

## APPENDIX

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TOTAL OZONE TREND SIGNIFICANCE  
FROM SPACE AND TIME  
VARIABILITY OF DAILY DOBSON DATA

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## Abstract

Assessing the significance of apparent total ozone trends is equivalent to assessing the standard error of the means. Standard errors of time (area) averages depend on the temporal (spatial) variability and correlation of the averaged parameter. Trend detectability is discussed, both for the present network and for satellite measurements, using statistics from daily observations at Dobson stations from 40° and 60°N.

## 1. Introduction

For several years much interest has been attached to detection of possibly anthropogenic trends in total ozone, either at single stations or station groups. Significance of trends or, equivalently, the standard errors of point- or area-means, is properly derived from knowledge of variances and of data independence, i.e., knowledge of temporal and spatial autocorrelations (e.g., Lieth, 1973; Jones, 1975). In general, authors who report ozone trends (e.g., Angell and Korshover, 1973, 1976; Komhyr et al., 1971, 1973; London and Kelley, 1974; Hill et al., 1975, 1977) use only monthly mean data, and are not explicit about how they assessed the standard error of the monthly averages, or, where used, of the area averages. The purpose of this note is to present estimates of standard errors of total ozone time and area means, as derived from ozone's natural temporal and spatial variability and autocorrelation in middle latitudes determined from daily Dobson data. The use of this information in assessing detectability of total ozone changes, at single stations and over areas, will be demonstrated.

## 2. Method

a. Data. Daily total ozone data for each of 26 Dobson stations between 40° and 60°N for the period 1957-1972 were obtained from the World Data Center for Ozone, Toronto, and were checked for gross errors before processing. For our purposes, a trend is defined as a change, of time scale at least one year, which is not explained by deterministic variations. Trends must be detected against a backdrop of non-deterministic variability, and a primary task is to describe this variability. In order to do this, the mean, a trend over the entire period-of-record, an average 29-month quasi-biennial oscillation, and the first three harmonics of the annual variation were subtracted from the data (see Wilcox et al., 1977) and the study proceeded using the residuals. These residuals primarily contain a somewhat persistent ("reddish") synoptic scale variability, but with assumed "white" contributions from smaller scale processes and from observational error. Any unremoved deterministic periodicity will increase the correlations, but this effect is thought to be quite small. The residuals are undoubtedly more seriously affected by slow calibration drifts and by changes in wavelengths used, and there needs to be a comprehensive, continuous program to check calibrations and observation techniques, as well as to recompute published values as necessary. The subtracting of a trend from the data can help remove slow calibration drifts, and it is hoped that any remaining non-random observation error is relatively small.

b. Standard error of time averages. For an atmospheric variable whose autocorrelation,  $R$ , is approximated by a "red-noise" model  $\hat{R} = \exp(-b\tau)$ , where  $\tau$  is lag, Lieth (1973) has shown that

$$\frac{\sigma_T^2}{\sigma^2} = \frac{2}{bT} \{1 - \frac{1}{bT} [1 - \exp(-bT)]\} \quad (A1)$$

Here,  $\sigma_T$  is the standard error of the mean,  $\sigma$  the standard deviation of the unaveraged time series, and  $T$  the averaging interval.

To find this ratio for total ozone, correlations at lags from 1 to 16 days were computed from the residuals for the four seasons (winter is December through February). "Zonal mean" correlation coefficients were estimated by weighting the station correlation coefficients by the square root of the number of observation pairs at that station relative to the total pairs of all 26 stations. The resulting correlation coefficients for lags 1-7 days are shown in Fig. A1. The average number of pairs for any lag at a station was 465 in winter and slightly larger in the other seasons. For the purpose of assessing the significance of the zonal mean autocorrelations it is estimated that, of the 26 available stations, nine are independent (more on this in the next section). Effectively, then,

