## MODELING AND ANALYSIS

 OF
## POWER PROCESSING SYSTEMS (MAPPS)

FINAL REPORT
VOLUME I - TECHNICAL REPORT
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Prepared for:
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LEWIS RESEARCH CENTER
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CONTRACT NAS3-2105I

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| 16. Abstract <br> Compute' aided design and analysis techniques are applied to Power Processing Equipment. This project is a further continuation of the modeling and analysis of power processing systems reported in CR-134686, CR-135173, and CR-135174. <br> This report, Volume I, covers work performed on the following Power Processing Modeling and Analys is topics: <br> (a) Discrete Time Domain analysis of ewitching regulators for performance analysis. <br> (b) Design Optimization of Power Converters using Augmented Lagrangian Penaity Function Technique. <br> (c) Investiyation of Current-Injected Multiloop Controlled Switching Regulators. <br> (d) Application of Optimization for Navy VSTOL Energy Power System <br> The discussion includes the generation of the mathematical models and the development and application of computer alded design techniques to solve the different mathematical models. <br> Recommendations are made for future work that would enhance the application of the computer aided design techniques for Power P.ocessing Systems. <br> Volume II contains the supporting appendices. |  |  |  |  |
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### 1.0 INTRODUCTION

"Modeling and Analysis of Power Processing Systems," (Contract NAS3-21051), provides basic analytical tools, in the form of mathematical models, and the computer-aided techniques required to solve those models, to gulde engineers in the various aspects of power proressing equipment and system design. Early work in this area included a feasibility study done by TRW Systems ${ }^{[1]}$ and General Electric ${ }^{[2]}$, and the initial modeling and analysis performed by TRW Systems ${ }^{[3]}$ and California Institute of Technology ${ }^{[4]}$. The present effort, performed by TRW Systems and Virginia Polytechnic Institute and State University, is an extension of that early work.

The four tasks completed on this contract include the following:

- Performance Analysis of Buck, Boost, and Buck-Boost DC-DC Converters using dual-loop feedback control system ${ }^{[5],[6]}$.
- Design Optimization for Boost and Buck-Boost DC-DC Converters. (Design Optimization for Buck DC-DC Converters is contained in Refneence [3].)
- Investigation of Current-Injected, Multiloop-Controlled Switching Regulators.
- Application of Design Optimization to U.S. Army VSTOL Emergency Power System.

A technical summary is presented in the following chapters on each of these tasks.

### 2.0 DC-DC CONVERTERS.

### 2.1 Introduction.

The following sections are both tutorial, and application oriented. Because of the variety of operating converter power and control schemes, the tutorial is necessary to understand the various analytical procedures and their use. Once understood, the material may be applied to a designers specific needs through the various analysis and simulation subprograms which are provided. Starting with a description of the basic characteristics of $D C-D C$ converters, a general evaluation of discreet time domain aralysis, and the performance analysis of the buck, boost, \& buck-boost converters is given.

Due to the finite flux capacity of the inductive elements, a $D C-D C$ converter must be oscillatory in nature. The oscillation is achieved by cyclically operating the power switct of the converter in conduction and non-conduction state. Consequently, the converter control system must be able to accept an analog signal obtained from the sensing circuit and the reference, and to convert it into discrete time intervals in controlling the conduction and non-conduction of the power switch.

The electrical performance of a DC-DC converter depends primarily on the quality of its control system. The performance characteristics of interest to a converter designer include stability as well as the converteroutput response to step and sinusoidal disturbances, both from the line and the load.

### 2.1 Introduction. (Cont.)

Functionally, a $D C-D C$ regulated converter can be divided into two parts: A power circuit, and a control circuit. By definition, the power circuit handles the energy transfer from the source to the load. Three most commonly used power circuits are the buck, the boost, and the buckboost.

The control circuit manages the rate of the source-load energy transfer as a function of the load demands. During nominal steady-state and transient operations, the control objectives are associated with (A) the tracking of a certain controlled quantity in accordance with a given reference, and ( $B$ ) the compliance to converter specifications such as the system response to step or sinusoidal line and load disturbances, and to external command signals. During transicnt operations, the control objective is to limit electrical-stress for all the elements associated with the converter, providing effective protection against catastrophic/degradation types of fallures. A control circuit thus serves the multiple functions of regulation, command, and protection.

A generalized standardized control module (SCM) has been developed to implement the above control functions (Reference $5,6,7$; . For the purpose of this report, the SCM Control circuit has been selected to accommodate the power circuits mentioned above.

A SCM-controlled DC-DC converter is shown in Fig. 2.1. The power circuit, occupying the upper half of the block diagram, processes the transfer of energy from a raw input $V_{i}$ to a regulated ourput $V_{0}$. Three basic power stages are shown here: Buck, boost, and buck-boost. The control circuit regulates the rate of energy transfer. It receives an analog signal ( $V_{0}$ ) from the power-stage output, and delivers a discrete-time interval signal(d) to achieve the required on-off control of the switch in the power stage. The discrete-time voltage or current pulses generated in the power stage are averaged by an LC filter having a much longer time constant than the discrete-time pulse intervals. The averaged output therefore contains negligible switching-frequency components., and can be regarded as an analog signal containing only lower-frequency information.


### 2.1 Introduction (Cont.)

The SCM, occupying the lower half of the block diagram, performs the control function within the converter. It contains an Analog Signal Processor (ASP) and a Digital Signal Processor (DSP). Implementations of both ASP and DSP are standardized: They combine to provide the required analog signal to discrete-t: ine interval conversion. The key feature of the SCM is the utilization of an inherent AC switching-frequency signal within the power stage. This utilization is in addition to the conventional DC sensing of output $V_{0}$. The sensed $A C$ signal and the $D C$ error are processed by the ASP. As a result, an adaptive stability is obtained which is independent of the filter parameter changes. The SCM control function is completed by the DSP, which processes the control-signal output from the ASP in conjunction with a prescribed control law, and operates the "ON-OFF" of the power switch via duty-cycle signal, $d$.

As stated previousiy, the control-circuit functions also include command and protection. The command function generally requires the converter to respond to an external signal capable of overriding control signal(d) in determining the on-off of the power switch. The protection function includes power-component peak-stress limiting and the converter shutdown in the event of a sensed abnormality such as overvoltage, undervoltage, or -vercurrent beyont a predetermined, tolerable level and duration. These functions are performed within SCM by the DSP $[5,6,7]$.

The three basic functional blocks of an SCM-controlled converter are shown in the block diagram of Figure 2.2.


Fig. 2.2. switching regulator block diagram

### 2.2 Performance Analysis and Simulation Technigues.

### 2.2.1 Nonlinear Operation of Switching Regulator.

A dc-dc switching-regulated converter is inherently a nonlinear device. The first major nonlinearity exists in the power stage, and is due to the operation of the power switch. Different circuit topologies correspond to the respective on and off time intervals in the switching cycle. The second major nonlinearity exists in the Digital Signal Processor (DSP). Hamonic frequencies, which are multiples of the input disturbance frequency, are contained in the DSP output. Because of such system nonlinearities, difficulties are encountered in reaching performance assessments of various system performances such as stability, attenuation of sinusoidal/step line disturbances, and response to sinusoidal/step load disturbances.

The Power Stage nonlinearity will be elaborated here, and various analytical approaches capable of treating this type of nonlinearity will be discussed. In light of the stated objective of this program, which is the performance analysis and simulation of dc-dc switching regulators, a specific approach will then be selected as the basic analytical tool for the entire program.

### 2.2.2 Power Stage Nonlinearity.

Each of the power stages shown in Figure 2.1 can be divided as a function of the output filter inductor MMF status. In Figure 2.3(A), often referred to as "continuous-conduction" or Mode 1 operation, the MMF ascends during on time $T_{\text {on }}$ when the power switch in $O N$ and the diode is OFF, and descends during $T_{F 1}$ when power switch is OFF and the diode is $O N$. Notice that the MMF never vanishes in the output inductor. In Figure 2.3(B). often referred to as "discontinuous-conduction" or Mode 2 operation, thie MMF ascends from zero MMF at the beginning of $T_{o n}$, and descends back to zero during $T_{F 1}$. An additional off time $T_{F 2}$ exists when both the power switch and the diode are OFF, during which the inductor MMF remains zero, and load current is supplied entirely by the output-filter capacitor.

FIG. 2.3 INDUCTOR CURRENT WAVEFORMS (A) CONTINUOUS MWF OPERATION
(B) DISCONTIMUOUS CURRENT OPERATION

### 2.2.2 Power Stage Nonlinearity - (Con't)

Topologies of the buck, boos \%, and buck-boost power stages correspond to $\mathrm{T}_{\text {on }} \mathrm{T}_{\mathrm{Fl}}$, and $\mathrm{T}_{\mathrm{F} 2}$, are lllustrated in Figures 2.4 to 2.6 , respectively. Even though a given power stage is inear for each time interval, the combination of all different 1 inear circuits for the purpose of analyzing a complete cycle of switching-regulator operation becomes a piecewise-1inear nonlinear analysis problem.

The difficulty is integrating these different topologies and collectively evaluating their responses to various line/load disturbances having a much longer time period than individual $\mathrm{T}_{\mathrm{on}}$, $\mathrm{T}_{\mathrm{F} 1}: \mathrm{T}_{\mathrm{F} 2}$.

The basic modeling approaches for conductin! performance analysis and simulation include the following:

- Discrete Time-Domain Analysis
- Average Time-Domain Analisis
.... Small Signal Analysis
- Exact Frequency-Domain Analysis)
- Discrete Time-Domain Simulation .... Large Signal Analysis

Techniques for all four approaches have been established in the previous MAPPS program phases ${ }^{[1,3,4]}$. Their respective utility for a given application depends on the analysis objective, the desirad accuracy, the control-circuit type, the nature of the disturbance, and perhaps most important, the user's analytical backgoround. A summary description of all four approaches is presented in Table 2-1.

The exact frequency domain analysis will not be pursued in the proposed program due to its difficulty in incorporating the input filter which often causes major complications in the design for the required performances in regulator stability as well as in audiosusceptibility and output impedarice.


FIGURE 2.4 BUCK PONER STAGE TOPOI.OGIES


FIGURE 2.5 BOOST POWER STAGE TOPOLOGIES

(A)

(B)
${ }^{T}$ F2


FIGURE 2.6 BUCK-BOOST POWER STAGE TOPOLOGIES

## ORIGINAL PAGE IS OF POOR QUALITY

TABLE 2-1 Major Performance Analysis Approaches


### 2.2.2 Power Stage Nonlinearity - (Con't.) <br> The performance requirements associated with the analysis and

 simulation effort are:- Regulator control-loop stability (local)
- Audio susceptibility (attenuation of source small signal disturbance)
- Transient response (response to source/load large-signal step change)

For the first two performance categories, the nature of the disturbances is such that the regulator can be regarded as a time-invariant system without a significant loss in analytical accuracy, and linearized about its equilibrium staf: to obtain a linear analytical model for small-signal performance evaluations. For the last category, the generally varying duty cycle subsequent to a step line/load change represents a time-varying nonlinear system. Since any practical system is likely to be higher than second-order, performance evaluation is through tactics closely identified with simulation techniques.

### 2.2.3 The Discrete-Time Domain Analysis.

Realizing that a switching regulator is inherently a highly nonlinear circuit containing analog-to-discrete-time conversion, it is only natural that it can be more accurately analyzed through discrete timedomain modeling and analysis. Therefore, discrete time-domain analysis has been selected as the approach to be utilized for the performance, analysis and simulation program.

In this approach, state-space techniques are employed to characterize regulators exactly through the formulation of nonlinear discrete timedomain equations in vector form. Newton's iteration is then used to solve for the equilibrium state of the regulator. The system is then linearized about its equilibrium state to arrive at a linear discrete time model. The closed-loop regulator is thus modeled as a single entity rather than the three separated functional blocks shown in Figure 2.1. The stability is studied by examining the eigenvalues of the linear system. The analysis can be extended, through $Z$-transform, to detemine frequency-related performance characteristics such as audiosusceptibility. The modeling and analysis approach makes extensive use of the digital computer, making automation in regulator analysis possible.

### 2.2.4 Di ete Time-Domain Analysis Objective.

Discrete time-domain analysis is the most accurate and straight forward of the different mathematical modeling techniques available. The user need only be proficient in the use of state-space analysis. Discrete timedomain analysis is applicable to all types of power and control circuit configurations, operating in either continuous or discontinuous mode. Thus, it clearly stands as the best approach avallable.

The performance analysis and simulation objective is twofold: (1) the creation of generally applicable, praciical, analysis subprograms and (2) a tutorial role of providing power processing designers with an effective analytical tool.

### 2.2.4 Discrete Time-Domain Analysis Objective - (con't)

While time-domain analysis readily fulfills the second objective, the subprograms can only be applied by the prospective user if the user's regulator power and control circuitry is identical to the circiit configuration upon which the subprograms are based. Consequently, in pursuing time-domain analysis, the objective is to achieve the following:
(1) To present clearly the tutorial information needed, by a user conducting time-domain analysis, for adopting the analysis to his specific application.
(2) To create subprograms for regulators with standardized mul-tiple-loop feedback control.

### 2.2.5 General Description of Performance Analysis and Simulation Techni ques.

Time-domain analysis may be applied to energy-storage converters to yield transient and steady-state solutions, stability, and audiosusceptibility. This approach has been applied here to buck, boost, and buck/boost converters with constant frequency and constant OFF time control. By way of introduction, a general step-by-step procedure for performing time-domain analysis of energy storage converters will now be presanted.

Step 1: Select the $n \times 1$ system state variable vector $\underset{x}{ }=\left[x_{1}, \ldots, x_{n}\right]^{\top}$. Normally, the state variables are the voltage across the capacitors and currents through the inductors. However, for the convenience of each individual problem, state variables can be chosen differently.
S. ' 2 : Write the system equations according to the modes of operation of the converter which are defined as foliows:

Mode 1 Operation: The current through the inductor is always greater than zero. The period of each switching cycle can be clearly divided into two time intervals. $T_{O N}$ and $T_{F I}$. During $T_{O N}$, the power transistor is "ON" and the diode is "OFF", and during $T_{F l}$, the power transistor is "OFF" and the diode is "ON".

Mode 2 Operation: The current through the inductor reduces to zero and resides at zero for a time interval $\mathrm{T}_{\mathrm{F} 2}$. During this time interval, both the transistor and the diode are "OFF". The time intervals $T_{O N}$ and $T_{F l}$ defined in the Model operation also exist in the Mode 2 operation.

The system representation for the Move 1 operation is


The column vector $\underline{u}$ is a (mxl) input vector, containing the input voltage $E_{I}$, the reference $E_{R}$, the saturation voltage drop across the power transistor, and the forward voltage drop across the diode. The nxn matrices F1 and F2 and the nxm matrices G1 and G2 are constant matrices containing various circuit parameters.

In the Mode 2 aperation, equation (2-3), must be added to the system representation:

$$
\begin{equation*}
\underline{\underline{x}}=F 3 \underline{x}+63 \underline{n} \cdot \ldots \text {. . during } T_{F 2} \tag{2-3}
\end{equation*}
$$

The dimensions of F3 and G3 are not necessarily the same as those of F1 and G1, respectively.

The converters, which are basically nonlinear switching circuits, are ascurately described by the piecewise-linear representations $(2-1)$ to $(2-3)$.

Step 3 The general solution of the linear differential equation

$$
\begin{equation*}
\underline{\dot{x}}=F_{1} \underline{x}+G_{1} \underline{u} \quad 1=1,2,3 \tag{2-4}
\end{equation*}
$$

Is

$$
\begin{align*}
& \underline{x}\left(t_{k}+T\right)=0_{i}(T) \underline{x}\left(t_{h}\right)+D_{i}(T) \underline{u}  \tag{2-5}\\
& \text { where } \bullet_{i}(T)=e^{F_{i} T} \quad 1=1,2,3 \\
& D_{i}(T)=e^{F_{i} T}\left[\int_{0}^{T} e^{\left.-F_{i} S_{d s}\right] G_{i} \quad 1=1,2,3}\right.
\end{align*}
$$

The terms $\varphi_{1}(T)$ and $D_{i}(T)$ can be computed either analytically or numerically. If they are computed numerically, the following Taylor series expansion may be used:

$$
F_{1} T=1+F_{1} T+\frac{\left(F_{1} T\right)^{2}}{2!}+\frac{\left(F_{1} T\right)^{3}}{3!}+\ldots . .1=1,2,3
$$

Step 4 Write the discrete-tine-domain equation for the converter. The discrete-time-domain equation can be expressed as

$$
\begin{equation*}
\underline{x}\left(t_{k+1}\right) \triangleq \underline{x}\left(t_{k}\right)+V \underline{u} \tag{2-6}
\end{equation*}
$$

where $t_{k}$ and $t_{k+1}$ correspond to instants of time at the beginning of the $k$ th cycle and the $k+1$ th cycle, respectively.

Mode 1 Operation
It may be shown, for Mode 1 operation, that the terms and $V$ of equation (2-6) are given by -

$$
\begin{align*}
& \bullet \Delta \theta_{2}\left(T_{F l}^{k}\right) \bullet_{1}\left(T_{O N}^{k}\right)  \tag{2-7}\\
& v \triangleq \theta_{2}\left(T_{F l}^{k}\right) D 1\left(T_{O N}^{k}\right)+D 2\left(T_{F l}^{k}\right) \tag{2-8}
\end{align*}
$$

where $T_{O N}^{k}$ and $T_{F I}{ }^{k}$. represent the $T_{O N}$ and $T_{F l}$ intervais during the $k$ th cycle. The time intervals $T_{O N}{ }^{k}$ and $T_{F l}{ }^{k}$ can be determined through the following two conditions:

Condition 1 A threshold condition, which is determined by the particular type of digital control signal processor empl yyed in the converter. and may be expressed as -

$$
\begin{equation*}
\varepsilon_{1}\left(\underline{x}\left(t_{k}\right), T_{O N}^{k}, T_{F l}^{k}\right)=0 \tag{2-9}
\end{equation*}
$$

Condition 2 A condition which specifies whether the converter is operating at a constant frequency, or a constant ON time, or constant OFF time, or a constant voltage-second, and is expressed as -

$$
\begin{equation*}
G_{2}\left(T_{O N}^{k}, T_{F 1}^{k}\right)=0 \tag{2-10}
\end{equation*}
$$

Mode 2 Operation
For mode 2 operation, the terms and $V$ of eq. (2-6) are given by -

$$
\begin{align*}
& \bullet \Delta \bullet_{3}\left(T_{F 1}^{k}\right) \bullet_{2}\left(T_{F 1}^{k}\right) \bullet_{1}\left(T_{O N}^{k}\right)  \tag{2-11}\\
& v \Delta \bullet_{3}\left(T_{F 2}^{k}\right) \bullet_{2}\left(T_{F 1}^{k}\right) D_{1}\left(T_{O N}^{k}\right) \\
&  \tag{2-12}\\
& \quad+\bullet_{3}\left(T_{F 2}^{k}\right) D_{2}\left(T_{F 1}^{k}\right)+D_{3}\left(T_{F 2}^{k}\right)
\end{align*}
$$

$$
\begin{align*}
& \text { In order to detemine the time intervals } \\
& T_{O N}^{k}, T_{F 1}^{k} \text { and } T_{F 2}^{k} \text {, othird condition, in } \\
& \text { addition to }(2-9) \text { and }(2-10) \text {, should be } \\
& \text { included to detect the } t \text { ime instant when } \\
& \text { the inductor current reduced to zero. } \\
& \text { This condition may be written - } \\
& \xi_{3}\left(\underline{x}\left(t_{k}\right), T_{O N}^{k}, T_{F 1}^{k}, T_{F 2}^{k}\right)=0  \tag{2-13}\\
& \text { of course, in the Mode } 2 \text { operation, the time } \\
& \text { interval } T_{F 2}^{k} \text { should also be a parameter in (2-9) } \\
& \begin{array}{l}
\text { and }(2-10) \text {. Thus, for Mode } 2 \text { operation, }
\end{array} \\
& 5_{1}\left(\underline{x}\left(t_{k}\right), T_{O N}^{k}, T_{F 1}^{k}, T_{F 2}^{k}\right)=0 \text { ans } \xi_{2}\left(T_{O N}^{k}, T_{F 1}^{k}, T_{F 2}^{k}\right)=0
\end{align*}
$$

Equations (2-6 to 2-13) are the complete representation of the converters.

Step 5 Solve for the approximate steady state $\overline{\mathrm{x}}$.*
The apprcximate solution is employed later as an initial guess toward solving the exact steady state through Newton's iteration method. In the steady state, equation (2-6) may be written as

$$
\begin{equation*}
\underline{x}^{*}=-\underline{x}^{*}+V \underline{y} \tag{2-14}
\end{equation*}
$$

The matrix anc $V$ matrices can be computed for the given $T_{O N}$. $T_{F 1}$, and $T_{F 2}$. For given input-output requirements of the converter, the approximate time intervals, $T_{O N}, T_{F 1}$ and $T_{F 2}$ can be determined. Reference 1 gives a detalled list of duty cycle formulae for different power stages and different control schemes.

If the minix (l-0) is non-singular, equation (2.14) may be solved for $\dot{x}$ *

$$
\begin{equation*}
\underline{x}^{*}=(1-0)^{-1} v \underline{u} \tag{2-15}
\end{equation*}
$$

However, in many cases, the matrix (1-0) is singular. In order to solve for $\bar{x}^{*}$, equation (2-6) together with (2.9) is required.

Step 6 Solve for the exact steady state $\underline{x}^{*}$.
Newton's iteration method is employed to find the steadystate solution with the initial guess $\underline{x}^{*}$. Equations (2-6) through (2-13) are used in the iteration process until a specified statematching condition is satisfied. The state-matching condition can be defined as

$$
\begin{equation*}
\sqrt{\sum_{i=1}^{n}\left[x_{i}\left(t_{k+1}\right)-x_{i}\left(t_{k}\right)\right]^{2}}<c \tag{2-16}
\end{equation*}
$$

for an arbitrarily small positive number, or it can be defined sim'y as

$$
\begin{equation*}
\left|x_{1}\left(t_{k+1}\right)-x_{1}\left(t_{k}\right)\right|<c \tag{2-17}
\end{equation*}
$$

Step 1 Linearize the discrel time-dsmain equation, Eq. (2-6), about the equilibrium state $\underline{x}^{*}$ for studying stability, audiosusceptibility, and transient response to a small step change of the input or the load.

Equation (2-6) may be written in the form

$$
\begin{equation*}
\underline{x}\left(t_{k+1}\right)=f\left(\underline{x}\left(t_{k}\right), u_{1}, T_{O N}, T_{F 1}, T_{F 2}\right) \tag{2-18}
\end{equation*}
$$

where $u_{f}$ is the input voltage.

Form the term $\delta \underline{x}\left(t_{k}+1\right)$ by taking partial derivatives of Eq. (2-18) with respect to $\underline{x}$ and $u_{1}$ and writing

$$
\begin{align*}
\delta \underline{x}\left(t_{k+1}\right) & =\frac{\partial f}{\partial \underline{x}} \delta \underline{x}\left(t_{k}\right)+\frac{\partial f}{\partial u_{1}} \delta u_{1} \\
& \triangleq \delta \underline{x}\left(t_{k}\right)+r \delta u_{1} \tag{2-19}
\end{align*}
$$

where $\downarrow$ is a (nxn) matrix and $r$ is a (nxi) column matrix.

The differentiation of (2-18) can be performed analytically, if the problem is simple, or numerically by difference quotients.

Step 8 Analyze the stability of the converter.

The linearized system (2-19), is stable if and only if all the eigenvalues $\lambda_{i}$ of the matrix $\psi$ are absolutely less than unity, i.e.,

$$
\left|\lambda_{1}\right|<1 \quad 1=1 \ldots n
$$

The $c$ envalues are evaluated by the computer. Changes of eigenvalues as a function of system parameters can be plotted in the complex plane, the $\lambda$-plane. The location of the eigenvalues in the $\lambda$-plane indicates not only the stability but also the transient behavior of the system, i.e., damping and rapidity of response.

Step 9 Analyze the Audio-Susceptibility of the converter.
Audiosusceptibility may be defined as the frequency response of the output voltage $V_{0}$ to a small amplitude sinusoldal perturbation of the input voltage $u_{1}$. The $Z$-transformation may be used with Equation (2-19) to derive the audiosusceptibility as a frequency domain transfer function.

The output voltage $v_{0}$ may be expressed as

$$
\begin{equation*}
v_{0}\left(t_{k}\right)=C \underline{x}\left(t_{k}\right) \tag{2-20}
\end{equation*}
$$

where $C$ is a constant ( $1 x n$ ) row matrix.

The $Z$ transformation of (2-19) is

$$
\begin{equation*}
6 x(2)=(12-v)^{-1} \text { roun }(z) \tag{2-21}
\end{equation*}
$$

The frequency-domain transfer function can be derived after replacing $Z$ with $e^{j w T_{p}}$ in (2-21) and combining (2-20) and (2-21).

$$
\begin{equation*}
G(j w)=\frac{\delta v_{0}(j w) / E_{R}}{\delta u_{1}(j w) / E_{1}}=\frac{E_{1}}{E_{R}} C\left(I e^{j w T_{p}}-\psi\right)^{-1} r \tag{2-22}
\end{equation*}
$$

where $E_{I}$ and $E_{R}$ are the dc average of the input voltage and the output voltage, respectively.

Step 10 Study the transient response of the Converter.
The linearized system remain: valid for a small step change of input voltage or load, since the system still continues to operate about its equilibrium state. The behavior of the transfent with respect to damping, oscillatory nature, decay time, and overshoot of the equilibrium position is governed by the location of the eigenvalues on the $\lambda$-plane.

### 2.2.6 Performance Analysis and Simulation Objectives

Since the principles and procedures for performing the time-domain analysis are the same regardless of the circuit configuration used, both objective: stated in the previous section can be achieved by conducting the time-domain analysis on regulators using SCM multiple-loop control. Certain Perfomance Analysis Subprograms (PAS's) using this control were generated in the initial MAPPS phase II effort under NAS3-19690 ${ }^{[4]}$. A summary on what has been done is presented in Table 2-II.

In Table 2-1I, the three basic power stages, buck, boost, and buc':-boost, are separated into Mode I and Mode II operations with constant on time, constant volt-second, constant off time, and constant frequency control. To cover all possible categories, there are a total of twenty-four different power/control configurations. For the particular multiple-loop control configurations marked by " $x$ ", time-domain analysis has been applied and completed through previous contracts. Those marked by "NA" are configurations incompatible with the control implementation, and therefore are not recommended for hardware design or time-domain analysis.

Multipln-loop control senses the rectangular voltage across the incuctor, and integrates it to form a triangular ramp output. The triançular ramp possesses a negative slope during on time $T_{N}$ and a positive slope during off time $T_{F}$. Since the regulator control determines the point at which the ramp intersects the fixed threshold level, the following conclusions become apparent:
(a) In constant on time or constant voit-second control, regulation is achieved by controlling off time $T_{F}$. The threshold level therefore prescribes the peak of the triangular ramp as the intersection oi the ascending ramp with the threshold level marks the end of off-time interval $T_{F}$.
TABLE 2-1I Status of Multiple-Loop Contral time Uomain Analysis

| Power Stage | Inductor MMF | Duty Cycle Control Schedule |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Constant On-Time | Constant Volt-Second | Constant Off-Time | Constant Frequency |
| Buck | Continuous | X | X | $X$ | X |
|  | Discontinuous | M | $M$ | $X$ | $\chi$ |
| Boost | Continuous | 0 | 0 | 0 | $\chi$ |
|  | Ofscontinuous | MA | $M$ | : | $\chi$ |
| Buck Boost | Continuous | 0 | 0 | $X$ | $X$ |
|  | Discontinuous | $M$ | $M$ | $\chi$ | X |

[^0]
### 2.2.6 Performance Analysis and Simulation Objectives (Cont.)

(b) In constant off time or constant frequency control, regulation is achieved by controlling the on time $T_{O N}$. The threshold level therefore prescribes the valley of the triangular ramp, as the intersection of the descending ramp with the threshold level marks the end of the on-time interval $T_{o n}$.

This rule governing the ramp-threshold interface is further illustrated in Figure 2.7 for continuous inductor-MMF operations. Here, the inductor voltage and the integrator output voltage are shown for constant-on-time control. Notice the difference in the relative position of the threshold level with respect to the triangular ramp.

When the inductor MMF becomes discontinuous, in Figure 2.7(B), the inductor voltage vanishes for a certain interval. During this interval, the integrator output extibits a flat top. In the constant-on-time control, this flat top coincides with the threshold level. Any slight noise in either the flat top or the threshold level is likely to 1 rigger the on-off control, making the constant-on-time control highly suscept: ${ }^{\text {a }}$ to noise when engaging in discontinous-MMF operation. Conversely, in the constant-off-time control, the flat top caused by the zero inductor voltage level is above the threshold level, thereby eliminating this noise susceptibility problem. Since the MMF is discontinuous when the regulator output load is light, the constant-on-time and the constant-volt-second control, shown in Figure 2-II, are considered to have limited utility, and therefore are not to be included in the analytical effort.

Hence, this effort concentrates on the time-domain analysis of multiple-loop control based on constant off time and constant frequency. The analysis includes all three power stages, with both continuous and discontinuous MMF operations.


### 2.2.6 Performance Analysis and Simulation Objectives (Cont.)

## Perform Discrete Time-Domain Analysis

In performing the discrete time-domain analysis, three subprograms, one far each of the three basic power stages, are create:. Each subprogram includes both continuous and discontinuous-MMF operations as well as the constant off time and constant frequency control schemes. Each subprogram (buck, boost, and buck-boost) has the capabllity of:
(1) Either constant-off-time or constant-frequency, control in one composite subprogram, or
(2) Either continuous- or discontinuous-MMF operation in the same subprogram.
In discontinuous-MMF operation, the exiscence of a third time interval during which the inductor MMF vanishes complicates the composite subprogram to a certain extent. Two separate computer subroutines are needed to compute the exact equilibrium state of the system for both continuous and discontinuous modes. The information is then fed into a common linearization subroutine to numerically derive the linearized system for small-signal analysis.

An information flow chart is presented in Figure 2.8 to show how the converter, w. .n both continuous and discontinuous current operations together with two types of duty-cycle controllers, can be implemented in a single computer program. The flow chart presented here is self-explan'ry, therefore no description will be given.


Figure 2.8 Information Flow Chart on Composite Subprogram

### 2.3 Formulation of State Equotions

Mathematical models have been generated for the Buck, Boost, and Buck-Boost $D C-D i$ convertions using the constant frequency and constant off-time pulse-width-modulation techniques, and are discussed in detall in the following sections.

### 2.3.1 Buck DC-DC Converter

### 2.3.1.1 Constant Frequency Buck Regulator

The buck dc to dc power converter topology is shown in Figure 2.9. The analysis approach is based on the ensuing mathematical model and the capability to consider both continuous and discontinuous inductor current operation.

FIGURE 2.9 BUCK REGULATOR


F19.2-10 Waveform of $e_{i}$ for Discontinuous InductorCurrent Operation.

## EQUIVALENT DISCRETE TiME SYSTEM

The waveform of $e_{i}$ vs. time shown in Fig. 2.10 is used to establish some notation regarding the time instant $t_{k}$ when each cycle starts and each switching action occurs :

In steady state operation,
fit $t_{k}, t_{1+1}, t_{k+2}$. . . . The clock pulse turns the power switch
At $c_{1}^{k}, t_{1}^{k+1}, t_{1}^{k+2}$..... the threshold condition turns the power
At $t_{2}^{k}, t_{2}^{k+1}, t_{2}^{k+2} \ldots$. . . the zero inductor current condition turns off the power diode.

The time intervals $t_{1}^{k}-t_{k} \cdot t_{2}^{k}-t_{1}^{k}$, and $t_{k+1}-t_{2}^{k}$ are defined as $T_{O N}^{k}$. $T_{F 1}^{k}$ and $T_{F 2}^{k}$, respectively. These time intervals may vary from cycle to cycle. However. the time interval between $t_{k}$ and $t_{k+1}$ is a constant equal to the period of oscillation $T_{p}$, i.e.,

$$
\begin{equation*}
t_{k+1}-t_{k}=T_{p} \quad \text { for all } k \tag{2-23}
\end{equation*}
$$

The system equations for Figure 2.9 are:

$$
\begin{array}{ll}
\underline{k}=F 1 \underline{X}+G 1 \underline{U} & t_{k} \leq t<t_{1}^{k} \\
\underline{\underline{x}}=F 2 \underline{X}+G 2 \underline{U} & t_{1}^{k} \leq t<t_{2}^{k} \\
\underline{k}=F 3 \underline{X}+G 3 \underline{U} & t_{2}^{k} \leq t<t_{k+1} \tag{2-26}
\end{array}
$$

where F1, F2, F3, G1, G2, and G3 are ( $3 \times 3$ ) constant matrices determined by the system parameters.

F1-F2 =

$$
F 3=
$$

GI $=$

$$
\begin{aligned}
& {\left[\begin{array}{c}
-\frac{1}{C_{0}\left(R_{5}+R_{L}\right)}-\frac{R_{5} R_{1}}{L_{0}\left(R_{5}+R_{L}\right)} \\
-\frac{1}{L_{0}}
\end{array}\right.} \\
& \frac{R_{L}}{C_{n}\left(R_{5}+R_{L}\right)}-\frac{R_{0} R_{5} R_{L}}{L_{0}\left(R_{5}+R_{L}\right)} \\
& -{ }_{-}^{R_{0}} \\
& \left.\begin{array}{ll}
1 \\
1 \\
0
\end{array}\right] \\
& \left.\left.\frac{n}{R_{4} C_{1}}-\frac{k_{d}}{R_{3} C_{1}}+\frac{C_{2}}{C_{1} C_{0}\left(R_{5}+R_{L}\right)}+\frac{C_{2} R_{5} R_{L}}{C_{1} L_{0}\left(R_{5}+R_{L}\right.}: C_{1} C_{0} R_{0} R_{5} R_{L} R_{5}+R_{L}\right)-\frac{m^{R_{0}}}{R_{4} C_{1}}+\frac{R_{1} C_{2}}{C_{1} C_{0}\left(R_{5}+R_{L}\right)}: 0\right] \\
& {\left[-\frac{1}{C_{0}\left(R_{5}+R_{L}\right)}\right.} \\
& {\left[\begin{array}{c:c:c}
R_{5} R_{1} \\
L_{0}\left(R_{5} \frac{R_{L} T}{}\right) & 0 \\
\frac{1}{L_{0}} & & 0 \\
-\frac{n}{R_{4} C_{1}}-\frac{C_{2} R_{5} R_{L}}{C_{1} L_{0}\left(R_{5}+R_{L}\right)} & \frac{k_{d}}{R_{3} C_{1}}
\end{array}\right]}
\end{aligned}
$$

and.

$$
G_{2}=G_{3}=\left[\begin{array}{cc}
0 & 0 \\
0 & 0 \\
0 & \frac{k_{d}}{R_{3} L_{1}}
\end{array}\right]
$$

The vectors $\underline{X}$ and $\underline{U}$ are state variable and forcing function vectors, respectively.

$$
\begin{aligned}
& \underline{x} \triangleq \quad\left[\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3}
\end{array}\right]-\left[\begin{array}{l}
e_{0} \\
1 \\
e_{c}
\end{array}\right] \\
& \underline{v} \triangleq \quad\left[\begin{array}{l}
u_{1} \\
u_{2}
\end{array}\right]\left[\begin{array}{l}
E_{1} \\
E_{R}
\end{array}\right]
\end{aligned}
$$

The constraints of the system are governed by the threshold condition at $t=t_{1}^{k}, t_{2}^{k} \ldots$.

$$
\begin{equation*}
x_{3}\left(t_{k}^{\prime}\right)=E_{\uparrow} . \tag{2-27}
\end{equation*}
$$

the zero inductor current condition at $t=t_{2}^{k}, t_{2}^{k+1} \ldots$

$$
\begin{equation*}
x_{2}\left(t_{k}^{2}\right)=0 . \tag{2-28}
\end{equation*}
$$

and the constant frequency condition . . . .

$$
\begin{equation*}
T_{O N}^{k}+T_{F 1}^{k}+T_{F 2}^{k}=T_{p} \quad \text { for all k. } \tag{2-29}
\end{equation*}
$$

Each of the linear equations (2-24) to (2-26) admits a closed fonm solution of the form

$$
\begin{align*}
& \underline{x}\left(t_{k}^{1}\right)=\underline{x}\left(t_{k}+T_{O N}^{k}\right)=01\left(T_{O N}^{k}\right) \underline{x}\left(t_{k}\right)+D 1\left(T_{O N}^{k}\right) \underline{u}  \tag{2-30}\\
& \underline{x}\left(t_{k}^{2}\right)=x\left(t_{k}^{1}+T_{F 1}^{k}\right)=02\left(T_{F 1}^{k}\right) \underline{x}\left(t_{k}^{1}\right)+02\left(T_{F 1}^{k}\right) \underline{u}  \tag{2-31}\\
& \underline{x}\left(t_{k+1}\right)=\underline{x}\left(t_{k}^{2}+T_{F 2}^{k}\right)=03\left(T_{F 2}^{k}\right) \underline{x}\left(t_{k}^{2}\right)+D 3\left(T_{F 2}^{k}\right) \underline{u}  \tag{2-32}\\
& \text { where } 01(T)=e^{F 1 T} \quad 1=1.2 .3  \tag{2-33}\\
& D 1(T)=e^{\left.F 1 t_{[ } \int_{0}^{T}-F 1 S_{d S}\right] 61 \quad 1=1.2 .3} \tag{2-34}
\end{align*}
$$

The structures of the matrices of and of for $1=1,2,3$ have the following forms:

$$
\begin{aligned}
& 01=\left[\begin{array}{lll}
01_{11} & 01_{12} & 0 \\
01_{21} & 01_{22} & 0 \\
01_{31} & 01_{32} & 1
\end{array}\right] \\
& 01=\left[\begin{array}{ll}
d 1_{11} & 0 \\
d 1_{21} & 0 \\
d 1_{31} & d 1_{32}
\end{array}\right] .
\end{aligned}
$$

$$
D 1=\left[\begin{array}{ll}
0 & 0 \\
0 & 0 \\
0 & 11_{32}
\end{array}\right] \quad \text { for } 1: 1,2,3
$$

Using equation (2-30) in equation (2-31), and the result in equation (2-32) the equivalent discrete time system for the constant frequency buck converter is given by:

$$
\begin{align*}
\underline{x}\left(t_{k+1}\right) & =\bullet 3\left(T_{F 2}^{k}\right) \quad 02\left(T_{F 1}^{k}\right) \bullet 1\left(T_{O N}^{k}\right) \underline{x}\left(t_{k}\right)+\Delta 3\left(T_{F 2}^{k}\right) \bullet 2\left(T_{F 1}^{k}\right) 01\left(T_{O N}^{k}\right) \underline{u} \\
& +\bullet 3\left(T_{F 2}^{k}\right) \quad 02\left(T_{F 1}^{k}\right) \underline{u}+03\left(T_{F 2}^{k}\right) \underline{u} \tag{2-35}
\end{align*}
$$

Also required in this description are the threshold conditions derived from (2-27)

$$
\begin{align*}
\bullet 1_{31}\left(T_{O N}^{k}\right) x_{1}\left(t_{k}\right) & +\oplus 1_{32}\left(T_{O N}^{k}\right) x_{2}\left(t_{k}\right)+\oplus 1_{33}\left(T_{O N}^{k}\right) x_{3}\left(t_{k}\right)+d 1_{31}\left(T_{O N}^{k}\right) u_{1} \\
& +d 1_{32}\left(T_{O N}^{k}\right) u_{2}=E_{T} . \tag{2-36}
\end{align*}
$$

the zero inductor current condition, derived from (2-28)

$$
\begin{equation*}
02_{21}\left(T_{F 1}^{k}\right) x_{1}\left(t_{k, 1}\right)+02_{22}\left(T_{F 1}^{k}\right) x_{2}\left(t_{k, 1}\right)=0 \tag{2-37}
\end{equation*}
$$

and the constant frequency condition

$$
\begin{equation*}
T_{F 2}^{k}=T_{P}-T_{O N}^{k}-T_{F 1}^{k} \tag{2-38}
\end{equation*}
$$

## Equilibrium Solutions

In steady-state, the following conditions prevail:

$$
\begin{align*}
& \underline{x}\left(t_{k+1}\right)=\underline{x}\left(t_{k}\right)=\underline{x}=\text { constant }  \tag{2-39}\\
& \text { for all } k \\
& T_{O N}^{k+1}=T_{O N}^{k}=T_{O N}=\text { constant }  \tag{2-40}\\
& T_{F 1}^{k+1}=T_{F 1}^{k}=T_{F 1}=\text { constant }  \tag{2-41}\\
& T_{F 2}^{k+1}=T_{F 2}^{k}=T_{F 2}^{*}=\text { constant }=T_{P}-T_{O N}^{*}-T_{F 1} \tag{2-42}
\end{align*}
$$

The Approximate Steady State
 formulae:

$$
\begin{align*}
& T_{O N}=\sqrt{\frac{2 L_{0} T_{D} P_{0}}{E_{1}\left(E_{1}-E_{R}\right)}}  \tag{2-43}\\
& T_{F 1}=\sqrt{\frac{2 L_{0} T_{P}^{P} P_{0}\left(E_{1}-E_{R}\right)}{E_{1} E_{R}^{2}}}  \tag{2-44}\\
& T_{F 2}=T_{D}-T_{0 N}^{-T} T_{F 1} \tag{2-42}
\end{align*}
$$

where $L_{0}$. the energy storage inductance
$P_{0}=$ output power
$E_{l}=$ input voltage
$E_{R}=$ output voltage

Substituting (2-39 to 2-42) into (2-35), one obtains

$$
\begin{align*}
& \Delta \cdot\left(T_{O_{N}}, T_{F 1}, T_{F_{2}}\right) \underline{x}+\underline{v}\left(T_{O_{N}}, T_{F_{1}}, T_{F 2}\right) \underline{U} \tag{2-45}
\end{align*}
$$

where $\left(T_{O_{N}}, T F_{1}, T F_{2}\right)-03\left(T_{F_{2}}\right)-2\left(T_{F}\right)-1\left(T_{Z_{N}}\right)$ and $\underline{V}\left(T_{O_{N}}, T F_{1}, T_{F 2}\right)$

- $03\left(T_{F}\right) \bullet 2\left(T_{F}\right) D 1\left(T_{F_{N}}\right)+03\left(T_{F 2}\right) 02\left(T_{F}\right)+D 3\left(T_{F 2}\right)$.


Equation (2-45) may now be solved for the state vector $\underline{X}^{*}$. Using this solution we may now solve for the states $X_{1}^{*}, X_{2}^{*}$, and $X_{3}^{*}$. Solution begins by arbitrarily setting $X_{2}^{*}=0$. Then:

$$
\begin{align*}
& +\left(3_{11} 1^{d 2_{11}}+3_{12}{ }^{d 2_{21}}\right) E_{1} \\
& +\mathbf{d 3}_{11} \mathrm{E}_{1} \tag{2-46}
\end{align*}
$$

Equation (2-16) may be solved for $X_{1}^{*}$

$$
\begin{align*}
& \left.\left.x_{1}=\frac{1}{1-\left[03_{11}\left(0211^{01} 11^{+02} 12^{01} 21\right)+{ }^{03} 12^{(02} 21^{01} 11^{+02_{22}}{ }^{01} 21\right.}\right)\right] \\
& \left\{+3_{11}\left[+2_{11} d 1_{11}++2_{12} d l_{21}+d 2_{11}\right]+3_{12}\left[+2_{21} d 1_{11}++2_{22}{ }^{d l_{21}}\right.\right.  \tag{2-47}\\
& \left.+d 2_{21}\right]+d 3_{11} J E_{1}
\end{align*}
$$

Using this result, the state $X_{3}^{*}$ may now be derived from equation(2-36):

$$
\begin{equation*}
x_{3}^{\star}=E_{T}-\oplus 1_{31} x_{1}^{*}-\oplus 1_{32} x_{2}^{*}-d l_{31} E_{1}-d 1_{32} E_{R} \tag{2-48}
\end{equation*}
$$

In this approximation, the threshold condition where the inductor current $x_{2}^{\star}$ equals zero may not be satisfied. This approximation is merely employed as a starting point in order to search for the exact steady state.

## The Exact Steady Stat.e

Define the system state with the power switch off as

$$
\begin{align*}
& \underline{Y}\left(t_{k}\right) \triangleq \underline{x}\left(t_{k}+T_{O N}^{k}\right)  \tag{2-49}\\
& \underline{z}\left(t_{k}\right) \triangleq \underline{x}\left(t_{k}+T_{O N}^{k}+T_{F I}^{k}\right) \tag{2-50}
\end{align*}
$$

where, using Fig. 2-32, it is clear that

$$
\begin{align*}
& Y_{3}\left(t_{k}\right)=E_{T} \quad \text { for all } k  \tag{2-51}\\
& Z_{2}\left(t_{k}\right)=0 \tag{2-52}
\end{align*}
$$

In steady-state operation

$$
\begin{align*}
& \underline{Y}^{\star}=\varnothing 1\left(T_{O N}^{*}\right) \underline{X}{ }^{\star}+D 1\left(T_{O N}^{*}\right) \underline{U}=f 1\left(T_{O N}^{*}, \underline{X}^{\star}\right) \tag{2-53}
\end{align*}
$$

and

$$
\begin{equation*}
\underline{X}^{*}=\Phi 3\left(T_{\vec{F} 2}^{\star}\right) Z^{*}+D 3\left(T_{F 2}^{*}\right) \underline{U} \tag{2-55}
\end{equation*}
$$

 threshold conditions (2-51) and (2-52).

If $T_{O N}^{*}$ and $T_{F}^{\prime}$ ] are the exact steady state values, then the steady-state $X *$ calculated from ( 2.46 to $2-48$ ) has to satisfy the following two matching conditions:
(1) the zero-inductor-current condition

$$
\begin{equation*}
B_{\text {match }}\left(\underline{x}^{*} \cdot T_{0 N} \cdot T_{F}\right)=Q_{21}\left(T_{F_{1}}\right) Y_{1}+2_{22}\left(T_{F}\right) Y_{2}=0 \tag{2-56}
\end{equation*}
$$

(2) the state matching condition

$$
\begin{gather*}
S_{\text {match }}\left(X * T_{0 N} \cdot T_{F 1}\right)=03_{d 1}\left(T_{F 2}\right) Z_{1}+03_{32}\left(T_{F 2}\right) Z_{2}+E_{T} \\
+d 3_{32}\left(T_{F 2}\right) U_{2}-x_{3}^{*}=0 \tag{2-57}
\end{gather*}
$$

Newton's method may now be used 'o find the $T_{0}^{\star} N$ and $T_{F}^{\star}{ }_{F}$ which satisfy the matching conditions.

