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The Rigid Shell Component of Superrotation in Planetary Atmospheres: Angular Momentum Budget , Mechanical Analogy and Simulation of the Spin Up Process

H. G. Mayr and I. Harris

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National Aeronautics and Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

THE RIGID SHELL COMPONENT OF SUPERROTATION IN PLANETARY ATMOSPHERES: ANGULAR MOMENTUM BUDGET, MECHANICAL ANALOGY, AND SIMULATION OF THE SPIN UP PROCESSES

H. G. Mayr and I. Harris
NASA/Goddard Space Flight Center
Greenbelt, MD 20771

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Abstract

The concept of local dynamical equilibrium in the form of geostrophic or cyclostrophic equilibria, which is valid at high altitudes above the solid surface, may be adequate to describe the wide range of superrotation rates observed in planetary atmospheres. It does not address the problem of "exactly how this superrotation ... is produced and maintained against the tendency for friction to oppose differential motions between the atmosphere and the underlying planet" (Hide et al., Nature, 1980). Our analysis, to some extent pedagogical, is based on the concept of a thermally driven zonally symmetric circulation (e.g. Leovy, 1973; Gierasch, 1975). On planets, with rotation axis perpendicular to the orbital plane, the equatorial region is heated preferentially. The atmosphere expands, and large scale motions in the form of Hadley cells develop which produce a redistribution of mass and angular momentum. To provide a basic understanding of the phenomenon simple analogies are proposed, where mechanical devices with variable moment of inertia are frictionally coupled to a rotating turntable. Using a three dimensional zonally symmetric spectral model and Laplace transformation, the time evolution of a fluid leading from corotation under globally uniform heating to superrotation under globally nonuniform heating is simulated. For high viscosities atmospheric superrotation can be understood in analogy with a pirouette. During spin up angular momentum is transferred to the planet. For reasonably low viscosities, however, this process is reversed. The increased tendency toward geostrophy, combined with the increase of surface pressure toward the poles (due to meridional mass transport), induces the atmosphere to subrotate temporarily at lower altitudes. The resulting viscous shear near the surface thus permits angular momentum to flow from the planet into the atmosphere where it propagates upwards and, combined with the change in moment of inertia, produces large superrotation rates at higher altitudes.

INTRODUCTION

The phenomenon of atmospheric superrotation is ubiquitous in our solar system. For Earth, there exists evidence suggesting that the rotation rate of the thermosphere is as much as a factor of $\Gamma = 1.4$ larger than that of the surface (e.g., King Hele, 1964). Superrotation is observed on Mars (e.g., Conrath et al., 1973), Jupiter (e.g., Smith et al., 1979) and Saturn (e.g. Smith et al., 1981), the most striking example being that of Venus (e.g., Schubert et al., 1980) where the rotation period of the visible cloud top is $\Gamma = 60$ times shorter than that of the solid body (243 days). Each of these atmospheres has a distinctive rigid shell (global average) component of superrotation, irrespective whether the planet rotates in the prograde (e.g., Earth) or retrograde (e.g., Venus) direction.

The difficulty of understanding superrotation has recently been emphasized in a paper by Hide et al., 1980 from which we quote: "The density of the atmosphere decreases rapidly with height and more than 95% of the mass of the atmosphere is contained within the troposphere, which extends from the ground to an altitude of ~ 10 km, and the lower stratosphere. These regions of the atmosphere rotate faster on average than the underlying solid Earth with a typical relative speed of about 10 m s^{-1}. Exactly how this superrotation (a term used for the generally more rapid relative motions found in the very tenous regions at heights around 300 km) is produced and maintained against the tendency for friction to oppose differential motions between the atmosphere and the underlying planet is a central problem of dynamical meteorology."

Numerous theoretical models have been proposed to describe the effect (e.g. Schubert and Whitehead, 1969; Thompson, 1970; Gierasch, 1970; Hide, 1966, 1971; Gold and Soter, 1971; Gierasch and Stone, 1968; Gierasch, 1975;

Leovy, 1973; the review of Rishbeth, 1972), but none is fully accepted. While different processes are probably important in different regions of the atmospheres and in various planetary settings, one feature all share in common is differential heating by solar radiation which has long been known to produce pressure variations and zonal wind velocities (e.g. Lorenz, 1967). It is the purpose of this paper to examine the angular momentum budget of this mechanism and to provide a mechanistic interpretation that may contribute to a basic understanding of the average or rigid shell component of superrotation. The important problem of differential rotation and equatorial superrotation will be discussed in a subsequent paper (Mayr and Harris, 1981).

Leovy (1973), Gierasch (1975) and others (e.g. Turikov and Chalikov (1971), Kalvay de Rivas (1975)) proposed models for the zonally symmetric circulation with superrotation resulting from differential heating. It was suggested by Leovy and shown by Gierasch that superrotation is maintained in a balance between upward transport through the meridional cell and downward transport through eddy processes. With a steady state solution of the nonlinear energy, continuity and momentum equations, Gierasch described the superrotation in the viscous atmosphere of Venus from the planetary surface upwards. The zonal wind field thereby rigorously satisfied slip-free and stress-free boundary conditions. Due to the neglect of the viscous stress in the meridional momentum equation, however, the model is not entirely self consistent in that the meridional winds could not be made to vanish at the surface. Our results, which do not suffer from this deficiency nevertheless confirm the importance of the meridional circulation for maintaining superrotation.

What remains then to be shown is how, precisely, the flow is built up or superrotation is produced and if and how the atmosphere receives angular momentum in this process? In answering these questions we shall discuss in the first part of this paper simple mechanical analogies that elucidate the basic process. In the second part we shall present a spectral model which describes in the linear regime the thermally driven zonally symmetric atmospheric circulation and superrotation. The approach is taken there to study the hypothetical spin up of a planetary atmosphere when differential solar heating is suddenly applied and follow the time evolution until a steady state is reached. Solutions obtained over a wide range of parameters thus provide insight into the processes that establish and maintain superrotation. We find that, consistent with the mechanical analogies, two aspects are important: With high viscosity the angular momentum for superrotation is primarily derived from the change in the moment of inertia due to mass redistribution by the meridional circulation. With reasonable low visocisity, however, this mass redistribution and the resulting pressure build-up in the polar region at lower altitudes causes the atmosphere to subrotate temporarily during the spin-up process so that the atmosphere can pick up the angular momentum from the planet giving rise to superrotation.

LOCAL DYNAMICS

We consider simplified equations to describe the zonally symmetric component of horizontal momentum conservation

$$2\omega_p \sin\Theta U + \frac{U^2}{r}\, tg\Theta + \eta_\Theta \frac{\partial^2 V}{\partial r^2} = -\frac{1}{r\rho}\frac{\partial p}{\partial \Theta} \tag{1}$$

$$2\omega_p \sin\Theta V + \frac{UV}{r}\, tg\Theta + \frac{V}{r\cos\Theta}\frac{\partial U}{\partial \Theta} - \eta_r \frac{\partial^2 U}{\partial r^2} = 0 \tag{2}$$

with U, zonal velocity

V, meridional velocity

ρ, mass density

η_r, η_θ viscosity coefficients in the vertical and horizontal direction

p, kinetic pressure

ω_p, rotation rate of the planet

Θ, latitude

Assuming $|V| << |U|$ and with a given pressure field, p(r, Θ), the solutions of (1) and (2) yield in the limit of small but finite viscosity (Mayr et al., 1981) the equation for gradient flow

$$U = -\omega_p r\cos\Theta + \frac{\omega_p}{|\omega_p|}\sqrt{\omega_p^2 r^2 \cos^2\Theta - \frac{ctg\Theta}{\rho}\frac{\partial p}{\partial \Theta}} \tag{3}$$

It reduces to

$$U = \frac{-\frac{\partial p}{\partial \Theta}}{2r\rho\omega_p \sin\Theta} \quad (|U| << |\omega_p r|) \tag{4}$$

and

$$U = +\frac{\omega_p}{|\omega_p|}\sqrt{-\frac{ctg\Theta}{\rho}\frac{\partial p}{\partial \Theta}} \quad (|U| >> |\omega_p r|) \tag{5}$$

in the geostropic and cyclostrophic appromināations respectively.

Formally, the above solutions represent a local balance between inertial forces and pressure gradients. They represent the simplest and perhaps most powerful concept for describing superrotation. On most planets in the solar system the rotation axis is perpendicular to the orbital plane, the equatorial region is heated preferentially and the pressure tends to increase toward the equator ($\frac{\partial p}{\partial \Theta} < 0$) at higher altitudes. The propensity toward superrotation in planetary atmospheres is thus explicable in terms of Equations (3) through (5). The largest superrotation rates are likely to occur at altitudes where the viscous constraint in the meridional force balance is not important, and the above solutions are thus approximately valid. An outstanding example is the work by Leovy (1973) where the large superrotation rate on Venus was attributed to cyclostrophic balance.

Using the same horizontal pressure gradient, $\frac{1}{p}\frac{\partial p}{\partial \Theta} = -0.3$, for different planets, we note that the concept of local dynamical equilibrium produces a wide range of values $\frac{\Delta\omega}{\omega_p} = 53$ (Venus), 0.20 (Earth) and 0.0014 (Jupiter) which match the observed average rotation rates within a factor of two. The zonal velocity is here expressed in the form of $r\Delta\omega$.

ANGULAR MOMENTUM BUDGET

The concept of local dynamical equalibrium does not provide a connection between superrotation and source mechanism. Moreover, it is restricted to that part of the atmosphere where, formally, viscosity is unimportant and thus it cannot provide a mechanistic connection between the superrotating atmosphere and the rotating planet.

The concept of angular momentum conservation is fundamental for our understanding of superrotation. With the primary emphasis on elucidating the physics, we shall describe here a scenario of atmospheric superrotation which heuristically evolves from fluid dynamical elements that can be readily understood to a more realistic configuration containing these elements. In support of this interpretation we shall make use of mechanical analogies.

As reference, a globally uniform atmosphere with mass density $\rho_o(r)$ is chosen. It is heated with a source, $Q_o(r)$, representing the global average of solar input. Due to viscous interaction this atmosphere corotates with the angular velocity of the planet, ω_p. Without adding an external torque to the entire atmosphere-planet system, we change the heat source to $Q(r,\Theta) = Q_o(r) + \Delta Q(r,\Theta)$ which in turn changes the density distribution to $\rho(r,\Theta) = \rho_o(r) + \Delta\rho\ (r,\Theta)$. The question we ask is how ΔQ maintains an atmospheric rotation rate $\omega(r,\Theta) = \omega_p + \Delta\omega(r,\Theta)$ with a rigid shell component different from that of the planet?

