NASA Contractor Report 3498

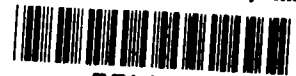# Numerical Studies of Laminar and Turbulent Drag Reduction

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NASA

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Numerical Studies of Laminar and Turbulent Drag Reduction

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SUMMARY

Computer codes have been developed to analyze flow over wave shaped surfaces. The results obtained by these codes for laminar flow over wavy surfaces agree very well with available experimental results. The computer simulations have also been applied to turbulent flow over wavy surfaces: here, the conclusions based on comparisons with experiments are that up to wave amplitudes that cause incipient flow separation, a low order turbulent closure is adequate to predict the physical quantities of engineering interest. Results of computer simulations for flow over unsymmetric waveforms are also presented. The computer simulations have been found to be cost effective (in comparison with wind tunnel experiments), so that an intense effort at computer simulations of flow over a large variety of unsymmetrical waveforms has been initiated. Preliminary results of these simulations are given in this report. The effect of propagating, traveling waves on the surface has also been studied using a modified version of the algorithms and results are reported.

Finally, this report describes work in progress in three-dimensional turbulent flow simulations. A brief discussion of the development of parallel flow-compressible stability analysis program (COSAL) is also given.

SECTION I

Introduction

This report presents results of computer simulations of flow over wavy surfaces. The purpose of these simulations is to develop capabilities that complement ongoing experimental investigations of turbulent flow over wavy walls at NASA/Langley Research Center. The primary motive behind these investigations is to identify surface geometries that have lower total drag than a smooth flat surface under the same flow conditions.

The problem of flow over wavy surfaces has been a major area of research in fluid mechanics for over a century. There have been a number of experiments of turbulent flow over wavy surfaces and a few experiments of laminar flow over rigid waves. For purposes of this report we cite only the experiments of Kendall[1] and Sigal[2] as representative turbulent flow experiments and the experiments of Kachanov, Kozlov, Kotjolkin, Levchenko, and Rudintsky[3] as the representative laminar flow experiment. (Each of these experiments have much of experimental data available for comparison with theoretical prediction; furthermore, the uncertainty of experimental errors is clearly bracketed in these experiments). There have also been a large number of theoretical examinations of the wavy wall flow experiments. For purposes of discussion we shall categorize these theoretical works into two main categories, as discussed below.

(i) Asymptotic Analyses

Here, the wavy wall flow is considered a perturbation of an undisturbed flat plate flow and the velocity profile over the wave is written as

$$U(\xi,\eta) = U(\eta) + f(\eta)e^{ik\xi} \tag{1}$$

where U is the velocity over the wave at a point (ξ,η); $U(\eta)$ is the undisturbed flat plate profile, f is the disturbance velocity and k denotes the wave number.

Miles[4] and Benjamin[5] developed theoretical calculations

for flow over waves whose nondimensional amplitudes Ka<<1, using a high Reynolds number approximation. There is a large discrepancy between wall pressure and skin friction measurements and the predictions for wavy walls of the Miles-Benjamin theory with k ≃ 0.1. A number of extensions of the Miles-Benjamin theory have been made, but, in general, the experimental measurements are still not satisfactorily accounted for.

Some of the more exotic works in this area include the effect on turbulence due to the presence of the wall through pressure gradient/curvature effects (6,7,8) and in some cases good agreement with specific experiments have been obtained.

(ii) Navier-Stokes Simulations

Only a limited number of computer simulations have been published to date. Recently, Gent and Taylor[9], Markatos[10] and Chalikov[11] have developed simulation programs. The work of Gent and Taylor[9] and Chalikov[11] are similar in philosophy. Here the wavy surface is approximated through an analytic conformal map and the computational domain has an upper lid. The steady state Navier-Stokes equations are solved (using a closure scheme for turbulent flows) through an iteration scheme. The boundary condition appropriate for this problem is an imposed stress boundary condition at the upper and lower walls. This presupposes a constant stress layer (log region of the velocity profile). The implication of these assumptions is that a solution is obtained that is not really representative of the wavy wall flow in a wind tunnel. Markatos[10] uses periodic boundary conditions in the flow direction, an upper computational lid, an edge worth expansion to map an approximating wave surface to a flat computational domain and solves the steady state equations in finite difference form. His pressure predictions for channel flow with lower wavy surface are unreliable due mainly to numerical difficulties and his paper does not provide any firm comparisons with experimental measurements.

The present work is an outgrowth of the need for a computational program that can provide reasonable simulations of wavy

wall experiments. The basis of the present work has been pre-
sented in detail in references 12 and 13. The Navier-Stokes
codes developed here differ from the works cited above in several
key areas:

(i) A fast numerical conformal mapping technique is used that
accurately maps the domain bounded by wave surface to a semi-
infinite rectangle. The mapping is accurate up to Ka = 1 and
can be done in N ℓn(N) operations where N is the number of
discretisation points in the wave direction.

(ii) Spectral methods are used to obtain high accuracy while
using relatively few points for approximating the flow domain.

(iii) The present solutions have an accuracy (Δt^2, Δx^N) (where
N is the number of points in the spatial direction).

(iv) The splitting method adopted in the code permits large
time steps (Courant number >1) thereby enabling accurate simu-
lation at low cost.