The step-by-step procedure is described as follows:
 the approximate state $\dot{\chi}^{*}$ from (2-46) to (2-48).

Step 2 Find a new $T_{\mathrm{F} \mid}^{*}$ by Newton's method such that for the given $\bar{X}^{*}$ and $\tilde{T}_{O}^{\star} N$ together with the new $T_{F}^{*}$, the zerocurrent condition $B_{\text {match }}\left(\tilde{T}_{O N}, \underline{\tilde{x}}^{*}, \dot{T}_{F}^{*} p\right)=0$ will be satisfied.

$$
T_{F 1}=\dot{T}_{F 1}=\frac{\left.B_{\text {match }}\left(\dot{\Gamma}_{O N}^{*}\right), \underline{x_{1}}, \bar{i}_{F 1}^{*}\right)}{\left[\partial B_{\text {match }} / \partial T_{F 1}\right]}
$$

Step 3 Check if $S_{\text {match }}=0$ is satisfied.

Step 4 If $S_{\text {match }}=0$ is not satisfied, modify $T_{O}^{\circ} \mathrm{N}$ according to the equation

$$
T_{O N}^{*}=\bar{T}_{O N}^{*}-\frac{S_{\text {match }}\left(\bar{T}_{O N}^{*}, T_{F 1}\right)}{\left[\partial S_{\text {match }} / \partial T_{O N}\right]_{T_{O N}^{*}}}
$$

 to derive a new approximate state $\underline{X}^{*}$. Then to go to Step 2 and repeat the process until the state matching condition, $S_{\text {match }}=0$, is satisfied .

A flow diagram for determining the steady state is presented in Fig. 2.11(A) and (B). A subroutine $B_{\text {match }}$ is developed to search for a proper $T_{F 1}$ to satisfy the zero-inductor condition shown in (2-56)
This subroutine is contained in another subroutine $S_{\text {match }}$ which ultimately computes the state matching condition given in (2-57)


FIGURE 2.11A FLON DIAGRAM FOR DETERMINING THE STEADY STATE


FIGURE 2.11B FLOW DIAGRAM FOR DETERMIN'YG THE STEADY STATE

## Analysis of Linearized Discrete Time System

The analysis of stability, audio susceptibility, and transient response due to step change in the input voltage and the load, is presented. The analysis is based on a discrete system linearized about its equilibrium state.

## Derivation of the Linearized System

The linearized system can be derived by perturbing the system at the kith cycle. After the perturbation the nonlinear discrete time system cation $(2,35)$ can be rewritten as:

$$
\left.{\underset{k}{x}+1}^{f} \underline{f}_{\underline{k}}, \underline{u}\right)
$$

where $£\left(\underline{x}_{k}, \underline{v}\right)=03\left(T_{F 2}^{k}\right) \bullet 2\left(T_{f}^{k}\right) \bullet 1\left(T_{O N}{ }^{k}\right) \underline{x}$
$+\bullet 3\left(T_{F 2}^{k}\right) \bullet 2\left(T_{F 1}^{k}\right)$ DI $\left(T_{O N}^{k}\right) \underline{U}$
$+03\left(T_{F 2}^{k}\right) D 2\left(T_{F 1}^{k}\right) \underline{U}$
$+03\left(T_{F 2}^{k}\right) \underline{U}$

This system can be linearized about its equilibrium state. ${ }^{*}$.
If the following two terms are defined:

$$
\begin{aligned}
& \delta \underline{x}\left(t_{k}\right)=\underline{x}\left(t_{k}\right)=\underline{x}^{*} \\
& \delta J_{1}\left(t_{k}\right)=U_{1}\left(t_{k}\right)-U_{1} *
\end{aligned}
$$

and
it follows that:

$$
\begin{equation*}
\delta \underline{x}\left(t_{k}+1\right)=\Delta \underline{x}\left(t_{k}\right)+r_{\delta 1},\left(t_{k}\right) \tag{2-56}
\end{equation*}
$$

where $\psi=\left.\frac{\partial}{\partial x} £\left(\underline{x}_{k}, \underline{u}_{1}\right)\right|_{\underline{x}^{*}, \underline{u}^{*}}$
and $r=\left.\frac{\partial}{\partial U_{1}} £\left(\underline{x}_{k}, \underline{\underline{u}}\right)\right|_{\underline{x}^{*}, \underline{u}^{*}}$
The matrix $\psi$ is $3 \times 3$ and the matrix $r$ is $3 \times 1$.
The partial derivatives may be approximated by difference quotients, and evaluated numerically.

For sufficiently small $\Delta x_{i},(1=1, \ldots, 3)$,

$$
\psi=\left.\frac{\partial f}{\partial \underline{x}}\right|_{\underline{x}^{*}, \underline{u}^{*}}=
$$

$$
\left[\begin{array}{cc}
\frac{f_{1}\left(x_{1}^{* *+\Delta x_{1}}\right)-f_{1}\left(x_{1}^{*}\right)}{\Delta x_{1}} \cdots \frac{f_{1}\left(x_{3}^{\left.* * \Delta x_{3}\right)-f_{1}\left(x_{3}^{*}\right)}\right.}{\Delta x_{3}}  \tag{2-59}\\
\vdots \\
\frac{f_{3}\left(x_{1}^{* *+\Delta x_{1}}\right)-f_{3}\left(x_{1}^{*}\right)}{\Delta x_{1}} \cdots \frac{f_{3}\left(x_{3}^{\left.*+\Delta x_{3}\right)-f_{3}\left(x_{3}^{*}\right)}\right.}{\Delta x_{3}}
\end{array}\right]
$$

Since $\underline{x}$ does not appear explicitly in $\underline{f}$, the change of $T_{o n}, T_{F 1}$, and $T_{F 2}$ due to a change of $\Delta x_{i}, i=1 \ldots 3$, must be determined first in evaluating (2-59). The new $T_{o n}$ and $T_{F 1}$ are computed according to the threshold conditions (2-36) and (2-37).

Similarly,

$$
r=\left.\frac{\partial f}{\partial u_{1}}\right|_{\underline{x}^{*}, \underline{u}^{*}}=\left[\begin{array}{c}
\frac{f_{1}\left(u_{1}+\Delta U_{1}\right)-f_{1}\left(u_{1}\right)}{\Delta U_{1}} \\
\vdots \\
\frac{f_{3}\left(U_{1}+\Delta U_{1}\right)-f_{3}\left(u_{1}\right)}{\Delta U_{1}}
\end{array}\right]
$$

It is important to select the appropriate increments $\Delta x_{j}$ and $\Delta U_{j}$. Some experimentation with the increment size is advisable, since the accuracy of the partial derivatives depends on it. If the linearized system shows high sensitivity, that is, changes its behavior rather rapidly as it moves away from equilibrium, then the results obtained for the linearized system are valid only for very small perturbations about the equilibrium state.

## The Stability of the Linearized System

The IInearized system

$$
\delta \underline{x}\left(t_{k+1}\right)=\psi \delta \underline{x}\left(t_{k}\right)+\Gamma \delta U_{1}\left(t_{k}\right),
$$

is stable if and only if all the eigenvalues of $\psi$ are absolutely less than unity, i.e..

$$
\begin{equation*}
\left|\lambda_{1}\right|<? \quad i=1,2,3,4 \tag{2-61}
\end{equation*}
$$

The eigenvalues are evaluated by the computer. Changes of eigenvalues as a function of system parameters can be plotted in the complex plane. The location of the eigenvalues in the complex plane indicates not only the stablifty but ' ; o the transient behavior of the system. l.e., damping and rapidity of response.

### 2.3.1.2 Buck Converter with Constant Off-Time Duty Cycle Control

The basic structure, and analysis approach, for the buck PAS program with constant off-time duty cycle control are the same as for the constant frequency buck PAS program. The differences between the two converter schemes exist in the procedures for the computations of the converter switching times.

Constant off-time, $T_{F}$, as defined in the context of previous converter analyses, is the total time that the switching transistor remains off during a switching cycle. Therefore,

$$
\begin{equation*}
T_{F} \equiv T_{F 1}+T_{F 2} \equiv \text { constant } \tag{2-62}
\end{equation*}
$$

Arbitrarily, $T_{F}$ has been assigned the value of the sum of $T_{F 1}$ and $T_{F 2}$. determined for the constant frequency buck converter operating in MODE 1.

The expression of conservation of power for the buck converter is given as:

$$
\begin{equation*}
T_{Q N}^{2}=\frac{2 L_{0} T_{P} P_{0}}{E_{1}\left(E_{I}-E_{R}\right)} \tag{2-63}
\end{equation*}
$$

Substituting $T_{P}=T_{O N}+T_{F}$ in the above equation results in the following quadratic equation for $T_{O N}$ :

$$
\begin{equation*}
E_{I}\left(E_{I}-E_{R}\right) T_{Q N}^{2}-2 L_{0} P_{0} T_{B N}-2 L_{0} P_{0} T_{F}=0 \tag{2-64}
\end{equation*}
$$

Solving this quadratic equation for $T_{O N}$ gives the following expression:

$$
\begin{equation*}
T_{Q N}=\frac{L_{0} P_{0}+\sqrt{\left(L_{0} P_{0}\right)^{2}+E_{I}\left(E_{I}-E_{R}\right)\left(2 L_{0} P_{0} T_{F}\right)}}{E_{I}\left(E_{I}-E_{R}\right)} \tag{2-65}
\end{equation*}
$$

Where the "+" sign has been chosen before the square root term since

$$
\begin{equation*}
L_{0} P_{0}<\sqrt{\left(L_{0} P_{0}\right)^{2}+E_{I}\left(E_{I}-E_{R}\right)\left(2 L_{0} P_{0} T_{F}\right)} \tag{2-66}
\end{equation*}
$$

As in the constant frequency buck control, the clock pulse initiates the $T_{O N}$ period. Following the computation of $T_{O N}$, the other switching times can be readily computed:

$$
\begin{align*}
& T_{F 1}=\frac{\left(E_{I}-E_{R}\right)}{E_{R}} * T_{Q N}  \tag{2-67}\\
& T_{F 2}=T_{F}-T_{F 1}  \tag{2-68}\\
& T_{P}=T_{B N}+T_{F} \tag{2-69}
\end{align*}
$$

The sequence of testing for the duty cycle scheme and inductor MMF mode of operation is illustrated in the computational flow chart presented in Figure 2.12. As in the constant frequency buck PAS program, the error criterion for MODE 2 operation is that $T_{F 2}$ is greater than EPS $=1 . E-6$.

The buck PAS program (Appendix A, Volume II) is written such that one computer program package nay be used to analyze both duty cycle control schemes operating in either continuous or discontinuous inductor MMF mode.


### 2.3.2 Boost DC-DC Converter

### 2.3.2.1 Boost Converter with Constant Frequency Duty Cycle Control

Given the boost circuit of Figure 2.13, and the state and input vectors defined above, the state equations can be readily determined. To facilitate the development of the state equations, two dummy variables, the voltage $e_{i}$ and the current ${ }_{j}$, as defined in Figure 2.13, are introduced. The resulting generalized state equations for all modes of operation within each switching cycle can be expressed in terms of the state and dummy variable and inputs.

$$
\begin{align*}
\dot{v}_{C} & =-\frac{1}{C_{0}\left(R_{L}+R_{S}\right)} \cdot v_{C}+\frac{R_{L}}{C_{0}\left(R_{L}+R_{S}\right)} \cdot 1_{D}  \tag{2-70}\\
\frac{d i}{d t} & =-\frac{R_{0}}{L_{0}} \cdot 1-\frac{1}{L_{0}} \cdot e_{1}+\frac{1}{L_{0}} \cdot E_{I}  \tag{2-71}\\
\dot{e}_{R} & =-\frac{1}{C_{2} R_{5}} \cdot e_{R}+\frac{1}{C_{2} R_{C}} \cdot \frac{R_{L}}{\left(R_{L}+R_{S}\right)} \cdot v_{C}+\frac{1}{C_{2} R_{5}} \cdot\left(\frac{R_{S} R_{L}}{R_{L}+R_{S}}\right) \cdot i_{D} \text { (2-72) }  \tag{2-72}\\
\dot{e}_{C} & =\left[-\frac{1}{C_{1} R_{5}}-\frac{1}{C_{1} R_{2}}\left(\frac{R_{2}}{R_{L}+R_{S}}\right)\right] \cdot\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \cdot v_{C}+\left(\frac{n R_{0}}{C_{1} R_{4}}\right) \cdot 1+\left(\frac{1}{C_{1} R_{5}}\right) \cdot e_{R} \\
& +\left[-\frac{1}{C_{1} R_{5}}-\frac{1}{C_{1} R_{3}}\left(\frac{R_{2}}{R_{1}+R_{2}}\right)\right] \cdot\left(\frac{R_{L} R_{S}}{R_{L}+R_{S}}\right) \cdot i_{D}+\left(\frac{n}{C_{1} R_{4}}\right) \cdot e_{1} \\
& +-\frac{n}{C_{1} R_{4} E_{I}}+\frac{1}{C_{1} R_{3}} \cdot E_{R} \tag{2-73}
\end{align*}
$$

Similarly, the output voltage, $e_{0}$, can be written:

$$
\begin{equation*}
e_{0}=\left(\frac{R_{L} R_{S}}{R_{L}+R_{S}}\right) \cdot i_{D}+\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \cdot v_{c} \tag{2-74}
\end{equation*}
$$

The general form of the state equations is:

$$
\underline{\dot{x}}=F_{1} x+\underline{G_{1}} \underline{u}
$$

where $i=1,2$ and 3 refer to the times $T_{o n}, T_{F_{1}}$, and $T_{F_{2}}$, respectively.


FIGURE 2.13 BOOST CONVERTER

During $T_{\text {on }}$, the power switch $S_{1}$ is on and the power diode $S_{2}$ is off. Therefore, the dummy variables are set to the following values:

$$
\begin{aligned}
& e_{1}=E_{Q} \\
& i_{D}=0
\end{aligned}
$$

Consequently, FI and Gl can be expressed as follows:

$$
\begin{align*}
& F 1=\left[\begin{array}{llll}
f 1_{11} & 0 & 0 & 0 \\
0 & f 1_{22} & 0 & 0 \\
f 1_{31} & 0 & f 1_{33} & 0 \\
f 1_{41} & f 1_{42} & f 1_{43} & 0
\end{array}\right]  \tag{2-75}\\
& G 1=\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
g 1_{21} & 0 & g 1_{23} & 0 \\
0 & 0 & 0 & 0 \\
g 1_{41} & g 1_{42} & g 1_{43} & 0
\end{array}\right] \tag{2-76}
\end{align*}
$$

where

$$
\begin{aligned}
& f 1_{11}=\frac{1}{-C_{0}\left(R_{S}+R_{L}\right)} f f_{22}=-\frac{R_{0}}{L_{0}} \quad f 1_{31}=\frac{1}{C_{2} R_{5}}\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \\
& f l_{3,3}=-\frac{1}{C_{2} R_{R}} \quad f f_{41}=-\left[\frac{1}{C_{1} R_{5}}+\frac{1}{C_{1} R_{3}}\left(\frac{R_{2}}{R_{1}+R_{2}}\right)\right]\left(\frac{R_{L}}{R_{L}+R_{5}}\right) \\
& f l_{42}=\frac{n R_{0}}{C_{1} R_{4}} \quad f f_{43}=\frac{1}{C_{1} R_{5}} \quad g 1_{21}=-g l_{23}=\frac{1}{L_{0}} \\
& g l_{41}=-\frac{n}{C_{1} R_{4}} \quad g l_{42}=\frac{1}{C_{1} R_{3}} \quad g l_{43}=\frac{n}{C_{1} R_{4}}
\end{aligned}
$$

The output voltage becomes:

$$
\begin{equation*}
e_{0}=\frac{R_{L}}{R_{L}+R_{S}} \cdot v_{c} \tag{2-77}
\end{equation*}
$$

During $T_{F_{1}}$, the power switch $S_{1}$ is off and the power diode $S_{2}$ is in. Therefore, $e_{i}$ and $i_{D}$ are:

$$
\begin{align*}
& e_{1}=\frac{R_{L}}{R_{S}+R_{L}} \cdot v_{C}+\frac{R_{S} R_{L}}{R_{S}+R_{L}} \cdot 1+E_{D}  \tag{2-78}\\
& { }^{1} D=1 \tag{2-79}
\end{align*}
$$

The resulting F2 and G2 are:

$$
\begin{align*}
& F 2=\left[\begin{array}{llll}
f 2_{11} & f 2_{12} & 0 & 0 \\
f 2_{21} & f 2_{22} & 0 & 0 \\
f 2_{31} & f 2_{32} & f 2_{33} & 0 \\
f 2_{41} & f 2_{42} & f 2_{43} & 0
\end{array}\right]  \tag{2-80}\\
& G 2=\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
g 2_{21} & 0 & 0 & g 2_{24} \\
0 & 0 & 0 & 0 \\
g 2_{41} & g 2_{42} & 0 & g 2_{44}
\end{array}\right] \tag{2-81}
\end{align*}
$$

where

$$
\begin{aligned}
& f 2_{11}=-\frac{1}{C_{0}\left(R_{L}+R_{S} T\right.} ; \quad f 2_{12}=\frac{1}{C_{0}} \frac{R_{L}}{\left(R_{L}+R_{S}\right)} ; \quad f 2_{21}=\frac{R_{L}}{L_{0}\left(R_{S}+R_{L}\right)} \\
& f 2_{22}=-\left(R_{0}+\frac{R_{S} R_{L}}{R_{L}+R_{S}}\right) \frac{1}{L_{0}} ; \quad f 2_{31}=\frac{1}{C_{2} R_{5}}\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \\
& f 2_{32}= \frac{1}{C_{2} R_{5}}\left(\frac{R_{S} R_{1}}{R_{L}+R_{S}}\right) ; \quad f 2_{33}=-\frac{1}{C_{2} R_{5}} \\
& f 2_{41}=-\left[\frac{1}{C_{1} R_{3}}\left(\frac{R_{2}}{R_{1}+R_{2}}\right)+\frac{1}{C_{1} R_{5}}-\frac{n}{C 1 R_{4}}\right]\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \\
& \quad-54-
\end{aligned}
$$

$$
\begin{aligned}
& f^{2}{ }_{42}=\frac{n R_{0}}{C_{1} R_{4}}-\left[\frac{1}{C_{1} R_{3}}\left(\frac{R_{2}}{R_{1}+R_{2}}\right)+\frac{1}{c_{1} R_{5}}-\frac{n}{c_{1} R_{4}}\right]\left(\frac{R_{5} R_{L}}{R_{L}+R_{S}}\right) \\
& f^{2}{ }_{43}=\frac{1}{c_{1} R_{5}} ; g^{2} 2_{21}=-g_{24}=\frac{1}{C_{0}} ; g^{2} 2_{41}=\frac{n}{c_{1} R_{4}} \\
& g^{2}{ }_{42}=\frac{1}{c_{1} R_{3}} ; g^{2}{ }_{44}=\frac{n}{c_{1} R_{4}}
\end{aligned}
$$

The output voltage $e_{0}$ is now

$$
\begin{equation*}
e_{0}=\left(\frac{R_{L}}{R_{S}+R_{L}}\right) \cdot v_{c}+\left(\frac{R_{S} R_{L}}{R_{S}+R_{S}}\right) \cdot i \tag{2-82}
\end{equation*}
$$

During $T_{F_{2}}$, the inductor current has gone to zero and both the power switch $S_{1}$ and the power diode $S_{2}$ are off. The dummy variables take on the following values.

$$
\begin{aligned}
& e_{i}=E_{I} \\
& i_{D}=i=0
\end{aligned}
$$

The F3 and G3 matrices discribing this mode of circuit operation are:

where

$$
\begin{aligned}
& f 3_{11}=-\frac{1}{C_{0}\left(R_{L}+R_{S}\right)} ; \quad f 3_{31}=\left(\frac{1}{C_{2} R_{5}}\right) \cdot\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \\
& f 3_{33}=-\frac{1}{C_{2} R_{5}} ; \quad 93_{41}=-\left[\left(\frac{1}{C_{1} R_{3}}\right) \cdot\left(\frac{R_{2}}{R_{1}+R_{2}}\right)-\frac{1}{C_{1} R_{5}}\right]\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \\
& f 3_{43}=\frac{1}{C_{1} R_{5}} ; \quad 93_{42}=\frac{1}{C_{1} R_{3}}
\end{aligned}
$$

The output voltage, $e_{0}$, is the same as during $T_{o n}$ :

$$
\begin{equation*}
e_{0}=\frac{R_{L}}{R_{L}+R_{S}} \cdot v_{c} \tag{2-85}
\end{equation*}
$$

By transforming the state equations into the state transition format (Ref.) the proceeding equation sets may be represented in standard form;

$$
\underline{x}_{k+1}=\underline{\Phi 1} \underline{x}_{k}+\underline{D}_{1} \underline{u}
$$

During each time duration within a switching cycle, the following state transition and control distribution matrices are generated

For Ton'

$$
\Phi 1=\left[\begin{array}{cccc}
\phi 1_{11} & 0 & 0 & 0 \\
0 & \phi 1_{22} & 0 & 0 \\
\phi 1_{31} & 0 & \phi 1_{33} & 0 \\
\phi 1_{41} & \phi 1_{42} & \phi 1_{43} & 1
\end{array}\right] \quad D 1=\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
d 1_{21} & 0 & d 1_{23} & 0 \\
0 & 0 & 0 & 0 \\
d 1_{41} & d 1_{42} & d 1_{43} & 0
\end{array}\right]
$$

For $T_{F_{1}}$.

$$
\left.\Phi 2=\left[\begin{array}{llll}
\$ 2_{11} & \phi 2_{12} & 0 & 0 \\
\phi 2_{21} & \phi 2_{22} & 0 & 0 \\
\phi 2_{31} & \phi 2_{32} & \phi 2_{33} & 0 \\
\phi 2_{41} & \phi 2_{42} & \phi 2_{43} & 1
\end{array}\right] \quad D 2=\left\lvert\, \begin{array}{llll}
d 2_{11} & 0 & 0 & \overline{d 2} 14 \\
d 2_{21} & 0 & 0 & d 2_{24} \\
d 2_{31} & 0 & 0 & d 2_{34} \\
d 2_{41} & d 2_{42} & 0 & d 2_{44}
\end{array}\right.\right]
$$

$$
\begin{aligned}
& \text { For } \mathrm{T}_{\mathrm{F}} \text {, } \\
& \phi 3=\left[\begin{array}{cccc}
\phi 3_{11} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\phi 3_{31} & 0 & \phi 3_{33} & 0 \\
\phi 3_{41} & 0 & \phi 3_{43} & 0
\end{array}\right] \quad D 3=\left[\begin{array}{llll}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & d 3_{42} & 0 & 0
\end{array}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
& \text { Di }=e^{F i T} \\
& D i=e^{F i T}\left[\int_{0}^{T} e^{-F i s} d s\right] G i \quad i=1,2,3
\end{aligned}
$$

By combining the above state transition matrices, the nonlinear discrete state transition equation for a complete switching cycle is obtained.

$$
\underline{x}\left(t_{k+1}\right)=\underline{\underline{x}}\left(t_{k}\right)+V \underline{u}
$$

where $t_{k}$ and $t_{k+1}$ correspond to the time instances at the beginning of the kith cycle and the $(k+1)$ th cycle, respectively. The $\Phi$ and $V$ matrics have the following structures:

$$
\Phi=\left[\begin{array}{llll}
\phi_{11} & \phi_{12} & 0 & 0 \\
\phi_{21} & \phi_{22} & 0 & 0 \\
\phi_{31} & \phi_{32} & \phi_{33} & 0 \\
\phi_{41} & \phi_{42} \phi_{43} & 1
\end{array}\right] \quad v=\left[\begin{array}{llll}
v_{11} & 0 & v_{13} & v_{14} \\
v_{21} & 0 & v_{23} & v_{24} \\
v_{31} & 0 & v_{33} & v_{34} \\
v_{41} & v_{42} & v_{43} & v_{44}
\end{array}\right]
$$

From the structure of $\phi$. it is apparent that the next values of $x_{1}, x_{2}$ and $X_{3}$ are independent of past values of $x_{4}$, a property which is useful in determining the steady state solution.

## Analysis

In order to approximate the steady-state soiution necessary for the linearized system analysis, the approximate switching times $\mathrm{T}_{\mathrm{ON}}$. TFI, and TF2 must be computed. The following formulae may be used:

$$
\begin{align*}
& T_{O N}=\sqrt{\frac{2 L_{0} T_{D} P_{0}\left(E_{0}+E_{D}-E_{1}\right)}{n E_{1}\left(E_{1}-E_{Q}\right)\left(E_{0}+E_{D}-E_{Q}\right)}}  \tag{2-86}\\
& T_{F 1}=\sqrt{\frac{2 L_{0} P_{p} P_{0}\left(E_{1}-E_{Q}\right)}{n E_{1}\left(E_{0}+E_{D}-E_{1}\right)\left(E_{0}+E_{D}-E_{Q}\right)}}  \tag{2-87}\\
& T_{F 2}=T_{P}-T_{O N}-T_{F 1} \tag{2-88}
\end{align*}
$$

where

$$
\begin{aligned}
& L_{0}=\text { the energy storage inductance } \\
& P_{0}=\text { output power } \\
& n=\text { efficiency } \\
& E_{1}=\text { input voltage } \\
& E_{0}=\text { output voltage } \\
& E_{Q}=\text { saturation voltage drop of the power transistor } \\
& E_{D}=\text { forward voltage drop of the diode }
\end{aligned}
$$

The expressions for $T_{O N}$ and TFI are generated based on the flux and energy conservation principles given below (refer to Figure 2.13):

## Flux Conservation

$$
\begin{equation*}
\operatorname{TON}(E I-E Q)=\operatorname{TFI}(E O+E D-E I) \tag{2-89}
\end{equation*}
$$

## Energy Conservation

$$
\begin{equation*}
\frac{n E 1(E I-E Q) T O N^{2}}{2 \backslash Y_{p}}+\frac{n^{2} E I(E O+E D-E I) T F I^{2}}{2 \mid T_{p}}=P_{0} \tag{2-90}
\end{equation*}
$$

For the computation of the approximate steady-state solution, $\underline{\chi}^{*}$, the following state transition equation is solved:

$$
\underline{x}^{*}=\underline{x}^{*}+\underline{v} \underline{v}
$$

Therefore:

$$
x^{\star}=[1-0]^{-1} \underline{v} \underline{u}
$$

By imposing the property of the state traisition matrix, 0 , noted above, the state variables $X_{1}, X_{2}$ and $x_{3}$ can be computed independently of $X_{4}$. Therefore, the solution of the steady-station expression may be partitioned such that only a $3 \times 3$ matrix need be inverted to solve for the $X_{1}, X_{2}$, and $X_{3}$ states. The existing voltage restriction on $X_{4}\left(t_{k}\right)$ [capacitor $C_{1}$ voltage in Figure 2.13] is used to solve for $X_{4}$. The following expressions are the closed form solutions for the approximate steady-state vector:

$$
\begin{align*}
& X 1=\left[\left(1-\Phi_{22}\right) Y U_{1}+\Phi_{12} V U_{2}\right] / D E N  \tag{2-9}\\
& X_{2}=\left\{\begin{array}{ll}
{\left[\begin{array}{ll}
\phi_{21} & V U_{1} \\
0 & \text { MODE } \left.=1-\phi_{11}\right) V U_{2}
\end{array}\right] / D E N} & \text { MODE }=2
\end{array}\right\}  \tag{2-92}\\
& \left.x 3=\left[\left(\phi_{21} \phi_{32}+\phi_{31}\left(1-\Phi_{22}\right)\right) \underline{v u}_{1}+\left(\phi_{32}\left(1-\Phi_{11}\right)+\phi_{12} \phi_{31}\right) \underline{V u_{2}}\right)\right] / D E T \\
& +\mathrm{VU}_{3} /\left(1-\text { 中 }_{3}\right)  \tag{2-93}\\
& X_{4}=E T-1_{41} \times 1-1_{42} \times 2-\$ I_{43} \times 3-d l_{41} U 1-d l_{42} U 2 \\
& -\mathrm{dl}_{43} \mathrm{U3}-\mathrm{dl}_{44} \mathrm{U4} \quad \text { The voltage restriction }
\end{align*}
$$

where

$$
\begin{aligned}
\underline{V U} & =\underline{V} \underline{U} \\
D E N & =\left(1-\phi_{12}\right)\left(1-\phi_{22}\right)-\phi_{12} \phi_{21} \\
D E T & =\left(1-\phi_{33}\right) D E N \\
M O D E & = \begin{cases}1 & \text { continuous indictor current operation } \\
2 & \text { discontinuous inductor current operation }\end{cases}
\end{aligned}
$$

The performance of PAS2 (Appendix B), Volume II, with these computation procedures was checked out.

## Continuous Inductor Current Mode of Operation

In order to make it complete in its capability of analyzing converters, PAS2 was modified to incorporate the continuous inductor current mode of operation. The appropriate subroutines requiring modifications were:

1) PAS2
2) STATE2
3) PSIM2
4) FFUNC2
5) GAMM2

## Linearized System Analysis

The eiqenvalues of the Jacovian matrix characterize the linearized system stability and indicate the system transient behavior, i.e., damping and response time.

The system eigenvalues, as functions of the circuit parameters, are computed numerically. In order to characterize the stability boundaries (if relevant) of the linearized system, these eigenvalues are parametically plotted on the Z-plane and analyzed with respect to stability and response criterion.

Eigenvalues near the positive real axis of the Z-plane indicate that the system has a long time constant; eigenvalues equal to zero in discontinuous current mode indicate that the inductor current state variable sampled at wiy cycle is insensitive to the state variable in the previous cycle. The indictor current translates to zero following any small signal perturbation.

### 2.3.2.2 Boost Converter with Constant Off-Time Duty Cycle Control

The primary structure and analysis approacin of the boost PAS program for constant off-time duty cycle control are the same as for the constant frequency boost PAS program except for differences in the evaluation of the converter switching times.

Off-time, $T_{F}$, as defined in the context of previous converter analyses, is the total time that the switching transformer remains off during a switching cycle. Therefore:

$$
\begin{equation*}
T_{F}=T_{F 1}+T_{F 2} \tag{2-95}
\end{equation*}
$$

In constant off-time control, $T_{F}$ is held constant while $T_{F 1}$ and $T_{F 2}$ are allowed to vary. Arbitrarily, $T_{F}$ has been assigned the value determined for the constant frequency harst converter operating in MODE 2.

The conservation of power for the boost converter may be represented by the equation:

$$
\begin{equation*}
T_{0 N}^{2}=\frac{2 L_{0} P_{0} T_{D}\left(E_{0}+E_{D}-E_{I}\right)}{\left.n E_{I}\left(E_{I}-E_{Q}\right) E_{0}+E_{D}-E_{Q}\right)} \tag{2-96}
\end{equation*}
$$

Substituting the expression $T_{P}=T_{O N}+T_{F}$ in the above equation results in the following quadratic equation for $T_{O N}$ :

$$
\begin{gather*}
n E_{1}\left(E_{I}-E_{Q}\right)\left(E_{0}+E_{D}-E_{Q}\right) T_{D N}^{2}-2 L_{0} P_{0}\left(E_{D}+E_{D}-E_{I}\right) T_{O N}  \tag{2-97}\\
-2 L_{0} P_{0}\left(E_{0}+E_{D}-E_{I}\right) T_{F}=0
\end{gather*}
$$

Therefore, solving this quadratic equation for $T_{Q N}$ gives the following solution:

$$
\begin{equation*}
T_{O N}=\frac{Q B+\sqrt{O B^{2}+4 Q A Q C}}{2 Q A} \tag{2-98}
\end{equation*}
$$

where

$$
\begin{aligned}
& Q A=n E_{1}\left(E_{1}-E_{Q}\right)\left(E_{0}+E_{D}-E_{Q}\right) \\
& Q B=-2 L_{D} P_{D}\left(E_{D}+E_{D}-E_{1}\right) \\
& O C O B \cdot T_{F}
\end{aligned}
$$

and where the positive sign has been chosen before the square root since

$$
Q B<\sqrt{\left(Q B^{2}\right)+4 \cdot Q A \cdot Q C}
$$

As in constant frequency boost control, the clock pulse initiates the $T_{0 N}$ period. Following the computation of $T_{\text {ON }}$, the other switching times can be readily computed:

$$
\begin{align*}
& T_{F 1}=T_{P N} \cdot \frac{\left(E_{I}-E_{Q}\right)}{E_{0}+E_{D}-E_{I}}  \tag{2-99}\\
& T_{F 2}=T_{P}-T_{B N}-T_{F 1}  \tag{2-100}\\
& T_{P}=T_{P N}+T_{F} \tag{2-101}
\end{align*}
$$

The testing sequence for the constant off-time duty cycle scheme and the different induc. .or MMF modes of operation are illustrated in the computational flow chart presented in Figure 2-14. As in the revised constant frequency boost converter PAS program, the error criterion for MODE 2 operation is that $T_{\text {F2 }}$ must be greater than $1 \%$ of the switching period.

The boost PAS2 program (Appendix B, Volume II) is written such that one computer program package may be used to analyze both duty cycle control schemes operating in either continuous or discontinuous inductor MMF mode.


### 2.3.3 Buck Bnost DC-DC Converter

### 2.3.3.1 Buck Boost Converter with Constant Frequency Duty Cycle Control

Figure 2-15 prese, , he circuit configuration of the buck-boost converter with a constant frequency duty cycle scheme. As opposed to the inductor current state variable representation utilized in the previous BUCK and BOOST power converter analysis programs, the inductor flux, which is continuous, replaces the discontinuous inductor current as a system state variable. Therefore, the state and input vectors, $\underline{X}$ and $\underline{U}$ are defined as:

$$
\underline{x}=\left[\begin{array}{l}
x_{1}  \tag{2-102}\\
x_{2} \\
x_{3} \\
x_{4}
\end{array}\right] \cdot\left[\begin{array}{l}
v_{c} \\
x_{4}
\end{array}\right]\left[\begin{array}{l}
f_{10 x} \\
e_{R} \\
e_{c}
\end{array}\right]
$$

$$
\underline{v}=\left[\begin{array}{l}
U_{1}  \tag{2-103}\\
U_{2} \\
U_{3} \\
U_{4}
\end{array}\right]=\left[\begin{array}{l}
E_{1} \\
E_{R} \\
E_{0} \\
E_{D}
\end{array}\right]
$$



The constraints on this system are:

1) $x_{4}\left(t_{k}^{\prime}\right)=e_{c}\left(t_{k}^{l}\right)=E_{T}$
2) $x_{2}\left(t_{k}^{2}\right)=\operatorname{Flux}\left(t_{k}^{2}\right)=0$
3) $T_{O N}^{k}+T_{F 1}^{k}+T_{F 2}^{k}=T_{P}$

In order to utilize the inductor flux as a state variable, the magnetization characteristics of the inductor have been quantified. The flux is expressed by the following equation:

$$
\text { Flux }=\frac{\mu A N i}{\ell}
$$

where $\mu \equiv$ permeability or the ratio of the flux density to the magnetizing force
$A \equiv$ cross-sectional area
$\ell \equiv$ length
$N \leq$ number of turns
1 玉 current

For convenience in deriving the state equations, constant $k$ is defined:

$$
k=\frac{\mu A}{l}
$$

Therefore, the expression for Flux may be written:

$$
\text { Flux }=k N i=\frac{L 1}{N}
$$

The following expression for $k$ can then be utilized for the resulting equation set:

$$
k=\frac{L}{N^{2}}
$$

where $k$ is in units of webers per amp-turns.
In the derivation of the state equations, it is convenient to define the current $i$ in the following manner:

$$
1=K_{p} \text { flux, where } K_{p}: \frac{K^{-1}}{N}
$$

During the $T_{O N}$ period, when the switching transistor is on and the diode is off, the following set of equations describes the power converter operation:

$$
\begin{align*}
\dot{v}_{c}= & -\frac{1}{C_{0}\left(R_{5}+R_{L}\right)^{v}}  \tag{2-107}\\
\text { Flux }= & =\frac{R_{0}}{L_{0}} F \text { lux }+\frac{E_{1}}{N_{1}}-\frac{E_{Q}}{N_{1}}  \tag{2-108}\\
\dot{e}_{R}= & \frac{1}{C_{2} R_{5}} \cdot \frac{R_{L}}{R_{L}+R_{5}} \cdot v_{c}-\frac{1}{C_{2} R_{5}} \cdot e_{R}  \tag{2-109}\\
\dot{e}_{c}= & -\left[\frac{1}{C_{1} R_{5}}+\frac{R_{2}}{C_{1}\left(R_{1} R_{2}+R_{3}\left(R_{1}+R_{2}\right) T\right] \frac{R_{L}}{R_{L}+R_{5}} v_{c}+\frac{N_{3} R_{0}}{C_{1} R_{4} L_{0}} F l u x}\right. \\
& +\frac{1}{C_{1} R_{5}} e_{R}=\frac{1}{E_{1} R_{4}} \frac{N_{3}}{N_{1}} \cdot E_{1}+\frac{1}{C_{1}\left(R_{3}+R_{1} R_{2}\right.} \frac{R_{1}+R_{2}}{R_{R}} \\
& +\frac{1}{C_{1} R_{4}} \frac{N_{3}}{N_{1}} E_{Q} \tag{2-110}
\end{align*}
$$

$$
\begin{equation*}
v_{0}=\frac{R_{L}}{R_{L}+R_{S}} v_{c} \tag{2-111}
\end{equation*}
$$

During the $T_{F l}$ period when the switching transistor is off and the power diode is on, the following equation set describes the system:

$$
\begin{align*}
& \dot{v}_{c}=-\frac{1}{C_{0}\left(R_{L}+R_{S}\right)} v_{c}+\frac{R_{L}}{C_{0}\left(K_{L}+R_{S}\right)} K_{p} \text { Flux } \\
& \text { Flux }=-\left(\frac{R_{L}}{R_{L}+R_{S}}\right) \frac{1}{N_{2}} v_{c}-\left(\frac{R_{S} R_{L}}{R_{L}+R_{S}}+R_{0}\right) \frac{1}{L_{0}} \text { Flux }-\frac{1}{N_{2}} E_{D} \\
& \dot{e}_{R}=\frac{1}{C_{2} R_{5}}\left(\frac{R_{L}}{R_{L}+R_{S}}\right) v_{c}+\frac{1}{C_{2} R_{5}}\left(\frac{R_{S} R_{L}}{R_{L}+R_{S}}\right) K_{p} \cdot F 1 u x-\frac{1}{C_{2} R_{5}} e_{R} \quad(2-114) \\
& \dot{e}_{c}=-\left(\frac{1}{C_{1} R_{5}}+\frac{R_{2}}{C_{1}\left(R_{1} R_{2}+R_{3}\left(R_{1}+R_{2}\right)\right.}-\frac{1}{C_{1} R_{4}} \frac{N_{3}}{N_{2}}\right)\left(\frac{R_{L}}{R_{5}+R_{L}}\right) \cdot v_{c} \\
& -\left[\left(\frac{1}{C_{1} F_{5}}+\frac{R_{2}}{C_{1}\left(R_{1} R_{2}+R_{3}\left(R_{1}+R_{2}\right)\right.}-\frac{1}{C_{1} R_{4}} \frac{N_{3}}{N_{2}}\right)\left(\frac{R_{5} R_{L}}{R_{L}+R_{S}}\right)-\frac{1}{C_{1} R_{4}} \frac{N_{3}}{N_{2}} R_{0}\right] K_{p} \cdot F l u x \\
& +\frac{1}{C_{1} R_{5}} e_{R}+\frac{1}{C_{1}\left(R_{3}+R_{1} R_{2}\right)} \frac{R_{1}+R_{2}}{R_{1}}+\frac{1}{C_{1} R_{4}} \frac{N_{3}}{N_{2}} E_{D}  \tag{2-115}\\
& v_{0}=\frac{R_{L}}{R_{L}+R_{S}} \cdot v_{c}+\frac{R_{S} R_{L}}{R_{L}+R_{S}} K_{p} \cdot \text { Flux }
\end{align*}
$$

where

$$
k_{p}=\frac{k^{-1}}{N_{2}}
$$

During the $T_{F 2}$ period when both the switching transistor and the power diode are off, the following state equations describe the converter system operation.

$$
\begin{equation*}
\dot{v}_{c}=-\frac{1}{C_{0}\left(R_{L}+R_{S}\right)} v_{c} \tag{2-117}
\end{equation*}
$$

$$
\begin{equation*}
\text { flux }=0 \tag{2-118}
\end{equation*}
$$

$$
\begin{align*}
\dot{e}_{R}= & \frac{1}{C_{2} R_{5}}\left(\frac{R_{L}}{R_{L}+R_{S}}\right) v_{c}-\frac{1}{C_{2} R_{5}} e_{R}  \tag{2-119}\\
\dot{e}_{c}= & -\left[\frac{1}{C_{1} R_{5}}+\frac{R_{2}}{\left.C_{1}\left(R_{1} R_{2}+R_{3}\left(R_{1}+R_{2}\right)\right)\right]\left(\frac{R_{L}}{R_{L}+R_{S}}\right) v_{C}+\frac{1}{C_{1} R_{5}} e_{R}} \begin{array}{rl} 
& +\frac{1}{C_{1}\left(R_{3}+R_{1} R_{2}\right)} \\
R_{1}+R_{2}
\end{array}\right. \\
v_{0}= & \frac{R_{L}}{R_{L}+R_{S}} v_{C} \tag{2-120}
\end{align*}
$$

In matrix representation, the above state equations for each time period may be expressed in the general form:

$$
\underline{\dot{x}}=\underline{F i} \underline{X}+\underline{G i} \underline{U} \quad 1=1,2,3
$$

where for $T_{O N}$

$$
F 1=\left[\begin{array}{cccc}
f 1_{11} & 0 & 0 & 0 \\
0 & f_{22} & 0 & 0 \\
f f_{31} & 0 & f 1_{33} & 0 \\
f f_{41} & f f_{42} & f f_{43} & 0
\end{array}\right],\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
g 1_{21} & 0 & g 1_{23} & 0 \\
0 & 0 & 0 & 0 \\
g 1_{41} & 01_{42} & g 1_{43} & 0
\end{array}\right]
$$

and for $T_{F}$

$$
F 2=\left[\begin{array}{cccc}
f 2_{11} & f 2_{12} & 0 & 0 \\
f 2_{21} & f 2_{22} & 0 & 0 \\
f z_{31} & f 2_{32} & f 2_{33} & 0 \\
f 2_{41} & f 2_{42} & f 2_{43} & 0
\end{array}\right],\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & g 2_{24} \\
0 & 0 & 0 & 0 \\
0 & g 2_{42} & 0 & 92_{44}
\end{array}\right]
$$

and for $\mathrm{T}_{\mathrm{F} 2}$

$$
F 3=\left[\begin{array}{cccc}
f 3_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
f 3_{31} & 0 & f 3_{33} & 0 \\
f 3_{41} & 0 & f 3_{43} & 0
\end{array}\right],\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 93_{42} & 0 & 0
\end{array}\right]
$$

The state transition equations are of the form:

$$
x_{k+1}=1 x_{k}+D 1 u \quad 1=1,2,3
$$

where

$$
\phi=\left[\begin{array}{cccc}
\phi 1_{11} & 0 & 0 & 0 \\
0 & 1_{22} & 0 & 0 \\
\phi 1_{31} & 0 & \phi 1_{33} & 0 \\
01_{41} & \phi 1_{42} & \phi 1_{43} & 1
\end{array}\right] \quad D 1=\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
d 1_{21} & 0 & d 1_{23} & 0 \\
0 & 0 & 0 & 0 \\
d 1_{41} & d 1_{42} & d 1_{43} & 0
\end{array}\right]
$$

and

$$
\otimes 2=\left[\begin{array}{llll}
\$ 211 & \$ 2_{12} & 0 & 0 \\
\$ 2_{21} & \$ 2_{22} & 0 & 0 \\
\$ 2_{31} & \$ 2_{32} & \phi 2_{33} & 0 \\
\$ 2_{41} & \$ 2_{42} & \Delta 2_{43} & 1
\end{array}\right] \quad\left[\begin{array}{cccc}
0 & 0 & 0 & d 2_{14} \\
0 & 0 & 0 & d 2_{24} \\
0 & 0 & 0 & \mathrm{~d}_{34} \\
0 & d 2_{42} & 0 & \mathrm{~d}_{44}
\end{array}\right]
$$

and

$$
\bullet 3=\left[\begin{array}{cccc}
13_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
3_{31} & 0 & 3_{33} & 0 \\
13_{41} & 0 & 3_{43} & 1
\end{array}\right]\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & d 3_{42} & 0 & 0
\end{array}\right]
$$

## Switching Times Analysis

The approximate steady-state switching times may be derived for the constant frequency control case. The boundary condition inherent with this system is that the time $T_{p}$ is a constant, that is:

$$
\begin{equation*}
T_{P}=T_{O N}+T_{F 1}+T_{F 2}=\text { constant } \tag{2-122}
\end{equation*}
$$

Due to the conservation of flux in the inductor, the following restriction applies:

$$
\begin{equation*}
v_{0}+E_{D}=\frac{N_{2}}{N_{1}}\left(E_{1}-E_{Q}\right) \frac{T_{O N}}{T_{F 1}} \tag{2-123}
\end{equation*}
$$

Finally, an expression involving the conservation of power is needed to complete the necessary information for calculating the approximate times:

$$
\begin{equation*}
\frac{T_{O N}^{2} E_{I}\left(E_{I}-E_{Q}\right)}{2 L T_{P}}=\frac{P_{0}}{n} \tag{2-124}
\end{equation*}
$$

Using these expressions, it is possible to derive the following relationships:

$$
\begin{align*}
& T_{O N}=\sqrt{\frac{2 L T_{P} p_{0}}{n E_{I}\left(E_{I}-E_{Q}\right.}}  \tag{2-125}\\
& T_{F 1}=\frac{N_{2}}{N_{1}\left(\frac{E_{1}-E_{Q}}{V_{O}+E_{D}}\right) T_{O N}}  \tag{2-126}\\
& T_{F 2}=T_{P}-T_{O N}-T_{F 1} \tag{2-127}
\end{align*}
$$

As in the previous material, Mode 1 is the condition that $\mathrm{T}_{\mathrm{F} 2}=0$ (inductor current is continuous), and Mode 2, the condition that $T_{F 2} \neq 0$ (inductor current is discontinuous). Assume that the circuit now operates with a load resistance such that $T_{F 2}=0$. Allow the load to vary uniformly in a direction where inductor current at the end of $T_{F}$ approaches zero. That value of load resistance at which the inductor current at the end of $T_{F 1}$ first becomes zero is a critical resistance. (Equivalently, the value of resistance that first gives a non-zero $T_{F 2}$ is a critical resistance). This critical resistance may be found in the following way:

From the power conservation expression, we have:

$$
\begin{equation*}
T_{O N}^{2}=\frac{2 L P_{0}}{n E_{1}\left(E_{1}-E_{Q}\right)}\left(T_{O N}+T_{F 1}+T_{F 2}\right) \tag{2-128}
\end{equation*}
$$

From this expression, and using the expression for $T_{F I}$, we have:

$$
\begin{equation*}
T_{F 2}=\left[\frac{n E_{I}\left(E_{1}-E_{Q}\right) R_{L} T_{O N}-2 L V_{0}^{2}\left(1+\frac{N_{2}}{\left.N_{1}\left(\frac{E_{1}-E_{Q}}{V_{0}+E_{D}}\right)\right)}\right.}{2 L V_{0}^{2}}\right] \cdot T_{O N} \tag{2-129}
\end{equation*}
$$

The critical load resistance may now be found:

$$
\begin{equation*}
R_{C}=\text { critical } R_{L}=\frac{2 L V_{0}^{2}\left(1+\frac{N_{2}}{N_{1}}\left(\frac{E_{1}-E_{Q}}{V_{0}+E_{D}}\right)\right)}{\eta E_{1}\left(E_{I}-E_{Q}\right) T_{O N}} \tag{2-130}
\end{equation*}
$$

For MODE 2 , the switching times are given above. For MODE 1 . however, the following times are more easily calculated using the equations:

$$
\begin{align*}
& T_{O N}=\frac{T_{P}}{1+\frac{N_{2}}{N_{1}}\left(\frac{E_{1}-E_{0}}{V_{0}+E_{D}}\right)}  \tag{2-131}\\
& T_{F 1}=T_{P}-T_{O N}  \tag{2-132}\\
& T_{F 2}=0 \text { (by definition) } \tag{2-133}
\end{align*}
$$

For the actual PAS program, the criterion for MODE 2 operation is given by the inequality constraint:

$$
T_{F 2}>0.01 T_{P}
$$

that is, $T_{F 2}$ must be greater than $1 \%$ of the switching period.

