMOSPHERIC MODEL DENSITY RATIOS

SECTION 2 - COMPARATIVE STUDY STRUCTURE

All the results presented in Section 3 of this paper are based on Goddard Trajectory Determination System (GTDS) Bayesian weighted least squares differential correction solutions. Nine different series of six GTDS Differential Correction (DC) Program runs were made for each of the three atmospheric models. Three different satellites, with perigee heights between 310 and 560 kilometers, were studied; other orbital parameters for these satellites are given in Table 2. The nine series of orbit determination arcs are listed in Table 3.

Each series contains six 30-hour-arc solutions. The solutions are used to generate 30-hour ephemerides that overlap adjacent ephemerides by 6 hours. The ephemerides are then compared in order to determine the maximum position differences (in the orbital reference frame) during the overlap periods. The 162 DC Program solutions produce 135 maximum overlap position differences. These differences are used to evaluate the consistency and accuracy obtained when each of the three atmospheric models is used.

Each differential correction solution is made up of seven numbers: three position coordinates, three velocity coordinates, and the drag variation parameter (ρ_1), which is a scaling factor in the drag acceleration equation, i.e.,

$$\vec{A}_{drag} = -\frac{1}{2}\frac{C_D}{m} A\, \rho\left(1 + \rho_1\right) |\vec{V}|\, \vec{V}$$

This scaling factor is applied during generation of the ephemeris that uses the differential correction solution.

TABLE 2. SATELLITE ORBITAL ELEMENTS

SATELLITE	DATE	PERIGEE HEIGHT (kilometers)	APOGEE HEIGHT (kilometers)	INCLINATION (degrees)
AE-3	AUGUST 1, 1978	331	341	68
MAGSAT	OCTOBER 31, 1979	352	561	97
	MARCH 1, 1980	323	471	97
SAGE	FEBRUARY 19, 1979	560	655	55

8347/81

TABLE 3. COMPARATIVE STUDY SERIES

SERIES NUMBER	SATELLITE	TIMESPAN
1	AE-3	AUGUST 1–6, 1978
2	AE-3	AUGUST 14–19, 1978
3	AE-3	SEPTEMBER 2–8, 1978
4	MAGSAT	OCTOBER 31–NOVEMBER 5, 1979
5	MAGSAT	DECEMBER 1–6, 1979
6	MAGSAT	JANUARY 1–6, 1980
7	MAGSAT	FEBRUARY 1–6, 1980
8	MAGSAT	MARCH 1–6, 1980
9	SAGE	FEBRUARY 19–25, 1979

8347/81

Spacecraft attitude is not considered, since a spherical model is employed. Furthermore, no aerodynamic forces (e.g., lift) other than drag are modeled. The spherical approximation is crude for all three satellites, and it is possible that other aerodynamic forces are nonnegligible. However, it is reasonable to expect that both assumptions have a negligible effect on the results of this study, because the results are obtained by applying each of the three atmospheric models to the same arcs with the same observation sets. Simply stated, unmodeled aerodynamic forces should perturb the solutions for all three atmospheric models in a similar manner.

SECTION 3 - COMPARATIVE STUDY RESULTS

This section summarizes the results of this comparative study of atmospheric density models in the context of short-arc (30-hour) orbit determination. A detailed, run-by-run presentation of these results is available in Reference 5. Two cautionary remarks are appropriate.

First, these results should not be interpreted as a comparison of atmospheric models; conclusions about the relative merits of the models must be limited to this highly specialized context--short-arc orbit determination in which an average drag scaling factor is solved for.

Second, any series of orbit determination and ephemeris comparison runs may contain a few sporadic large overlap differences and a few differential corrections with large RMS residuals. Some of the runs included in this study show such large differences and/or high RMSs.

The average weighted RMSs and the average maximum position differences for the three AE-3 series are given in Table 4. The averages over all three series are also given, along with the ranges of the EUV heating index ($F_{10.7}$) and the solar particle flux index (K_p). The averages show that the Jacchia-Roberts overlap differences are about 11.5 percent (24 meters) smaller than the Harris-Priester averages and that the MSIS averages are about 19 percent (38 meters) larger than the Harris-Priester averages. The 62-meter difference between the Jacchia-Roberts and MSIS averages cannot be considered either large or significant.

