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## INTRODUCTION

A scaling law for the prediction of the overall flyover noise of a single-stream shock-free circular jet from static experiments has been presented in references 1 and 2. In reference 3, it was extended to coannular jets. In contrast to the earlier scaling laws of Ffowcs Williams (ref. 4) and Ribner (refs. 5 and 6), the whole flow field and the coherence length scale are assumed to be axially stretched in flight. The results are not restricted to isothermal jets since the density terms of the source function are included. These terms are important for hot jets as has been demonstrated in reference 7. As a consequence, the effect of flight on jet noise is considerably different for hot and cold jets. The boundary layers on the inner surface of the nozzle and about the outside of the engine nacelle are likely to influence the flyover jet noise, as briefly discussed in reference 3. This effect is, however, neglected in the present paper.

Whereas overall sound pressure levels are adequate for studying the physics of noise generation, perceived noise levels are more important for aircraft noise certification. They require the determination of the power spectral densities or one-third-octave spectra, for which we must distinguish between flyover and wind tunnel conditions. As a consequence of the motion of the aircraft relative to the observer in the flyover case, we must consider a Doppler frequency shift.

In the following, we first derive the scaling law for the power spectral density of the sound pressure in the far field. This is done in a coordinate system that is fixed with respect to the jet nozzle (wind tunnel coordinate system). The result is then transformed into a coordinate system that is fixed with respect to the ambient fluid (flyover coordinate system). We then derive the corresponding scaling laws for one-third-octave spectra in both coordinate systems. Finally, the scaling law is compared with measurements of one-third-octave spectra of the J85 engine in the Aérotrain (ref. 8) and with unpublished wind tunnel simulation experiments at the National Aerospace Laboratory (NLR), Netherlands, performed by R. Ross. Both experiments were carried out with a short nacelle and therefore thin boundary layers about the outside of the nozzle.

## POWER SPECTRAL DENSITY OF JET NOISE IN FLIGHT

The scaling laws of references 1, 2, and 3 are based on the solution of the convective Lighthill equation in a coordinate system fixed with respect to the jet nozzle. The solution for the sound pressure at the far-field point $x_i$ is given by

$$p'(x_i,t) = \frac{1}{4\pi r_o}\frac{a_o}{a_f^2}\left[\frac{1}{a_f}\int \left.\frac{\partial^2 q_1(y_i,t)}{\partial t^2}\right|_{t_r} dy_i + \int \left.\frac{\partial q_2(y_i,t)}{\partial t}\right|_{t_r} dy_i\right] \tag{1}$$

where $q_1$ and $q_2$ are source terms, $y_i$ is the source point, $r_o$ is wave normal distance from the source (fig. 1), and $a_o$ is ambient velocity of sound. (A list of

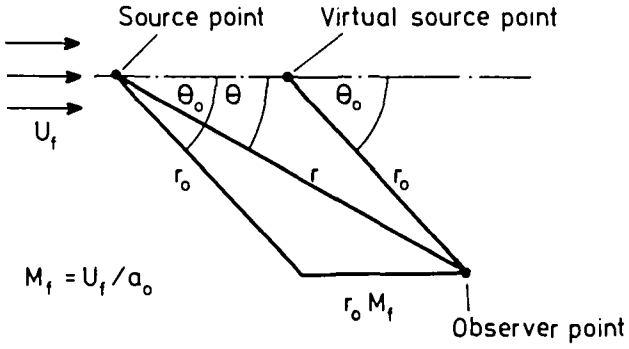


Figure 1.- Relation between observer angle $\theta$, observer distance $r$, emission angle $\theta_o$, and wave normal distance $r_o$ for a source in a moving stream.

symbol definitions is included after the references.) The integrands are evaluated at the retarded time $t_r$ defined by

$$t_r = t - \frac{r_o}{a_o} + \frac{y_{ro}}{a_f} \tag{2}$$

The apparent speed of sound propagation is

$$a_f = a_o(1 + M_f \cos \theta_o) \tag{3}$$

where $M_f$ is flight Mach number. We assume that entropy is conserved along particle path lines, or that the influence of viscosity and thermal conductivity on the sound production is negligible. We also assume that $\left|p'/(\rho_o a_o^2)\right| << 1$ and obtain for $q_1$ and $q_2$

$$q_1 = \rho_o u_{ro}^2 - \left(1 - \frac{\rho_o}{\rho}\right)p' \tag{4}$$

$$q_2 = p' \frac{\partial}{\partial y_{ro}}\left(\frac{\rho_o}{\rho}\right) \tag{5}$$

where $\rho$ is density of the flow and $\rho_o$ is ambient density. The source quantity $q_2$ depends on the density variations in the flow field and is therefore important in hot jets. The subscript on $u_{ro}$, $y_{ro}$, and $\partial/\partial y_{ro}$ indicates the components of $u_i$, $y_i$, and the gradient $\partial/\partial y_i$ taken in the wave normal direction $\theta_o$ of emission which is different from the direction at which the observer point is geometrically located in the far field (fig. 1). The wave normal distance $r_o$ in equations (1) and (2) is also different from the distance $r$ between the coordinate origin and the far-field observer point.

Since the solution (eq. (1)) is stationary random in the chosen coordinate system, we can derive the power spectral density of the pressure fluctuations in the far field of a jet by autocorrelating equation (1) and Fourier transforming the result. This yields

$$W_{WT}(x_i,f) = \frac{a_o^2}{\left(4\pi r_o a_f^2\right)^2} \iint W_{FF}(y_i,\eta_i,f) \exp(-i2\pi f\tau)\, d\eta_i\, dy_i \tag{6}$$

with the cross spectral density

$$W_{FF}(y_i,\eta_i,f) = \int_{-\infty}^{\infty} F(y_i,t)\, F(y_i+\eta_i,t+\tau) \exp(i2\pi f\tau)\, d\tau \tag{7}$$

of a source function

$$F(y_i,t) = \frac{1}{a_f} \frac{\partial^2 q_1}{\partial t^2} + \frac{\partial q_2}{\partial t} \tag{8}$$

where $f$ is frequency and $\eta_i$ is the separation vector between two source points. The source terms $q_1$ and $q_2$ are defined by the equations (4) and (5). The difference between retarded times,

$$\tau = \frac{\eta_{ro}}{a_f} \tag{9}$$

that appears in the interference function (the exponential term in eq. (6)) is due to the difference in retarded times for two different source points. Equation (6) is valid in a coordinate system fixed with respect to the jet nozzle. The observation point $x_i$ in the far field is independent of time $t$. The result, therefore, describes the problem in the wind tunnel coordinate system as denoted by the subscript WT.

## NORMALIZED POWER SPECTRAL DENSITY OF JET NOISE IN FLIGHT

To derive a scaling law that relates the power spectral density of jet noise in flight to the power spectral density of static jet noise, it would be necessary to find a formulation of equation (6) in which the double integral is independent of flight velocity $U_f$. The effect of flight would then be taken into account by a scaling factor outside the integral.

In the present formulation of equation (6), the flight velocity $U_f$ influences the integral via $q_1$, $q_2$, $a_f$, and the sizes of the integration volumes. To reduce the effect of flight on the integral, we normalize with the nozzle diameter $D$, the relative velocity or specific thrust of the jet $\Delta U = U_j - U_f$, and the jet density $\rho_j$ and obtain

$$q_1 = \rho_j \, \Delta U^2 \, \tilde{q}_1 \tag{10}$$

$$q_2 = \frac{\rho_j \, \Delta U^2}{D} \, \tilde{q}_2 \tag{11}$$

$$F = \frac{\rho_j \, \Delta U^3}{D^2} \, \tilde{F} \tag{12}$$

$$W_{FF} = \left(\frac{\rho_j \, \Delta U^3}{D^2}\right)^2 \frac{D}{\Delta U} \, \tilde{W}_{FF} \tag{13}$$