## Steady-State Solution

Propagating the state variables through a complete switching cycle results in the following expression:

$$
\begin{align*}
\underline{x}\left(t_{k+1}\right) & =\underline{\underline{0} \underline{\underline{2}} \underline{0} \underline{x}\left(t_{k}\right)+[\underline{03} \underline{02} \underline{01}+\underline{03} \underline{02}+\underline{03}] \underline{u}}  \tag{2-134}\\
& =\underline{\underline{x}}\left(t_{k}\right)+\underline{v} \underline{\underline{u}}
\end{align*}
$$

Since $\underline{x}\left(t_{k+1}\right)=\underline{x}\left(t_{k}\right)=\underline{x}$. the approximate steady-state $\underline{x}^{*}$ may be calculated using the following procedure:

$$
\underline{x}^{*}=\left[1-\underline{\theta^{-1}} \underline{v} \underline{v}\right.
$$

where $\underline{x}^{*}=\left[x_{1}^{*} x_{2}^{*} x_{3}^{*}\right]^{\top}$ and

$$
x_{4}^{*}=E_{T}-\phi 1_{41} x_{1}^{*}-\phi 1_{42} x_{2}^{*}-\phi 1_{43} x_{3}^{*}-d 1_{41} u_{1}-d l_{42} u_{2}-d l_{43} u_{3}
$$

The exact steady-state solution for the state variables can be computed using a Newton-Raphson iteration algorithm to find the proper $T_{O N}$ and $T_{F l}$, and hence, $\underline{x}$, which satisfy the state matching conditions defined below. The matching condition on the integrator output voltage, $S_{\text {match }}^{k}$, and on the inductor flux, $B_{\text {match }}^{k}$, defined below:

$$
\begin{aligned}
& s_{\text {match }}^{k}=x_{4}^{k+1}-x_{4}^{k}=03_{41}^{k} 2_{1}^{k}+03_{43}^{k} z_{3}^{k}+2_{4}^{k}+d 3_{42}^{k} u_{2}=x_{4}^{k} \\
& 8_{\text {match }}^{k}=z_{2}^{k}=02_{21}^{k} y_{1}^{k}+02_{22}^{k} y_{2}^{k}+02_{24}^{k} u_{4}
\end{aligned}
$$

Iterating on $T_{O N}$ and $T_{F 1}$ in order to drive these matching conditions to zero results in the proper exact steady-state solution for $\underline{x}$.

Therefore, by defining the following variables

$$
\begin{aligned}
& \underline{y}\left(t_{k}\right) \equiv \underline{x}\left(t_{k}+T_{O N}^{k}\right)=\underline{\theta I_{k}} \underline{x}\left(t_{k}\right)+\underline{D 1_{k} \underline{u}} \\
& \underline{z}\left(t_{k} ; E \underline{y}\left(t_{k}+T_{F 1}^{k}\right)=\underline{0 I_{k}} \underline{y}\left(t_{k}\right)+\underline{D 2_{k}} \underline{u}\right. \\
& \underline{x}\left(t_{k+1}\right) E \underline{z}\left(t_{k}+T_{F 2}^{k}\right)=\underline{03} \underline{z}\left(t_{k}\right)+\underline{D 3_{k}} \underline{u}
\end{aligned}
$$

the state variable boundary conditions may be expressed as:

$$
\begin{align*}
y_{4}\left(t_{k}\right)= & d 1_{41} x_{1}\left(t_{k}\right)+d 1_{42} x_{2}\left(t_{k}\right)+141_{43} x_{3}\left(t_{k}\right)+x_{4}\left(t_{k}\right) \\
& +d 1_{41} u_{1}+d 1_{42} u_{2}+d 1_{43} u_{3}=E_{T}  \tag{2-135}\\
z_{2}\left(t_{k}\right)= & 2_{21} y_{1}\left(t_{k}\right)+1352_{22} y_{2}\left(t_{k}\right)+d 2_{24} u_{4}=0 \\
T_{F 2}^{k}= & T_{p}=T_{O N}^{k}-T_{F 1}^{k}
\end{align*}
$$

### 2.3.3.2 Buck-Boost Converter with Constant Off-Time Duty Cycle Control

The basic structure and analysis approach of the buck-boost PAS program for constant off-time duty cycle control is the same as for the constant frequency buck-boost PAS program. The differences between the two converter schemes exist in the procedures for the computations of the converter switching times.

Constant off-time, $T_{F}$ as defined previously, is the total time that the switching transistor remains off during a switching cycle. Therefore

$$
\begin{equation*}
T_{F} \equiv T_{F 1}+T_{F 2} \equiv \text { constant } \tag{2-138}
\end{equation*}
$$

Arbitrarily, $T_{F}$ has been assigned the value of the sum of $T_{F 1}$ and $T_{F 2}$ determined for the constant frequency buck-boost converter operating in MODE 1.

Conservation of power for the buik-boost converter is expressed in the equation:

$$
\begin{equation*}
T_{Q N}^{2}=\frac{2 L_{0} T_{p} P_{0}}{n E_{L}\left(E_{I}-E_{Q}\right)} \tag{2-139}
\end{equation*}
$$

Substituting for $T_{p}$ in the above equation results in the following quadratic equation for $T_{O N}$ :

$$
\begin{equation*}
n E_{I}\left(E_{I}-E_{Q}\right) T_{Q N}^{2}-2 L_{0} P_{0} T_{Q N}-2 L_{0} P_{0} T_{F}=0 \tag{2-140}
\end{equation*}
$$

Solving t'nis quadatric equation for $T_{\text {ON. }}$ gives:

$$
\begin{equation*}
T_{O N}=\frac{L_{0} P_{0}+\sqrt{\left(L_{0} P_{0}\right)^{2}+n E_{1}\left(E_{1}-E_{Q}\right)\left(2 L_{0} P_{0} T_{F}\right)}}{n E_{1}\left(E_{1}-E_{Q}\right)} \tag{2-141}
\end{equation*}
$$

Where the " + " sign has been chosen before the square root because

$$
\begin{equation*}
L_{0} P_{0}<\sqrt{\left(L_{0} P_{0}\right)^{2}+n E_{1}\left(E_{1}-E_{Q}\right)\left(2 L_{0} P_{0} T_{F}\right)} \tag{2-142}
\end{equation*}
$$

As in the constant frequency buck-boost control, the clock pulse initiates the $T_{O N}$ period. Following the computation of $T_{O N}$. the other switching times can be readily computed:

$$
\begin{align*}
& T_{F 1}=\frac{\left(E_{I}-E_{Q}\right)}{*} * T_{Q N}  \tag{2-143}\\
& T_{F 2}=T_{F}-T_{F 1}  \tag{2-144}\\
& T_{P}=T_{Q N}+T_{F} \tag{2-145}
\end{align*}
$$

The sequence of testing for the duty cycle scheme and inductor MMF mode of operation are illustrated in the computational flow chart presented in Figure 2-16. As in the constant frequencij buck-boost PAS program, the threshold criterion for MODE 2 operation is that $T_{F 2}$ is greater than $1 \%$ of the switching period.

The buck-boost PAS program (Appendix C, Volume II) is written such that one computer program package may be used to anlayze both duty cycle control schemes operating in efther continuous or discontinuous inductor MMF mode.


### 2.4 Computer Programs.

Computer programs have been generated using the three mathematical models described. Figure 2-17 shows the general information flow chart for a composite subprogram including both continuous and discontinuous inductor current operation, and constant frequency or constant off-time control.

The performance analysis computer flow diagrams are included as part of Appendix A through C. Volume II, for the buck, boost, and buck-boost regulators, respectively. Although the programs are similar, that for the buck-boost is most complex. In review of the presently avallable data, it seems reasonable that the buck-boost computer program can be modified so that it is capaile of performing the buck, boost and buck-boost DC-DC converter performance analysis.


Figure 2-17 Information Flow Chart on Composite Subprogram

### 2.5 Computer Performance, Analysis Results.

The performance analysis computer programs provide the following results:

- Linearized stability analysis.
- Root locus analysis - up to 10 optional system parameters.
- Audio-susceptibility analysis.
- Transient response analysis - linearized system.
- Discontinuous or continuous inductor MMF mode of operation.
- Load change analysis - linearized system.
- Analysis of a non-linear transition response to a step input voltage.
- Constant frequency or constant off-time duty cycle control.

The computer programs contained in Appendix A through C (Volume II) have been verified experimentally for the buck, boost and buck-boost regulators, respectively.

A brief example of the computer results for a buck-boost converter is presented to fllustrate the type of data available.

Figures 2-18 to 2-21 show the root locus results of a buck-boost. DC-DC converter where circuit parameters are varied in order to establish the relative cability of the converter.

| - Figure 2-18 | C2 varied |
| :--- | :--- | :--- |
| - Figure 2-19 | R5 varied |
| - Figure 2-20 | R4 varied |
| - Figure 2-21 | C0 varied |

Figure 2-22 and 2-23 show the small signal open-1nop gain and phase relationsilp. Figure 2-24 shows the output transient response when the input voltage is changed from 24 VDC to 40 VDC.

Figure 2-25 shows the output trans,ent response when the output load resistance is changed from 49 ohms to 600 ohms.

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Figure 2-22 Constant $T_{p}$ Frequency Response (Gain)
$1-3 \mathrm{COW}$ Of-7t $\mathbf{0 t - 1 3}$

## 

-85-
Figure 2-23 Constant $T_{p}$ Frequency Response (Phase)
EI-40 M. -40 neocel

# Figure 2-24 Constant $\mathrm{T}_{\mathrm{p}}$ Transition Response 



CI-24-40 RL-60 N-I-2
Figure 2-25 Step Load Change Transient Response
COO-LIMETH Of-TH OF-13 ton
-88-

### 3.0 DESIGN OPTIMIZATION OF POWER CONVERTER

### 3.1 Introduction.

In converter design practice, it is of ten attempted to find the smallest possible magnetic core to accommodate the necessary windings in order to satisfy a given set of design requirements [8]. It is hoped, in return, that the weight and size of the overall converter would be reduced. One effective way to reduce the weight and size of the magnetic components (a way which many designers are pursuing nowadays) is to increase the switching frequency of the converter. To a certain extent, this approach is viable. Equations which govern the selections of design variables (such as magnetic components and capacitors), however, are nonlinear and interdependent due to the complex nature of various functions of power converter design, Employing the conventional design approach, only a piecemeal, ruboptimum converter design at best can be achleved. For example, when the size of the energy storage inductor is reduced, the ac switching current component is invariably increased. Consequently, a penalty is imposed on the weight and size of the frout filter design in order to attenuate the ac switching current component which reflects back into the source. Similarly, as the ac switching current component is increased, larger output filter capacitors should be used to limit the output ripple valtage component. To give further illustration of the complex interrelations: When the switching frequency is incraased beyond a certain range, the gain of weight- and size-saving of magnetic components is diminished because the magnetic core losses and the semiconductor switching losses are increased as a function of switching frequency. Thus, higher losses and heavier overall system design can result due to the increase of weight and size of package and heatsink. The goal of minimum weight converter
design is seldom achieved, despite the extensive and time-conauming trial and error design procese.

In power-converter design, the key to implementing a euccessful deeign optimization reste largely on the availability of suitable mathematical and computer programming technique Handicapped by a lack of suitable design and optimization tools, the tendency has been for a designer to rely on time-consuming intuitive and empirical methods resulting in a suboptimum design. Such inadequacies invariably lead to penalties involving equipment weight, operating efficiency or other performances.

The philosophy of design optimization [8] of a power converter is briefly described in the following:

### 3.1.1 Design Optimization

A non-optimum design (illustrated in Fig. 3.1.1) generally involves four design ingredients First, a set of performance requirements such as output ripple factor and frequency-dependent source conducted EMI level, $r=\left(r_{1}, r_{2}, \ldots, r_{m}\right)$, is given to guide the design. Second, a set of defign constants, such as transistor switching times and maximum magnetic operating flux density, $k=\left(k_{1}, k_{2}, \ldots, k_{\ell}\right)$, is employed. These constants are known to a designer either through manufacturers' data sheet, or designer's common sense and experience. Third, the objective of the design is to pinpoint numerically all the unknown design variables such as the detailed magnetic core size, mean magnetic path length and other component sizes, $x=\left(x_{1}, x_{2}, \ldots, x_{n}\right)$. These three design ingredients are then Integrated together with the fourth ingredient, the norilinear design constraints such as out ut ripple voltage constraint, EMI constraint, window area constraints and $f$ lux density constraints, $\boldsymbol{f}_{j}(x, k, r)=0$.

## DESIGN APPROACH WITHOUT OPTIMIZATION



$$
\begin{aligned}
n>j \longrightarrow \quad & \text { INFINITE SET OF SOLUTIONS TO SATISFY } \\
& \text { ALL CONSTRAINTS } g_{j}(x, k, r)=0 \\
& \text { AND PERFORMANTE REQUIREMENTS }
\end{aligned}
$$

Fig. 3.1.1 Conventional design approach

Due to the deaign complaxities of the suitching power convarter, the number of unknown design variables generally exceoda that of the constraints. Therefore there exists a virtually infinite set of solutions which satisfy both the constraints and performance requirements. Different desis.ers may come out with completely different designs based on individual design experience and approaches. For example: Having selected a power circuit configuration, the designer picks a power-converter switching frequency through intuition or experience; then, using the pe:formance requirements, he proceeds to obtain input filter design, output filter design, and energy storage inductor design. The same procedure may be repeated several times for different switching frequencies betore a comparatively lower weight power-circuit degign is achleved. Such a design ia characterfzed by the designer's subjective fudgement. Despite the time consuming irerations, optimization of the overall power converter is seldom achieved.

The optimum design approach takes advantage of powerful, computeraided, nonlinear programing that integrates all the design ingredients In the objective lunction - that is, the function to be optimized-defined by the designer. The objective function can be the converter weight, elficiency, or any other realizable physical quantity. The philosophy of design optimization (shown in Fig. 3.1.2) is obtained through the injection of an objective iunction $f(x, k)$ into the $\quad, ~ o n a l$ design approach. The essence of the design optimization is chen io realize a set of design variables using nonlinear programmin ique. This set will satisfy all constraints $g_{j}=0$ and requirements, , and concurrently optimize a certain converter characteristics, $f(x, k)$, defined by the aesigner. By introducing the objective function ints the design rocess, the infinite set of design solutions presented in Fig. 3.1. 's reduced to a single solution set that is an optimum of the objective function.

by introduring the objective function into the design PROCESS, SINGLE SET OPTIMUM SOLUTION $\left(x_{1}, x_{2} \ldots \ldots, x_{n}\right)$ CAN BE PINFOINTED TO SATISFY ALL CONSTRAINTS $9_{j}=0$ AND CONCURRENTLY OPTIMIZE THE OBJECTIVE FUNCTION $f(x, k)$

Fig. 3.1.2 Optmum design approach

### 3.1.2 Powar Converter Optimization

The purpose of this report is to demonstrate the usefulness of an Augmented Lagrangian Multiplier (ALAG) based nonlinear programming technique. This is used for minimum weight dasign of the booat and buck-boost power converters. Previous experiences of the authors with the use of AliAG based programming techniques for power converter design optimizations [15] have encouraged such an approach. According to expectations, reliable results have also been obtained for boost and buck-boost converters.

At the beginning, mathematical models are presented for the boost and buck-boost converters. Various design requirements and physical operating chatacteristics of these converters are summarized in the form of equality and inequality constraints. The minimum weight design requirement is formalated as the objective function.
A. Problem Formulation

The circuit schematic, the objective function, and set of constraints are briefly discussed here for the boost aid the buck-boost converters shown in Fig. 3.1.3.
A.1. Objective Function: The objective function is formulated as a sum of various component weights winich include:
(1) Core weight.
(ii) Winding waight.
(1i1) Capacitor weight.


Fig. 3.1.3 Schematic of the (a) Boost Converter, (b) Buck/Boost Converter
(iv) Source weight.
(v) Package and heat sink weights.
A. 2 Constraint are: A number of equality and inequality constraints form the constraint set:
(1) The loss constraint which is composed of input filter copper 108s, conduction and switching losses of transistor and diode, two-windinginductor copper and core loss, and output filter capacitor ESR loss.
(1i) Operating flux density constraint.
(1i1) Window area constraint.
(iv) Parasitic resistance constraint.
(v) Input filter peaking constraint.
(vi) Frequency dependent source EMI constraint.

## B. Solution Methodology

The stringent requirements for modeling the power converter design Rive rise to a set of very complicated nonlinear equations and an objective function. Obvi wisly, such a model does not lend itself to a closed form solution; but one may use numerical techniques to arrive at an optimum solution. There are several nonlinear programming algorithms which provide convergence from a reasonable set of initial guesses. The selection of such an algorithm depends on the characteristics of the problem at hand, and the availability or non-availibility of a feasible starting solution.

In the course of this research project two nonlinear programming algorithms were found to be appropriate for use in the minimum weight design optimization study. Both of these algorithms are based on transforming a constrained optimization problem into a sequence of unconstrained problems. The successive solutions of unconstrained
problems converge to a colution of the constrained problem. The Sequential Unconatrained Minimizatior. Technique T) $[10,11]$ and the Augmented Lagrangian Multiplier Tecinique (ALAG) [1\%, 13, 14] are two popular paramatric transformation techniques that have been examined during this study. The ALAG algorithm has been found to be the faster and easier technique for the problem at hand [15]. The deaign optimization regulte that are presented in this report are obtained using the ALAG algorithm.

The followirg optimization approach is proposed for finding the converter minimum weight design:
(1) Fix the switching frequency.
(11) Find all the circuit parameters which give the minimum
(11i) Change the system frequency over a certain desirable range and repeat the process.
C. Advantages of Computer Aided Design (CAD)

Usirg the Computer-Aided-Design (CAD) approach, designers no longer have to relay on subjective and brute-force trial and error methods. Computer-aided design not only provides the optimum solution but also offers the following advantages:
(1) The CAD approach is cos, effective, since the switching frequency, circuit components and optimum mas, netic designs, (down to the detalls of core size, mean magnetic path length, elc.) can be obtained in one computer run. This capability has a unique distinction over the conventional piecemeal suboptimum design. The CAD
approach integrates the interdependent nature of the various functions of the power converter.
(2) By treating the awitching frequency as a parametric conistant In the simulation process, the overwhelming computation time and convergence difficulties which otherwise redult can be reduced to a minimum.
(3) Assessment of tradeoffs between converter weight and loss as function of switching frequency is immediately possible through the proposed approach.

## 3.2 bOOST CONVERTER OPTMMIZATION - OBJECTIVE FUNCTION AND CONSTRAINTS

In this chapter, the formulation of objective functions and deagn constraints for the boost power converter are presented:
(1) To use thie practical example in order to illustrate the power converter design optimization using the nonlinear programming techniques; and (2) To demrnstrate the minimum weight design of the awitching power converter and its weight/loss tradeoffs. The circuit schematic with a two-stagic input filter is shown in Fig. 3.2.1. The energy storage inductor $L_{5}$ stores the energy when the transistor $Q$ is on, then releases the energy to the load and recharges the output filter capacitor when transistor is of $f$. The key operating waveforms of this circuit are showr in Fig. 3.2.2. In this figure:
$I_{1}=$ input average DC current, $I_{0}$ - output average load current, 2d = peak to peak ripple current in $L_{5}$.

The waveforms are employed to facilitate drivations of the objective function and the constraints.

### 3.2.1 Unknown Design Variables.

There are 22 design variables for this boost power converter including RLC component values, and details of magnetic design such as core cross-sectional area, mean magnetic path length, number of turas and winding area. The transitor switching frequency and converter overall operating efficiency are two other important variables.


Fig. 3.2.1 Boost converter power circuit

$i_{5}$



Fig. 3.2.2 Important waveforms for the Boost converter


### 3.2.2 Design constants.

Design constants are obtained either through manufacturer's specifications or desizners' own experiences. Numerical values in MKS units are given in the parenthesis.

| ${ }^{5} \mathrm{C}$ | : Winding pitch factor - $\frac{\text { mean length per turn }}{\text { core circumference }}$ : (1.9). |
| :---: | :---: |
| $F_{\text {W }}$ | Core window fill factor: (0.4). |
| $\rho$ | : conductor resistivity: ( $0.172 \times 10^{-7}$ ) . |
| $D_{1}$ | Core density: (7800). |
| ${ }^{\text {c }}$ | : Conductor density: (8900). |
| ${ }^{B}$ S | : Maximum operating flux density: (0.4). |
| $\mathrm{D}_{\mathrm{K}}$ | : Weight per farad: ( $\mathrm{D}_{\mathrm{K} 3}, \mathrm{D}_{\mathrm{K4}}, \mathrm{D}_{\mathrm{K} 6} / 210,1100,72$ ). |
| $\mathrm{V}_{\text {ST }}$ | : Transistor saturation voltage drop: ( 0.25 V ). |
| $V_{B E}$ | : Transistor emitter-to-base voltage drop: (0.8V). |
| $\mathrm{T}_{\text {SR }}$ | : Transistor turn-on rise time: (0.15 $\boldsymbol{0}$ ) . |


| $\mathrm{T}_{\text {SF }}$ |  | Transistor turnooff fall t |
| :---: | :---: | :---: |
| $V_{D}$ | : | Diode conduction voltage drop: ( 0.9 V ) . |
| ${ }^{\text {FND }}$ | : | Diode turnoon rise tima: ( 0.03 Hs). |
| $\mathrm{T}_{\text {PD }}$ | : | Diode turn-off fall time: (0.05 $\mu \mathrm{E}$ ). |
| $\mathrm{T}_{\text {RE }}$ | : | Diode turn-off recovery time: ( $0.03 \mu \mathrm{H}$ ) |
| $K_{\text {H }}$ | : | Heat sinm weight density: ( $15.4 \mathrm{w} / \mathrm{kg}$ ). |
| ${ }^{\text {K }}$ | : | Source weight denaity: ( $30.8 \mathrm{w} / \mathrm{kg}$ ) . |

3.2.3 Power Converter Performance Requiremente.

The performance requirements apecified below will be amployed in the next section to formulate dasign constrainte.
$E_{1}:$ Input voltage: (28V).
$\mathrm{E}_{0} \quad: \quad$ Output voltage: (37.5V).
$\mathrm{P}_{0} \quad: \quad$ Output power: (70W).
S : Frequency dependent source conducted interference: (0.1A). This specification imits the maximum percentage of the switching carrent being reflected back to the source ensuring that the source is not significantly disturbed by the switching action downstream. Referenced here is mil - sta 461, whose characteristic curve is shown as follows:

-102-
$V_{R} \quad$ : outpht ripple factor: (1X).
The output ripple facter in defined as output ripple factor: (?\%) A. peak-to-peak output. rippic voltare. .
nominal dc output voltage
PEI
: Input-filter resonant peaking limit: (2). The input filter peaking at ite resonant frequency should be ilmited in order not to degrade the etability and the audiosusceptibility of the converter.

### 3.2.4 Objective Function.

The objecife function is defined as the tutal weight of the converter, which is the sum of various component weights including:
(n) Core Weight: WI $=D_{1}\left(A_{1} Z_{1}+A_{2} Z_{2}+A_{5} Z_{5}\right)$
(where AZ = core volume)
(b) Winding Weight: WTW $=4 F_{C} D_{C}\left(A_{C 1} N_{1} \sqrt{\Lambda_{1}}+A_{C 2} N_{2} \sqrt{\Lambda_{2}}\right.$

$$
\left.+A_{C 5} N_{5} \sqrt{A_{5}}\right)
$$

(where $4 \mathrm{~F} \sqrt{\Lambda_{1}}=$ mean length per curn of the winding)
(c) Capacitor Weight: $W C=D_{K_{3}} C_{3}+D_{\mathrm{K}_{4}} C_{4}+D_{\mathrm{K}_{6}} \mathrm{C}_{6}$
(d) Source Weight: wS $=\frac{\mathrm{P}_{0}}{\text { effK }}$
(where $\frac{\mathrm{P}_{\mathbf{0}}}{\text { eff }}$ = input power)
(e) Heat Sink Weight: $W H=\frac{P_{0}(1-e f f)}{e f f K_{H}}$

$$
\begin{gather*}
\text { (where } \frac{P_{0}}{\text { eff }}-P_{0}=\text { total loss) } \\
\text { Objective Function: } F=W I+W T W+W C+W S+W H \tag{3.2.1}
\end{gather*}
$$

### 3.2.5 Design Constraints.

In this study, the design effort is carried out using appropriate models which portray the physical characteristics of the boost power
converter. Some of the more eignificant characteriatice are: power loan, core willdow area, core flux density, and magnetic windisg reaistance. Inclusion of these tharacteris:ice reaulte in a very complicated set of nonlinear constrainte. Mathematical restrictionsfor chese contrainta can be found in Appendix $D$.
(a) Lose Constraint: $C(1)=0$

$$
\begin{equation*}
C(1)=P_{0}\left(\frac{1}{f f f}-1\right)-D I F-P Q-P D-P O R-P C A P \tag{3.2.2}
\end{equation*}
$$

where PIF - input filter copper lose

$$
-\left(\frac{P_{0}}{\operatorname{eff} E_{1}}\right)^{2}\left(R_{1}+R_{2}\right)
$$

$\mathrm{PQ}=\mathrm{Tranaletor}$ aturation lose + Base

## drive lose

+ transistor turn on switching loss
+ transistor turn off switching loss

$$
\begin{aligned}
& -\frac{P_{0} V_{S T}\left(E_{0}-E_{1}\right)}{\text { eff } E_{1} E_{0}}+0.1 \frac{P_{0} V_{B E}\left(E_{0}-E_{1}\right)}{\text { eff } E_{1} E_{0}} \\
& +\frac{T_{S R}{ }^{F}}{6}\left(E_{0}+V_{D}+2 V_{S T}\right)\left(\frac{P_{0}}{\text { eff } E_{1}}-\frac{E_{1}\left(E_{0}-E_{1}\right)}{2 L_{5} E_{0}{ }^{F}}\right)
\end{aligned}
$$

$$
\begin{equation*}
+\frac{T_{S F} F}{6}\left(E_{0}+V_{D}+2 V_{S T}\right)\left(\frac{P_{0}}{e f f} E_{i}+\frac{E_{i}\left(E_{0}-E_{i}\right)}{2 L_{5} E_{0} F}\right), \tag{3.2,3}
\end{equation*}
$$

$P_{D}=$ Diode conduction lose

$$
\begin{align*}
& + \text { Turn on loss } \\
& + \text { Turn off and recovery lose } \\
& -\frac{P_{0} V_{D}}{\text { ff } E_{0}}+\frac{E_{0} T_{N D} F}{12}\left[\frac{P_{0}}{C f f E_{I}}+\frac{E_{1}\left(E_{0}-E_{1}\right)}{2 L_{5} E_{0}^{F}}\right] \\
& +\frac{E_{0}\left(T_{F D}+3 T_{R E}\right) F}{12}\left[\frac{P_{0}}{e f f E_{i}}-\frac{E_{1}\left(E_{0}-E_{1}\right)}{2 L_{5} E_{0}^{F}}\right] \tag{3.2.4}
\end{align*}
$$

POF - Output filter copper and cora loss =

$$
\begin{align*}
& \quad\left[\left(\frac{P_{0}}{\text { eff } E_{1}}\right)^{2}+\frac{1}{12} \frac{E_{1}{ }^{2}\left(E_{0}-E_{1}\right)^{2}}{L_{5}{ }^{2} E_{0}{ }^{2} F^{2}}\right] R_{5} \\
& +  \tag{3.2.5}\\
& \frac{80 E_{1}\left(E_{0}-E_{1}\right) z_{5} \times 0.0022 \sqrt{F}}{E_{0} N_{5}},
\end{align*}
$$

PCAP = Output filter capacitor ESR loss =

$$
\begin{align*}
& \left(1-\frac{E_{1}}{E_{0}}\right)\left(\frac{P_{0}}{E_{0}}\right)^{2} R_{6}+\frac{E_{1}}{E_{0}}\left[\frac{E_{1}^{2}\left(E_{0}-E_{i}\right)^{2}}{12 L_{5}{ }^{2} E_{0}^{2}{ }^{2}}\right. \\
& \left.+\left(\frac{P_{0}}{\operatorname{eff} E_{i}}-\frac{P_{0}}{E_{0}}\right)^{2}\right] R_{6} . \tag{3.2.6}
\end{align*}
$$

(b) Operating flux density conctraint: $C(5)=C(6)-C(9)=0$

This constraint ensures that the magneric core will not exceed its intended maximum operating flux density. Notice that $L_{5}$ handle's both $D C$ and ripple componente.

$$
\begin{equation*}
C(5)=N_{1} A_{1}-\frac{L_{1} P_{0}}{\operatorname{eff} E_{1} B_{B 1}} \tag{3.2.7}
\end{equation*}
$$

$$
\begin{align*}
& C(6)=N_{2} A_{2}-\frac{L_{2} P_{0}}{\text { effE} E_{1} B_{22}}  \tag{3.2.8}\\
& C(9)=N_{5} A_{5}-\frac{L_{5}}{B_{55}}\left[\frac{P_{0}}{e f f}+\frac{E_{1}\left(E_{0}-E_{1}\right)}{2 L_{5} E_{0}{ }^{F}}\right] \tag{3.2.9}
\end{align*}
$$

(c) Window area constraint: $C(7)=C(8)=C(10)=0$

All the inductor windings must be accommodated within the physical confinement of the available core window area. All cores employ a toroidal configuration with square cross section area.

$$
\begin{align*}
& C(7)=\left(\frac{N_{1} A_{C 1}}{\pi P_{W}}\right\}^{0.5}-\frac{Z_{1}}{2 \pi}+\frac{A_{1}}{2}=0  \tag{3.2,10}\\
& C(8)=\left(\frac{N_{2} A_{C 2}}{\pi F_{W}}\right)^{0.5}-\frac{Z_{2}}{2 \pi}+{\frac{A_{2}}{2}}^{0.5}=0  \tag{3.2.11}\\
& C(10)=\left(\frac{N_{5} A_{C 5}}{\pi F_{W}}\right)^{0.5}-\frac{Z_{5}}{2 \pi}+\frac{A_{5}}{2}=0 \tag{3.2.12}
\end{align*}
$$

(d) Parasitic resistance for $L_{1}, L_{2}, L_{5}: C(2)=C(3)=C(12)=0$

$$
\begin{align*}
& C(2)=R_{1} A_{C 1}-4 \rho F_{C} \sqrt{A_{1}} N_{1}  \tag{3.2.13}\\
& C(3)=R_{2} A_{C 2}-4 \rho F_{C} \sqrt{A_{2}} N_{2}  \tag{3.2.14}\\
& C(12)=R_{5} A_{C 5}-4 \rho F_{C} \sqrt{A_{5}} N_{5} \tag{3.2.15}
\end{align*}
$$

(e) Input filter peaking constraint: $C(4)=0$

Thie conetralit is important in determinins the audiosuaceptibility performance and the control loop atability.[16]

$$
\begin{equation*}
c(4)=(\text { PEI })^{2}-\frac{1+\left(\frac{R_{3}{ }^{2} c_{3}}{L_{1}}\right)}{\left(\frac{C_{4}}{C_{3}}\right]^{2}+\left(R_{3}{ }^{2} \frac{C_{3}}{L_{1}}\right)\left[1-\frac{c_{4}}{c_{3}}-\frac{L_{2} C_{4}}{L_{1} C_{3}}\right]^{2}} \tag{3.2.16}
\end{equation*}
$$

(f) Output ripple constraint: $C$ (11) $=0$

Output ripple factor (in percentage) is expressed as
$V_{R}=\left[\frac{P_{0}}{e f f E_{1} E_{0}}+\frac{E_{1}\left(E_{0}-E_{1}\right)}{2 L_{5} E_{0}{ }^{2} F}\right] R_{6}+\frac{P_{0}\left(E_{0}-E_{1}\right)}{2 E_{0}{ }^{3} C_{6} F}$
$C(11)=V_{R}-\left[\frac{P_{0}}{\text { eff } E_{1} E_{0}}+\frac{E_{Y}\left(E_{0}-E_{i}\right)}{2 L_{5} E_{0}^{2} F}\right] R_{6}$

$$
-\frac{P_{0}\left(E_{0}-E_{1}\right)}{2 E_{0}{ }^{3} C_{6} F}
$$

(g) Frequency dependent source EMI constraint: C(13) $\geqslant 0$

This constraint limits the maximum percentage of the switching current being reflected back to the source.

The input filter must be designed to satisfy the following requirement:
Required attenuation at switching frequency
$-\frac{\text { EMI requirement }}{\text { Fundamental component of the switching current }}$

$$
\begin{align*}
C(13) & =\frac{S}{\left\lvert\, A V \sqrt{1+\left(\frac{F}{2000}\right)^{2}}\right.}-\left\{\left(\frac{L_{2} C_{4}}{L_{1} C_{3}}\right\}\left(2 \pi F \sqrt{L_{1} C_{3}}\right)^{3} \frac{1}{D}\right.  \tag{3.2.19}\\
& -\frac{C_{4}}{C_{3}}\left(2 \pi F{\left.\sqrt{L_{1} C_{3}}\right)^{2}}^{2}\right\}^{-1}
\end{align*}
$$

where

$$
\begin{aligned}
& A=\frac{-E_{0}}{\pi^{2} L_{5} F}\left(\sin \frac{\pi\left(E_{0}-E_{1}\right)}{E_{1}}\right)^{2} \\
& D=R_{3}\left(\frac{C_{3}}{L_{1}}\right)^{0.5}
\end{aligned}
$$

(h) Additional inequality constrainta: $C(14), C(15), C(16)$, $\mathrm{C}(17) \geqslant 0$

These constraints are needed to confine some of the variables in reasonable ranges in order to facilitate program convergence.
$C(14)=0.97-$ eff $>0$
$C(15)=R_{T}-R_{1}-R_{2} \geqslant 0$
$c(16)=c_{4}-1.0 \times 10^{-6} \geqslant 0$
$C(17)=c_{3}-1.0 \times 10^{-6} \geqslant 0$

### 3.3 INTRODUCTION TO NONLINEAR PROGRAMMING AND AUGMENTED LAGRANGIAN (alag) PEnalty function method

Most optimization problems arising from practical power converter applications are sufficiently complicated to defy closed-form solutions. To numerically realize an optimum design, one has to resort to nonlinear programing algorithms which can provide fast convergence to an optimum solution from a reasonable gunss of the starting point. The nonlinear programing problem (NLP) of extremizing (maximizing or minimizing) a function of $n$ variableq, while requiring other functions of the same variables to satisfy either equality or inequality constraint relationships, is called constrained NLP. The problem is to maximizing or minimizing a function of n variables without regard to side conditions or constraints is called an unconstrained NLP. While there exist numerous methods of nonlinear programming, the effectivaness of each method depends greatly on the particular multidimensional problem to which the method is applied. The availabilty of numerous efficlent numerical methods for solving the unconstrained optimization problem has motivatid the design of algorithms that transform a constrained problem into a sequence cf unconstrained problems such that the successive solutions of the unconstrained problems converge to a soiution of the constrained problem. These transformation methods implicitly incorporate all the constrainta into the objective function that is to be optimized. The algorithms based on the transformation approach are conceptually simpler and easier to implement than the algorithms that handle the constrainto directly because of the relative ease of extremizing an unconstrained problem compared to a constrained one.

### 3.3.1 Nonlinear Programaing Panalty Function Method.

The Penalty Punction Technique amploya the aforementioned traneformation method [11]. Let us define problem P1 as the original conatrained NLP problea and P2 as the cransformed unconstrained NLP problem.

P1: Minimize objective function $f(x)$ subject to:
1.) Inequaltiy constraints $g_{1}(x) \geqslant 0,1=1,2, \ldots, P$.
2.) Equality constraints $h_{j}(x)=0, j=1,2, \ldots, q$.

P2: Minimize $A\left(x, w^{m}, g, h\right), m=1,2, \ldots$
Where $x=$ vector of $n$ unknown variables.
$A\left(x, w^{m}\right)$ - new objective function formed by augmenting the original objective function $f(x)$ with weighted terms (penalty terms) that depend on the constraints $g$ and $h$. $w^{\text {m }} \quad$ - controlling weighting factor, on penalty term, a vector of Lagrange Multipliers.
m $\quad=$ number of iterations.
The essence of transforming the constrained NPL into an unconstrained NLP is that by gradually removing the effect of the conctraints in the new objective function (by controlling the weighting factor $w^{m}$ ) it is posaible to generate a sequance of uncorstrained problems that have solutions converging to the solution of the original constrained problem.

```
That 1s: \(\lim A\left(x, w^{m} g, h\right)-f\left(x^{*}\right)=0\), \(m \rightarrow \infty\)
```

After initerations, the variable $x$ approaches the optimum $x^{*}$., In effect, the influence of the constraints on the augmented objective function is relaxed and, in the limit, removed, and the aumented objective function $A\left(x, w^{m}, g, h\right)$ converges to the same optimum value $f\left(x^{*}\right)$ of the original object ${ }^{\text {ve }}$ function.

### 3.3.2 Sequentially Unconetrained Minimisation Technique (SUNT).

SUMT was developed, validated, extended and refined by Fiacco and McCormick $\{10,11\}$. This method replaces the conetrained problem Pl defined below by a sequence of unconetrained minimization problems ae defined in P2.

P1: Minimize $f(x, y)$ ubject to:
1.) $g_{1}(x, y, z) \geqslant 0,1=1,2, \ldots, p$
2.) $h_{j}(x, y, z)=0, j=1,2, \ldots, q$

P2: Minimize $P\left(x, r_{k}\right)=\varepsilon-r_{k} \sum_{i=1}^{p} \ln g_{i}+\frac{1}{r_{k}} \sum_{j=1}^{g} h_{j}^{2}$
where $P\left(x, r_{k}\right)$ is the penalized objective function, $r_{k}$ is a monotonically decreasing sequence tending to zero, and $x$ is an $n$-dimensional vector representing the design variables to be optimally selected. In the power converter design optimization, the componente of $x$ are values of $R, L, C$, and the design datails for magnetics such as core cross-section area, mean magnetic path length, wire size, number of turns on magnetic winding, etc. $y$ represants the vector of constants related to component characteristice such is winding and core densities, transistor and diode switching times, the intended maximum operating flux density of given magnetics, etc. $z$ represents the vector of performance requirements to be met by optimum design such as the maximum output ripple, EMI requirement, output power, input filter peaking limit, etc. $f(x, y)$ repremente the objective function (such as the total converter weight) to be minimized.

The basic idea of SUMT is to solve a sequence of unconstrained problems like P2 whose solution approaches the solution of P1. Considerable computational difficulcies have been experienced with the SUMT algorithm. The most serious handicaps are summarized below. The contours of $P\left(x, r_{k}\right)$
correspond to increaningly steep sided valleye ae the controling paramer decreses, and the Hassian of the function becomes progressivaly more 111conditioned as $r_{k}+0$ and the opeimum solution $x^{\boldsymbol{*}}$ is approsched. As a reault, the search directions may become misieading. The rate of convergence depends on the initial value of $r_{0}$ and the method of reducing $r_{k}$. Pinally, most of the information about the topology of $f(x, y)$ and $P\left(x, r_{k}\right)$ is discarded from one stage to the next even if some type of extrapolation is incorporated in the algorithm. The attempts to overcome these computational difificulties have resulted in several modifications of SUMT. The ALAG penalty function technique resulted from such efforts to improve the computational method. It has gained recognition as one of the most effective methods for solving constrained optimization problems. The algorithm based un this method converges at a superifnear rate; the computational effort per iteration falls off rapidly; the initial starting point need not be feasible; and the transiormation function is defined for all values of the parameters.

### 3.3.3 ALAG Penalty Function Technique.

The Augmented Lagrangian Penalty Function for P1 is obtained by combining the Powell-Hestenes $[12,13]$ penalty function and the Rockefellar penalty function [14] as in P3 in the equation below.

$$
\begin{aligned}
& \text { P3: Minimize } \psi(x, \lambda, \sigma) \\
& \text { where } \psi\left(x, \lambda_{f}, \sigma\right)=f(x, y)-\sum_{j=1}^{q}\left[\lambda_{j} h_{j}-\frac{1}{2} \sigma_{j} h_{j}^{2}\right] \\
& +\frac{1}{2} \sum_{i=1}^{P}\left[\sigma_{i}\left[g_{1}-\frac{\lambda_{1}}{\sigma_{1}}\right)_{-}^{2}-\frac{\lambda_{1}^{2}}{\sigma_{1}}\right] \\
& \text { where }\left(g_{1}-\frac{\lambda_{1}}{\sigma_{1}}\right)=\min \left[\left(g_{1}-\frac{\lambda_{1}}{\sigma_{1}}\right), 0\right], \sigma_{1}=\lambda_{1} \theta_{1}, \quad \forall_{1} \\
& \lambda_{1} \varepsilon E^{P+q}, \theta_{1} \varepsilon E^{P+q}, \sigma_{i} \varepsilon E_{+}^{P+q} \quad \forall 1
\end{aligned}
$$

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The $\theta_{1}^{\prime}$ e and $\sigma_{1}^{\prime}$ e are controlliug parameters, wheres the SUNT algoritha has only one controlling parameter $r_{k}$. The parameter $q_{1}$ is changed only when the rate of convergence is not atiatiactory and $\theta_{1}$ is changed in every iteration to enforce constraint eatisfaction. The important feature of this approach is that $1 / \sigma_{1}$, which corresponds to the controlling parameter $r_{k}$ in SUMT, is not required to tend to zero for convergence of the algo:ithm. In the penalty function $\psi$ defined above, the term $\lambda_{1} / \sigma_{1}$ represents a penalizing threshold for the ith inequality constraint. Increasing $\sigma_{1}$ to enforce faster convergence reduces the penalty threahold level and laads to closer conatraint atisfaction. When the inequality constraint $g_{1}>0$, then $\lambda_{1}$ (or $\theta_{1}$ ) is relaxed to zero; otherwise, it is changed to make the corresponding constraint active at the current solution $x$.

### 3.3.4 Comparison Between SUMT and ALAG

The SUMT package was utilized in the initial phase of this design optimization. Considerable effort was spent in computer coding and implementation of the SUMT package. Although optimum solutions were reached, the results were less satisfactory and were sensitive to initial guesses. Therefore another software package, ALAG, requiring only slight modification of the original SUMP Code, was adopted to explore an alternative means of design optimization. The triale on the ALAG algorithm were quite successful. The two algorithms are compared briefly below [15].
(1) Since SUMT requires first and second srdar derivatives for the constraints and the objective funceion, many hours of data praparation are needed. The ALAG algorithm needs only the first derivatives.
(2) The initial atarting point in SUST should be atrictiy feaaible with reapect to the inequality constraints in order to get proper convergence. The ALAG algorithm does not require the atarting point to be feasible.
(3) SUMT requires about ten times more computer atorage and CPU time than the ALAG algorithm.
(4) ALAG convergee much more readily than SUMP.
(5) Computational comparisons between SUMT and ALAG were obtained using Buck convertar an an example [15]. The cumparisons are shown in Table 3.3-1.

Table 3.3-I COMPARISONS BETWEEN SUNT AND ALAG USING BUCK CONVERTER DESIGN OPTIMIZATION PROGRAM

|  | SUMT | ALAG |
| :--- | :---: | :---: |
| COMPILING TIME <br> (SECOND) | 4.61(USER PROGRAM <br> ONLY) | 6.54 ENTIRE PROGRAM |
| EXECUTION TIME | 197 | 6.34 |
| TOTAL TIME | $201+?$ | 13 |
| INPUT CARDS | 2719 | 1299 |
| KILOBYTE SECONDS | 165216 | $\$ 4.29$ |
| TOTAL RUN CHARGES | $\$ 16.65$ |  |

Since the ALAG was found generally superior to SUMT in several aspects, it was adopted to implement the computer-aided design optimization for the remainder of the study.

### 3.4 IMPLEMENTATION OF COMPUTER CODE AND USER'S GUIDE

The nonlinear prograning package AlAG is designed with the ueer convenience in mind. There is one main program and nine subroutines in the whole package. A list of computer programe is given in the Appendix F. The nine subroutines are:

Alaga
ALAGZ
QNTA
MUDA
MUDB
MUDE
BQMA
BQMB
ALAGB $\rightarrow$ User's supplied subroutine

All subroutines except ALAGB are supplied in the ALAG package. The user also needs to supply the main program fincluding the objective function, constraints and their derivatives. The flow-chart of general optimization sequence using a multiplier algorithm is shown in Figure 3.4.1.

### 3.4.1 User Supplied Main Program and Parameters

The main program basically supplies the controlling parameters, the input data buch as design constants, performance requirement, initial starting point), variable and constraint scaling factors, and the output information with print-out format. Detailed descriptions of the main program and the subroutine ALAGB are given in this chapter and Appendix E. Both of these programs may serve as a limited user's application guideline.


