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# Prediction of Composite Hygral Behavior Made Simple

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# PREDICTION OF COMPOSITE HYGRAL BEHAVIOR MADE SIMPLE

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## ABSTRACT

A convenient procedure is described to determine the hygral (moisture) behavior (moisture expansion coefficients and moisture stresses) of angleplied fiber composites using a pocket calculator. The procedure consists of equations and appropriate graphs of ( $\pm\theta$ ) ply combinations. These graphs present reduced stiffness and moisture expansion coefficients as functions of ( $\pm\theta$ ) in order to simplify and expedite the use of the equations. The procedure is applicable to all types of balanced, symmetric fiber composites including interply and intraply hybrids. The versatility and generality of the procedure is illustrated using several step-by-step numerical examples.

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## 1.0 INTRODUCTION

Moisture expansion coefficients and moisture strains and stresses in angleplied laminates are frequently required for the initial sizing of structural components made from the fiber composites. These coefficients, strains and stresses are referred to as composite hygral behavior. The significance of composite hygral behavior is extensively discussed in reference 1. Moisture expansion coefficients and moisture strains and stresses are determined using composite mechanics and laminate theory usually in a computer code (refs. 2 and 3). A computer code was used effectively (ref. 1) to evaluate moisture stresses in angleplied laminates and thereby assess the effects of these stresses on the structural integrity of composites. It is generally recognized that the use of a computer code is expedient and quite general. However, it does not provide the user with insight and instant feedback of the laminate hygral behavior and capability as he proceeds with the design/analysis of the component.

A convenient procedure (method) is described in this paper which can be used to determine the hygral behavior of angleplied laminates. The procedure is suitable for hand calculations using a pocket calculator. It consists of simple equations and appropriate graphs of ( $\pm\theta$ ) ply combinations from the most frequently used composites. The procedure makes use of the well known transformation equations, and laminate theory equations. Its structure is similar to that in reference 5 and 6. The procedure can handle all types of composites including interply and intraply hybrids. The procedure is illustrated using

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step-by-step numerical examples. The discussion on this paper is limited to linear mechanical and hygral behavior of composites where the combined moisture temperature effects are neglected. This article presents simplified individual approaches to the hygrothermomechanical behavior of components. An integrated treatment is described in reference 1 including a review of the field up to 1977. Continuing interest in the moisture behavior of composites is evident from the research cited under Recent Relevant Articles.

The paper and numerical examples are written mainly in a tutorial manner in order to illustrate the step-by-step procedure. For this reason, the various sections are self contained as much as is practical at the expense of some duplication. The notation used is defined when it first appears, in general, and also summarized under symbols for convenient reference. Customary units are used throughout the numerical examples since these serve primarily to illustrate step-by-step numerical calculations. Appropriate conversion factors are given in the symbols.

## 2.0 MOISTURE EXPANSION COEFFICIENTS

The inplane moisture expansion coefficients (MECs) of  $[0/\pm\theta]_S$  angleplied laminates are determined from the following equations:

$$\beta_{cxx} = \frac{1}{E_{cxx}} \left\{ v_{p\theta} \left[ (Q_{\theta 11} - v_{cxy}Q_{\theta 21})\beta_{\theta 11} + (Q_{\theta 12} - v_{cxy}Q_{\theta 22})\beta_{\theta 22} \right] + v_{p0} \left[ (Q_{\ell 11} - v_{cxy}Q_{\ell 21})\beta_{\ell 11} + (Q_{\ell 12} - v_{cxy}Q_{\ell 22})\beta_{\ell 22} \right] \right\} \quad (2.1)$$

$$\beta_{cyy} = \frac{1}{E_{cyy}} \left\{ v_{p\theta} \left[ (Q_{\theta 21} - v_{cyx}Q_{\theta 11})\beta_{\theta 11} + (Q_{\theta 22} - v_{cyx}Q_{\theta 21})\beta_{\theta 22} \right] + v_{p0} \left[ (Q_{\ell 21} - v_{cyx}Q_{\ell 11})\beta_{\ell 11} + (Q_{\ell 22} - v_{cyx}Q_{\ell 21})\beta_{\ell 22} \right] \right\} \quad (2.2)$$

The composite moduli ( $E_{cxx}$  and  $E_{cyy}$ ) and the composite Poisson's ratios ( $v_{cxy}$  and  $v_{cyx}$ ) are given by:

$$\left. \begin{aligned} E_{cxx} &= Q_{cxx} - \frac{Q_{cxy}^2}{Q_{cyy}}; & E_{cyy} &= Q_{cyy} - \frac{Q_{cxy}^2}{Q_{cxx}} \\ v_{cxy} &= \frac{Q_{cxy}}{Q_{cyy}}; & v_{cyx} &= \frac{Q_{cxy}}{Q_{cxx}} \end{aligned} \right\} \quad (2.3)$$

The reduced laminate stiffness ( $Q_c$ 's) are given by:

$$\left. \begin{aligned} Q_{cxx} &= v_{p\theta}Q_{\theta 11} + v_{p0}Q_{\ell 11} \\ Q_{cyy} &= v_{p\theta}Q_{\theta 22} + v_{p0}Q_{\ell 22} \\ Q_{cxy} &= v_{p\theta}Q_{\theta 12} + v_{p0}Q_{\ell 12} = Q_{cyx} \end{aligned} \right\} \quad (2.4)$$

The  $Q_\theta$ 's are obtained from the appropriate figures 3 to 18. The  $Q_\ell$ 's are equal to  $Q_\theta$ 's at  $\theta = 0$  in figures 3 to 18. Note that  $Q_{\ell 21} = Q_{\ell 12}$ . The  $\beta_\theta$ 's and  $\beta_\ell$ 's are obtained from the same figures. The parameter  $V_{p\theta}$  denotes the thickness ratio of the  $\pm\theta$ -plies to the total laminate thickness while  $V_{p0}$  is the corresponding ratio for the 0-plies.  $V_{p\theta}$  and  $V_{p0}$  satisfy the identity:

$$V_{p\theta} + V_{p0} = 1 \quad (2.5)$$

The following procedure is convenient to calculate numerical values for  $\beta_{cxx}$  and  $\beta_{cyy}$  for a given  $[0/+\theta]_S$  APL using equations (2.1) and (2.2):

1. Obtain values for  $Q_{\theta 11}$ ,  $Q_{\theta 22}$ ,  $Q_{\theta 12} = Q_{\theta 21}$ ,  $Q_{\ell 11}$ ,  $Q_{\ell 22}$ ,  $Q_{\ell 12} = Q_{\ell 21}$ ,  $\beta_{\theta 11}$ ,  $\beta_{\theta 22}$ ,  $\beta_{\ell 11}$  and  $\beta_{\ell 22}$  from the appropriate figures. Note that ten values are needed.

2. Calculate values for  $V_{p\theta}$  and  $V_{p0}$ , respectively,

$$V_{p\theta} = \frac{\text{thickness of } \pm\theta\text{-plies}}{\text{thickness of APL}} \quad (2.6)$$

$$V_{p0} = \frac{\text{thickness of 0-plies}}{\text{thickness of APL}} \quad (2.7)$$

3. Calculate values for  $Q_{cxx}$ ,  $Q_{cyy}$  and  $Q_{cxy}$  using equations (2.4), using the information obtained in item (1) above and that obtained from equations (2.6) and (2.7).