With equations (10) through (13) along with equations (3) and (9), we obtain for equation (6)

$$W_{WT}(x_i, f) = \left(\rho_o a_o^2\right)^2 \frac{D}{a_o}(1 + M_f \cos \theta_o)\left(\frac{\rho_j}{\rho_o}\right)^2 \frac{(U_e/a_o)^5}{(4\pi \tilde{r}_o)^2}$$

$$\times \iint \tilde{W}_{FF} \exp\left(-i2\pi \tilde{f} \frac{U_e}{a_o} \tilde{\eta}_{ro}\right) d\tilde{\eta}_i \, d\tilde{y}_i \tag{14}$$

where $U_e$, the effective jet velocity, is

$$U_e = \frac{\Delta U}{1 + M_f \cos \theta_o} \tag{15}$$

The normalized quantities are

$$\tilde{q}_1 = \tilde{u}_{ro}^2 \frac{\rho_o}{\rho_j} - \left(1 - \frac{\rho_o}{\rho}\right)\tilde{p}' \tag{16}$$

$$\tilde{q}_2 = \tilde{p}' \frac{\partial}{\partial \tilde{y}_{ro}}\left(\frac{\rho_o}{\rho}\right) \tag{17}$$

$$\tilde{F} = \frac{U_e}{a_o} \frac{\partial^2 \tilde{q}_1}{\partial \tilde{t}^2} + \frac{\partial \tilde{q}_2}{\partial \tilde{t}} \tag{18}$$

$$\tilde{W}_{FF} = \int_{-\infty}^{\infty} \tilde{F}\left(\tilde{y}_i, \tilde{t}\right) \tilde{F}\left(\tilde{y}_i + \tilde{\eta}_i, \tilde{t} + \tilde{\tau}\right) \exp(i2\pi \tilde{f} \tilde{\tau}) \, d\tilde{\tau} \tag{19}$$

$$\tilde{f} = fD/\Delta U \qquad \tilde{u}_i = u_i/a_o \tag{20}$$

$$\tilde{t} = t\,\Delta U/D \qquad \tilde{\tau} = \tau\,\Delta U/D \tag{21}$$

$$\tilde{y}_i = y_i/D \qquad \tilde{\eta}_i = \eta_i/D \qquad \tilde{r}_o = r_o/D \tag{22}$$

$$\tilde{p}' = p' \Big/ \left(\rho_j\,\Delta U^2\right) \tag{23}$$

According to equation (18), the function $\tilde{F}$ depends on $U_e/a_o$. The relative importance of the two source functions $\tilde{q}_1$ and $\tilde{q}_2$ can be modeled correctly only for equal $U_e/a_o$. The interference function, which is the exponential term in the integral of equation (14), is also a function of $U_e/a_o$. We would therefore expect that the integral is primarily a function of $U_e/a_o$, although the source quantities $\tilde{q}_1$ and $\tilde{q}_2$ depend on $U_j/a_o$ and $M_f$ separately, too.

## INTRODUCTION OF CONTRACTED COORDINATES

In order to eliminate, approximately, the influence of $U_f$ on the spatial distribution of $\tilde{q}_1$ and $\tilde{q}_2$, a contracted coordinate system must be introduced as was first done in reference 2. It is defined by

$$\tilde{y}_{si} = \left(\tilde{y}_{s1}, \tilde{y}_{s2}, \tilde{y}_{s3}\right) = \left(\tilde{y}_1/\sigma, \tilde{y}_2, \tilde{y}_3\right) \tag{24}$$

where

$$\sigma = 1 + A\,\frac{U_f}{\Delta U} \tag{25}$$

is the stretching factor of the flow field and $A$ is a stretching parameter related to the reciprocal of the normalized phase velocity of the turbulent fluctuations

$$\tilde{u}_p = \frac{U_p - U_f}{\Delta U} \tag{26}$$

A recent stability analysis by Michalke and Hermann (ref. 9) and experimental results of Michel on turbulent structures in a jet indicate that in a spectral treatment of jet turbulence, the normalized frequency $\tilde{f}$ of a jet in forward motion and the Strouhal number $N_{St,s}$ of a static jet are related by

$$N_{St,s} = \frac{f_s D}{U_s} = \frac{\tilde{f}}{\sigma} \tag{27}$$

In addition, $A = 1.4$ seems to be an appropriate value in equation (25). Then in a first approximation, the flow remains similar for small $M_f$. The value, $A = 2$, which was used in reference 2, is only valid for a planar shear layer.

Equation (16) for the source quantity $\tilde{q}_1$ remains unaffected by the axial contraction, whereas $\tilde{q}_2$ (eq. (17)) is influenced by the transformation (eq. (24)) as follows:

$$\tilde{q}_2 = \tilde{p}' \frac{\partial}{\partial \tilde{y}_{sro}}\left(\frac{\rho_o}{\rho}\right) + \frac{1 - \sigma}{\sigma} \cos \theta_o \frac{\partial}{\partial \tilde{y}_{s1}}\left(\frac{\rho_o}{\rho}\right) \tag{28}$$

In heated jets, the axial density gradient is smaller than the radial one. In addition, $\left[(1 - \sigma)/\sigma\right] \cos \theta_o < 1$. We may therefore approximate equation (28) by equation (17). Then both source quantities $\tilde{q}_1$ and $\tilde{q}_2$ remain the same in the contracted coordinate system for small $M_f$. The same is true for the function $\tilde{F}$ (eq. (18)), provided that $U_e$ is kept constant.

The complex cross spectral density $\tilde{W}_{FF}$ of $\tilde{F}$ (eq. (19)) may be defined by its magnitude and its phase

$$\tilde{W}_{FF} = \left|\tilde{W}_{FF}\right| \exp i\psi \tag{29}$$

We assume that Michel's previously mentioned experimental results (see eq. (27)) for the cross spectral density of the turbulent pressure fluctuations can also be applied to $\tilde{W}_{FF}$. The dependence of $\left|\tilde{W}_{FF}\right|$ on the frequency may then be assumed to be universal for small $M_f$ and constant contracted coordinates $\tilde{y}_{si}$ and $\tilde{y}_{si} + \tilde{\eta}_{si}$ if $\left|\tilde{W}_{FF}\right|$ is plotted as function of $\tilde{f}/\sigma$. However, $\left|\tilde{W}_{FF}\right|$ may change in magnitude by a $U_f$-dependent factor. Therefore $\left|\tilde{W}_{FF}\right|$ in the flight case can be expressed by a corresponding static value:

$$\left|\tilde{W}_{FF}\left(\tilde{f}, \tilde{y}_i, \tilde{\eta}_i, \theta_o, U_j/a_o, M_f\right)\right| = \left|\tilde{W}_{FF}\left(N_{St,s}, \tilde{y}_{si}, \tilde{\eta}_{si}, \theta_o, U_e/a_o, 0\right)\right| \frac{\sigma_1}{\sigma} \tag{30}$$

where $\sigma_1$ can be interpreted as the ratio of the mean square values of $\tilde{F}$ in flight to the corresponding value for the equivalent static jet. This ratio seems to be fairly constant in the flow region that is important for jet noise.

In references 1, 2, and 3, the mean square values of $\tilde{F}$ were assumed to be equal in the flight and static cases leading to $\sigma_1 = 1$. The authors found that the peak fluctuation level of the axial velocity component in the experiment of Tanna and Morris (ref. 10) can best be approximated by $\sigma_1 = \sqrt{\sigma}$. The same result seems to fit Michel's experimental data best. Equal power spectral densities of $\tilde{F}$ for $\tilde{f}$ in the flight case and for $f_s D/U_e$ in the static case would lead to an increase in the mean square values of $\tilde{F}$, i.e., $\sigma_1 = \sigma$.