Figure 3.4.1 General Minimization Sequence Using o Multipller Algorlthm

The user supplied controlling parameters are explained in the

## tollowing

## Controlling Paraneters



### 3.4.2 User Supplied Subroutine AlacB.

ALAGB is the only user eupplied eubroutine. Thie subroutine sivea the information about constraint equations and their firet order derivatives. The following ntepe are used to prepare the computer code Ior this subroutine:

## Step 1

Define array variablee $x(1)$ to $x(N)$, were $N$ is to the number of the design unknowns.

## Scep 2

Manipulate the constraint equations such that they are simplified as much as posaible.

Step 3
Examine the constraints obtained in Step 2, and assign a name to (.. mon constant terme in order to further simplify the constraint equations, Step 2 and 3 will save computer data preparation time. Step 4

Now transform all the constraint equations in terms of the array variables $X(1)$ to $X(N)$ and the constant name created in Step 3. The computer progras layout is such that the inequality constraints come after the equality constraints. The constraints are designated irom $C(1)$ to $C(M)$, where $M$ 1s the total number of equality and inequality constrainta.

## Step 5

Take fi:st derivatives of the objective function and the constraints with respect to their corresponding variables. For example, assume constraint $\mathcal{C}(i)$ contains one variable $X(j)$, then the derivative will be designated as GC(f,i). For the objective function, the derivative will be designated as $G(j)$, where $f$ is one of the variables contained in the objective function.

## Scep 6

The computer cude for the subruutine Alage will have the following layouti COHEON STATETENT
EQUATE XI $=\mathrm{X}(1)$ (This enablas the users to use XI inatead of $X(1)$ )
OBJECTIVE FUNCTION $P$
CONSTRAINT EQUATION C(I)
derivatives of objective punction g(j)
derivatives of all the constraints cc $(\mathrm{j}, 1)$
RETURN
END
The user is referred to Appendix $B$ for more programing details.
Example
Consider the EMI constraf t:
$C(13)=\frac{S}{\sqrt{1+\left(\frac{F}{2000}\right)^{2}}} \frac{1}{A}-\left[\left(\frac{L_{2} C_{4}}{L_{1} C_{3}}\right)\left(2 \pi F \sqrt{L_{1} C_{3}}\right)^{3} \frac{1}{D}-\frac{C_{4}}{C_{3}}\left(2 \pi F \sqrt{L_{1} C_{3}}\right)^{2}\right]^{-1}$
where $A=\frac{E_{0}}{\pi^{2} L_{5} F} \sin \frac{\pi\left(E_{0}-E_{i}\right)}{E_{1}}$

$$
D=R_{3}\left(\frac{C_{3}}{L_{1}}\right)^{0.5}
$$

Step 1: Assign variable name:

$$
\begin{aligned}
& x_{7}=L_{1} \\
& x_{9}=C_{3} \\
& x_{10}=C_{4} \\
& x_{15}=R_{3} \\
& \frac{x_{7}}{P E I}=L_{2} \\
& x_{20}=L_{5}
\end{aligned}
$$

## Step 2: Mianipulate the equation:

$C(13)=\frac{S}{\sqrt{1+\left(\frac{F}{2000}\right)^{2}}}-\frac{1}{\frac{E_{0}}{\pi^{2} L_{5} F} \sin \frac{1}{\pi\left(E_{0}-E_{1}\right)}} \frac{E_{1}}{4 \pi^{2} F^{2} C_{4} L\left[\begin{array}{l}2 \pi F L_{1} \\ \overline{P E I R}_{3} \\ -1\end{array}\right]}$

Step 3: Assign constant name:
Let XM26 $=\frac{S}{\sqrt{1+\left(\frac{F}{2000}\right)^{2}}} \frac{1}{\frac{E_{0}}{\pi^{2} F} \operatorname{Sin} \frac{\pi\left(E_{0}-E_{i}\right)}{E_{i}}}$

$$
\begin{aligned}
& \mathrm{XX} 27=4 \pi^{2} \mathrm{~F}^{2} \\
& \mathrm{XM} 28=\frac{2 \pi \mathrm{~F}}{\mathrm{PEI}}
\end{aligned}
$$

Step 4: Constraint equation:
Now C(13) becomes
$-1+(\mathrm{XM} 26)(\mathrm{XM} 27) \mathrm{X}_{7} \mathrm{X}_{10} \mathrm{X}_{20}\left(\mathrm{XM} 28 \frac{\mathrm{X}_{7}}{\mathrm{X}_{15}}-1\right) \geqslant 0$.

Step 5: Take first derivatives

$$
\begin{aligned}
& \text { GC(7,13) }=2.0 \text { (XM26) (XM27) (XM28) } \frac{X_{7} X_{10} X_{20}}{X_{15}} \\
& -(\mathrm{XM} 26)(\mathrm{XM} 27) \mathrm{X}_{10} \mathrm{X}_{20} \\
& \operatorname{GC}(10,13)=(X M 26)(X M 27) X_{7} X_{20}\left(X M 28 \frac{X_{7}}{X_{15}}-1\right) \\
& \operatorname{Gc}(15,13)=-(X M 26)(x M 27)(x M 28) \frac{x_{7}^{2} x_{10} x_{20}}{x_{15}{ }^{2}} \\
& \operatorname{GC}(20,13)=(\mathrm{XM} 26)(\mathrm{XM} 27) \mathrm{X}_{7} \mathrm{X}_{10}\left(\text { XM }^{28} \frac{\mathrm{X}_{7}}{\mathrm{X}_{15}}-1\right)
\end{aligned}
$$

STEP 6: = Set up computer code for subroutine.ALAGB.

### 3.4.3 Initial Starting Point and Scaling Technique

### 3.4.3.1 Selection of initial starting point

For a complicated nonlinear optimization problam with 20 or more variables, proper selection of the initial starting point plays an important role in the apeed of convergence and accuracy of the solution, As a generial guideline, the initial starting point should be selected such that the value of each equality constraint is as small as possible. This point must also satisfy the inequality constraints in order to stay in the feasible region. A properly selected initial starting point will speed up the rate of convergence. The time and effort spent to choose a good initial point prior to running the program is well worth the result. For a practical problem, choice of a good initial starcing point can usually be based on the degigner's past experience, or on some simplified design guidelines and equations.

### 3.4.3.2 Variabie scaling technique and convergence

In a switching power converter design, the values of the design variables are scattered over a wide range. The capacitance may be in the order of $10^{-4}$, for example, and the switching frequency in the order of $10^{5}$. This wide scattering of values is one of the primary causes of convergence difficulty. Therefore, a variable scaling technique is provided in the computer program to scale all the variables between values of 1 and 10 . For example: if $x_{1}=0.5 \times 10^{-6}$ and $x_{2}=0.8 \times 10^{3}$, then one can use VSCAL (1) $=10^{-7}, \operatorname{VSCAL}(2)-10^{2}$, so that:

$$
\frac{x_{1}}{\operatorname{VSCAL}(1)}-5.0 \frac{x_{2}}{\text { VSCAL (2) }}=8.0
$$

where VSCAL (1), VSCAL (2) are scale factors for the respective variables.

For reasonably acceptable accuracy, the tolerance for variable convergence (EPS) is set around EPS $=10^{-6}$ to $10^{-7}$. For the program to exist from the iterative computation through the variable convergence criterion, it must satisfy the following requirement:
$\max \left|x_{1}^{(k)}-x_{1}^{(k-1)}\right|_{\leq E P S}$
This requirement states that the largest difference between two values of any variable from consecutive iterations must be less than the tolerance required. It should also be mentioned that the program can exist via a constraint convergence criterion.

### 3.4.3.3 Constraint scaling technique and convergence

It is very unlikely that the initial starting point can satisfy all the constraints to the extent that each equality constraint residual is smaller than the constraint tolerance and each inequality constraint is also satisfied. If the starting poing did satisify all the constraints, then, of course, the problem would already be solved. In reality, the constraint values based as the initial guess can vary over a wide range. Since conditions where certain constraint values may be so large that the effects of other constraints are obscured should be avoided, it is desirable to scale each constraint by such a factor that the effect of violating any given constraint is of the same order of magnitude as the effect of violating any other constraint. Unfortunately there are no universal guidelines for selecting the constraint scaling paramerters. It has been observed that faster convergence can be achieved by the proper selection of these parameters; however, improper use of constraint scaling can cause divergence problems. Experience shows
that by scaling the constraint values in a range between $10^{2}$ and $10^{-2}$, and setting the constraint tolerance around $10^{-3}$, the program can achieve a faster rate of convergence.

For an acceptable accuracy, the constraint tolerance AKMIN is set around $10^{-3}$ to $10^{-4}$ using the scaling technique. Whenever the maximum scaled constraint violation AKK ${ }^{(k)}$ is less than AKMIN, program convergence is reached. This stopping criteria can be put in a more concise way in the following.
$C_{i}{ }^{(k)}$ : Constraint value for 1 th constraint in iteration $k$
$S_{1}$ : Scale factor for the ith constraint
$W W_{i}(k)$ : Scaled constraint violation for ith constraint in iteration $k$

That is $\quad W W_{1}{ }^{(k)}=\frac{\left|C_{i}^{(k)}\right|}{S C_{1}}$
$A^{\prime}{ }^{(k)}$ : Largest scaled constraint violation in iteration $k$, that 18 , $A K K^{(k)}=\max _{1}\left\{W_{1}^{(k)}\right\}$

Whenever AKK ${ }^{(k)} \leqslant$ AKMIN, the convergence is reached and computation is terminated.

The program sometimes can also be run without using the constraint scaling technique. Experience shows, however, that by using the constraint scaling technique the program can be brought under better control.

### 3.4.4 Stopping Criteria for Computation

For normal exit, there are constraint convergence criterion and variable convergence criterion as mentioned in the previous subsections. In most cases, consiraint convergence is deemed more desirable. The accuracy of the result however depends on how to choose the constraint tolerance and variable tolerance. In some circumstances, the solution via variable
convergence criterion is sufficlently accurate to be acceptable. Infinite looping and abnormal exit are also possible as shown in the flowchart in Fig. 3.4.2.

## Discussion of flowchart

## EXIT 1

This exit means the objective function has been evaluated a number of times equal to the user's supplied paramteter MAXFN. The solutions from this exit are in most cases not accurate. The user may fincrease MAXFN to get proper convergence and more accurate solutions. EXIT 2

This exit means the largest scaled constraint violation is less than the constraint tolerance. The solution from this exit is deemed most desirable.

## EXIT 3

The exit means the largest difference of variables between consecutive iteration is less than the variable tolerance. Depending on the exit condition, the solution from this exit is often acceptable. LOOP 4

In this red cape loop, the program is never converged. The user must set execution time limit or printing page limit in case end less loop occurs.

### 3.4.5 Checklist for Computer Printout.

The following checklist is provided for the user to assure the final solution is accurate and acceptabl..
(1) Check if the solucion $C$ is in the feasible region, that is, if the inequality constraint residuals are all greater than zero.
(2) Check the accuracy of the solution obtained. That f.a: check if largest scaled constraint violation is less then AKMIN.
(3) Check if MAXFN IS REACHED, if the exit is normal or abnormal.
(4) Check if the solution $X$ is reasonable using common sense and previous experience.

If the solution obtained is not accurate enough then one can follow the flowchart as shown in Fig. 3.4.3, by changing the starting point (uaing the final result of the previous run) or readjusting the scaling factors and rerun the program.

where
MAIN = moln program
ALAGA $=$ subroutine that monoges the stopping algorithm
Figure 3.4.2 Flowchart of Computation Stopping Algorithm


Figure 3.4.3 Flowchart of effective Programming Approach

### 3.5 OPTIMIZATION RESULTS OF BOOST CONVERTER

The transistor switching frequancy is a critical parameter in the minimum weight design of switching power converter. In the course of optimization the frequency is held constant for a computer run and a sufficient number of runs are collected catrying the frequency over a certain range of interest. Several distinct advantages can be obtained in this approach:
(1) By treating the frequency as a constant in each computer run, the nonlinear optimization problem is simpler and reaches convergence easily.
(2) Important design insights can be obtained when the weight and loss are plotted against frequency. Instead of identifying a single optimum switching frequency and a minimum weight design as in our earlier optimization attempt [8], the curve presented here provides a range of frequencies in which the system weight is minimized in all practical sense. The curve will also provide information regarding sensitivity of the converter weight as a function of the switching frequency.
(3) The rade-offs between weight and loss as a function of switching frequency can be evaluated readily. This information can be used as a design guideline for the weight/efficiency optimization.

By treating the switching frequency as a constant in the optimization process, a set of design data as a function of switching frequency is obtained by varying the frequency between 20 KHz to 120 KHz in a 10 KHz step. The design parameters specified in Section 3.2 are employed to make these computer runs. Detailed design results including the detailed loss and weight breakdowns oí various components are collected and tabulated in Table 3.5-I. The minizum weight converter design data including the details of magnetic design for each chosen frequency are shown in each column. This helps in the














 Acs $0.398 \times 10^{-6} 0.329 \times 10^{-6} 0.295 \times 10^{-6} 0.259 \times 10^{-6} 0.101 \times 10^{-6} 0.912 \times 10^{-7} 0.816 \times 10^{-1} 0.396 \times 10^{-7} 0.367 \times 10^{-7} 0.291 \times 10^{-7} 0.285 \times 10^{-7}$ $250.340 \times 10^{-1} 0.309 \times 10^{-1} 0.282 \times 10^{-1} 0.265 \times 10^{-1} 0.199 \times 10^{-1} 0.194 \times 10^{-1} 0.185 \times 10^{-1} 0.161 \times 10^{-1} 0.154 \times 10^{-1} 0.141 \times 10^{-1} 0.130 \times 10^{-1}$ LS $0.477 \times 10^{-1} 0.331 \times 10^{-1} 0.304 \times 10^{-1} 0.341 \times 10^{-1} 0.1353 \quad 0.2648 \quad 0.1786 \quad 0.510 \quad 0.383 \quad 0.626 \quad 0.626$ LS $0.255 \times 10^{-4} 0.273 \times 10^{-4} 0.229 \times 10^{-4} 0.222 \times 10^{-4} 0.245 \times 10^{-4} 0.246 \times 10^{-4} 0.238 \times 10^{-4} 0.320 \times 10^{-4} 0.317 \times 10^{-4} 0.244 \times 10^{-4} 0.239 \times 10^{-4}$ c3 $0.133 \times 10^{-4} 0.322 \times 10^{-5} 0.318 \times 10^{-5} 0.201 \times 10^{-5} 0.200 \times 10^{-5} 0.200 \times 10^{-5} 0.200 \times 10^{-3} 0.200 \times 10^{-5} 0.199 \times 10^{-5} 0.205 \times 10^{-5} 0.200 \times 10^{-5}$ (1) $0.163 \times 10^{-1} 0.162 \times 10^{-1} 0.162 \times 10^{-1} 0.162 \times 10^{-1} 0.324 \times 10^{-2} 0.524 \times 10^{-2} 0.524 \times 10^{-2} 0.524 \times 10^{-2} 0.523 \times 10^{-2} 0.523 \times 10^{-2} 0.522=10^{-2}$ cs $0.655 \times 10^{-5} 0.161 \times 10^{-5} 0.159 \times 10^{-5} 0.101 \times 10^{-5} 0.100 \times 10^{-5} 0.100 \times 10^{-5} 0.100 \times 10^{-5} 0.100 \times 10^{-5} 0.996 \times 10^{-6} 0.103 \times 10^{-5} 0.100 \mathrm{~m} 10^{-5}$ C6 $0.109 \times 10^{-3} 0.760 \times 10^{-4} 0.678 \times 10^{-4} 0.995 \times 10^{-4} 0.511 \times 10^{-4} 0.468 \times 10^{-4} 0.442 \times 10^{-4} 0.390 \times 10^{-4} 0.376 \times 10^{-6} 0.389 \times 10^{-4} 0.371 \times 10^{-4}$
 RCAP $1.30 \times 10^{-1} 0.173 \times 10^{-1} 0.198 \times 10^{-1} 0.226 \times 10^{-1} 0.277 \times 10^{-1} 0.310 \times 10^{-1} 0.333 \times 10^{-1} 0.679 \times 10^{-1} 0.518 \times 10^{-1} 0.338 \times 10^{-1} 0.556 \times 10^{-1}$
 WI $0.691 \times 10^{-2} 0.479 \times 10^{-2} 0.409 \times 10^{-2} 0.302 \times 10^{-2} 0.249 \times 10^{-2} 0.148 \times 10^{-2} 0.124 \times 10^{-2} 0.063 \times 10^{-3} 0.199 \times 10^{-3} 0.573 \times 10^{-3} 0.354 \times 10^{-3}$ ww $0.113 \times 10^{-1} 0.834 \times 10^{-2} 0.757 \times 10^{-2} 0.928 \times 10^{-2} 0.136 \times 10^{-2} 0.108 \times 10^{-2} 0.925 \times 10^{-3} 0.603 \times 10^{-3} 0.554 \times 10^{-3} 0.443 \times 10^{-3} 0.634 \times 10^{-3}$ WC $0.179 \times 10^{-1} 0.798 \times 10^{-2} 0.730 \times 10^{-2} 0.501 \times 10^{-2} 0.519 \times 10^{-2} 0.489 \times 10^{-2} 0.470 \times 10^{-2} 0.432 \times 10^{-2} 0.422 \times 10^{-2} 0.436 \times 10^{-2} 0.424 \times 10^{-2}$ nac $0.182 \times 10^{-1} 0.111 \times 10^{-1} 0.117 \times 10^{-1} 0.8 ; 1 \times 10^{-2} 0.385 \times 10^{-2} 0.256 \times 10^{-2} 0.217 \times 10^{-2} 0.145 \times 10^{-2} 0.135 \times 10^{-2} 0.102 \times 10^{-2} 0.909 \times 10^{-3}$ $\begin{array}{llllllllllllllllll}W T & 2.7465 & 2.6794 & 2.6759 & 2.6741 & 2.7249 & 2.7502 & 2.7644 & 3.0437 & 3.0962 & 3.1898 & 3.1930\end{array}$

Table 3.5-I Boost Converter Optimization Results
ascesement of the optimum component deaign as function of owitching frequency. The deaignations utilized in the loes and weight breakdowas are shown below.


In order to gain more design insights, the total-weight/total-lose and the component weights/loss breakdowns are ploted againat the awitching frequency in Fig. 3.5.1 and Fig. 3.5.2, respectively. Summarized in the following are several important observations from the table and curves.
(1) The curve of total weight versus frequency exhibits U-shape characteristics. The converter weight is heavier at both low frequency end and high frequency end.
(2) The total weight of the converter reaches its minimum value in the frequency range from 30 KHz to 50 KHz .
(3) The U-shape curve is also observed by plotting the total losa characteristics against frequency. The rapid decrease of the total loss at lower switching frequencies is caused by the reduction of winding losses of the magnetics, meanwhile the increasing total loss at higher frequencies is caused by the higher owitching


Figure 3.5.1 Weight Breakdowns for Boost Converter


Figure 3.5.2 Loss Breakdowns for Boost Converter
losses and much rapidiy increasing magnetic losaes. As a result of the increase of total lose at high frequencios, the source weight and packaging weight also increase rapidly. The weight reduction due to the decreasing magnetic component weight as frequency increase is less pronounced than the increase of packaging and wource weight. Therefore a-shape curve of cotal weight vs. frequency is formed. The U-shaped curve implies that there exists an optimal switching frequency.

### 3.6 DESIGN OPTIMIRATION OF DUCK-BOOST CONVERTER

The Buck-Boost awitching power converter is chosen as a second example. The same procedure used for the design of Boost converter is now applied to Buck-Boost. The circuit schematic and problem formulation are atated in the following. The input-output relationship and derivationa of the constraints are given in Appendix $F$. The results of design optimization are demonstrated in Section 3.7.

### 3.6.1 Circuit Schematic, Design Variableb, and Waveforms

The circuit parameters and design unknowns are shown in Fig. 3.6.1. There are 24 unknown variables including the details of magnetic design. This circuit contains a two-stage input-filter, a two-winding energy storage inductor, a power transistor, a diode and an output filter. When the transistor is turned on, the energy from the source is stored in the two-winding energy storage inductor; the output filter capacitor $C_{5}$ supplies power to the load. When the transistor is switched off, the energy previously stc ed in the inductor is dumped out to the load where it also replenishes the output-filter-capacitor energy.

The operating waveforms are shown in Fig, 3.6.2. These waveforms are used in derivations of design constraints such as the output ripple factor, and EMI constraint, etc. The notations marked on the waveform are defined as follows:
$2 \mathrm{~d}=$ peak-peak ripple current through transistor $Q$
$I_{1} \frac{\text { Ton }}{T}=$ average input current from source $E_{i}$
$I_{0} \quad$ output DC current
$n \quad=$ primary-to-secondary turns ratio of energy storage inductor,
( $=1$ in the design example presented in Section 3.7).

Desion Unknowns: 24 variables $R_{1}, R_{2}, R_{D}: D C$ winding resistance of Inductor and transformer
$R_{3} \quad:$ Input filter resistor
$\mathrm{L}_{1}, \mathrm{~L}_{2} \quad:$ Input filter inductor
$L_{p} \quad: \quad$ Primary Inductance of two winding Inductor
$C_{3}, L_{4}, C_{5}$ : Filter capacitor
$A_{1}, A_{2}, A_{p}$ : Cross section area of inductor or transformer
$z_{1}, z_{2}, z_{p}$ : Mean magnet Ic path length $N_{1}, N_{2}, N_{D}$ : Number of turns of the winding $A_{C 1}, A_{C 2}, A_{C D}$ : Winding area per turn $F \quad: \quad$ Swltching frequency eff : こyerall efficiency

Figure 3.6.1 Circuit Schematic and Design Variables of Buck-Boost Con'erter


Figure 3.6.2 Operating Waveforms of Buck-Boost Converter

### 3.6.2 Objective Punction and Constraints

The notations used previously for the Boost converter are adopted here for Buck-Boost.

### 3.6.2.1 Objective function: total weight

$$
\begin{aligned}
& \text { Core Weight WI }=D_{1}\left(A_{1} Z_{1}+A_{2} Z_{2}+A_{p} Z_{p}\right) \\
& \text { where AZ }- \text { Core volume } \\
& \text { Winding weight WTW }-4 F_{C} D_{C}\left(A_{C 1} N_{1} \sqrt{\Lambda_{1}}+A_{C 2} N_{2} \sqrt{\Lambda_{2}}\right. \\
& \qquad
\end{aligned}
$$

where $4 \mathrm{~F}_{\mathrm{c}} \sqrt{\mathrm{A}_{1}}$ = nean length per turn of the winding Capacitor weight $={ }^{2} \mathrm{~K}_{3} \mathrm{C}_{3}+\mathrm{D}_{\mathrm{K} 4} \mathrm{C}_{4}+\mathrm{D}_{\mathrm{K} 5} \mathrm{C}_{5}$ Source weight $=$ WS $=\frac{P_{0}}{\text { eff } K_{s}}$, where $\frac{P_{0}}{\text { eff }}=$ input power Heat sink weight $W H=\frac{P_{0}(1-e f f)}{\text { eff } K_{n}}$,
where $\frac{P_{0}}{\text { eff }}-P_{0}=$ total power loss

Objective function $=$ WI + WTW + WC + WS + WH

### 3.6.2.2 Constraints

## Loss constraint $\dot{Q}(1)=0$

$$
\begin{equation*}
C(1)=P_{0}\left(\frac{1}{e f f}-1\right)-P I F-P Q-P D-P O F \tag{3.6.2}
\end{equation*}
$$

where
PIF = Input filter copper 1088
$=\left(\frac{P_{0}}{e f E_{I}}\right)^{2}\left(R_{1}+R_{2}\right)$
$P Q=$ Transistor saturation loss + Base drive loss

+ transistor turn on loss + transistor turn off loss

$$
=\frac{P_{0} V_{S T}}{\text { eff } E_{I}}+\frac{0.1 P_{0} V_{B E}}{\text { eff } E_{1}}
$$

$$
+\frac{T_{S R} F}{6}\left(\frac{E_{0}+V_{D}}{n}+E_{I}+2 V_{S T}\right)\left(\frac{P_{0}}{e f f E_{I}} \frac{E_{0}+n E_{I}}{E_{0}}\right.
$$

$$
\left.-\frac{E_{I} E_{0}}{2 L_{p}\left(E_{0}+n E_{I}\right) F}\right)
$$

$$
+\frac{T_{S F}}{6}\left(\frac{E_{0}+v_{D}}{n}+E_{I}+2 v_{S T}\right)\left(\frac{P_{0}}{\text { eff } E_{I}} \frac{E_{0}+n E_{I}}{E_{0}}\right.
$$

$$
\begin{equation*}
\left.+\frac{E_{1} E_{0}}{2 L_{p}\left(E_{0}+n E_{I}\right) F}\right) \tag{3.6.3}
\end{equation*}
$$

PD = Diode conduction loss + Turn on lose

+ Turn off loas

$$
\begin{align*}
& =\frac{P_{0} V_{D}}{e f f E_{0}}+\frac{\left(n E_{I}+E_{0}\right)}{12}\left[\frac{P_{0}\left(E_{0}+n E_{I}\right)}{e f f n E_{I} E_{0}}+\frac{E_{I} E_{0}}{2 n L p\left(E_{0}+n E_{I}\right) F}\right] \\
& =\frac{\left(n E_{I}+E_{0}\right)\left(T_{S D}+3 T_{R E}\right) F}{12}\left[\frac{P_{0}\left(E_{0}+n E_{I}\right)}{e f E n E_{I} E_{0}}\right. \\
& \left.-\frac{E_{I} E_{0}}{2 n L p\left(E_{0}+n E_{I}\right) F}\right] \tag{3.6.4}
\end{align*}
$$

POF = Two-winding inductor $T$ copper and core losses

$$
\begin{align*}
& =\frac{E_{0}}{\left(E_{0}+n E_{I}\right)}\left[\left(\frac{p_{0}\left(E_{0}+n E_{I}\right)}{e f f E_{I} E_{0}}\right)^{2}+\frac{E_{I}^{2} E_{0}^{2}}{12 L_{p}^{2}\left(E_{0}+n E_{I}\right)^{2} F^{2}}\right] R_{p} \\
& +\frac{n E_{I}}{E_{0}+n E_{I}}\left[\frac{E_{I}^{2} E_{0}^{2}}{12 n^{2} L_{p}^{2}\left(E_{0}+n E_{I}\right)^{2} F^{2}}\right. \\
& \\
& \left.\quad+\frac{\left(E_{0}+n E_{I}\right)^{2} P_{0}^{2}}{e f f^{2} n^{2} E_{I}^{2} E_{0}^{2}}\right]^{2} R_{p}  \tag{3.6.5}\\
& +\frac{E_{I} E_{0}}{\left(E_{0}+n E_{I}\right) N_{p}}\left(B 0 Z_{p} \sqrt{F}\right)(0.0022)
\end{align*}
$$

PCAP = Output filter capacitor ESR loss

$$
\begin{align*}
& =\frac{E_{0}}{E_{0}+n E_{I}}\left(\frac{P_{0}}{E_{0}}\right)^{2} R_{5}+\frac{n E_{I}}{E_{0}+n E_{1}}\left[\frac{E_{I}^{2} E_{0}^{2}}{12 n^{2} L_{p}^{2}\left(E_{0}+n E_{I}\right)^{2} F^{2}}\right. \\
& \left.+\left(\frac{\left(E_{0}+n E_{X}\right) P_{0}}{n E_{I} E_{0} e_{f f}}-\frac{P_{0}}{E_{0}}\right)^{2}\right] R_{5} \tag{3.6.6}
\end{align*}
$$

Parasitic resistance for $L_{1}, L_{2}, T, C(2)=C(3)=$
$C(12)=0$
$C(2)=R_{1} A_{C 1}-4 P F_{C} N_{1} \sqrt{\lambda_{1}}$
$C(3)=R_{2} A_{C 2}-4 P F_{C} N_{2} \sqrt{\Lambda_{2}}$
$C(12)=R_{p} A_{C p}-4 P F_{C} N_{p} \sqrt{\Lambda_{p}}$

Input filter peaking constraint $C(4)=0$
$C(4)=(\text { PEI })^{2}-\frac{1+\frac{R_{3}{ }^{2} C_{3}}{L_{1}}}{\left(\frac{C_{4}}{C_{3}}\right]^{2}+\left(\frac{R_{3}{ }^{2} C_{3}}{L_{1}}\right]\left[1-\frac{C_{4}}{C_{3}}-\frac{L_{2}}{L_{1}} \frac{C_{4}}{C_{3}}\right]^{2}}$

Operating flux density constraint $C(5)-C(6)-C(9)=0$
$C(5)=N_{1} A_{1}-\frac{L_{1} P_{0}}{e f f E_{1} B_{S 1}}$
$C(6)=N_{2} A_{2}-\frac{L_{2} P_{0}}{\text { eff } E_{1} B_{S 2}}$
$c(9)=N_{p} A_{p}-\frac{L_{1}}{{ }^{B_{S p}}}\left[\frac{P_{0}\left(E_{0}+n E_{I}\right)}{\operatorname{eff} E_{I} E_{0}}+\frac{E_{1} E_{0}}{2 L p\left(E_{0}+n E_{I}\right) F}\right]$

## Window area conatraint $C(7)=C(8)=C(10)=0$

$c(7)=\left\{\frac{N_{1} A_{C 1}}{F_{W}}\right\}^{0.5}-\frac{Z_{1}}{2 \pi}+\frac{\sqrt{A_{1}}}{2}$
$C(8)=\left(\frac{N_{2} A_{C 2}}{F_{W}}\right)^{0.5}-\frac{z_{2}}{2 \pi}+\frac{\sqrt{\Lambda_{2}}}{2}$
$C(10)=\left(\frac{2 N_{p} A_{C P}}{F_{W}}\right)^{0.5}-\frac{Z_{p}}{2 \pi}+\frac{\sqrt{A_{p}}}{2}$

Output ripple factor constraint $C(11)=0$

$$
\begin{align*}
c(11)= & V_{R}-\left[\frac{P_{0}\left(E_{0}+n E_{I}\right)}{e f f E_{I} E_{0}{ }^{2} n}+\frac{E_{I}}{2 L P\left(E_{0}+n E_{I}\right) n F}\right] R_{5} \\
& -\frac{P_{0}}{2 E_{0}\left(E_{0}+n E_{I}\right) C_{5} F} \tag{3.6.17}
\end{align*}
$$

Frequency dependent source EMI constraint $C(13) \geqslant 0$

$$
\begin{align*}
c(13) & =\frac{S}{\sqrt{1+\left(\frac{F}{2000}\right)^{2}}} \frac{1}{\sqrt{A^{2}+B^{2}}}-\left[\left(\frac{L_{2} C_{4}}{L_{1} C_{3}}\right)\left(2 \pi F \sqrt{L_{1} C_{3}}\right)^{3}\left(\frac{1}{D}\right)\right. \\
& \left.-\frac{C_{1}}{C_{3}}\left(2 \pi F \sqrt{L_{1} C_{3}}\right)^{2}\right]^{-1} \tag{3.6.18}
\end{align*}
$$

where

$$
A=\frac{2 P_{0}\left(E_{0}+n E_{I}\right)}{\pi e f f E_{I} E_{0}} \sin \frac{\pi E_{0}}{E_{0}+n E_{I}}
$$

$B=\frac{E_{1} E_{0}}{\pi L p\left(E_{0}+n E_{1}\right) F}\left[\cos \frac{\pi E_{0}}{E_{0}+n E_{1}}-\frac{i n \frac{\pi E_{0}}{E_{0}+n E_{1}}}{\frac{\pi E_{0}}{E_{0}+n E_{1}}}\right]$
$D=R_{3}\left(\frac{C_{3}}{L_{1}}\right)^{0.5}$
Other inequality constraint $C(14), C(15), C(16), C(17) \geqslant 0$

$$
\begin{aligned}
& C(14)=0.97-\text { eff } \geqslant 0 \\
& C(15)=R_{T}-R_{1}-R_{2} \geqslant 0 \\
& C(16)=C_{3}-1.0 \times 10^{-6} \geqslant 0 \\
& C(17)=C_{4}-1.0 \times 10^{-6} \geqslant 0
\end{aligned}
$$

## 3.7: OPTIMIZATION RESULTS OF BUCK-BOOST CONVERTER

By treating the awitching frequency as a constant in each optinization run, aet of converter peramater data is obrained.

This set of parameter data represents the optimum converter design for the specified switching frequency, A sufficient number of runs are executed by varing the írequency between 20 KHz to 120 KHz in a 10 kHz step. Detailed optimization results, following the aforedescribed design process are collected and tabulated in Table 3.7-I. To facilitate comparison of optimal converter designs between the boost converter and the buck/boost converter, the same input-output requirements, design constants, and converter performance specifications are used. (Reference to Section 3.2 for detailed information,) The turn-ratio $n=N_{P} / N_{S}=1$. The designations employed in Table 3.7.1 are the same as those in Table 3.5-I. To provide more design insighta, the weight and loss breakdowns are plotted against the switching frequency in Fig. 3.7.1 and Fig. 3.7.2, respectively.

The difference between converter optimization results for the boost converter and buck/boost converters are sumnarized as follows:
(1) The Buck-Boost converter is heavier than the Boost converter. In order to have the mininum weight design the Buck-Boost converter has to operate at a higher frequency than Boost power converter.
(2) Switching losses of semiconductor devices are higher for the Buck-Boost converter. This is logical since the switching current amplitude is considerable higher than that of the boost converter for the same input and output voltase and.the same power level.
(3) The magnetic component for the buck/boost converter are generally larger in size and heavier in weight.
(4) The magnetic losses (PMAGecore lose + winding loss) for the buck/boost converter are dominated by the winding loss in low frequencies. The PMAG lose characteriatic falls rapidly we the switching frequency increases. The high magnetic losses in low frequencies cause severe weight penalty. It is clearly demonstrated in Fig. 3.7.1 and 3.7.2 that in order to minimize the converter weight/ loss, it is desirable to operate the converter frequency about 80KHz ~ 100KHz. For the minimum weight/loss boost converter design, however, the optimal frequency reat about $40 \mathrm{KHz} \sim 60 \mathrm{KHz}$.

## ORIGINAL PAGE IS OF POOR QUALITY

| 1 | 20 | 0 N | 40\% | 301: | 60k | 710 | MIK | 'ME | l/NJK |  | 120k |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| . 1 | 0.124al0 $0^{-4}$ | 11.2Ad. $10^{-4}$ | 1. $254 \times 111^{-4}$ | $0.226 \times 10^{-4}$ | $0.202 \times 10^{-6}$ | $0.176 \times 10^{-4}$ | $11.41 \mathrm{M} \times 1 \mathrm{c}^{-1}$ | : $2.942 \times 10^{-3}$ | $0.454 \times 10^{-5}$ | 0.41 | $0,911 \times 10^{-5}$ |
| N1 | 30.111 | 44.1044 | 41.493 | 40.842 | 40.050 | 45.1171 | 41. 340 | \$1.774 | 17.123 | 31. | 44.437 |
| N: | $0.343 \times 10^{-6}$ | $0.444 \times 10^{-6}$ | 0. $34 \mathrm{Hm} 10^{-6}$ | 0. $\tan \times 210^{-6}$ | $0.314 \times 10^{-6}$ | 11. $1: 5 \times 10^{-6}$ | $0.381 \times 10^{-6}$ | 0. $204 \times 10^{-6}$ | $0.844 \times 10{ }^{\text {-h }}$ | 0.212 | $0.251 \times 10^{-6}$ |
| 4 | $0.473 \times 10^{-1}$ | $0.417 \times 10^{-1}$ | $0.347411^{-1}$ | U. $360 \times 10^{-1}$ | $0.346 \times 14{ }^{-1}$ | 0, $\mathrm{Mn}_{n} 10^{-1}$ | $0.145 \times 10^{-1}$ | $0.331 \times 10^{-1}$ | $0.315 \times 10^{-1}$ | 0.306: | 0.291.10 $0^{-1}$ |
| H | 0.640x10-1 | $0.694 \times 10^{-1}$ | $0.645 \times 10^{-1}$ | . $094 \times 10^{-1}$ | $0.102 \times 10^{-1}$ | $0.729 \times 10^{-1}$ | 0.766n10 ${ }^{-1}$ | $0.752 \times 10^{-1}$ | $0.756 \times 10^{-1}$ | 0.734x | $0.768 \times 10^{-1}$ |
| 1.1 | $0.217 \times 10^{-3}$ | $0.167810^{-5}$ | $0.161 \times 10^{-3}$ | $0.123 \times 10^{-3}$ | $0.104 \times 10^{-3}$ | $0.103 \times 10^{-3}$ | $0.205 \times 10^{-6}$ | $0.763 \times 10^{-64}$ | $0.112 \times 10^{-6}$ | $0.671 \times 1$. | $0.610 \times 10^{-4}$ |
| . 1. | $0.203 \times 11^{-1}$ | $0.106 \times 10^{-64}$ | $0.165 \times 10^{-6}$ | $0.123 \times 10^{-4}$ | $0.125 \times 10^{-4}$ | $0.161 \times 10^{-4}$ | $0.121 \times 10^{-6}$ | $0.113 \times 10^{-6}$ | $0.103 \times 10^{-6}$ | $0.866 \times 10^{-5}$ | $0.0966 \times 10^{-5}$ |
| N2 | 27.1n7 | 25.335 | 26.420 | 24.121 | 21.513 | 14.812 | 13.952 | 16.099 | 14.440 | 16.539 | 19.377 |
| Ac: | $0.534 \times 10^{-6}$ | $0.432 \times 10^{-6}$ | $0.400 \times 10^{-6}$ | $0.370 \times 10^{-6}$ | 0. $334 \times 10^{-6}$ | $0.295 \times 10^{-6}$ | 0. 310. $10^{-6}$ | $0.297 \times 10^{-6}$ | $0.284 \times 10^{-6}$ | $0.2^{\prime \prime} 4 \times 10^{-6}$ | $0.211 \times 10^{-6}$ |
| $\% 1$ | 0. $353 \times 10^{-1}$ | $0.314 \times 10^{-1}$ | $0.295 \times 10^{-1}$ | $0.274 \times 10^{-1}$ | $0.262 \times 10^{-1}$ | $0.246 \times 10^{-1}$ | $0.214 \times 10^{-1}$ | $0.220 \times 10^{-1}$ | $0.222 \times 10^{-1}$ | 0.217w10-1 | . $212 \times 10^{-1}$ |
| K 6 | 0. $100 \times 10^{-}$ | 0.302 $\times 10^{-1}$ | $0.304 \times 10^{-1}$ | . $305 \times 10^{-1}$ | $0.297 \times 10^{-1}$ | $0.271 \times 10^{-1}$ | $0.234 \times 10^{-1}$ | $0.238 \times 10^{-1}$ | $0.263 \times 10^{-1}$ | 0.245nio ${ }^{-1}$ | $0.231 \times 10^{-1}$ |
| 1.6 | $\frac{1.12+11^{-4}}{}$ | $0.554 \times 10^{-6}$ | $0.409 \times 10^{-6}$ | $0.408 \times 10^{-4}$ | 0.3s8m10 $0^{-4}$ | $0.342 \times 10^{-4}$ | $0.2611 \times 10^{-6}$ | $0.254 \times 10^{-4}$ | $0.231 \times 10^{-6}$ | $0.224 \times 10^{-4}$ | $0.204 \times 10^{4}$ |
| A | $0.327 \times 11^{-4}$ | $0.244 \times 10^{-6}$ | $0.207 \times 10^{-4}$ | $0.180 \times 10^{-4}$ | $0.165 \times 10^{-4}$ | $0.140 \times 10^{-4}$ | $0.160 \times 10^{-4}$ | $0.149 \times 10^{-4}$ | $0.146 \times 10^{-4}$ | $n .131 \times 10^{-4}$ | $0.806 \times 10^{-3}$ |
| N1 | 6H.9.) | 61.842 | 39.1157 | 54.411 | 54.317 | 50.082 | 21.324 | 21.8H3 | 29.712 | 10.354 | 44.12 |
| 1, 1 | $0.172 \times 10^{-6}$ | $0.142 \times 10^{-6}$ | $0125 \times 10^{-6}$ | $0.115 \times 10^{-6}$ | $0.111 \times 10^{-6}$ | $0.135 \times 10^{-6}$ | $0.179 \times 10^{-6}$ | $0.179 \times 10^{-6}$ | $0.188 \times 10^{-6}$ | $0.101 \times 10^{-6}$ | $0.175 \times 10^{-6}$ |
| 1 | $0.452 \times 10^{-}$ | 11, 119 $\times 10^{-1}$ | 0. $158 \times 10^{-1}$ | $0.115 \times 10^{-1}$ | $0332 \times 10^{-1}$ | 0. $3 \times 2 \times 10^{-1}$ | 0. $111 \times 10^{-1}$ | $0.297 \times 10^{-1}$ | 0. $107 \times 10^{-1}$ | $0.244 \times 10^{-1}$ | $0.314 \times 10^{-1}$ |
| $\mathrm{H}^{\prime}$ | 0.301 | 0.294 | 0.281 | 0.264 | 0.261 | 0.182 | $0196 \times 10^{-1}$ | $0.807 \times 10^{-T}$ | $0.791 \times 10^{-1}$ | - $0.79 \mathrm{ha} 10^{-7}$ | $0.344 \times 10^{-7}$ |
| 1.1 | $0.940 \times 10^{-4}$ | $0.642 \times 10^{-4}$ | $0345 \times 10^{-4}$ | $0.461 \times 10^{=4}$ | $0.428 \times 10^{-4}$ | $0.326 \times 10^{-4}$ | $0.148 \times 10^{-4}$ | 1, 15 1 ¢ $10^{-4}$ | $0.188 \times 10^{-4}$ | $0.172 \times 10^{-6}$ | $0.179 \times 10^{-4}$ |
| c ${ }^{\text {d }}$ | $0.103 \times 10^{-1}$ | $0.779 \times 10^{-4}$ | $0.621 \times 10^{-4}$ | $0.519 \times 10^{-4}$ | $0.453 \times 10^{-4}$ | $0.389 \times 10^{-4}$ | $0.438 \times 10^{-4}$ | $0.353 \times 10^{-4}$ | $0.128 \times 10^{-4}$ | $0.115 \times 10^{-4}$ | $0.774 \times 10^{-5}$ |
| NJ | 1.128 | 1.106 | 1.136 | 1.166 | 1.173 | 1.228 | 1.129 | 1.330 | 1408 | 0.474 | 0.431 |
| 44 | $0.162 \times 10^{-4}$ | $0.112 \times 10^{-4}$ | $0.898 \times 10^{-5}$ | $0.760 \times 10^{-5}$ | $0.672 \times 10^{-3}$ | 0. $3 \times 3 \times 10^{-5}$ | $0.816 \mathrm{cc10} 0^{-3}$ | $0.788 \times 10^{-5}$ | $0.720 \times 10^{-5}$ | $0.605 \times 10^{-5}$ | $0.399 \times 10^{-5}$ |
| 1.4 | $0.148 \times 10^{-3}$ | $0.121 \times 10^{-1}$ | 3. $0.107 \times 10^{-1}$ | $0.985 \times 10^{-4}$ | $0.907 \times 10^{-4}$ | $0.892 \times 10^{-4}$ | $0.112 \times 10^{-3}$ | $0.101 \times 10^{-3}$ | $0.881 \times 10^{-4}$ | $0.768 \times 10^{-4}$ | $0.019 \times 10^{-4}$ |
| 1. | 0.8208 | 0.8238 | 0.8271 | 016295 | 0.8319 | 0.0567 | 0.8751 | 0.8761 | 0.8183 | 0.0751 | 0.8691 |
| PI | 1.4674 | 1 H46 | 1.9036 | 21187 | 2. 3232 | 2.4924 | 2.8014 | 2.9771 | 3,1171 | 3. 3441 | 3.5315 |
| 111 | 2.0944 | 2.1174 | 2.1389 | 2.1650 | 21994 | 2.1445 | 1.9662 | 2.0273 | 2.1182 | 2.1492 | 2.2235 |
| Plar | 11.1938 | 0.2275 | 0.2489 | 0.2637 | 0.2736 | 0.2761 | 0.3397 | 0.3108 | 0.2964 | 0. 3016 | 0. 3018 |
| PMati | 11.5281 | 10.9421 | 10.3432 | 9.8383 | 9.3489 | 6.7959 | 4.8831 | 4.5803 | 4.1676 | 41959 | 4.4609 |
| PT | 15.2802 | 14.9736 | 14.6346 | 14.3857 | 14.1471 | 11.7030 | 9.9904 | 9.9035 | 9.6993 | 9.9915 | 10.5377 |
| WS | 2.7689 | 2.7589 | 27479 | 2.7398 | 2.7320 | 2.6529 | 25471 | 2.5943 | 2.5876 | 2.5971 | 2.6149 |
| WH | 0.9925 | 0.9723 | 0.9503 | 0.9341 | 0.9186 | 0.7603 | 0.6487 | 0.6431 | 0.6298 | 0.6486 | 0.6843 |
| $W$ | 0.02992 | 0.02114 | 0.01678 | 0.01394 | 0.01214 | 0.01118 | $0.847 \times 10^{-2}$ | $0.793 \times 10^{-2}$ | $0.762 \times 10^{-2}$ | $0.691 \times 10^{-2}$ | $0.583 \times 10^{-2}$ |
| WW | 0.02419 | 0.01639 | 0.01275 | 0.01068 | $0.912 \times 10^{-2}$ | $0.857 \times 10^{-2}$ | $0.790 \times 10^{-2}$ | $0.735 \times 10^{-2}$ | $0.705 \times 10^{-2}$ | $0.652 \times 10^{-2}$ | $0.650 \times 10^{-2}$ |
| WC | 0.0501 | 0.03744 | 0.03065 | 0.02634 | 0.02342 | 0.12074 | 0.02646 | 0.02330 | 0.01694 | 0.0153 | 0.0119 |
| wrati | 0.0534 | 0.03733 | 0.02952 | 0.2462 | 0.02126 | 0.01945 | 0.01638 | 0.01528 | 0.01467 | 0.01364 | 0.01234 |
| WT | 3.8050 | 3.8062 | 3.7583 | 3.7249 | 3.6954 | 3.4534 | 3.2886 | 3.2759 | 3.2491 | 3.2741 | 3. 3234 |

## TABLE E.7-I Buck-Boost Converter Optimization Results



Figure 3.7.1 Welght Breakdowns for Buck-Boost Converter
ORICINAL PACE IS


Figure 3.7.2 Loss Breakdowns for Buck-Boost Converter

### 3.8 CONCLUSIONS AND SUGGESIED FUTURE WORKS

### 3.8.1 Conclusions

Nonlinear programming techniques have been successfully employed to $1 m$ plement the mininum-weight design of switching power converters. Two different computational algorithm: - ALAG and SUMT-based on the penalty function method were compared and their figure of mertis were assessed. For power converter optimization, the ALAG package was deemed more effective than its counterpart the SUMT package, when the computation time, ease of coding, and rate of convergence are concerned.