4. Calculate values for  $E_{cxx}$ ,  $E_{cyy}$ ,  $\nu_{cxy}$  and  $\nu_{cyx}$  using the information obtained in item (3) above. We use the well known relationship

$$\nu_{cyx} = \nu_{cxy} \frac{E_{cyy}}{E_{cxx}} \quad (2.8)$$

to check our numerical values. In summary, we need to look-up ten quantities from the graphs and calculate a minimum of seven others in order to determine the moisture expansion coefficients (MECs)  $\beta_{cxx}$  and  $\beta_{cyy}$  using equations (2.1) and (2.2).

It is worth noting in equations (2.1) and (2.2) that the APL MECs depend on the properties of the  $\pm\theta$ -plies ( $Q_\theta$ 's), the 0-plies ( $Q_\ell$ 's) and the APL (integrated) properties  $E_c$  and  $\nu_c$ . Also, the APL Poisson's ratios ( $\nu_c$ ) restrain the MECs of the APL since these quantities are preceded by a minus sign and, therefore, subtract from the total. Furthermore, the shear moduli contribute to the MECs through the  $Q_\theta$ 's. In addition, equations (2.1) and (2.2) can be easily extended for more than one set of  $\pm\theta$ -ply combinations. Similar terms

$V_{p\theta_1}[\ ] + V_{p\theta_2}[\ ] + \text{etc.}$ , are added to accommodate this case as will be described with a numerical example later.

Example 2.1. - Calculate the MECs of the  $[\pm 30/0_2]_S$  APL made from AS/E composite: (1) Following the procedure outlined in item (1) above, we obtain the Q's from figure 7 and the  $\beta$ 's from figure 8 both at  $\theta = \pm 30$  and 0 (within curve reading accuracy):

$$Q_{\theta 11} = 11.3 \times 10^6 \text{ psi}$$

$$Q_{\lambda 11} = 18.7 \times 10^6 \text{ psi}$$

$$Q_{\theta 22} = 2.9 \times 10^6 \text{ psi}$$

$$Q_{\lambda 22} = 2.0 \times 10^6 \text{ psi}$$

$$Q_{\theta 21} = Q_{\theta 12} = 3.7 \times 10^6 \text{ psi}$$

$$Q_{\lambda 21} = Q_{\lambda 12} = 0.6 \times 10^6 \text{ psi}$$

$$\beta_{\theta 11} = -0.008 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{\lambda 11} = 0.006 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{\theta 22} = 0.060 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{\lambda 22} = 0.129 \times 10^{-2} \text{ in/in/\%M}$$

Note the MECs are given in dimensionless form in in/in/% moisture in the composite.

(2) Following item (2), we calculate  $V_{p\theta}$  and  $V_{p0}$

$$V_{p\theta} = 4/8 = 0.5$$

$$V_{p0} = 4/8 = 0.5$$

(3) Following item (3), we calculate the  $Q_c$ 's using equations (2.4) and carrying the units selectively for convenience

$$Q_{cxx} = V_{p\theta} Q_{\theta 11} + V_{p0} Q_{\lambda 11}$$

$$= 0.5 \times 11.3 \times 10^6 + 0.5 \times 18.7 \times 10^6 = 15.0 \times 10^6 \text{ psi}$$

$$Q_{cyy} = V_{p\theta} Q_{\theta 22} + V_{p0} Q_{\lambda 22}$$

$$= 0.5 \times 2.9 \times 10^6 + 0.5 \times 2.0 \times 10^6 = 2.45 \times 10^6 \text{ psi}$$

$$Q_{cxy} = Q_{cxy} = V_{p\theta} Q_{\theta 12} + V_{p0} Q_{\lambda 12}$$

$$= 0.5 \times 3.7 \times 10^6 + 0.5 \times 0.6 \times 10^6 = 2.15 \times 10^6 \text{ psi}$$

(4) Following item (4), we calculate  $E_{cxx}$ ,  $E_{cyy}$  and  $\nu_{cxy}$

$$E_{cxx} = Q_{cxx} - \frac{Q_{cxy}^2}{Q_{cyy}} = 15.0 \times 10^6 - \frac{(2.15 \times 10^6)^2}{2.45 \times 10^6} = 13.1 \times 10^6 \text{ psi}$$

$$E_{cyy} = Q_{cyy} - \frac{Q_{cxy}^2}{Q_{cxx}} = 2.45 \times 10^6 - \frac{(2.15 \times 10^6)^2}{15.0 \times 10^6} = 2.14 \times 10^6 \text{ psi}$$

$$\nu_{cxy} = \frac{Q_{cxy}}{Q_{cyy}} = \frac{2.15 \times 10^6}{2.45 \times 10^6} = 0.878$$

$$\nu_{cyx} = \frac{Q_{cxy}}{Q_{cxx}} = \frac{2.15 \times 10^6}{15.0 \times 10^6} = 0.143$$

Check:  $\nu_{cyx} = \nu_{cxy} \frac{E_{cyy}}{E_{cxx}}$

$$0.143 = 0.878 \frac{2.14 \times 10^6}{13.1 \times 10^6} \Rightarrow 0.143 = 0.143 \text{ O.K.}$$

(5) Using the above information in equations (2.1) and (2.2) and cancelling  $10^6$  with  $10^{-2}$  within the braces:

$$\begin{aligned} \beta_{cxx} &= \frac{1}{E_{cxx}} \left\{ \nu_{p\theta} \left[ (Q_{\theta 11} - \nu_{cxy} Q_{\theta 21}) \beta_{\theta 11} + (Q_{\theta 12} - \nu_{cxy} Q_{\theta 22}) \beta_{\theta 22} \right] \right. \\ &\quad \left. + \nu_{p\phi} \left[ (Q_{\ell 11} - \nu_{cxy} Q_{\ell 21}) \beta_{\ell 11} + (Q_{\ell 12} - \nu_{cxy} Q_{\ell 22}) \beta_{\ell 22} \right] \right\} \\ &= \frac{10^4}{13.1 \times 10^6} \left\{ 0.5 \left[ (11.3 - 0.878 \times 3.7)(-0.008) + (3.7 - 0.878 \times 2.9)(0.060) \right] \right. \\ &\quad \left. + 0.5 \left[ (18.7 - 0.878 \times 0.6)(-0.008) + (0.6 - 0.878 \times 2.0)(0.129) \right] \right\} \\ &= \frac{10^4}{13.1 \times 10^6} \left\{ 0.5 \left[ -0.064 + 0.069 \right] + 0.5 \left[ 0.145 - 0.149 \right] \right\} \end{aligned}$$

$$\beta_{cxx} = -0.011 \times 10^{-2} \text{ in/in/in}^2$$

(Cont'd.)