In reality, the rotation rate of the planet also changes during the spin up process. For the purpose of discussing superrotation, however, we are justified to neglect this effect since the moment of inertia of the atmosphere is much smaller than that of the planet.

Under conservation of angular momentum, L, the physical conditions for uniform and nonuniform heating, representing end states of equilibria, are related

$$L_a\ (Q) = L_a(Q_o) - \Delta L, \tag{6}$$

where

$$\Delta L = L_p(Q) - L_p(Q_o) \tag{7}$$

represents the total amount of angular momentum that is transferred from the atmosphere to the planet (positive) or visa versa (negative) during the spin up process. Subscripts a and p refer to atmosphere and planet respectively. Assuming $\Delta\rho \ll \rho_o$, the explicit form of Eq. (6) is

$$\int \Delta\omega\rho_o d\psi = -\omega_p \int \Delta\rho d\psi - \Delta L. \tag{8}$$

With the definition $d\psi = r^2\cos^2\Theta d\tau$, the integrations are taken over the entire volume of the atmosphere (with element $d\tau$). Equation (8) states that three components contribute to the conservation of the total angular momentum: (a) atmospheric superrotation, $\Delta\omega$, (b) changes in the atmospheric moment of inertia which are associated with changes in mass density, $\Delta\rho$, and (c) the above defined exchange of angular momentum between atmosphere and planet, ΔL, during the transition from one (Q_o) to the other (Q) equilibrium state.

A simple thought experiment illustrates the nature of ΔL. Starting out with an atmosphere characterized by Q_o, ρ_o, and ω_o, corotating with the planet, we lower the heat source uniformly over the globe $Q(r) = Q_o(r) - \Delta Q(r)$. Under mass conservation the atmosphere contracts, $\rho(r) = \rho_o(r) - \Delta\rho(r)$ and its moment of inertia decreases. During this change, the atmosphere tends to maintain its angular momentum and initially there is superrotation. This motion produces viscous interaction with transfer of angular momentum to the planet, until eventually a new static equilibrium is established in which atmosphere and planet corotate with the angular velocity, ω. Of course, for this particular case the solution is simply

$$\Delta L = I_p\,(\omega - \omega_o) > 0 \qquad (9)$$

with

$$\omega = \frac{\omega_o[I_p + \int \rho_o d\psi]}{[I_p + \int \rho d\psi]}. \qquad (10)$$

where I_p refers to the planet's moment of inertia. Considering $\Delta\omega = 0$ another form of the solution can be obtained from Eq. 8,

$$\Delta L = -\omega_p \int \Delta\rho(r)\, d\psi. \qquad (11)$$

This means that the atmosphere transfers an amount of angular momentum equivalent to the <u>entire</u> change in its moment of inertia; thus none remains in the atmosphere to maintain superrotation.

PIROUETTE

For a realistic atmosphere with globally nonuniform source the conservation of angular momentum is complicated, as will be shown in numerical simulations of the spin up process. To provide a fundamental understanding of superrotation it is therefore helpful to discuss first a simple model which proves to be valid for very large kinematic viscosities.

The process is illustrated in Figure 1. On a planet where the equatorial region is heated preferentially the compressible viscous atmosphere expands. Thus large scale motions in the form of Hadley cells are set up (e.g. Lorenz, 1967, Leovy, 1973, Gierasch, 1975) that effectively transport mass from the equator toward the poles closer to the rotation axis. As a result the atmospheric moment of inertia decreases

$$\Delta I = \int \Delta\rho d\psi \quad \propto \quad - \int \Delta Q d\psi < 0 \qquad (12)$$

which tends to decrease the angular momentum. Under Newton's law such a change is met with inertia and the angular velocity must increase (superrotation) to conserve angular momentum. We <u>assume</u> that during the initial phase of the spin up process the atmosphere superrotates all the way down to the planetary surface and "reacts" by transferring, through a viscous torque, angular momentum to the planet.

Unlike the case for a <u>uniform</u> source where this momentum transfer corresponds to the <u>entire</u> change in moment of inertia (Eq. 11), only <u>part</u> of it is transferred to the planet ($\Delta L > 0$) under nonuniform heating; this superrotation can be sustained, which is restated in mathematical form

$$\langle\Delta\omega\rangle I_a \equiv \int \Delta\omega\rho_o \, d\psi = -\omega_p \int \Delta\rho d\psi - \Delta L > 0. \qquad (13)$$

Apart from the poleward transport of mass (term containing $\Delta\rho$ in Eq. 13) which is driving superrotation, the meridional winds are also important in "isolating" the superrotating atmosphere from the planet. The associated inertial forces (which change sign as one vertically transverses a Hadley cell) are in balance with the viscous forces from the zonal wind field, so that in equilibrium superrotation can be maintained with stress free boundaries. Solar heating and the resulting meridional circulation thereby continually supply the energy that is expended through viscous dissipation. During the spin up process (non-equilibrium) the inertial forces effectively limit the transfer of angular momentum to the planet.

The magnitude of this momentum transfer is proportional to the viscosity and it has the effect of damping superrotation. A more viscous atmosphere is tighter coupled to the planet which tends to constrain it into corotation.

In support of this interpretation a mechanical model is presented which is closely related to the spin up in a pirouette. This analogy is crude by comparison with the dish pan experiments (e.g. Schubert and Whitehead, 1969) providing a fluid dynamic simulation of superrotation. However, it contains some of the important elements to describe the process and has the advantage that the physics is more transparent.

The discrete phases of the pirouette experiment are illustrated in Figure 2. Sitting on a rotating turntable, representing the solid planet, is a mechanical device referred to as "figure" which represents the atmosphere; its configuration and moment of inertia (with masses in shaded areas) can be changed by pulling a string. The angular velocity of the figure is schematically shown as a function of time. In the first picture frame the string is relaxed and no external force is applied. The figure has the maximum moment of inertia attainable and corotates with the turntable, which is the analogy of an atmosphere under globally uniform heating.

In the second frame the string is pulled. Expending energy against gravity, inertial forces (e.g. centrifugal and Coriolis) and friction, the figure straightens out, its moment of inertia decreases and it superrotates. Locked in this position, the new figure would continue to superrotate indefinitely if there were no frictional interaction or momentum exchange with the turntable (thin solid line in the velocity diagram). Under realistic conditions, however, friction causes a slowdown (heavy solid line in the velocity diagram) which eventually would lead to corotation. To prevent this the figure must be spun up periodically. This is accomplished by relaxing the string so that the figure assumes its original moment of inertia. All of the potential energy and part of the kinetic (inertial) energy previously expended are thereby released. In that position the figure subrotates and picks up angular momentum from the turntable. By pulling the string the figure is then spun up again, and a new cycle begins.

The rotation rates in Figure 2 illustrate the steady state condition during an experiment cycle and indicate that the net result of the procedure is superrotation. A simple analysis of the mechanism can readily verify this.

The equation of motion has the form

$$\frac{d}{dt}(I\omega) = -\frac{\alpha}{m} I\omega, \qquad (14)$$

where $I(t)$ is the time varying moment of inertia, ω is the difference between the angular velocities of the figure (with mass m) and the turntable, and α is the friction coefficient. Over the experiment cycle, in steady state, the net amount of momentum exchange with the turntable must be zero

$$\int_0^{\tau} \frac{d}{dt} (I\omega)\, dt = - \frac{\alpha}{m} \int_0^{\tau} I\omega dt = 0. \tag{15}$$

$\bar{\omega}_1$ and $\bar{\omega}_2$ being the average angular velocities during the two phases of the cycle and t_1, I_1, and t_2, I_2 being the corresponding time periods ($\tau = t_1 + t_2$) and moments of inertia, Equation (15) becomes

$$I_1 \bar{\omega}_1 t_1 + I_2 \bar{\omega}_2 t_2 = 0. \tag{16}$$

It follows then immediately that the average superrotation rate is

$$\bar{\omega} \equiv \frac{1}{\tau} \int_0^{\tau} \omega dt \equiv \frac{1}{\tau} (\omega_1 t_1 + \omega_2 t_2) =$$

$$\frac{1}{\tau} \bar{\omega}_1 t_1 \left(1 - \frac{I_1}{I_2}\right) = \frac{1}{\tau} \frac{m}{\alpha} (\omega_{11} - \omega_{12}) \left(1 - \frac{I_1}{I_2}\right) > 0 \tag{17}$$

which is positive, since $\omega_{11} > \bar{\omega}_1 > \omega_{12} > 0$ and $I_1 < I_2$. The angular velocities ω_{11} and ω_{12} are defined in Figure 2. The figure superrotates because of the cyclic change in the moment of inertia. Energy is thereby continually fed into the system where it is dissipated through friction. The superrotation rate increases inversely proportional to friction.

Relative to the initial state of corotation (with ω_o), the subsequent state of superrotation has a <u>lower</u> (average) angular momentum. The difference, $\Delta L > 0$ (Eq. 13) is transferred to the turntable during the spin up process. It increases with friction and decreases superrotation.

In summary, we note the analogies with the superrotating atmosphere. The oscillatory motions within one cycle of the pirouette experiment simulates the Hadley cell. The higher the frequency is in the experiment cycle, the shorter is the turnover time in the Hadley cell and the more energy is going through the superrotation engine. In the former, mass and momentum exchange take place sequentially, while in the latter they occur simultaneously in the upper and lower legs of the circulation. The external force on the string represents the horizontal pressure gradient due to differential heating. These forces drive the radial motions of the pirouette experiment and the meridional motions in the atmosphere, thereby controlling the moments of inertia in both systems. They are primarily balances by Coriolis and centrifugal forces which provide the link with friction and viscous dissipation where energy is ultimately dissipated. Thus closing the energy chain we obtain a basic understanding of superrotation which is consistent with the interpretations of Leovy, 1973 and Gierasch, 1975.

One easily sees that the concept of this mechanical analogy can also be applied to individual particles of air moving with a Hadley cell. In a fluid, at any given time, every phase of this motion is equally represented and thus justifies the time averaging procedure as a means of representing the dynamic system in steady state.

DOUBLE PIROUETTE

The problem with the above analogy is that the entire atmosphere is portrayed as a body which can be deformed horizontally but not vertically. Such a picture is essentially valid when large viscosity provides enough rigidity. Under more realistic conditions with reasonably low viscosity, however, the situation is more complicated.

