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# Development of an Efficient Procedure for Calculating the Aerodynamic Effects of Planform Variation

J. E. Mercer and E. W. Geller

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# Development of an Efficient Procedure for Calculating the Aerodynamic Effects of Planform Variation

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## SUMMARY

This report describes numerical procedures that can be used to compute the gradients of aerodynamic forces and moments with respect to wing planform changes. Two basic procedures were investigated, one which computes the aerodynamic increments directly and one which computes the perturbed case which is then differenced from the base case to obtain the increments. The study showed that the direct calculation of the increments does not work because of the approximate representation of the pressure singularity at the planform leading edge. Proper representation of the singular behavior might eliminate this problem; however, this was not attempted in this study.

This investigation showed that the perturbed-shape aerodynamic calculation can use information saved from the base solution if the planform perturbation can be modeled by changing the panels adjacent to the perturbed edge of the planform. In that case, most of the influence coefficients in the base solution and in the perturbed solution are identical. This time-saving procedure was demonstrated using two aerodynamic panel method codes, USSAERO and the Vortex Spline scheme.

Results of the investigation showed that the Vortex Spline Code offered computing speed advantages over the USSAERO Code and that a single aerodynamic gradient could be calculated in about 20 seconds on a CDC Cyber 175 once the base solution was obtained. Improvements in the Vortex Spline Code are suggested for further reduction in gradient computation. With the suggested improvements, the time for calculating the influence coefficients could be

decreased by an order of magnitude and, thus, the total calculation time demonstrated in this report could be cut in half.

## INTRODUCTION

One requirement in the application of computer optimization procedures to aircraft structures is the calculation of the change of aerodynamic loadings with configuration perturbations. Previously, these gradients were calculated by perturbing the configuration and recalculating the flow for the new configuration, then differencing the perturbed result from the base result. This procedure is very costly since every perturbation requires a complete aerodynamic evaluation. To find more efficient ways to perform these gradient calculations, a study was undertaken. The study was limited to the linearized flow where lifting-surface theory could be applied. Also, panel methods were used to solve the governing integral equation for lifting-surface theory. For this application, the panel methods construct the flow about a wing mean surface by distributing elemental horseshoe vortex singularities over a planar approximation to the surface. The singularities are associated with panels into which the surface is divided. In some schemes, the singularity is limited to a single panel. In others, the singularity is distributed over several panels and several singularities share common panels. This overlapping is used to preserve some degree of continuity in the global singularity distribution.

The strengths of these singularities are determined such that the net flow is tangent to the wing mean surface. This boundary condition is enforced at a finite number of points on the surface. The points, known as control points, are associated with the panels used to define the singularities. For the col-

location method, the number of control points is chosen to be equal to the number of unknown singularity strengths, and the solution is deterministic. If there are more control points than singularity strengths, the system of equations is overdetermined and the solution must satisfy the boundary conditions in a least-square error sense. By weighting the match at the control points by some associated area, the least-square error solution can approximate a least-square integral match to the boundary conditions.

In either case (the collocation or the least-square method), the resulting set of equations (one for each control point) is of the form

$$\bar{A} p = w \quad (1)$$

where

- p = the vector of unknown singularity strengths (or the wing pressure loading)
- w = the vector of normal velocities at control points (or the streamwise slope of the wing mean surface)
- $\bar{A}$  = the influence coefficient matrix.

The alternative interpretations of p and w indicated above in parentheses are the result of linearizing approximations, and in the case of p, the result of choosing the singularities to be elemental horseshoe vortices. The element  $A_{ki}$  of the matrix  $\bar{A}$  is the normal velocity induced at the "kth" control point by the "ith" singularity with unit strength.

Equation (1) is a numerical approximation for the governing integral equation relating pressure to camber shape as obtained from lifting-surface theory (see refs. 1 and 2). It is the basis of two codes developed by Woodward<sup>3</sup> and by Mercer et al.<sup>4</sup> which have been used during this study and which are



referred to as the USSAERO Code and the Vortex Spline Code, respectively. These codes are described in the following section.

The aerodynamic gradients needed for structural optimization codes are  $\partial p / \partial s_i$  where  $s_i$  are the parameters that define the planform (e.g., aspect ratio, quarter-chord sweep, taper ratio, span). An alternative to obtaining  $\partial p / \partial s_i$  by direct differencing of solutions to Equation (1) is to solve the derivative of Equation (1),

$$\bar{A} \frac{\partial p}{\partial s_i} = - \frac{\partial \bar{A}}{\partial s_i} p + \frac{\partial w}{\partial s_i} \quad , \quad (2)$$

for  $\partial p / \partial s_i$ . This approach, and its inherent potential for reducing computing time, as well as the direct differencing approach are discussed in this report.

Because of the presence of the last term, Equation (2) shows that the gradients  $\partial p / \partial s_i$  depend upon the camber and twist distribution. For the sample calculations of this study, the convenient choice of a flat plate was made for the wing mean surface. This simplification does not weaken the conclusions of the investigation regarding the relative efficiency of the various procedures for calculating the aerodynamic gradients. These conclusions are valid for wings with nonflat mean surfaces. When treating such wings, it will of course be necessary to use a nonconstant  $w$  distribution for Equation (1), but this generalization does not introduce significant complications.

We also present in this report a repaneling technique for the perturbed planform that provides a large reduction in computing time. Using this technique, a large fraction of the elements of  $\bar{A}$  are unchanged for the perturbed planform if the direct differencing approach is used. The same large fraction of the elements of  $\partial \bar{A} / \partial s_i$  vanish if Equation (2) is used.

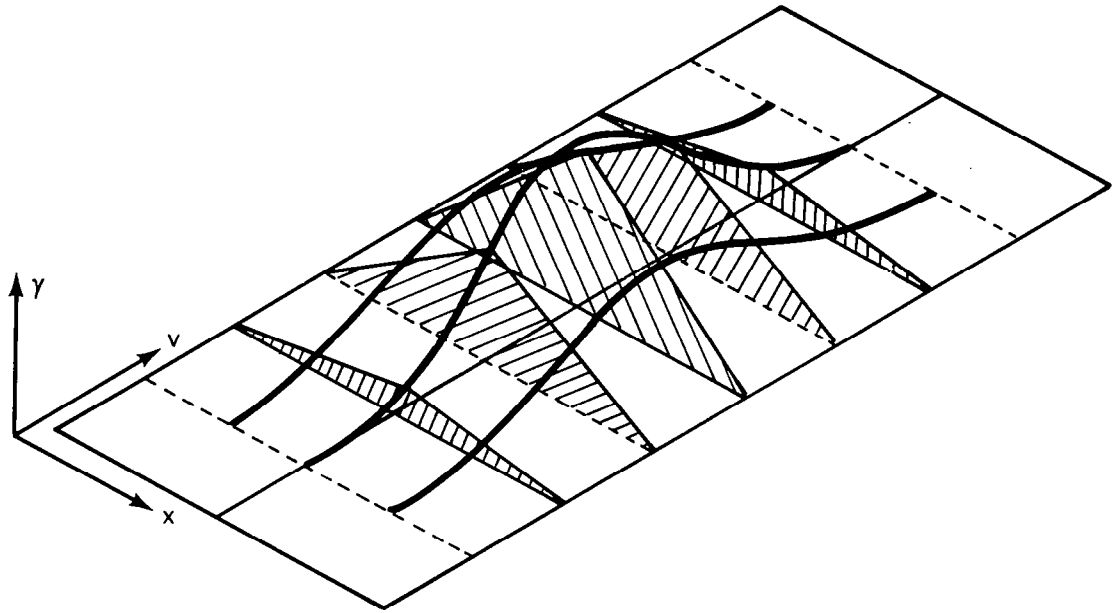
The computing time saved with the approaches presented in this report was investigated using the Vortex Spline Code and the USSAERO Code. This investigation is discussed and recommendations to give further speed improvements are presented. The results of this work are briefly summarized and conclusions and recommendations are given in the final section of this report.