Developments of the periodic code (Code I of Ref. 12) and
inflow-outflow code (Code II) were carried out during the con-
tract period. Key comparisons were performed with available
experimental data. Results for laminar flows were presented
in references 12 and 13. The present work describes (see Sec.
II) the results of turbulent flow simulations and the details
of the closure models used for simulating the turbulence.

During the progress of this work some added merits of the
periodic code in comparison with the inflow-outflow code were
noted. These aspects are discussed in Sec. III.

A primary motive of the present computer simulations is
to identify surface geometries that have lower total drag than
a smooth plate. Therefore work was directed towards the problem
of waveforms with complex shapes (asymmetric waves, groove shapes,
etc.). Some results of these investigations are presented in
Sec. IV.

The effect of a propagating waveform on the surface is
described in Sec. V along with some results.

Through funding provided under the present contract, three-

dimensional turbulent shear flows have been studied. These investigations have uncovered some new physics of transitional and turbulent flows as described briefly in Sec. VI.

Finally, technical and consulting support has been provided to NASA/LaRC personnel in developing (i) the stability code (COSAL) for compressible boundary layer flows and (ii) analysis of compliant wall geometries. A brief description of tasks that were performed is given in Sec. VIII.

Sec. VIII deals with conclusions and suggestions for future areas of research in this exciting field.

SECTION II

Turbulent Flow Simulations

The time average Navier-Stokes equations for turbulent flow over wavy surface can be written as

$$
\left.
\begin{aligned}
U_{i,i} &= 0 \\
U_{i,t} + U_j\, U_{i,j} &= -p_{,i} + \tau_{ij,j} \\
\tau_{ij} &= \nu(U_{i,j} + U_{j,i}) - \overline{U'_i\, U'_j}
\end{aligned}
\right]
\tag{2}
$$

Turbulent closure is required to evaluate the Reynolds stress term $-\overline{U'_i\, U'_j}$. There are a large number of closure models that have been suggested to describe turbulence. Unfortunately the state-of-the-art of turbulence modeling can only be described at best as chaotic. For simple boundary layer type flows, (i.e., flows without geometrical complexities, nonreacting flows, nonseparated flows, flows without too much imposed pressure gradient, etc.) the various closure models give a degree of accuracy comparable to the known data base. However, as one enters a new problem where the model's ability has not been tested, questionable results may emerge.

In the majority of complex flows, recent published calculations (14,15) indicate that lowest order closure models seem to do as well or better in predicting the flow than higher order models.

The turbulent closure scheme adopted in this work is based on the concept of mixing length models which is described below.

<u>Mixing length models</u>. - Here, the Reynolds stress $-\overline{U'_i\, U'_j}$ is determined based on the mean-velocity gradients in the flow. The concept of mixing length is originally due to Prandtl. The mixing length models are also called local equilibrium models because they are applicable only strictly for flows which are locally in equilibrium. For a thin shear layer the turbulent kinetic energy equation reduces (under the assumption that transport terms are negligible) to

$$
-\overline{U'_1\, U'_2}\ \frac{\partial U_1}{\partial X_2} = -\varepsilon
\tag{3}
$$

where 'ε' is the turbulent dissipation rate. Furthermore, if a length scale can be defined by,

$$L = (-\overline{U'_1 U'_2})^{3/2}/\varepsilon \tag{4}$$

we get, on substituting and rearranging in (3)

$$-\overline{U'_1 U'_2} = L^2 \left(\frac{\partial U_1}{\partial X_2}\right)^2 \tag{5}$$

which is the Prandtl mixing length formula. In the log-region of the flow, (i.e., near a solid surface) where, local equilibrium is a good approximation,

$$L = \kappa y \simeq 0.41y \quad , \tag{6}$$

Elsewhere, the flow is not in local equilibrium and L is not quite equal to the mixing length 'ℓ'; the mixing length in these regions is defined as,

$$-\overline{U'_1 U'_2} = \ell^2 \left(\frac{\partial U_1}{\partial X_2}\right) \left|\frac{\partial U_1}{\partial X_2}\right| \tag{7}$$

The sign of the Reynolds stress thus depends on the sign of the velocity gradient $\frac{\partial U_1}{\partial X_2}$. The usefulness of the mixing length concepts is limited in view of the following.

(i) For some complex flows $-\overline{U'_1 U'_2}$ and $\frac{\partial U_1}{\partial X_2}$ may have opposite sign (e.g., near the velocity peak in a wall jet).

(ii) Where $\frac{\partial U_1}{\partial X_2}$ is very small, the transport terms neglected in arriving at (3) are no longer small compared to mean flow generation terms such as energy production rate $-\overline{U'_1 U'_2} \frac{\partial U_1}{\partial X_2}$.

The mixing length formula has the advantage over other local equilibrium methods in that the reasons for its short-comings and inaccuracies may be easily assessed.