For axially displaced positions ($\tilde{\eta}_1 \neq 0$), the phase $\psi$ of $\tilde{W}_{FF}$ is mainly determined by the axial convection of turbulence. This leads to an almost linear dependence of $\psi$ on $\tilde{\eta}_1$:

$$\psi\left(\tilde{f},\tilde{y}_i,\tilde{\eta}_i,\theta_o,U_j/a_o,M_f\right) = \psi_1\left(\tilde{f},\tilde{y}_i,\tilde{\eta}_i,\theta_o,U_j/a_o,M_f\right) + 2\pi\tilde{f}\tilde{\eta}_1 \frac{\Delta U}{U_p} \tag{31}$$

It is reasonable to assume and also confirmed by experiments that the normalized phase velocity $\tilde{u}_p$ of equation (26) is no explicit function of $U_f$ or $U_j$:

$$\tilde{u}_p = \tilde{u}_p\left(\tilde{f}/\sigma,\tilde{y}_{si},\tilde{\eta}_{si}\right) \tag{32}$$

Since the convection effect is considered by the separate second term of equation (31), we assume $\psi_1$ to be independent of $U_f$ and $U_j$ in the contracted coordinate system:

$$\psi_1\left(\tilde{f},\tilde{y}_i,\tilde{\eta}_i,\theta_o,U_j/a_o,M_f\right) = \psi_{1s}\left(\tilde{f}/\sigma,\tilde{y}_{si},\tilde{\eta}_{si},\theta_o\right) \tag{33}$$

With equations (26) and (30) through (33) the cross spectral density $\tilde{W}_{FF}$ can be written as

$$\tilde{W}_{FF} = \frac{\sigma_1}{\sigma}\left|\tilde{W}_{FF}\left(N_{St,s},\tilde{y}_{si},\tilde{\eta}_{si},\theta_o,U_e/a_o,0\right)\right| \times \exp\left\{i\left[\psi_{1s}\left(\tilde{f}/\sigma,\tilde{y}_{si},\tilde{\eta}_{si},\theta_o\right) + 2\pi\frac{\tilde{f}}{\sigma}\tilde{\eta}_1 \frac{\sigma}{\tilde{u}_p\left(\tilde{f}/\sigma,\tilde{y}_{si},\tilde{\eta}_{si}\right) + \frac{U_f}{\Delta U}}\right]\right\} \tag{34}$$

Considering that with equation (24), $d\tilde{y}_i = \sigma\, d\tilde{y}_{si}$ and $d\tilde{\eta}_i = \sigma\, d\tilde{\eta}_{si}$, we obtain from equation (14) and the foregoing results

$$W_{WT}\left(\tilde{r}_o,\theta_o,\tilde{f},U_j/a_o,M_f\right) = \sigma_1\sigma(1 + M_f \cos\theta_o)W \tag{35}$$

where

$$W = \left(\rho_o a_o^2\right)^2 \frac{D}{a_o}\left(\frac{\rho_j}{\rho_o}\right)^2 \frac{(U_e/a_o)^5}{(4\pi\tilde{r}_o)^2}\tilde{W} \tag{36}$$

$$\tilde{W} = \iint \left| \tilde{W}_{FF}\left(N_{St,s}, \tilde{y}_{si}, \tilde{\eta}_{si}, \theta_o, U_e/a_o, 0\right) \right| \exp\left(i\psi_{1s} + i2\pi \frac{\tilde{f}}{\sigma} \tilde{\tau}_1\right) d\tilde{\eta}_{si}\, d\tilde{y}_{si} \qquad (37)$$

The time difference $\tilde{\tau}_1$ is determined by the exponential term in equation (14) and the second part of the exponential term in equation (34)

$$\tilde{\tau}_1 = \tilde{\eta}_1 \frac{\sigma}{\tilde{u}_p + \frac{U_f}{\Delta U}} - \frac{U_e}{a_o} \tilde{\eta}_{ro}\sigma \qquad (38)$$

where $\tilde{\eta}_{ro}$ may be expressed in terms of $\tilde{\eta}_1$ and $\tilde{\eta}_n$ where $\tilde{\eta}_n$ is the component of $\tilde{\eta}_i$ that is normal to $\tilde{\eta}_1$:

$$\tilde{\eta}_{ro} = \tilde{\eta}_1 \cos\theta_o + \tilde{\eta}_n \sin\theta_o \qquad (39)$$

The transformation (eq. (24)) yields for $\tilde{\tau}_1$ with equation (39)

$$\tilde{\tau}_1 = \sigma\left[\tilde{\eta}_{s1}\left(\frac{\frac{1}{\tilde{u}_p}\frac{\sigma}{1 + \frac{1}{\tilde{u}_p}\frac{U_f}{\Delta U}} - \sigma\frac{U_e}{a_o}\cos\theta_o}{}\right) - \tilde{\eta}_{sn}\frac{U_e}{a_o}\sin\theta_o\right] \qquad (40)$$

While the $U_f$-dependence of $\tilde{q}_1$ and $\tilde{q}_2$ was eliminated with the introduction of the contracted coordinate system (eq. (24)), this is impossible for the $U_f$-dependence of the time difference $\tilde{\tau}_1$.

The relative phase velocity $\tilde{u}_p$ depends on frequency and position and varies between approximately 0.6 and 0.8. In spite of this variation, the ratio $\sigma\Big/\left[1 + U_f/(\tilde{u}_p\, \Delta U)\right]$ is nearly 1 for $A = 1.4$ in equation (25). Equation (40) may then be simplified to

$$\tilde{\tau}_1 = \sigma\left[\tilde{\eta}_{s1}\left(\frac{1}{\tilde{u}_p} - \sigma\frac{U_e}{a_o}\cos\theta_o\right) - \tilde{\eta}_{sn}\frac{U_e}{a_o}\sin\theta_o\right] \qquad (41)$$

For the equivalent static case, $\sigma = 1$ and we obtain

$$\tilde{\tau}_1 = \tilde{\tau}_{1,s} + (\sigma - 1)\left\{\tilde{\eta}_{s1}\left[\frac{1}{\tilde{u}_p} - (1 + \sigma)\frac{U_e}{a_o}\cos\theta_o\right] - \tilde{\eta}_{sn}\frac{U_e}{a_o}\sin\theta_o\right\} \qquad (42)$$

## SCALING LAW FOR THE POWER SPECTRAL DENSITY OF JET NOISE IN FLIGHT

The scaling law can be derived if equation (35) is once evaluated for the flight case at the frequency $\tilde{f}$ and once for the static case with the effective velocity $U_e$ given by equation (15) and the Strouhal number $N_{St,s} = \tilde{f}/\sigma$ given by equation (27). This yields

$$W_{WT}\left(\tilde{r}_o, \theta_o, \tilde{f}, U_j/a_o, M_f\right) = \sigma_1 \sigma B (1 + M_f \cos\theta_o)\, W_{WT}\left(\tilde{r}_o, \theta_o, N_{St,s}, U_e/a_o, 0\right) \tag{43}$$

Here B, the influence of source coherence in flight, is the ratio of $\tilde{W}$ of equation (37) for the flight and static cases:

$$B = \frac{\iint \left|\tilde{W}_{FF}\left(N_{St,s}, \tilde{y}_{si}, \tilde{\eta}_{si}, \theta_o, U_e/a_o, 0\right)\right| \exp\left(i\psi_{1s} + i2\pi N_{St,s}\tilde{\tau}_1\right) d\tilde{\eta}_{si}\, d\tilde{y}_{si}}{\iint \left|\tilde{W}_{FF}\left(N_{St,s}, \tilde{y}_{si}, \tilde{\eta}_{si}, \theta_o, U_e/a_o, 0\right)\right| \exp\left(i\psi_{1s} + i2\pi N_{St,s}\tilde{\tau}_{1,s}\right) d\tilde{\eta}_{si}\, d\tilde{y}_{si}} \tag{44}$$

Numerator and denominator differ only in the time difference $\tilde{\tau}_1$ and $\tilde{\tau}_{1,s}$. In a jet, $|\tilde{W}_{FF}|$ decays to very small values within a certain separation distance $\tilde{\eta}_i$. We would have $B = 1$ if inside of this distance the condition

$$\left|2\pi N_{St,s}\, \Delta\tilde{\tau}\right| \ll 1 \tag{45}$$

would be fulfilled where, according to equation (42), the difference $\Delta\tilde{\tau}$ in the retarded times between the flight and the static cases is

$$\begin{aligned} \Delta\tilde{\tau} &= \tilde{\tau}_1 - \tilde{\tau}_{1,s} \\ &= (\sigma - 1)\left\{\tilde{\eta}_{sl}\left[\frac{1}{\tilde{u}_p} - (1 + \sigma)\frac{U_e}{a_o}\cos\theta_o\right] - \tilde{\eta}_{sn}\frac{U_e}{a_o}\sin\theta_o\right\} \end{aligned} \tag{46}$$

This difference may vanish for a given $\tilde{\eta}_{si}$ at a certain angle $\theta_o$. If we neglect the radial extension of the jet ($\tilde{\eta}_{sn} \approx 0$), this angle would be given by

$$\cos\theta_o = \frac{1}{\tilde{u}_p(1 + \sigma)\frac{\Delta U}{a_o} - M_f} \tag{47}$$

For the radially extended jet, we might expect the smallest deviation from $B = 1$ in the direction of this angle.