$$\begin{aligned} \beta_{cyy} &= \frac{1}{E_{cyy}} \left\{ \nu_{p\theta} \left[ (Q_{\theta 21} - \nu_{cyy} Q_{\theta 11}) \beta_{\theta 11} + (Q_{\theta 22} - \nu_{cyy} Q_{\theta 21}) \beta_{\theta 22} \right] \right. \\ &\quad \left. + \nu_{p0} \left[ (Q_{\ell 21} - \nu_{cyy} Q_{\ell 11}) \beta_{\ell 11} + (Q_{\ell 22} - \nu_{cyy} Q_{\ell 21}) \beta_{\ell 22} \right] \right\} \\ &= \frac{10^4}{2.14 \times 10^6} \left\{ 0.5 \left[ (3.7 - 0.143 \times 11.3)(-0.008) + (2.9 - 0.143 \times 3.7)(0.060) \right] \right. \\ &\quad \left. + 0.5 \left[ (0.6 - 0.143 \times 18.7)(0.006) + (2.0 - 0.143 \times 0.6)(0.129) \right] \right\} \\ &= \frac{10^4}{2.14 \times 10^6} \left\{ 0.5 \left[ -0.0167 + 0.142 \right] + 0.5 \left[ -0.012 + 0.247 \right] \right\} \end{aligned}$$

$$\beta_{cyy} = 0.084 \times 10^{-2} \text{ in/in/\%M}$$

Example 2.2. - Calculate the MECs of the  $[0/\pm 45/90]_5$  APL made from AS/E composite. This APL is commonly called pseudo-isotropic or quasi-isotropic laminate meaning that the laminate behaves like an isotropic material with respect to its inplane elastic and hygral properties. Again we follow the procedure outlined in items (1) through (5) and extend it to three different ply configurations (0,  $\pm 45$ , 90):

- (1) Obtain from figures 7 and 8 (within curve-reading accuracy) and using the first subscript  $\theta$  to denote all three conditions:

<u>Q and <math>\beta</math></u>	<u><math>\theta = \pm 45</math></u>	<u><math>\theta = 0</math></u>	<u><math>\theta = 90</math></u>
$Q_{\theta 11}$ ( $10^6$ psi)	6.1	18.7	2.0
$Q_{\theta 22}$ ( $10^6$ psi)	6.1	2.0	18.7
$Q_{\theta 21} = Q_{\theta 12}$ ( $10^6$ psi)	4.8	0.6	0.6
$\beta_{\theta 11}$ ( $10^{-2}$ in/in/%M)	0.011	0.006	0.129
$\beta_{\theta 22}$ ( $10^{-2}$ in/in/%M)	0.011	0.129	0.006

Note the 0-ply and 90-ply properties are complementary as expected.



(2) The respective thickness ratios for this 8-ply APL are:

$\theta$	Number of plies	$V_{p\theta}$
$\pm 45$	4	$4/8 = 0.50$
0	2	$2/8 = 0.25$
90	2	$2/8 = 0.25$

(3) The corresponding  $Q_c$ 's are:

$$Q_{c_{xx}} = V_{p\pm 45}Q_{\pm 4511} + V_{p0}Q_{011} + V_{p90}Q_{9011}$$

$$= (0.50 \times 6.1 + 0.25 \times 18.7 + 0.25 \times 2.0) \times 10^6 = 8.22 \times 10^6 \text{ psi}$$

$$Q_{c_{yy}} = V_{p\pm 45}Q_{\pm 4522} + V_{p0}Q_{022} + V_{p90}Q_{9022}$$

$$= (0.50 \times 6.1 + 0.25 \times 2.0 + 0.25 \times 18.7) \times 10^6 = 8.22 \times 10^6 \text{ psi}$$

$$Q_{c_{yx}} = Q_{c_{xy}} = V_{p\pm 45}Q_{\pm 4512} + V_{p0}Q_{012} + V_{p90}Q_{9022}$$

$$= (0.5 \times 4.8 + 0.25 \times 0.6 + 0.25 \times 0.6) \times 10^6 = 2.70 \times 10^6 \text{ psi}$$

(4) The APL moduli and Poisson's ratio are:

$$E_{c_{xx}} = Q_{c_{xx}} - \frac{Q_{c_{xy}}^2}{Q_{c_{yy}}} = \left( 8.22 - \frac{2.70^2}{8.22} \right) \times 10^6 = 7.33 \times 10^6 \text{ psi}$$

$$E_{c_{yy}} = Q_{c_{yy}} - \frac{Q_{c_{yx}}^2}{Q_{c_{xx}}} = \left( 8.22 - \frac{2.70^2}{8.22} \right) \times 10^6 = 7.33 \times 10^6 \text{ psi}$$

$$\nu_{c_{xy}} = \frac{Q_{c_{xy}}}{Q_{c_{yy}}} = \frac{2.70}{8.22} = 0.328$$

$$\nu_{c_{yx}} = \frac{Q_{c_{yx}}}{Q_{c_{xx}}} = \frac{2.70}{8.22} = 0.328$$

Check:  $\nu_{c_{yx}} = \nu_{c_{xy}} \frac{E_{c_{yy}}}{E_{c_{xx}}}$

$$0.328 = 0.328 \frac{7.33}{7.33} \Rightarrow 0.328 = 0.328 \text{ O.K.}$$

The calculations for  $E_{cyy}$ ,  $\nu_{cyx}$  and for the "check" were carried out for completeness since it is obvious that for this laminate  $E_{cyy} = E_{cxx}$  and  $\nu_{cyx} = \nu_{cxy}$ .

(5) The MECs for this APL are calculated from equations (2.1) and (2.2) but are generalized to include more than two different ply combinations. The form of the equations using the summation sign are:

$$\beta_{cxx} = \frac{1}{E_{cxx}} \sum_{\theta = \pm 45, 0, 90} \nu_{p\theta} [(Q_{\theta 11} - \nu_{cxy} Q_{\theta 21}) \beta_{\theta 11} + (Q_{\theta 12} - \nu_{cxy} Q_{\theta 22}) \beta_{\theta 22}]$$

$$\beta_{cyy} = \frac{1}{E_{cyy}} \sum_{\theta = \pm 45, 0, 90} \nu_{p\theta} [(Q_{\theta 21} - \nu_{cxy} Q_{\theta 11}) \beta_{\theta 11} + (Q_{\theta 22} - \nu_{cxy} Q_{\theta 21}) \beta_{\theta 22}] \quad (2.9)$$

where the sum is taken over  $\theta = \pm 45, 0$  and  $90$ . Using the values calculated previously in equations (2.9) and combining the  $10^6$  term with the  $10^{-2}$  within the braces we have:

$$\beta_{cxx} = \frac{10^4}{7.33 \times 10^6} \left\{ 0.50 [(6.1 - 0.328 \times 4.8)(0.011) + (4.8 - 0.328 \times 6.1)(0.011)] \right. \\ \left. + 0.25 [(18.7 - 0.328 \times 0.6)(0.006) + (0.6 - 0.328 \times 2.0)(0.129)] \right. \\ \left. + 0.25 [(2.0 - 0.328 \times 0.6)(0.129) + (0.6 - 0.328 \times 18.7)(0.006)] \right\} \\ = \frac{10^4}{7.33 \times 10^6} \left\{ 0.50 [0.050 + 0.031] + 0.25 [0.111 - 0.007] + 0.25 [0.233 - 0.033] \right\}$$

$$\beta_{cxx} = 0.017 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{cyy} = \frac{10^4}{7.33 \times 10^6} \left\{ 0.50 [(4.8 - 0.328 \times 6.1)(0.011) + (6.1 - 0.328 \times 4.8)(0.011)] \right. \\ \left. + 0.25 [(0.6 - 0.328 \times 18.7)(0.006) + (2.0 - 0.328 \times 0.6)(0.129)] \right. \\ \left. + 0.25 [(0.6 - 0.328 \times 2.0)(0.129) + (18.7 - 0.328 \times 0.6)(0.006)] \right\} \\ = \frac{10^4}{7.33 \times 10^6} \left\{ 0.050 [0.031 + 0.050] + 0.25 [-0.033 + 0.233] + 0.25 [-0.007 + 0.111] \right\}$$

$$\beta_{cyy} = 0.017 \times 10^{-2} \text{ in/in/\%M}$$

as expected. The reader may find it instructive to note that: (1) the values within the various parentheses for  $\beta_{cx'y'}$  and  $\beta_{cy'y'}$  are complementary.