A logical extension of the mechanical model is the "double pirouette" experiment which is significantly different from the earlier single pirouette experiment in that is has an additional degree of freedom for the vertical momentum distribution. Instead of one mechnical device, two are used on top of each other, separated by a friction plate (Figure 3). Pulling the string at the upper figure (F_u) supplies the energy. As F_u is stretched and its moment of inertia decreases, the lower figure (F_1) is compressed and its moment of inertia increases. Springs in F_1 may provide a restoring force due to the compressibility of the atmosphere.

The analytical solution for this thought experiment is straightforward. But depending on the conditions, the results may differ significantly, and a detailed discussion would go beyond the scope of this paper. Adopting here a simplified description, we consider the corotating (ω_o) double pirouette with moments of inertia I_u^d and I_1^d representing the atmosphere under globally uniform heating. The string is pulled during the "energization phase" and the moments of inertia change to I_u^e, I_1^e; it is relaxed during the "dissipation phase" and the moments of inertia return back to I_u^d, I_1^d. Subscripts u and 1 refer to the figures F_u and F_1, while superscripts e and d indicate the energization and dissipation phases respectively. During the energization phase, the time average rotation rates (relative to ω_o) are $\bar{\omega}_u^e > 0$ and $\bar{\omega}_1^e < 0$ in F_u and F_1 respectively.

Conservation of angular momentum with a system of two coupled equations corresponding to (15) yields

$$\bar{\omega}_u^d \, t^d = \frac{t^e}{(\gamma-1)} \left[\bar{\omega}_u^e \left(\frac{I_1^e}{I_1^d} - \gamma\frac{I_u^e}{I_u^d}\right) - \bar{\omega}_1^e \, \gamma \left(\frac{I_1^e}{I_1^d} - \frac{I_u^e}{I_u^d}\right) \right] \qquad (18)$$

$$\bar{\omega}_l^d \, t^d = \frac{t^e}{(\gamma-1)} \left[\bar{\omega}_l^e \left(\frac{I_u^e}{I_u^d} - \gamma \frac{I_l^e}{I_l^d}\right) - \bar{\omega}_u^e \left(\frac{I_u^e}{I_u^d} - \frac{I_l^e}{I_l^d}\right) \right] \qquad (19)$$

where

$$\gamma = \frac{\alpha_o + \alpha}{\alpha},$$

α_o, friction coefficient between turntable and pirouette device

α, friction coefficient between both pirouette devices.

Since $I_u^e < I_u^d$ ($\bar{\omega}_u^e > 0$) and $I_l^e > I_l^d$ ($\bar{\omega}_l^e < 0$), it follows from (18) and (19) that the average rotation rates for both figures are positive

$$\bar{\omega}_u = \frac{1}{\tau} (\bar{\omega}_u^e \, t^e + \bar{\omega}_u^d \, t^d) > 0 \qquad (20)$$

$$\bar{\omega}_l = \frac{1}{\tau} (\bar{\omega}_l^e \, t^e + \bar{\omega}_l^d \, t^d) > 0 \qquad (21)$$

Examing the angular momentum budget (Eqs. 8 and 13)

$$0 < \bar{\omega} I_o \equiv \bar{\omega}_u \, I_u^d + \bar{\omega}_l \, I_l^d = \omega_o \, (I_u^e - I_u^d + I_l^e - I_l^d) - \Delta L \qquad (22)$$

produces an interesting result. Suppose we chose $I_u^e = I_l^d$ and $I_l^e = I_u^d$ which is the case illustrated in Figure 3. Under this condition the total change in the moment of inertia is $\Delta I \equiv (I_u^e - I_u^d + I_l^e - I_l^d) = 0$. Knowing from (20) and (21) that there is superrotation, Eq. 22 then implies that the double pirouette device is capable of picking up angular momentum, $\Delta L < 0$, from the turntable during the transition from (uniform) corotation to superrotation.

This result is completely different from that obtained with the single pirouette experiment where the decrease in moment of inertia causes superrotation and, additionally, permits angular momentum to be transferred to the turntable during the spin up process. Depending on the amount of friction, the total change in moment of inertia, ΔI, and or the (initial) exchange of angular momentum with the turntable (planet), ΔL, may thus contribute to the angular momentum for superrotation. Under atmospheric conditions with reasonably low viscosity it is the latter which is usually more important.

The particular form that was used here to energize the double pirouette experiment is not unique. An external force applied to the lower figure would lead to virtually the same conclusions.

On the basis of this analysis which is consistent with the results from our numerical simulation of atmospheric superrotation in the second part of this paper, the following conclusions are drawn:

Viscosity. In the case of identically zero viscosity or friction, the equations (20) and (21) cannot be applied. But assuming that initially the mechanical device (or the atmosphere under uniform heating), somehow, uniformly corotates, one can easily show that $\bar{\omega}_u > 0$ but $\bar{\omega}_l < 0$. The angular momenta in regions of super-and sub-rotation add up to zero implying that the average change in the rotation rate is also zero. Superrotation cannot be understood without viscosity. To sustain this motion the system must be capable of converting kinetic energy back into thermal energy.

Mass Transport. Without an external torque, conservation of angular momentum requires that the moment of inertia must change in order to sustain superrotation. This is accomplished through mass redistribution by the meridional circulation of the atmosphere induced by solar heating and pressure gradients, or by the oscillatory motions due to external forcing in the priouette experiments. To understand superrotation it is important that the change in moment of inertia is not maintained statically but through transport processes. The mass carrying meridional circulation of the atmosphere and the oscillatory motions of the mechanical analogies continually supply the energies that are expended by viscous dissipation associated with superrotation.

Momentum exchange with the planet. During the transition from corotation under globally uniform heating to superrotation under differential heating the atmosphere exchanges angular momentum with the planet. In simplest form, one expects a behavior similar to that of the single pirouette experiment where, through viscous interaction, angular momentum is transferred to the turntable thus slowing down superrotation. However, under more realistic conditions with reasonably low viscosity, a secondary circulation develops temporarily at lower altitudes and the analogy with the double pirouette experiment is appropriate. Initially, during the spin up process of the atmosphere or during the energization phase of the mechanical analogy, subrotation prevails near the surface, so that the systems can actually receive angular momentum from the planet or the turntable.

Analogy with subrotation. The double pirouette experiment represents a mechanical model for an atmosphere, preferentially heated at the equator. Our simplified description of this analogy, aimed toward a basic understanding of superrotation, however, cannot be adopted for subrotation. In reality, the changes in the moment of inertia due to the oscillatory motions of the pirouette device are continuous and the Coriolis force must be considered. Under this condition the analysis is more complicated but shows that external compression against the action of the Coriolis force indeed leads to subrotation. This analogy was used to illustrate the subrotation on the sun whose atmosphere is preferentially heated at the poles (Schatten et al., 1981). Analogous to the fluid (e.g. Eq. 3), the mechanical experiment is not symmetrical for large values of super- and sub-rotation when the centrifugal force becomes important.

FLUID DYNAMIC MODEL

In the framework of a spectral model, we shall concentrate here on the lowest order harmonic or rigid shell component of superrotation which is unique in that it alone pocesses a global angular momentum; the angular momenta of the higher order vector spherical harmonics are identically zero. A subsequent paper will discuss the problem of differential rotation and the acceleration of the equatorial region (Hide, 1971) by extending the analysis to higher order modes under consideration of mode coupling and viscous flow (Mayr and Harris, 1981).

In the following sections the model is presented, and boundary conditions employed at the planetary surface are discussed. Steady state solutions are then considered followed by a presentation and discussion of time evolution simulations. The thrust of this study is toward a general understanding of superrotation rather than to provide quantitative results for a specific planetary atmosphere.

To describe atmospheric superrotation, a three dimensional spectral model (Mayr and Volland, 1972; Harris and Mayr, 1975; Mayr et al., 1978, referred to as MH) is adopted whose properties are summarized here: (1) Energy advection and convection associated with large scale circulation are considered. (2) Heat conduction and dissipative processes due to ion drag and viscosity in both the zonal and meridional force balances are included. (3) Vertical and horizontal mass transport including momentum transfer between the major species of the atmosphere are considered in self consistent form. (4) Perturbation theory is applied relative to a globally uniform atmosphere which corotates with the underlying planet. The physical quantities including temperature, densities and winds are represented in an expansion of spherical harmonics and time functions such as Fourier harmonics, which effectively separate latitude and time dependences from altitude variations. (5) The equations of energy, mass and momentum conservation are integrated from the planetary surface up to the exosphere. At the lower boundary, the vertical, meridional and zonal velocities relative to the rotating planet are set to zero, and adiabatic conditions are assumed to prevail. For the upper boundary conditions, the properties of molecular heat conduction and viscosity require that the vertical gradients of temperature and horizontal winds vanish.

In the zonally symmetric circulation (m=0) we consider the "fundamental mode" of superrotation

$$\left.\begin{matrix} \Delta Q_2 \\ \Delta T_2 \\ \\ \rho_2 \\ \vec{W}_2 \end{matrix}\right\} = x_2\,(r)\; P_2^o\,(\Theta) = x_2\,(r)\; {}^1\!/_2\,(3\,\sin^2\Theta - 1) \qquad (23)$$

$$\vec{V}_2 = U_2(r)\,\vec{B}_2^{\,o}\,(\Theta) = \vec{j}_\theta\; U_2(r)\,\sin\Theta\,\cos\Theta \qquad (24)$$

$$\vec{U}_1 = V_1(r)\,\vec{C}_1^{\,0}\,(\Theta) = \vec{j}_\phi\; \Delta\omega r\cos\Theta \qquad (25)$$

where ΔQ_2 is the external heat source, and ΔT_2, $\Delta\rho_2$, $\vec{W}_2$, $\vec{V}_2$, $\vec{U}_1$ are the resulting perturbations in temperature, density and velocity components in the vertical, meridional and zonal directions respectively. Standard notations are used for the scalar- (P_2^0) and vector- ($\vec{B}_2^0$, $\vec{C}_1^0$) spherical harmonics. Due to the orthogonality of spherical harmonics, the contributions to the global angular momentum budget (Eq. 8) from the zonal wind field and the mass redistribution (or changes in the moment of inertia) are

$$\int \rho_o (r) \sum V_n C_n^0 r \cos\theta \, d\tau \equiv \int \rho_o \sum V_n C_n^0 C_1^0 \, d\tau$$
$$= \int \rho_o V_1 (C_1^0)^2 \, d\tau = \int \rho_o \Delta\omega r \cos\theta C_1^o d\tau \qquad (26)$$

$$\omega_p \int \sum \Delta\rho_n P_n^o r^2 \cos^2\theta \, d\tau \equiv \omega_p \int r^2 \sum \Delta\rho_n P_n^o \frac{2}{3} (2P_0^o - P_2^o) \, d\tau$$
$$= -\frac{2}{3} \omega_p \int r^2 \Delta\rho_2 (P_2^0)^2 \, d\tau \qquad (27)$$

Apart from the corotating globally uniform atmosphere, only the fundamental mode or the rigid shell component of rotation (angular velocity independent of latitude) has non-vanishing angular momenta due to $\Delta\omega$ and $\Delta\rho_2$. For higher order modes which represent differential rotation (variable angular velocity as a function of latitude) the global angular momenta are identically zero at each height.