Adopting the ALAG rountine, a cost-effective computer-aided design approach is presented which provides a minimum-weight converter design down to the details of component level and concurrently meets all powercircuit performance requirements. This computer-aided design approach provides important design insights which helps to assess the following important design concerns:
(1.) The trade-offs between weight and loss ns the switching frequency is increased.
(2) The ontimum converter design down to the detalls of component level.s.
(3) The optimum component designs as a function of the switching frequency and their relationships to the overall system optimization.
(4) The significance of the U-shape curves representing total-wight/ total-loss -rsus frequency as observed in the collected suboptimization runs. This allows the designer to easily identify the optimum switching frequency or a range of frequencies over which the total weight/loss is minimum in the partical sense.
(5) Impact of various critical component characteristics, such as magnetic losses, switching losses of semiconductor devices, to the overall system.
(6) The optimal converter topology for a given application.

Employing the nonlinear-program based optimization technique, the power converter designer can conceive the overall optimum aystem design taking Into consideration the powi :-circuit relate performance requirements with the design objective of either minimizing weight, loss, or any other physical realizable quantity. It thus sets the stage for a more scientific design approach instand of subjective brute-force, trial-and-error, piecemeal design.

### 3.8.2 Suggested Future Works

The investigations of complex converter uptimization problems using nonlinear programming techniques have shown marked succesa. Demonstration of the buck converter optimization in the previous modeling and analysis phases sponsored by NASA, the half-bridge rwverter optimization sponsored by NAVY, together with the boost and buck/boost converter optimization presented in this report have collectively provided clear evidence that a large scale converter optimization is feasibie using NLP techniques; yet, the divelopment of sich a tool has not reached the stage of maturity where it can be widely used. Presently, it takes a person with considerable insight to the nonlinear programming algorithms, and with sufficient converter design experience, to make the program converge. It is our bellef, however, that the afore-described NLP techniques could be made easier and more systematic than they are now. The following tasks are suggested as means fur improving the NLP techniques to make them a more universal converter design tool with wide user applicability.
(1) Systematic way of improving initial starting point.
(2) Means of optimizing variable scalinge and conetraint scalinge.
(3) Means of optimizing convergence stopping criteria.
(4) Improved method of formulating nonlinear constraints to enhance convergence.
(5) Eatablish conditions for convergence.
(6) Program cransportability.

### 4.0 INVESTIGATION OF CURRENT-INJECTED MULTILOOP CONTROLLED SWITCHING REGULATORS.

### 4.1 INTRODUCTION

In recent years, vast amounts of interest and research in universities and industries have been directed teward development of a multi-100p, multistate control scheme which could be applied to switching regulators. This collaborated effort has resulted in astonishing improvements of stability and dynamic performance of switching regulators.

A host of control schemes has emerged many of which employ the principle of current-injected control $[17,18,19,20,21]$. These control schemes share, the common property of transforming a switching converter from a voltage source into a current source. This control concept has exhibited many desirable properties such as inherent over-load protection, stable and equal load sharing when several power converter modules are in parallel, and $1 . \quad$ system response.

Illustrated in Fig. 4.1.1 is a buck/boost converter employing currentinjected control. The control is implemented by sensing the output voltage vo of the converter and the instantaneous current ip through the power switch. The duty cycle signal is terminated when switching current ascends and intersects the threshold voltage $v_{x}$ (dc error signal) determined by subtracting $v_{0}$ from the reference voltage $E_{R}$. Since the switch current wavaform sensed by the current transformer contains both the dc bias current component, and the small amplitude ac modulation signal (to be used for additional error compensation), the control thus provides inherent transistor peak-current protection (from the dc-current component) and improved dynamic performances (from the ac modulation signal).

fig. 4.1.1 Circuit Schematic of Current-Inject Controlled Buck/Boost Regulator

Presented in this report is the modeling and analysis of the currentinfected control system. The modeling approach employed in the present paper, a departure from previous efforts [17,20], provides additional insight to the current-injected control characteristics which failed to be manifested in the previous modeling and analysis efforts. To facilitate comparison between the method presented in this report and the approach employed in the earlier attempts, a brief review of the earlier work is provided. The concept of major loop and minor loop was employed in the previous works $[17,20]$. By considering the dc-feedback and compensation network being the major loop, and the ac-(switch current) feedback as being the minor loop, the minor loop was lumped into the power stage in the process of modeling. The multi-loop converter was thus reauced to a single loop system as shown in Fig. 4.1.2. The ac feedback loop which contains the switching current information is embedded in the "new" power stage. The transfer function $c_{i}$ the new power stage $\hat{v}_{0} / \hat{v}_{x}$ has a surprisingly simple form (only a single pole and a single zero). The authors feel that while this modeling approach offers a way to examine certain small signal characteristics of the system, little information is provided regarding the relative stability of the system (the concept of the gain margin and the phase margin). Even though the de lonp can be opened, the ac loop is inherently closed in the "new" power stage model. A true open loop characteristic, where both the dc and the ac loop are opened, is thus not accessible in this modeling approach.

The modeling and analysis approach of the multi-loop current infected control presented in this report eliminates the aforementioned modeling dilemma. Following an approach similar to that described in the
author's previous work $\{22,23,24,25\}$, the amall aignal medel of the converter in Fig.4.1.1 is derived as shown in Fis. 4.1.3. The seall aignal model has the following features:
(1) The power stage has three inputs and two outputs.

The three inputs are:

| line disturbance | $\hat{v}_{1}$ |
| :--- | :--- |
| load disturbance | $\hat{v}_{0}$ |
| auty cycle disturbance | $\hat{d}$ |

The two outputs are:
the output voltage $\quad \hat{v}_{0}$
the switch current $\quad \hat{i}_{p}$
(2) The error processor senses the two modulation signals $\hat{i}_{0}$ and $\hat{v}_{0}$. The transfer function $F_{A C}$ represents the gain of the ac loop and $F_{D C}$ represents the combined gain of the dc loop and the compensation network.
(3) The duty cycle modulator is represented by a describing function $\mathrm{F}_{\mathrm{M}}$.

Employing the above described small signal model one can readily examine the following performance characteristics:
(1) The control-to-output characteristics $\hat{v}_{0} / \hat{v}_{x}=F_{M} F_{D 1}\left(\Delta+F_{M} F_{A C} F_{D 2}\right)$ where $\Delta=s^{2}+25 W_{0} S+W_{0}^{2}$. (This is the characteristic examined in the previous papers $[17,20]$ which exhibits a single-pole and and single-zero).
(2) The open dc loop characteristic
$G_{D L}=F_{M} F_{D C} F_{D 1} /\left(\Delta+F_{M} F_{A C} F_{D 2}\right)$.
(3) The open loop characteristics (open both dc loop and ac loop) $G_{T}=\frac{1}{\Delta} F_{M}\left(F_{D C} F_{D 1}+F_{A C} F_{D 2}\right)$. The open loop characteristic $G_{T}$ is used to examine the relative

fig. 4.1.2 Discussion of CALTECH Modeling Approach of CurrentInjected Control

POWER STAGE

fig. 4.1.3 Small Signal Model for the Current-Injected Control Buck/Boost Regulator

## atability of the syatem.

(4) The audioausceptibility characterietic $G_{A}-\hat{v}_{0} / \hat{v}_{1}$

$$
G_{A}=\frac{1}{1+G_{T}}\left\{\frac{1}{\Delta^{2}} F_{U 11}\left(\Delta+F_{M_{A C}} A_{D 2}\right)-F_{D 1} F_{M^{F} A C} F_{U 21}\right\} .
$$

(5) The output impedance characteristic $Z_{0}=\hat{v}_{0} / i_{0}$

$$
Z_{0}=\frac{1}{1+G_{T}}\left\{\frac{1}{\Delta^{2}} F_{U 12}\left(\Delta+F_{M}{ }^{F} A C{ }^{F} F_{D 2}\right)-F_{U 22} F_{A C} F_{M}{ }^{F} D 1\right\} .
$$

Modeling of the power-stage, error processor, and pulse modulation is presented in chapters $4.2,4.3$, and 4.4 , respectively. Various open and closed-loop performance characteristics are evaluated in chapters 4.5 and 4.6. Effects of dc-loop and ac-loop gain, and of compensation networks, are also discussed. Finally, guidelines for selecting control circuit parameters are provided.

### 4.2.0 CURRENT-INJECTED BUCK/BCOST POWIE STAGE MODEL

### 4.2.1 Power Circuit Description

The function of the dc-dc converter is to procese and transfer electric power from an unregulated input $v_{1}$ to a regulated output $v_{0}$. The output voltage of the two-winding buck-boost converter thom in Fig. 4.2.1.1 can be either greater than or less than the input voltage, depending on the duty cycle of the switch and the turn ratio of the storage inductor. The magnetically-coupled windinge, provided by the energy-atorage inductor, allow input/output iselation and mul:iple outputs. Proper choice of the inductor's turn ratio also can alleviate the difficulon jf implamenting the extreme duty-eycle condition due to wide ranges of input and output voltages.

The energy exchange transpires in the power stage (Fig. 4.2.1.1(a)) in the following fashion. The state of the duty-cycle drive $d(t)$, shown in Fig. 4.2.1.1.(b), determines the instantaneous position of the switch. The high-level of $d(t)$ indicates the conduction, or on-state, and the low-level determines the off-time, or off-state, of the power switch. During $T_{\text {on }}$, a voltage approximately equal to $v_{I}(t)$ (neglecting the losses due to the parasitic resistance in the primary winding) is established across $N_{p}$ which causes a current $i_{p}(t)$ to increase as illustrated in Figure 4.2.1.1.(c). Occurring aimultancousiy, a voltage $v_{S}(t)$ is induced across $N_{s}$ by transformer action, but no conduction is allowed because of the reverse biased diode. As the off-time, Toff Is initiated, the energy associated with $N_{p}$ is transferred to $N_{s}$ by an ampere-turn redistribution. As a result, the current $i_{p}(t)$ is reduced


FIG. 4.2.1.1 Dc-dc two-winding buck-boost converter (a) equivalent circuit, (b)-(e) waveforms
to zero and $1_{S}(t)$ is forced to magnitude needed to maintain a continuous MMF flow through the inductor. The output voltege in Figure 4.2.1.1.(e) is kept nearly constant due to the large capacitive filter at the output which absorbe the pulsating current $i_{S}(t)$ and delivers a dc current with minimal ripple to the load.

### 4.2.2 Analytical Implerw cation

The switched dc-to-dc converter, assumed to be nondisaipative, is nonlinear in nature. The basic dc-to-dc voltage conversion is achieved by repetitive switching between a number of linear networks switches and diodes. The number of linear networks in one switching cycle is determined by the mode of operation of the inductor's magnetomotive force, MMF. If the MMF is continuous as shown lif Fig. 4.2.2.1.(a), the power stage model has two linear networks corresponding to a mode 1 type operation. For a discontinuous inductor MMF operation as shown in Figure 4.2.2.1.(b) the power stage model is composed of three linear networks corresponding to a mode 2 type operation.

Each linear circuit model is described by a set of linear state.space equations. For the current-injected buak-boost power stage the state variables (independent variables) are customarily the magnetic flux, 0 , and the capacitor voltage, $v_{C}$. The total number of storage elements determines the order of the system.

### 4.2.2.1 State Space Averaging Technique

To derive a linear model for the power stage, the averaging technique is used. [27] Employing the mode 1 operation as an example, the


Fig. 4.2.2.1 Inductor magneto-motive force.
power stage is modelled by two intervals of operation, $T_{\text {on }}$ and $T_{F 1}$, respectively.

$$
\text { (1) } \begin{aligned}
& \text { interval } T_{o n} \\
& \underline{\dot{x}}=A_{1} \underline{x}+B_{1} \underline{u} \\
& \underline{y}=c_{1} \underline{x}+E_{1} \underline{u}
\end{aligned}
$$

(ii) interval $\mathrm{T}_{\mathrm{Fl}}$

$$
\begin{aligned}
& \underline{\dot{x}}=A_{2} \underline{x}+B_{2} \underline{u} \quad \text { (4.2.2.1) } \\
& \underline{y}=C_{2} \underline{\underline{x}}+E_{2} \underline{u}
\end{aligned}
$$

The p:incipal of the average method is to replace the atate-space description of the two linear circuits by a aingle atate-space deacription which represents the approximate behavior of the system through one cycle of operation. Takirg the average of both intervals and summing the results yields the following linest time-varying continuous system:

$$
\begin{align*}
& \underline{\dot{x}}=d\left(A_{1} \underline{x}+B_{1} \underline{u}\right)+d^{\prime}\left(A_{2} \underline{\underline{x}}+B_{2} \underline{u}\right)  \tag{4.2.2.2}\\
& \underline{y}=d\left(C_{1} \underline{x}+E_{1} \underline{u}\right)+d^{\prime}\left(C_{2} \underline{x}+E_{2} \underline{u}\right) \\
& d=\frac{T_{o n}}{T_{p}} \quad d^{\prime}=\frac{T_{F 1}}{T_{p}}
\end{align*}
$$

where $T_{p}$ is the period of the switching cycle.
The basic requirement for the average method is that the effective filter corner frequency of the awitching converter be much lower than the switching frequency [27].

The linear time-varying equations (4.2.2.2) can be rewritten in the following form:

$$
\begin{align*}
\underline{\dot{x}} & =A \underline{x}+\underline{B} \underline{u}  \tag{4.2.2.3}\\
\underline{y} & =C \underline{x}+\underline{E} \underline{1} \\
\text { where } A & =d A_{1}+d^{\prime} A_{2} \\
B & =d B_{1}+d^{\prime} B_{2} \\
C & =d C_{1}+d^{\prime} C_{2} \\
E & =d E_{1}+d^{\prime} E_{2}
\end{align*}
$$

### 4.2.2.2. Perturbation

To study the mall-signal behavior, the linear timavarying equations (4.2.2.3) are perturbed. The introduction of input variation and duty-cycle variations in turn perturb the output and state vectore. The perturbed

Input vectors are:

$$
\begin{equation*}
u=\underline{U}+\underline{\hat{u}} \quad \text { and } \quad d=D+\hat{d} \tag{4.2.2.5}
\end{equation*}
$$

where $U$ and $D$ are the steady-state values and $\hat{u}$ and $\hat{d}$ are small perturbations. These perturbations in turn lead to following:

$$
\underline{x}=\underline{x}+\underline{x} \quad \text { and } \quad \underline{y}=\underline{y}+\hat{y}
$$

wherc $\underline{i}+\underline{y}$ are the steady-state values and $\hat{x}$ and $\hat{y}$ are mall parturbations. With the corresponding perturbations substituted into equation (4.2.2.3) the basic model becomes:

$$
\begin{aligned}
& \dot{\hat{x}}=A \underline{X}+\underline{B} \underline{\underline{U}}+\hat{A} \underline{\underline{x}}+\hat{B} \underline{u}+\left[\left(A_{1}-A_{2} \underline{X}+\left(B_{1}-B_{2}\right) \underline{U}\right] \hat{d}\right. \\
& \text { (dc term) (line } \begin{array}{c}
\text { variation) (duty ratio variation) }
\end{array} \\
& +\left[\left(A_{1}-A_{2}\right) \underline{\underline{x}}+\left(B_{1}-B_{2}\right) \hat{\mathbf{u}}\right] \hat{d} \\
& \text { (nonlinear secund order) } \\
& \underline{Y}+\hat{y}=\mathbf{C X}+E \underline{U}+\hat{C} \underline{\underline{x}}+\underline{E} \underline{\underline{u}}+\left[\left(C_{1}-C_{2}\right) \underline{X}+\left(E_{1}-E_{2}\right) \underline{U}\right] \hat{d}(4.2 .2 .8) \\
& \text { (dc term) (line (dute ratio var lation) } \\
& \text { variation) } \\
& +\left[\left(C_{1}-C_{2}\right) \hat{x}+\left(E_{1}-E_{2}\right) \underline{\hat{u}}\right] \hat{d} \\
& \text { (nonlinear second order) }
\end{aligned}
$$

The perturbed state-space description is nonlinear owing to the presence of the product of time-dependent quantities $\hat{x}$ and $\hat{u}$ with $\hat{d}$.

### 4.2.2.3. Linearization

Since the ac variations are very small in magnitude compared to their steady-state value, the following small-signal approximations can be made:

$$
\begin{equation*}
\frac{\hat{u}_{1}}{u_{1}}, \frac{\hat{u}_{2}}{u_{2}} \ll 1 ; \frac{d}{D} \ll 1 ; \frac{\hat{x}_{1}}{X_{1}}, \frac{\hat{x}_{2}}{x_{2}} \ll 1 ; \frac{\hat{y}_{1}}{Y_{1}}, \frac{\hat{y}_{2}}{Y_{2}} \ll 1 \tag{4.2.2.9}
\end{equation*}
$$

where $\underline{\hat{u}}=\left(\hat{u}_{1}, \hat{U}_{2}\right), \underline{v}=\left(U_{1}, U_{2}\right), \underline{x}=\left(\hat{x}_{1}, \hat{x}_{2}\right), e^{\text {. }}$
Using the approximations (4.2.2.9), the nonlinear sacond order terma in equations ( 4.2 .2 .7 ) and ( 4.2 .2 .8 ) can be neglected, reaulting in a inear system. Separating the steady-state (dc) and dynamic (ac) parts of the linearized system, the final state-space model is acquired.

Steady-state (dc) model:

$$
\begin{align*}
\underline{X} & =-A^{-1} \underline{B U}  \tag{4.2.2.10}\\
\underline{Y} & =C \underline{C}+\underline{E} \underline{U}  \tag{4.2.2.11}\\
& =\left(E-C A^{-1} B\right) \underline{U}
\end{align*}
$$

Linear Dynamic (ac) model:

$$
\begin{align*}
& \dot{\hat{x}}=\hat{A \hat{x}}+\hat{B} \underline{\hat{u}}+\left[\left(A_{1}-A_{2}\right) \underline{x}+\left(B_{1}-B_{2}\right) \underline{u}\right] \hat{d}  \tag{4.2.2.12}\\
& \hat{y}=\hat{C} \underline{\hat{x}}+\hat{E} \underline{\hat{u}}+\left[\left(C_{1}-C_{2}\right) \underline{x}+\left(E_{1}-E_{2}\right) \underline{u}\right] \hat{d} \tag{4.2.2.13}
\end{align*}
$$

### 4.2.2.4. Transfer Function Representations

In small-signal analysis, particular input/output relations (transfer functions) are needed to construct the basic building blocks necessary to full; describe the power stage model. To find the input-to-state variable $\hat{\underline{x}}$ and input-to-cutput transfer functions, one assumes the ac duty-ratio variation is zero. The dynamic model described in equations (4.2.2.12) and (4.2.2.13) can be simplified as follows:


Taking the Laplace transformation of the previous equations, the following relations are obtained:

$$
\underline{\underline{x}}(\mathrm{~s})-\underline{\hat{X}}(0)-\hat{A} \underline{\underline{x}}(\mathrm{~s})+\hat{\underline{\hat{u}}}(\mathrm{~s})
$$

And

$$
\begin{align*}
\hat{y}(s) & =C[s \underline{\underline{I}}-A]^{-1} \hat{B} \hat{u}(s)+D \hat{\underline{u}}(s) \\
& =\left[C[s \underline{I}-A]^{-1} B+D\right] \underline{\hat{u}}(s) \tag{2.2.14}
\end{align*}
$$

 To find the duty cycle-to-state variable $\hat{x}$ and duty cycie-to-output transfer functions, one assumes the ac variation of $\underline{u}$ is zero. Equations (4.2.2.12) and (4.2.2.13) yield the following:

$$
\begin{align*}
& \dot{\dot{x}}=\hat{\hat{x}}+\left[\left(A_{1}-A_{2}\right) \underline{x}+\left(B_{1}-B_{2}\right) \underline{\underline{U}}\right] \dot{d} \\
& \hat{y}=\hat{\underline{x}}+\left[\left(C_{1}-C_{2}\right) \underline{x}+\left(E_{1}-E_{2}\right) \underline{U}\right] \hat{d} \tag{4.2.2.16}
\end{align*}
$$

And the resulting duty ratio modulation $\hat{d}$ to state-variable $\underline{\hat{x}}$ and duty ratio modulation $\hat{d}$ to output $\overline{\mathbf{y}}$ transfer functions are:

$$
\begin{gather*}
\frac{\hat{\underline{x}}(s)}{\hat{\hat{d}}(s)}=(S I-A)^{-1}\left[\left(A_{1}-A_{2}\right) \underline{X}+\left(B_{1}-B_{2}\right) \underline{U}\right]  \tag{4.2.2.17}\\
\hat{\underline{y}(s)}=C(S I-A)^{-1}\left[\left(A_{1}-A_{2}\right) \underline{X}+\left(B_{1}-B_{2}\right) \underline{U}\right]+  \tag{4.2.2.18}\\
\overline{\hat{d}(s)}= \\
{\left[\left(C_{1}-C_{2}\right) \underline{X}+\left(E_{1}-E_{2}\right) \underline{U}\right]}
\end{gather*}
$$

These transfer functions will be used as building blocks to construct the power stage transfer functions to be presented in the following sections.

### 4.2.3 Power Stage Analytical Model

The objective of the power stage model is to develop a group of transfer functions that describe the low-frequency behavior of the switching circuit. The model developed is comprised of three inputs and two outputs. From Fig. 4.2.3.1 the three inputs are the supply


Fig. 4.2.3.1 Power stage input/output relationship.
voltage $\hat{v}_{1}$, the output current $\hat{i}_{0}$, and the duty ratio $\hat{d}^{\text {. . The two outputa }}$ supplied are the output voltage $\hat{v}_{0}$, and the awitching current $\hat{i}_{p}$.

The two-winding buck/boost power stage is shown in Figure 4.2.3.2(a). The two linear equivalent circuit models for the continuous MMF case are 11lustrated in Fig. 4.2.3.2(b) and (c). The power stage model contains an ideal switch and a diode. The storage inductor is a linear core circumscribed by a primary and a secondary winding with inductances $L_{P}$ and $L_{S}$ respectively, where $L_{P}=\left(N_{P} / N_{s}\right)^{2} L_{s}$. Also described by the model is the winding resistances $R_{p}$ and $R_{s} /$ The output filter is represented by a capacitance and an equivalent series resistance, ESR. On the output a current source $\hat{i}_{o}$ is employed to represent a disturbance injested to the converter from the load.

The power stage model is composed of two independent variables. The state variables for both linear circuit equivalents are the magnetic flux $\phi$ of the core shared by the primary and secondary windings $N_{p}$ and $N_{s}$, and the capacitor voltage $v_{c}$. During the interval $T_{o n}$, the power stage is described by the following:

$$
\begin{array}{rlr}
\underline{\dot{x}}=A_{1} \underline{x}+B_{1} \underline{\underline{u}} & \underline{y}=C_{1} \underline{x}+E_{1} \underline{u} \\
\text { where, } \underline{x}=\left[\begin{array}{l}
\phi \\
v_{C}
\end{array}\right], \underline{u}=\left[\begin{array}{l}
v_{1} \\
1_{0}
\end{array}\right], \underline{y}=\left[\begin{array}{l}
v_{0} \\
i_{p}
\end{array}\right]
\end{array}
$$



Fig. 4.2.3.2 (a) Two-winding buck/boost
(b) Equivalent circuit model during $T_{O N}$
(c) Equivalent circuit model during $\mathrm{T}_{0} \mathrm{FF}$

$$
\begin{aligned}
& \Lambda_{1}=\left[\begin{array}{cc}
-\frac{R_{P}}{L_{P}} & 0 \\
0 & -\frac{1}{\left(R_{C}+R_{L}\right) C}
\end{array}\right] \quad B_{1}=\left[\begin{array}{cc}
\frac{1}{N_{P}} & 0 \\
0 & -\frac{R_{L}}{\left(R_{C}+R_{L}\right) C}
\end{array}\right] \\
& C_{1}=\left[\begin{array}{cc}
0 & \frac{R_{L}}{R_{C}+R_{L}} \\
\frac{N_{P}}{L_{P}} & 0
\end{array}\right] \quad D_{1}=\left[\begin{array}{cc}
0 & R_{C} / / R_{L} \\
0 & 0
\end{array}\right]
\end{aligned}
$$

For the interval $\mathrm{T}_{\mathrm{FI}}$ the power stage takes the following form

$$
\begin{equation*}
\underline{\dot{x}}=A_{2} \underline{\underline{x}}+B_{2} \underline{\underline{u}} \tag{4.2.3.2}
\end{equation*}
$$

$$
y=c_{2} \underline{\underline{x}}+E_{2} \underline{\underline{u}}
$$

where,

$$
\begin{aligned}
& A_{2}=\left[\begin{array}{cc}
-\left(\frac{R_{S}+R_{C} / / R_{L}}{L_{S}}\right) & -\frac{R_{L}}{R_{C}+R_{L}} \cdot \frac{1}{N_{S}} \\
\frac{R_{L}}{R_{C}+R_{L}} \cdot \frac{N_{B}}{L_{S} C} & \frac{-1}{R_{C}+R_{L}} \cdot \frac{1}{C}
\end{array}\right] \\
& B_{2}=\left[\begin{array}{cc}
0 & -R_{C} / / R_{L} \cdot \frac{1}{N_{S}} \\
0 & \frac{R_{L}}{R_{C}+R_{L}} \cdot \frac{1}{C}
\end{array}\right] \\
& C_{2}=\left[\begin{array}{cc}
R_{c} / / R_{L} \cdot \frac{N_{S}}{L_{S}} & \frac{R_{L}}{R_{C}+R_{L}} \\
0 & 0
\end{array}\right] D_{2}=\left[\begin{array}{ll}
0 & R_{C} / / R_{L} \\
0 & 0
\end{array}\right]
\end{aligned}
$$

Appedix A gives the derivation of the two equivalent linear models used to describe the power stage during $T_{\text {on }}$ and $T_{F 1}$.

### 4.2.3.1 Average Model.

Continuing the process of characterizing the small-signal model needed to describe the dynamic, ac behavior of the power stage, the average model is:

$$
\begin{equation*}
\dot{\underline{x}}=A \underline{x}+B \underline{u} \quad \underline{y}=C \underline{x}+E \underline{u} \tag{4,2,3,3}
\end{equation*}
$$

$A=\left[\begin{array}{ll}\frac{-D R_{P}}{L_{P}}-D^{\prime}\left(\frac{R_{S}+R_{C} / / R_{L}}{L_{B}}\right) & \frac{-D^{\prime}\left(R_{C} / / R_{L}\right)}{N_{S} R_{C}} \\ \frac{D^{\prime}\left(R_{C} / / R_{L}\right)}{R_{C} L_{S}} & \cdot \\ \frac{-D}{\left(R_{C}+R_{L}\right) C}-\frac{D^{\prime}}{\left(R_{C}+R_{L}\right) C}\end{array}\right]$
$B=\left[\begin{array}{cl}\frac{D}{N_{P}} & \frac{-D^{\prime}}{N_{S}}\left(R_{C} / / R_{L}\right) \\ 0 & \frac{D\left(R_{C} / / R_{L}\right)}{R_{C} C}+\frac{D\left(R_{C} / / R_{L}\right)}{R_{C}}\end{array}\right]$
$C=\left[\begin{array}{lll}\frac{D^{\prime} N_{S}\left(R_{C} / / R_{L}\right)}{L_{S}} & \frac{D\left(R_{C} / / R_{L}\right)}{R_{C}}+\frac{D^{\prime}\left(R_{C} / / R_{L}\right)}{R_{C}} \\ \frac{D N_{P}}{L_{P}} & 0\end{array}\right]$
$E=\left[\begin{array}{cc}0 & \left(D+D^{\prime}\right) R_{C} / / R_{L} \\ 0 & 0\end{array}\right]$
where $D$ is the steady-state duty ratio and $D^{\prime}-1$ - $D$.
To simplify our matrix expressions, assume:
(i) $R_{C} \ll R_{L} \quad$ so that $\quad \begin{aligned} & R_{C} / / R_{L} \approx R_{C} \\ & R_{C}+R_{L} \approx R_{C}\end{aligned}$
(ii) $\frac{R_{P}}{L_{P}}=\frac{R_{S}}{L_{S}}$

Also by definition the following conditions are used:
(1) $L_{e} \Delta \frac{L_{S}}{D^{\prime 2}}$
(ii) $R_{e} \Delta \frac{R_{S}}{D^{\prime 2}}$
(iii) $\quad \omega_{0}^{2} \Delta \frac{1}{L_{e} C} \quad \zeta \Delta \frac{\omega_{0}^{0}}{2}\left[\frac{L_{e}}{R_{L}}+\left(R_{e}+\frac{R_{C}}{D^{\prime}}\right) C\right]$
(iv) $D \triangleq \frac{T_{0}}{T_{P}}$ and $D^{\prime} \triangleq \frac{T_{F 1}}{T_{P}}$ where $T_{P}$ is the switching period,
so that $D+D^{\prime}=\frac{T_{\text {on }}+T_{F 1}}{T_{P}}=\frac{T_{P}}{T_{P}}=1$
Applying these simplifications the average model becomes:
$A=\left[\begin{array}{cc}\frac{-R_{S}-D^{\prime} R_{C}}{L_{e}\left(D^{\prime}\right)^{2}} & -\frac{D^{\prime}}{N_{S}} \\ \frac{N_{S} \omega_{o}{ }^{2}}{D^{\prime}} & \frac{-1}{R_{L} C}\end{array}\right] \quad B=\left[\begin{array}{cc}\frac{D}{N_{P}} & \frac{-D^{\prime} R_{C}}{N_{S}} \\ 0 & 1 / C\end{array}\right]$

$$
C=\left[\begin{array}{cc}
\frac{N_{s} R_{C}}{L_{e} D^{\prime}} & 1 \\
\frac{D N_{P}}{L_{p}} & 0
\end{array}\right] \quad E=\left[\begin{array}{ll}
0 & R_{C} \\
0 & 0
\end{array}\right]
$$

### 4.2.3.2. Linearized Power Stage Model

Line voltage variation $\hat{v}_{1}$, duty cycle variations $\hat{d}_{\text {, }}$ and output currint source disturbances $\hat{i}_{0}$, are now introduced into the circuit such that

$$
\begin{array}{ll}
v_{1} \Delta v_{I}+\hat{v}_{1} & d \Delta D+\hat{d}  \tag{4.2.3.5}\\
i_{0} \Delta 0+\hat{i}_{0} & d^{\prime} \Delta_{D^{\prime}}-\hat{d}
\end{array}
$$

where $V_{I}$ is the dc input voltage and $D$ and $D^{\prime}$ is the steady-state on-time and off-time duty ratios, Employing the small-signal approximation as discussed in Section 4.2.2.2, the second-order nonlinear term may be neglected and a linear system obtained.
4.2.3.2. (a) Steady-state (dc) model.

Using method of section 4.2.2.3, the steady state output vector $\underline{Y}$ may be seen to be:

$$
\begin{align*}
\underline{Y}=\left[\begin{array}{l}
v_{0} \\
I_{P}
\end{array}\right] & =\left(E-C A^{-1} B\right) \underline{U} \\
& =\left[\begin{array}{c}
\frac{D N_{S}}{D^{\prime} N_{P}} \\
\frac{D^{2} L_{\epsilon}}{L_{P}^{R} L}
\end{array}\right] \quad V_{I} \tag{4.2.3.6}
\end{align*}
$$

### 4.2.3.2(b) The Input-to-output transfer function.

The input-to-output transfer function due to an input variation 쓸asuming the duty ratio $\dot{d}$ (ac) variation is zero is as follows:

$$
\begin{align*}
& \hat{y}(s)=\left[\begin{array}{l}
\hat{v}_{0}(s) \\
\hat{i}_{p}(s)
\end{array}\right]=\left[C[S I-A]^{-1} B+E\right] \hat{\hat{u}}(s) \\
& {\left[\begin{array}{ll}
\frac{N_{s}}{N_{P}} \frac{D}{D^{\prime}} \omega_{0}^{2}\left(R_{C} C s+1\right) & R_{C}\left(s^{2}+z_{1} s+\omega_{0} z_{2}\right) \\
\frac{D^{2}}{L_{P}}\left(s+\frac{1}{R_{L} C}\right) & \frac{-D D^{\prime} N_{P}}{L_{P} N_{S} C}\left(R_{C} C s+1\right)
\end{array}\right]\left[\begin{array}{l}
\hat{v}_{1} \\
\hat{i}_{0}
\end{array}\right]} \\
& =-s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2} \tag{4,2.3.7}
\end{align*}
$$

where $z_{1}=2 \zeta \omega_{0}+\frac{1}{R_{C} C}-\frac{R_{C}}{L_{e}} \quad, \quad z_{2}=\frac{R_{e}}{R_{L}}+\frac{D}{D^{\prime}}$
4.2.3.2(c) Duty cycle-to-output transfer function

The duty cycle-to-output transfer function assuming $\hat{u}=0$ is:

$$
\frac{\underline{y}(s)}{d(s)}=\left[\begin{array}{c}
\hat{v}_{0}(s) \\
\frac{\hat{d}_{0}(s)}{d(s)} \\
\frac{\hat{i}_{p}(s)}{\hat{d}(s)}
\end{array}\right]=C(S I-i)^{-1}\left[\left(A_{1}-A_{2}\right) X+\left(B_{1}-B_{2}\right) U\right]+
$$

For detailed derivations of the transfer functions, refer to Appendix 1.

Further simplifcations are made by letting $\Delta=s^{2}+2 \zeta \omega_{0} s+\omega_{0}{ }^{2}$ and then forming gain expressions. The gain blocks in matrix form due to a variation $\underline{\hat{u}}=\left[\begin{array}{ll}\hat{v}_{1} & \hat{i}_{0}\end{array}\right]^{T} \quad(4.2 .3 .7)$ results in the following:

$$
\hat{\dot{y}}(\mathrm{~s}) \Delta \frac{1}{\Delta}\left[\begin{array}{ll}
F_{\mathrm{u} 11} & F_{u 12}  \tag{4.2.3.9}\\
F_{u 21} & F_{u 22}
\end{array}\right]\left[\begin{array}{l}
\hat{v}_{1} \\
\hat{i}_{0}
\end{array}\right]
$$

$$
=\frac{1}{\Delta}\left[\begin{array}{lll}
\frac{N_{B}}{N_{P}} \frac{D}{D^{\prime}} & \omega_{0}^{2}\left(R_{C} C s+1\right) & R_{C}\left[s^{2}+z_{1} s+\omega_{0}^{2} z_{2}\right] \\
\frac{D^{2}}{L_{P} R_{L} C} & \left(R_{L} C s+1\right) & \frac{-D N_{s} \omega_{0}{ }^{2}}{D^{\prime} N_{P}}\left(R_{C} c s+1\right)
\end{array}\right] \text {. }
$$

$$
\left[\begin{array}{l}
\hat{v}_{1}  \tag{4,2,3.10}\\
\hat{i}_{0}
\end{array}\right]
$$

From the duty cycle-to-output transfer function (4.2.3.8), the matrix simplification yields the following:

$$
\begin{aligned}
& \hat{y}(s)=\left[\begin{array}{l}
\dot{v}_{0} \\
\dot{i}_{p}
\end{array}\right] \quad \Delta \quad \frac{1}{\Delta}\left[\begin{array}{l}
F_{D 1} \\
F_{D 2}
\end{array}\right] \hat{d}(s) \\
& =\frac{1}{\Delta}\left[\begin{array}{lll}
\omega_{0}^{2} \frac{v_{0}}{D D^{\prime}} & \left(R_{C} C s+1\right)\left(1 \cdot \frac{D}{R_{L}}\left(\frac{S}{\omega_{0}^{2} C}+R_{0}+\frac{R_{C}}{D^{\prime}}\right)\right. \\
\frac{v_{0} N_{S}}{R_{L} N_{P}}\left(\frac{\left(R_{L} C S+D+1\right)}{L_{S} C}\right. & \left.+\frac{\Delta}{D^{\prime}}\right)
\end{array}\right] \hat{d(a)} \\
& \text { (4.2.3.12) }
\end{aligned}
$$

The block diagram for the emall-aignal approximation of the power stage is shown in Fig. 4.2.3.3. The analytical expressions for the $\mathrm{F}_{\mathbf{u}}=$ and $F_{D-}$ functions are given in equations (4.2.3.10) and (4.2.3.12).


Fig. 4.2.3.3. Small signal averaged power stage block diagram.

### 4.3.0 ERROR PROCESSOR MODEL

### 4.3.1 Introduction

The error processor, EP, is a feedback compensation network, It processes multiple-input control signals derived from the power stage, and delivers the required analog information to the pulse modulator. Fig. 4.3.1.1. illustrates the analog-signal error processor employed in the present analysis. From a small-signal viewpoint, the EP is a linear network consisting of two control loops. The signals sensed in these two loops are the converter output-voltage $v_{0}$, and the primary switching-current $1_{p}$.

The loop sensing $v_{0}$ is the same as any conventional dc loop, where the sensed $v_{0}$ is processed by an amplifier with reference voltage $E_{R}$, to generate a dc error signal $v_{X}$. The dc error voltage after processing through an integral plus lead-1ag compensation network establishes as the threshold level for tie switching-current information derived from the ac luup.

The integral plus lead-lag compensation network is employed to shape the frequency response to improve the converter stability and dynamic response.

The ac loop which senses the collector current of the power Lransistor serves two functions. The first function is to transform the primary switching-current into a proportional voltage signal. This voltage signal $\mathbf{v}_{S W}$ is then compared with $\mathbf{v}_{X}$ resulting in a control that turns off the power switch when $v_{S W}=v_{X}$. The second function of the ac loop is to derive the low-frequency modulation signal or error signal for additional loop compensation.


## 4:3.2 Error Processor Ac and Dc Loop Gaine

Fig. 4.3.2.1 is a functional equivalent of the efrcuit shown in Fig. 4.3.1.1. In Fig. 4.3.2.1. the two error aignale, namely $\mathbf{V}_{\mathrm{X}}$ and $\mathbf{V}_{\mathrm{SW}}$ are subtracted to form the positive input to the threshold detector. The negative input of the error processor is now replaced by a zero reference.

The authors feel that the modified error processor better aerves to perceive the modeling effect for the following reasons:
(1) As stated earlier, the ac loop contains two types of information: The large amplitude awitching-current vaveform is used to implement the analog-to-digital conversion. Such a function is considered part of the pulse modulator instead of the error processor. The small-amplitude low-frequency modulation signal (aimilar to the modulation signal sensed by the dc loop) is brought together with the error signal from the dc loop and compensation lonp to provide the total state-feedback compensation for improved stability and dynamic responses. For the purpose of modeling of the EP, the large-amplitude switching-current information should be extracted from the ac loop and incorporated into the pulse modulation model (presented in Chapter 4.4).
(2) Modeling the EP is difficult because of the unconventional circuit implementation of the threshold detector. In the conventional design, the threshold detector is implemented with one fixed threshold voltage input, the other input containing the error

information. With the current-injected mode of control, both inputs to the threshold detector contain error signals or wi.. dulation signals. In order to model the whole converter system, the pulse modulator should take the combined error signal from the EP output and convert it into a pulsewidth modulator dutycycle signal. In order to implement such a "single-input singleoutput" pulse modulator model, various error signals derived from the multiple feedback paths should be combined to form a composite error signal containing error information from the dc, ac and compensation loops.
(3) For reasons mentioned above, the original EP circuit is modified to that of Fig. 4.3.2.1, where the positive input, $\mathbf{v}_{\mathrm{T}}$, to the threshold detector contains error signals from all three feedback loops. The error voltage $v_{T}$, is the output of the multi-loop EP and also serves as the input to the pulse modulator. The functional equivalence cf Fig. 4.3.1.1. and Fig. 4.3.2.1. can be Justified in the following way. Mathematicali (:\% any $\mathbf{v}_{X}$ and $\mathbf{v}_{\mathbf{S W}}$ waveform, the block diagram of the circuit in Fig. a.3.2.2.(a) is equivalent to the analytical model in Fig. 4.3.2.2(b). The equivalence is proven through the following arguement. For the pulse modulator control, the following equations hold true:

$$
\begin{align*}
& \text { If } v_{X}>v_{S W} \text {, then } v_{T H}=1  \tag{4.3.2.1}\\
& \text { If } v_{X}<v_{S W}, \text { then } v_{T H}=0
\end{align*}
$$


(b)

Fig. 4.3.2.2. (a) Actual circuit implementation, (b) Equivalent analytical model.

To form a composite error signal as the input to the threshold detector, the dc error voltage $v_{S W}$ and voltage $v_{X}$ are combined." The following relations hold for the modified circuit of Fig. 4.3.1.2:

If $v_{X}-v_{S W}>0$, then $\mathbf{v}_{\mathbf{T H}}=1$
If $v_{X}-v_{S W}<0$, then $v_{T H}=0$
Equations (4.3.2.1) and (4.3.2.2) are exactly the same; therefore, Fig. 4.3.1.1 and 4.3.1.2 are functionally equivalent.

In performing the small-signal analysis of the network shown in Fig, 4.3.1.2, the reference $E_{R}$ is replaced by a short circuit. Applying Kirchhoff's current law at node A yields the following:

$$
\begin{equation*}
\hat{v}_{a}\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}\right)=\frac{\hat{v}_{o}}{R_{1}}+\frac{\hat{v}_{b}}{R_{3}} \tag{4,3.2,3}
\end{equation*}
$$

where $\hat{v}_{a}$ is the voltage across $R_{2}, \hat{v}_{b}$ is the differential input voltage of the operational amplifier, and $\hat{v}_{0}$ is the output voltage.

As for ncde $B$ the Kirchhoff equation is

$$
\begin{equation*}
\hat{v}_{b}\left(\frac{1}{z_{X}}+\frac{1}{R_{3}}+s C_{3}\right)=\hat{v}_{x}\left(\frac{1}{2_{X}}+g C_{3}\right)+\frac{\hat{v}_{a}}{R_{3}} \tag{4.3.2.4}
\end{equation*}
$$

where $z_{X}=R_{4}+\frac{1}{s C_{1}}$.

The chen-loop response of the operational amplifier is $A(s)$, where $\hat{v}=-A(s) \hat{v}_{b}$,

$$
\begin{equation*}
A(s)=\frac{-K}{1+s / W n} \tag{4.3.2.7}
\end{equation*}
$$

Substituting equation (4.3.2.6) into (4.3.2.3), and (4.3.2.4) results in the following set of equations:

$$
\begin{equation*}
\hat{v}_{a}\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}\right)-\frac{\hat{v}_{0}}{R_{1}}-\frac{\hat{v}_{x}}{A(s) R_{3}} \tag{4.3.2.8}
\end{equation*}
$$

$$
\begin{equation*}
-\hat{v}_{x}\left[\frac{1}{A(s)}\left(\frac{1}{z_{x}} \quad+\frac{1}{R_{3}}+B C_{3}\right)+\left(\frac{1}{z_{x}}+B C_{3}\right]\right]=\frac{\hat{v}_{a}}{R_{3}} \tag{4.3.2.9}
\end{equation*}
$$

Eliminate the node voltages $\hat{\mathbf{v}}_{a}$ and $\hat{v}_{b}$ by aimultaneously melving (4.3.2.8) and (4.3.2.9) for $\hat{\dot{v}}_{0}$ and $\hat{v}_{x}$. A single expression in terms of $\hat{v}_{0}$ and $\hat{v}_{x}$ results:
$\hat{v}_{x}\left(\frac{1}{A(s) R_{3}}\left(\frac{1}{1+\frac{R_{3}}{R_{1}}+\frac{R_{3}}{R_{2}}}\right)-\frac{1}{A(8)}\left(\frac{1}{z_{X}} \quad+\frac{1}{R_{3}}+8 C_{3}\right)-\left(\frac{1}{2_{X}}+B C_{3}\right)\right)=$

$$
\frac{1}{\left(\frac{R_{3}}{R_{1}}+\frac{R_{3}}{R_{2}}+1\right)} \frac{\hat{v}_{0}}{R_{1}}
$$

Since the ac and dc expressions are low-frequency models, the operational amplifier gain is $A(s)=-K$. With a very large $K$, equation (4.3.2.10) can be simplified to the following form:

$$
\begin{equation*}
\hat{v}_{x}=\frac{-\left(\frac{1}{\left(\frac{R_{3}}{R_{1}}+\frac{R_{3}}{R_{2}}+1\right) R_{1}}\right)}{\left(\frac{1}{z_{x}}+s C_{3}\right)} \tag{+3.2.11}
\end{equation*}
$$

The ac loop is described by the following:

$$
\begin{equation*}
\hat{i}_{a w}=\frac{\hat{i}_{p}}{n} \tag{4.3.2.12}
\end{equation*}
$$

where $n$ is the turn ratio of the current transformer.

$$
\begin{equation*}
\hat{v}_{\mathrm{s}}=\hat{i}_{\mathrm{s}} R_{\mathrm{SW}}=\hat{i}_{p} \frac{R_{S W}}{n} \tag{4.3.2.13}
\end{equation*}
$$

From Figure 3.1.2 the following equation in derived:

$$
\hat{v}_{t}=\hat{v}_{x}-\hat{v}_{\mathrm{s}}
$$

Substituting equationa (4.3.2.11) and (4.3.2.13) Into equation (4.3.2.14) yislds the following:

$$
\begin{equation*}
\hat{v}_{t}=-\frac{\left(\frac{1}{\left({ }^{R_{3} / R_{1}}+{ }^{R_{3} / R_{2}}+1\right) R_{1}}\right)}{\left(\frac{1}{z_{x}}+s C_{3}\right)} \hat{v}_{0}-\frac{R_{S W}}{n} i_{p} \tag{4.3.2.15}
\end{equation*}
$$

The dc loop gain for the EP is defined by the following:

$$
\begin{align*}
F_{D C} & =\left.\frac{\hat{v}_{t}}{\hat{v}_{0}}\right|_{\hat{i}_{p}}=0 \\
& =\frac{1}{\left(C_{1}+C_{3}\right) s}\left(\frac{1+s R_{4} C_{1}}{1+s R_{4}\left(\frac{C_{1} C_{3}}{C_{1}+C_{s}}\right)}\right)\left[\frac{G P}{R_{1}}\right] \tag{4.3.2.16}
\end{align*}
$$

where GP $\left.=\frac{1}{\left(1+\frac{R_{3}}{R_{1}}+\frac{R_{3}}{R_{2}}\right.}\right)$
The ac loop gain is given by the following:

$$
\begin{align*}
F_{A C} & =\left.\frac{\hat{v}_{T}}{I_{p}}\right|_{\hat{v}_{0}}=0 \\
& =\frac{R_{S W}}{n} \tag{4.3.2.17}
\end{align*}
$$

Fig. 4.3.2.3 illustrates the error processor block diagram.