Eddy viscosity. - The concept of eddy viscosity, in the simplest form, is closely related to the mixing length

$$-\overline{U'_1 U'_2} = \nu_t \frac{\partial U_1}{\partial X_2} \tag{8}$$

For a thin shear boundary layer,

$$\nu_t = \ell^2 \left| \frac{\partial U_1}{\partial X_2} \right| \tag{9}$$

The eddy viscosity concept can be modified for two-dimensional flows as

$$-\overline{U'_i U'_j} = \nu_t \left(\frac{\partial U_\ell}{\partial X_m} + \frac{\partial U_m}{\partial X_\ell} \right) - \frac{2}{3} \delta_{ij} k \tag{10}$$

where δ_{ij} is the Kronecker delta and k is the turbulent kinetic energy. In the form described above $\nu_t = \nu_{t,ij\ell m}$ i.e., a fourth order tensor. However it is usual to assume $i=\ell$, $j=m$ making ν_t a second order tensor.

The simplest form for the eddy viscosity, as given by (9), has been used in this work in predicting the wavy wall flow through zero equation models. The algebraic mixing length formulas used in this report are

(i) Simple model incorporating van Driest damping

$$\frac{\ell}{\delta} = \left(\frac{\ell}{\delta} \right)_{max} \tanh \left[\frac{\kappa}{(\ell/\delta)_{max}} (y/\delta) \right] (1 - \exp(\frac{-y}{A}) \tag{11}$$

where δ is the boundary layer thickness, A is the van Driest damping coefficient (effective sublayer thickness) and $(\ell/\delta)_{max} = 0.09$ (based on success in flat plate boundary layers). For flat plate flows with no imposed pressure gradients the nondimensional effective sublayer thickness A^+ ($A = A^+\nu/u_\tau$) is assumed to be

$$A^+ = 26 . \tag{12}$$

For flows with imposed pressure gradients in boundary layers formulae have been presented of the form

$$\left. \begin{array}{l} A^+ = f(p^+) \\ = g(\beta) \end{array} \right] \tag{13}$$

where both p^+ and β are the pressure gradient parameters $\left[p^+ \right.$ is nondimensionalized in wall units and β in outer units; i.e., $p^+ = \frac{\nu}{\rho U_\tau^3} \frac{dP}{dX_1}$; $\beta = \delta^*/\tau_\omega (dP/dX) \left. \right]$

Balasubramanian et al. (16) and Cary, Weinstein and Bushnell (17) have presented turbulence modeling for wavy walls based on

modification to A^+, k and ℓ/δ by pressure gradient curvature and nonequilibrium effects. The approach adopted in this report is to note that neither p^+ nor β is an appropriate parameter for wavy wall flows. The pressure gradients arising on a wavy surface have large variations extending over a disturbance height $= \frac{\lambda}{2\pi}$ (i.e., height where $p = 0.36 \, P_w$); when the ratio of boundary layer thickness to the wavelength $\delta/\lambda \leq O(1)$, the disturbance height extends over the entire boundary layer. Thus, data obtained on flows subjected to imposed pressure gradients can no longer be applied ad hoc, to wavy wall flows. Similar arguments serve to discard inclusion of curvature effects on mixing length (such as the Bradshaw-Ferris-Atwell model.) Thus the results on zero equation modeling that are presented elsewhere use the model (11) with

$$(\ell/\delta)_{max} = 0.09, \quad = 0.41; \quad A^+ = 26 \ . \tag{14}$$

The intermittent behavior of the boundary layer outer flow has led to measurements and correlation of the intermittency factor. It has been an accepted practice to introduce these intermittency factors while computing the eddy viscosity. Elsewhere, the effect of intermittency has been studied and reported.

(ii) Cebeci-Smith model.

In the inner region,

$$0 \leq y \leq y_c \quad \nu_t = \ell^2 \, (\frac{\partial U_1}{\partial X_2})$$

with $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (15)

$$\ell = k \, y \, (1 - \exp(^{-y/A}))$$

and in the outer layer

$$y_c \leq y \leq \delta \quad \nu_t = 0.0168 \, U_e \delta^*$$

The effect of the imposed pressure gradient is included in A, through the relation

$$A^+ = 26/[1 - 11.8 \, p^+]^{\frac{1}{2}} \tag{16}$$

The reason for testing the Cebeci-Smith model on the wavy wall problem was to find whether it had any added merits over the simple model.

(iii) Two-equation turbulent transport models

As can be noted from equation (10) the eddy viscosity is the product of a length scale and a velocity scale. The typical velocity scale is $k^{\frac{1}{2}}$ as a means for incorporating the energy containing eddies. A range of eddy viscosity transport models exist for which

$$\nu_t = C_\mu \, k^{\frac{1}{2}} \, \ell \qquad (17)$$

A measure of the energy transfer to the small scale eddies is $k^{3/2}/\ell$ defined as the dissipation rate 'ϵ'. Thus,

$$\nu_t = C_\mu \, k^2/\epsilon \qquad (18)$$

Jones and Launder (18) have proposed a two equation transport model for k and ϵ, as

$$\frac{Dk}{Dt} = \frac{\partial}{\partial X_2}\left[(\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial X_2}\right] + \nu_t \left(\frac{\partial U_1}{\partial X_2}\right)^2 - \epsilon - 2\nu \left(\frac{\partial k^{\frac{1}{2}}}{\partial X_2}\right)^2 \qquad (19)$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial X_2}\left[(\nu + \frac{\nu_T}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial X_2}\right] + C_1 \frac{\epsilon}{k}\nu_T \left(\frac{\partial U_1}{\partial X_2}\right)^2 - C_2 \frac{\epsilon^2}{k}$$

$$+ 2\nu \, \nu_T \left(\frac{\partial^2 U_1}{\partial X_2^2}\right)$$

The various constants C_1, C_2, C_μ, σ_k, σ_ϵ appearing in these equations as suggested by Jones and Launder are

$$C_\mu = 0.09, \ C_1 = 1.55, \ C_2 = 2.0, \ \sigma_k = 1.0, \ \sigma_\epsilon = 1.3 \qquad (20)$$