We emphasize again that in this paper no attempt is made to simulate a particular scenario of atmospheric (rigid shell) superrotation which would require a thorough analysis of the energy sources (due to e.g. solar radiation, auroral particles and Joule heating) and external forces (due to e.g. electric fields of solar wind-magnetospheric origin) that are important. For illustrative purposes we chose a global average atmosphere with T_o, ρ_o which corotates with the planetary rotation rate ω_p representing the conditions on Venus. To drive superrotation, a specific heat source ΔQ_2 is adopted which peaks at an altitude of 50 km. With these inputs fixed, the height independent vertical eddy diffusion coeffecient K and the corresponding coefficient of eddy viscosity are varied to examine the effects on atmospheric superrotation. Since we consider here only the fundamental mode or the rigid shell component of superrotation the horizontal eddy diffusion coefficient does not enter the analysis.

BOUNDARY CONDITIONS

Disregarding the complexities due to composition, eight boundary conditions are required to solve the ordinary differential equations that describe the vertical structure of temperature, density and velocity fields. Of these, seven conditions are firmly established: (a) the vanishing vertical and horizontal (zonal and meridional) velocities at the lower boundary, (b) the requirement that the energy equation is satisfied under neglect of heat conduction at the lower boundary and (c) the assumption that the heat conduction-and momentum-fluxes vanish at the upper boundary.

In steady state, one must further require for the rigid shell component of superrotation that the viscous torque or the momentum transfer vanishes at the planetary surface r_o

$$\left(\frac{\partial \Delta\omega}{\partial r}\right)_{r_o} = 0; \quad \left(\frac{\partial}{\partial t} = 0\right) \qquad (28)$$

In implementing the stress free boundary condition (28) it is important to realize that mass redistribution (and the resulting change in moment of intertia) is one of the three elements that balance the steady state angular momentum budget (see Eq. 8). This mass redistribution also produces a pressure variation at the surface, Δp_2, which, a priori, is not known. Also contributing to the momentum budget is the atmospheric momentum exchange with the planet, ΔL, during the spin up process which is related to the surface stress. We require, therefore, that the surface pressure or density amplitudes associated with P_2^o are chosen such that the stress free boundary condition (Eq. 28) is satisfied in steady state

$$(\Delta p_2)_{r_o} + (\frac{\partial \Delta \omega}{\partial r})_{r_o} = 0 \qquad (29)$$

In our computer model an iterative procedure is used combined with binary search to enforce condition (29).

It turns out that for a slowly rotating planet such as Venus this rigorous approach to the boundary value problem produces results reasonably close to those obtained with the (redundant) assumption of hydrostatic equilibrium. This indicates that without constraint the system tends to find its natural state and in turn justifies using the hydrostatic condition for the time dependent solution.

The concept of influencing the angular momentum budget by varying the surface pressure distribution proves valuable in still another way. Thus a computer experiment can be performed in which the surface pressure distribution is chosen so that the atmospheric angular momentum is invariant during the transition from corotation to superrotation

$$(\Delta p_2)_{r_o} + \Delta L = 0. \qquad (30)$$

STEADY STATE

For a fixed heat source, ΔQ_2, which peaks at the equator, Figure 4 shows computer results of rigid shell superrotation in steady state ($\frac{\partial}{\partial t} = 0$). Two different values of the vertical kinematic viscosity corresponding to eddy diffusion coefficients of $K = 10^4$ and 10^9 are chosen. Solid lines represent realistic conditions for which the viscous stress or the vertical gradient of the zonal velocity is forced to zero at the planetary surface (Eq. 29). Under this condition, rigid shell superrotation prevails for both viscosities throughout the atmosphere.

In analyzing the angular momentum budget we evaluate the integrals of superrotation and mass redistribution from Eq. 8, or Eqs. 26 and 27

$$S = \int \Delta\omega(r)\rho_o r^2 \cos^2\Theta d\tau \qquad (31)$$

$$M = -\omega_p \int \Delta\rho_2(r) \overset{0}{P}_2(\Theta) r^2 \cos^2\Theta d\tau \qquad (32)$$

where $\Delta L = M-S$. The ratios $R = \frac{M}{S}$ are given in Figure 4 and reveal that in the case of large viscosity the atmosphere has lost angular momentum to the planet $\Delta L > 0$; while for the smaller value of viscosity the atmosphere has gained angular momentum from the planet $\Delta L < 0$ during the spin up process.

Retaining the input parameters, we show for comparison a second set of solutions (dashed lines) which are artificial and diagnostic in nature. Here the surface pressure amplitude is adjusted (Eq. 30) so that, under differential heating, $\Delta L=0$. For large viscosity superrotation prevails all the way down to the surface. In that state the velocities are consistently larger than those under the stress free boundary condition (solid line), and a surface torque $(\frac{\partial\Delta\omega}{\partial r})_{r_o} > 0$ tends to slow down the excess superrotation while transferring angular momentum to the planet. Obviously, such a condition cannot prevail in steady state. Initially, during the transition from corotation under uniform heating to superrotation under nonuniform heating the atmosphere tends to conserve its angular momentum. Solutions for $\Delta L=0$ are therefore interpreted as heuristic representations of the transient state which, combined with the true steady state solutions $(\frac{\partial\Delta\omega}{\partial r})_{r_o} = 0$, may serve to illustrate the mechanics of the spin up process.

For small viscosity the situation is reversed. With $\Delta L = 0$ the velocities are now consistently lower than those under the stress free boundary condition. The atmosphere actually subrotates (minus) at lower altitudes. In this state a negative surface torque $(\frac{\partial \Delta\omega}{\partial r})_{r_o} < 0$ induces the atmosphere to pick up angular momentum from the planet which contributes to the larger superrotation.

TIME EVOLUTION

Steady state solutions contain no information on causality and therefore cannot afford a complete understanding. An example is the Hadley cell: Mass transport in the upper leg of the circulation away from the source and in the lower leg of the circulation toward the source are in balance. But in the transient process of establishing the circulation, it is precisely the inbalance of mass flow which is building up the pressure in the polar region at lower altitudes, and it is that pressure force which keeps the atmosphere in motion.

We shall find that in principle the situation is similar for superrotation. There is, however, an important additional complication in that the atmosphere is coupled to a planet with virtually unlimited angular momentum. While the atmosphere's mass budget that governs the Hadley cell formation is closed, its angular momentum budget which governs superrotation is not.

In describing the spin up process we assume that up to the time $t = 0$ the atmosphere is heated with a globally uniform source, $\Delta Q_2 = 0$, and corotates with the planet, $\Delta\omega = 0$. A globally nonuniform source is introduced as a step function

$$\begin{aligned} \Delta Q_2 (t) &= 0; \; t < 0 \\ \Delta Q_2 (t) &= \text{const.}; \; t > 0. \end{aligned} \tag{33}$$

Using Laplace transformations, the time dependences of the heat source (33) and the physical variables X(t) (Eqs. 23, 24, 25) are described in the form

$$\Delta q_2 (f) = \int_0^\infty e^{-ft} \Delta Q_2 (t) \, dt = \frac{1}{f} \qquad (34)$$

and

$$q_2 (f) \, x(f) = \int_0^\infty e^{-ft} X(t) \, dt, \qquad (35)$$

respectively. With the time dependence e^{ft}, f being a variable time constant, the MH model is used to compute the transfer function x(f) for unit heat input. The integral equation (35) is solved with the Gauss-Legendre technique of Bellman et al, 1966. Using six Gaussian points, the calculations are carried out over various timescales to provide the necessary resolution.

For the same input values that produced the results of Figure 4, the time evolution of atmospheric superrotation is simulated and is given in Figures 5, 6, and 7. For large viscosity the atmosphere superrotates at all altitudes throughout the spin up process. Immediately above the surface (2 km altitude) a transient velocity maximum developes. In time, this velocity returns to a small value near zero. The corresponding (transient) surface stress has the effect of decelerating and accelerating the atmosphere and the underlying planet respectively. That momentum transfer is also reflected in the steady state superrotation rates throughout the atmosphere which are lower than those during the initial phase of the spin up process. This confirms our earlier interpretation of the solutions in Figure 4. Judging from the time evolution of the height integrated superrotation rate we find that the atmosphere spins up like a solid body and transfers angular momentum to the planet, in analogy with the single pirouette experiment (Figure 2).

For the smaller viscosity the results are entirely different (Figure

6). The increased tendency toward geostrophy, combined with the increase of surface pressure toward the poles (due to meridional mass transport) induce the atmosphere to subrotate temporarily during the initial phase of the spin up below a critical level at about 40 km. Near the surface, the resulting viscous shear thus allows the atmosphere to pick up angular momentum from the planet which is transported upward and, in time, during spin up, contributes to the large superrotation rates that eventually prevail at high altitudes in steady state. Leovy (1973) has suggested this process by invoking the concept of geostrophic adjustment. The phase of this wavelike phenomenon propagates downwards while energy and momentum are propagating upwards. We believe that below the critical level the angular momentum is carried upward by viscosity and gravity waves. Above that level, however, the downward propagating mode of viscosity waves may be more important while gravity waves continue traveling upwards until the spin up process is completed. The location of the critical level depends on the source distribution, and moves downwards as the viscosity increases.

The time evolution in the height distribution of mass flow is shown in Figure 7. Unlike the case with large viscosity where the change in moment of inertia (Eq. 32) is important for the angular momentum budget ($R \lesssim 1$), the rotational component (Eq. 31) dominates for $R>1$. Thus, a short period (2 months) after energization the inertia of the system still tends to conserve angular momentum, and the regions of super- and sub-rotation are almost in balance. This condition is heuristically described with the $\Delta L = 0$ solution in Figure 4b. As time progresses the atmosphere accumulates, through viscous stress, an increasingly larger amount of angular momentum from the planet until eventually, in steady state, a stress free boundary condition is established and superrotation, $\Delta\omega \geq 0$, prevails all the way down to the surface.