For $\theta_o = 180^o$, the direction of flight, $\Delta\tilde{\tau}$ reaches a maximum given by

$$\Delta\tilde{\tau}_{max} = \tilde{\eta}_{sl}(\sigma - 1)\left[\frac{1}{\tilde{u}_p} + (1 + \sigma)\frac{\Delta U}{a_o(1 - M_f)}\right] \tag{48}$$

The values $A = 1.4$, $\tilde{u}_p = 1/A$, $M_f = 0.24$, and $\Delta U/a_o = 1.06$ yield $\theta_o = 49^o$ from equation (47) and $\Delta\tilde{\tau}_{max} = 1.47\tilde{\eta}_{sl}$ from equation (48). Inserting the latter value in equation (45), we obtain $\tilde{\eta}_{sl} = \eta_{sl}/D \approx 0.11N_{St,s}$ for the maximum allowable separation with nonzero $|\tilde{W}_{FF}|$. This condition can be fulfilled only with a "small eddy" turbulence. Forward of the angle with smallest deviation from $B = 1$, the interference in the numerator of equation (44) is greater than in the denominator leading to $B < 1$. In the rearward direction, on the other hand, we would expect the opposite, $B > 1$.

According to equation (43), we can determine the power spectral density of the far-field jet noise in flight from the power spectral density of a static jet within ±1 dB by neglecting the influence of $B$ as long as $0.79 \leq B \leq 1.26$. The jet velocity of the equivalent static jet is determined by equation (15) and the Strouhal number $N_{St,s}$ by equation (27) with $U_s = U_e$. These two equations also define the frequency $f_s$ at which the noise of the equivalent static jet has to be analyzed:

$$f_s = \frac{f}{\sigma(1 + M_f \cos\theta_o)} \tag{49}$$

Whereas the Strouhal number of the equivalent static jet is equal to $\tilde{f}/\sigma$ for all angles, the velocity and the frequency vary with emission angle.

We might mention here that Morfey (ref. 11) proposed to introduce an effective emission angle $\theta_e$ via the definition $\cos\theta_e = \sigma\cos\theta_o$ in order to eliminate the influence of $U_f$ on the retarded time. From equation (41), it can now be seen that for the present paper this definition would have to be adjusted to $\cos\theta_e = \sigma^2\cos\theta_o - (\sigma - 1)/(\tilde{u}_p U_e/a_o)$ as a consequence of the frequency shift according to equation (27). The radial extension of the jet would have to be neglected. The double integral of equation (37) would be a function of both angles $\theta_e$ (via $\tilde{\tau}_1$) and $\theta_o$ (via $|\tilde{W}_{FF}|$). In order to eliminate the parameter $\theta_o$, we would have to evaluate $|\tilde{W}_{FF}|$ at the angle $\theta_e$ instead of $\theta_o$ and neglect the error of this procedure. This might be permissible for small differences $\theta_e - \theta_o$. Unfortunately, $\theta_e - \theta_o$ becomes large for stretching factors $\sigma$ up to a value of 2. In addition, $\theta_e$ is not defined outside a limited range of $\theta_o$.

## SCALING LAW FOR THE POWER SPECTRAL DENSITY OF FLYOVER JET NOISE

For flyover jet noise, we must consider the Doppler shift of the frequency. This is accompanied by a change in the level of the power spectral density since the same power is distributed over a different frequency bandwidth. We, therefore, must consider the relation

$$W_{FO}\left(\tilde{r}_o,\theta_o,\tilde{f},U_j/a_o,M_f\right) = (1 + M_f \cos\theta_o)\, W_{WT}\left(\tilde{r}_o,\theta_o,\tilde{f}_o,U_j/a_o,M_f\right) \tag{50}$$

where the wave normal frequency is

$$\tilde{f}_o = \tilde{f}(1 + M_f \cos\theta_o) \tag{51}$$

In the forward arc ($\theta_o > 90^o$), $\tilde{f}_o < \tilde{f}$ and $W_{WT} > W_{FO}$. If we assume that $B = 1$, the scaling law for the flyover noise that corresponds to equation (43) is

$$W_{FO}\left(\tilde{r}_o,\theta_o,\tilde{f},U_j/a_o,M_f\right) = \sigma_1\sigma(1 + M_f \cos\theta_o)^2\, W_{FO}\left(\tilde{r}_o,\theta_o,N_{St,s,o},U_e/a_o,0\right) \tag{52}$$

Compared to equation (43) for the wind tunnel case, equation (52) now contains the square of the factor $(1 + M_f \cos\theta_o)$. The frequency at which the equivalent static experiment has to be analyzed is

$$f_{s,o} = f_s(1 + M_f \cos\theta_o) = \frac{f}{\sigma} \tag{53}$$

and the Strouhal number of the equivalent static jet is

$$N_{St,s,o} = \frac{f_{s,o}D}{U_e} = \frac{fD}{\Delta U}\,\frac{1 + M_f \cos\theta_o}{\sigma} \tag{54}$$

In the flyover case, it is the frequency of the equivalent static jet that is independent of the emission angle $\theta_o$, while $U_e/a_o$ and $N_{St,s,o}$ vary.

## SCALING LAWS FOR ONE-THIRD-OCTAVE SPECTRA

One-third-octave spectra of the sound pressure describe the mean square pressure in frequency bands with constant relative bandwidth $\Delta f/f_c$ where $f_c = \sqrt{f_1 f_2}$ is the geometric center frequency and $\Delta f = f_2 - f_1$ is the difference between the two limits of the frequency band. For one-third-octave spectra, $f_2/f_1 = 2^{1/3}$.

By integrating the power spectral density in wind tunnel coordinates (eq. (43)) with $B = 1$ between the limits $f_1 = \sigma f_{s1}$ and $f_2 - \sigma f_{s2}$, we obtain the scaling law for the mean square pressure in one-third-octave bands:

$$P_{WT}\left(\tilde{r}_o,\theta_o,\tilde{f}_c,U_j/a_o,M_f\right) = \sigma_1\sigma^2(1 + M_f \cos\theta_o)^2\, P_{WT}\left(\tilde{r}_o,\theta_o,N_{St,c,s},U_e/a_o,0\right) \tag{55}$$

where with equations (35) through (37)

$$P_{WT}\left(\tilde{r}_o,\theta_o,N_{St,c,s},U_e/a_o,0\right) = \left(\rho_o a_o^2\right)^2\left(\frac{\rho_j}{\rho_o}\right)^2 \frac{\left(U_e/a_o\right)^6}{\left(4\pi\tilde{r}_o^2\right)^2} \times \int_{N_{St,s1}}^{N_{St,s2}} \tilde{W}\left(\theta_o,N_{St,s},U_e/a_o\right)\, dN_{St,s} \tag{56}$$

Note that the exponents of $\sigma$ and of $(1 + M_f \cos\theta_o)$ both have changed to 2 in equation (55) as compared to equation (43) for the power spectral density. The filter frequency $f_{c,s}$ and the Strouhal number $N_{St,c,s}$ of the equivalent static jet are

$$f_{c,s} = \frac{f_c}{\sigma(1 + M_f \cos\theta_o)} \tag{57}$$

$$N_{St,c,s} = \frac{f_{c,s}D}{U_e} = \frac{f_c D}{\Delta U}\frac{1}{\sigma} \tag{58}$$

The corresponding scaling law for the one-third-octave spectra for flyover noise can be derived from equation (52):

$$P_{FO}\left(\tilde{r}_o,\theta_o,\tilde{f}_c,U_j/a_o,M_f\right) = \sigma_1\sigma^2(1 + M_f \cos\theta_o)^2\, P_{FO}\left(\tilde{r}_o,\theta_o,N_{St,c,s,o},U_e/a_o,0\right) \tag{59}$$

where

$$f_{c,s,o} = \frac{f_c}{\sigma} \tag{60}$$

and

$$N_{St,c,s,o} = \frac{f_{c,s,o}D}{U_e} = \frac{f_c D}{\Delta U}\frac{1 + M_f \cos\theta_o}{\sigma} \tag{61}$$

Unlike the power spectral densities, the scaling laws for one-third-octave spectra are equal in wind tunnel coordinates (eq. (55)) and flyover coordinates (eq. (59)), except that we must consider the Doppler frequency shift in the flyover case.

