Spin-Axis Attitude Estimation and Magnetometer Bias Determination for the AMPTE Mission*

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Abstract

Algorithms are developed for the determination of magnetometer biases and spin-axis attitude for the AMPTE mission. Numerical examples of the performance of the algorithm are given.

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I - Introduction

This paper describes methods for determining spin-axis attitude (i.e., the direction in space of the spacecraft spin axis) and magnetometer biases which are being investigated for ground support of the Active Magnetospheric Particle Tracer Explorer (AMPTE) mission.

The AMPTE mission will consist of two spacecraft.[1] The first is the Ion Release Module (IRM), provided by the Federal Republic of Germany, which will be placed in a highly elliptical orbit with apogee at approximately 19 Earth radii in order to release lithium tracer ions outside the magnetosphere. This spacecraft will be spin stabilized at a rate of 30 rpm. The second spacecraft is the Charge Composition Explorer (CCE), which will detect the tracer ions inside the magnetosphere at altitudes of from 300 km to 7.5 Earth radii. The CCE will be spin stabilized at 10 rpm.

Estimation of spin-axis attitude for both AMPTE spacecraft will be based on the measurements of the geomagnetic field and the projection of the Sun line on the spacecraft spin-axis, which we take nominally to be the symmetry axis $\hat{\underset{\sim}{Y}}_A$ of the spacecraft bus.

For the purpose of this study, the attitude sensors are assumed to consist of a three-axis magnetometer and a Sun sensor which measures the angle between the Sun line and $\hat{\underset{\sim}{Y}}_A$. For simplicity it is assumed likewise that one axis of the magnetometer is along $\hat{\underset{\sim}{Y}}_A$. The other two axes of the magnetometer define $\hat{\underset{\sim}{X}}_A$ and $\hat{\underset{\sim}{Z}}_A$.

The measured quantities are taken to be

$\underset{\sim}{M}$ = magnetic field vector in body coordinates

$\cos\beta$ = $\hat{\underset{\sim}{S}}\cdot\hat{\underset{\sim}{Y}}_A$, where $\hat{\underset{\sim}{S}}$ is the unit vector directed from the spacecraft to the Sun (β is the "Sun angle").

Attitude determination activities fall into two areas:

- Determination of spin-axis attitude
- Determination of the magnetometer biases

Because the orbit-apogee distance for these two spacecraft is so great, accurate geomagnetic field data for attitude estimation is available only for the segment of the orbit near perigee. This is due to the poor accuracy of the magnetic-field model at such high altitudes resulting from both the small magnitude of the geomagnetic field as well as from fluctuations in the field caused by extraterrestrial phenomena. However, because of the large spacecraft angular momenta, it can be assumed for both spacecraft that the spin-axis attitude at apogee will not differ markedly from that at perigee of the same orbit.

Algorithms for spin-axis attitude and magnetometer bias determination are now being investigated. These are:

- attitude-independent estimation of three-axis magnetometer biases and

- estimation of spin-axis attitude from measurements of the Sun and geomagnetic field angle.

Each of these algorithms are batch estimators utilizing a long segment of magnetometer and Sun data. The algorithms are developed in succeeding sections and then tested using simulated AMPTE data.

II - Magnetometer Bias Determination

The attitude of the spacecraft is usually not known before the magnetometer biases must be determined. Here an algorithm is developed which determines the magnetometer bias vector by minimizing a loss function which is independent of the attitude.

The quantities used throughout this section are defined as follows:

$H_j(i)$ = jth component of the model magnetic field in the geocentric inertial (GCI) system at time i

$M_j(i)$ = jth magnetometer reading at time i

B_j = jth component of the magnetometer bias vector, which is taken to be independent of the spacecraft position

For the ith point, an error $\delta(i)$ is defined by the following equation:

$$\delta(i) = |\underset{\sim}{H}(i)|^2 - |\underset{\sim}{M}(i) - \underset{\sim}{B}|^2 \tag{1}$$