## TWO COMPUTER PROGRAMS FOR CALCULATING WING LOADINGS

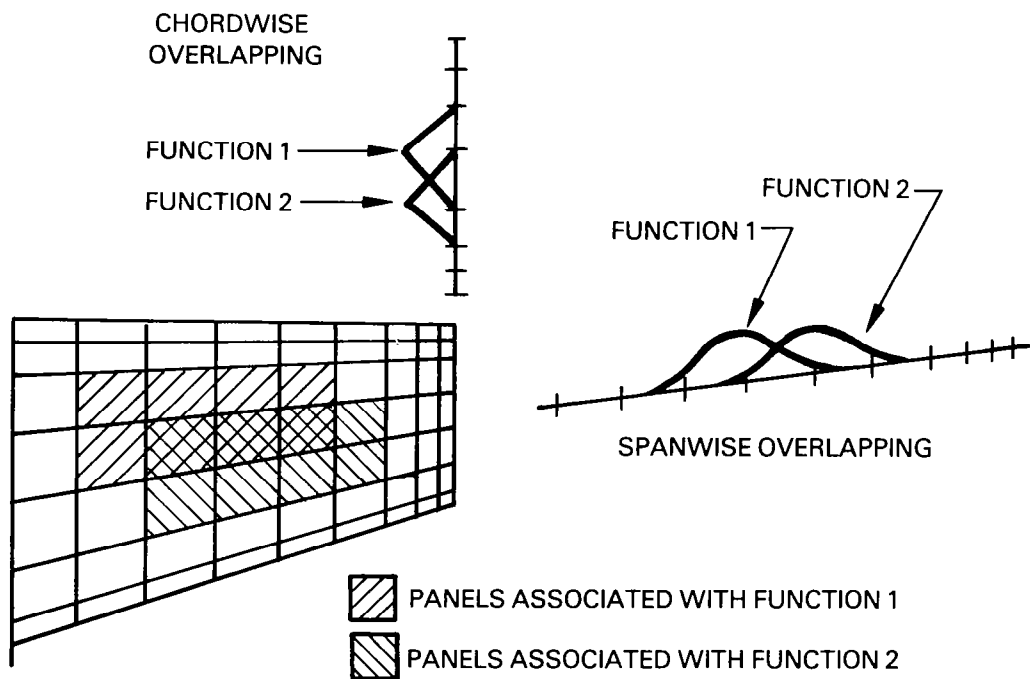
In this section two computer programs for computing aerodynamic loads are briefly described. They are based on linearized lifting-surface theory and solve the integral equation that expresses the downwash due to vorticity distributed on the wing planform and wake. The vorticity distribution is constructed by superposition of fundamental "building blocks," which are associated with a paneling into which the wing planform is divided. The two computer codes described in the following differ mainly in the type of vorticity distribution used for these building blocks.

### The Vortex Spline Computer Program

The basis for the Vortex Spline computer program is given in Reference 3. The code is applicable to both linearized subsonic and linearized supersonic flows. The basic vorticity distribution, which is called the vortex spline, covers eight panels as depicted in Figure 1. The intensity varies quadratically in the spanwise direction and linearly in the chordwise direction. The magnitude of the building block is defined as the maximum intensity or some equivalent measure. A distribution of vorticity over the entire wing planform



(a) Surface Spline Distribution of Vorticity



(b) Overlapping of Surface Distributions

**Figure 1. The Vortex Spline**

is constructed by superimposing these fundamental splines as shown in Figure 1. The mathematical problem is to find the strengths of these splines such that the vertical velocity components that they "induce" at a finite number of points, called control points, take on prescribed values or match these values in a least-square error sense. According to linearized lifting-surface theory, these downwash velocities, when normalized by the free-stream velocity, are equal to the streamwise slope of the wing mean surface (and are therefore known quantities), and the vorticity is proportional to the aerodynamic loading (in the lift direction) on the wing. These equivalences, downwash with mean surface slope and loading with vorticity, allow the mathematical problem defined above to be expressed as the solution of Equation (1).

Unlike the USSAERO Code described in the following, the Vortex Spline Code enforces the boundary condition in a least-square sense at a number of control points greater than the number of unknown vortex spline strengths. The procedure is as follows. Equation (1) is not square (the number of equations is greater than the number of unknowns) but is made square by multiplication by the transpose of the matrix  $\bar{A}$  and a weighting factor "a". The result is

$$(\bar{A}^T a \bar{A})p = \bar{A}^T a w \quad . \quad (3)$$

The weighting factor is selected to be the wing area associated with a control point, so that this formulation solves the problem in an integral sense in that the error over the entire planform area is minimized. With this least-square error formulation, Equation (2) is replaced with

$$\left( \bar{A}^T a \bar{A} \right) \frac{\partial p}{\partial s_i} = \bar{A}^T a \left( - \frac{\partial \bar{A}}{\partial s_i} p + \frac{\partial w}{\partial s_i} \right) . \quad (4)$$

This can be verified by differentiating Equation (3) and collecting terms.

## The USSAERO Code

The basis for the USSAERO Code is given in Reference 4. This code is also applicable to both linearized subsonic and linearized supersonic flows. It is distinguished from the Vortex Spline method in two ways.

First, the basic vorticity building block has constant intensity in the spanwise direction and covers one panel in that direction. The chordwise treatment is essentially the same linear variation as for the vortex spline.

The second difference is that the number of control points is equal to the number of singularities. Thus, the matrix in Equation (1) is square and the equation can be solved directly.

### COMPUTATION OF AERODYNAMIC GRADIENTS DUE TO PLANFORM PERTURBATION

The difference  $\Delta p$  of two solutions of Equation (1), one for a base planform and one for a perturbed planform corresponding to a change in a planform parameter  $\Delta s_i$ , provides the desired gradient according to

$$\frac{\partial p}{\partial s_i} = \frac{\Delta p}{\Delta s_i} \quad . \quad (5)$$

An alternative to this direct differencing approach is to solve Equation (1) for  $p$  and then to solve the derivative of Equation (1),

$$\bar{A} \frac{\partial p}{\partial s_i} = - \frac{\partial \bar{A}}{\partial s_i} p + \frac{\partial w}{\partial s_i} \quad , \quad (2)$$

for  $\frac{\partial p}{\partial s_i}$ . A comparison of the two approaches is given below, where it is shown that the second approach is more efficient.