## SCALING LAW FOR THE OVERALL SOUND PRESSURE

The overall sound pressure $\overline{p'^2}$ is obtained if equation (43) is integrated over the whole frequency range. As in equations (55) and (59), this yields the same results in wind tunnel coordinates and flyover coordinates for B = 1:

$$\overline{p'^2}\left(\tilde{r}_o,\theta_o,U_j/a_o,M_f\right) = \sigma_1\sigma^2(1 + M_f \cos\theta_o)^2\,\overline{p'^2}\left(\tilde{r}_o,\theta_o,U_e/a_o,0\right) \tag{62}$$

This equation differs from the corresponding scaling laws in references 1, 2, and 3 by the factor $\sigma_1$. In these references, the mean square values of the normalized turbulent flow quantities were assumed to remain independent of $U_f$ in a contracted coordinate system. This assumption has now been relaxed by allowing the mean square values to increase in flight by the factor $\sigma_1$. This seems to agree better with Michel's experimental results and with those of reference 10.

## THE DIFFERENCE BETWEEN THE SOUND POWER LEVEL IN ONE-THIRD-OCTAVE BANDS AND THE OVERALL SOUND POWER LEVEL

Prediction schemes such as that in reference 12 relate the sound power in one-third-octave bands to the overall sound power:

$$\Delta SPL(f_c) = SPL(f_c) - OASPL = (10\text{ dB})\log_{10}\frac{P(f_c)}{\overline{p'^2}} \tag{63}$$

Equation (55) in wind tunnel coordinates as well as equation (59) in flyover coordinates yield the following result for the relative one-third-octave spectra:

$$\Delta SPL\left(\theta_o,\tilde{f}_c,U_j/a_o,M_f\right) = \Delta SPL\left(\theta_o,N_{St,c,s},U_e/a_o,0\right) \tag{64}$$

where the Strouhal number $N_{St,c,s}$ is defined by

$$N_{St,c,s} = \frac{f_{c,s}D}{U_e} = \begin{cases} \tilde{f}_c/\sigma & \text{(Wind tunnel)} \quad (65) \\ \tilde{f}_c(1 + M_f \cos\theta_o)/\sigma & \text{(Flyover)} \quad (66) \end{cases}$$

The relative one-third-octave spectra of the equivalent static jets are shifted on the logarithmic frequency axis according to equations (65) and (66). Their shapes remain unchanged, however. The scaling law (eq. (64)) is especially independent of the actual values of $\sigma$ and $\sigma_1$ in equations (55), (59), and (62).

There is considerable experimental evidence that for a given emission angle $\theta_o$, for a constant temperature ratio between jet and ambient air, and for jet velocities $U_e < 1.5a_o$, the one-third-octave spectrum $\Delta SPL$ of a static jet is a function of its Strouhal number $N_{St,c,s}$ only. We may then neglect the parameters $U_j/a_o$ and $U_e/a_o$ in equation (64):

$$\Delta SPL\left(\theta_o,\tilde{f}_c,M_f\right) = \Delta SPL\left(\theta_o,N_{St,c,s},0\right) \tag{67}$$

Usually comparisons between static jet noise and jet noise in flight are made for a fixed jet velocity $U_j$. It may be concluded from equation (67) that the peak frequency $f_{p,f}$ for $U_f > 0$ and $f_{p,o}$ for $U_f = 0$ have to fulfill the condition $N_{St,c,o} = f_{p,o}D/U_j = \tilde{f}_{c,f}$. We then obtain with equations (65) and (66)

$$\frac{f_{p,o}D}{U_j} = \begin{cases} f_{p,f}D\Big/\left[\sigma(U_j - U_f)\right] & \text{(Wind tunnel)} \\ f_{p,f}D(1 + M_f \cos\theta_o)\Big/\left[\sigma(U_j - U_f)\right] & \text{(Flyover)} \end{cases} \tag{68}$$

The ratio of the peak frequencies is, therefore,

$$\frac{f_{p,f}}{f_{p,o}} = \begin{cases} \sigma(U_j - U_f)/U_j & \text{(Wind tunnel)} \\ \sigma(U_j - U_f)\Big/\left[U_j(1 + M_f \cos\theta_o)\right] & \text{(Flyover)} \end{cases} \tag{69}$$

Equation (25) further yields

$$\sigma \frac{U_j - U_f}{U_j} = 1 + (A - 1)\frac{U_f}{U_j} \tag{70}$$

From this equation we can infer that the frequency of the spectral peak at $\theta_o = 90^\circ$ is not reduced in flight, as one would expect from the reduced relative velocity $U_j - U_f$, but instead is weakly increased.

## COMPARISONS WITH EXPERIMENTS

In reference 2, the overall sound pressure levels measured with the Aérotrain (ref. 8) were shown to compare well with the scaling law (eq. (62)) with $\sigma_1 = 1$. The stretching parameter $A = 2$ was assumed to determine $\sigma = 1 + AU_f/\Delta U$. According to our new theoretical and experimental results, $A = 1.4$ seems to be more appropriate for the circular jet. This reduction of $\sigma$ is partly compensated in equation (62) by the factor $\sigma_1 > 1$. The appropriate value for $\sigma_1$ has yet to be determined, but for small $U_f/\Delta U$, $\sigma_1 = \sigma$ and $A = 1.4$ give results nearly identical to those in reference 2.

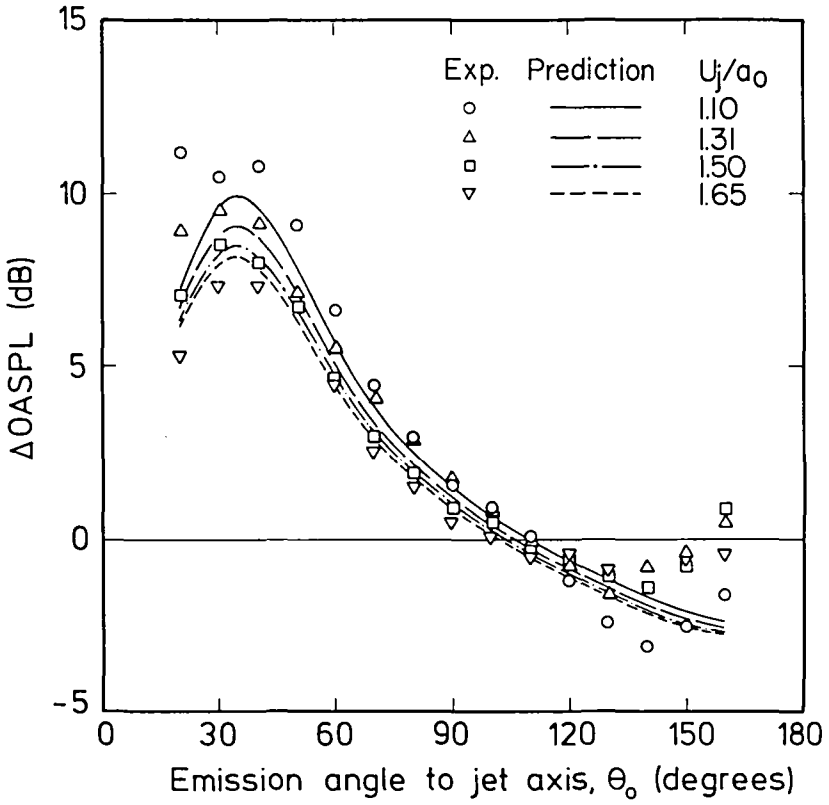


Figure 2.- Measured jet noise reduction levels (ref. 8) compared with predictions using equation (62) and the jet velocity power law extrapolation of the measured static jet noise data. $M_f = 0.24$; $A = 1.4$.

Figure 2 demonstrates how well the Aérotrain "flyover" measurements can be scaled from a power law for the static jet for $A = 1.4$ and $\sigma_1 = \sigma$. The figure displays the measured and predicted reductions in OASPL between flight and static noise:

$$\Delta\text{OASPL} = (10\text{ dB})\ \log_{10} \frac{\overline{p'^2}\left(\tilde{r}_o, \theta_o, U_j/a_o, 0\right)}{\overline{p'^2}\left(\tilde{r}_o, \theta_o, U_j/a_o, M_f\right)} \tag{71}$$

A positive value indicates a noise reduction with increasing flight velocity. For angles $\theta_o > 150^o$, the measured reduction levels are higher than the predicted ones. This somewhat too high "forward arc lift" of the prediction might be explainable with equation (44). As discussed earlier, $B < 1$ is expected for large $\theta_o$ if a small scale model does not completely describe jet turbulence with respect to noise generation.