Fig. 4.3.2.3. Error processor block diagram.


#### Abstract

4.4. PULSE MODULATOR CHARACTERIZATION 4.4.1 Pulse Modulator Descriptions

The pulse modulator converts the analog error eignal from the output of the error processor into modulated pulae-train in order to provide proper duty-ratio control of the power ewitch. The pulse nodulator analyzed in the present chapter $u * 111 z e s$ a constant off-tine control.


Fig. 4.4.1.1 illustrates the pulse modulator (PM) in oimplified block diagram form. The two inputs of the threshold comparator are the dc error voltage $V_{X}$, and the ac voltage signal $v_{S W}$ (proportional to the current through the power switch). The dc error voltage $v_{X}$ is a floating threshold level proportional to the difference between the desired and the measured output voltage. As the error between the desired and actual voltage increases, the level of the voltage $v_{X}$ rises. Since the ac voltage $v_{S W}$ is proportional to the current waveform through the power switch, peak current protection is an inherent feature of the current-injected control.

When the condition $v_{S W}>v_{X}$ is satisfied, the chreshold detector ourpit $V_{T}$ is momentarily driven low. This signal instructs the digital signal processor (DSP) to turn off the power switch. Therefore by limiting the level of $v_{X}$, the peak current protection of the power switch can be achieved.

For constant off-time implementation the DSP output, D, is latched low for a predetermined amount of time. At the end of the off-ilme period, the power switch is comanded on and the cycle repeats.


Fig. 4.4.1.1. Pulse modulator.

### 4.4.2 Seell-8ignal Model

To atudy the small-aignal behavior of the pulse modulator, an analytical model is developed using the deecribing function technique.

Fig. 4.4.2.1 show the block diagram of the analytical model at steady-state without a low frequency dieturbance. As shown in Fig. 4.4.2.1.(b), when $\mathbf{v}_{\mathrm{SW}}$ is greater than $\mathrm{v}_{\mathrm{X}}, \mathrm{T}_{\mathrm{OFF}}$ comences. $\mathbf{s}_{\mathrm{N}}$ is the rising elope of the awitching-current eignal $v_{\text {Sw }}$ during the period $T_{O N} \cdot S_{F}$ is the falling slope determined by the secondary current of the two winding inductor reflected back to the primary eide during the period $\mathrm{T}_{\text {OFF }}$. The trajectory formed by $\mathrm{S}_{\mathrm{N}}$ and $\mathrm{S}_{\mathrm{F}}$ is proportional to the inductor MMF and is used to jetermine continuous or discontinuous current operation.

Fig. 4.4.2.2 shows the equivalent circuit waveforms for the perturbed case. The small-signal disturbance is represented by an ac generator placed in the de loop as illustrated 1n, Fig. 4.4.2.2(a). For the perturbed case, the magnitude of the low-frequency ac signal is assumed to be sufficiently small and the rising slope $S_{N}$, and falling slope $S_{F}$, are considered constant and unaffected by the disturbance. With this assumption, the perturbed switchingcurrent information $v_{S W}$, alone with the arror voltage $\mathrm{in}_{\mathrm{y}}$, can be modeled.

The development of Fig. 4.4.2.2(b) 1s, follows:
(1) Given a slope $S_{N}$ determined by tine por... de'/ supply voltage
$v_{I}$, the prifary inductance $L p$, and an initial de ztarting jnilue large enough to guarantee continuous-current eperation. From the starting point a line is drawt with $t$ a slope


Fig. 4.4.2.1. Unperturbed analytical PM waveforms.
-190-

$$
c-3
$$

$$
\begin{aligned}
& * ? \\
& 0
\end{aligned}
$$



Fig. 4.4.2.2. Perturbed analytical PM waveforms.
$S_{N}$. At the intersection of the perturbed $V_{X}$ and the awitchingcurrent information $v_{S W}$, $T_{O F F}$ begins. After act span of time equal to the period $T_{0 F P}$ expires, the cycle repeats.
(ii) The initial point for the next cycle is determined by aubtracting the magnitude $S_{n} T_{O F F}$ from $v_{X}$ at that particular point in time.
(iii) From the new inicial values (i) and (ii) are repeated.

Fig. 4.4.2.2(:) shows the input $v_{T}$ to the analytical threshold detector. Using the relation $\mathbf{v}_{\mathrm{T}}=\mathbf{v}_{\mathrm{X}}-\mathbf{v}_{\mathrm{SW}}$, the development of Fig. 4.4.2.2(d) can be understood. Fig. 4.4.2.2(d) 111ustrates the perturbed duty-cycle.

The small-signal behavior of the PM can now be modeled. A set of equations are developed for the darkened trajectory $\mathbf{v}_{T}^{\prime}$ of F18. 4.4.2.2(c). $v_{T}^{\prime}$ is used because the trajectory describes the low-frequency modulation affecting the PN gain. The aditional information in $v_{T}$ that deals with the dc component of the switching waveform of the converter is not relevant in the modulation of the dutv cycle signal. and therefore has no contribution to the PM small-signas gain.

### 4.4.3 Formulation of the PM Transfer Function

The output of the FM can be expressed in a Fourier serips in the

## form:

$d(t)=D+a_{1} \sin \omega t+b_{1} \cos \omega t+\cdots$
The input of the PM can aloo be expressed in a Fourier series in the form:
$v_{\underline{T}}^{\prime}(t)=V+c_{1} \sin \omega t+d_{1} \cos \omega t+\cdots$
The describing function $F_{M}$ of the pulse modulator is defined as:

$$
\begin{equation*}
F_{M} \triangleq \frac{\partial(t)}{\dot{v}_{t}^{\prime}(t)}=\frac{\left(a_{1}^{2}+b_{1}^{2}\right)^{\frac{1}{2}}}{\left(c_{1}^{2}+d_{1}^{2}\right)^{b_{1}}} \cdot 1\left(\tan ^{-1} \frac{b_{1}}{a_{1}} \cdot \tan ^{-1} \frac{d_{1}}{c_{1}}\right) \tag{4,4,3.1}
\end{equation*}
$$

The resultant waveform $v_{T}^{\prime}$ is a function of $S_{N}$ and $S_{F}$ (both asaumed positive) and the ainusoidal disturbance $A$ sin wt. During the pariod $T_{\text {OFF }}$ at ( $N$ - Dth $^{\text {checle }} \mathrm{v}_{\mathrm{T}}^{\prime}$ has the form:

$$
\begin{equation*}
v_{T}^{\prime}(t)=S_{F}\left(t-t_{2 n-1}\right)+A \sin \omega t-A \sin \omega t_{2 n-1} \tag{4.4.3.2}
\end{equation*}
$$

During the period $T_{O N}$ at the $N^{t h}$ cycle $v_{T}^{\prime}$ has the form:

$$
\begin{equation*}
v_{T}^{\prime}(t)=S_{N}\left(t_{2 n+1}-t\right)+A \sin \omega t-A \sin \omega t_{2 n+1} \tag{4.4.3.3}
\end{equation*}
$$

The on-time $\Delta T_{N}$ at the $n t h$ cycle $\left[t_{2 n}, t_{2 n+2}\right]$ can be expressed through the following development: $\Delta \mathrm{T}_{\mathrm{N}}$ can be derived from equations (4.4.3.2) and (4.4.3.3) by evaluating the equations at $t=t_{2 n}$, where $v_{T}^{\prime}\left(t_{2 n}\right)=S_{F}\left(t_{2 n}-t_{2 n-1}\right)+A \sin \omega t_{2 n}-A \sin \omega t 2 n-1$
or
$v_{T}^{\prime}\left(t_{2 n}\right)=s_{N}\left(t_{2 n+1}{ }^{-t}{ }_{2 n}\right)+A \sin \omega t_{2 n}-A \sin \omega t_{2 n+1}$.
From Figure 4.2.2, $T_{F} \triangleq T_{\text {OFF }}=t_{2 n}{ }^{-t}{ }_{2 n-1}$.
Let $\subset T_{N} \Delta t_{2 n+1} t_{2 n-1}$.
Set (4.3.3) equal to (4.3.4) and solve for $\Delta T_{N}$.
$S_{F} T_{F}+A \sin \omega t_{2 n}-A \sin \omega t_{2 n-1}=S_{N}\left(t_{2 n+1}{ }^{-t} 2 n\right)+$
A $\sin \omega t_{2 n}-A \sin \omega t 2 n+1$
$\left(S_{F}+S_{N}\right) T_{F}-S_{N} \Delta T_{N}=-2 A \sin \frac{\omega \Delta T_{N}}{2} \cos \omega\left(t_{2 n-1}+\frac{\Delta T_{N}}{2}\right)$

$$
\begin{align*}
\Delta T_{N} & =\frac{\left(S_{F}+S_{N}\right) T_{F}}{S_{N}-A \omega \cos \omega t_{2 n-1}} \\
& =\left(1+\frac{S_{F}}{S_{N}}\right) T_{F} \frac{1}{1-\frac{\Lambda \omega}{S_{N}} \cos \omega t_{2 n-1}}
\end{align*}
$$

The duty-cycle output $d$, may be expressed in the following form:

$$
\begin{equation*}
d(t) \Delta D+a_{1} \text { in } \omega t+b_{1} \cos \omega t+\cdots \tag{4.4.3.9}
\end{equation*}
$$

The coefficient of sin wit in equation (4.4.3.9) can be expressed as:
$a_{1}=\frac{\omega}{\pi}\left[\int_{0}^{t_{1}} \sin \omega t d t+\int_{t_{2}}^{t_{3}} \sin \omega t d t+\cdots+\int_{t_{2 n}}^{t_{2 n+1}} \sin \omega t d t+\cdots\right]$
$=\frac{\omega}{\pi} \frac{1}{A}\left[\int_{0}^{t_{1}} A \sin \omega t d t+\int_{t_{2}}^{t_{3}} A \sin \omega t d t+\cdots\right]$
$=\frac{1}{\pi} \frac{1}{A} \sum_{n=n}^{N} A_{2 n+1}$
where $A_{2 n+1}=\frac{\Delta T_{N}-T_{F}}{2}\left(A \sin \omega t 2 n+A \sin \omega t t_{2 n+1}\right)$

$$
\begin{align*}
& =\left(\Delta T_{N}-T_{F}\right) A \sin \omega\left(t_{2 n-1}+\frac{T_{F}+\Delta T_{N}}{2}\right) \cos \omega\left(\frac{\Delta T_{N}-T_{F}}{2}\right) \\
& =h\left(\Delta T_{N}-T_{F}\right)\left\{\sin \omega t_{2 n-1}\left[\frac{1}{2} \cos \omega \Delta T_{N}+\frac{1}{2} \cos \omega T_{F}\right]\right. \\
& \quad+\cos \omega t 2 n-i^{\left.\left\{y s i n \omega \Delta T_{N}+\frac{1}{2} \sin \omega T_{F}\right]\right\}} \tag{4.4.3.11}
\end{align*}
$$

A first-order approximation for $A_{2 n+1}$ is:

$$
\begin{equation*}
A_{2 n+1}=A\left(\Delta T_{N}-T_{F}\right) \sin \omega t_{2 n-1}+\frac{A}{2}\left(\Delta T_{N}^{2}-T_{F}^{2}\right) \cos \omega t_{2 n-1} \tag{4,4,3,12}
\end{equation*}
$$

Eliminating second order effects:

$$
A_{2 n+1}=\Delta T_{N} A\left(1-\frac{T_{F}}{\Delta T_{N}}\right) \text { in } w t_{2 n-1}
$$

From equation (4.3.7),

$$
\begin{equation*}
a_{1}=\frac{1}{\pi} \frac{1}{A} \sum_{n=0}^{K} A\left(1-\frac{T_{F}}{\Delta T_{N}}\right) \sin \omega t_{2 n-1} \Delta T_{N} \tag{4.4.3.14}
\end{equation*}
$$

where $K$ is the ratio between the switching frequency and the modulation frequency. In ordar to simplify the analysis, it is assumed that $K$ is an integer.

If $\Delta T_{N}$ is amall compared to the disturbance period, i.e. if $K$ is very large, it follows that

$$
\begin{aligned}
a_{1} & =\frac{1}{A \pi} \int_{0}^{2 \pi / \omega} A\left(1-\frac{T_{F}}{\Delta T_{N}}\right) \sin \omega t 2 n-1 d t \\
& =\frac{1}{A \pi}\left\{\int_{0}^{2 \pi / \omega} A \sin \omega t d t-\int_{0}^{2 \pi / \omega} A T_{F} \frac{\left(1-\frac{A \omega}{S_{N}} \cos \omega t\right)}{\left(1+\frac{S_{F}}{S_{N}}\right) T_{F}} \cdot \sin \omega t d t\right\}
\end{aligned}
$$

$$
=\frac{1}{\pi} \frac{1}{1+\frac{S_{F}}{S_{N}}} \frac{A \omega}{S_{N}} \int_{0}^{2 \pi / \omega} \cos \omega t \sin \omega t d t=0
$$

Equation 4.4.3.15 is orthogonal. Therefore the coefficient $a_{1}$ is zero.
The coefficient of cos wt in equation (4.3.6) is:

$$
\begin{aligned}
b_{1}= & \frac{\omega}{\pi}\left[\int_{0}^{t_{1}} \cos \omega t d t+\int_{t_{2}}^{t_{3}} \cos \omega t d t+\cdots+\int_{t_{2 n}}^{t_{2 n+1}} \cos \omega t d t+\cdots\right] \\
= & \frac{1}{A \pi}\left[A \sin \omega t_{1}+A \sin \omega t_{3}-A \sin \omega t_{2}+\cdots+A \sin \omega t_{2 n+1}\right. \\
& \left.-A \sin \omega t_{2 n}+\cdots\right]
\end{aligned}
$$

$$
-\frac{1}{A n}\left[\sum_{n=0}^{N} B_{2 n+1}\right]
$$

where,

$$
\begin{align*}
B_{2 n+1} & =A \sin \omega t t_{2 n+1}-A \sin \omega t_{2 n} \\
& =2 A \cos \frac{\omega}{2}\left(t_{2 n+1}+t_{2 n}\right) \sin \frac{\omega}{2}\left(t_{2 n+1}-t_{2 n}\right) \\
& =2 A \cos \omega\left(t_{2 n-1}+\Delta T_{N}-\frac{T_{F}}{2}\right) \sin \omega\left(\frac{\Delta T_{N}-T_{F}}{2}\right) \tag{4.4.3.17}
\end{align*}
$$

A first order approximation for equation (4.3.15) is:

$$
\begin{align*}
& B_{2 n+1}=2 A\left(\omega-\frac{N^{\prime}-T_{F}}{2}\right) \cos \omega t_{2 n-1} \\
& =\frac{A \omega}{\Delta T_{N}}\left(\Delta T_{N}-T_{F}\right) \cos \omega t_{2 n-1} \Delta T_{N} \tag{4.3.18}
\end{align*}
$$

From equation (4.4.3.13)

$$
\begin{equation*}
b_{1}=\frac{1}{A \prime \prime}\left[\sum_{n=0}^{N} \frac{A \omega}{\Delta T_{N}}\left(\Delta T_{N}-T_{F}\right) \cos \omega t_{2 n-1} \Delta T_{N}\right] \tag{4.4.3.19}
\end{equation*}
$$

With $\Delta \mathrm{T}_{\mathrm{N}}$ small compared to the modulation period equation (4.4.3.19) can be expressed as follows:

$$
\begin{align*}
b_{1} & =\frac{1}{A \pi} \int_{0}^{2 \pi / \omega} \Lambda \omega\left[1-\left(\frac{T_{F}}{S_{F}} \bar{T}^{2}+\left(1-\frac{A \omega}{S_{N}} \cos \omega t\right)\right] \cos \omega t d t\right. \\
& =\frac{\omega}{\pi} \frac{1}{1+\frac{S_{F}}{S_{N}}} \int_{0}^{2 \pi / \omega} \frac{A \omega}{2 S_{N}}(1+\cos 2 \omega t) d t \\
& =\frac{\lambda \omega}{S_{N}+S_{F}}
\end{align*}
$$

Again, when modeling the analog voltage $V_{T}$, the information that needs to be extracted is contained in $v_{T}^{\prime}$. $v_{T}^{\prime}$ containe the low-frequency information necessary without the high-frequency awitching aignal of $\mathbf{v}_{\mathbf{T}}$. From equation (4.3.1) the amall-signal input voltage to the pulee modulator 1s:

$$
\begin{align*}
v_{T}^{\prime}\left(t_{2 n}\right) & =S_{F} T_{F}-A \sin \omega t_{2 n-1}+A \sin \omega t_{2 n} \\
& =S_{F} T_{F}+2 A \cos \frac{\omega}{2}\left(t_{2 n}+t_{2 n-1}\right) \sin \frac{\omega}{2} T_{F} \tag{4.4.3.21}
\end{align*}
$$

In equation (4.4.3.21) sin $\frac{\omega T_{F}}{2}$ can be approximated by $\frac{\omega T_{F}}{2}$ reaulting in:

$$
\begin{equation*}
v_{T}^{\prime} \cong S_{F} T_{F}+A \omega T_{F} \cos \omega\left(t_{2 n-1}+\frac{T_{F}}{2}\right) \tag{4.4.3.22}
\end{equation*}
$$

From equation(4.4.3.22) the fundamental of the small-signal average of $\mathbf{v}_{\mathrm{T}}^{\prime}$ can be taken.

$$
\begin{equation*}
\hat{v}_{t}^{\prime}=\frac{1}{2} A \omega T_{F} \cos \omega\left(t+\frac{T_{F}}{2}\right) \tag{4.4.3.23}
\end{equation*}
$$

The describing function $F_{M}$ for the pulse-width modulator is:

$$
\begin{equation*}
F_{M} \Delta \frac{\partial(t)}{\hat{v}_{t}^{\prime}}=\frac{\left(a_{1}^{2}+b_{1}^{2}\right)^{\frac{1}{2}}}{\hat{v}_{t}^{\prime}} e^{-j \tan ^{-1}\left(b_{1} / a_{1}\right)} \tag{4.4.3.24}
\end{equation*}
$$

Substituting equations (4.4.3.15), (4.4.3.20) and (4.4.3.23) into (4.4.3.24) the PM gain 1s:

$$
\left|F_{M}\right|=\frac{\frac{A \omega}{S_{N}+S_{F}}}{\frac{1}{2} A \omega T_{F}}=\frac{2}{\left(S_{L l}+S_{F}\right) T_{F}}
$$

Phase $\angle F_{M}=\frac{W_{F}}{2}$,

$$
F_{M}=\frac{2}{\left(S_{N}+S_{F}\right) T_{F}} e^{j \omega T_{F}}
$$

## The slopes $\mathrm{S}_{\mathrm{N}}$ and $\mathrm{S}_{\mathrm{F}}$ can be defined by the aystem parametera.

During $T_{o n,}$,

$$
\begin{equation*}
v_{I}=I_{P} R_{P}+L_{P} \frac{d i_{P}}{d t} \tag{4.4.3.27}
\end{equation*}
$$

Let $\frac{d 1_{p}}{d t} \Delta S_{N}^{\prime}$ where $S_{N}^{\prime}$ is the slope of $i_{p}$ during $T_{f n}$. Then:

$$
V_{I}=I_{P} R_{P}+L_{P} S_{N}^{\prime}
$$

and

$$
\begin{equation*}
S_{N}^{\prime}=\frac{V_{1}-I_{P} R_{P}}{L_{P}} \tag{4.4.3.28}
\end{equation*}
$$

The magnitude of $I_{p} R_{p}$ is much less than $V_{I}$, so (4.4.3.28) is approximated by:

$$
\begin{equation*}
s_{N}^{\prime}=\frac{v_{1}}{L_{p}} \tag{4.4.3.29}
\end{equation*}
$$

The current transformer used to sense the switching-current reduces the sensing current by a $1: n$ turns ratio resulting in the following expressions:

$$
\begin{equation*}
s_{N}^{\prime \prime}=\frac{v_{I}}{n L_{P}} \tag{4,4,3,30}
\end{equation*}
$$

The signal sensed by the comparator is a voltage proportional to the current passing through $\mathrm{R}_{\mathrm{SW}}$. Therefore

$$
S_{N}=\frac{V_{I} R_{S W}}{n L_{P}} \text {, where, } S_{N}=R_{S W} \cdot S_{N}^{\prime \prime}
$$

During $\mathrm{T}_{\mathrm{OFF}}, \mathrm{S}_{\mathrm{F}}$ may be found through the following:

$$
\begin{align*}
& \left|\Delta 1 T_{O N}\right|=\left|\Delta 1 T_{O F F}\right|  \tag{4.4.3.32}\\
& \left|\Delta 1 T_{O N}\right|=s_{N}^{\prime} T_{O N} \\
& \left|\Delta 1 T_{O F F}\right|=s_{F}^{\prime} T_{O P F} \tag{4.4.3.34}
\end{align*}
$$

The subetitution of equations (4.4.3.33) and (4.4.43.34) into equation. (4.3.32) resulta in the expressions:

$$
\begin{align*}
& s_{N}^{\prime} T_{O N}=s_{F}^{\prime} T_{O F F}  \tag{4,4,3,35}\\
& s_{F}^{\prime}=\frac{T_{O N}}{T_{O F F}} s_{N}^{\prime} \tag{4.4.3.36}
\end{align*}
$$

Applying equation (4.4.3.31) to $\mathrm{S}_{\mathrm{F}}$, we obtain

$$
\begin{equation*}
S_{F}^{\prime}=\frac{T_{O N}}{T_{O F F}} v_{I} \frac{R_{S W}}{n L_{P}}-\frac{N_{P}}{N_{S}} \frac{V_{O}}{n L_{P}} R_{S W} \tag{4.4.3.37}
\end{equation*}
$$

Substituting equations (4.4.3.31) and (4.4.43.37) into the PM describing function the following can be obtained:

$$
\begin{align*}
& F_{M}=\frac{2}{\frac{V_{I} R_{S W}}{n L_{P}}\left(\frac{T_{O N}}{T_{O F F}}+1\right) T_{O F F}} e^{j \frac{\omega T_{F}}{2}}  \tag{4.4.3.38}\\
& e^{j \frac{\omega T_{F}}{2}}=e^{j \frac{\pi T_{F}}{T_{M}}}=1 \tag{4.4.3.39}
\end{align*}
$$

where ${ }^{1} F / T_{M} \ll 1 . T_{F}$ is the off-time period of the switch and $T_{M}$ is the period of the low-frequency modulation. Therefore the $F_{M}$ gain may be approximated by:

$$
F_{M} \neq \frac{2 n L_{P}}{V_{I} R_{S W}} \frac{1}{T_{O N}+T_{O F F}}
$$

Expressing the $F_{M}$ describing function in terms of the duty-ratio instead of $\mathrm{T}_{\mathrm{ON}}$ and $\mathrm{T}_{\mathrm{OFF}}$, equation (4.4.3.40) becomes:

$$
F_{M}=\frac{2 n L_{P}}{V_{I} R_{S W}} \frac{1}{T_{O F F}\left(1+\frac{N_{P}}{N_{S}} \frac{V_{0}}{V_{I}}\right)}
$$

Fig. 4.4.2.3 1s the block diagram for the amall-aigna gain from the composite error proceseor signel $V_{T}$ to the pulse modulator output $d$.


Fig. 4.4.2.3. Pulae modulator gain block

### 4.5.0 perporvance ahalyses and test verifications

### 4.5.1 imall-signa. Block Diagram

The objective of this eection 1s to incorporate the power atage, analog error proceasor, and pulee modulator amall-aignal modelis into a closed-loop block diagram. The block diagram model will be used to examine the open-loop and closed-loop performinces of the awitching regulator. Pig. 4.5.1.1 shows the block diagran for the buck/boost regulator derived from Fig. 4.2.3.3, Fig. 4.3.2.3, and Fig. 4.4.2.3.

The amall-aignal analyaie will include control-to-output response, de open-loop behavior, oystem open-loop responce atability, closed-loop response audiosusceptibility, and output impedance $\frac{\hat{v}_{0}(s)}{\hat{i}_{0}(s)}$

In this chapter, a Bode plot showing the gain and phase of each particular transfer function is presented. The analytical curves are compared with the experimental measurements to verify the amall-signal model. The analytical curves are presented by solid lines and the experimental results by the following:
(i) Gain curve - $x$
(ii) Phase curve - -

The gain and phase plots for the small-aignal analysis employ the following converter parameters and operating conditions:


Fig. 4.5.1.1. Generalized small-signal block diagram for buck/boont converter.

## Converter Parameters

$$
\begin{aligned}
& L_{p}=464.94 \text { microhenries } \\
& L_{S}-101.64 \text { microhenries } \\
& R_{S}=.293 \text { ohms } \\
& L_{e}=549.70 \text { microhenriec } \\
& R_{c}=1.58 \text { ohms } \\
& \text { C = } 441 \text { microfarada } \\
& R_{C}=.07 \text { ohms } \\
& R_{L}=15 \text { ohms } \\
& N_{p}=85 \text { turns } \\
& N_{S}=40 \text { turns } \\
& R_{1}=11.47 \mathrm{kilo} \text {-ohns } \\
& R_{e}=10.0 \mathrm{kilo}-\mathrm{ohms} \\
& R_{3}=25.3 \mathrm{kilo} \text { ohms } \\
& R_{4}=105.0 \text { kilo-ohms } \\
& C_{1}=11.8 \text { nanofarads } \\
& C_{3}=22.6 \text { picofarads } \\
& \text { CTN - } n=200 \text { turns } \\
& R_{S W}=250 \text {. ohms } \\
& \omega_{0}=2021 \mathrm{rad} / \mathrm{sec} \\
& V_{1}=18.5 \text { volte } \\
& v_{0}=11.4 \text { volte } \\
& \text { D }-.57 \\
& D^{\prime}=.43 \\
& \text { TOFF }=11.0 \text { microsecs }
\end{aligned}
$$

### 4.5.2 Control-to-Output Response

The control-tomoutput reaponae is the open-loop gal.n from $\hat{v}_{X}(a)$ to the output $\hat{v}_{0}(s)$ in Fig.4.5.1.1. The oontrol-to-output transfer function denotes the power stage characteristics plus ac feecback.

Figure 4.5.2.1 illustrates the gain block diagram for the control-to-output response, derived from the system block diagram.

From Fig. 4.5.2.1 the control-to-output transfer function is derived, yielding the following results:

$$
\begin{equation*}
\frac{\hat{v}_{0}(B)}{\hat{v}_{x}^{(s)}}=\frac{+F_{M} F_{D 1}}{\Delta+F_{M} F_{A C} F_{D 2}} \tag{4.5.2.1}
\end{equation*}
$$

Where $\Delta=s^{2}+25 \omega s+\omega{ }^{5} 2$
Substituting equafions (4.2.3.12), (4.3 2.17) and (4.4.3.41) into the controi-
to-outpst transfer function the following equations can be generated:

$$
\begin{equation*}
\frac{\hat{v}_{0}(s)}{\hat{v}_{x}(s)}=\frac{F_{M} \frac{v_{0} \omega_{0}^{2}}{D D^{\prime}}\left(R_{C} C s+1\right)\left[-\left(\frac{D L_{e}}{R_{L}} s+\left(R_{e}+\frac{R_{c}}{D^{\prime}}\right) \frac{D}{R_{L}}-1\right)\right]}{\left(s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}\right)+F_{M} F_{A C} \frac{V_{0}^{N_{s}}}{R_{L} N_{P}}\left(\frac{1}{L_{s} C}\left(R_{L} C s+D+1\right)+\frac{\left(s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}\right)}{D^{\prime}}\right)} \tag{4.5.2.2}
\end{equation*}
$$

Employing the inequality $\left(R_{e}+\frac{R_{c}}{D^{\dagger}}\right) \frac{D}{R_{L}} \ll 1$, equation (4.5.2.2) is simplified

$$
\begin{equation*}
\frac{\hat{v}_{0}(s)}{v_{x}(s)}=\frac{-F_{M} v_{0} \omega_{0}^{2}}{D D^{\prime}} \frac{\left(R_{C} C s+1\right)\left(\frac{D L_{e}}{R_{L}} s-1\right)}{\left(1+\frac{F}{D^{\prime}}\right) s^{2}+\left(2 \zeta \omega_{0}+F\left(\frac{R_{L}}{L_{S}}+\frac{2 \zeta \omega_{0}}{D^{\prime}}\right)\right) s+\omega_{0}^{2}+F\left(\frac{D+1}{L_{s} C^{C}}+\frac{\omega_{0}^{2}}{D^{\prime}}\right)} \tag{4,5.2.3}
\end{equation*}
$$

where $F=F_{M} F_{A C} \frac{V_{0} N_{s}}{R_{L} N_{P}}$.


#### Abstract



Fig. 4.5.2.1. Control-to-output response.


Equation (4.5.2.3) may be further simplified to read

$$
\begin{equation*}
\frac{\hat{v}_{0}(s)}{\hat{v}_{x}(B)}=\frac{-\dot{F}_{M_{0}} V_{0}^{\omega_{0}}}{\left(D^{\prime}+F\right) D} \frac{\left(R_{C} C a+1\right)\left(\frac{D L_{e}}{R_{L}}-1\right)}{s^{2}+\left(2 \zeta \omega_{0}+\frac{F D^{\prime}}{F+D^{\prime}} \frac{R_{L}}{L_{S}}\right) s+\omega_{0}^{2}+\frac{F D^{\prime}}{F+D^{\prime}} \frac{(D+1)}{L_{S} C}} \tag{4.5.2.4}
\end{equation*}
$$

Equations $(4.5 .2 .4)$ is the control-to-output cransfer function with the angle expressed in phass delay.

### 4.5.2.1 Test Verification for Control-to-Output Response

Fig. 4.5.2.2 demonstrates the method used to measure the control-to-output response. An ac signal, A sin wt, is injected into the de loop. The magnitude $A$ is adjusted to provide the optimal signal-to-noise ratio ( a clear and stable read out). Excessive amplitude of the infected signal could result in distortion of the modulation waveform and should be avoided. Channels $A$ and $B$ are connected as shown in Fig. 4.5.2.2. Channel Bminus Channel A results in the desired control-tooutput response...

Fig. 4.5.2.3 shows the gain and ghase of equation (4.5.2.4) for the dc-dc converter. The solid-line projection represents the analytical solutions for the control-to-out.put response.

### 4.5.3 DC Open-Loop Behavior

The dc open-loop response is the open-loop gain derived from opening the dc feedback loop. Due to the nature of the currentinjected mode of control, the true system open-loop response cannot be measured. The dc open-loop response can be used to indicate the


Fig. 4.5.2.2. Measurement technique for the control-to-output characteristics.


Fig. 4.5.2.3. Theory end measurement of control-to-output characteristics ( $\hat{v}_{0} / \hat{v}_{x}$ ).
relative stability of the system. Pig. 4.5.3.1 illustrates the block diagram frir the dc open-loop behavior.

Referring to Fig. 4.5.3.1 the dc opan-loop tranafer function can be expressed in the following form:

$$
\begin{equation*}
-G_{D L}=\frac{-F_{M} F_{D C} F_{D 1}}{\Delta+F_{M}{ }^{F} A C^{F}{ }^{F}} \tag{4.5.3.1}
\end{equation*}
$$

The general expreasion can be formulated by aubatituting equations (4.2.3.12), (4.3.2.16), (4.3.2.17) and (4.4.3.41) into equation (4.5.3.1). The following formulation yields:

$$
\begin{align*}
& \text { With }\left(R_{e}+\frac{R_{c}}{D^{\prime}}\right) \frac{D}{R_{L}} \ll 1 \text { and } F=F_{M} F_{A C} \frac{V_{0} N_{S}}{R_{L} N_{P}} \text {, equation (4.5.3.2) becomes: } \\
& \frac{F_{M^{V}} 0_{0}{ }^{\left(D_{0}\right.}}{\left(D^{\prime}+F\right) D\left(C_{1}+C_{3}\right)}\left(R_{C} C_{s+1}\right)\left(\frac{D L_{e}}{R_{L}} s-1\right)\left(\frac{1+8 R_{4} C_{1}}{1+8 R_{4}\left(\frac{C_{1} C_{3}}{C_{1}+C_{3}}\right)}\right) \quad\left[\frac{G P}{R_{1}}\right] \\
& G_{D L}=-\frac{\left.s^{2}+\left(2 \zeta \omega_{0}+\frac{F D^{\prime}}{F+D^{\prime}}, \frac{R_{L}}{L_{S}}\right) s+\omega_{0}^{2}+\frac{F D^{\prime}}{F+D^{\prime}} \frac{(D+1)}{L_{S}}\right)}{s}
\end{align*}
$$


Fig. 4. 5.3.1. DC open-loop block diagram.

Equation (4.5.3.3) 1s the de open-loop tranafer function for the current-injected converter.

### 4.5.3.1 Teat Verification for DC Open-Loop

The dc-open-loop measurenents are taken by injecting an ac signal Into the dc loop. After traveraing the loop, gain and phase of the ac eignal is measured. Fig. 4.5.3.2 illustrates the measuring technique employed for the dc-open-loop response.

The analytical and experimental waveforms for the dc open-loop gain and phase are shown in Fig. 4.5.3.3. The analytical waveforme are represented by the solid ine and the experimental gain measurements are denoted by the "x" sign. The phase measurements are represented by the "." sign.
4.5.4 System Open-Loop Response

Thי system open-loop response characterizes the converter's stability. Because of the multiple feedback paths of the converter, the loop is opened at a node common to all the feedback paths. Analytically, the loop can be opened in this fashion. However, experimentally this characteristic can not be measured. In current-injected control the ac aignal and de information are fed into a comparator. Phyaically, the only location common to the dc and ac loop is the output of the threshold detector. Unfortunately the comparator oulput is a


Fig. 4.5.3.2. Mensarement technique for dc open loop response.


Fig. 4.5.3.3. Theory and measurement of de open loop characteriatics.
logic level signal. Even for the modified equivalent circuit as shown in Fig. 4.3.2.1, the aignal $v_{T}^{\prime}$ containe both digital and analog inforaation. By opening the loop at $v_{T}$ of Fig. 4.3.1.1 or $v_{T}^{\prime}$ of Fig. 4.3.2.1 the measurement of amall-signal gain and phase would lose its phyaical meanirg. For this reason, only the analytical soiution for the syatem open-loop responce is given. Fig. 4.5.4.1 showe the block diagram for the aystem openloop responne.

From the Fig. 4.5.4.1 the aystem open-loop response can be derived. The resulting equation is given below:

$$
G_{O L}=\frac{F_{M}}{\Delta}\left[F_{D C} F_{D 1}+F_{A C} F_{D 2}\right]
$$

Substituting the converter gain expressions (4.2.3.12), (4.3.2.16), (4.3.2.17) and (4.4.3.41) into (4.5.4.1), the following formulation results:
$G_{O L}=\frac{F_{M}}{s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}}\left\{\frac{1}{\left(C_{1}+C_{3}\right) s}\left(\frac{1+s R_{4} C_{1}}{1+s R_{4}\left(\frac{C_{1} C_{3}}{C_{1}+C_{3}}\right)}\right) \quad\left[\frac{G P}{R_{1}}\right)\right.$

$$
\begin{align*}
& {\frac{V_{0} \omega_{0}^{2}}{D D^{\prime}}\left(R_{C}^{C s+1}\right)}^{\left.-1-\frac{D}{R_{L}}\left(L_{e}^{s+R_{e q}}\right)\right]+F_{A C} \frac{V_{0} N_{S}}{R_{L} N_{P}}\left[\frac{1}{L_{S} C}\left(R_{L} C+D+1\right)\right.} \\
& \left.\left.+\frac{s^{2}+2 \zeta \omega_{0} s+\omega_{o}^{2}}{D^{\prime}}\right]\right\} \tag{4.5.4.2}
\end{align*}
$$

with $\frac{\mathrm{DR}_{e q}}{R_{L}} \ll 1$, equation (4.5.4.2) is simplifled yielding:


$$
G_{O L}=\frac{F_{M} V_{0} \omega_{0}^{2}}{D D^{\prime}} \frac{1}{e^{2}+25 \omega_{0}+\omega_{0}^{2}}\left\{\frac{-G P}{\left(C_{1}+C_{3}\right) R_{1} S}\left(\frac{1+\Delta R_{4} C_{1}}{1+\Delta R_{4}\left(\frac{C_{1} C_{3}}{C_{1}+C_{3}}\right)}\right)\right.
$$

$$
\begin{equation*}
\left.\left(\frac{D L_{e}}{R_{L}} s-1\right)+\frac{F_{A C^{N} S^{D}}}{R_{L} N_{P} \omega_{0}^{2}}\left(s^{2}+\left(\frac{R_{L} D^{\prime}}{L_{S}}+2 \zeta \omega_{0}\right)=+\frac{2 D^{\prime}}{L_{S} C}\right)\right\} \tag{4.5.4.3}
\end{equation*}
$$

Equation (4.5.4.3) is the system op a-loop tranafer function in general form. The expresion will be further aimplified in chapter six.

The analytical waveforms for the gain and phase response are shown in Fig. 4.5.4.2. It is interesting to point out that the system open-loop gain approaches a constant value and the phase delay approaches zero as frequency increases to a high value. This characteristic has made the current injected control unique in comparison with any other type of control. High-gain, wide-bandwidth and stable operation could be accomplished simultaneously. In the analytical model, the pulse modulator gain is assumed constant with no phase delay. However, it is pointed out in reference [ ] that the reduction of pulse modulator gain together with an increase of the phase delay was observed in laboratory measurement. Having incorporated the non-constant gain and phase delay of pulse modulator model in the system open-loop characteristic, the gain characteristic would carry a certain slorc and the phase delay would not be approaching zero.


Figure 4.5.4.2

### 4.5.5 Audiosusceptibility

Audiosusceptibility is the closed-loop, input-to-output response for the dc-dc converter. The audiosusceptibility characteristic is used to evaluate the rejection rate of the propagation of a sinusoidal disturbance from converter input to output. Figure 4.5.5.1 illuatrates the block diagram for the closed-loop analysis of the current-injected regulator. The expression obtained by examining Fig. 4.5.5.1 is the following:
$\frac{\hat{v}_{0}(s)}{\hat{v}_{1}(s)}=\frac{\frac{F_{u 11}}{\Delta}\left[1+\frac{F_{M} F_{A C}{ }^{F} D 2}{\Delta}\right]-\frac{F_{D 1}{ }^{F_{M}{ }^{F} A C^{F}{ }^{\prime}}}{\Delta^{2}}}{1+\frac{F_{M}{ }^{F} A C^{F}{ }^{F} D_{2}}{\Delta}+\frac{F_{D 1} F_{M}{ }^{F} D C}{\Delta}}$
In equation (4.5.5.1), the denominator can be expressed in terms of the system open-loop response $G_{O L}$. After substituting equation (4.5.4.1) Into (4.5.5.1) the following expression is given:
$\frac{\hat{v}_{o}(s)}{\hat{v}_{i}(s)}=\frac{F_{u 11}\left(\Delta+F_{M} F_{A C} F_{D 2}\right)-F_{D 1} F_{M} F_{A C} F_{u 21}}{\Delta^{2}\left(1+G_{O L}\right)}$

The general form for the closed-loop transfer function can be expressed by substituting equations (4.2.3.12), (4.3.2.16), (4.3.2.17) (4.4.3.41) and (4.5.4.3) into equation (4.5.5.2).

$$
\begin{aligned}
& \frac{\hat{v}_{0}(s)}{\hat{v}_{i}(s)}=\left\{\frac{N_{S} D}{N_{P} D}, \omega_{0}^{2}\left(R_{C} C s+1\right)\right. \\
& \Gamma^{s}+2_{\zeta} \omega_{0} s+\omega_{0}^{2}+F_{M} F_{A C} \frac{V_{0} N_{s}}{R_{L} N_{P}}\left(\frac{1}{L_{s} C}\left(R_{L} C s+D+1\right)+\right. \\
&\left.\left.\left.\frac{s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}}{D^{\prime}}\right)\right]+\frac{F_{H} F_{A C} V_{0} D \omega_{0}^{2}}{D^{\prime} L_{P} R_{L} C}\left(R_{C} C s+1\right)\left(\frac{D L_{e}}{R_{L}} s-1\right)\left(R_{L} C s+1\right)\right\} .
\end{aligned}
$$

$\stackrel{5}{3}$

$i_{i,(5)}$

$$
\begin{align*}
& \Delta^{2}+\frac{F_{M} V_{0} \omega_{0}^{2}}{D D^{\prime}}\left(s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}\right)\left\{\frac{-G P}{\left(C_{1}+C_{3}\right) R_{1} S}\left(\frac{1+B R_{4} C_{1}}{1+8 R_{4}\left(\frac{C_{1} C_{3}}{C_{1}+C_{3}}\right)}\right) \quad\left[\frac{D L^{e}}{R_{L}}\right] \cdot\left(R_{c} C S+1\right)\right. \\
& \left.+\frac{F_{A C^{N} S^{D}}}{R_{L} N_{P} \omega_{0}^{2}}\left\{s^{2}+\left(\frac{R_{L^{\prime}}}{L_{S}}+2 \zeta \omega_{0}\right) s+\frac{2 D^{\prime}}{L_{S} C^{\prime}}\right)\right\}
\end{align*}
$$

Equation (4.5.5.3) will be simplified in chapter six.
4.5.5.1 Test Verification for Audiosusceptibility

The audiosusceptibility measurements are taken by injecting an ac signal in series with the input voltage of the converter, acconiplished by injecting the perturbation through a transformer whose secondary winding is in series with the supply voltage. Figure 4.5.5.2 demonstates how the measurement technique is implemented.

Fig. 4.5.5.' illustrates the gain curves for equation (4.5.5.3) for the current-injected regulator. The analytical curves are portrayed by the solid lines and the experimental result is represented by the $\times$ cuive.

### 4.5.6 Output Impedance

The output impedance is employed to measure the dynamic parformance of a switching regulator subjected to sinusoidal load disturbance. A switching regulator with zero output impedance represents an ideal voltage source. In a linear system, the output impedance is of ten used to analyze transient response. When a switching regulator is subjected to a small step-change in load, the output voltage normally varies only



Fig. 4.5.5.3. Theory and measurement of audiosusceptibility characteristics.
silghtly. If the duty ratio is regarded as constant during the load transient, the linear average model that characterizes the converter at a given quiescent point remaine valid. \{24,25,26\}.

The output impedance of the converter is defined as the ratio $\hat{v}_{0}(\mathrm{~s})$ $\hat{i}_{0}(s)$, where $\hat{i}_{0}(s)$ is the sinusoidal disturbance at the converter output. The output impedance characteristic of an open-loop regulator usually has its maximum value at the output filter resonance frequency. This undeairable result can be reduced by effectively deaigning the feedback parameters of the control loop as will be shown in chapter six. The block diagram for the output impedance is illustrated in Fig. 4.5.6.1.

Using Mason's gain formula or a block reduction method the ratio $\frac{\hat{v}_{0}(s)}{\hat{1}_{0}(s)}$ reduces to the following form:

$$
\begin{equation*}
\frac{\hat{v}_{0}(s)}{\hat{i}_{0}(s)}=\frac{F_{u 12}\left(\Delta+F_{M} F_{A C} F_{D 2}\right)-F_{M} F_{A C} F_{U 22} F_{D 1}}{\Delta\left(\Delta+F_{M}\left(F_{D 1} F_{D C}+F_{A C} F_{D 2}\right)\right)} \tag{4.5.6.1}
\end{equation*}
$$

The general form for the output impedance transfer functions can be evaluated by subatituting equations (4.2.3.12), 4.3....16), (4.3.2.17), (4.3.41), and (4.5.4.3) into equation (4.5.6.1).

$$
\begin{align*}
& \frac{\hat{v}_{0}(s)}{i_{0}(s)}=\frac{R_{C}\left(s^{2}+z_{1} s+z_{2} \omega_{0}^{2}\right)\left[\Delta+F_{M} F_{A C} \frac{V_{0} N_{S}}{R_{L} N_{P}}\left(\frac{R_{L} C s+D+1}{L_{s} C}+\frac{D}{D^{\prime}}\right)\right.}{\Delta^{2}+\frac{F_{M} V_{0} \omega_{0}^{2}}{D D^{\prime}}\left[\frac{-G P}{\left(C_{1}+C_{3}\right) R_{1} s}\left[\frac{1+S R_{4} C_{1}}{1+S R_{4}\left(C_{1} / / C_{3}\right)}\right]\left(R_{C} C s+1\right)\left[\frac{D L_{e}}{R_{L}} s-1\right]\right.} \\
& \left.-F_{M} F_{A C} \frac{V_{0} N_{S}}{D^{\prime 2} N_{P}} \omega_{0}^{4}\left(R_{C} C_{s+1}\right) \cdot\left(\frac{D_{2}}{R_{L}}{ }_{s-1}\right]\right] \\
& \left.+\frac{F_{A C^{N}} S^{D}}{{ }_{R_{L}} N_{P} \omega_{0}^{2}}\left(s^{2}+\left[\frac{R_{L} D^{\prime}}{L_{S}}+2 \zeta \omega_{0}\right] s+\frac{2 D^{\prime}}{L_{S} C}\right)\right\} \tag{4.5.6.2}
\end{align*}
$$



Fig. 4.5.6.1. Output impedance blcck diagram.

### 4.5.6.1 Test Varification for Output Impedance

Fig. 4.5.6.2 illustrates the method implemented to measure the output impedance. A voltage source with a one-hundred ohm resistor in series etitutes the current source equivalent. Channel A reade the input disturbance current $\hat{i}_{0}$, and channel $B$ measures the perturbed reaponce $\dot{v}_{0}$. In generating the theoretical reaponse, the symble RLOAD ${ }^{\text {is }}$ used for the load impedance instead of $R_{L}$. RLOAD is defined by the following:

$$
\begin{equation*}
R_{\text {LOAD }}=R_{L} / / 101.0 \tag{4.5.6.3}
\end{equation*}
$$

The theoretical and measured response are shown together in Fig. 4.5.6.3. The solid curve is the theoretical output impedance and the curve marked " $x$ " is the measured response.

Fig. 4:5.6.2. Measurement technique for the output impedance.

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Figure 4.5.6.3 Theory and measurement of the output Impedance characteristic.

### 4.6. PERPORMANCE EVALUATIONS AND CONTROL DESIGN

### 4.6.1 Introduction

The stability and dynumic performance of the current-injected switching regulator are examined. A critical look at the amall-aignal, open-loop, and cloaed-loop responses allows for a qualitative and quantitative underatanding of the control-to-output transfer function $\frac{v_{0}}{\hat{v}_{x}}$, dc operrloop response $G_{D L}$, system open-loop characteristics $G_{O L}$, and the close-loop behavior Also presented in this chapter are parametric studies on the behavior of syatem responses by varying certain "key" control variables in the feedback loops. These studies in turn egnerate information on how the ac and dc feedbackloops effect the stability, audiosusceptibility, andother important design features. After collating all the above mentioned information, some design guidelines are set forth for the dc and ac feedback-loop parameters.