Both approaches require the solution of two sets of equations. For the

first approach, they are the solution of

$$\bar{A}_1 p_1 = w_1 \quad (6a)$$

$$\bar{A}_2 p_2 = w_2 \quad (6b)$$

for  $p_1$  and  $p_2$ , where the subscripts 1 and 2 refer to evaluation for the base planform and for the perturbed planform, respectively. For the second approach, they are the solution of

$$\bar{A}_1 p_1 = w_1 \quad (7a)$$

for  $p_1$  and then

$$\bar{A}_1 (p_2 - p_1) = - (\bar{A}_2 - \bar{A}_1) p_1 + (w_2 - w_1) \quad (7b)$$

for  $(p_2 - p_1)$  using the known  $p_1$  on the right-hand side. The advantage of the second approach is a consequence of the fact that the matrix multiplying the unknowns is the same for both the first and second set of equations. Thus, information from the solution of the first set can be used to reduce the computation required for solution of the second. For example, if matrix inversion is used to solve the first set according to

$$p_1 = \bar{A}^{-1} w_1, \quad (8)$$

then the inverted matrix  $\bar{A}^{-1}$  can be saved to solve the second set. Note that in both approaches the matrices  $\bar{A}_1$  and  $\bar{A}_2$  need to be calculated.

The preceding arguments were posed in terms of Equation (1). The same arguments are applicable starting with Equation (3), the equation used by the Vortex Spline Code, in which case Equations (7a) and (7b) become

$$(\bar{A}_1^T a \bar{A}_1) p = \bar{A}_1^T a w_1 \quad (9a)$$

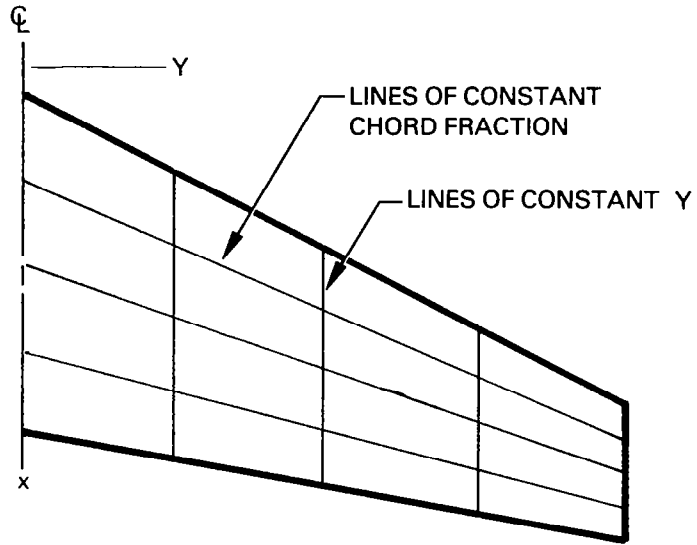
$$(\bar{A}_1^T a \bar{A}) (p_2 - p_1) = \bar{A}_1^T a \left[ -(\bar{A}_2 - \bar{A}_1) p_1 + (w_2 - w_1) \right]. \quad (9b)$$

## A REPANELING TECHNIQUE FOR EFFICIENT COMPUTATION

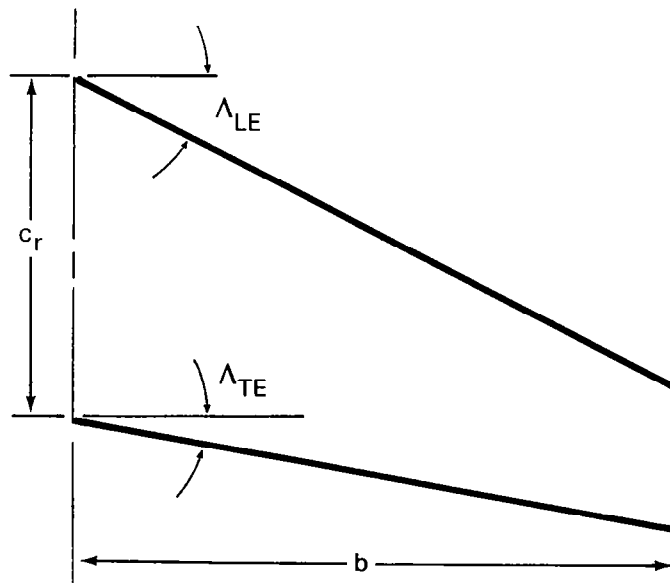
For either choice of obtaining the derivatives  $\partial p/\partial s_i$ , direct differencing or the method of Equations (7b) and (9b), it is necessary to calculate the matrix  $\bar{A}$ , first for the base planform and second for the perturbed planform (see  $\bar{A}_1$  and  $\bar{A}_2$  in Equations (6) and (7)). In repaneling for the perturbed planform it is desirable to keep as many of the panels as possible unchanged, since elements of the matrix associated with unchanged panels will not change. We now describe a procedure for which most of the panels are not changed. It can be effected when the wing parameterization and the wing paneling are "congruent."

### Congruence

We define congruence to exist when a change in one and only one of the parameters can be realized by altering one of the rows or one of the columns of panels. We illustrate the concept for the case of a simple wing with straight leading and trailing edges. Traditionally, the wing might be defined in terms of the four parameters: aspect ratio AR, quarter-chord sweep  $\Lambda_{1/4}$ , span b, and taper ratio  $\lambda$ . These parameters are not congruent with the typical paneling scheme illustrated in Figure 2. A set of congruent parameters ( $c_T$ , b,  $\Lambda_{LE}$ , and  $\Lambda_{TE}$ ) is shown in Figure 3. Choose any one of the defining parameters in Figure 3, and it is possible to change it without changing the other parameters by perturbing one of the rows or columns of the paneling shown in Figure 2. For example, we can change parameter  $c_T$  without changing b,  $\Lambda_{LE}$ , or  $\Lambda_{TE}$  by increasing the chord of each trailing-edge panel by the same



**Figure 2. Traditional Paneling Scheme for a Simple Wing Planform**



**Figure 3. A Parametric Definition that is Congruent to the Paneling in Figure 2**



amount without changing the panel leading-edge positions as shown in Figure 4a. The remaining perturbations that are congruent are also shown in Figure 4. Note that once the loading gradients with respect to  $b$ ,  $c_T$ ,  $\Lambda_{LE}$ , and  $\Lambda_{TE}$  have been obtained, it is possible to calculate from them the gradients with respect to  $AR$ ,  $b$ ,  $\Lambda_{1/4}$ , and  $\lambda$ , if desirable. Thus, choosing the parameterization of the wing definition to be congruent to the paneling scheme does not exclude computing changes due to classical parameters. In fact additional parameters such as leading- and trailing-edge planform breaks can also be modeled by altering a part of a row of panels.