The reader will obtain valuable practice and insight by using the procedure to calculate MECs of APL with ply configuration of his choice and a different composite system.

### 3.0 TRANSFORMATION OF MOISTURE EXPANSION COEFFICIENTS

The moisture expansion coefficients (MECs) about any  $x'-y'$  coordinate axes of an orthotropic angleplyed laminate APL with material symmetry about the  $x-y$  coordinate axes are given by:

$$\begin{aligned}\beta_{cx'x'} &= \beta_{c_{xx}} \cos^2\phi + \beta_{c_{yy}} \sin^2\phi \\ \beta_{cy'y'} &= \beta_{c_{xx}} \sin^2\phi + \beta_{c_{yy}} \cos^2\phi \\ \beta_{cx'y'} &= (\beta_{c_{yy}} - \beta_{c_{xx}}) \sin 2\phi\end{aligned}\tag{3.1}$$

where the notation in equation (3.1) is as follows:  $\beta_{cx'x'}$ ,  $\beta_{cy'y'}$  and  $\beta_{cx'y'}$  are the MECs about the new coordinate system  $x'-y'$ ;  $\beta_{c_{xx}}$  and  $\beta_{c_{yy}}$  are the MECs about the  $x-y$  coordinate system and are calculated as described in Section 2;  $\phi$  is the angle that the  $x'$  axis makes with the  $x$  axis. Note,  $\beta_{cx'y'}$  is a shear-type moisture deformation which is present along any  $x'-y'$  coordinate system  $x'-y'$  located at some angle  $\phi$  where  $0 < \phi < 90$  since  $\sin 2\phi \neq 0$  in this range.

To perform the calculations using equations (3.1) we need the MECs  $\beta_{c_{xx}}$ ,  $\beta_{c_{yy}}$  and the angle  $\phi$ . The MECs for the APL of interest are either known or can be computed using the procedure and examples described in the previous section. The angle  $\phi$  is known once the coordinate  $x'-y'$  axes has been selected.

Example 3.1. - Calculate the MECs of the  $[\pm 30/0_2]_S$  APL made from AS/E composite about an  $x'-y'$  coordinate axes where the  $x'$  axis is located by  $\phi = 15^\circ$  from the  $x$ -axis. From example (2.1)  $\beta_{c_{xx}} = -0.011 \times 10^{-2}$  in/in/%M and  $\beta_{c_{yy}} = 0.084 \times 10^{-2}$  in/in/%M. Using these values in equations (3.1) we have:

$$\begin{aligned}\beta_{cx'x'} &= \beta_{c_{xx}} \cos^2\phi + \beta_{c_{yy}} \sin^2\phi \\ &= (-0.011 \cos^2 15^\circ + 0.084 \sin^2 15^\circ) \times 10^{-2} \text{ in/in/%M} \\ &= -0.005 \times 10^{-2} \text{ in/in/%M}\end{aligned}$$

(Cont'd.)

$$\begin{aligned}
\beta_{cy'y'} &= \beta_{cxx} \sin^2 \phi + \beta_{cyy} \cos^2 \phi \\
&= (-0.011 \sin^2 15^\circ + 0.084 \cos^2 15^\circ) \times 10^{-2} \text{ in/in/\%M} \\
&= 0.078 \times 10^{-2} \text{ in/in/\%M} \\
\beta_{cx'y'} &= (\beta_{cyy} - \beta_{cxx}) \sin 2\phi \\
&= \left\{ [0.084 - (-0.011)] \sin 2(15^\circ) \right\} 10^{-2} \text{ in/in/\%M} \\
\beta_{cx'y'} &= 0.0475 \times 10^{-2} \text{ in/in/\%M}
\end{aligned}$$

As can be seen, the shear-type MEC  $\beta_{cx'y'}$  is substantial. Restraining this APL along the  $x'$ - $y'$  coordinate axes will induce considerable in-plane shear stresses.

#### 4. LAMINATE MOISTURE STRAINS AND STRESSES

Along the laminate material axes. - The equations for calculating moisture strains  $\epsilon_{cxy}$  and  $\epsilon_{cyy}$  along the laminate  $x$ - $y$  coordinate axes are:

$$\begin{aligned}
\epsilon_{cxy} &= (M - M_0) \beta_{cxy} \\
\epsilon_{cyy} &= (M - M_0) \beta_{cyy}
\end{aligned} \tag{4.1}$$

where  $M$  is the present and  $M_0$  is the reference moisture (usually zero); and  $\beta_{cxy}$  and  $\beta_{cyy}$  are the MECs which are either known or can be determined as described previously. Use of equations (4.1) requires that the MECs be independent of moisture within the range  $M - M_0$ .

Example 4.1. - Calculate the moisture strains for the  $[\pm 30/0_2]_S$  APL made from AS/E composite (Example 2.1) where  $M = 1\%$  and  $M_0 = 0$ . The values for the MECs from Example 2.1 are:

$$\beta_{cxx} = -0.011 \times 10^{-2} \text{ in/in/\%M}; \beta_{cyy} = 0.084 \times 10^{-2} \text{ in/in/\%M}$$

Substituting these values in equation (4.1), we obtain:

$$\begin{aligned}
\epsilon_{cxx} &= (M - M_0) \beta_{cxx} = (1.0 - 0) (-0.011 \times 10^{-2}) = -110 \times 10^{-6} \text{ in/in} \\
\epsilon_{cyy} &= (M - M_0) \beta_{cyy} = (1.0 - 0) (0.084 \times 10^{-2}) = 840 \times 10^{-6} \text{ in/in}
\end{aligned}$$

The equations to calculate corresponding moisture stresses, assuming that the laminate is completely restrained from moisture expansion, are:

$$\begin{aligned}
\sigma_{cxx} &= \Delta M (Q_{cxx} \beta_{cxx} + Q_{cxy} \beta_{cyy}) \\
\sigma_{cyy} &= \Delta M (Q_{cyy} \beta_{cyy} + Q_{cxy} \beta_{cxx})
\end{aligned} \tag{4.2}$$

where  $\Delta M = (M - M_0)$ ; the  $Q_c$ 's and  $\beta_c$ 's are determined as described previously.