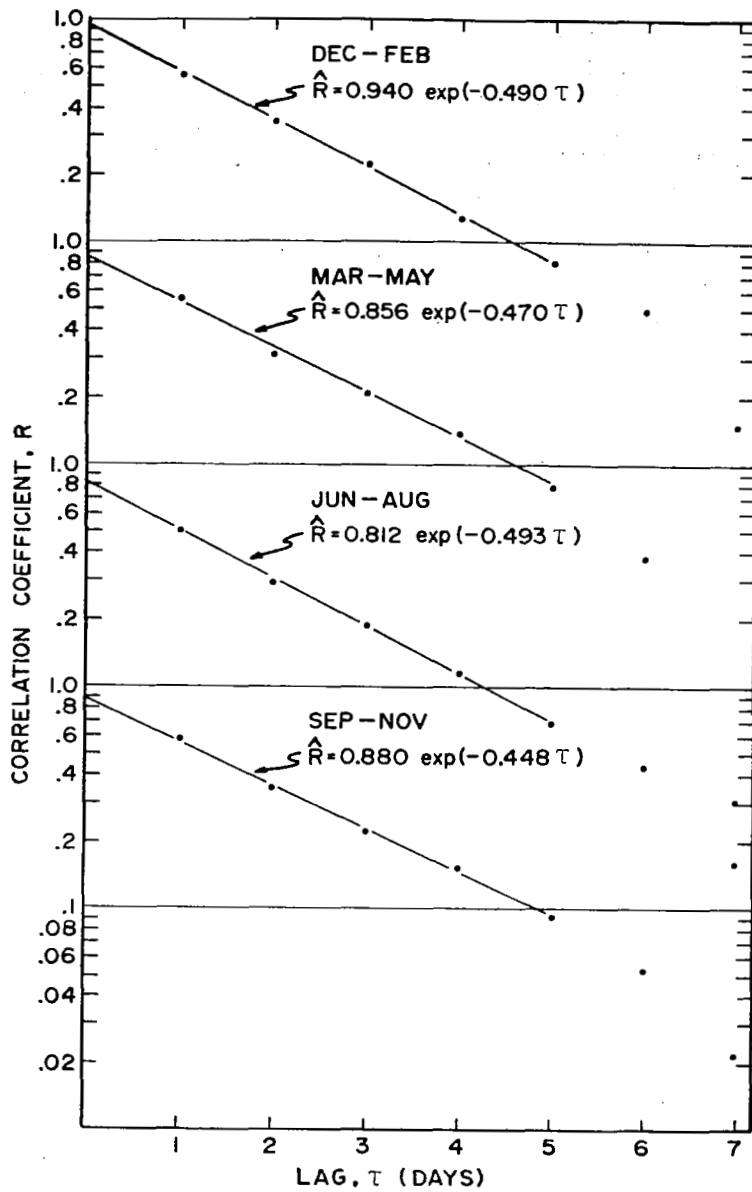


Figure A1. Total ozone temporal autocorrelations, zonally averaged, from the Dobson stations between 40° and 60°N, for lags 1 to 7 days.

we have about 4200 pairs at each lag, which implies that any "zonal mean" correlation coefficient above about 0.04 in absolute value is significant at the 99% level.

The computed coefficients for lags 1-5 days were fitted with the simple red-noise model  $\hat{R}(\tau) = a_\tau \exp(-b_\tau \tau)$ . The coefficients  $a_\tau$  and  $b_\tau$ , for each season, are shown in Table A1. The "zonal mean" temporal standard deviations, both day-to-day,  $\sigma_d$ , and year-to-year,  $\sigma_a$ , were also obtained by weighting the variances at stations by the square root of the number of observations.

The difference of  $a_\tau$  from unity is mostly due to observation error (see e.g., Julian & Thiebaux, 1975), and the value  $\sigma_\epsilon^2 = \frac{1}{2} (1 - a_\tau^2) \sigma_d^2$ , whose square root is given in Table A1, may be interpreted as variance due to observation error. Therefore,  $\sigma_d^2$  reduced by this amount is  $\sigma^2$ , the "true" day-to-day variance. Using the values of  $b_\tau$  from Table A1, equation (A1) yields  $\sigma_T/\sigma = .21, .21, .21, .22$  for winter, spring, summer, and fall, respectively (90-day means). For example, a single winter season average at a typical mid-latitude station will have a standard error (from the true season mean) of  $.21\sigma$ , or 8.2 D.U. Calculations for other averaging times and/or seasons can easily be made.