#### Computing Efficiency Obtained With Congruence

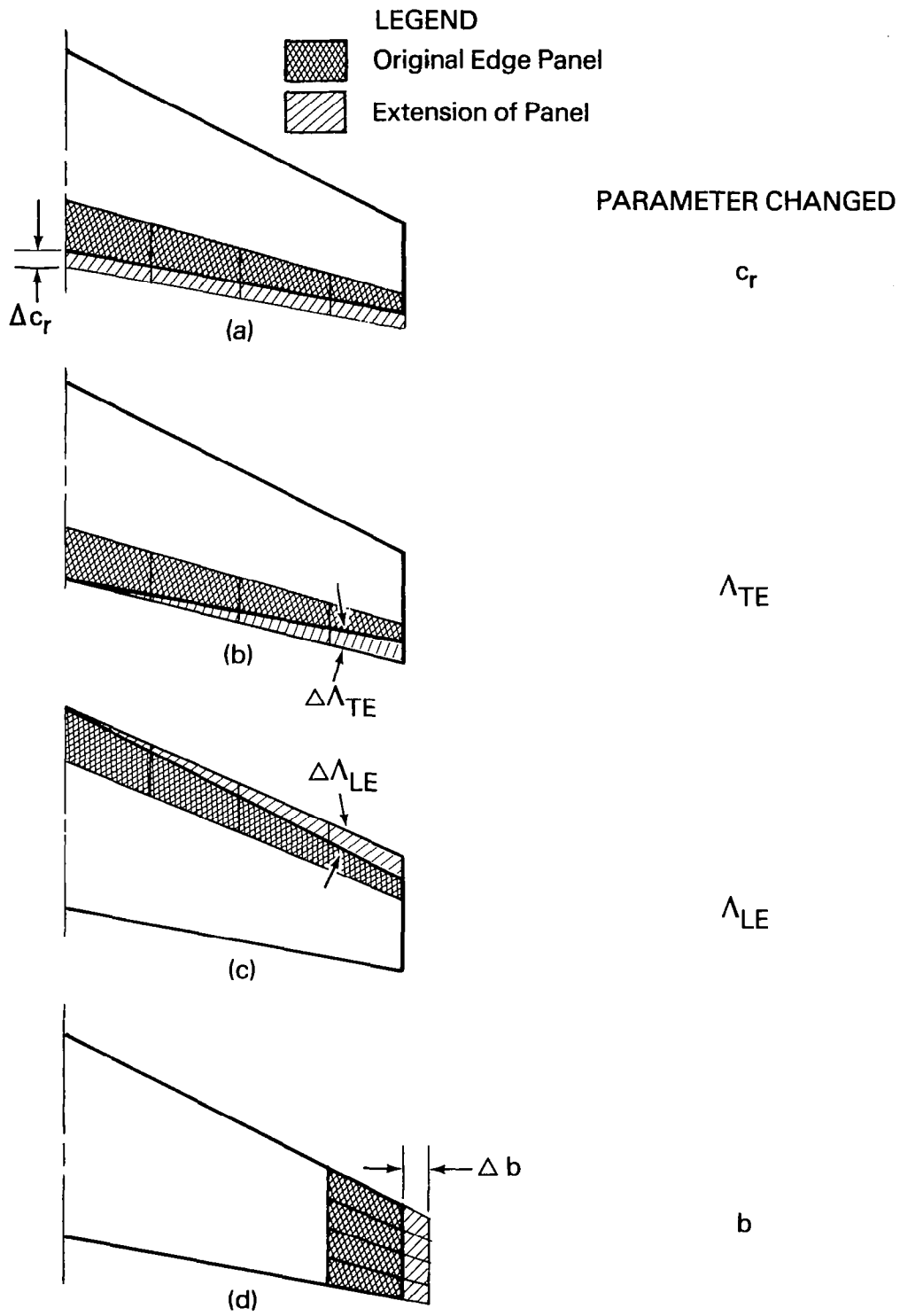
The preceding discussion of a congruent parameterization and paneling suggests a potential reduction in computing time for obtaining the matrix  $\bar{A}$  for the perturbed planform. When the perturbed paneling is made according to that which demonstrates the congruence (e.g., Figure 4), only a fraction of the elements of the matrix  $\bar{A}$  are changed since only a fraction of the panels are changed. The fraction of influence coefficients that needs to be changed for a typical perturbation is shown in the following example.

The total number of influence coefficients for the USSAERO Code is

$$N = N_C^2 \cdot N_S^2 \quad (10)$$

where  $N_C$  is the number of chordwise panels and  $N_S$  is the number of spanwise panels. For the Vortex Spline Code, the total number is

$$N = N_C^2 \cdot (N_S + 1) \cdot N_S \cdot N_p \quad (11)$$



**Figure 4. Planform Perturbations Demonstrating the Congruence of the Paneling and the Parameters Shown in Figures 2 and 3**

where  $N_C$  and  $N_S$  are defined as above and  $N_P$  is the number of control points per panel.

For the case where the leading edge is perturbed, there are actually two chordwise sets of singularity functions changed. For the USSAERO Code, the number of influence coefficients changed is

$$N_{\Delta} = N_S^2 (3 \cdot N_C - 2) \quad . \quad (12)$$

For the Vortex Spline Code, this number is

$$N_{\Delta} = N_S \cdot N_P \cdot (3 \cdot N_C \cdot N_S + 3N_C - 2N_S - 2) \quad . \quad (13)$$

Typically, this means that 40 percent or more of the influence coefficients can be saved and do not have to be evaluated.

An example of the time savings represented by only having to calculate a fraction of the influence coefficient matrix is presented in the next section.

#### Accuracy

One consideration to be made when using the repaneling technique suggested above is the accuracy compared to that obtained if a complete repaneling was used. In this regard, the higher the order of the singularity distribution, the less sensitivity to the "smoothness" of the paneling. In the spanwise direction, the singularity strength is constant over the panel for the USSAERO Code as opposed to quadratically varying for the Vortex Spline Code. Therefore, for spanwise repaneling, the latter is not as susceptible to error due to the nonuniform division that can occur when the repaneling technique is used. In fact, the results given in the next section will show that significantly fewer panels are needed for the Vortex Spline Code than for the USSAERO

Code for the same accuracy. This means that the matrix problem being solved is of a much lower order and is consequently significantly faster to compute.

#### TESTING THE PROPOSED PROCEDURES USING THE VORTEX SPLINE AND USSAERO CODES

For evaluating the proposed procedures, we made calculations on wings with flat mean surfaces. As mentioned in the Introduction, this simplification does not affect the validity of the investigation and its conclusions. The procedures were demonstrated on a rectangular wing with an aspect ratio of 2. Although this planform is relatively simple, it does have significant three-dimensional effects, so that small perturbations in shape can make noticeable changes in the aerodynamic loading.

In order to investigate the method outlined in Equations (5) through (9) with a minimum amount of code modification, the following procedure was used. Two runs were made with the Vortex Spline Code, one for the base planform and one for the perturbed planform. The influence coefficient matrix  $\bar{A}$  from each run and the solution for the loading vector  $p$  for the base planform were stored on a disk. The right-hand side of Equation (9b) was then calculated using this stored information. Equation (9b) was then solved for the perturbation in loading,  $(p_2 - p_1)$ , by using that part of the Vortex Spline Code which solves equations of the form of Equation (9a) with the quantity in brackets from Equation (9b) replacing  $w_1$  in Equation (9a).

