OPTICAL SPATIAL INTEGRATION METHODS FOR

AMBIGUITY FUNCTION GENERATION

P.N. Tamura, J.J. Rebholz, O.T. Daehlin, and T.C. Lee

Honeywell Corporate Technology Center

Bloomington, MN. 55420

SUMMARY

An optical spatial integration approach to ambiguity function generation has been developed. It uses acousto-optic Bragg cells as input transducers and a special passive optical element called a space variant linear phase shifter to create timedelayed versions of one of the signals. Real time processing has been achieved.

INTRODUCTION

The ambiguity function $\chi(v,\tau)$ for two given signals $f_1(t)$ and $f_2(t)$ is defined by:

$$\chi(\nu,\tau) = \int_{-\infty}^{\infty} f_1(t) f_2^*(t-\tau) \exp(-j2\pi\nu t) dt.$$
 (1)

The ambiguity function is widely used in the processing of radar and sonar signals. A received signal $f_2(t)$ will differ from a reference signal $f_1(t)$ by a time delay and a Doppler shift corresponding to the range and radial velocity component of a target. When the v and τ variables in (1) take on the appropriate values to compensate for the delay and Doppler differences between $f_1(t)$ and $f_2(t)$, the ambiguity function will yield a peak value at the specific position (v, τ) in the range-Doppler plane.

Equation (1) imposes a severe computational load on digital signal processing hardware, especially in surveillance applications where timely computation of large numbers of ambiguity functions calls for near real time processing. Consequently, many optical schemes to perform the ambiguity calculation have been investigated over the years. These include both spatial integration approaches¹⁻⁶, and more recently, time integrating architectures that utilize acousto-optic Bragg cells as input transducers.⁷

In this paper, a coherent spatial integration optical processor architecture that computes the ambiguity function will be described. The results of real time processing will also be shown.

PROCESSING ARCHITECTURE

Inspection of equation (1) shows that the operations to be performed by the

sign is opposite.

Therefore, the space variant linear phase shifter can be accurately fabricated by cementing a cylindrical lens and a spherical lens of opposite power together, and orienting them at 45°. The focal length of the cylindrical lens should be half that of the spherical lens.

ACOUSTO-OPTIC BRAGG CELLS

The key element in any optical processing system is the input transducer which converts an electronic signal in time to an optical signal in space. The fastest such device known is the acousto-optic Bragg cell, which receives a temporal signal modulated on an RF carrier which propagates through the crystal as a shear wave, and space-modulates an incident optical beam. Speed is a critical factor in ambiguity function calculation, and since digital signal processor throughput will soon approach the video rates at which most two-dimensional spatial light modulators operate, Bragg cells, which have a frame time of 50 μ s, are a very appropriate choice as input transducers. In addition to having a fast frame time, Bragg cells also have the advantages of electronic data composing and high diffraction efficiency.

The Bragg cells used for all experiments were TeO₂ devices that operate in the slow shear mode with 50 μ s frame times. The center frequency was 56.5 MHz with 40 MHz bandwidth at the 3 dB points, giving a time-bandwidth product of 2000. They were designed to operate at 514.5 nm.

REAL TIME PROCESSING

The most important design criterion in the real time system is the system timebandwidth product (TBW). An acoustic shear wave in a TeO₂ Bragg cell will propagate with a speed of 617 meters/sec, while the practical physical window size is 30 mm. It takes only 48 μ sec for the acoustic wave to propagate through the window. A sufficient system TBW, therefore, requires a fast data rate and high speed DACs. Fast 8 bit DACs commercially available operate at a 50 MHz data rate, and a digital time base compression (TBC) buffer of comparable speed must be constructed to operate with it.

The real time system is designed around the optical ambiguity function generator that utilizes two Bragg cells and the LIPS element. The front end electronics consisting of a TBC buffer, two digital-to-analog converters (DACs), and two RF driver/modulators, receives the input signals from the HP 9825 calculator in digital form and converts them to analog signals which cause index of refraction modulations in the Bragg cells which are proportional to the input signals. The rear end electronics, consisting of a two-dimensional detector array camera and a video memory, captures the ambiguity function, displays it on a CRT, and digitizes and stores it for examination. A block diagram of the real time system is seen in Figure 3.