It is a well-known result of sampling theory that

$$\sigma_T^2 = \sigma^2/N \quad (A2)$$

where  $N$  is the effective sample size. Since the asymptotic value of  $\sigma_T^2/\sigma^2$ , from equation (A1), is  $2/b_\tau T$ , the effective sample size  $N = T/T_0$  where

$$T_0 = \int_{-\infty}^{\infty} R(\tau) d\tau = 2/b_\tau \quad (A3)$$

is a characteristic time between effectively independent observations (Lieth, 1973). Values of  $T_0$  are shown in Table A1.

c. Standard error of area averages. The preceding method may also be applied to the determination of errors in spatial averages and equation (A1) again applies, with  $T$  now understood to refer to an averaging distance. Correlation coefficients of nearly simultaneous residuals at different stations were computed, and, in order to include large lags while keeping some uniformity in the data set, pairs were restricted to those whose orientations were more east-west than north-south. Figure A2 shows these correlation coefficients (to save space, only those pairs whose longitude separations are 90 degrees or less are shown). Similar computations have been made by Fabian (1967) for a set of European Dobson stations and by Nastrom (1977) for aircraft measurements of ozone concentration near the tropopause.

Table A1. Total ozone temporal variability and correlation statistics, zonally averaged, from Dobson stations between 40° and 60°N.

Season	Model $\hat{R} = a_{\tau} e^{-b_{\tau}\tau}$		"Day-to-day" standard deviation	"Year-to-year" standard deviation	observation error	$\sigma_d$ corrected for obs. error	Time between independent observations
	$a_{\tau}$	$b_{\tau}$	$\sigma_d$	$\sigma_a$	$\sigma_{\epsilon}$	$\sigma$	$T_0$
Dec-Feb	.940	.490	40 D.U.	15 D.U.	9.7 D.U.	39 D.U.	4.1 days
Mar-May	.856	.470	36	11	13.1	34	4.3
Jun-Aug	.812	.493	22	9	9.2	20	4.1
Sep-Nov	.880	.448	25	9	8.4	23	4.5
Annual Average	.872	.475	31	11	10.1	29	4.3