The method as suggested by Jones and Launder was applied to the wavy wall flow problem by the authors. It was found that the effective time step that can be used for solving the Navier-Stokes equation in conjunction with these transport equations was excessively restrictive. Chien (19) suggested a suitable modification of equation (19) such as

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \, [1 - \exp(-C_3 \, y^+) \qquad (21)$$

with a suggested value of $C_3 = 0.0115$ and replacing $2\nu \, \nu_T \left(\frac{\partial^2 U_1}{\partial X_2^2}\right)$

by $-2\nu\, \dfrac{\varepsilon}{y^2}\, \exp(-C_4\, y^+)$ where C_4 was suggested to be $C_4 = 0.5$.
Furthermore, the term $2\nu\,(\dfrac{\partial k^{\frac{1}{2}}}{\partial X_2})^2$ which appears in the turbulent
kinetic energy equation to incorporate wall effects was to be
replaced by $\dfrac{2\nu k}{y^2}$ from arguments that $k \sim y^2$ near the wall.
Since Chien was successful in analyzing simple flows with his
modified J-L model it was thought best to consider this form of
the K-ε model on the wavy wall flow.

Results and Comparison with Experiments

Both the zero equation and two equation models were tested
for flat plate flows in order to assess, for example, the validity
of various modeling constants. In Fig. (1) and (2) the skin
friction distribution over a flat plate is presented. Fig. (1)
shows the results for the mixing-length model. It was observed
that an intermittency function needs to be included in order to
adjust the level of C_f properly. The results were obtained using
the periodic code (Code I) where the turbulence modeling was
included in the code. Fig. (2) shows the result of calculations
using the Chien's model. Here there is no need for use of
intermittency functions for eddy viscosity. However the suggested
value of $C_3 = 0.0115$, was found to be unsatisfactory. Computer
simulations with various values of C_3 indicated that $C_3 = 0.01$
gave the best results where C_f was compared to the Spalding-Chi
chart. Consequently $C_3 = 0.01$ was used in all subsequent calculations.

For wavy walls, extensive comparisons can be conducted using
the present code. The experiment of Sigal (2) was chosen for
comparison purposes. In Fig. (3) the distribution of the pressure
coefficient is compared with experimental measurements. It
should be observed that the pressure prediction using the various
turbulence models is in excellent agreement with data. In Fig. 4
the skin friction distribution at the wall is compared with
measurement. The two layer Cebeci-Smith model seems to give
the poorest agreement with measurements. The two equation model
and the mixing length model yield about the same agreement.
The slight disagreement between calculations and measurements can
be ascribed to various factors such as, (i) the starting profile

used had boundary layer thickness slightly different than that of the experiment. While the experimental data were obtained with a tripped boundary layer, the predictions were based on profiles which had to be refined by evolving the initial profile through a few time steps. Thus, a mismatch in the thickness of the profiles may be possible. (ii) The experimental measurements were made using a preston tube where a law of the wall type analysis had to be invoked. Calibration of skin friction was done in a zero pressure gradient field and the experimental results may indeed have some errors of this nature. (iii) It is also possible that the modeling of turbulence that was used does not adequately represent the situation for wavy walls of amplitude ka $\approx$ O(0.2) very well.

In Fig. 5 the static pressure distribution normal to the wave is compared with the experimental predictions. Again, it is clear that the simulation reproduces faithfully the experimental measurements.

SECTION III

Philosophy of Periodic Versus Inflow-Outflow Code

In general, it is believed that periodic boundary conditions are appropriate only for flow conditions where nonparallel effects are negligible. A common flow situation is that of high Reynolds number flow in which boundary layer growth is negligible. Therefore a large number of computational algorithms use periodic boundary conditions.

It appears that the periodic code is actually an effective tool for studying a large class of problems. In order to avoid stress boundary conditions at the upper lid, computations were allowed to proceed up to ∞ in the normal direction. Under these conditions a truly unsteady periodic code represents the physics of flow as well as any inflow-outflow code.

Experience with simulations using the periodic code indicates that for a flat plate in both laminar and turbulent flow, computation time in the periodic analysis should be identified with downstream locations for inflow-outflow analysis. So, the periodic code gives the solution of the flow at an x location corresponding to $x \simeq U_\infty t$, where x is the downstream location from the point of starting calculation.

Thus, when one is interested in performing a computation where the downstream computational distance from the inflow is large, it is easy to see why a periodic code can be more economical to use than an inflow-outflow code. For the latter it would be necessary to use a larger number of grid points in the downstream direction, thereby increasing the computational effort. Current comparisons between inflow-outflow and periodic code indicate the great advantage of the periodic code; consequently, all calculations on disturbed boundary layers were performed using the periodic code.