Various tests were made to be sure that there were no errors in the coding. Several sample test cases were computed to test the analysis. The first test

case was a 3-percent increase in span. The lift coefficient perturbation (for  $w = 1.0$ ) was computed by differencing the base case from the repaneled perturbed case (all fractions of chordwise and spanwise paneling divisions were kept the same). This value was 0.048 (1.9-percent increase). By direct computation (Equation (9b)), perturbing only the tip panels, the value was 0.035 (1.4-percent increase), about a 27-percent error. The second test case was a 3-percent perturbation of the chord. The repaneled lift coefficient perturbation showed a decrease of 0.048 (1.9 percent). By perturbing the trailing-edge panels, the direct perturbation computation value was 0.054 (2.2-percent decrease), about a 12-percent error.

This same chordwise perturbation case was repeated, only the leading-edge panels were perturbed. This time the direct calculation provided a decrease of 0.242, which is completely erroneous. Other runs were made perturbing the second row from the leading edge (about a 20-percent error) and perturbing the first two rows of panels (about a 75-percent error). These runs seemed to point out that the perturbation of the leading-edge panels could not be made without introducing a great deal of error. This is probably due to the singular nature of the pressure on these panels and the fact that the linear vortex representation is not adequate for the direct perturbation calculation. Perhaps if the proper singular representation was made at the leading edge, the direct perturbation solution technique would work. In any case, special treatment of singular regions will probably have to be made to obtain a good perturbation solution.

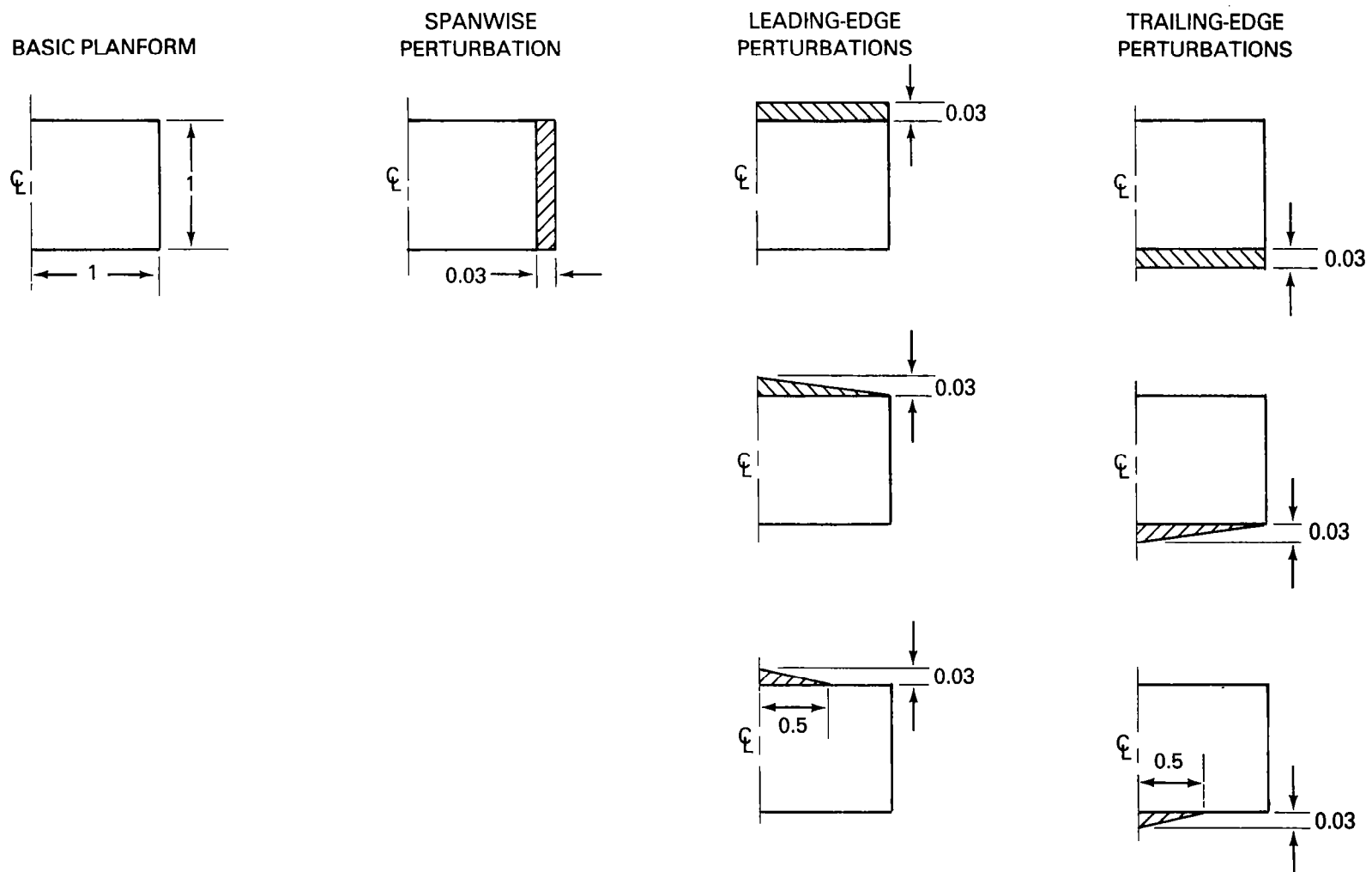
Although these test cases showed that the direct method could not be applied to existing codes, the idea of perturbing a small fraction of the wing panels still can be applied. Since this latter concept provides most of

the computer savings, the inapplicability of the direct perturbation solution technique does not constitute a big loss.

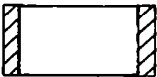





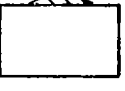
One of the concerns associated with the panel-perturbing technique described in the previous section is the sensitivity of the solution to changes in the distribution of the panel division lines. If maximum accuracy in calculating the solution perturbation is the objective, we expect that the best way to repanel is to keep the same distribution of panel division lines (e.g., keeping the span fraction the same for streamwise division lines).

Several test cases were run using the Vortex Spline Code to test the program's sensitivity to paneling. A series of test calculations was made with the planform perturbations defined in Figure 5. The changes in lift and moment coefficients ( $C_L$  and  $C_M$ , respectively) were calculated in four different ways; the results are given in Figure 6. For the results in the first column, the Vortex Spline Code was used with 48 panels (6 spanwise by 8 chordwise) and with the perturbed calculation made with a complete repaneling using the same chord fraction and span fraction at the division lines. These calculations were repeated with only the edge panels changed according to the repaneling technique presented in the previous section, and the results are shown in the second column of Figure 6. For some of the planform perturbations, calculations were made with the USSAERO Code, and the results are given. For these runs, the repaneling was accomplished by changing the edge panels according to the repaneling technique. Not all cases could be run with the USSAERO Code because of input specification restrictions. These restrictions are not fundamental so that this problem could be overcome by a coding change.