Example 4.2. - Calculate the restrained moisture stresses in Example 4.1. Referring to Examples 2.1 and 4.1, we have:

$$\begin{aligned} Q_{c_{xx}} &= 15.0 \times 10^6 \text{ psi} & \beta_{c_{xx}} &= -0.011 \times 10^{-2} \text{ in/in/\%M} \\ Q_{c_{yy}} &= 2.45 \times 10^6 \text{ psi} & \beta_{c_{yy}} &= 0.084 \times 10^{-2} \text{ in/in/\%M} \\ Q_{c_{yx}} = Q_{c_{xy}} &= 2.15 \times 10^6 \text{ psi} & \Delta M &= (1.0 - 0) = 1\% \end{aligned}$$

Using these values in equations (4.2) we obtain (combining  $10^6$  with  $10^{-2}$ ):

$$\begin{aligned} \sigma_{c_{xx}} &= -\Delta M(Q_{c_{xx}}\beta_{c_{xx}} + Q_{c_{xy}}\beta_{c_{yy}}) \\ &= -1.0 [15.0 \times (-0.011) + 2.15 \times 0.084] \times 10^4 \\ &= -1.0 (-0.165 + 0.181) \\ &= -160 \text{ psi} \approx -0.16 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \sigma_{c_{yy}} &= -\Delta M(Q_{c_{yx}}\beta_{c_{xx}} + Q_{c_{yy}}\beta_{c_{yy}}) \\ &= -1.0 [2.15 \times (-0.011) + 2.45 \times 0.084] \times 10^4 \\ &= -1.0 (-0.024 + .206) \times 10^4 \\ &= -1660 \text{ psi} \approx -1.7 \text{ ksi} \end{aligned}$$

Two points are worth noting in connection with the above values of these moisture stresses:

1. The moisture stresses  $\sigma_{c_{xx}}$  and  $\sigma_{c_{yy}}$  are relatively small (0.2 and 7.7 percent, respectively) compared to the corresponding compressive failure stresses of the laminate  $S_{c_{xxc}} = 83 \text{ ksi}$  and  $S_{c_{yyc}} = 22 \text{ ksi}$ , based on first ply failure (ref. 5).

2. The magnitude of these moisture stresses may be sufficiently high to cause panel buckling. For example, a 20 in. x 10 in. x 0.04 in. panel from this 8-ply APL has buckling stresses of about  $\sigma_{c_{xx}} = -8.2 \text{ psi}$  and  $\sigma_{c_{yy}} = -82 \text{ psi}$  (calculated using the equation in ref. 7) or approximately 5 percent of the restrained moisture stresses which are relatively low. A panel with this geometry will buckle at an increase in moisture of about 0.05%. The important conclusion from the discussion in this example is that moisture stresses need be considered carefully by the designer/analyst in situations where restraints may be present. It is noteworthy to compare these buckling stresses with those of the same panel made from aluminum. The aluminum panel will buckle at about twice the stress and be twice as heavy. The two panels are equivalent on the buckling stress to weight basis.

## 5.0 PLY MOISTURE STRAINS AND STRESSES

It is instructive to describe the ply moisture strains and stresses in the plies of an APL by breaking them down into three "commonly thought-of" types.

These types are:

1. Restrained - APL is restrained from moisture expansion.
2. Free - APL is free to undergo moisture expansion.
3. Combined - Combinations of free and restrained.

The ply moisture strains and stresses to be described are those along the ply material axes 1, 2, 3, figure 1. For convenience, the strains along the 1-direction (fiber direction) are defined by  $\epsilon_{l11}$  and the stresses by  $\sigma_{l11}$ ; those along the 2-direction (transverse to the fiber direction) are defined by  $\epsilon_{l22}$  and  $\sigma_{l22}$ ; and those in the 1-2 plane (intralaminar shear) are defined by  $\epsilon_{l12}$  and  $\sigma_{l12}$ .

Restrained APL. - The ply moisture strains for this case are given by

$$\epsilon_{l11} = \epsilon_{l22} = \epsilon_{l12} = 0 \quad (5.1)$$

The corresponding ply stresses are given by

$$\begin{aligned} \sigma_{l11} &= -\Delta M(Q_{l11}\beta_{l11} + Q_{l12}\beta_{l22}) \\ \sigma_{l22} &= -\Delta M(Q_{l21}\beta_{l11} + Q_{l22}\beta_{l22}) \\ \sigma_{l12} &= 0 \end{aligned} \quad (5.2)$$

where  $\Delta M$  equals the change in moisture. Where the  $Q_l$ 's are the reduced ply stiffness and the  $\beta_l$ 's are ply moisture expansion coefficients (ply MECs). The ply reduced stiffness  $Q_l$ 's and the MECs can be estimated from figures 3 to 18 at  $\theta = 0^\circ$ . Equations (5.2) show that the ply material axes moisture stresses in an APL restrained from moisture expansion depend only on ply properties. Also, there is no intralaminar shear stress for this case.

Example 5.1. - Calculate the ply moisture stress in the plies of the  $[\pm 30/0]_S$  APL, made from AS/E composite, where the moisture is increased from 0% to 1% and the APL is restrained from moisture expansion. The numerical values we need for these calculations are determined as follows: Figures 7 and 8 at  $\theta = 0^\circ$

$$Q_{l11} = 18.7 \times 10^6 \text{ psi}, \quad Q_{l22} = 2.0 \times 10^6 \text{ psi}$$

$$Q_{l21} = Q_{l12} = 0.6 \times 10^6 \text{ psi}$$

$$\beta_{l11} = 0.006 \times 10^{-2} / \%M; \quad \beta_{l22} = 0.129 \times 10^{-2} / \%M$$

and  $\Delta M = 1.0\%$  consistent with the previous definition. Using these numerical values in equations (5.2), we calculate

$$\begin{aligned}\sigma_{\ell 11} &= -\Delta M(Q_{\ell 11}\beta_{\ell 11} + Q_{\ell 12}\beta_{\ell 22}) \\ &= -1.0(18.7 \times 0.006 + 0.6 \times 0.129) \times 10^4 = -1896 \text{ psi} \quad -1.9 \text{ ksi}\end{aligned}$$

$$\begin{aligned}\sigma_{\ell 22} &= -\Delta M(Q_{\ell 21}\beta_{\ell 11} + Q_{\ell 22}\beta_{\ell 22}) \\ &= -1.0(0.6 \times 0.006 + 2.0 \times 0.129) \times 10^4 = -2616 \text{ psi} \quad -2.6 \text{ ksi}\end{aligned}$$

Free APL. - The ply moisture strains for this case are given by:

$$\begin{aligned}\epsilon_{\ell 11} &= \Delta M(\beta_{c_{xx}}\cos^2\theta + \beta_{c_{yy}}\sin^2\theta - \beta_{\ell 11}) \\ \epsilon_{\ell 22} &= \Delta M(\beta_{c_{xx}}\sin^2\theta + \beta_{c_{yy}}\cos^2\theta - \beta_{\ell 22}) \\ \epsilon_{\ell 12} &= \Delta M(\beta_{c_{yy}} - \beta_{c_{xx}})\sin^2\theta\end{aligned}\quad (5.3)$$

The corresponding ply stresses are given by:

$$\begin{aligned}\sigma_{\ell 11} &= \Delta M [Q_{\ell 11}(\beta_{\ell 11}(\beta_{c_{xx}}\cos^2\theta + \beta_{c_{yy}}\sin^2\theta - \beta_{\ell 11}) + Q_{\ell 12}(\beta_{c_{xx}}\sin^2\theta + \beta_{c_{yy}}\cos^2\theta - \beta_{\ell 22}))] \\ \sigma_{\ell 22} &= \Delta M [Q_{\ell 21}(\beta_{c_{xx}}\cos^2\theta + \beta_{c_{yy}}\sin^2\theta - \beta_{\ell 11}) + Q_{\ell 22}(\beta_{c_{xx}}\sin^2\theta + \beta_{c_{yy}}\cos^2\theta - \beta_{\ell 22})] \\ \sigma_{\ell 12} &= \Delta M [Q_{\ell 33}(\beta_{c_{yy}} - \beta_{c_{xx}})\sin^2\theta]\end{aligned}\quad (5.4)$$

Also, when the ply moisture strains are calculated using equations (5.3), the corresponding ply stresses are given by:

$$\begin{aligned}\sigma_{\ell 11} &= Q_{\ell 11}\epsilon_{\ell 11} + Q_{\ell 12}\epsilon_{\ell 22} \\ \sigma_{\ell 22} &= Q_{\ell 21}\epsilon_{\ell 11} + Q_{\ell 22}\epsilon_{\ell 22} \\ \sigma_{\ell 12} &= Q_{\ell 33}\epsilon_{\ell 12}\end{aligned}\quad (5.5)$$

where the strains ( $\epsilon_{\ell}$ ) are calculated from equation (5.3).