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SECTION IV

Results for Nonsymmetric Waveforms

Using the computer algorithm developed herein a number of
waveforms were analyzed for purposes of drag reduction studies.
Table I summarizes the results of some of these runs and com-
pares the results with the unpublished experimental measurements
of Weinstein, L. M. and Cary, Jr., A. M. of NASA/Langley
Research Center.

SECTION V

Effect of a Propagating Surface Wave

The laminar and turbulent codes can be suitably modified to study the effect of propagating a traveling waveform on the boundary. The effect of propagation velocity on the coefficient of pressure for a sine wave and the corresponding skin friction values are shown in Table II. It is seen that a propagation speed in the direction of flow has a beneficial effect on the drag.

SECTION VI

Three-Dimensional Navier-Stokes Computer Codes

The compressible turbulent computer code, originally developed for STAR-100A, has been optimized for CYBER-203. This involved significant changes in the version of the code and revisions from STAR-100A, including techniques to make effective use of the speed of scalar operations in CYBER-203 and a major effort to take advantage of the improved scatter/gather operations of CYBER-203. Some details of these modifications are given in Ref. (19).

The present version of the code is highly vectorized and makes effective use of the hardware capabilities available. This has permitted the initial investigation of some new and apparently significant physics of turbulent flows outlined as below:

1) A new fundamental three-dimensional mechanism is identified by which vorticity can be produced. This mechanism is responsible for the generation of small scale turbulence. It appears that this three-dimensional mechanism applied generally to shear flows, and explains many features of transition and turbulence. Results for planar shear flows have been described in Ref. (20) and for pipe flows in Ref. (21). To summarize the physics uncovered, it is noted that "Two-dimensional flow states are generally stable or at worse weakly unstable to two-dimensional perturbations, but they are explosively unstable for three-dimensional perturbations". This instability occurs by a subtle interplay between vortex stretching and vortex tilting mechanisms. Both these mechanisms are essential to explain the explosive growth of three-dimensional instability. However, the vortex tilting mechanism seems to explain the localized, spotty nature of turbulence in thin shear flows. It is apparently the first quasi-analytic description of turbulent spots.

Work continues on the direct simulation of turbulent shear flows using "STAR" codes developed with this contract and

additional results on the structure of turbulence are anticipated soon.

SECTION VII

Compressible Stability Analysis Code

Under this contract the contractors provided assistance
in developing a compressible stability analysis code (COSAL)
which uses a local eigenvalue search procedure. This code
is computationally more efficient than any existing compressible
stability code. The details of the COSAL code and some results
of calculations for an LFC swept wing are presented in Ref. (22).

SECTION VIII

Conclusions and Directions of Future Research

A computational code to analyze flow over complicated geometry has been developed. Comparisons with experimental results indicate that nonseparating turbulent flow over wavy surfaces can be adequately predicted using this code. The computer code is capable of analyzing flow over unsymmetric waveforms as well as traveling waveforms.

An effective way of designing drag reducing surfaces is to exercise the computer algorithm, rather than arbitrarily testing out waveforms. Whereas the construction of a single wavy wall itself costs today about $8000, the computer program can generate complete solution of flow over this geometry for less than $600; i.e., for more than an order of magnitude less cost. Previous drag-reduction studies at LaRC on wavy walls has been on a hit-or-miss basis; i.e., a wave shape is suggested and after testing the model in the wind tunnel conclusive evidence of drag reduction may be established. The present code can be exercised to isolate an array of shapes where a possibility for drag reduction exists and these candidate surfaces can then be tested for drag reduction. This approach substantially improves the odds for success and will be pursued in further research.

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Table 1. Results of computer predictions for a variety of waveforms and their comparison to unpublished measurements of Weinstein, L. M. and Cary, A. M.

MODEL	WAVEFORM	U_∞ m/s	δ mm	a mm	λ mm	a/λ	h^+	COMPUTED c_d/c_{d_0}	c_d/c_{d_0} AVERAGE
1		22.68	16.5	0.127	25.4	0.005	8.7	1.016	1.013
2		22.68	16.5	2.54	1	0.01	17.5	1.056	1.090
3		22.68	16.5	5.08	1	0.02	35.	1.23	1.290
VA		22.68	16.5	0.1397	1	0.0055	9.6	1.03	1.05
VB		22.68	16.5	0.1397	1	0.0055	9.6	1.03	1.05
VB*		22.68	16.5	0.2794	1	0.011	19.2	1.054	*
VB*		22.68	16.5	0.2794	1	0.011	19.2	1.055	*
VA*		22.68	16.5	0.5588	1	0.022	38.5	1.200	*
IIIA		22.68	16.5	0.1524	32.38	0.0047	10.	1.02	1.025
IIIB		22.68	16.5	0.1524	1.275	0.0047	10.	1.02	1.025

* No data from tunnel available

Table 2. Results of computer predictions of propagating
waveforms on the boundary for laminar flow.