The lift perturbations show that agreement to about 10-percent can be obtained between the single-row or column-of-panels perturbation and complete



**Figure 5. Set of Perturbations Used to Produce Planform Gradients.**






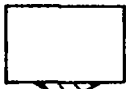

PLANFORM	COMPLETE REPANEL		REPANELING ACCORDING TO TECHNIQUE OF PREVIOUS SECTION	
	$\Delta C_L = C_{L\text{TOTAL}} - C_{L\text{BASE}}$ REPANEL		$\Delta C_L = C_{L\text{EDGE}} - C_{L\text{BASE}}$ REPANEL	
	VORTEX SPLINE (6 × 8 PANELS) $C_{L\text{BASE}} = 0.2488$	VORTEX SPLINE (6 × 8 PANELS) $C_{L\text{BASE}} = 0.2488$	USSAERO (14 × 14 PANELS) $C_{L\text{BASE}} = 0.2495$	
	0.0048	0.0053	0.0049	
	-0.0048	-0.0052	-0.0051	
	-0.0048	-0.0049		
	-0.0021	-0.0023	*	
	-0.0021	-0.0021	*	
	-0.0010	-0.0011	*	
	-0.0010	-0.0009	*	

(a) Lift Coefficient Results

Figure 6. Calculated Loading Perturbations Due to Planform Changes

\* NOTE: THE USSAERO INPUT FORMAT WOULD NOT ALLOW A SINGLE ROW OF PANELS TO BE MODIFIED TO MODEL THESE PERTURBATIONS.



PLANFORM	COMPLETE REPANEL		REPANELING ACCORDING TO TECHNIQUE OF PREVIOUS SECTION	
	$\Delta C_M = C_{M_{TOTAL}} - C_{M_{BASE}} \text{ REPANEL}$		$\Delta C_M = C_{M_{EDGE \text{ REPANEL}}} - C_{M_{BASE}}$	
	VORTEX SPLINE (6 × 8 PANELS) $C_{M_{BASE}} = -0.0534$	VORTEX SPLINE (6 × 8 PANELS) $C_{M_{BASE}} = -0.0534$	USSAERO (14 × 14 PANELS) $C_{M_{BASE}} = -0.0535$	
	-0.0014	-0.0014	-0.0013	
	0.0013	0.0017	0.0016	
	0.0013	0.0010		
	0.0005	0.0007	*	
	0.0047	0.0045	*	
	0.0002	0.0003	*	
	0.0025	0.0024	*	

(b) Moment Coefficient Results

Figure 6. Concluded

\* NOTE: THE USSAERO INPUT FORMAT WOULD NOT ALLOW A SINGLE ROW OF PANELS TO BE MODIFIED TO MODEL THESE PERTURBATIONS.

repaneling. The moment agreement is not as good. One reason may be that some of the perturbation values are so small that the basic accuracy of the influence coefficient calculation may be responsible. Another reason is that the moment is obtained by a single integration scheme which employs the value of the pressure at a panel control point and the moment arm to that point. Clearly, near the leading edge where gradients are large, this scheme will produce substantial inaccuracies. Better agreement could be expected if a more accurate integration scheme was employed. It should be noted that the sign of the moment increment currently being obtained is correct, as well as the order of magnitude of the perturbation. Experience with optimization schemes in the past has shown that this is often sufficient accuracy.

Some idea of the number of panels required for a given accuracy can be obtained from Figure 7 which shows the lift coefficient calculated for a rectangular wing of aspect ratio 2 as obtained from a series of computer runs (with both the Vortex Spline Code and the USSAERO Code). All runs were made at an angle of attack of  $5.73^\circ$ . If we take the correct solution to be 0.2475, as indicated in Figure 7 by the asymptote for the USSAERO curve, it is seen that 48 panels for the Vortex Spline calculation and 196 panels for the USSAERO calculation are required to obtain a solution within 1 percent of the correct one. (Note that the vertical scale for  $C_L$  on Figure 7 is highly magnified as indicated by the percent error scale.)

The computational time required on the Langley Research Center Cyber 175 Computer is shown in Figure 8 for the number of panels that were determined above to reduce the error to less than 1 percent. When the panels are rectangular, the USSAERO Code uses a special algorithm for efficiently calculating the influence coefficients, resulting in a total time about the same as for the

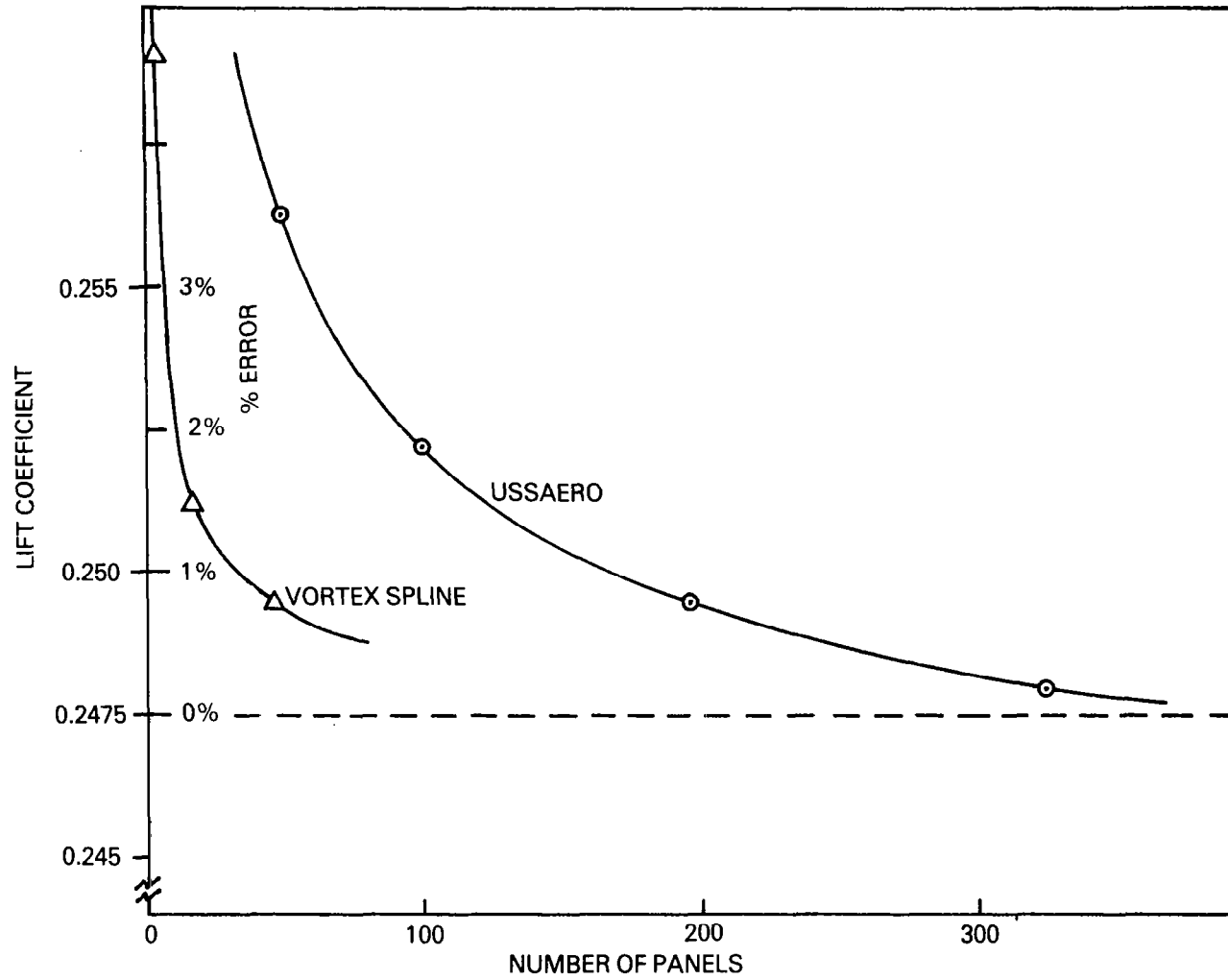


Figure 7. The Convergence of Lift Coefficient with Increasing Panel Density

	VORTEX SPLINE (48 Panels)	USSAERO* (196 Panels)
Time to Calculate Influence Coefficients	225	520
Time to Solve Equations	9	23
Total Time	234	543