The objective of this equation is to minimize the quantity $\delta(i)$ by adjusting the bias vector $\underset{\sim}{B}$ to its optimal value. Thus, the loss function to be minimized is given by

$$L(\underset{\sim}{B}) = \sum_{i=1}^{N} \omega(i)|\delta(i)|^2 \tag{2}$$

where $\omega(i)$ is the weight associated with the ith data point. The weights are assumed to be normalized to unity, that is,

$$\sum_{i=1}^{N} \omega(i) = 1 \tag{3}$$

Determining the minimum value of $L(\underset{\sim}{B})$ first requires that its derivatives with respect to the components of the bias vector be set equal to zero:

$$\frac{\partial L}{\partial B_m} = 0 \qquad m=1,2,3 \tag{4}$$

where

$$\frac{\partial L}{\partial B_m} = -4 \sum_{i=1}^{N} \omega(i) \left[|\underset{\sim}{H}(i)|^2 - |\underset{\sim}{B} - \underset{\sim}{M}(i)|^2 \right] (B_m - M_m(i)) \tag{5}$$

Combining Eqs. (3-5) leads to the following results:

$$\sum_{k=1}^{3} G_{mk} B_k = b_m + F_m(\underset{\sim}{B}) \tag{6a}$$

or in matrix form,

$$G \underset{\sim}{B} = \underset{\sim}{b} + \underset{\sim}{F}(\underset{\sim}{B}) \tag{6b}$$

where

$$G_{mk} = \delta_{mk}(\langle|\underset{\sim}{H}|^2\rangle - \langle|\underset{\sim}{M}^2|\rangle) - 2 \langle M_m M_k\rangle \tag{7a}$$

$$b_m = \langle(|\underset{\sim}{H}|^2 - |\underset{\sim}{M}|^2)M_m\rangle \tag{7b}$$

$$F_m(\underset{\sim}{B}) = |\underset{\sim}{B}|^2 \langle B_m - M_m\rangle - 2 \underset{\sim}{B}\cdot\langle\underset{\sim}{M}\rangle B_m \tag{7c}$$

The bracket denotes the weighted average

$$\langle A\rangle = \sum_{i=1}^{N} \omega(i)A(i) \tag{8}$$

δ_{mk} is the Kronecker delta defined as unity when m=k and zero otherwise.

Eq. (6) can be solved directly to obtain the best value for the bias vector $\underset{\sim}{B}$.

General Description of the Iterative Solution

Eq. (6) is nonlinear in $\underset{\sim}{B}$ and must be solved iteratively. The zero-th order (trial) solution to Eq. (6), is obtained by dropping the nonlinear terms in comparison to the linear terms. This approximation is valid only when the bias is small in comparison with the actual magnetic field. This point is not critical, as the iteration scheme constructs an accurate solution even when the trial solution is not close to the true solution. This will be discussed in more detail in the treatment of the numerical example.

The trial solution is given by

$$\underset{\sim}{B}^{(0)} = G^{-1}\underset{\sim}{b} \tag{9}$$

where G^{-1} = inverse of the matrix G

$\underset{\sim}{B}^{(0)}$ = trial solution

This solution may be iterated as

$$\underset{\sim}{B}^{(j)} = \underset{\sim}{B}^{(0)} + G^{-1}\underset{\sim}{F}(\underset{\sim}{B}^{(j-1)}) \qquad j \geq 1 \tag{10}$$

The iteration continues until

$$\left| \frac{B_k^{(j)} - B_k^{(j-1)}}{B_k^{(j)}} \right| < \varepsilon \tag{11}$$

where ε = some arbitrarily small value depending on the accuracy desired.

Numerical Examples

The AMPTE engineering data simulator[2] was used to generate biased magnetometer data for the purpose of investigating the convergence properties of the iterative solution. Two cases were considered:

$$B/H \ll 1$$

and

$$B/H \gg 1$$

The first case considered was $B/H \ll 1$; in this case, 200 data points were used in the calculation. Data at the perigee point, at which the magnetic field attains its maximum value, was included. The magnetic field can be resolved into a component along the AMPTE spin axis, $H_{\parallel}$, and a component perpendicular to the spin axis, $H_{\perp}$. The maximum or perigee values for these components are $H_{\perp}^{MAX}$ = 240 milligauss (mG) and $H_{\parallel}^{MAX}$ = 90 mG. The input biases were chosen to be 5 mG, 10 mG, and 15 mG along the x, y, and z axes, respectively. The results of the bias determination calculation are shown in Table 1 taken from Reference 3. Rapid convergence and very high accuracy is obtained. The trial solution $B^{(0)}$ (iteration 0) initially was not accurate in the y component and needed to be iterated to obtain satisfactory results. Investigation of the case in which $B \gg H$ used a subset of the data used in the first test. Here, 100 data points well outside the perigee region were used. For this test, $H_{\perp}^{MAX}$ = 5 mG and $H_{\parallel}^{MAX}$ = 2 mG. As before, the input biases are 5 mG, 10 mG, and 15 mG. These results[3] are presented in Table 2. In this case, convergence is very slow and incomplete. Improved convergence cannot necessarily be obtained by using standard Newton-Raphson techniques.