SURFACE CHARACTERISTIC	FRICTION DRAG	PRESSURE DRAG	TOTAL DRAG	NET CHANGE
Flat Plate	6.042×10^{-3}	————	6.042×10^{-3}	————
Sinewave (a=.015" $\lambda = 1.0$") $C/U_\infty = 0$	6.007×10^{-3}	0.318×10^{-3}	6.325×10^{-3}	4.66%
Sinewave (a=0.015" $\lambda = 1.0$") $C/U_\infty = 1.0$	5.884×10^{-3}	-0.309×10^{-3}	5.575×10^{-3}	-7.8%
Sinewave (a=.015" $\lambda = 1.0$") $C/U_\infty = 2.0$	5.862×10^{-3}	-0.628×10^{-3}	5.134×10^{-3}	-15.2%

Comparisons are made for laminar flow at $Re_x = 12070.$; Free stream velocity = 21.6 in/sec

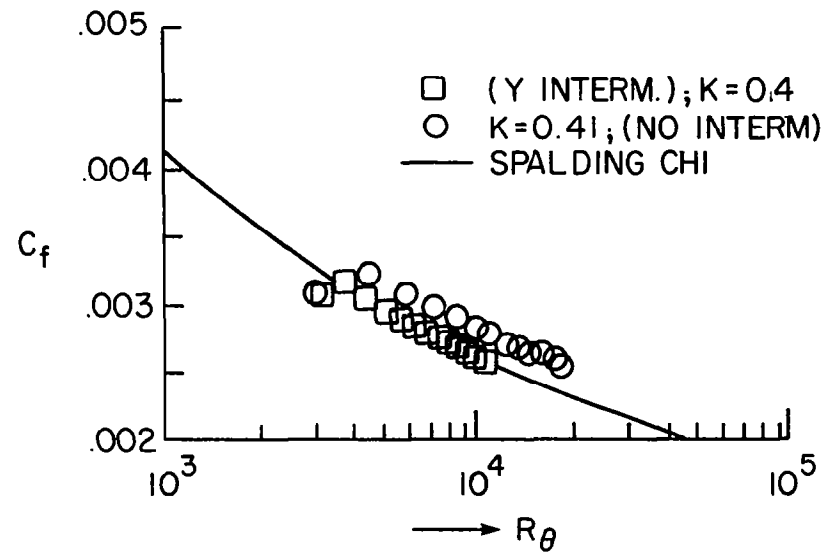## SKIN FRICTION DISTRIBUTION OVER FLAT PLATE

Figure 1. Prediction of skin friction over a flat plate using
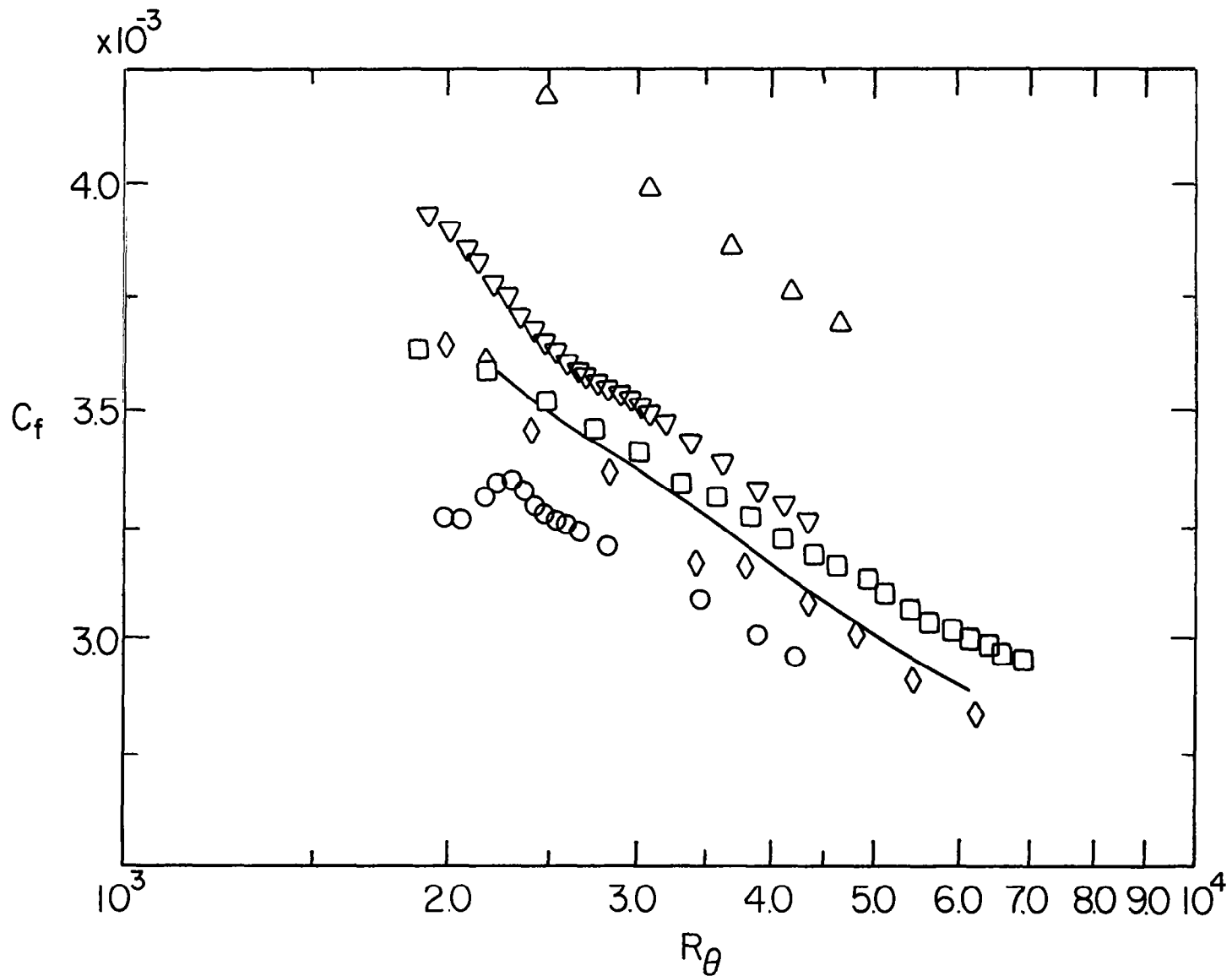zero-equation turbulence modeling.