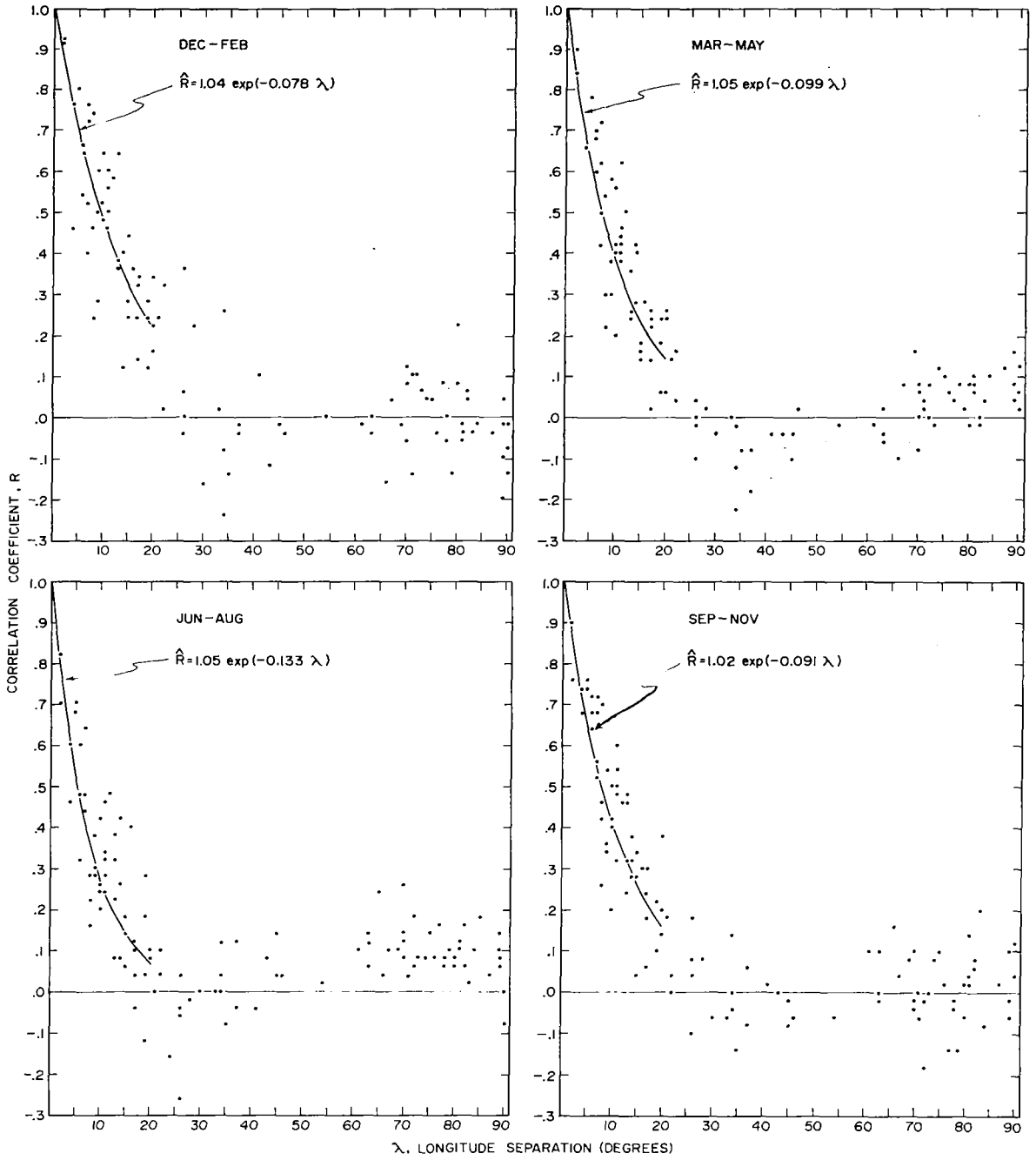


Figure A2. Total ozone spatial autocorrelations for separations (lags) 0 to 90 degrees longitude. Only Dobson stations between 40° and 60°N were used, and the pairs were restricted to those whose orientations were more east-west than north-south.

An average of about 400 observation pairs goes into each correlation coefficient in Figure A2. However, the autocorrelation within the time series at each station affects the significance of the cross-correlation between the two stations. Mitchell's (1963) approximation for effective sample size using purely persistent series implies that a correlation coefficient must be above about 0.18 in absolute value to be significant at the 99% level.

The red-noise model  $\hat{R} = a_\lambda \exp(-b_\lambda \lambda)$ , where  $\lambda$  is longitudinal separation (lag), was fitted to various sub-interval averages of the measured correlations from the range of lags 0 to 20 degrees. In computing the sub-interval averages, the individual correlations were weighted by the square root of the number of observation pairs. The time difference between observations at both stations of a pair was generally less than about 2 hours for these small separations, and the associated temporal variability was neglected. (Note that the taking of sub-interval averages effectively lowers the 0.18 significance threshold somewhat in the range 0-20.) The values of  $a_\lambda$  being generally greater than 1 indicates that a better model would have used  $\lambda$  raised to a power slightly greater than 1; however, this refinement in the present study does not seem warranted. Inserting values of  $b_\lambda$  in equation (A1) yields, for zonal means,  $\sigma_T/\sigma = .26, .23, .20,$  and  $.24$  for winter, spring, summer, and fall respectively. Application of these results will be demonstrated presently.

An effective length between independent observations,  $L_0 = 2/b_\lambda$ , is also given in Table A2. These values were used in subjectively estimating, in the previous section, that of the 26 available stations only about 9 were independent. Extrapolation would indicate that on the order of 100 effectively independent daily values are possible from an ideal global network.

Table A2. Total ozone spatial variability and correlation statistics, zonally averaged, from Dobson stations between 40° and 60° N.

Season	Model $R = a_\lambda e^{-b_\lambda \lambda}$		Longitude separation of independent observations $L_0$
	$a_\lambda$	$b_\lambda$	
Dec-Feb	1.04	.078	26 deg
Mar-May	1.05	.099	20
Jun-Aug	1.05	.133	15
Sep-Nov	1.02	.091	22
Annual Average	1.04	.100	21

### 3. The significance of point- and area-average total ozone trends.

A common method for determining trends uses monthly or seasonal deviations from the long-term normal (e.g., Angell and Korshover, 1973, 1976; Pittock, 1974; London and Kelley, 1974). Significance of single station trends is determined by the size of the standard error of these deviations, which can now be determined from equation (A1), using  $T = 30$  or 90 days, and  $\sigma$  from Table A1. If long-term means are required, the problem is only slightly different. Here, it seems reasonable to assume that little interannual correlation exists in the residuals (e.g., Hill et al., 1975), except possibly for a small amount due to a sunspot cycle. Therefore, the standard error  $\sigma_{LT}$  of a long-term seasonal (90-day) mean as an estimate of a climatic mean is given by equation (A3) substituted in equation (A2), i.e.,

$$\sigma_{LT} = \frac{b_{\tau} (\sigma^2 + \sigma_a^2)^{1/2}}{2(90Y)} \quad (A4)$$

where  $Y$  is the number of years considered, and the inclusion of  $\sigma_a$  accounts for the effect of interannual variability.