Example 5.2. - Calculate the ply moisture strains in the plies of the  $[\pm 30/0_2]_S$  APL, made from the AS/E composite, where the moisture is increased by 1% and the APL is not restrained from moisture expansion. We will calculate the ply moisture strains in the  $+30^\circ$  plies, in the  $-30^\circ$  plies and in the  $0^\circ$ -plies by using equations (5.3). The numerical values we need are:

$$\Delta M = 1.0\%$$

$$\theta = 30^\circ, -30^\circ, 0^\circ$$

$$\beta_{c_{xx}} = -0.011 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{c_{yy}} = 0.084 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{\ell_{11}} = 0.006 \times 10^{-2} \text{ in/in/\%M}$$

$$\beta_{\ell_{22}} = 0.129 \times 10^{-2} \text{ in/in/\%M}$$

(These values are taken from Example 2.1)

30°-ply. - Substituting these numerical values and  $\theta = 30^\circ$  in equations (5.3) we calculate:

$$\epsilon_{\ell_{11}} = \Delta M (\beta_{c_{xx}} \cos^2 \theta + \beta_{c_{yy}} \sin^2 \theta - \beta_{\ell_{11}})$$

$$= 1.0 (-0.011 \times \cos^2 30^\circ + 0.084 \times \sin^2 30^\circ - 0.006) \times 10^{-2} \text{ in/in}$$

$$= 67.5 \times 10^{-6} \text{ in/in or } \approx 0.007\%$$

$$\epsilon_{\ell_{22}} = \Delta M (\beta_{c_{xx}} \sin^2 \theta + \beta_{c_{yy}} \cos^2 \theta - \beta_{\ell_{22}})$$

$$= 1.0 (-0.011 \times \sin^2 30^\circ + 0.084 \times \cos^2 30^\circ - 0.129) \times 10^{-2} \text{ in/in}$$

$$= -688 \times 10^{-6} \text{ in/in } \approx -0.069\%$$

$$\epsilon_{\ell_{12}} = \Delta M (\beta_{c_{yy}} - \beta_{c_{xx}}) \sin 2\theta$$

$$= 1.0 [0.084 - (-0.011)] \times 10^{-2} \sin 60^\circ$$

$$= 822 \text{ in/in } \approx 0.082\%$$

-30°-ply. - The moisture strains in the -30° ply are the same as those in the +30° plies for  $\epsilon_{\ell_{11}}$  and  $\epsilon_{\ell_{22}}$  and opposite sign for  $\epsilon_{\ell_{12}}$ . The reader can readily verify that this is the case by inspection of the appropriate equations.

0°-ply. - Substituting the above numerical values and  $\theta = 0^\circ$  in equations (5.3) we calculate:



$$\begin{aligned}\epsilon_{\ell 11} &= \Delta M(\beta_{c_{xx}} \cos^2 \theta + \beta_{c_{yy}} \sin^2 \theta - \beta_{\ell 11}) \\ &= 1.0(-0.011 \times \cos^2 0^\circ + 0.084 \times \sin^2 0^\circ - 0.006) \times 10^{-2} \text{ in/in} \\ &= -170 \times 10^{-6} \text{ in/in} = -0.017\%\end{aligned}$$

$$\begin{aligned}\epsilon_{\ell 22} &= \Delta M(\beta_{c_{xx}} \sin^2 \theta + \beta_{c_{yy}} \cos^2 \theta - \beta_{\ell 22}) \\ &= 1.0(-0.011 \sin^2 0^\circ + 0.084 \times \cos^2 0^\circ - 0.129) \times 10^{-2} \\ &= -450 \times 10^{-6} \text{ in/in} = -0.045\%\end{aligned}$$

$$\begin{aligned}\epsilon_{\ell 12} &= \Delta M(\beta_{c_{yy}} - \beta_{c_{xx}}) \sin 2\theta \\ &= 1.0 [0.084 - (0.011)] \sin 2(0) \quad (0^\circ) \\ &= 0.0\end{aligned}$$

The reader will find it instructive to compare the corresponding moisture strains in the  $30^\circ$  and  $0^\circ$  plies. Both normal strains  $\epsilon_{\ell 11}$  and  $\epsilon_{\ell 22}$  in the  $30^\circ$  ply are about the same as those in the  $0^\circ$  plies while the shear strain  $\epsilon_{\ell 12}$  has the largest magnitude in the  $30^\circ$  plies and is "zero" in the  $0^\circ$  plies.

Example 5.3. - Calculate the corresponding moisture stresses in the plies in the APL of Example 5.2. To calculate the corresponding moisture stresses we can use either equations (5.4) or (5.5) since we already calculated the strains in Example 5.2. We will use equations (5.3) for convenience. In order to use equations (5.5), we need numerical values for the reduced ply stiffnesses  $Q_\ell$  and corresponding moisture strain values  $\epsilon_\ell$  from Example 5.2. We tabulate these numerical values for convenience.

Reduced Ply Stiffnesses in $10^6$ psi (fig. 7 at $\theta = 0^\circ$ or from Example 5.1)	Moisture Strains in $10^{-6}$ in/in from Example 5.1 for $\theta =$		
	$30^\circ$	$-30^\circ$	$0^\circ$
$Q_{\ell 11} = 18.7$	$\epsilon_{\ell 11} \quad 67.5$	$67.5$	$-170$
$Q_{\ell 22} = 2.0$	$\epsilon_{\ell 22} \quad -688$	$-688$	$-450$
$Q_{\ell 21} = Q_{\ell 12} = 0.60$	$\epsilon_{\ell 12} \quad 822$	$-822$	$0$
$Q_{\ell 33} = 0.56$			

Using corresponding values in equations (5.5) we have (cancelling  $10^6$  with  $10^{-6}$  for convenience)

$$\begin{aligned} \text{+30}^\circ\text{-ply: } \sigma_{\ell 11} &= Q_{\ell 11} \epsilon_{\ell 11} + Q_{\ell 12} \epsilon_{\ell 12} \\ &= 18.9 \times 67.5 + 0.6 \times (-688) = 863 \text{ psi} \end{aligned}$$

$$\begin{aligned} \sigma_{\ell 22} &= Q_{\ell 21} \epsilon_{\ell 11} + Q_{\ell 22} \epsilon_{\ell 22} \\ &= 0.6 \times 67.5 + 2.0 \times (-688) = -1336 \text{ psi} \end{aligned}$$

$$\sigma_{\ell 12} = Q_{\ell 33} \epsilon_{\ell 12} = 0.56 \times 822 = 460 \text{ psi}$$

-30<sup>o</sup>-ply:  $\sigma_{\ell 11}$  and  $\sigma_{\ell 22}$  are the same as for the +30<sup>o</sup> ply;

$\sigma_{\ell 12}$  has opposite sign or  $\sigma_{\ell 12} = -460$  psi

$$\begin{aligned} \text{0}^\circ\text{-ply: } \sigma_{\ell 11} &= Q_{\ell 11} \epsilon_{\ell 11} + Q_{\ell 12} \epsilon_{\ell 12} \\ &= 18.9 \times (-170) + 0.60 \times (-450) = -3483 \text{ psi} \end{aligned}$$

$$\begin{aligned} \sigma_{\ell 22} &= Q_{\ell 21} \epsilon_{\ell 11} + Q_{\ell 22} \epsilon_{\ell 22} \\ &= 0.60 \times (-170) + 2.0 \times (-450) = -1002 \text{ psi} \end{aligned}$$

$$\sigma_{\ell 12} = Q_{\ell 33} \epsilon_{\ell 12} = 0.56 \times (0) = 0$$

The interesting point to be noted from the numerical values of the ply moisture stresses is that the transverse ply stresses  $\sigma_{\ell 22}$  are compressive and are about 3 percent of the compressive ply strength ( $S_{\ell 22C}$  equals about 35 ksi at 1% moisture).