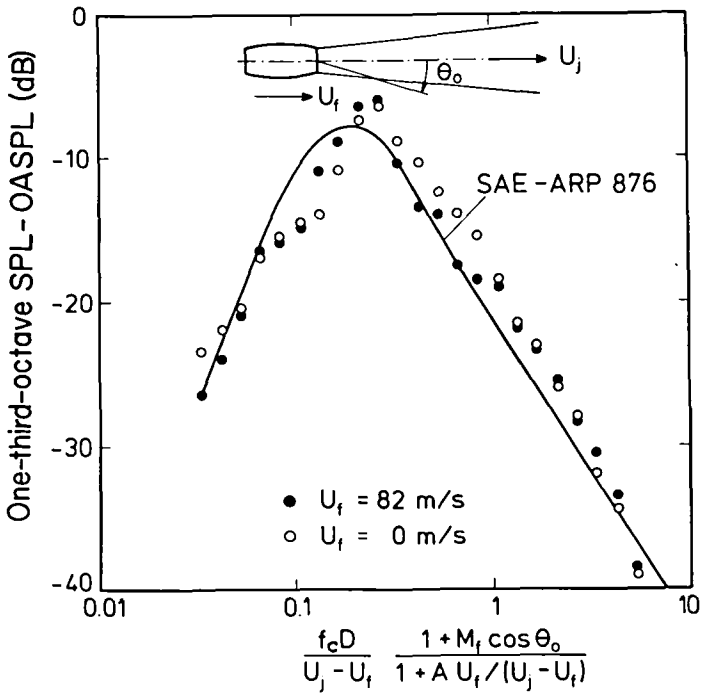


Figure 3.- Relative one-third-octave spectra of static and flyover jet noise (ref. 8) for an emission angle $\theta_o = 30^o$ relative to the jet axis. A = 1.4; $U_j$ = 440 m/s.

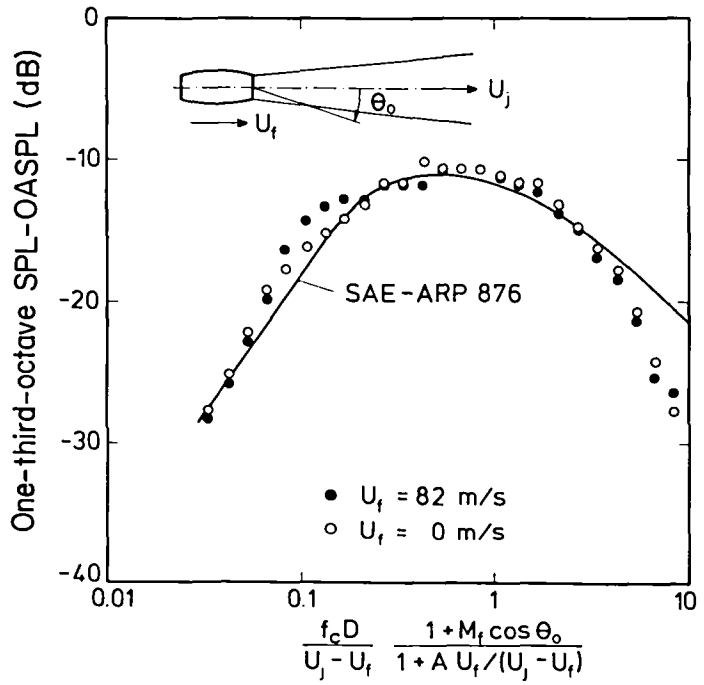


Figure 4.- Relative one-third-octave spectra of static and flyover jet noise (ref. 8) for an emission angle $\theta_o = 90^o$ relative to the jet axis. A = 1.4; $U_j$ = 440 m/s.

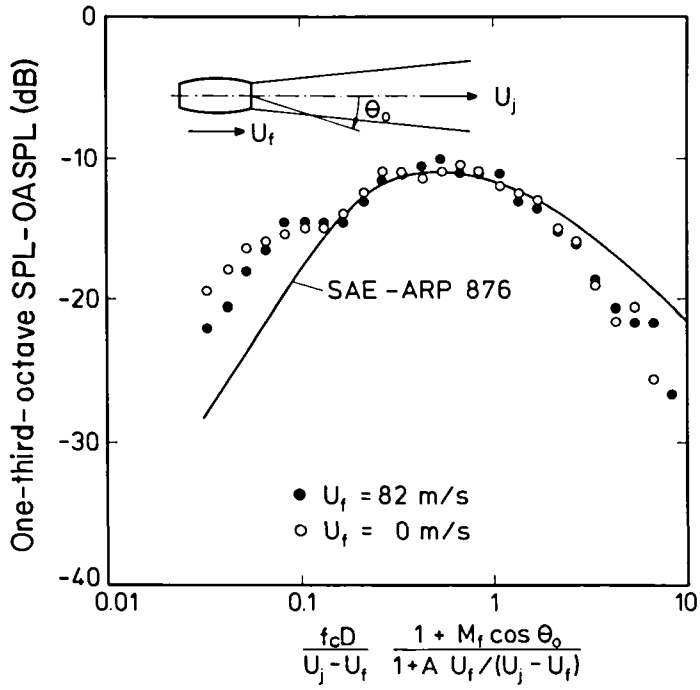


Figure 5.- Relative one-third-octave spectra of static and flyover jet noise (ref. 8) for an emission angle $\theta_o = 150^o$ relative to the jet axis. $A = 1.4$; $U_j = 440$ m/s.

The comparison is now extended to the spectra. Figures 3, 4, and 5 display relative one-third-octave spectra measured with the Aérotrain. Comparisons are made between spectra for static and "flyover" noise at three emission angles: $\theta_o = 30^o$, $90^o$, and $150^o$. According to equation (61), the spectra are plotted as a function of $f_c D(1 + M_f \cos \theta_o)/(\Delta U\, \sigma)$. This expression is the Strouhal number for the static jet.

The figures also include the predicted spectra according to SAE ARP 876 (ref. 12). All figures demonstrate that the flyover spectra can be directly derived from the static spectra.

In figures 6 through 9, the same comparison is made for the relative spectra measured in a wind tunnel flight simulation experiment. The data were supplied by R. Ross. The experimental setup consists of a short nacelle enclosing the nozzle. A hot jet is generated with liquid $H_2O_2$ that reacts with a catalyzer. To our knowledge, this is the first flight simulation experiment with thin boundary layers on the inside and about the outside of the jet nozzle.

From equations (68) and (70), we would expect that the peak frequency would remain nearly unaffected by the forward motion at all angles. In the figures the spectra are plotted as a function of $fD/(\Delta U\, \sigma)$ as indicated by equation (58). Figure 6 for $\theta_o = 30^o$ shows a collapse of the spectral peaks. At high frequency, the flight levels are elevated, however. This might be a consequence of noise scattering in the tunnel shear layer. The other three figures for $\theta_o = 60^o$, $90^o$, and $125^o$ display a very good collapse of the spectra. This even includes some dips which are unusual for jet mixing.

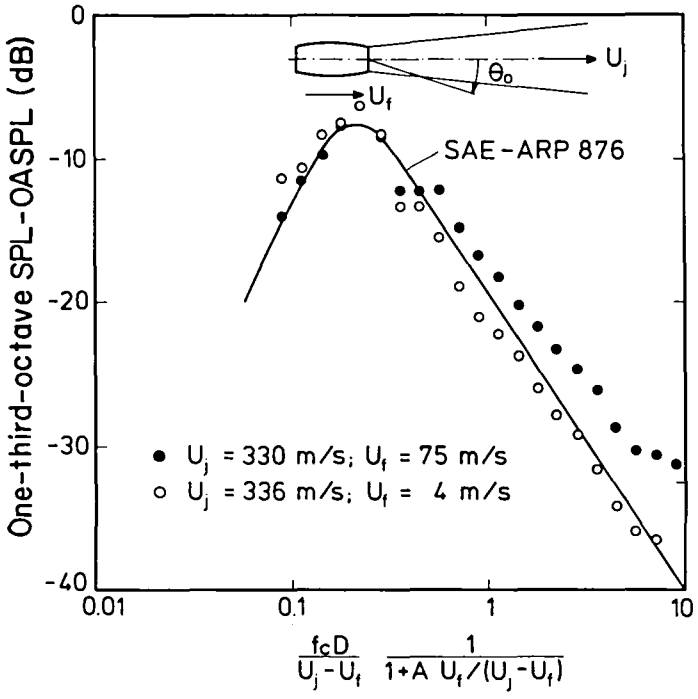


Figure 6.- Relative one-third-octave spectra of jet noise in an NLR wind tunnel flight simulation for an emission angle $\theta_o = 30^o$ relative to the jet axis. A = 1.4.