Considering that for low viscosity a large part of the atmosphere's rotational energy comes from the planet, a scale analysis yields

$$K \frac{\partial \Delta\omega}{\partial r} \tau \propto \Delta \omega \propto \frac{1}{K} \qquad (36)$$

where the symbols K, $\frac{\partial \Delta\omega}{\partial r}$ and τ stand for the viscosity, the vertical velocity gradient near the surface during the spin up process and the spin up time respectively. A decrease in viscosity is overcompensated by an increase in the gradient and spin up time so that the atmosphere can receive an increasingly larger amount of angular momentum. The meridional circulation transports angular momentum upwards and its time response to the solar radiative input is almost instantaneous. By comparison, the viscous flow of angular momentum from the planet is slow and slows further down as the viscosity decreases. Thus with decreasing viscosity, larger velocity shears must be built up and a longer spin up time is required before the viscous flow can match the convective momentum transport.

The dependence of atmospheric superrotation on viscosity is illustrated in more detail with Figure 8. For a fixed heat source and a wide range of eddy diffusion (viscosity) coefficients, the velocities are shown at various altitudes. To characterize the angular momentum budget, the ratio between mass and rotational momenta, R, is presented. We identify two regimes: (a) For lower viscosities and $R < 1$ ($\Delta L < 0$) the atmosphere receives an increasingly larger amount of angular momentum as K decreases. The velocity is inversely proportional to viscosity. (b) At the higher viscosities $K > 2 \times 10^8$ the atmosphere is losing angular momentum to the planet and the velocity decrease is more precipitous. For $\Delta L > 0$ and $\Delta L < 0$ the analogies with the single and double pirouette experiments are appropriate, respectively.

CONCLUSION

The spin up process leading to superrotation is discussed for the linear regime of the thermally driven zonally symmetric circulation. Depending on the state of turbulence the atmosphere can go through two different modes. For large viscosity ($K > 2x10^8$, in this study) it behaves almost like a solid body, tightly coupled to the planet. The superrotation rates are small and the spin up times are short. The angular momentum of superrotation is "driven" by the decrease in the moment of inertia due to mass redistribution by the meridional circulation. During the spin up time the atmosphere transfers angular momentum to the planet. This process has an analogy in the single pirouette experiment.

For lower values of viscosity there is an important additional degree of freedom in the vertical momentum distribution, and the analogy with the double pirouette experiment is appropriate. Mass can more freely return from the poles in the lower leg of the meridional circulation. The change in the moment of inertia is therefore not as large. This return flow combines with the increased tendency toward geostrophy to cause subrotation in the lower region of the atmosphere during the initial phase of the spin up process. Through viscous shear the planet applies a torque to the atmosphere which propagates up and in time supplies the angular momentum for large super-rotation, consistent with a suggestion by Leovy (1973). The spin up times in this process are long. Below a critical altitude level which is about 40km for $K=10^4$ the momentum flow during spin up is apparently carried by upward propagating gravity and viscosity waves. For a critical viscosity of about 2 x 10^8, in this analysis, the atmosphere does not exchange angular momentum with the planet.

It is again emphasized that no attempt was made here to describe differential rotation and equatorial jets, such as observed on Jupiter and Saturn. By expanding the spectral model to include higher order modes, the concept of mode coupling is used to describe differential rotation in an atmosphere with zonal and meridional viscous stresses (Mayr and Harris, 1981). Momentum is thereby allowed to cascade from lower order modes (which "absorb" most of the solar energy) to higher order modes, and the feedback on the rigid shell component of superrotation is considered. At altitudes above the center of the meridional circulation, where the specific heat input is relatively large, angular momentum is effectively advected poleward, except near the equator where the meridional winds and the Coriolis force vanish and where the horizontal momentum transport must be carried by horizontal diffusion (viscous flow). This causes the atmospheric rotation rate to decrease at low latitudes and increase at high latitudes, relative to the rigid shell component of superrotation; inducing a tencency for the equatorial region to subrotate. However, below the center of the meridional circulation and below the maximum energy input, energy and angular momentum are advected from high to low latitudes. Thus the equatorial region can be accelerated to the extent that positive jets develop. Again, near the equator, angular momentum must be carried by horizontal diffusion; the equatorial minima in the angular velocities of Jupiter and Saturn being evidence for down gradient momentum flow toward the equator.

In this model the tropospheres of Jupiter and Saturn are assumed to be convectively unstable with a superadiabatic temperature lapse rate accounting for global scale energy diffusion toward higher altitudes where radiation to space becomes important. Under this condition, the upward motions in the direct and indirect atmospheric circulations imposed by insolation can then supply dynamic energy for "localized" equatorial heating which in turn drives equatorial superrotation. Preferential solar heating at low latitudes induces the ordered structure in the atmospheric motions. The efficiency of this process is determined by the (in)stability of the troposphere, the energy conversion in the interior and the radiative loss from the upper troposphere and stratosphere. Assuming zonal symmetry and thermal forcing by solar differential heating, we believe that four factors are important in understanding the atmosphere dynamics of these planets: (1) Their large size and rotation rate produce through mode coupling a multiple Ferrel-Thomson type circulation with upward motions at low latitudes. Due to the lower mass, size and rotation rate of Saturn its equatorial jet is much broader than that of Jupiter. (2) Energy from the interior is diffusing radially outwards and maintains a supeadiabatic lapse rate which provides the condition for trapping some energy that is advected toward the equator in the lower leg of the meridional circulation. (3) The eddy diffusivity is sufficiently low to require large temperature and velocity gradients for the energy and momentum balances, the atmosphere of Saturn being perhaps less turbulent than that of Jupiter. (4) The observed temperature differences on both planets (Hanel et al., 1981) cause the cloud layers, the tracers of wind systems, to be formed higher up below the tropopause on Jupiter where the upper and lower legs of the meridional cell produce a complex interference pattern in the angular momentum budget but deeper down in the convection region of the troposphere on Saturn where meridional energy and momentum advection toward th equator dominate.

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Figure Captions

Figure 1: Planet with rotation axis penpendicular to the orbital plane around the Sun. Schematic diagram illustrating the energy deposition in the equatorial region, the Hadley cell and the resulting mass relocation toward the poles. Under the tendency to conserve angular momentum the dynamically induced decrease in the moment of inertia produces superrotation.

Figure 2: Single pirouette experiment as analogy for an atmosphere with high viscosity. A mechanical device (figure), its moment if inertia being cyclically changed by an external force (representing solar differential heating and pressure gradient), is frictionally coupled to a rotating turntable. Averaged over an "experiment cycle" the figure superrotates receiving the angular momentum from the (average) decrease in moment of inertia. During the spin up process, angular momentum, ΔL, is frictionally transferred to the turntable which slows down superrotation.

Figure 3: Double pirouette experiment as an analogy for an atmosphere with (reasonably) low viscosity. Compared to Figure 2 a second figure provides the added degree of freedom for the vertical momentum distribution. By alternatingly changing the moments of inertia of both figures they superrotate in the time average. But this can now be accomplished without changing the total moment of inertia, thus the device is capable of picking up the angular momentum for superrotation from the turntable.

Figure 4: Computed vertical distribution for the rigid shell component of "superrotation", $U\cos\Theta$. Velocities are zero at the surface. In solid lines steady state solutions are presented with stress free

boundary conditions. In dashed lines an artificial condition is shown for which the angular momentum of the atmosphere is conserved, $\Delta L = 0$. It is interpreted as a heuristic representation of the transient state during spin up. The results are given for two viscosities corresponding to eddy diffusion coefficients of $K = 10^9$ (Figure 4a) and $K = 10^4$ (Figure 4b). Plus and minus signs indicate super- and sub-rotation respectively.

Figure 5: Simulation of the time evolution leading to superrotation for $K = 10^9$. The latitude dependent component of solar heating which causes superrotation is turned on abruptly as indicated in the dashed line. Note that the atmosphere super-rotates throughout the spin up, with the positive velocity above the surface providing the viscous stress for momentum transfer to the planet during that process.

Figure 6: Results for the same heat source as in Figure 5 except that $K = 10^4$. The atmosphere at lower altitudes subrotates during the initial phase of the spin up. The negative velocity above the surface is picking up a torque form the faster rotation planet which provides most of the momentum for superrotation. Note that in both results (Figures 5 and 6) superrotation prevails all the way down to the planetary surface in steady state. With low viscosity, the spin up times are longer and the superrotation rates are larger compared to the corresponding values for large viscosity.

Figure 7: The results from Figure 3 are shown for mass flow, $\rho_o U$, at various (approximate) times during the spin up process. Note that initially, about 2 months after energization, the angular momenta

in regions of super- and sub-rotation almost balance each other, thus justifying our interpretation of the $\Delta L = 0$ solutions in Figure 4.

Figure 8: Dependence of the steady state superrotation rate on viscosity. Also shown is the ratio R between the total angular momenta due to changes in the atmospheric moment of inertia and rotation rate. For $R \gg 1$ superrotation is primarily driven by the mass redistribution (Figure 4a and 5) with angular momentum being transferred to the planet during spin up. For $R \ll 1$ superrotation is primarily driven by the angular momentum picked up from the planet during spin up (Figures 4b, 6 and 7). For $R > 1$ ($\Delta L > 0$) an analogy with the single pirouette can be made, while for $R \leq 1$ ($\Delta L \leq 0$) the analogy with the more complicated double pirouette must be invoked. For the critical value of $K = 2 \times 10^8$ the atmosphere does not exchange angular momentum with the planet.