### 4.6.2 Discussion of the Control-To-Output Characteristic

Referring to equation (4.5.2.4), the following expression can be written:

where $F_{P}=\frac{1}{\underbrace{}_{D^{\prime}}+\frac{R_{L} T_{F}}{2 L_{S}}\left(1+\frac{D^{\prime}}{D}\right)} \frac{\Delta D^{\prime}}{F+D^{\prime}}$

Equation (4.6.2.1) is reduced in the following manner, assume the second order polynomial of equation (4.6.2.1) is represented by:
$(s+\alpha)(s+\beta)=s^{2}+(\alpha+\beta) \cdot s+\alpha Q$

If $\alpha \gg B$ then equation 4.6 .2 .2 ylelds the following approximation:

$$
\begin{equation*}
(a+a)(B+B)=\varepsilon^{2}+\alpha \beta+\alpha \beta \tag{4.6.2.3}
\end{equation*}
$$

With the resulting approximation, the eecond order polynomial eimply fectore Into the following:

$$
\begin{align*}
& =+a=c+2 \zeta_{0}+\frac{F_{P} R_{L}}{L_{S}}  \tag{4.6.2.4}\\
& \left.z+B=+\frac{\omega_{0}^{2}\left(1+F_{P} \frac{(D+1)}{D_{D}^{2}}\right.}{2 \zeta_{\omega_{0}}+\frac{F_{P} R_{L}}{L_{S}}}\right) \tag{4.6.2.5}
\end{align*}
$$

Subatituting equations ( 4.6 .2 .4 ) and (4.6.2.5) for the second order polynominal generates the simplified version given $b y$,

$$
\begin{equation*}
\frac{\hat{v}_{0}(s)}{\hat{v}_{x}(s)}=\frac{-F_{P} R_{L} N_{P} \omega_{0}{ }^{2}}{F_{A C}{ }^{N} S} \quad \frac{\left(R_{C} C s+1\right)\left(\frac{\partial L_{e}}{R_{L}} s-1\right)}{\left(s+2 \zeta \omega_{0}+\frac{F_{P} R_{L}}{L_{S}}\right)\left(s+\omega_{0}^{2}\left(1+F_{P} \frac{(D+1)}{\left.2 \zeta \omega_{0}+\frac{F_{P} R_{L}}{R_{S}}\right)}\right)\right.} \tag{4.6.2.6}
\end{equation*}
$$

The justification for the approximation is proven by substituting the parameter values of the tested model into equation (4.6.1.6)
$\frac{\hat{v}_{0}(s)}{\hat{v}_{x}(s)}=.021 \quad \frac{(8+32.394 K)(s-47.873 K)}{(s+49.376 K)(s+304.86)}$
with $\alpha=49.376 \mathrm{~K}$ and $\beta=304.86, \alpha \gg B$, equation (4.6.2.6) is a good approximatina. The pole/zero plot for equation (4.6.2.7) is given in Fig. 4.6.2.1. For amallaignal analyais the power stage parameters are given in Chapter 4.5. The control-to-output characteristic does not include the dc feedback-loop parameters and come ion network parameters: however, it does include the

POLE/ZERO PLOT
${ }^{\mathrm{v}} \mathrm{o}_{\hat{v}_{\mathrm{x}}}$
Pig. 4.6.2.1
-230-
ac feedbsck loop parameters. The control-tomoutput transfer function consists of two poles and two zeros. It in interesting to compare this transfer function with that developed at Caltech (following a different modeling approach) which consists only of a single pole and single zero [20]. Although these two transfer functions are similar in the qualitative nature, yet there exists an important difference which can be illustrated by examining the ayymptotic curves of the bode plots. In Fig. 4.6.2.2, notice the dip in the gain curve around 6 KHz before leveling off as the frequency increases. This second-order effect could become more pronounced with a greater pole/zero separation. A similar effect can be seen in the phase plot of the control-to-output expression. This second order effect is demonstrated as the load parameter $R_{L}$ increases forcing the current to approach the discontinuous mode: a noticeable change in the phase response of the control-to-output response occurs. Fig. 4.5.2.3 illustrates the control-to-output response for $R_{L}=15 \Omega$. With $R_{L}=40 \Omega$ the theoretical and phase as shown in Fig. 4.6 .2 .4 definitely has a second zero which 13 pulling the phase up. This second order effect around six to eight kilohertz cannot be explairied by a single zero/aingle pole system. From the above discussion, one can conciude that the first-order approximation may be valid for a given range of parameter values. It however, cculd produce gross error under other parameter values or operating conditions.

### 4.6.3 DC Open Loop Characteristic and Compensation Network

The dc open-ioop response is simply the control-to-output response multiplied by the dc loop gain ( $F_{D C}$ ). The model for $F_{D C}$ given by equation (4.3.2.16) is a general integral-plus-lead/lag network.

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Fig. 4.6.2.3 (a) Gain and
(b) Phase Response for The Control- InputOutput Characteristics

Illustrated in Pigures 4.6.3.1 (a) and (b) are two commonly used integral-plus-lead/lag networks derived from Fig. 4.3.1.1. Fig. 4.6.3.1 (a) and (b) are quivalent dc loop compensation networks. The transfer function for compensation network $A$, as derived from the gencralized expression, is

$$
\begin{align*}
&\left.F_{D C}\right|_{A}=\frac{G P}{R_{1}} \frac{1}{\left(C_{1}+C_{3}\right) S} \frac{1+8 R_{4} C_{2}}{1+8 R_{4} C_{2} / / C_{3}}  \tag{4.6.3.1}\\
&= \frac{G P}{R_{1} C_{1} s} \frac{1+8 R_{4} C_{i}}{1+B R_{4} C_{3}} \\
& \text { if } C_{3} \ll C_{2} \\
& \text { Where } G_{P}=\left(1+\frac{R 3}{R_{1}}+\frac{R 3)^{-1}}{R_{2}}\right.
\end{align*}
$$

and compensation network $B$ can be modeled by the following expression:

$$
\begin{align*}
\left.F_{D C}\right|_{B} & =\frac{1+s C_{2}\left(\frac{R_{1}}{G_{P}}+R_{5}\right)}{R_{1} C_{1} s}  \tag{4.6.3.2}\\
& \frac{1+s R_{5} C_{2}}{1+s C_{2} \frac{R_{1}}{G_{P}}} \\
& =\frac{G P}{R_{1} C_{1}}
\end{align*}
$$

$$
\text { if } \frac{R_{1}}{G P} \gg R_{5}
$$

With network A the lead/lag corner frequencies are independently adjusted by changing the capacitance values $C_{1}$ and $C_{3} ;$ in network $B$ the same is accomplished by varying the resistive values $\frac{1}{G P}$ and $R_{5}$. Neither network has any particular advantage over the other. For the discussion of the dc open loop characteristics network $A$ is chosen.

Compensation network $A$ has a single-zero/two-pole transfer characteristic which provides an integral plus lead/lag compensation. Fig. 4.6.3.2 shows the gain response of network $A$ with respect to parameter variationa in resistor $R_{4}$ and capacitors $C_{1}$ and $C_{3}$. Capacitor $C_{1}$ alters the lead--235-


COMPENSATION NETWORK B


Fig. 4.6.3.1. Current-injected control modeling approach.


Fig. 4.6.3.2. Dc loop compensation.
corner frequency. Larger values of $C_{1}$ generate a lower crossover frequency as indicated by the arrow in Fig.4.6.3.2. Increasing $R_{4}$ forces the mid-frequency gain to increase while decreasing values of $C_{3}$ reduces the lag cros nover frequency. Similarly, increasing $G_{P}$ (or reducing $R_{3}$ ) causes the overall dc loop gain to increase. The dc loop response can be reshaped to any deairable characteristic by varying these parameters. The relationship of the open dc loop characteristic to the closed-loop system response will be discussed in section 4.6.6.

Using equation (4.6.3.1) as the expression for $F_{D C}$, the dc open-loop reuponse described by equation (4.5.3.2) is simplified as follows:

where const $=\frac{F_{P} \omega_{o}^{2} G P R_{L} N_{P}}{F_{A C} D^{\prime} R_{1} C_{1} N_{S}}$
The pole/zero plot for the $D C$ open loop transfer function with the converter parameter values given in section 4.5 .1 is shown below in Fig. 4.6.3.3.
4.6.4 System Stability and Its Interpretation

As discussed in section 4.5 .4 , the true system open-loop response'cannot be experimentally measured, but the dc open loop response can be measured. In Caltech's modeling approach [20], the ac loop is lumped into the power stage and a small-signal system model is derived which consists of a "new" power stage and a dc feedback loop as shown in Fig.4.6.4.1. While the dc feedback loop remains in its original forn, the "new" power stage in effect incorporates both the original power stage and the current-irujected loop.


Fig. 4.6.4.1. Current-injected control modeling approach.

Caltech's modeling approach was legitimate and was experimentally verified. Nevertheless, the interpretation of the model has to be exercised with great caution. It is obvious that the open loop response of Fig. 4.6.4.1 is merely the dc open loop characteristics of the original system and not the true open-loop response since the ac loop is embedded In the "new" power stage model. The effect of onening the ac loop cannot be examined using their model. To illustrate, fig. 4.6.4.2 demenstrates a two-loop system illustrated in a general block diagram fashion. assume that the loop containing $H_{1}$ is the ac feedback loop and the loop containing $H_{2}$ is the dc feedback loop. The following transfer functions can be easily derived.

Close loop gain: $\frac{v_{0}}{v_{I}}=\frac{\mathrm{GH}_{2}}{1+\mathrm{G}\left(\mathrm{H}_{1}+\mathrm{H}_{2}\right)}$

$$
\begin{align*}
& \begin{array}{l}
\text { Open loog gain } \\
\text { at A: }
\end{array}  \tag{4.6.4.2}\\
& \begin{array}{l}
G=G\left(H_{1}+H_{2}\right) \\
\text { Open loop gain } \\
\text { at } B \text { : }
\end{array} \\
& G_{B}=G H_{2} /\left(1+G H_{1}\right)
\end{align*}
$$

The system is unstable when the following Nyguiat stability criterio: is satisfied:

$$
\begin{equation*}
G\left(H_{1}+H_{2}\right)=-1 \tag{4.6.4.4}
\end{equation*}
$$

The left-hand side of (4.6.4.4) is identical to the open-loop gain at point A. Examining equation (4.6.4.2), $G_{A}$ provides both the conditon for stability, and information concerning the relative stability of the system (the phase margin and the gain margin). Examining the opendc loop characteristic $G_{B}$, only when $G_{B}=-1$ is the condition for instability as given in (4.6.4.4) revealed. Therefore, only at the point of stability is the following expression valid:

$$
G_{A}=G_{B}=-1
$$



Fig. 4.6.4.2, Generalized two-loop control scheme.

If, however, one employed the open-ic-isop gain $G_{B}$ to investigate the relative atability of the syatem (gain margin and phase margin) the conclusions could be very misleading. The point becomes obvious if one examines the dissimilarity of the open-dc-loop response vs. the aystem open loop response as illustrated in Fig. 4.5.3.3 and Yig. 4.5.4.2, raspectively. In conclusion, although the system open loop response could not be measured experimentally, it provides the only proper way of measuring the relative stability of the system via Bode analysis. It ahould be noted that, the characteristic equation for system open loop and open-dc loop are, however, the same 1.e.

$$
\begin{align*}
1+G_{A} & =0 \\
1+G_{B} & =0 \tag{4.6.4.6}
\end{align*} \quad(4.6 .4 .5)
$$

Equation (4.6.4.5) and (4.6.4.6) are identical. Therefore the atability could be examined via characteristic roots or eignevalues using either (4.6.4.5) or (4.6.4.6).

### 4.6.5 System Open Loop Response

The system open loop chacacteristics can be examined by the small-signal model, but experimental verification of that response is not permissible with the control hardware currently used. The problem is due to the inability to retrieve the ac-modulation-signal information from a "logic level" signal produced by the output of the comparator in the error processor with currently available instrumentation. The comparator output, determined by the control inputs (switching current and the de error voltage), is the only location where both dc and ac loops can be opened to examine the system response. To check the system open-loop characteristics, the various gain blocks formulating the analytical expression are examined via the control-to-output response and the dc open loop behavior. The e..pression for the system open loop response $G_{O L}$ as given by equation (4.6.5.1) utilizes the compensation network "A: represented by equation (4.6.3.1). -242-

$$
\begin{align*}
& C_{O L}=\frac{F_{M}}{\Delta}\left(F_{D C}{ }^{P_{D 1}}+F_{A C}{ }^{F_{D 2}}\right) \\
& -\frac{F_{M}{ }^{\nu} 0_{0}^{\omega_{0}}}{D D^{\prime}}\left(\frac{\frac{-G P}{\left(C_{1}+C_{3}\right) R_{1} s}}{s^{2}+25 \omega_{0} e+1}\right)\left(\frac{1+\Delta R_{4} C_{1}}{1+s R_{4}\left(C_{1} / / C_{3}\right)}\right)\binom{D L_{0}}{\frac{R_{L}}{R_{2}}-1} \\
& +\frac{{ }_{F} C^{N_{S}} S}{R_{L} N_{P} \omega_{0}^{2}}\left(\frac{e^{2}+\left(\frac{R_{L} D^{\prime}}{L_{S}}+2 \zeta \omega_{0}\right) e+\frac{2 D^{i}}{L_{S} C^{C}}}{s^{2}+2 \zeta \omega_{0} s+\omega_{0}^{2}}\right) \tag{4.6.5.1}
\end{align*}
$$

A unique characteristic of equarion (4.6.5.1) is that the number of zeroes is always equal to the number of poles regardless of the form of compensation network used. Equation (4.6.5.1) has four poles and four zeroes; but if the order of the system changes due to the addition of a pole or a zero in the compensation network, the open loop transfer function $G_{0 L}$ will carry five poles and five zeroes since the second term of (4.6.5.1), $\frac{{ }^{F} A C^{F} D 2}{\Delta}$, has an equal number of zeroes and poles (two zeroes and two poles). This second order expression is determined solemnly by the power atage and is independent of the form of the dc feedback loop. Equation (4.6.5.1) can be expressed in the following form:
$G_{O L}=\frac{F_{M}{ }^{F} A C V_{0} N_{S} R_{4} C_{3}}{D^{\prime} R_{C} N_{P}} \cdot \frac{s^{4}+\alpha B^{3}+\beta B^{2}+\gamma B+\mu}{B\left(R_{4} C_{3} B+1\right)\left(s^{2}+2 \zeta \omega_{0} S+\omega_{,}{ }^{2}\right)}$
where,
$a=\frac{R_{2} D^{\prime}}{L_{S}}+\frac{1}{C_{3}}\left(\frac{1}{R_{4}}-\frac{N_{P}}{N_{S}} \frac{G P R_{C}}{F_{A C} R_{1}}\right)$

$$
\begin{aligned}
& B=\frac{G P N_{P} R_{C}}{F_{A C} R_{1} N_{S} C_{1}}\left(\frac{R_{1}}{D L_{2}}-\frac{1}{R_{4} C_{1}}-\frac{1}{R_{C} C}\right)+\frac{R_{L} D^{\prime}}{R_{4} C_{3} L_{S}}+\frac{2 D}{L_{S} C^{\prime}} \\
& \left.\gamma=\frac{1}{R_{4} C_{3} L_{2}} \int_{-}^{G_{1} C_{1} F_{A C} N_{S} D}\left(\frac{D R_{2}}{R_{C}}-R_{C} C-R_{4} C_{1}\right)+\frac{2}{D^{\prime}}\right] \\
& \mu=\frac{G P R_{2} N_{P} \omega_{0}^{2}}{F_{A C} D N_{B} R_{1} C_{1} R_{4} C_{3}}
\end{aligned}
$$

In the general form, equation (4.6.5.2) is too difficult to factorize. Possibly if bounds are given on parameter values, some simplification can be done. Due to the complexity of the fourth order expression and not to take from the generality, a parametic study of the syftem open-loop response along with other system behaviors will be discussed in the next section.

Another salient feature of the system open-loop characteristic is the gain which approaches a constant value at high frequency. A simplified expression for the constant gain at high frequency can be attained. From equation (4.6.5.2) assuming $\rightarrow \infty$, equation (4.6.5.3) yields:

$$
\begin{equation*}
\left.G_{O L}\right|_{s \rightarrow \infty}=\frac{F_{M} V_{O} N_{S}}{D^{\prime} R_{L} N_{I}}{ }^{F_{A C}} \tag{4.6.5.3}
\end{equation*}
$$

In reality, however, the gain $1 . s$ expected to decrease at high frequency. This is because the pulse modulator gain can no longer be regarded constant at such high frequencies. The analytical nodel presenced here fails to take the high frequency effect of the pulse modulator into account. Due to the complicated describing function modeling employed here, the pulse modulation is simply represented by a constant gain.

### 4.6.6 Parametric Evaluations

A parametric study is performed on the small-signal behavior of the current-injected converter. The purpose of the parametric examination is to key on particular salient features of the control characteristics and design strategy for the current-Infected control. Due to the complexity of each small-signal models, especially the audiosusceptibility and the output impedance characteristics, the analyticsl expressions could nut be simplified without a loss in generaifty. For this reason, parametric study by ways of changing certain key design parameters including the duty-cycle variation, $\frac{T_{O N}}{T_{P}}$, ac loop-gain, $F_{A C}$, dc loop-gain via $G P$ and the dc loop-gain via $R_{4}$ are applied to the small-signal expressions. These results can be used to establish design guidelines for the controi loops in order to optimize the regulator performances. The small-signal characteristics examined are the following: control-to-output response, dc open-loop characteristic, $G_{D L}$; system open-loop characteristic $G_{O L}$; audiosusceptibility $\hat{\mathrm{v}}_{\mathrm{C}} / \hat{v}_{1}$; and output impedance, $z_{0}$.

### 4.6.6.1 Duty-cycle variation

Figure 4.6.6.1 and 4.6.6.2 demonstrate the control-tomoutput and dc open-loop response with respect to changes in the steady-state duty-cycle ratio. As expected, an increase in the duty-cycle ratio forces the control-tu-output gain and the dc open-loop gain downward.

As shown in Fig. 4.6.6.3 the duty-cycle has no drastic effect on the syetem open-loop phase delay. Therefore, a wide variation in duty cycle operation should not disturb the stability of the system. The duty-cycle
does have a more appreciable effect on the high frequency gain of the system upen-loop remponse. The larger duty-cycle forces the crousoverfrequenry to a higher value. Cauition must be exercised when examining the high frequency behavior blice the accuracy of the model degenerates when the modulation frequency approaches one-half the switching frequency.

The quality of the system's audionusceptibility demonstrates the converter's ability to reject input noise. Figure 4.6 .6 .4 shows the lower the duty-cycle the better the audiosusceptibility. Intuitively thio is understandable, because a lowir duty-cycle allows a smaller percentage of the total input noise to propagate to the output.

Similar effect is shown when one examines the output impedance characteristic its a function of duty-cycle. The output impedance reduces with the lowering of the duty-cycle as show in Fig. 4.6.6.5.

### 4.6.6.2 Variation in ac loop gain

The ac-loop-gain block is comprised of a resistor, $R_{s w}$, and a turn ratio $n$, a current transformer in series with the power switch. $F_{A C}=R_{S W / n}$. In Fig. 4.6.6.6 the ac loo, gain, $F_{A C}$, forces the control-to-output gain to reduce for larger values in $R_{s w} / n$. With either lower or higher $F_{A C}$ the effect of two-pole/two-zero characteristic becomes more pronounced. Examining the phase curve for $F_{A C}=3.0$, one notices the phase curve rises after approaching $-90^{\circ}$. This phenomenon will not be observed in the simplified single-zer,/single-pole approximation with the present model, a second zero in the left-half-plane causes the phase to rise after reaching $-90^{\circ}$ and then the second pole cancels the LHP zero's effect allowing the phase to continue downward to $-180^{\circ}$.

Increasing the ac loop gain decreases the dc open-loop gain as shown
in Fig. 4.6.6.7. The higher dc-open-loop gain, in general, results to better closed-loop responses such as audinsusceptibility and output impedance as will be discussed later.

Illustrated in Fig. 4.6.6.8 is the effect of the ac-loop gain to the system open-loop characteristics.
'is ac loop gain $F_{A C}$ has a very dominating effect on the system open-loop characteristic at high frequencies and virtually no effect at frequencies below the output filter resonant frequency. By decreasing the ac feedback two of the systen open-loop zeros move from the LHP toward the imaginary axis. Further reduction in the ac gain results In a complex conjugate pair of zeros as exemplified by the second-order effect of the gain curve with $F_{A C}=0.3$. Continuing even further the conjugate pair changes into two positive zeros. The open-loop phase drastically changes when the zeros migrate to the right-half-plane. This detrimental phase characteristic which approaches $-360^{\circ}$ has a very m desirable effect on the system stability. With positive zeros adding to the phase delay the phase margin is reduced to a negative value which results to an unstabie system.

The audiosusceptibility characteristic as a function of the ac loop gain is plotted in Fig. 4.6.6.9. The ac feedback loop provides an excellent parameter to adfust the low-frequency response of the audiosusceptibility. Since the passive filters in the regulator generally can provide adequate attenuation of disturbances at higher frequencies, the lower frequency range within, say, zero to ten-times the output filter resonant frequency is the frequency range to control. For this reason the ac-loop gain will play a significant role in optimizing the regulators ability to attenuate a
small-signal sinusoidal disturbance propagating from the regulator input to its output.

For smaller values of $\mathrm{F}_{\mathrm{AC}}$, the audio response reduces particularly at low frequencies. The ac feedback gain can be reduced to the point where second order peaking effect appears around $5-6 \mathrm{KHz}$. The second-order peaking effect is caused by the emergence of two poles of the closed-loop response into a complex-conjugate pair. This condition causes severe degradation in the audiosusceptibility performance of the system. As discussed earlier, the system becomes unstable under this condition. The optimal condition for the audiosusceptibllity response occurs when $F_{A C}=0.6$ prior to the occurrence of the two zeros forming a complex conjugate pair. Similar effect is shown when the output impedance characteristic is plotted as a function of $\mathrm{F}_{\mathrm{AC}}$ in Fig. 4.6.6.10.

### 4.6.6.3 DC loop gain variation vla $C P$

GP contains the de gain resistor, $R_{3}$, as shown in the following expression:

$$
\mathrm{GP}=\frac{1}{1+\mathrm{R}_{3 / \mathrm{R}_{1}}+\mathrm{R}_{3 / \mathrm{R}_{2}}}
$$

As $R_{3}$ decreases, GP increases, forcing the dc-open-loop gain upward as demonstrated in Fig. 4.6.6.11. The dc open-loop phase is not effected.

Variations in GP have a significant effect on the low frequency end of the system open-loop gain as shown in Ftg.4.6.6.12. As GP approaches its theoretical limit of one, the system gain increases. The second-order peaking effect occurred around 30 KHz exceeds the theoretical bound of the model, since the model is only grod up to half of the switching frequency $(40 \mathrm{KHz} / 2)$. Examining the phase characteristics of the system open-loop In Fig. 4.6.6.12 no zero migration to the right-half plane is noticed as
mentioned previously with a variation in the ac-loop gain.
Approaching the theoretical limit of GP improves the audiosusceptibility. Again CP has notable effect on the low frequency section of the audio. Fig.4.6.6.13 show \& 25 db improvement over the range of GP. Therefore by decreasing the dc gain resistor, $R_{3}$, the attenuation of noise from the input to the output is improved.

Fig.4.6.6.14 shows the output impedance characteristics as a function of GP.

### 4.6.6.4 DC-loop galn variation via $R_{4}$

The dc-loop gain can also be varied by changing the gain feedback resistor, $R_{4}$. But when $R_{4}$ is varied the corner frequencies of the pole and zero produced by the transfer function, $F_{D C}$, changes. Therefore, the parametric study of $R_{4}$ has been constrained to only take in the gain effect and not a shift the corner frequencies. This is done by holding the expressions $\mathrm{K}_{4} \mathrm{C}_{1}$ and $\mathrm{R}_{4} \mathrm{C}_{3}$ constant for any change in $\mathrm{R}_{4}$. Two sets of data are provided for variations in $R_{4}$. The first set varies $R_{4}$ from $10 K \Omega$ to $150 \mathrm{~K} \Omega$ while the second set examines variations of $R_{4}$ from $100 \mathrm{~K} \Omega$ to $2 \mathrm{M} \Omega$.

Figure 4.6.6.15(a) and (b) demonstrate for an increase in $R_{4}$ the $d c-$ open-loop gain fincreases. The phase does not change because the constraints set previously inhibit the corner frequency of the compensation network from changing.

From Fig. 4.6.6.16(a) the variation in $R_{4}$ has no degrading effect on the phase and affects the system open-loop gain only in the low frequency range. Apparently, the variation from 10 K to 150 K of $\mathrm{R}_{4}$ has a negligible effect on the system's stability. As $R_{4}$ is increased further ( $R_{4} \rightarrow 1 M \Omega$ ),
two zeros migrate to the RHP forming a complex-conjugate pair and forcing the phase to approach $-360^{\circ}$ instead of $0^{\circ}$ as shown in Fig. 4.6.6.16(b). This arastic change of the phase has detrimental effect on the system atability. In fact, for large $R_{4}$, the branch formed by $R_{4} C_{1}$ can be regarded as open circuit. Figure $4.6 .6 .17(a)$ shows the switch current waveform of the stable converter employing the set of circuit parameter values given in Chapter 5, page 53. Figure 4.6.6.17(b) 111ustrates the switch current waveform of the unstable system when the branch $R_{4} C_{1}$ is opened.

The effect of $R_{4}$ to the audiosusceptibility curves is given in Fig. 4.6.6.18(a) and (b). Notice as $R_{4}$ increases the audiosusceptibility steadlly improves until the two positive zeros are felt causing second= order peaking in the high frequency range. The complex zeros in the RHP eventually turn into real zeros. $A s R_{4}=2 M \Omega$, the peaking effect is no longer shown and the audiosusceptibility is greatly improved. Nevertheless, the system becomes unstable due to the excessive phase delay. Similar characteristics are displayed for the output impedance curves as shown in Fig. 4.6.6.19(a) and (b).


Fig. 4.6.6.1 Control-to-output gain and phase characteristics for $D=0.25,0.3,0.4,0.5,0.6,0.65,0.7,0.75$.


Fig. 4.6.6.2 Dc-open-loop gain and phase characteristics for D $-0.25,0.3,0.4,0.5,0.6,0.65,0.7,0.75$.

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Fig. 4.6.6.3 System open-loop gain and phase response for $\mathcal{D}=0.25$, $0.3,0.4,0.5,0.6,0.65,0.7,0.75$.


Fig. 4.6.6.4 Audiosusceptibility gain and phase characteristics for $D=0.25,0.3,0.4,0.5,0.6,0.65,0.7,0.75$.


Fig. 4.6.6.5 Output impedance characteristics for $D=0.25,0.3,0.4,0.5,0.6,0.65,0.7,0.75$.


Fig. 4.6.6.6 Control-to-output gain and phase response for $\mathrm{F}_{\mathrm{AC}}=0.05$, $0.1,0.3,0.6,1.0,1.5,2.5,3.0$.
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Fig. 4.6.6.7 Dc-open-gain and phase characteristics for $\mathrm{F}_{\mathrm{AC}}=0.05$, $0.1,0.3,0.6,1.0,1.5,2.5,3.0$.


Fig. 4.6.6.8 System open-loop gain and phase characteristics for

$$
F_{A C}=0.05,0.1,0.3,0.6,1.0,1.5,2.5,3.0
$$



Fig. 4.6.6.9 Audiosusceptibility gain response for $F_{A C}=0.05,0.1$, $0.3,0.6,1.0,1.5,2.5,3.0$


Fig. 4.6.6.10 Output impedance characteristics for

$$
\mathrm{F}_{\mathrm{AC}}=0.05,0.1,0.3,0.6,1.0,1.5,2.5,3.0
$$

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Fig. 4.6.6.11 Dc-open-loop gain and phase characteristics for $G P=0.05,0.075,0.1,0.3,0.5,0.7,0.9,0.95$.



Fig. 4.6.6.12 System open-loop gain and phase characteristics for $G P=0.05,0.075,0.1,0.3,0.5,0.7,0.9,0.95$.


Fig. 4.6.6.13 Audiosusceptibility characteristics for $G P=0.05$, $0.075,0.1,0.3,0.5,0.7,0.9,0.95$


Fig. 4.6.6.14 Output impedance characteristics for GP $=0.05,0.075,0.1,0.3,0.5,0.7,0.9,0.95$.

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Fig. 4.6.6.15(a) Dc-open-ioop gain and phase characteristics for $\mathrm{R}_{4}=10 \mathrm{~K}, 30 \mathrm{~K}, 50 \mathrm{~K}, 7 \mathrm{KK}, 90 \mathrm{~K}, 110 \mathrm{~K}, 130 \mathrm{~K}, 150 \mathrm{~K}$.



Fig. 4.6.6.15(b) DC-open-loop gain and phase characteristics for $R_{4}=100 \mathrm{~K}, 200 \mathrm{~K}, 300 \mathrm{~K}, 400 \mathrm{~K}, 500 \mathrm{~K}, 700 \mathrm{~K}, 900 \mathrm{~K}, 2 \mathrm{M} \Omega$.



Fig. 4.6.6.16(a) System open-loop gain and phase characteristics for $\mathrm{R}_{4}=10 \mathrm{~K}, 30 \mathrm{~K}, 50 \mathrm{~K}, 70 \mathrm{~K}, 90 \mathrm{~K}, 110 \mathrm{~K}, 130 \mathrm{~K}, 150 \mathrm{~K} 3$.


Fig. 4.6.6.16(b) System open-loof gain and phase characteristics for $R_{4}=100 \mathrm{~K}, 200 \mathrm{~K}, 300 \mathrm{~K}, 400 \mathrm{~K}, 500 \mathrm{~K}, 700 \mathrm{~K}, 900 \mathrm{~K}, 2 \mathrm{M}$ ?.

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(a)

(b)

Fig. 4.6.6.17(a) Transistor collector-current waveform for a stable system with $R_{4} C_{1}$ branch. Vertical: $1 A / d i v .$, Horizontal: $20 \mathrm{~ms} / \mathrm{div}$.
(b) Transistor collector-current for an unstable system when $\mathrm{R}_{4} \mathrm{C}_{1}$ is removed. Vertical: lA/div., Horizontal: $0.1 \mathrm{~ms} / \mathrm{div}$.


Fig. 4.6.6.18(a) Audiosusceptibility characteristics for $R_{4}=10 \mathrm{~K}, 30 \mathrm{~K}$, $50 \mathrm{~K}, 70 \mathrm{~K}, 90 \mathrm{~K}, 110 \mathrm{~K}, 130 \mathrm{~K}, 150 \mathrm{~K}$. The product $\mathrm{R}_{4} \mathrm{C}_{1}$ and $R_{4} C_{3}$ remain constant.

$\begin{aligned} & \text { Fig. 4.6.6.18(b) Audiosusceptibility characteristics for } R_{4}=100 \mathrm{~K}, \\ & 200 \mathrm{~K}, 300 \mathrm{~K}, 400 \mathrm{~K}, 500 \mathrm{~K}\end{aligned}$ $200 \mathrm{~K}, 300 \mathrm{~K}, 400 \mathrm{~K}, 500 \mathrm{~K}, 700 \mathrm{~K}, 900 \mathrm{~K}, 2 \mathrm{M} \Omega$.


Fig. 4.6.6.19(a) Output impedance characteriatics for $\mathrm{K}_{4}=10 \mathrm{~K}, 30 \mathrm{~K}, 50 \mathrm{~K}, 70 \mathrm{~K}, 90 \mathrm{~K}, 110 \mathrm{~K}, 130 \mathrm{~K}, 150 \mathrm{~K}$. The product $R_{4} C_{1}$ remain constant.


Fig. 4.6.6.19(b) Output impedance characteristics for $\mathrm{R}_{4}=100 \mathrm{~K}, 200 \mathrm{~K}, 300 \mathrm{~K}, 400 \mathrm{~K}, 500 \mathrm{~K}, 700 \mathrm{~K}, 90 \mathrm{~K}, 2 \mathrm{M} \Omega$.

### 4.7 CONCLUSIONS

Modeling and Analyeis of a buck/boost regulator employing the currentinjected (current-programmed) control iv presented. The objective is to examiue the small-signal dynamic performance of the awitching regulator including:

- Open-loop characteristics and syatem stabilitiy.
- Closed-loop characteristics and audiosusceptibility.
- Response due to load disturbances and output impedance characteristics.
- Features of the current-injected control and design guidelines.

In order to gain insights of the performance characteristics of various parts of the awitching regulator employing the current-injected control, the regulator is modeled according to the three basic functional blocks: power stage, error processor, and duty-cycle pulse modulator. The power stage model presented in Chapter 4.2 consists of three inputs: disturbances from the line $\hat{v}_{i^{\prime \prime}}$ the loal $\hat{\mathrm{i}}_{0}$, and the duty-cycle control loop $\hat{d}_{\text {; }}$ and two outputs: the output voltage $\hat{v}_{o}$, and the switch current $\hat{i}_{p}$. The error processor (EP) model as presented in Chapter 4.3 contains the dc feedback, current-injected loop and the compensation network. The EP processes and compensates the signals from both the dc and ac feedback loops and provides the necessary error signal to the duty cycle pulse modulacor (PM). The PM takes the analog error aignal from the EP and converts the error signal into a series of pulse-width modulated dutycycle signals. Presented in Chapter 4.4 is the low-frequency model of the duty-cycle pulse modulator employing the describing function techniques.

In Chapter 4.5, the mall-signal model for the power atage, the error processor and pulse modulator, derived from Chapters 4.2,4.3 and 4.4, respectively, are assembled. Rmploying this small aignal model, the following open- and closed-lop characteristics are examined:

- The control-to-output characteristic $\hat{\mathbf{v}}_{0} / \hat{\mathbf{v}}_{\mathrm{x}}$.
- The dc open-loop characteristic. (The dc feedback loop is opened, while the current-injected loop is intacted.)
- The system open-loop characteristics. (Both the dc feedback loop and the current-injected loop are opened.)
- The closed-loop input-to-output response - audiosusceptibility.
- The closed-loop response due to a load disturbance - output impedance characteristic.

Lxperimental verifications are also provided for the above described analytical models with good correlations.

In Chapter 4.6, various regulator open-loop and closed-loop performance characteristics are examined in further detall. Effects of various key control parameters to the regulator performance characteristics are investigated. Features of the cirrent-injected control are discussed. In particular, comparisons are made brtween the results derived in the current modeling and analysis efforts and the earlier works presented by Caltech (which followed a different modeling approach).

Many interesting characteristics are unveiled upon careful examination of the small-signal model derived in this report. Among those findings, the following ones are part cularly noteworthy:
(1) The ac feedback loop is imbedded in the control-to-outy, fer function which exhibjts two-pole and two-2ers chay ifc instead of a single-pole single-zero. However, the swo-pole
two-zero transfer function can be approximated by aingla-pole single-zero transfer function without encounter ing gross errors under most circumstances.
(2) When the system open-loop characteristic is examined with both dc and ac loops opened, the two-pole two-zero (or singlepole single-zero approximation) characteristic ceases to exist. In fact the system open-loop transfer function exhibits astonishingly different characteristics. Very little resemblance can be found when one compares the open-loop characteristic $G_{T}$ with the open-dc-loop characteristic $G_{D L}$.
(3) The stability characteristic for the current-injected control is quite unique somparing to other types of multi-loop control systems $[22,23]$. The stability margin could extend beyond $90^{\circ}$ with very high cross-over frequency. The atability of the system is particularly sensitive to certain control parameter values. A positive zero could be observed in the openloop characteristic if the control paraweters are not properly selected.

### 5.0 MAPPS DEMONSTRATION PROBLEM FOR VSTOL EMERGENCY POWER SYSTEM

### 5.1 INTRODUCTION

The VSTOL Emergency Power System is shown in Figure 5.1. A. Ni-Cd battery source with an effective series resistance $R_{e}$ is processed by a boost converter to provide the bus power for emergency use when the generator falled. The battery valtage level depends on the number of Ni -Cd cells connected in series, with the number yet to be determined. The boost-converter power circuit consists of inductance $L$ with winding resistance RL, power switch $Q$, power diode $D$, and output capacitor $C$ having an equivalent series resistance (ESR) RC. Since the losses in the converter are supplied from the battery source, and since the converter packaging weight (heat sink included) increases with converter losses, it follows that the combined battery and converter mechanicalstructure weight becomes heavier if more converter loss is allowed for a given output power. On the other hand, the converter component weights, f.e., those of magnetics and capacitors, tend to diminish with more allowable losses. Consequently, for a given output power, there must exist an optimum converter efficiency at which the combined system weight including battery, packaging, and converter components, is at its minimum. The essence of the optimization is to identify the battery voltage level along with the detailed boost converter design so that the total system weight of the battery and the packaged converter can be minimized.

The optimization effort starts with the identification of key operating waveforms of the boost converter as functions of line/load conditions, from which the voltage and current expressions are derived for all powerhandling components. These expressions are then used to formulate component losses as well as other converter design constraints including input EMI, output ripple, and the proper design of the inductor. The optimization rbjective of minimizing the system weight is then defined analytically.

At this juncture, two approaches become possible. One is to rely on an established nonlinear programming routine such as the SUMT (Sequential Unconstrained Minimization Technique) to numerically seek the optimum design and the attendant minimum system weight. Two basic variables in this approach are the input voltage to, and the switching frequency of, the converter. They are treated as unknowns, to be determined along with other variables in a single, large-scale numerical optimization. A second approach, less ambitious and thiss perhaps more practical, breaks the single large-scale optimization into many smaller optimizations. The two aforementioned variables are given as known values, thereby greatly simplifying the problem and allowing a closed-form optimum VSTOL design for all other variables. Without resorting to nonl inear programming, optimum designs for other sets of known input voltages and switching frequencies can then be similarly calculated, from which the true global optimum corresponding to a specific set can be identified among calculated optimum designs by mapping the data of optimum designs for different sets of voltages and frequencies. A decision was made to adopt the second approach to the VSTOL system optimization, with the purpose of establishing an alternate means to nonlinear programming in achieving optimization. The chosen approach was successfully implemented, and closed-form optimum system designs for any given set of input voltage and switching frequency were obtained. A computer program was generated to perform the straightforward numerical calculations of the analyticallyderived closed-form solutions. A 3000-Watt, 0.5-hour emergency application was used as a demonstration example. The closed-form solutions, the computer program, tre calculated results and graphical representations, the discussion and the concliarion regarding the VSTOL emergency power svstem. are presented.

### 5.2. CURRENT AND VOLTAGE WAVEFORMS

The relevant boost converter waveforms are given in Figure 5.2.The power switch operates with an on-time TN and an off-time TF. The switching between the two time intervals are exaggerated in length for both switch $Q$ and diode $D$ to illustrate for the switching-loss modeling to be formulated in a later section.

The power-switch current iQ and diode current iD are shown in figures 5.2 A and 5.2 B . The midpoint of each pulse current has a level of If. The


Figure 5.1 VSTOL Emergency Power System
average diode current is identical to the output load current Io. The composite waveform of Figure 5-2A and 5-2B is given in Figure 5-2C which reresents the inductor current iL as well as the converter input current. The average converter input current is therefore II. The difference between iD and $I_{0}$ becomes the capacitor current $\mathbb{I} C$, as is shown in Figure 5-2D. The switch and diode voltage waveforms are given in Figures 5-2E and 5-2F.

### 5.3. BASIC VOLTAGE AND CURRENT EXPRESSIONS

Let the following designations be made:
D: duty cycle defined as (TN)/T
di: peak to peak ac component in the inpot current
F: switching frequency
ic: output capacitor current
1D: diode current
1L: input (or inductor) current
1Q: power-switch current
Ii: the midpoint puise current show in figure 5-2A
L : inductance in boost converter
PL: totai loss in the converter
PO: output power to the load
TN: on-time of the power switch
VI: input voltage to the converter
VQ: output voltage to the load

The following expressions can be formulated:

$$
\begin{align*}
& \text { (iD)ave a } \frac{P O}{V O}  \tag{5-1}\\
& I i=\frac{P O}{V O *(1-D)} \tag{5-2}
\end{align*}
$$



Figure 5.2 Boost Converter Key Waveforms

$$
\begin{equation*}
\text { (1Q)ave }=\frac{P O^{* D}}{V O^{*}(1-D)} \tag{5-3}
\end{equation*}
$$

$$
\begin{align*}
& \text { (IL)ave }=11=\frac{P 0}{V^{*}(1-D)} \\
& d t=\frac{V I * D}{L^{* W}} \tag{5-5}
\end{align*}
$$

$$
\begin{equation*}
\left(i L ; \text { rms }=\sqrt{\left[\frac{P O}{V D^{*}(I-D)}\right]^{2}+\left[\frac{V I * D}{[* F}\right]^{2}}\right. \tag{5-6}
\end{equation*}
$$

$$
(1 C) \mathrm{rms}=\left[(I 1)^{2} \star(1-D)-(P 0 / V O)^{2}\right]^{0.5}=\frac{P 0}{V 0} \star\left[\frac{D}{-0}\right]^{0.5}
$$

Since the total input power (VI) (Ii) is equal to the sum of output Po and loss PL,

$$
\begin{equation*}
P O+P L=\frac{P O^{*} V I}{V O^{*}(\mid-D)} \tag{5-8}
\end{equation*}
$$

From which one obtains:

$$
\begin{align*}
& D=1-\frac{P O^{* V I}}{V O^{*}(P O+P L)}  \tag{5-9}\\
& 1-D=\frac{P O^{* V I}}{V O^{* 2}(P O+P L)} \tag{5-10}
\end{align*}
$$

Substituting (5-9) and (5-10) into (5-2) through (5-7) one has:

$$
\begin{align*}
& I 1=\frac{P O+P L}{V I}  \tag{5-11}\\
& (1 Q) \text { ave }=\frac{P O+P L}{V I} *\left[1-\frac{P O^{* V I}}{V O^{*}(P O+P L)}\right]
\end{align*}
$$

$$
\begin{aligned}
& \text { (1L)ave }=\frac{P O+P L}{V I}
\end{aligned}
$$

$$
\begin{align*}
& (1 L) \text { rus }=\left[(1 L)_{\text {ave }}^{2}+(d 1)^{2}\right]^{0.5} \underset{\sim}{P O+P L}  \tag{5-15}\\
& (i C)_{\text {rms }}=\frac{P 0}{V O} *\left[\frac{V *(P O+P L)}{V I * P O}-1\right]^{0.5} \tag{5-16}
\end{align*}
$$

### 5.4. FORMULATION OF CONVERTER LOSSES

Let: A Cross sectional area of indoctor core
AC. - Winding area per turn of inductor
BDC = Intended maximom dc operating flox density level ac flux-densfty excursfon in the inductor core

C = Capacitance at output
F - Swftching frequency
FC $=$ Pitch factor of intactor winding
FW = Inductor core fill factor
L - Indoctance
$N=$ Number of thmes used in inductor
RC E ESR of C
RHO = Resistivity of indactor winding
RI = Peak-peak allowable ripple at the outpot
TDF = Fall the of diode doring switching
TDR = Rise the of diode during switching
TSF = fall time of power swliteh during swritching
TSR = Rise time of power switch doring switching

## VB - Forward base-emitter voltage of transistor

vo - Forward drop of the diode
VS : Saturation voltage drop of the power switch
$z$ : Mean length of inductor core

With these descriptions defined, the following losses can be formulated.

### 5.4.1 INDUCTOR LOSS

The inductor loss consists of copper loss and iron loss. 8ased on the assumption of a toroid inductor and a square cross section for the core, the copper loss becomes:

$$
\begin{align*}
\text { PLC } & =(I i)^{2} * \text { inductor winding resistance } \\
& =\left[\frac{P O+P L}{V I}\right]^{2} * 4 * R H O * \frac{F C \star N A A}{A C} \tag{5-17}
\end{align*}
$$

The iron loss per switching cycle can be expressed by the area of the minor BH loop. Such a characteristic is shown in Figure 5-3. The flux excursion, $\emptyset$, can be expressed as:

$$
\begin{align*}
0 & =(V O-V I) *(I-D) /(N * F) \\
& =\frac{(V O-V I) * P O^{*} V I}{V O^{*} N^{*} F^{*}(P O+P L)}=B A C * A \tag{5-18}
\end{align*}
$$

where $\quad B A C=\frac{(V O-V I) * P O * V I}{V O^{*}(P O+P L)^{* N^{*} * * A}}$
The ampere-turns Ni corresponding to the BH loop-width $H$ of the inductor core can be expressed as Ni Hz , where H is normally expressed in oersteds.

Since $H$ is a nonlinear far: etion of the switching frequency $F$, it can be


Figure 5-3
Inductor Case Characteristic
Shaded Area Represents Energy Loss Per Cycle

Since $H$ is a nonlinear function of the switching frequency $F$, it can be related to $F$ as the following:

$$
\begin{equation*}
H=0.089 * F^{0.6} \quad \text { (For Permalloy powder core onlv) } \tag{5-20}
\end{equation*}
$$

where $H$ and $F$ are in amy-turns/meter and hertz, resnectively. The loop width is 2 Ni . or

$$
\begin{equation*}
2 N 1=2 * H * Z \tag{5-21}
\end{equation*}
$$

The tron loss becomes:
PLI = 2*(Ni)*Q*F =2*BAC*H*A*Z*F
where ( $B A C$ ) and $H$ are given in 5-19 and 5-20, respectively.

The total inductor loss is therefore:
PL =PLC + PL!
where PLC arid PLI are respecifively given in (5.17) : 12).