Figure 2. Variation of coefficient of skin friction for a flat plate boundary
layer vs R_θ. Here the solid curve is obtained from the two-equation
model (II.6)-(II.10) with $c_3 = 0.010$. The other points are:
$\triangle$ Model solution with $c_3 = 0.02$; $\triangledown$ Model solution with $c_3 = 0.0115$,
as used by Chien; $\square$ One equation model; $\bigcirc$ $c_3 = 0.008$;
$\diamond$ Experimental results due to Weighardt.

COMPARISON OF PRESSURE PREDICTION
WITH SIGAL'S MEASUREMENTS

——— ZERO EQUATION MODEL WITH
　　　EQUILIBRIUM CONSTANTS

————— CEBECI-SMITH 2-LAYER MODEL

O SIGAL'S DATA

———— TWO EQUATION MODEL

C_p
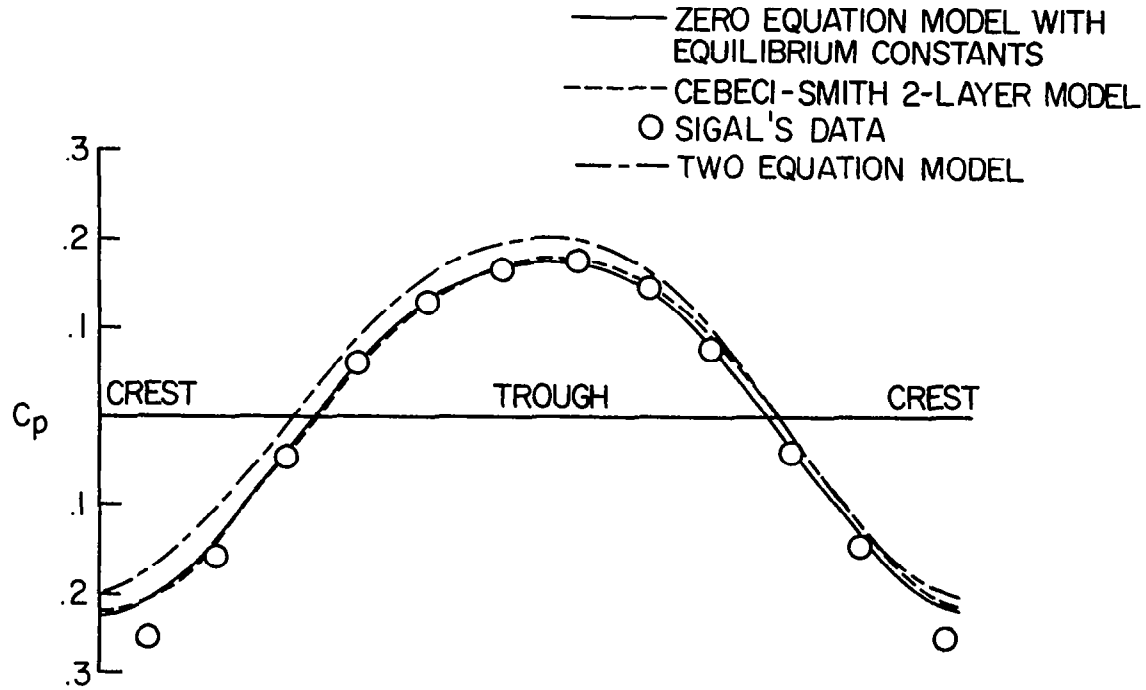
.3
.2
.1

CREST　　　　　　TROUGH　　　　　　CREST

.1
.2
.3

Figure 3.　Comparison of pressure predictions with the
experimental results of Ref. (2).