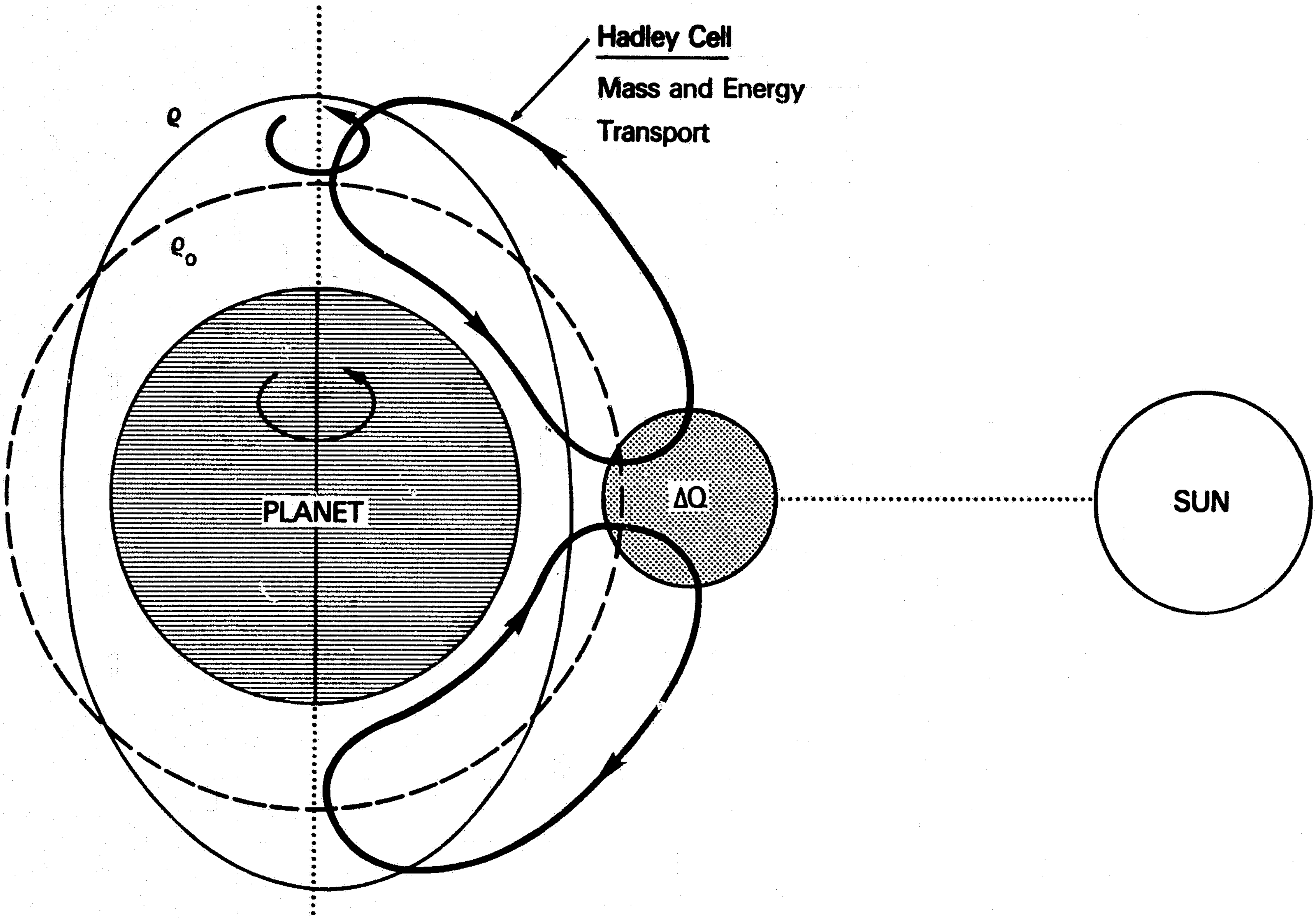

Hadley Cell
Mass and Energy
Transport
ϱ
ϱ_o
PLANET
ΔQ
SUN

Figure 1

PIROUETTE

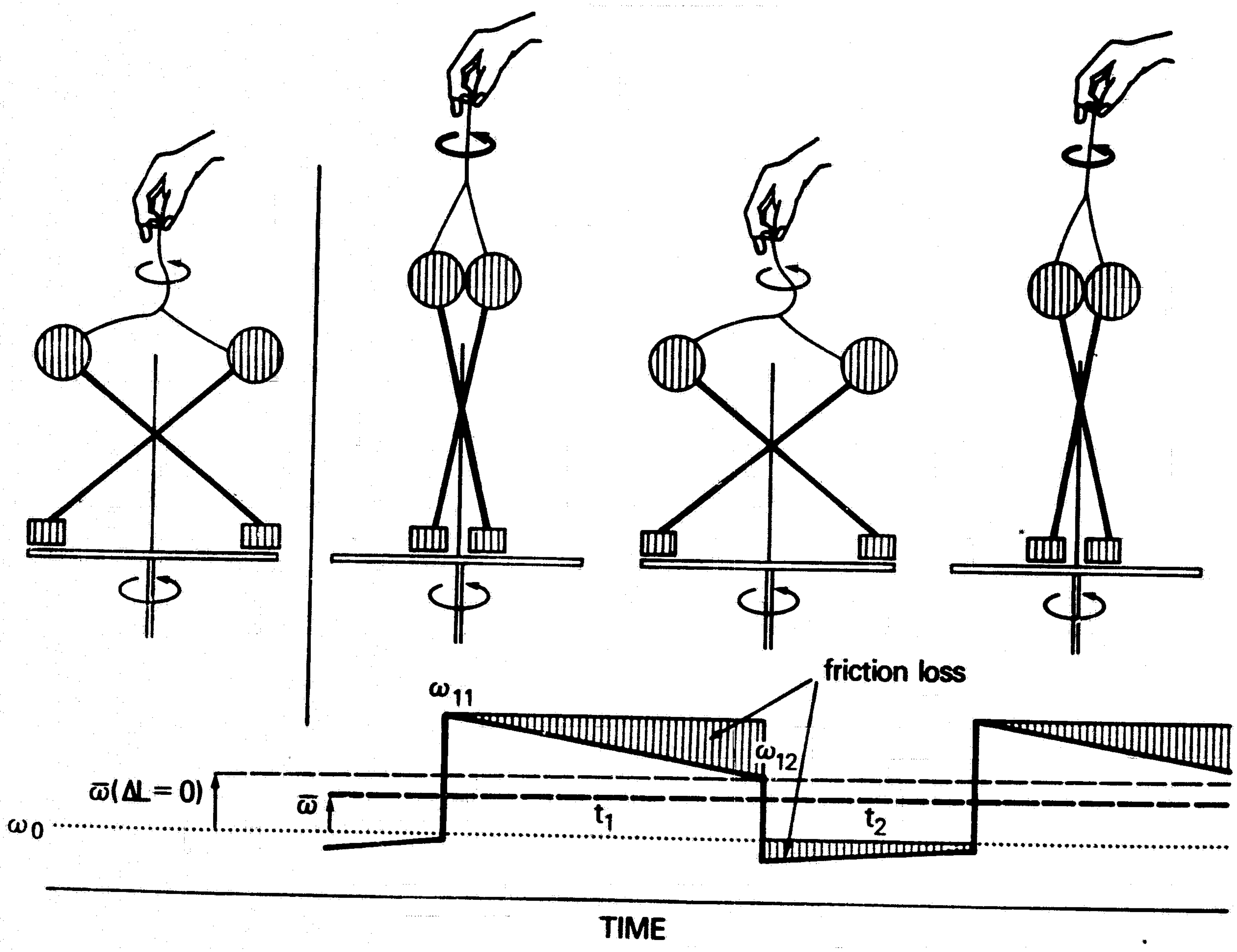


Figure 2

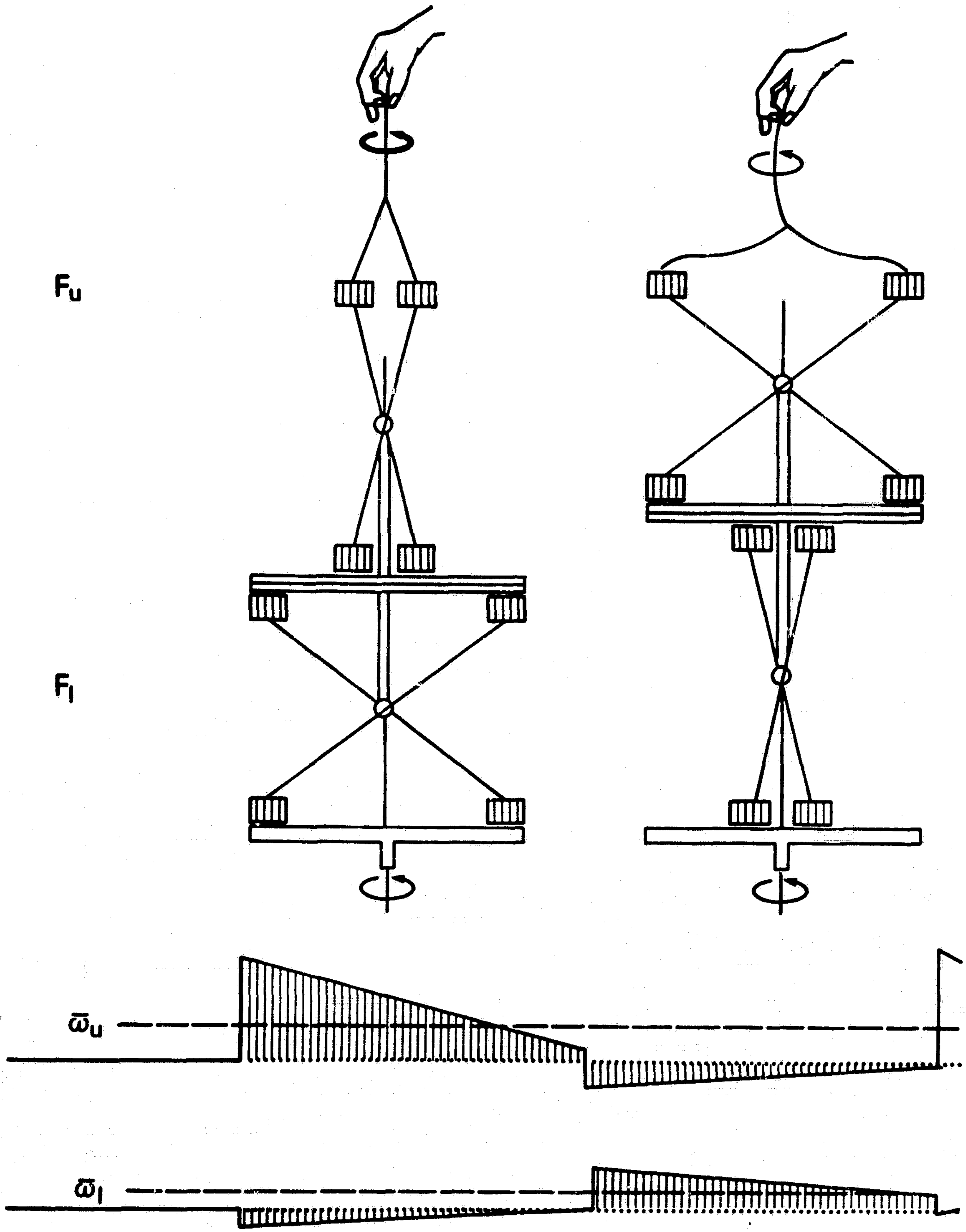
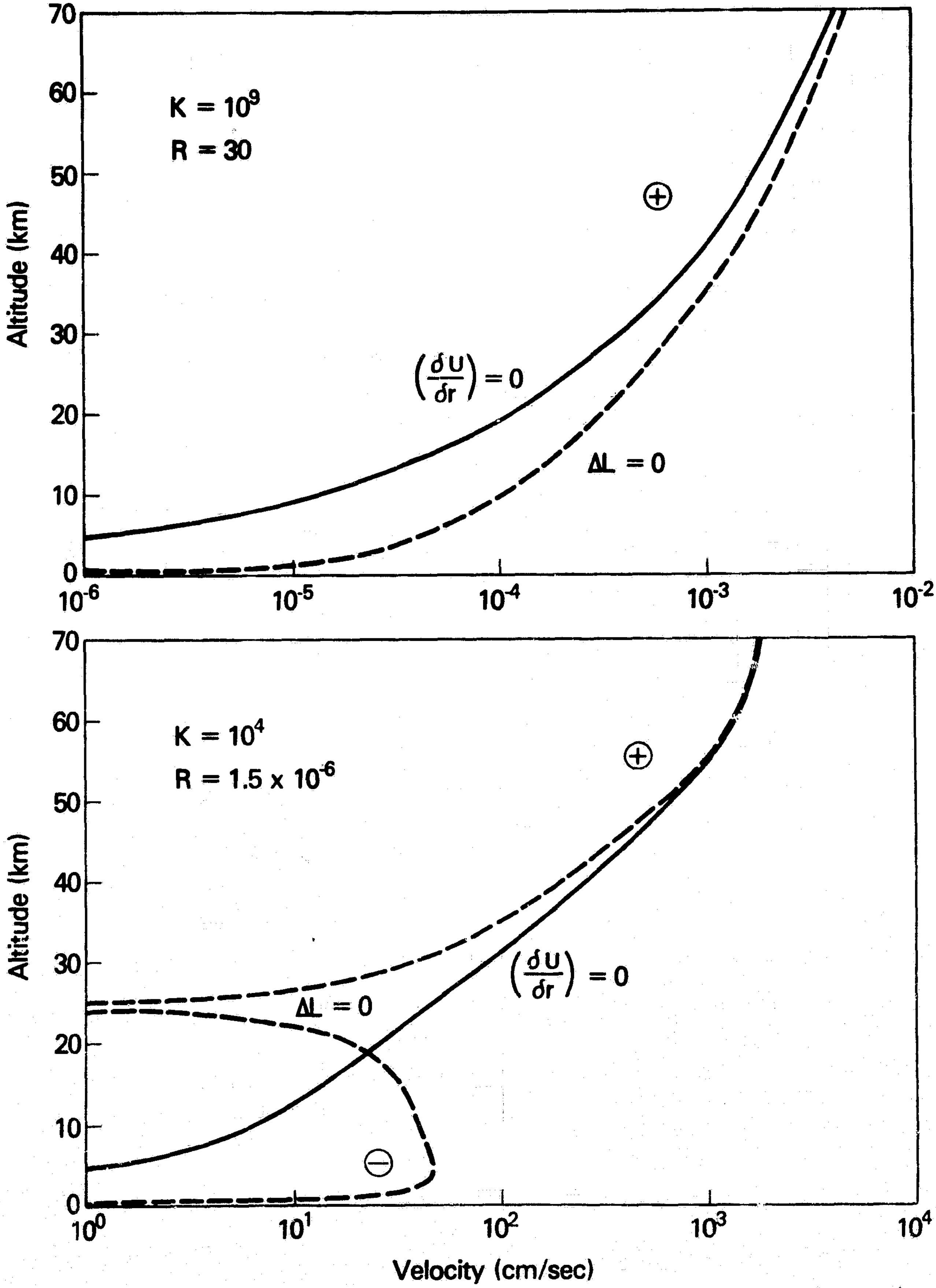


Figure 3