## 6.0 CONCLUSIONS

A convenient procedure is described to determine the hygral behavior (moisture expansion coefficients and stresses) of angleplied fiber composites. The procedure consists of equations and appropriate graphs of (+ $\theta$ ) ply combinations. These graphs consists of reduced stiffness and moisture expansion coefficients of frequently used composites and hybrids as functions of + $\theta$  in order to simplify and expedite the use of the equations. The procedure is applicable to all types of balanced, symmetric fiber composites including interply and intraply hybrids. The versatility and generality of the procedure is illustrated using several step-by-step numerical examples. The step-by-step numerical examples are set up so that the calculations can be made using a pocket calculator. Some of the numerical examples were selected to illustrate significant implications of composite hygral behavior in design applications. One percent moisture change induces negligible ply stress in angleplied laminates. These moisture ply stresses can be neglected in general. However, thin angleplied laminates restrained from free moisture expansion need be checked for possible buckling even at low moisture levels.

## 7.0 SYMBOLS

APL	angleplied laminate
AS/E	AS-graphite-fiber/epoxy-matrix composite
B/E	boron-fiber/epoxy-matrix composite
$E_c$	laminate modulus - subscripts x,y denote structural axes directions
$E_Q$	ply modulus - subscripts 1,2 denote ply material axes directions
HMS/E	high modulus graphite-fiber/epoxy-matrix composite
K/E	Kevlar-fiber/epoxy matrix composite
M	Moisture, % by weight in composite
$\Delta M$	moisture change, % by weight in composite
MEC	moisture expansion coefficient
$Q_c$	reduced laminate stiffness - subscripts x,y denote structural axes directions
$Q_\ell$	reduced ply stiffness - subscripts 1,2 denote ply material axes directions
$Q_\theta$	reduced stiffness for $\pm\theta$ symmetric laminate - subscripts 1,2 denote material axes directions
$S_\ell$	ply strength - subscripts 1,2 denote ply material axes directions; - subscripts T, C, S denote type
S-G/E	S-glass-fiber/epoxy-matrix composite

$V_p$	ply thickness ratio - subscripts 0, 0, 90 denote ply designation to which the ratio applies
$x, y, z$	structural axes coordinate directions
1, 2, 3	material axes coordinate directions - 1 taken along the fiber direction, 2 transverse to the fiber and 3 through the thickness
$[-/-/-]_S$	laminate configuration designation - numbers in the blanks denote ply stacking sequence and orientation - subscript S denotes symmetry about ply in last blank space
$\beta_c$	laminate MEC - subscripts $x, y$ denote laminate structural axes directions
$\beta_l$	ply MEC - subscripts 1, 2 denote ply material axes directions
$\beta_\theta$	$\pm\theta$ laminate MEC - subscripts 1, 2 denote material axes directions
$\epsilon_c$	laminate strain - subscripts $x, y$ denote structural axes directions
$\epsilon_l$	ply strain - subscripts 1, 2 denote material axes directions
$\theta$	ply orientation angle measured from the x-laminate structural axes to the 1-ply material axes and taken positive
$\nu_c$	laminate Poisson's ratio - subscripts $x, y$ denote structural axes directions
$\nu_l$	ply Poisson's ratio - subscripts 1, 2 denote ply material axes directions
$\sigma_c$	laminate stress - subscripts $x, y$ denote structural axes directions
$\sigma_l$	ply stress - subscripts 1, 2 denote material axes directions
$\phi$	laminate coordinate axes $x', y', z'$ orientation other than the structural axes $x, y, z$ measured from the $x$ axis to the $x'$ -axis and taken positive.

## Conversion factors:

$$\text{MPa} = 6.89 \text{ ksi}$$

$$\text{ksi} = 0.145 \text{ MPa}$$

$$^{\circ}\text{C} = 5/9 (^{\circ}\text{F} - 32)$$

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L. J. Broutman, "Diffusion Mechanisms and Degradation of Environmentally Sensitive Materials, Dept. of Energy Report COO-444-9, 1981.

TABLE I. - TYPICAL PROPERTIES OF UNIDIRECTIONAL FIBER COMPOSITES AT ROOM TEMPERATURE

Properties	Units	Boron/ epoxy	Boron/ polyimide	Scotchply/ epoxy	Modmor I/ epoxy	Modmor I/ polyimide	Thornel 300/ epoxy	Kevlar 49/ epoxy	Graphite As/epoxy
1. Fiber volume ratio	-----	0.50	0.49	0.72	0.45	0.45	0.70	0.54	0.60
2. Density	lb/in <sup>3</sup>	0.073	0.072	0.077	0.056	0.056	0.058	0.049	0.057
3. Longitudinal thermal coefficient	10 <sup>-6</sup> in/ in/°F	3.4	2.7	2.1	-----	0.0	0.01	-1.60	0.40
4. Transverse thermal coefficient	10 <sup>-6</sup> in/ in/°F	16.9	15.8	9.3	18.5	14.1	12.5	31.3	16.4
5. Longitudinal modulus	10 <sup>6</sup> psi	29.2	32.1	8.8	27.5	31.3	26.3	12.2	16.0
6. Transverse modulus	10 <sup>6</sup> psi	3.15	2.1	3.6	1.03	0.72	1.5	0.70	2.2
7. Shear modulus	10 <sup>6</sup> psi	0.78	1.11	1.74	0.9	0.65	1.0	0.41	0.72
8. Major Poisson's ratio	-----	0.17	0.16	0.23	0.10	0.25	0.28	0.32	0.25
9. Minor Poisson's ratio	-----	0.02	0.02	0.09	-----	0.02	0.01	0.02	0.34
10. Longitudinal tensile strength	psi	199 000	151 000	187 000	122 000	117 000	218 000	172 000	220 000
11. Longitudinal com- pressive strength	psi	232 000	158 000	119 000	128 000	94 500	247 000	42 000	180 000
12. Transverse tensile strength	psi	8100	1600	6670	6070	2150	5850	1600	8000
13. Transverse compres- sive strength	psi	17 900	9100	27 300	28 500	10 200	35 720	9400	36 000
14. Intralaminar shear strength	psi	9100	3750	6500	8900	3150	9800	4000	10 000
15. Longitudinal moist- ure coefficient	10 <sup>-2</sup> in/ in/%M	0.003	0.003	0.014	0.003	0.003	0.006	0.008	0.006
16. Transverse Moist- ure coefficient	10 <sup>-2</sup> in/ in/%M	0.168	0.168	0.128	0.129	0.129	0.129	0.151	0.129

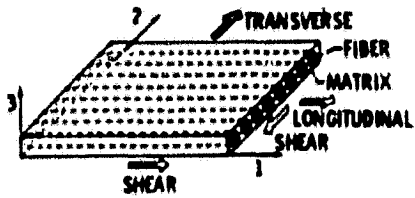


Figure 1. - Schematic of single ply.