ITERATION NUMBER	LOSS FUNCTION	B_x (mG)	B_y (mG)	B_z (mG)
0	54621.0	5.00288	12.0278	15.0213
1	5153.0	4.98344	9.38109	14.9473
2	370.0	5.00481	10.1647	15.0152
3	29.0	4.99870	9.95352	14.9959
4	2.0	5.00037	10.0128	15.0012
5	0.2	4.99990	9.99635	14.9997
6	0.01	5.00003	10.0009	15.0001

Table 1

Bias Determination Calculation for B/H $\ll$ 1

ITERATION NUMBER	LOSS FUNCTION	B_x (mG)	B_y (mG)	B_z (mG)
0	24100.0	1.8	2.8	5.3
10	1460.0	3.7	5.5	11.0
20	501.0	4.1	6.1	12.4
30	240.0	4.4	6.3	13.1
40	133.0	4.5	6.5	13.6
50	81.0	4.6	6.6	13.9

Table 2

Bias Determination Calculation for B/H $\gg$ 1

III - Spin-Axis Attitude Determination

Once the magnetometer biases have been chosen properly, data from the Sun sensor and the magnetometers may be used to determine the spin-axis attitude. It is assumed that the spin axis is not varying over the data interval examined.

The spin axis is denoted by $\hat{\underset{\sim}{a}}$. The data are

$\beta(i)$ = measured Sun angle at time i $\qquad i=1,\ldots,N_S$

$\underset{\sim}{M}(i)$ = measured magnetic field at time i, $\qquad i=1,\ldots,N_M$

$\hat{\underset{\sim}{S}}(i)$ = (true) Sun vector in GCI at time i, measured from the spacecraft to the sun $\qquad i=1,\ldots,N_S$

$\underset{\sim}{H}(i)$ = (true) geomagnetic field at time i, $\qquad i=1,\ldots,N_M$

Note that there will be no requirement of simultaneous Sun-sensor and magnetometer data.

The spin-axis (attitude) vector, $\hat{\underset{\sim}{a}}$, is subject to the following constraint:

$$\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{a}} = 1 \tag{12}$$

The spin-axis vector is chosen to minimize the following loss function:

$$L(\hat{\underset{\sim}{a}}) = \frac{1}{2}\sum_{i=1}^{N_S} \omega_S(i)\left|\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{S}}(i) - \cos\beta(i)\right|^2 + \frac{1}{2}\sum_{i=1}^{N_M} \omega_M(i)\left|\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{M}}(i) - \cos\eta(i)\right|^2 - \frac{1}{2}\lambda\,\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{a}} \tag{13}$$

where

λ = Lagrange multiplier chosen to satisfy the constraint equation

$\omega_S(i)$ = weight assigned to the jth magnetic field measurement

$\omega_M(j)$ = weight assigned to the jth magnetic field measurement

The quantity η is the angle between the geomagnetic field and the spacecraft spin axis given by

$$\eta = \cos^{-1}(M_y/|\underset{\sim}{M}|) \tag{14}$$

The weights are normalized to unity

$$\sum_{i=1}^{N_S} \omega_S(i) + \sum_{i=1}^{N_M} \omega_M(i) = 1 \tag{15}$$

The spin-axis vector $\hat{\underset{\sim}{a}}$ is chosen to minimize the loss function

$$\frac{\partial L(\hat{\underset{\sim}{a}})}{\partial a_m} = 0 \tag{16}$$

The derivative of the loss function is given by

$$\begin{aligned}\frac{\partial L}{\partial a_m} &= \sum_{i=1}^{N_S} \omega_S(i)\ (\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{S}}(i) - \cos\beta(i))\ S_m(i) \\ &+ \sum_{i=1}^{N_M} \omega_M(i)\ (\hat{\underset{\sim}{a}}\cdot\hat{\underset{\sim}{M}}(i) - \cos\eta(i))\ M_m(i) - \lambda a_m.\end{aligned} \tag{17}$$