The processing rate of the real time system is limited by the detector array read out time to 500 frames/sec. Also, the HP 9825 cannot reload the TBC buffer fast enough to create 500 different frames each second. Instead, the same frame is

optical signal processor are a conjugation operation, a multiplication of $f_1(t)$ by $f_2(t-\tau)$, and a one dimensional Fourier transform along the Doppler axis. These operations can easily be performed in the space domain by Fourier optics techniques, and an optical architecture capable of performing them is shown in Figure 1.

The first operation, conjugation, is performed by placing each signal on a carrier frequency and passing the opposite diffracted order from each Bragg cell. This heterodyning with a carrier is necessary anyway since the signal data is invariably in complex form from a previous basebanding operation. The second operation, multiplication, is performed by imaging Bragg cell I telecentrically on to Bragg cell II. Finally, lens S5 takes a one-dimensional Fourier transform along the frequency axis to yield the ambiguity function. This architecture has been analyzed in formal Fourier mathematics in a previous paper⁸.

The time-shifted version of $f_2(t)$ is created by using a passive optical element called a space variant linear phase shifter (LPS). The action of this LPS element can be understood by considering that its horizontal cross section is that of a prism. The result of imaging Bragg cell I onto Bragg cell II through a prism is to cause a position shift of the image. Since this is a spatial integration approach, a misregistration in the space domain is equivalent to a time shift in the time domain, and a delayed version of $f_2(t)$ will be created. By continually varying the wedge angle of the prism along the vertical axis, a continuous range of delay values can be created.

The feasibility of implementing the optical architecture of figure 1 depends heavily on the manufacturability of the linear phase shifter element. It is essentially an optical wedge; its wedge angle changes linearly with height. The complex transmissivity function of this component is given in rectangular coordinates by

$$g(x,y) = e^{j\alpha x y}$$
⁽²⁾

where α is a constant.

Conventional manufacturing processes such as grinding and polishing a glass piece would be difficult if not impossible to apply to the fabrication of such an element. We have invented a method to fabricate this component out of conventional optics, hence high accuracy of the transmitted wavefront is possible.

By modifying equation (2), we have

$$g(x,y) = e^{j\frac{\alpha}{2}} (x+y)^{2} e^{-j\frac{\alpha}{2}} (x^{2}+y^{2})$$
(3)

where the decomposition of equation (3) is essentially the same as that used in the chirp-z algorithm. If we define $r = x^2 + y^2$ and introduce a coordinate system (x',y') that is rotated from (x,y) by 45° (figure 2), equation (3) can be rewritten as

$$g(x,y) = e^{j\alpha x'^2} e^{-j\frac{\alpha}{2}r^2}$$
 (4)

The first exponent in equation (4) is the complex transmissivity function of a cylindrical lens oriented parallel to the x' axis. The second exponent is a spherical lens. The cylindrical lens is twice as powerful as the spherical lens, and the repetitively read to the Bragg cells to demonstrate high speed processing.

The test signals used to demonstrate the real time processing capability of this system were synthetically produced complex random signals with varying amounts of noise added. The time bandwidth product was selected to be 256. The output of the 100 x 100 detector array camera was plotted in an isometric display format. One such plot for an input signal-to-noise ratio of 0 dB (signal power equal to noise power) is shown in figure 4. The optically produced results compare favorably with digital calculations, and their resolutions along both the delay and Doppler axes are in agreement with theoretical predictions.

CONCLUSIONS

A coherent spatial integration approach to ambiguity function generation has been described. It uses one-dimensional acousto-optic Bragg cells in conjunction with a space variant linear phase shifter optical element to generate the twodimensional ambiguity function in one exposure. A real time implementation of this system has been demonstrated.

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Figure 1.- Optical layout for ambiguity function generation.



Figure 2.- Construction of the space variant linear phase shifter.







FREQUENCY (NU) Figure 4.- Real time results, SNR = OdB.