The same information is given for Magsat in Table 5. This study includes five series of arcs. The Magsat results show that both the Jacchia-Roberts and MSIS average differences are about 9 percent larger than the Harris-Priester average

TABLE 4. COMPARATIVE ATMOSPHERIC DENSITY MODEL STUDY RESULTS FOR AE-3 (AUGUST AND SEPTEMBER 1978)

SERIES	RANGE OF $F_{10.7}$ $\left(10^{-22}\ \text{watt}/(\text{m}^2\text{-Hz})\right)$	RANGE OF K_p	HARRIS-PRIESTER MODEL		JACCHIA-ROBERTS MODEL		MSIS MODEL	
			AVERAGE WEIGHTED RMS	MAXIMUM POSITION DIFFERENCE (meters)	AVERAGE WEIGHTED RMS	MAXIMUM POSITION DIFFERENCE (meters)	AVERAGE WEIGHTED RMS	MAXIMUM POSITION DIFFERENCE (meters)
AUGUST 1-6	106.0-117.6	0-6	4.9	191	5.2	175	8.4	265
AUGUST 14-19	115.6-134.9	0-6	7.3	225	7.8	217	8.5	324
SEPTEMBER 2-8	159.8-181.1	0-6	7.3	209	8.4	163	7.2	164
AVERAGES	–	–	6.5	208	7.2	184	8.0	251

8226/81

TABLE 5. COMPARATIVE ATMOSPHERIC DENSITY MODEL STUDY RESULTS FOR MAGSAT (NOVEMBER AND DECEMBER 1979; JANUARY, FEBRUARY, AND MARCH 1980)

PERIOD	RANGE OF $F_{10.7}$ VARIATION (10^{-22} watts m^{-2} Hz^{-1})	RANGE OF Kp VARIATION	HARRIS - PRIESTER RESULTS		JACCHIA - ROBERTS RESULTS		MSIS RESULTS	
			WEIGHTED RMS	MAXIMUM POSITION DIFFERENCE (meters)	WEIGHTED RMS	MAXIMUM POSITION DIFFERENCE (meters)	WEIGHTED RMS	MAXIMUM POSITION DIFFERENCE (meters)
OCT. 31 NOV. 5, 1979	207.5–214.9	0–4	8.3	204	7.8	176	8.0	190
DEC. 1–6, 1979	152.2–223.4	0–4	12.4	204	11.5	175	12.8	255
JAN. 1 6, 1980	188.9–212.4	1–5	9.4	213	9.5	166	11.3	288
FEB. 1- 6, 1980	212.6–231.7	0–4	12.7	326	12.5	298	13.8	313
MAR. 1 6, 1980	170.2–176.7	0–3	9.8	161	13.4	396	10.0	169
AVERAGES	--	--	10.6	222	10.9	242	11.2	243

8226/81

differences. As in the case of AE-3, the Magsat results demonstrate that the three atmospheric density models are comparable in the context of this study.

The average RMSs and overlap position differences for the series of SAGE arcs are given in Table 6. Both the RMSs and the overlap differences agree to within 3 percent; all three atmospheric models produce essentially equivalent errors.

TABLE 6. COMPARATIVE ATMOSPHERIC DENSITY MODEL STUDY RESULTS FOR SAGE (FEBRUARY 19–25, 1979)

ATMOSPHERIC DENSITY MODEL USED	AVERAGE WEIGHTED RMS	AVERAGE MAXIMUM POSITION DIFFERENCE (meters)
HARRIS–PRIESTER	10.9	108
JACCHIA–ROBERTS	11.2	114
MSIS	11.0	112

8226/81

NOTE: DURING THIS PERIOD, $F_{10.7}$ VARIED FROM 196.0 TO 237.7 X 10^{-22} WATTS METER^{-2} HERTZ^{-1}, AND K_p VARIED FROM 1 TO 7.

SECTION 4 - CONCLUSION

The results presented in this paper support the conclusion that, for satellites above 300 kilometers, the Harris-Priester, Jacchia-Roberts, and MSIS atmospheric density models all produce roughly similar density profiles and essentially comparable orbit determination results when the drag variation parameter is solved for and orbit quality is measured by adjacent arc overlap comparisons. It is impossible to predict which of the three models will produce the best fit or best predictions for any given orbit determination arc. However, for some problem arcs, switching atmospheric models may result in marked solution improvements.

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