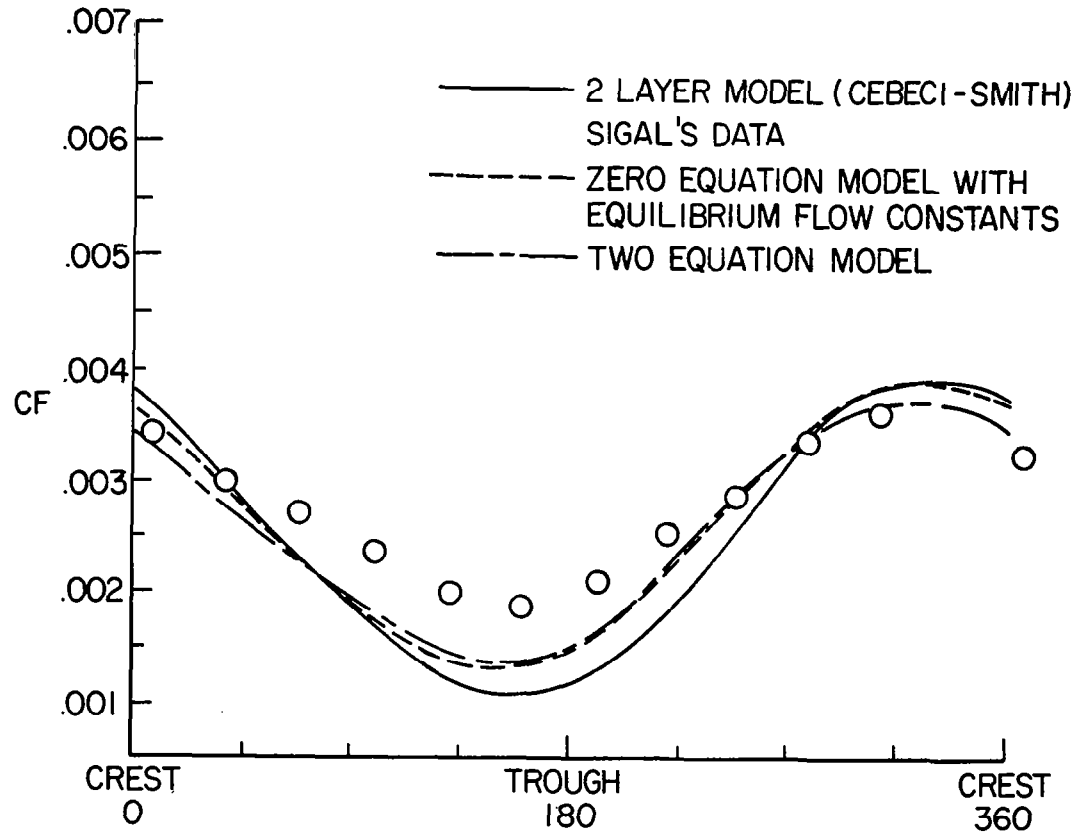## SKIN FRICTION DISTRIBUTION OVER A WAVE

Figure 4. Comparison of skin friction distribution with the experimental measurements of Ref. (2).

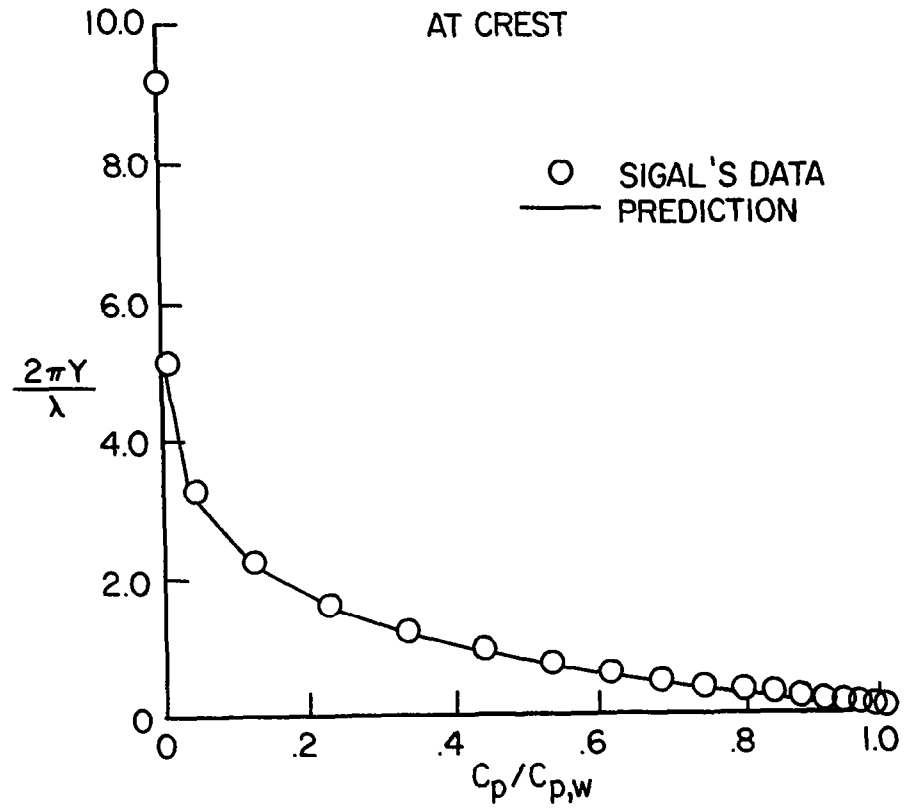## STATIC PRESSURE VARIATION ACROSS THE BOUNDARY LAYER

Figure 5. Comparison of static pressure distribution with the experiment of Ref. (2).

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16. Abstract

Two-dimensional incompressible flow over wavy surfaces is studied numerically by spectral methods. Turbulence effects are modeled. Results for symmetric and asymmetric wave forms are presented. Effect of propagating surface waves on drag reduction is studied. Comparisons between computer simulations and experimental results are made.

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