### 5.4.2 POWER SHITCH

Losses in the transistor include the cond . less, the base loss, and the switching loss. The conduction loss PTC is:

$$
\begin{equation*}
P T C=I 1 * V S * D=\frac{P O+P L}{V I} * V S *\left[1-\frac{P O * V I}{V O *(P O+P L)}\right] \tag{5-24}
\end{equation*}
$$

Assuming a 10 to 1 base current drive, the base loss is:

$$
\begin{equation*}
P T B=0.1 * I 1 * D * V B=0.1 * \frac{P O+P L}{V I} * V B E *\left[1-\frac{P O * V I}{V O^{*}(P O+P L)}\right] \tag{5-25}
\end{equation*}
$$

The switching losses during turnon and turnoff can be formulated with aid of Figure 5-20. During turnon, the voltage across the power switch decreases from ( $V_{0}+V D$ ) to VS, and the current through the switch increases from zero to [ (PO+PL.)/VI - (di/2) ]. Assuming the rate of voltage or current change during the switching interval TSR is constant, it can be shown easily that the turnon switching loss is:

$$
\begin{equation*}
P T R=(1 / 6) *(V O+V D-V S) *\left(\frac{P O+P L}{V I}-\frac{d i}{2}\right) * T S R * F \tag{5-26}
\end{equation*}
$$

During turnoff, the voltage across the power switch increases from VS to $\mathrm{VO}_{\mathrm{o}}+V D I$, and the current decreases from $[(\mathrm{PO}+\mathrm{PL}) / V I+d 1 / 2]$ to zero. Therefore, during switching interval TSF, one has:

$$
\begin{equation*}
P T F=(1 / 6) *(V O+V D-V S) *\left(\frac{P O+P L}{V I}+\frac{d y}{2}\right) * T S F * F \tag{5-27}
\end{equation*}
$$

Realizing the opposite signs associated with the di/2 term in (5-26) and (5-27), one makes the simplifying assumption of equal TSR and TSF, and the

$$
E-4
$$

sum of switching losses becomes:

$$
\begin{equation*}
P T R+P T F \quad=(1 / 6) *(V O+V D-V S) * \frac{P O+P L}{V I} *(T S R+T S F) * F \tag{5-28}
\end{equation*}
$$

By summing up (5-24) (5-25) and (5-28) the total loss in the power switch becnmes:

$$
\begin{aligned}
P T & =P T C+P T B+P T R+P T F \\
& =\frac{P O+P L}{V I}\left[1-\frac{P O * V I}{V O^{*}(P O+P L)^{*}}\right]^{*}(V S+0.1 * V B E)+(V O+V D-V S) *(T S R+T S F) * F / 6(5-29)
\end{aligned}
$$

### 5.4.3 DIODE LOSS

The diode conduction loss is:

$$
\begin{equation*}
P D C=I 1 * V D *(1-D)=P O * V D / V O \tag{5-30}
\end{equation*}
$$

Essentially following the same derivation for the power-switch switching loss, the diode switching loss becomes:

$$
\begin{equation*}
P D R+P D F=(1 / 6) *(V O+V D-V S) * \frac{P O+P L}{V I} *(T D R+T D F) * F \tag{5-31}
\end{equation*}
$$

The total diode loss is:

$$
\begin{align*}
P D & =P D C+P D R+P D F \\
& =\frac{P O * V D}{V O}+(V O+V D-V S) \frac{P O+P L}{V I} *(T D R+T D F) * F / 6 \tag{5-32}
\end{align*}
$$

### 5.4.4 CAPACITOR LOSS

The capacttor loss PC is:

$$
\begin{equation*}
P C=(I C)_{\text {ms }}^{2} * R C=\left(\frac{P O}{V O}\right)^{2} * R C *\left[\frac{V Q^{*}(P O+P L)}{V I \cdots P}-1\right] \tag{5-33}
\end{equation*}
$$

### 5.4.5 OTHER CONVERTER LOSSES

The total converter loss in the control circuit, housekeeping circuit, wires and connectors is assumed to be 1.5\% of the total output power, i.e..

$$
\begin{equation*}
\text { OT }=0.015 * P 0 \tag{5-34}
\end{equation*}
$$

### 5.4.6 TOTAL CONVERTER LOSS

The total converter loss can be obtained by summing up (5-17), (5-22), (5-29), (5-32), (5-33) and (5-34):

$$
\begin{align*}
P L= & \left(\frac{P O+P L}{V I}\right)^{2} * 4 * R H O * \frac{F C * N * A^{0.5}}{A C} \\
& +2 * \frac{(V O-V I) * P O \star V I}{V O^{*}(P O+P L) * N * A^{*} F} * 0.089 * F 1.6 * A * Z \\
& +\frac{P O+P L}{V I}\left[1-\frac{P O \star V I}{V O^{*}(P O+P L)}\right] *(V S+0.1 * V B E)+(V O+V D-V S) *(T S R+T S F) * F / 6 \\
& +\frac{P O * V D}{V O}+(V O+V D-V S) * \frac{P O+P L}{V I} *(T D R+T D F) * F / 6 \\
& +\left(-\frac{P O}{V}\right)^{2} *\left[\frac{V O \star(P O+P L)}{V I * P O}-1\right] * R C \\
& +0.015^{*} P O \tag{5-35}
\end{align*}
$$

With PO much greater than PL, very little error will be induced if the following simplifying approximations are made:

$$
\begin{align*}
& P O /(P O+P L)=0.93 \\
& (P O+P L)^{2}=1.156 * P O^{2} \tag{5-36}
\end{align*}
$$

Substituting these abpi'oximations into(5-17)and(5-19) eq.(5-32)becomes:

$$
P L=P L 1 / P L 2
$$

where:

$$
\begin{align*}
P L I= & 4.624 * R H O * \frac{F i * H * A^{0.5}}{A C} *\left(\frac{P O}{V I}\right)^{2}+\left(\frac{P O}{V I}-\frac{P O}{V O}\right) *(V S+0.1 * V B) \\
& +\frac{P O}{\sigma^{* V I} *(V O+V D-V S) *(T S R+T S F+T D R+T D F) * F+\frac{P O * V U}{V O}} \\
& +\left(\frac{P O}{V O}\right)^{2} * R C *\left(\frac{V O}{V I}-1\right)+0.015 * P O \\
& +0.165 * F^{0.6 * \frac{(V O-V I) * V I}{A * W V O} * A * Z} \\
P L 2= & 1-\frac{V S+0.1 * V B E}{V I}-\frac{P O * R C}{V O * V I} \\
& -\frac{V O+V D-V S}{V I * 6} *(T S R+T S F+T D R+T D F) * F \tag{5-37}
\end{align*}
$$

### 5.5. FORMULATION OF CONSTRAINTS

The following constraints are to be observed:

- The input current ac component mast be limited to below a certain peak-to-peak amplitude.
- The indoctor should not operate in the saturation region.
- The output voltage ripple must be limited to telow a certain peak-to-peak amplitude.
- The core must have sufficient window area to accomodate all the windings.

These constraints are formulated as follows:

### 5.5.1 PEAK-TO-PEAK INPUT AC COMPONENT

The peak-to-peak ac componant has bisen expressed as di in eq(5-14). Let the maximum anplitude be specified as IAC, then,

$$
\begin{equation*}
I A C \geqslant \frac{V I}{T * F} *\left[1-\frac{P O * V I}{\left(P O^{\prime}+P L\right)^{*} V O}\right] \tag{5-38}
\end{equation*}
$$

### 5.5.2 BELOW INDUCTOR SATURATION FLUX DENSITY

This constraint specifies that at the peak inductor current, the corresponding flux density in the core should be below a certain predetermined level, (BDC). Consequently,

$$
\begin{align*}
& N * A-L * I P / B D C \geq 0 \\
& I P=I 1+d I / 2=\frac{P O+P L}{V I}+\frac{V I}{2 * L F} *\left[1-\frac{P O * V I}{V O^{*}(P O+P L)}\right] \tag{5-39}
\end{align*}
$$

where $N$ and $A$ are the turns and the area of the inductor core.

### 5.5.3 OUTPUT VOLTAGE RIPPLE

The peak-to-peak output-voltage ripple is caused by two components: the capacitive component due to the ampere-second procissed by $C$ and the resistive component due to the ESR RC of C. It can sie shown that the constraint concerning the sum of these two components can be expressed as:

$$
\begin{aligned}
& R I \geq(I 1+d 1 / 2) * R C+P O * D /\left(2 * V O^{*} C * F\right) \\
& R I \geq \frac{P O+P L}{V I} * R C+\frac{1}{2^{\star} F} *\left[1-\frac{P O^{*} V I}{V O^{*}(P O+P L)}\right] *\left(\frac{V I \star R C}{L}+\frac{P O}{V O^{*} C}\right)
\end{aligned}
$$

Where RI is the peak-to-peai ripple specification. In actual design, series-parallel combination of capacitors may be needed in order to meet the voltage and ripple requirements. However, the product ( $R C * C$ ) will always remain a constant value regardless of the combination used. For example, three different combinations of capacitors are given in Figure 5-4. Capacitance $C$ and resistance RC are associated with each individual capacitor. It can be easily proved that the equivalent RC product for all three connections are identical, and are equal to $R C * C$. Let this constant value be designated $G$ for a given capacitor type, one has:

$$
\begin{equation*}
R I \geqslant \frac{P O+P L}{V I} * \frac{G}{C} * \frac{1}{\delta^{*} F} *\left[1-\frac{P O^{*} V I}{V O^{*}(P O+P L}\right] *\left(\frac{V[* G}{L * C}+\frac{P O}{V 0^{*} C}\right) \tag{5-40}
\end{equation*}
$$


$\begin{array}{ll}\text { Figure 5-4 } & \text { Different Capacitor ranections with } \\ & \text { identical time constant } R C * C .\end{array}$

### 5.5.4 SUFFICIENT WINDOW AREA

Assuming a torold core and a square cross-sectional core area, then, for a mean length $Z$, the window radius will be ( $\left.Z /\left(2^{*} 3.1416\right)-A^{0.5} / 2\right)$.

The window area becomes $3.1416 \star\left[2 /(2 \star 3.1416)-A^{0.5} / 2\right]^{2}$. This area, multiplied by window fill factor $F \boldsymbol{F}$, most be sufficient for all the inductor windings, which has a total area of $N \star A C$. Consequently,

$$
\begin{align*}
& N * A C \leqq \pi * F W *\left(\frac{Z}{2 * \pi}-\frac{A^{0.5}}{2}\right)^{2} \\
& \left(\frac{N * A C}{\pi^{*} F W}\right)^{0.5}-\frac{2}{2^{* \pi}}+\frac{A^{0.5}}{2} \leq 0 \tag{5-41}
\end{align*}
$$

### 5.6. SIMPLIFICATION OF CONSTRAINTS FOR CLOSED-FORM OPTIMIZATION

In equations (5-38) to (5-40), power loss PL is always associated with an output power PO. Admittedly, PL is a highly complicated function of many variables, as indicated in eq. (5-37). If one seeks a closedform optimization using the aforementioned constraints, the parameter PL must not be treated in accordance with (5-37). Rather, and indeed

Portunately so, little error will be introduced to these constraints if $P L$ is given a reasonable numerical valoe, and used to derive variables such as $L, C, R C, N, A$, etc. These known values can then be substituted into(5-37) to represent a more precise PL. In assence, an iteration of PL based on a reasonable approximation will be utilized for the sake of enhancing closed-form optimization. Numerically, the iterative approximation is Justified by considering that from ( $5-38$ ) to ( $5-40$ ), PL is always assuctated with PO in the form of ( $\mathrm{PO}+\mathrm{PL}$ )/PO, which is close to unity due to the fact that PO is generally much greater than PL. For example, a $10 \%$ error in assigning a numerical PL would only result in about $1 \%$ error for ( $P O+P L$ )/nO. Consequently, the variables to be thus obtained are only subjected to a rather insignificant error, which in turn, will introduce only a relatively small error for the precise expression for PL when the varlous variables are eventaally deployed in(5-37). Consequently, a numerical PL $=0.07$ * PO will be used in(5-38)to(5-41) in quest for closed-form optimization, and equations(5-38)to(5-40)become:

$$
\begin{equation*}
L \geq \frac{V I *[1-V I /(1.07 * V O)]}{I A C * F} \tag{5-42}
\end{equation*}
$$

$$
\begin{equation*}
N * A \geq \frac{L}{B O C} *\left[\frac{1.07 * P O}{V I}+\frac{V I}{2 *[* F} *\left(1-\frac{V I}{1.07 * V O}\right)\right] \tag{5-43}
\end{equation*}
$$

$$
\begin{equation*}
c \geqq \frac{1}{R I} *\left[\frac{1.07 * P O * M}{V I}+\frac{1}{2 * F} *\left(1-\frac{V I}{1.07 * V O}\right) *\left(\frac{V I * M}{L}+\frac{P 0}{V 0}\right)\right] \tag{5-44}
\end{equation*}
$$

In eqs. (5-42) to (5-44), the terms IAC, BDC, and RI, Voltage VO, and power $F O$, are specified parameters. Consequently, given an input voltage VI and frequency $F$, both $L, N * A, C$, and $R C$ can be determined numerically. Substituting these values into (5-37) reduces it into the following:

$$
\begin{equation*}
P L=\frac{K 1 * N^{*} A^{0.5}}{A C}+K 2 * A * Z+K 3 \tag{5-45}
\end{equation*}
$$

Where constants K1, K2, and K3 are:

$$
\begin{equation*}
K I=\frac{4.624 * R H O *(P O / V I)^{2} * F C}{K 4} \tag{5-46}
\end{equation*}
$$

$K 2=\frac{0.165 * F^{0.6} *(V O-V I) \star V I /(V O \star A * N)}{K 4}$

$$
\begin{equation*}
+\frac{P O * V D}{V O}+\left(\frac{P O}{V O}\right)^{2} \neq R C *\left(\frac{V O}{V I}-1\right) \tag{5-47}
\end{equation*}
$$

$$
K 3=\frac{\left(\frac{P O}{V I}-\frac{P O}{V O}\right) \star(V S+0.1 \star V B)+\frac{P O}{6 \star V I}(T S R+T S F+T D R+T D F) \star F *(V O+V D-V S)+0.015 \star P O}{K 4}(5-48)
$$

$K 4=1-\frac{V S+0.1 * V B E}{V I}-\frac{V O+V D-V S}{6 * V I} *(T S R+T S F+T D R+T D F) * F-\frac{P O * R C}{V O * V I}$

### 5.7. BATTERY WEIGHT

In order to formulate battery weight, let the following parameters be defined:
$n$ : Number of NI-Cd cells in series within the batcery
RE : Total effective resistance internal to battery
re : Effective resistanca of each cell, i.e., RE-n(re)
$T$ : Anticipated emergency power utilization time
VB : Battery voltage before the internal drop across RE
Vk : Individual cell voltage, f.e., VBen(Vk)

With reference to Fig. 5-1,

$$
\begin{align*}
& V B=V I+\frac{P U+P L}{V I} * n * r e=n * V k  \tag{5-50}\\
& n=\frac{V I}{V k} *\left[1-\frac{r e(P O+P L)}{V I * V k}\right]^{-1}
\end{align*}
$$

The total power drawn from the battery becomes

$$
\begin{align*}
P B & =P O+P L+\left(\frac{P O+P L}{V I}\right)^{2} * n * r \\
& =P O+P L+\frac{(P O+P L)^{2} * r e}{V I * V} *\left[1-\frac{(P O+P L) * r e}{V I * V}\right]^{-1} \tag{5-51}
\end{align*}
$$

The voltage required from the Eattery is:

$$
\begin{align*}
& V B=P B / I 1-V I+\frac{(P O+P L) * r e}{V k} *\left[1-\frac{(P O+P L) * r e}{V I * V k}\right]^{-1} \\
&=V I+\frac{0.015 \hbar^{*} m^{*} V I}{V k} *\left[1-\frac{0.015^{\star} m}{V k}\right]^{-1}  \tag{5-52}\\
& \text { (see eq. (5-56) later) }
\end{align*}
$$

The battery ampere-hour capacity required is:

$$
A H \geqslant \frac{P B \div T}{V W m} \quad \begin{align*}
& m=1.0 \text { for } T=1  \tag{5-53}\\
& m=0.9 \text { for } T=0.5
\end{align*}
$$

where $m$ represents the reduction of cell capacity due to more than IC discharge. Using the General Electric Ni-Cd Handbook as a reference, the individual cell weight is essentially a linear function of its ampere-hour (AH) rating. Let Wk be the individual cell weight, then,

$$
\begin{equation*}
W k=K * A H=\frac{K \dot{T} \star P B}{V T \cdots m} \tag{5-54}
\end{equation*}
$$

where $K \simeq 42$ grams/(amp-hr) for Ni-Cd celts. Combining(5-50)and (5-54) the total battery weight is therefore:

$$
W B=n * W_{k}=\frac{K^{*} T^{*} P B}{V J^{*}} \cdot\left[1-\frac{(P O+P L)^{\star r} e_{9}}{V I^{\star} k}\right]^{-1}
$$

Sincy it is alou known that resistance (re) is inversely proportional to the cell size, and is within e renge of $10-15$ millivolts per $C$ rate of discharge, one has

$$
\begin{equation*}
\mathrm{re} \text { y } \frac{0.015^{* V I} I^{* m}}{P_{B}} \tag{5-56}
\end{equation*}
$$

Substituting (5-56)to(5-55)gives the total cell weight as:

$$
\frac{K^{\star} T^{* P B}}{m^{\star} V k} \quad\left[1-\frac{0.015^{*} m^{*}(P O+P L)}{V k^{\star} P B}\right]^{-1}
$$

The total battery weight is greater than the cell weight by an amount depending on battery packaging, with the difference likely to increase with the number of cells used in series. Modeling the packaging weight by by $n^{0.02}$, the battery weight becomes:

$$
\begin{equation*}
W B=\frac{k^{\star} 1 / P B}{m^{\star} V k} *\left[1-\frac{0.015^{*} m^{\star}(P O+P L)}{V k^{\star} P B}\right]^{-1} * n^{0.02} \tag{5-57}
\end{equation*}
$$

### 5.8. OPTIMIZATION OBJECTIVE FUNCTION

As previously stated, the objective of the optimization is to minimize the weight of the battery-converter system shown in Figure 5-1. The total system consists of the following thrise weight contributors:

- Converte: magnetic and capacitive components
- Battery
- Converter mechanical structure incorporating thermal design


### 5.8.1 CONVERTER COMPONENT WEIGHT

The converter magnetic weight is:

$$
\begin{equation*}
W M=4 * F C * A C * D C * N * A^{0.5}+D I * A * 2 \tag{5-58}
\end{equation*}
$$

where these parameters were all defined in Section 4, with the two terms on the right hand side of $(5-58)$ representing copper weight and iron weight of the inductor, respectively. The converter capacitor weight is:

$$
\begin{equation*}
W C=K C A P * C \tag{5-59}
\end{equation*}
$$

Where KCAP is a constant expressed as grams per microfarad of capacitance for a specified capacitor type ind voltage rating at a prescribed temperature. For this application, foil-tantalum capacitor is tentatively chosen. Taking into consideration the 400 V rating required to provide a proper derating for a $270 V$ working dc voltage, the KCAP for this application is taken as 3 grams /microfarad. From (5-58) and (5-59) the total converter component weight becomes:

$$
\begin{equation*}
W C O N=4 \star F C * A C \star D C \star N \star A A^{0.5}+D I \star A \star Z+K C A P \star C \tag{5-60}
\end{equation*}
$$

### 5.8.2 BAT TERY WEIGHT

The battery weight, WB, was defined in (5-57).

### 5.8.3 CONVERTER MECHANICAL STRUCTURE WEIGHT

For the first order of approximation, the converter mechanice? structure weight, taking thermal design into consideration, is assumed to be directly proportional to the converteroutput power PO and loss PL.

$$
\begin{equation*}
W P=K M E C \star P O+K T H E \star P L \tag{5-61}
\end{equation*}
$$

KMEC and KTHE are proportionality constants. For this application, KMEC is estimated at 5 grams/watt, and KTHE at 10 grams/watt.

### 5.8.4 OPTIMIZATION OBJECTIVE FUNCTION

The objective function "OBJF" is the sum of eqs.(5-57) (5-60) and (5-6)). Analytically,

$$
\begin{align*}
& \text { OBJF = WB + WCON + WP = WT } \\
& =\frac{K^{\star} T^{*} \cdot P B}{m^{*} V k} *\left[1-\frac{0.015 \hbar^{*} m^{\star}(P O+P L)}{V_{k}^{*} F_{B}}\right]^{-1} * n^{0.02} \\
& +4 * F C \star A C \star D C * N^{\star} A^{0.5}+D I * A \star Z+K C A P \star C \\
& \text { + KMEC*PD + KTHE*PL } \tag{5-62}
\end{align*}
$$

Strictly for convenience, the parameters in(5-62)are clariffed again as follows:
$K$ : Proportionality constant between individual cell weight and its ampere-hour inting (42 grams/ampere-hour).

KTHE: Weight constant for thermal design ( 10 grams/watt of loss).
$T$ : Anticipated emergency power utilization time (hours).
PO : Required emergency output power (watts).
PL : Converter losses (watts).
$m$ : Cell capacity reduction when discharging nore than $I C$, where $C$ in amperes is equivalent to the ampere-hours rating of the cell. For a half-hour discharge application, $m=0.9$.

Vk : Individual cell voltage (volts, 1.00 V for a Ni-Cd cell).
FC : Winding pitch factor (assumed to be 2.0).
AC : Inductor winding area (square meters).
$D C$ : Conductor density ( $8900 \mathrm{~kg} / \mathrm{m}^{3}$ for copper, or, $8.9 \times 10^{6} \mathrm{~g} / \mathrm{m}^{3}$ )
$N$ : Number of turns of the inductor winding (dimensionless).
A : Cross-sectional area of inductor core (square meters).
DI : Core density ( $7800 \mathrm{~kg} / \mathrm{m}^{3}$ for iron, or, $7.8 \times 10^{6} \mathrm{~g} / \mathrm{m}^{3}$ )
$Z$ : Mean core length (meters).
KCAP: Weight constant for capacitors (3 grams/microfarad).
C : Output capacitance (microfarads).
KMEC: Weight constant for mechanical structure ( $5.0 \mathrm{grams} /$ watt of converter output power).

Having clarified these parameters, they can be catagorized as follows:

- Specified system requirements: T, PO, m.
- Given celi characteristics: K, VK.
- Assuried constants: FC, DC, DI, KCAP, KMEC.
- Previoosly calculated parameter: C (see eq. (5-44).
- Unknowns : PL, AC, Y, A, Z.

Separating (62) into known and unknown terms, one has:

$$
\begin{equation*}
W T=\left[\text { CONST 1] }+\left[\text { CONST 2]*PL }+4 \star F C \star A C * D C \star N \star A^{0.5}+D I \star A \star Z\right.\right. \tag{5-64}
\end{equation*}
$$

where CONST 1 and CONST 2 are two constants representing, respectively, the quantities in the two square brackets on the right-hand side of (63). It is recalled that PL of (5-63) was expressed as a function of the other unknowns in (5-45). Combining(5-45) and(5-64) and simplifying the resultant equation, one has:

$$
\begin{aligned}
W T= & (\text { CONST } 1+K 3 * \text { CONST 2) } \\
& + \text { CONST } 2 * K I * N \star A 0.5 / A C \\
& +4 \star F C * D C \star A C \star N \star A 0.5+(D I+K 2 \star A \star 2 \star C O N S T 2)
\end{aligned}
$$

where $K 1, K 2$, and $K 3$ were given in (5-46) to (5-49). Equation (5-65) is the objective function to be optimifzed. The unknown in (65) are $N, A, A C$, and $Z$. For simplicity in programming, (PO+PL)/PB in (5-63)is treated as unity.

### 5.9. REVIEN OF OPTIMIZATION CONSTRAINTS

It is recalled that in Section 5.5 there were four constraints.

$$
\begin{align*}
& +\left[\frac{K^{\star} T^{\star n} n^{0.02}}{m^{*} V k}\left(1-\frac{.015^{\star} m^{r} \cdot(P O+P L)}{V k^{*} P B}\right)^{-1}+K T H E\right] * P L  \tag{5-63}\\
& +4 \star F C * A C * D C * N \star A .5+D I * A * Z
\end{align*}
$$

For a given VI and $F$, one constraint concerning the input current ripple resulted in the numerical identification of inductance $L$ in (5-42). Another constraint regarding the output-voltage ripple made possible the design for the capacitance C in(5-44). The two remaining constraints on core window and output ripple are listed below for convenience:

$$
\begin{equation*}
\left(\frac{N^{\star A C}}{\pi^{\star} \hbar W}\right)^{0.5}-\frac{2}{2^{\star \pi}}+\frac{A^{0.5}}{2} \leq 0 \tag{5-42}
\end{equation*}
$$

$N^{*} A \geqslant \frac{L^{*} I P}{B D C}$
(repeat of (5-43))

Here, the known parameters are:
FW: Core window fill factor (assumed 0.35).
$B D C$ : Maximum operating flux density ( 0.35 Weber/meter ${ }^{2}$ ).
L : Inductance in Henries as calculated from the equal sign of (5-42).

VI : Converter input voltage in volts.
PO : Converter output power in watts.
F : Converter switching frequency in Hertz.
IP : Peak inductor current in amperes defined in eq. (5-39).

The unknowns are $N, A, A C$, and $Z$, which are the same ones in (5-65). Notice that the permeabllity of the ifiductor core has yet to be defined. The reason for this seeming remission is that it is really not an independent variable;

$$
\begin{equation*}
\text { Permeability }=U=\frac{L * Z}{N^{2} * A} \tag{5-66}
\end{equation*}
$$

### 5.10. CLOSED-FORM OPTIMIZATION SOLLUTIONS

Concisely stated, for a given set of VI and $F$, the purpose of this optimization is to identify all variables $N, A, A C$, and $Z$ such that eqs. (5-41) and (5-43)can be satisfied, and concurrently the OBJF of eq. (5-65) can be minimized. The identification of these variables numerically defines the particular power loss PL per(5-45)that will result in a minimum emergency power system weight for a given set of VI and F. The calculation can be repeated for other sets of VI and F, from which the global minimum for one particular set of VI and $F$ can be numerically obtained along with the specific PL corresponding to this set. Substituting these values into(5-52) and(5-53)will then yield the optimum battery voltage level VB and the precise minimum battery ampere-hour rating AH.

For simplification purposes, let

$$
\begin{align*}
& K A=\text { CONST } 1+K 3 * \text { CONST } 2  \tag{5-67}\\
& K B=K I * \text { CONST } 2  \tag{5-68}\\
& K C=4 * F C * D C  \tag{5-69}\\
& K D=D I+K 2 * \text { CONST } 2  \tag{5-70}\\
& K E=L * I P / B D C \tag{5-71}
\end{align*}
$$

$$
\begin{align*}
& K H=0.5  \tag{5-74}\\
& x=A^{0.5}, y=N^{0.5}, \quad V=(A C)^{0.5}, z=Z \tag{5-75}
\end{align*}
$$

Then, equation(5-65)oecomes.

$$
\begin{equation*}
W T=K A+K B * \frac{y^{2} * x}{v^{2}}+K C * \cdot x y^{2} v^{2}+K D * x^{2} z \tag{5-76}
\end{equation*}
$$

Equations(5-41)and(5-43)become:

$$
\begin{align*}
& K F * y v-K G * z+K H * x \leqq 0  \tag{5-77}\\
& x^{2} y^{2}-K E \geq 0 \tag{5-78}
\end{align*}
$$

Using the method of Lagrange Multipliers, the optimization function $h$ ( $x, y, z, v$ ) becomes:

$$
h(x, y, z, v)=K A+K \cdot S^{\star} y^{2} x v^{-2}+K C \star x y^{2} v^{2}+K D-x^{2} z-\alpha\left(x^{2} y^{2}-K E\right)-\beta\left(K F \star y v-K G \star z+K H^{\star} x\right)
$$

where $a$ and $B$ are the Lagrange Multipliers.

$$
\frac{\partial h}{\partial x}=K B * y^{2} v^{-2}+K C * y^{2} v^{2}+2 \star K D * x z-2 \star \alpha * x y^{2}-\beta \star K H=0
$$

$$
\frac{\partial h}{\partial y}=2 \star K B * y x v^{-2}+2 \star K C \star x y v^{2}-2 * \alpha * x^{2} y-\beta * K F \star v=0
$$

$$
\begin{equation*}
\frac{\partial h}{\partial z}=K D * x^{2}+B * K G=0 \tag{5-81}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial h}{\partial v}=-2^{\star} K B^{\star} y x v^{-3}+2^{\star} K C^{\star} x y v-A \star K F=0, \quad y \neq 0 \tag{5-82}
\end{equation*}
$$

The six equations, $(5-77)$ to $(5-82)$, can be used to solve for the six variables, $x, y, z, v, a$ ald $B$ :

$$
\begin{equation*}
N=\left(\frac{\pi \star L * I P * F W}{B O C * A C}\right)^{0.5} * S^{-1} \tag{5-83}
\end{equation*}
$$

$$
\begin{equation*}
A=\left(\frac{L^{*} I P * A C}{\pi^{*} B_{B C *}^{*} F W}\right)^{0.5} * S \tag{5-84}
\end{equation*}
$$

$$
\begin{equation*}
Z=2 * \pi *\left(\frac{L * I P * A C}{\hbar * B D C * F W}\right)^{0.25} *\left(\frac{S^{0.5}}{2}+S^{-0.5}\right) \tag{5-85}
\end{equation*}
$$

,

$$
U=2 * \pi *\left(\frac{A C}{\pi * F W}\right)^{0.75} *\left(\frac{B D C}{I P}\right)^{1.25} * L^{-0.25} * S *\left(\frac{S^{0.5}}{2}+S^{-0.5}\right)^{(5-86)}
$$

$$
\begin{align*}
& a=12^{\star} F C \star D C *\left(4^{\star} F C * D C-\frac{D I+K 2 \star C O N S T 2}{F W}\right)  \tag{5-89}\\
& b=C O N S T 2 \star K 1 *\left(\frac{D I+K 2 \star C O N S T 2}{F W}-24 * F C * D C\right)  \tag{5-90}\\
& c=3 *(\mathrm{KI} * \text { CONST } 2)^{2} \tag{5-9}
\end{align*}
$$

Values thus obtained for $N, A, Z$, and $A C$ can be used in eq(5-45) to numerically determine the power loss PL.

### 5.11. APPROACH FOR OVERALL OPTIMIZATION

At this juncture it is perhaps worthwhile to review briefly what has been accomplished. First, one has to realize that the optimization study so far has been predicated by a given set of VI and F. Based on suct a given set, the progression includes the following:

- The numerical identification of inductance $L$ through eq. (5-42).
- The numerical identification of capacitance $C$ througn eq. (5-44).
- The detalled design for inductance $L$ through eqs. (5-83) to (5-91). Detalls include $N, A, Z$, and $A C$.
- The determination of loss PL through eqs. (5-45) to (5-49). Such a PL will give a minimum total system weight.
- The battery voltage VB can be found from eq. (5-52).
- The battery ampere-hour rating AH can be found from eq. (5-53).
- The minimum total system weight is prescribed by eq. (5-64).

It is iterated that the minimu.." weight of (5-64) applies only to the given set of VI and F . Different minimum weights corresponding to different sets of VI and F can be similarly generated. By plotting this weight as a function of $F$, with VI as the varying parameter, a family of curves can be displayed to identify the global minimum for one specific set of VI and
F. The global minimum represents the optimum design for all bettery and converter parameters, and the corresponding battery voltage level can be determined from the specific VI.
5.12. COMPUTER PROGRAM FOR DESIGN OPTIMIZATION CALCULATION

The foregoing design optimization equations are transcribed into a computer program for numerical processing. The program is given as Appendix K. A total of 28 parameters are given to the program in statements 2200 to 2230. Their corresponding numerical values are provided in statements 2000 to 2030. For a cost-effective program,no tolerances are assumed for these parameter's as was indicated in statements 2100 to 2130 . The program is designated as "VCTOL" in 2300 . The output data of the design optimization program, as specified in 2400. include the following:
$L \quad:$ inductance in henries
C : capacitance in farads
RC : equivalent series resistance of $C$ in ohms
IP : peak inductor current in amperes
$N$ : number of turns of $L$
A : cross-section area of the core for $L$ in meter ${ }^{2}$
Z : mean core length in meter
$U$ : core permeabllity in gauss/oersted
$A C$ : winding area per turn in meter ${ }^{2}$
PL : total converter loss in watts
PLC : copper loss of inductor in watts
PLI : tron loss of inductor in watts
PM : total inductor loss in watts
PT : total loss of transistor switch in watts
PD : total diode loss in watts
PC : total capacitor loss in watts

```
P : Other converter losses in watts
NB : number of cells in series
VB : minimum battery voltage near end of discharge
        in volts
WM : inductor waight in grams
WP : converter packaging weight in grams (excluding
        power components)
WC : converter component and packaging weight in grams
WB : battery welght in grams
WCON : converter weight in grams
WT : total power system weight in grams
AH : ampere-hour capacity required of the battery
```

From statements 3000 to 3064, the program essentially reproduces the various equations presented in the text, with the following corresponc.ences:

| Statement in Program | Equation in Text |
| :---: | :---: |
| 3000 | $5-42$ |
| 3002 | $5-44$ |
| 3004 | RC*C=G |
| 3006 | $5-49$ |
| 3008 | $5-46$ |
| 3010 | $5-43$ |
| 3012 | $5-47$ |
| $3014,3016,3018$ | $5-48$ |

3020
3022
3024
3026
3028
3030
3032
3034
3036
3038
3040
3042
3044
3045
3046
:047
3048
3049
3050
3052
3053
3054
3056. 3058

3060
3062

```
CONST I of (5-04)
CONST 2 of (5-64)
    5-89
    5-90
    5-91
    5-87
    5-88
    5-39
    5-83
    5-84
    5-85
    5-66
    5-45
    5-17
    5-50, 5-56
    5-19
    5-52
                                    5-18,5-19 5-20, 5-21, 5-22
    5-58
    5-59
    5-65
    5-23
    5-29
    5-32
    5-33
```

The numerical inputs to the computer are given as follows:

| $v i$ | AO | 150 | 0 | TiR | 84 | .10E-O. | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| F | al | 5000 | 0 | 10f | 95 | - 95 -06 | 0 |
| vo | A2 | 270 | 0 | RHO | 80 | .17E8E-07 | 0 |
| PO | AJ | 3000 | 0 | Fic | 4 | 2 | 0 |
| $\dagger$ | Ad | . $\mathrm{JE}+00$ | 0 | F4 | 88 | . $35 \mathrm{E}+00$ | , |
| 0 | A5 | . 1884 -04 | 0 | 8DC | 89 | . 3EE+00 | 0 |
| Jac | As | 4 | 0 | K | co | 42 | 0 |
| RI | A7 | 6 | 0 | $\cdots$ | Cl | .9E100 | 0 |
| US | A8 | 1.8 | 0 | UK | c. 2 | 1 | 0 |
| UBE | :9 | 1.2 | 0 | kCAF | c3 | 3 | 0 |
| vi | 10 | 1.2 | 0 | hinc | c 4 | 5 | 0 |
| is | 11 | 3.1416 | 0 | OC | C5 | 6700000 | 6 |
| TSR | 12 | . $35 \mathrm{E}-\mathrm{D6}$ | 0 | 31 | C6 | ;30i003 | 0 |
| T85 | 33 | . 35E-06 | 0 | kthe | c7 | 10 | 6 |

As previously outlined, a set of VI and F will be assigned for each computer run, and the corresponding printout represents the optimum-weight design for that particular set of VI and $F$.

### 5.13. COMPUTER PRINTOUTS

A sample printout is shown in Figure 5-5. The aforementioned twenty-six parameters are printed under the heading of "X-AXIS", and their numerical values under "OUTPUT". Since tolerances have not been given to the program, the number " $0^{\circ}$ will appear under "+ TOL" and "- TOL", and the lower limit "- LIM" and upper limit "+ LIM" will exhibit identical numerical values as those displayed under "OUTPUT".

A summary of arguments and tolerances accompanies each printout. For example, in the sample printout the output voltage VO is designated A2, and has a specified value of 270 V with zero tolerance. The particular run is based on an input voltage VI of 150V, switching frequency $F$ of 10 kHz , output power PO of 3000 W , and power utilization time $T$ of 0.5 hour. (See underlined portion of Figure 5-5). The total optimum system weight, WT, is calculated to be 103.9 kilograms.

Computer printouts for other sets of $V \mathrm{I}, \mathrm{F}, \mathrm{PO}$, and F .

$$
\begin{aligned}
& V I=25,50,100,150,200 \mathrm{~V} \\
& F=5,10,20,30,40,50,100 \mathrm{kHz} \text { for each } \mathrm{VI} \\
& P O=3000 \mathrm{~W} \\
& T=0.5 \mathrm{hr} .
\end{aligned}
$$

The calculated results for total system weight $W$ T are presented in Figure 5-6. A family of curves showing weight versus frequency, with the input voltage as the varying index, are plotted. These results will be discussed in tie next section.

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## 5-14. DISCUSSION OF CALCULATED RESULTS

The following observations can be made based on Figure 5-6:

1. The weight vs. frequency curves generally exhibit $U$ shape characteristics, with the shape more pronounced at lower input voltages. Within quite a wide frequency range, from 20 to 60 kHz , the total system weight is relatively constant for a given input voltage.
2. For each frequency, the weight reduces with an increase of input voltage. The weight reduction is most pronounced for voltage increments at lower VI's. The reduction becomes diminished for voltage increment at higher VI's. For example, at 30 kHz , 16 kg reduction is reailized when VI is increased from 25 to 50 V . The corresponding reduction is only 4.5 kg for a 100 -to-200V increase.

These observations are discussed as follows:

## THE U-SHAPE WEIGHT VERSUS FREQUENCY

The $U$-shape weight versus frequency can be understood by plotting the corresponding loss versus frequency and weight distribution characteristics for all power components including the inductor, capacitor, transistor, and diode, which are part of the computer printouts. These characteristics are shown in Figures 5-7 and 5-8. It is clear from Figure 5-7 the loss versus frequency curve also exhibits the U-shape. The diminishing total loss at lower frequencies is caused by the copper loss of the inductor, which decreases as irequency increases due to the smaller inductor size for a given source-current ripple requirement. The increasing

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total loss at higher frequencies is caused by the higher switching losses associated with the transistor and diode switches. The opposing loss trends for the inductor and the semiconductor switches as a function of frequency leave a minimum total los: in the 10 30 kHz frequency range. The weight profiles shown in Figure $5-8$ is directly influenced by the corresponding losses. A monotonically reducing inductor weight as a function of frequency is argumented by increasing weights from both the battery and the converter packaging, thus explaining the aforementioned U-shape. Since the inductor losses and the semiconductor switching losses are more pronounced when the input voltage is low and the corresponding rms and peak current in the inductor as well as the semiconductors are high, the U-shape is also more pronounced aí lower input voltages. The opposing trends of weight versus frequency for the inductor and the battery plus converter packaging leave a relatively constant total system weight in the middie frequency range, from 20 to 60 kHz.

## WEIGHT REDUCTION WITH INPUT VOLTAGE

The converter loss and battery internal loss are much higher at lower input voltages. Consequently, greater weight reduction can be realized in all system components including magnetics, capacitors, battery, and converter packaging when a given voltage increment is added to a lower input voltage. As the loss-related system weight diminishes, the battery and converter packaging weights, which are constant functions of the output power, become dominant in their weight contributions to the system. Consequently, little weight saving can be realized by increasing VI when the efficiency gain is no longer an important factor. This fact is amply substantiated by curves exhibited in figure 5-5, where the weight reduction from VI= 100 V to $\mathrm{VI}=200 \mathrm{~V}$ is rather meager.

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## 5-15. IMPACT OF DIFFERENT, CORE MATERIAL ON TOTAL SYSTEM WEIGHT

In the anaiysis presented so far, the inductor core is assumed to be made of the molypermalloy powder material to achieve a minimum core loss, with an operating flux-density limit of 0.35 weber/meter ${ }^{2}$. However, in view of the high ration of copper loss PLC to core loss PLI, it becomes apparent that a different core material with a higher operating flux density level will result in saving of total system weight even though the attendent core loss will be higher. A Cut-C core of Deltamax (orthonol) laminations, with a conservative fluxdensity limit of 1.2 wever/meter ${ }^{2}$ is selected to replace the power core in the computer run to assess its impact on the total system weight. This change is accomplished in the computer program through the following program edition:

```
-E,2030
```



```
    2030 DATA 1.2.42..9, 1.3.5.a.9Eo.i.CEo.10
    E,3012
    SÛi2 ro=.165*A1".00*(AZ-A0)*AO/YSIAZ;YS
    3012 YS=4.84*A1".26:(A2-NO)*AO/Y3/AZ/Y5
< E.304%
    3047 X1=2* X04.089*A1*.6*29*KO*A1
```



The flux density BDC in statement 2030 is changed from 0.35 to 1.20. The core-loss description in statements 3012 and 3049 are also changed to reflect the core-loss profile of the new material. fiuns were executed. A weight versus frequency plot is given in Figure 5-9. Comparing the curve for a given VI in Figure 5-9 and Figure 5-6, it is clear that a 10 kg weight saving is accomplished for VI=25V when BDC is increased from $0.35 \mathrm{w} / \mathrm{m}^{2}$ to $1.2 \mathrm{w} / \mathrm{m}^{2}$ that the $U$-shape of weight versus frequency is not as pronounced at the low frequency end of Figure 5-9 as that of Figure 5-6. For a given VI, the inductor weight saving as a result of a higher frequency is less in Figure 5-9 in relation to that in Figure 5-6.


## 5-16. Justification For All Input Parameters

Justifications for numerical values of all input parameters to the computer are given in Appendix $K$. These parametrus are defined in the previous sections.

5-17. CONCLUSION
From analysis presented in this report, the following design-related conclusions can be made regarding the 'VSTOL emergency power system:

1) As expected, the total system weight reduces with the converter input voltage. However, a level of diminishing return is soon reached for VIz 100 V .
2) For a given watt-hour rating, connecting more cells in series to effect a higher battery voltage is more costly than connecting fewer series cells. In view of the significant weight saving that can be realized by raising VI from 25 V to 30 V , and the rather meager weight saving that is realizable by using a higher-than-loov input voltage VI, it is recommended that the battery voltage VB be set wtthin a 50-100V range.
3) Depending on the required output power PO, which determines the power-switch current for a given VI, a 25 V battery may be used in smaller power applications, e.g., POs1000 W.
4) With the present availability of power switches and magnetic core materials, the total system weight WT for a given VI stays nearly constant within a wide frequency range. The range covers from 20 to 60 kHz in Figure $5-6$ and 15 to 40 kHz in Figure 5-9. A recommendable operating frequency is in the vicinity of 30 kHz .
5) For a core with $B O C=1.2$ weber/meter ${ }^{2}$, the opt 1 mum system weight in kllograms can be read from Flyure $5-9$ as follows for $\mathrm{PO}=3 \mathrm{~km}$ and $\mathrm{T}=0.5 \mathrm{Hr}$ :

Switching Frequency (kHz)
Weight (kg)

|  | $V I=25 \mathrm{~V}$ | $\underline{50 \mathrm{~V}}$ | $\underline{100 \mathrm{~V}}$ | $\underline{150 V}$ |
| :--- | :--- | :--- | :--- | :--- |
| 5 | 127 kg | 113 kg | 105 kg | 102 kg |
| 10 | 121 | 109 | 103 | 101 |
| 20 | 118 | 108 | 103 | 101 |
| 30 | 119 | 108 | 103 | 101 |
| 40 | 121 | 109 | 103 | 101 |
| $5 u$ | 123 | 110 | 103 | 101 |
| 100 | $1 \vdots$ | 115 | 106 | 103 |

6) The use of higher flux-density core niaterial reduces the inductor copper loss by an amount much greater than the attendant increase in core loss, thus reducing the total system weight.
7) While the impact of core matertal is not to be overlooked, the major weight contributor in the system is the battery. Except at very low frequencies ( 5 kHz ), the battery generally is responstble for more than 80\% of the total system weight.

## 6. CONCLUSIONS ANO RECOMMENDATIONS

The Modeling and Analysis of Power Processing Systems (MAPPS) Project has developed the mathematical models and computer software program to perform the calculation for Buck, Boost, and Buck-Boost DC-DC Converters in order to determine:

- Performance Analysts
- Design Optimization

A computer-aided discrete time domain modeling and analysis technique for Performance Analysis has been presented which is applicable to all types of switching regulators using any type; of duty cycle controllers and operating with continuous, as well as discontinuous, inductor current. State space techniques are employed to characterize coverters exactly by the nonlinear discrete time domain equations in vector forms. Newton's iteration method is employed to solve for the exact equilibrium state of the converter. The system is then linearized abnut its equilibrium state to arrive at a linear discrete time model. The stability nature and transient responses are studied by examining the eigenvalues of the linear system. Changes in eigenvalues due to system parameter changes can be plotted in the complex z-plane yielding an excellent design tool very similar to conventional root-locus plots. The analysis is also extended to determining the frequency related performance characteristics such as the closed loop input-to-output transfer function used to determine the audiosusceptibility of the converter. The modeling and analysis approach makes extensive use of the digital computer as an analytical tool, replacing highly complex and tedious analyses by numerical method and making automation in power converter design and analysis possible.

## 6. CONCLUSIONS AND RECOMMENDATIONS (Cont'd)

In addition to its particular utility at analyzing high-frequency control-loop related phenomena, the analysis also serves as a useful design tool which provides design guidelines for such important control parameters as the dc loop gain, the ac loop gain, and the R-C compensation network of a two loop converter to optimize its transient response and to stabllize the system.

To those working with switching regulators, converters, and systems comprised of these equipment, certain design and analysis intricacies invariably make themselves felt throughout the equipment and system destgn and development stages. Empirical and intuitive reliances of ten intercede with the designer's desire to be "more scientific" and his commitment of belng "on schedule". Handicapped by a general lack of established modiling, analysis, design, and optimization tools, it has not been uncommon for a power processing designer to fulfill very little of the desire and/or the commitment.

The cost and schedule plights that most equipment and system designers find themselves in have to do with at least one of the following entities: power circuit weight/efficiency, controlrelated performance requirements, and trial-and-error power and control design iterations. While power processing as a technology has reached the level of sophistication where the modeling, analysis, design, and optimization of these entities should have been established, a survey of literatures conducted at the initiation of the MAPPS program had proved the contrary. In addition, the recent evoiving trend of higher power and equipment standardizatior has further heightened the need for analytically-based design and optimization.

In this regard, the program has accomplished the following objectives:

- The methodologies of power processing modelini, analysis, design, and optimization, are all established.

6. CONCLUSIONS AND RECOMMENDATIONS (Cont'd)

- Application-oriented analysis, design, and optimization subprograms for power-circuit design, control circuit design. and control performance analysis are becoming avallable.
- Cost-effective system configuration study and system disturbance propagation are now feasible.

Being government sponsored, all softwares ise available without proprietary complications.

Continued MAPPS effort will aim at the following goals:

- Analyze periormance for commonly-used power processing equipment and selected systems.
- Detailed power circuit design optimization to meet given powerrelated performance requirements for most commonly-used power circuit configurazions.
- Standardize control-circuit design to meet control-related performance requirements.
- Provide cost-effective tools for the identification of optinum system configurations and system failure-mode analysis.

The following is a list of basic tasks for future work:

- Application Handbook for Performance Analysis.
- Application Handbook for Design Optimization.
- Continued Current Injection Multiloop Control Modeling.
(a) Single Power Stage.
(b) Parallel Modular Power Stages.
- Continued Power Subsystem Optimization Techniques.
- Development of Performance and Design Optimization for Series Resonant Inverter Power.


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[^0]:    $x$ : Completed
    $M A$ : Configuration Not Recommended
    0 : Not yet performed