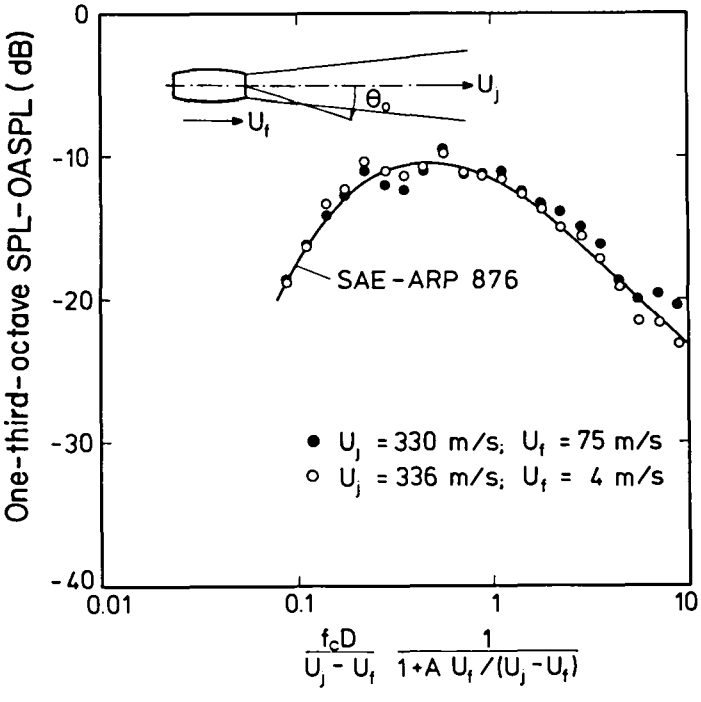


Figure 7.- Relative one-third-octave spectra of jet noise in an NLR wind tunnel flight simulation for an emission angle $\theta_o = 60^o$ relative to the jet axis. A = 1.4.

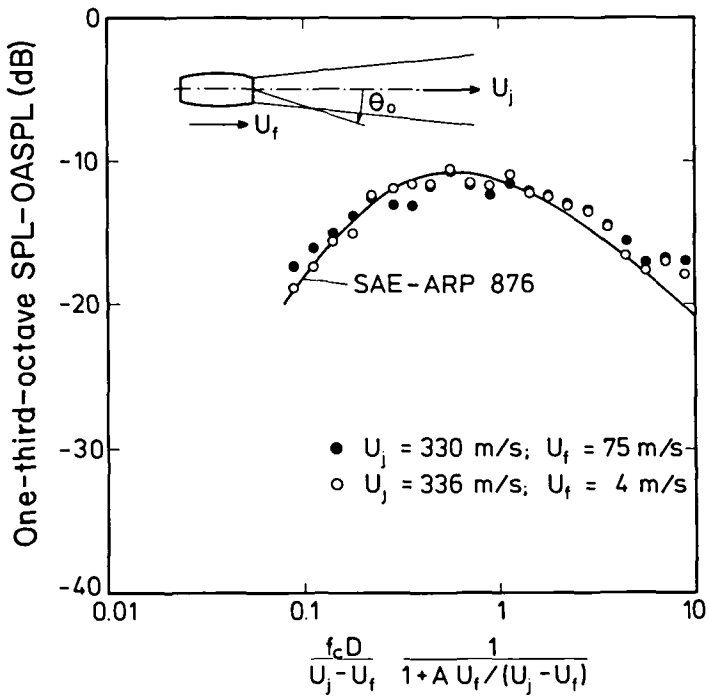


Figure 8.- Relative one-third-octave spectra of jet noise in an NLR wind tunnel flight simulation for an emission angle $\theta_o = 90^o$ relative to the jet axis. A = 1.4.

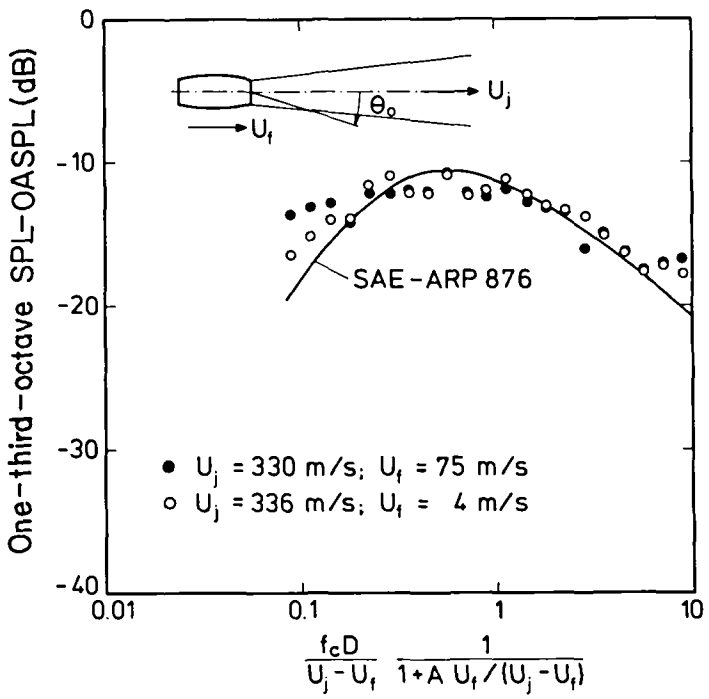


Figure 9.- Relative one-third-octave spectra of jet noise in an NLR wind tunnel flight simulation for an emission angle $\theta_o = 125^o$ relative to the jet axis. A = 1.4.

## DISCUSSION

In our earlier paper (ref. 2) on the prediction of overall jet noise, we had to introduce assumptions about the influence of flight on the source cross correlation. These had to be replaced in the present paper by assumptions on the source cross spectral density. The new assumptions were based on experimental results about the influence of flight on jet turbulence and on analytical results about the instability of a jet with an external flow (ref. 9). Both the analytical and experimental results indicate a shift of the frequency to higher values in flight by the same factor $\sigma$ by which the source region is axially stretched in flight. The experimental results also reveal an increase in the normalized mean square values of the turbulent fluctuations which is considered by the factor $\sigma_1$ in equation (62). The increase is offset by a somewhat smaller value for the stretching parameter A that determines $\sigma$ in equation (25). For small flight Mach numbers and $\sigma_1 = \sigma$, the new law gives approximately the same results as the old one. The favorable comparisons of the scaling law with experimental results for the overall sound intensity is therefore not affected by this correction.

Without the frequency shift of the source cross spectral density, a reduced frequency had been expected in flight due to the reduced relative velocity. With the frequency shift, we can now explain the surprising experimental result that the frequency of the spectral peak of the far-field jet noise at $\theta_o = 90^o$ remains nearly unchanged in flight.

It is known that the peak of a jet noise spectrum depends on the emission angle $\theta_o$. The peak frequencies are, however, generally different for flyover and static noise. The relation between these two frequencies was considered to be very complicated in reference 13. According to the present result (eq. (69)), the ratio of these frequencies is, surprisingly, inversely proportional to the Doppler factor $(1 + M_f \cos \theta_o)$.

In references 2 and 7, the good agreement between the results from experiments and from the flight effect scaling law indicated that Lighthill's approach can also be successfully used for hot jets. It is a new insight into the mechanisms of sound generation in jets that the pressure-density gradient term $q_2$, but not the Reynolds stress term $q_1$, dominates the noise of hot jets. A thorough discussion of this problem was given by Ribner (ref. 14) in his lecture.