\*NOTE: The USSAERO Code runs about 3 times faster than the values given here if the panels are rectangular. Since most planforms cannot be represented by rectangular panels, the nonrectangular values are used for comparison.

**Figure 8. Computing Time in Seconds on LRC Cyber 175**

Vortex Spline Code. Since rectangular panels are the exception, it is advisable to make the comparison on the basis of nonrectangular paneling, in which case two conclusions can be made. First, the Vortex Spline Code is much faster (about 3 times for an error of less than 1 percent) and second, for both codes the time to solve the set of equations is more than an order of magnitude less than the time required to calculate the matrix of influence coefficients. Another key issue is that the Vortex Spline solver is faster since the matrix size is considerably smaller than that for the USSAERO Code. This point becomes very important for the perturbation solution since the matrix solver could require about as much time as that required for influence coefficient recalculation in some cases. For example, if the leading-edge panels of a planform are perturbed, the Vortex Spline Code with 48 panels (6 spanwise by 8 chordwise) would require the following:

Number of influence coefficients changed--1848

Total influence coefficients--5376

Fraction of influence coefficients changed--0.344

Time to compute changed influence coefficients (Cyber 175)--77 seconds

Time to solve perturbation equations (Cyber 175)--9 seconds

Total time for perturbation problem--86 seconds

Now for the problem using the USSAERO Code with 196 panels:

Number of influence coefficients changed--7840

Total influence coefficients--38 416

Fraction of influence coefficients changed--0.204

Time to compute changed influence coefficients--106 seconds

Time to solve perturbation equations--23 seconds

Total time for perturbation problem--129 seconds

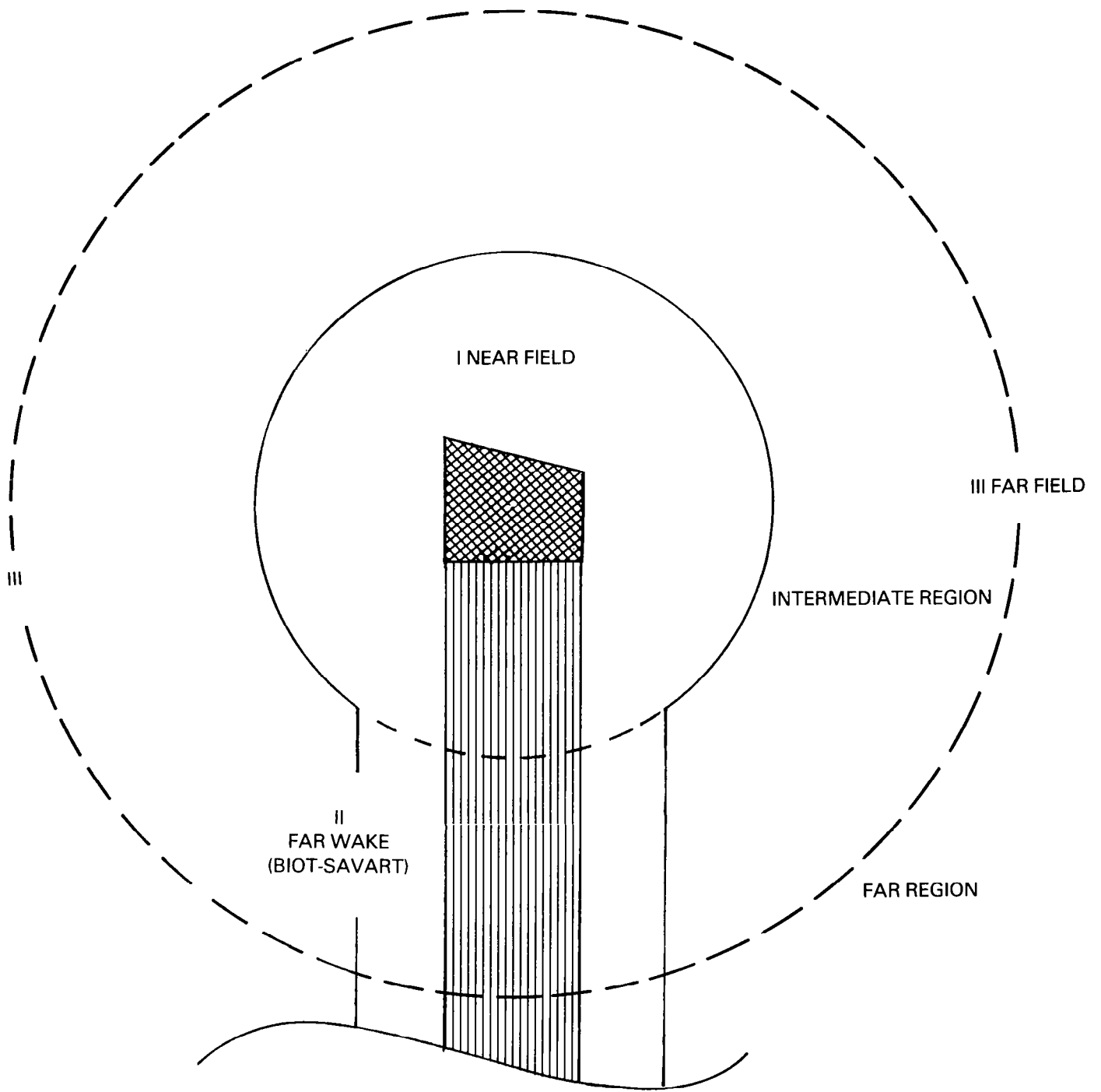
Comparison of these times shows that the perturbation problem for the Vortex Spline Code takes about two-thirds the time of that for the USSAERO Code. This comparison assumes the number of panels for the two codes has been adjusted to provide the same accuracy for the base solution. In the following section, we describe some modifications that could be made to the Vortex Spline Code which would bring the influence coefficient subroutines up to the state of the art of the USSAERO subroutines. These outlined improvements would make the perturbation solution about 10 times faster for the Vortex Spline Code.