The solution to Eq. (16) may now be written as:

$$\sum_{k=1}^{3} (A_{mk} - \lambda\delta_{mk})a_k = b_m \tag{18}$$

where

$$A_{mk} = \langle S_m S_k \rangle_S + \langle M_m M_k \rangle_M \tag{19a}$$

$$b_m = \langle \cos\beta\ S_m \rangle_S + \langle \cos\eta\ M_m \rangle_M \tag{19b}$$

and the brackets denote weighted averages over the magnetometer and Sun data. That is,

$$\langle C_j \rangle_S \equiv \sum_{i=1}^{N_S} \omega_S(i)\ C_j(i) \tag{20}$$

Eq. (18) may be written in matrix notation as

$$(A - \lambda I)\ \underset{\sim}{a} = \underset{\sim}{b} \tag{21}$$

where I is the unit matrix.

Attitude Solution

A general solution to Eqs. (18) and (19) is constructed in this section. The solution to these equations leads to the spin axis attitude in the Geocentric Inertial (GCI) coordinate system. Again an iterative procedure is developed to construct a numerical solution to the equations. An approximate solution to the problem is to take $\lambda = 0$, i.e., to relax the constraint that a be normalized to unity. Given this approximation, Eq. (18) may be solved to obtain

$$\underset{\sim}{a}^{(0)} = A^{-1}\,\underset{\sim}{b} \tag{22}$$

Note that this vector is not normalized. In practice this solution will be very close to having unit norm since even with $\lambda = 0$, $\hat{\underset{\sim}{a}}$ is overdetermined in general by Eq. (18). Thus, normalizing $\underset{\sim}{a}^{(0)}$ will lead to a very good approximation for $\hat{\underset{\sim}{a}}$ (see Ref. 4). An exact numerical solution is generated by solving for λ iteratively starting with a trial solution $\lambda = 0$ and $\underset{\sim}{a}^{(0)}$ given by Eq. (22).

Define the function $f(\lambda)$ by

$$f(\lambda) = \underset{\sim}{a}(\lambda)\cdot\underset{\sim}{a}(\lambda) - 1 \tag{23}$$

Given the numerical value of $\underset{\sim}{a}(\lambda)$, the Newton-Raphson method is used to determine λ. Differentiating Eq. (23) gives

$$\frac{\partial f}{\partial \lambda} = 2\underset{\sim}{a} \cdot \frac{\partial \underset{\sim}{a}}{\partial \lambda} \tag{24a}$$

and

$$\frac{\partial \underset{\sim}{a}}{\partial \lambda} = (A - \lambda I)^{-1}\,\underset{\sim}{a} \tag{24b}$$

The Newton-Raphson scheme gives

$$\lambda^{(j)} = \lambda^{(j-1)} - \frac{f(\lambda^{(j-1)})}{\frac{\partial f}{\partial \lambda}(\lambda^{(j-1)})} \tag{25a}$$

$$\underset{\sim}{a}^{(j)} = (A - \lambda^{(j)} I)^{-1}\,\underset{\sim}{b} \tag{25b}$$

Numerical Example

The spacecraft orbit in this example is of the AMPTE type, and the Sun and magnetometer data used covered the perigee point. The data is perfect (uncorrupted by random error) as generated by the AMPTE simulator. The "true" value of the right ascension, α, and declination, δ, were chosen to be

$$\alpha = 159.67 \text{ deg} \tag{26a}$$

$$\delta = 0.0 \text{ deg} \tag{26b}$$

The zero-order result as given by Eq. (22) was

$$\alpha = 159.55 \text{ deg} \tag{27a}$$

$$\delta = 0.073 \text{ deg} \tag{27b}$$

in very good agreement. After ten iterations, the values changed only slightly, as expected, namely

$$\alpha = 159.76 \text{ deg} \tag{28a}$$

$$\delta = 0.062 \text{ deg} \tag{28b}$$

IV - Conclusions

Efficient and reliable algorithms have been developed for spin-axis attitude and magnetometer bias determination for the AMPTE spacecraft. Using simulated numerical data it was demonstrated that the methods work well for AMPTE mission parameters. The present work does not address problems associated with noise, data rate, sensor misalignments and etc. These problems were investigated in references (3) and (5).

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