The extension of the scaling law to the spectra might now indirectly indicate the importance of large scale turbulence with respect to jet noise. Our scaling law has been derived with the assumption that the normalized turbulence structure is preserved in flight, whatever structure dominates the jet. To obtain the simple result of equation (43) with $B = 1$, we have had to neglect a phase difference determined by $\Delta\tilde{\tau}$ (eq. (46)). This difference is small for small $\theta_o$ (rear arc) but large for $\theta_o \to 180^o$ and should lead to an overprediction of flyover noise. It has been shown theoretically (ref. 15) and also found experimentally (ref. 16) that for angles $\theta_o < 45^o$ and $\theta_o > 135^o$, the large scale structure may play an important role for the noise radiation of subsonic jets. In fact, the observed overprediction of up to 2 dB of the Aérotrain measurements at the angle $\theta_o = 160^o$ (ref. 2) might be a consequence of that influence.

## CONCLUDING REMARKS

In the present paper, the theoretical analysis of the influence of flight effects on the overall jet noise radiation intensity as given in an earlier paper by the authors has been modified by introducing a new set of assumptions concerning the source cross spectral density. These new assumptions are based on experimental results about the influence of flight on jet turbulence and on analytical results about the instability of a jet with external flow. Under the new assumptions, the previously obtained agreement between the scaling law for the overall sound intensity and experimental results remain unchanged. In addition, this modified theory offers an explanation for the experimental observations that the far-field jet noise peak frequency at an emission angle $\theta_o$ of $90^o$ remains nearly unchanged in flight. Also, the peak frequency in flight at other angles is proportional to the static value with a simple Doppler proportionality factor, $(1 + M_f \cos \theta_o)^{-1}$, where $M_f$ is flight Mach number. The extension of the scaling law to the spectral domain indicates indirectly that large scale turbulence plays an important role in jet noise radiation. The phase difference for the source function affects the overall jet noise intensity, and is different for a jet in forward flight than for a static jet with the same relative jet velocity. The omission of this phase difference in the theory can lead to an overprediction of in-flight jet noise in the forward arc.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
November 10, 1981

## REFERENCES


1. Michalke, A.; and Michel, U.: Relation Between Static and In-Flight Directivities of Jet Noise. J. Sound & Vib., vol. 63, no. 4, Apr. 22, 1979, pp. 602-605.

2. Michalke, A.; and Michel, U.: Prediction of Jet Noise in Flight From Static Tests. J. Sound & Vib., vol. 67, no. 3, Dec. 8, 1979, pp. 341-367.

3. Michalke, A.; and Michel, U.: Prediction of Flyover Noise From Single and Coannular Jets. AIAA-80-1031, June 1980.

4. Ffowcs Williams, J. E.: The Noise From Turbulence Convected at High Speed. Phil. Trans. Roy. Soc. London, ser. A, vol. 255, no. 1061, Apr. 18, 1963, pp. 469-503.

5. Ribner, H. S.: Aerodynamic Sound From Fluid Dilatations - A Theory of the Sound From Jets and Other Flows. UTIA Rep. No. 86 (AFOSR TN 3430), Inst. Aerophys., Univ. of Toronto, July 1962.

6. Ribner, H. S.: The Generation of Sound by Turbulent Jets. Vol. 8 of Advances in Applied Mechanics, H. L. Dryden and Th. von Kármán, eds., Academic Press, Inc., 1964, pp. 103-182.

7. Michalke, A.; and Michel, U.: Importance of Jet Temperature on the Prediction of Jet Noise in Flight. Mechanics of Sound Generation in Flows, E.-A. Müller, ed., Springer-Verlag, 1979, pp. 256-263.

8. Drevet, P.; Duponchel, J. P.; and Jacques, J. R.: The Effect of Flight on Jet Noise as Observed on the Bertin Aérotrain. J. Sound & Vib., vol. 54, no. 2, Sept. 22, 1977, pp. 173-201.

9. Michalke, A.; and Hermann, G.: On the Inviscid Instability of a Circular Jet With External Flow. J. Fluid Mech., vol. 114, Jan. 1982, pp. 343-359.

10. Tanna, H. K.; and Morris, P. J.: In-Flight Simulation Experiments on Turbulent Jet Mixing Noise. J. Sound & Vib., vol. 53, no. 3, Aug. 8, 1977, pp. 389-405.

11. Morfey, C. L.: Propagation From Moving Sources in Flows. Special Course on Acoustic Wave Propagation, AGARD-R-686, Aug. 1979, pp. 11-1 - 11-13.

12. Gas Turbine Jet Exhaust Noise Prediction. ARP 876, Soc. Automot. Eng., Mar. 1978. (Supersedes AIR 876.)

13. Stone, James R.; and Montegani, Frances J.: An Improved Prediction Method for the Noise Generated in Flight by Circular Jets. NASA TM-81470, 1980.

14. Ribner, Herbert S.: Perspectives on Jet Noise. AIAA-81-0428, Jan. 1981.

15. Michalke, A.: On the Effect of Spatial Source Coherence on the Radiation of Jet Noise. J. Sound & Vib., vol. 55, no. 3, Dec. 8, 1977, pp. 377-394.

16. Dahan, C.; Elias, G.; Maulard, J.; and Perulli, M.: Coherent Structures in the Mixing Zone of a Subsonic Hot Free Jet. J. Sound & Vib., vol. 59, no. 3, Aug. 8, 1978, pp. 313-333.

## SYMBOLS

| | |
|---|---|
| $A$ | stretching parameter |
| $a$ | velocity of sound |
| $a_f$ | $= a_o(1 + M_f \cos \theta_o)$ |
| $B$ | influence of source coherence on jet noise in flight |
| $D$ | nozzle diameter |
| $F$ | source function |
| $f$ | frequency |
| $f_c$ | one-third-octave filter frequency |
| $\Delta f$ | filter bandwidth |
| $M_f$ | flight Mach number, $U_f/a_o$ |
| $N_{St,s}$ | Strouhal number, $f_s D/U_e$ |
| OASPL | overall sound pressure level in dB relative to 20 μPa |
| $P$ | mean square pressure in one-third-octave bands |
| $p'$ | sound pressure |
| $q_1$ | Reynolds stress source term |
| $q_2$ | pressure-density gradient source term |
| $r$ | distance from nozzle exit |
| $r_o$ | wave normal distance |
| SPL | sound pressure level in dB relative to 20 μPa |
| $t$ | time |
| $U$ | velocity |
| $\Delta U$ | relative jet velocity or specific thrust, $U_j - U_f$ |
| $u_i$ | velocity vector relative to ambient velocity, $i = 1,2,3$ |
| $U_p$ | phase velocity |
| $\tilde{u}_p$ | normalized phase velocity, $(U_p - U_f)/\Delta U$ |
| $W$ | power or cross spectral density |

$x_i$ far-field point, space coordinate, i = 1,2,3

$y_i$ source point coordinate, i = 1,2,3 ($y_1$ is parallel to the jet axis)

$y_{si}$ contracted source coordinate, i = 1,2,3

$\eta_i$ separation vector between two source points

$\eta_{si}$ contracted $\eta_i$

$\theta$ angle between far-field point, nozzle, and downstream jet axis

$\theta_o$ emission angle, wave normal direction

$\rho$ density

$\sigma$ stretching factor

$\sigma_1$ increase in normalized turbulent mean square values in flight

$\tau$ difference between retarded times

$\psi$ phase of cross spectral density $\tilde{W}_{FF}$

Subscripts:

e effective

f flight

FO in flyover coordinates

j jet exit condition

o ambient; wave normal

r retarded

ro component in the wave normal direction

s static; contracted

WT in wind tunnel coordinates

Superscripts:

$\bar{}$ time average

$\tilde{}$ normalized values

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15. Supplementary Notes

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16. Abstract

A scaling law is derived for predicting the flyover noise spectra of a single-stream shock-free circular jet from static experiments. The theory is based on the Lighthill approach to jet noise. Density terms are retained to include the effects of jet heating. The influence of flight on the turbulent flow field is considered by an experimentally supported similarity assumption. The resulting scaling laws for the difference between one-third-octave spectra and the overall sound pressure level compare very well with flyover experiments with a jet engine and with wind tunnel experiments with a heated model jet.

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