#### IMPROVEMENTS TO OPTIMIZATION CODE

The current version of the Vortex Spline Code uses Gaussian quadrature to integrate the kernel function. The order of the quadrature formula was determined by the most critical evaluation and does not change throughout the flow field. This evaluation is inefficient, and great savings in computer time could be realized if some simple approximations for the far field were implemented.

Figure 9 shows an isolated panel on a wing. The flow field is divided into three distinct regions. Region I is the near field, Region II is the far wake field, and Region III is the far field. The three regions can be treated independently so that the computation times for Regions II and III can be made significantly less than for Region I. Currently, the entire field is treated as in Region I.

In Region II, the flow behaves as one induced by an infinite or semi-infinite sheet of line doublets or elementary horseshoe vortices. The details of the surface distribution of vorticity on the panel do not have a noticeable



**Figure 9. Flow Field Regions of an Isolated Panel on a Wing**

influence on the induced downwash. The only apparent effect is the spanwise loading. Therefore, the panel can be treated as a lifting line, and an analytical expression for the downwash can be evaluated without need for numerical integration. The expression is obtained from the Biot-Savart integral.

The far field (Region III) can actually be divided into an intermediate region and a far region. In the far region, the influence appears as one emanating from a single line doublet with its strength determined by the net vorticity on the panel. This influence can be expressed entirely by an analytical expression involving the integrated vorticity and the kernel function:

$$w(x, y) = \kappa(y - \eta_c, x - \xi_c) \int_{Y_L}^{Y_R} \int_{X_{LE}(\eta)}^{X_{TE}(\eta)} \gamma(\xi, \eta) d\xi d\eta \quad (14)$$

where  $(\eta_c, \xi_c)$  are the coordinates of the center of the panel.

For the intermediate region, the basic kernel function integrand can be expanded:

$$w(x, y) = \int_{Y_L}^{Y_R} \int_{X_{LE}(\eta)}^{X_{TE}(\eta)} \gamma(\xi, \eta) \kappa(y - \eta, x - \xi) d\xi d\eta \quad (15)$$

where

$$\begin{aligned} \kappa(y - \eta, x - \xi) = & \kappa(y - \eta_c, x - \xi_c) + \kappa_\eta(y - \eta_c, x - \xi_c) \cdot (\eta - \eta_c) \\ & + \kappa_\xi(y - \eta_c, x - \xi_c) \cdot (\xi - \xi_c) + \dots \end{aligned} \quad (16)$$

Now we define

$$P \equiv \iint \gamma(\xi, \eta) d\xi d\eta \quad (17a)$$

$$P_{(\xi)} \equiv \iint \gamma(\xi, \eta) (\xi - \xi_c) d\xi d\eta \quad (17b)$$

$$P_{(\eta)} \equiv \iint \gamma(\xi, \eta) (\eta - \eta_c) d\xi d\eta \quad (17c)$$



The downwash,  $w$ , can then be expressed as

$$\begin{aligned}
 W = & P \cdot \kappa(y - \eta_c, x - \xi_c) + P_{(\xi)} \cdot \kappa_{\xi}(y - \eta_c, x - \xi_c) \\
 & + P_{(\eta)} \cdot \kappa_{\eta}(y - \eta_c, x - \xi_c) + \dots \dots \dots
 \end{aligned}
 \tag{18}$$

The first term is the same as the far-field expression given above (Equation (14)). The first three terms added together provide an intermediate field expression.

Use of the above approximations could provide as much as an order of magnitude reduction in computing time. Still more savings could be realized for the geometry variations by another expansion of the kernel function. For the planform perturbations, there are two types of changes in the influence coefficient matrix. One change involves the influence of the perturbed panels at all the control point locations. The second change involves the change in control point location on the perturbed panels. This latter change can be treated in the far field as

$$\begin{aligned}
 w(x + \delta x, y + \delta y) \approx & w(x, y) + \frac{\partial w}{\partial y} \delta y + \frac{\partial w}{\partial x} \delta x \\
 \approx & w(x, y) \left\{ 1 + \frac{1}{\kappa(x - \xi_c, y - \eta_c)} \left[ \kappa_x(x - \xi_c, y - \eta_c) \cdot \delta x \right. \right. \\
 & \left. \left. + \kappa_y(x - \xi_c, y - \eta_c) \cdot \delta y \right] \right\}.
 \end{aligned}
 \tag{19}$$

The merits of this expression over the previous far-field expression would have to be explored to see which would be better to use. The above expression avoids having to calculate the moments of the vorticity and may be valid closer to the influencing panel.

Using the above expressions, it is estimated that a planform gradient could be calculated in less than 2 seconds on a CDC Cyber 176 or less than 20 seconds on a CDC Cyber 175.

## CONCLUSIONS AND RECOMMENDATIONS

Procedures for reducing the time for computing the rate of change of aerodynamic loadings with respect to wing planform parameters were investigated using two existing computer codes, the USSAERO Code and the Vortex Spline Code. For both codes and for the required accuracy, we found that most of the computing time is spent setting up the equations (i.e., calculating the influence coefficients). Only about one-twentieth of the time is used in solving the set of equations. Since the technique of solving directly for the loading derivatives using the equation where these terms appear explicitly is a method aimed at reducing the time to solve the set of equations, this approach is not very effective in reducing the total computation time. Because of this and the fact that the direct technique does not work without proper representation of the loading singularities, the direct calculation method was abandoned. To increase the accuracy of the solution, the number of panels was increased and the equation-solving time became a larger fraction of the total time. However, it seems unlikely that any reasonable accuracy requirement will make the equation-solving time an important consideration.

It appears that the method of changing one strip of panels for calculating the perturbed planform does provide sufficient accuracy and significant time savings. The calculation shows that when using the Vortex Spline Code, this method gives lift coefficient perturbations within 10 percent of those obtained by complete repaneling. The corresponding moment coefficient comparison was not as good, the worst comparisons being for the constant chord increment leading-edge and trailing-edge perturbations. The 30-percent discrepancies are probably acceptable for optimization procedures. The moment

accuracy is affected by the crude scheme used to integrate the moment. Improvements might be seen if a more accurate moment integration scheme were incorporated into the code. More investigation is recommended to see whether or not this error could be reduced.

The computing study found that for a fixed accuracy, the Vortex Spline Code requires about one-fourth as many panels as the USSAERO Code and is therefore considerably faster.

On the basis of the preceding information, we recommend that aerodynamic force and moment gradients with respect to planform perturbation be calculated using the edge repaneling method regardless of the code used. To realize the potential savings in computer time, the code would need modification so that those influence coefficients associated with the edge strip of panels that are perturbed would be recalculated.

Furthermore, we recommend that the Vortex Spline Code be used in order to save additional computing time. Although the time to calculate one influence coefficient is greater than for the USSAERO Code, the smaller number of panels required more than compensates so that a net reduction in computing time is realized.

We recommend that the Vortex Spline Code be modified to fully automate the calculation of force perturbations with planform for a prescribed mean surface using direct differencing of the solutions for two different planforms, the second being obtained from the first by changing one strip of panels. These modifications should not be extensive. They mostly involve automating the repaneling and accommodating the specification of a camber and/or twist distribution. The set of planform variations will need to be determined to implement this code. The ones presented in this report should provide a good base.

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