Several authors (e.g., Angell and Korshover, 1973, 1976) compute means at groups of stations in order to estimate regional trends. To determine the standard error of such group means, one must account for both the temporal and spatial correlation. To fix ideas, consider the standard error of a group mean for several stations which can all be conveniently enclosed in a rectangle whose sides are lengths  $L_1$  and  $L_2$ . The standard error of the seasonal mean at each of the stations<sup>2</sup> is given by equation (A1), and as a first approximation we will consider that the mean at every point within the area is known within the same standard error. This is an optimistic view, but one which becomes more realistic as station density increases. To account for the spatial averaging, it is appropriate to apply equation (A1) using  $b_{\lambda}$  with  $T = L_1$ , and then apply it again using  $T = L_2$ . This assumes isotropy, which again is probably valid only to a first approximation (see, e.g., Buell, 1972; Julian and Thiebaux, 1975). Note also that the spatial standard deviation of the (instantaneous) field of total ozone is now required. This computation has not been carried out; however, oscillations in the residuals are likely due predominantly to truly transient eddies, and thus will affect all stations in the latitude band more or less equally. This being the case, the temporal standard deviation should be a reasonable approximation of the spatial standard deviation.

As an example, the standard error of yearly means for the group of North American stations between  $40^{\circ}$  and  $60^{\circ}\text{N}$  will be estimated (Churchill, Edmonton, Goose Bay, Caribou, Green Bay, Bismark, Bedford, Fort Collins, Boulder, and Toronto). Using the "zonal mean"



statistics which are already given in Table A1 (instead of a set derived specifically from these stations),  $\sigma_T/\sigma \approx 0.11$ . The longitudinal extent of this area is about 40 degrees, while the latitudinal extent is about 17 degrees, or roughly equivalent to 26 degrees longitude. Using equation (A1), these values of T yield ratios  $\sigma_T/\sigma = .61$  and  $.70$  respectively, and the result if  $\sigma_T = (.11 \times .61 \times .70)\sigma = .047 (29) \approx 1.4$  D.U. In other words, a detected  $2\sigma_T$  (i.e., 2.8 D.U.) change from one year to any other year in the annual mean total ozone in this region could be judged significant at the 95% level of confidence.

#### 4. Concluding remarks

It has been shown that standard errors of the mean for time- and space-averages are properly determined from time and space correlation and variability statistics. Sample statistics have been given, on a seasonal and zonal mean basis, for the Dobson stations between  $40^{\circ}$  and  $60^{\circ}$  N. Further research in this vein should aim at determining these statistics for all the specific regions and years where Dobson measurements are available. Also, since anisotropy is expected, the north-south statistics should be included.

Hill et al. (1977), have assessed the detectability of global total ozone trends at about 1%, assuming an (independent) 18-station network. The present work suggests that there probably exist at least that many stations whose monthly means are independent (but see Pittock, 1974). However, Hill et al., apparently assume that these 18 determine the means (to within the same standard error) at all points on the globe. This does not seem likely, as has been previously stated implicitly by Kohmyr et al. (1971); Pittock (1974); and Angell and Korshover (1976). However, the example in the preceding section points to the likelihood that trends can be calculated with high confidence for certain areas of high station density, such as the United States and southern Canada, Europe, and perhaps India and Japan. Trends over regions not presently well-sampled will apparently only be detected by future satellite observations of total ozone. Note that the assumption that one knows the mean to within the same standard error at all points within a region is well-satisfied with satellite observations. It is thus of some interest to determine, a priori, the space and time scales of averaging required to detect a change in ozone with a given level of significance from satellite observations. For example, let it be necessary to detect a regional 1% change in annual mean total ozone (i.e., 3.5 D.U.) at the 99% significance level. Such a confidence level requires that  $2.6\sigma_T < 3.5$  D.U., or  $\sigma_T \leq 1.3$  D.U. Averaging over a year (with  $b_T = .475$  and  $\sigma = 29$ ) yields  $\sigma_T = 3.1$  D.U. In addition, averaging over a square 3000 km on a side would decrease  $\sigma_T/\sigma$  by another factor of  $(.60)^2$  to  $\sigma_T = 1.1$  D.U., thus satisfying the requirement. Such preliminary

estimates of trend significance are important in the planning of observing programs. However, several years of satellite observations will have to be available before year-to-year changes can be recognized as either part of some periodicity or not.

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