K = 10^9
R = 30
(δU/δr) = 0
ΔL = 0
Altitude (km)
K = 10^4
R = 1.5 x 10^-6
(δU/δr) = 0
ΔL = 0
Altitude (km)
Velocity (cm/sec)

Figure 4

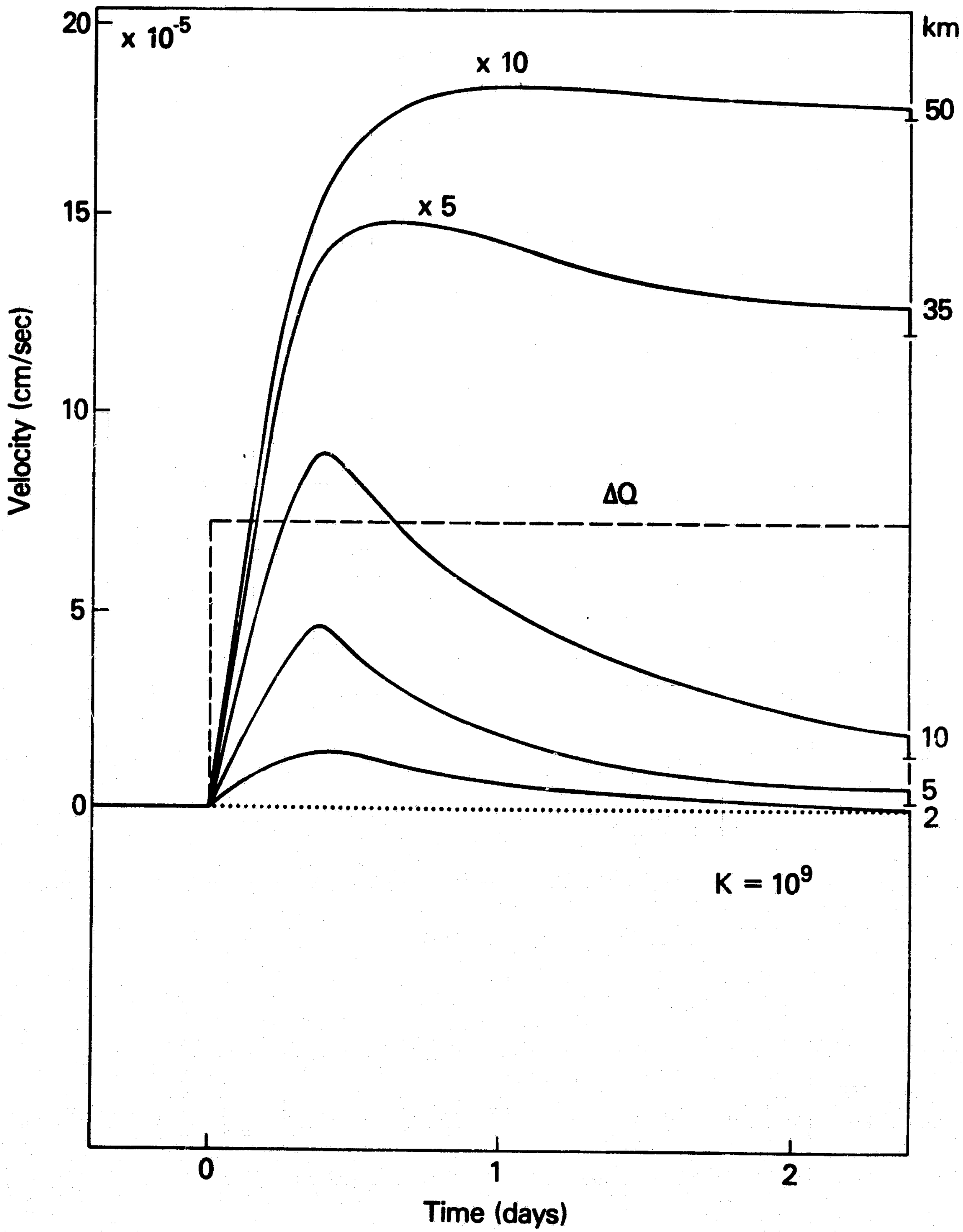

20
15
10
5
0
x 10-5
Velocity (cm/sec)
x 10
x 5
ΔQ
K = 10^9
km
50
35
10
5
2
0
1
2
Time (days)

Figure 5

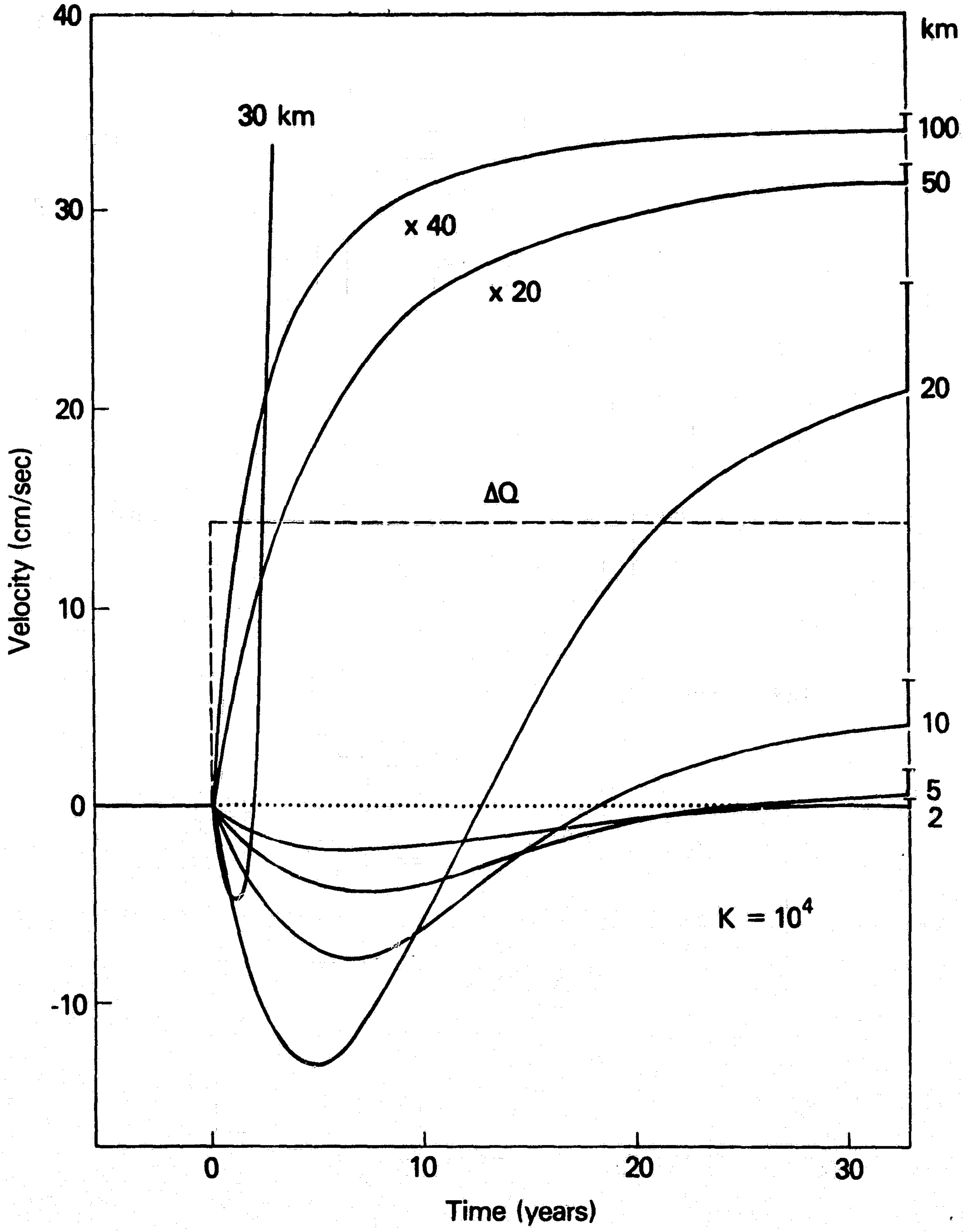
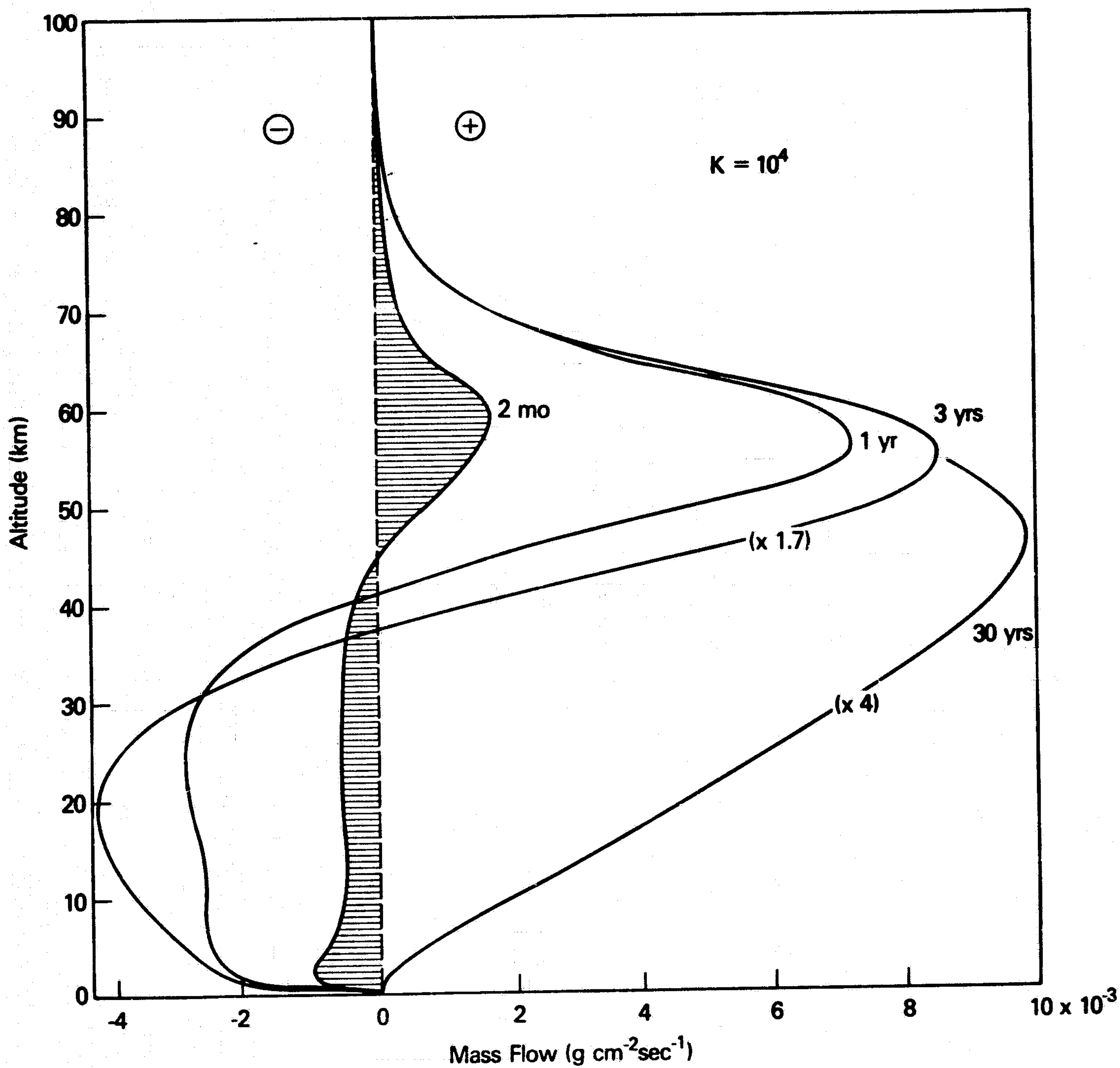

40
30
20
10
0
-10
Velocity (cm/sec)
30 km
x 40
x 20
ΔQ
K = 10^4
km
100
50
20
10
5
2
0
10
20
30
Time (years)

Figure 6



K = 10^4
⊖
⊕
2 mo
1 yr
3 yrs
30 yrs
(x 1.7)
(x 4)
Altitude (km)
100
90
80
70
60
50
40
30
20
10
0
-4
-2
0
2
4
6
8
10 x 10^-3
Mass Flow (g cm^-2 sec^-1)

Figure 7

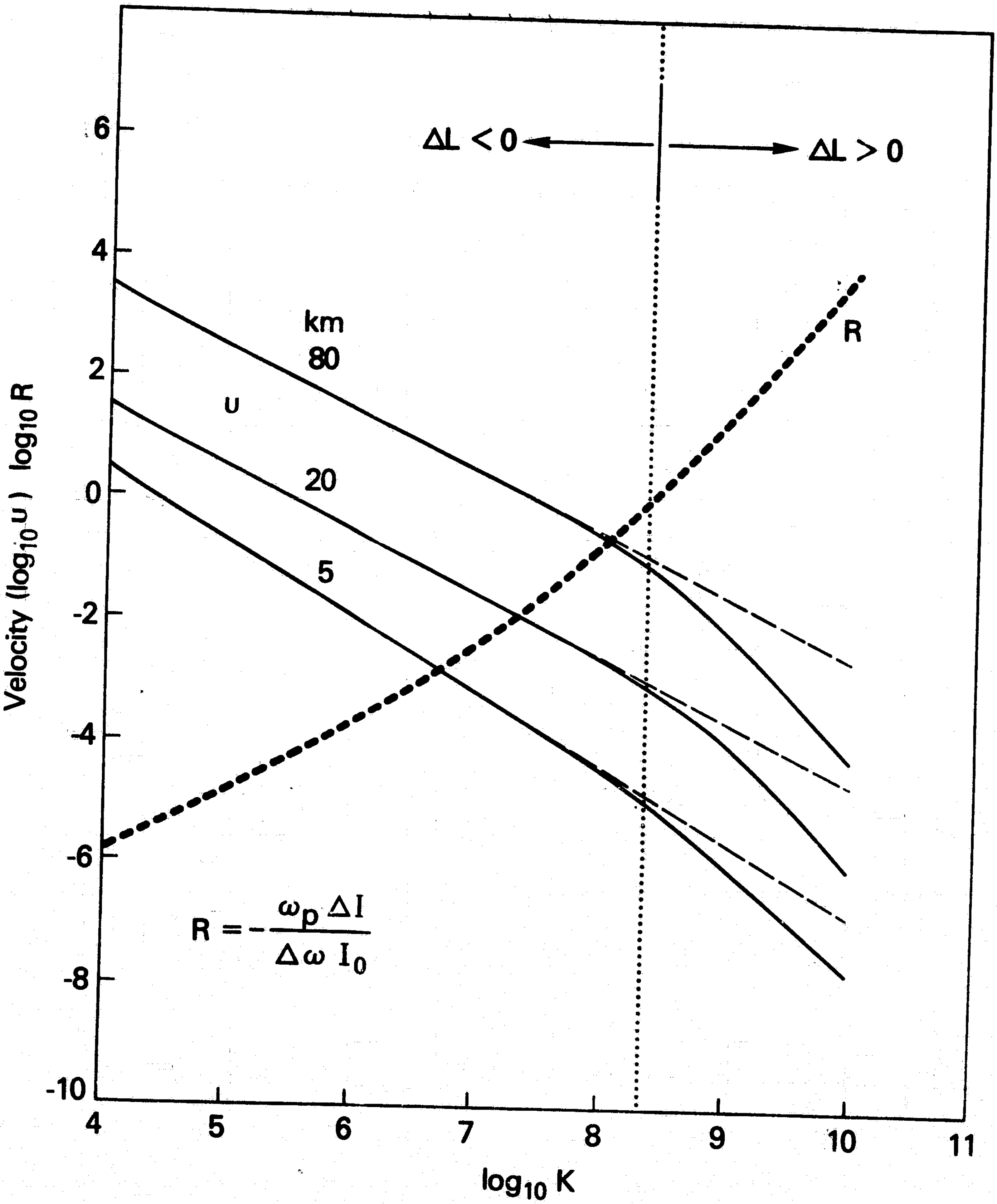

ΔL < 0
ΔL > 0
km
80
20
5
u
R
$R = -\frac{\omega_p \Delta I}{\Delta\omega\, I_0}$
Velocity ($\log_{10}u$) $\log_{10}R$
$\log_{10}K$
6
4
2
0
-2
-4
-6
-8
-10
4
5
6
7
8
9
10
11

Figure 8