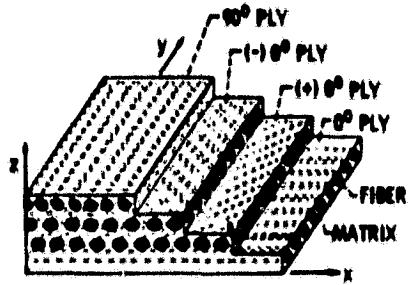


Figure 2. - Schematic of angle-ply laminate.

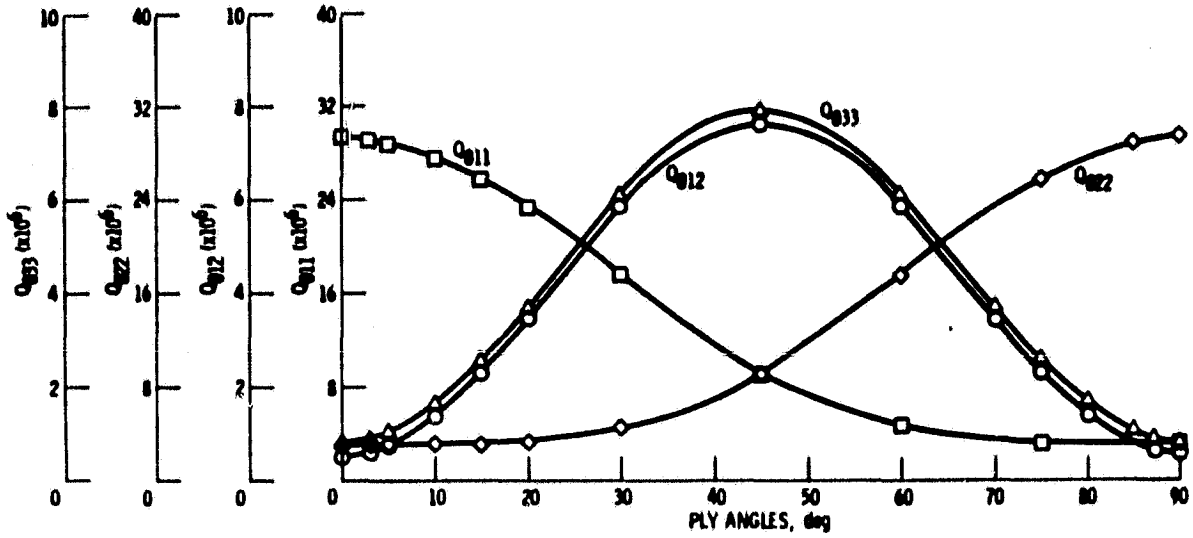


Figure 3. - Reduced stiffnesses of boron-fiber/epoxy (B/E)  $\pm\theta$  laminates.

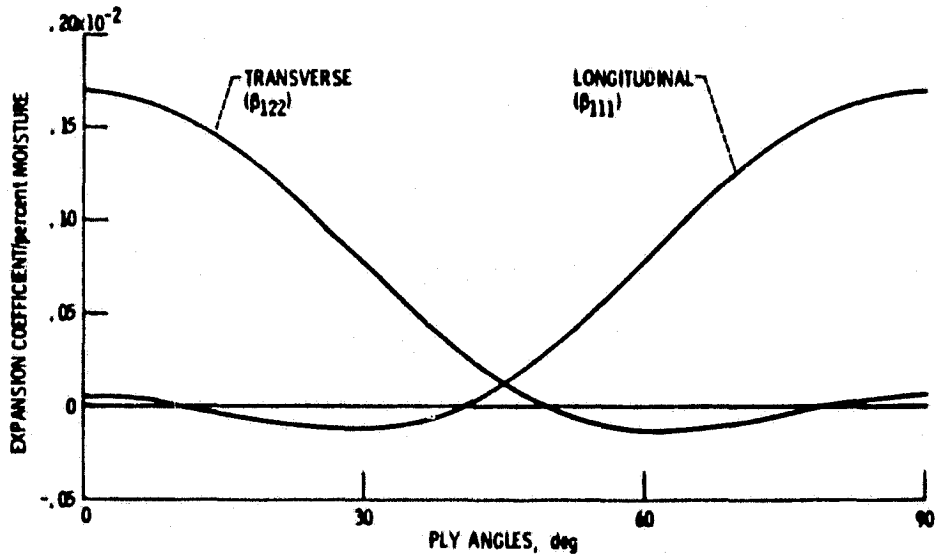


Figure 4. - Moisture expansion coefficients of boron-fiber/epoxy (B/E)  $\pm\theta$  laminates.



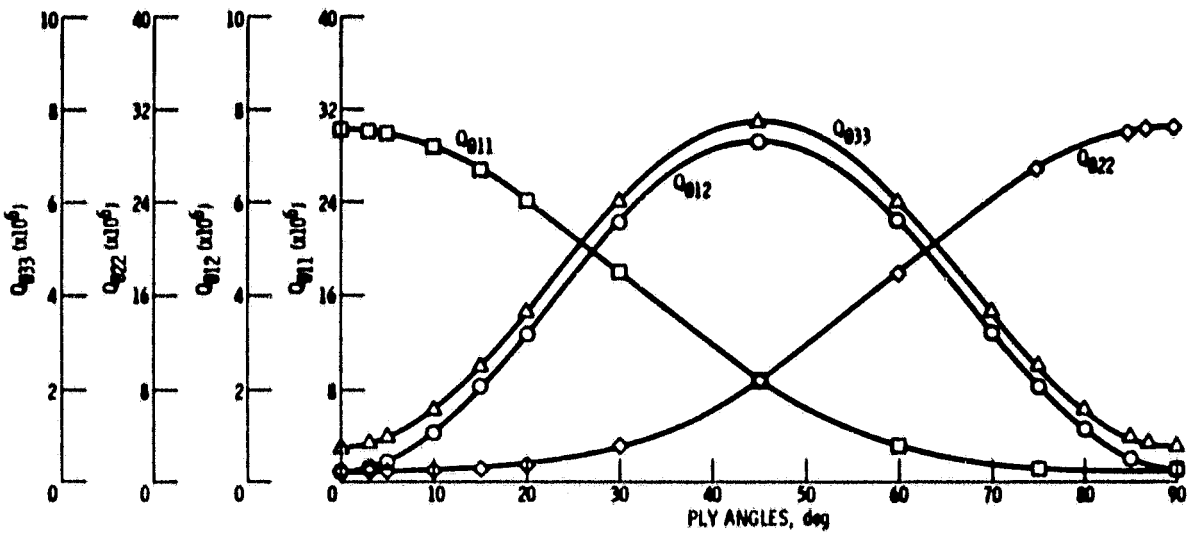


Figure 5. - Reduced stiffnesses of high modulus graphite-fiber/epoxy (HMG/E) ±θ laminates.

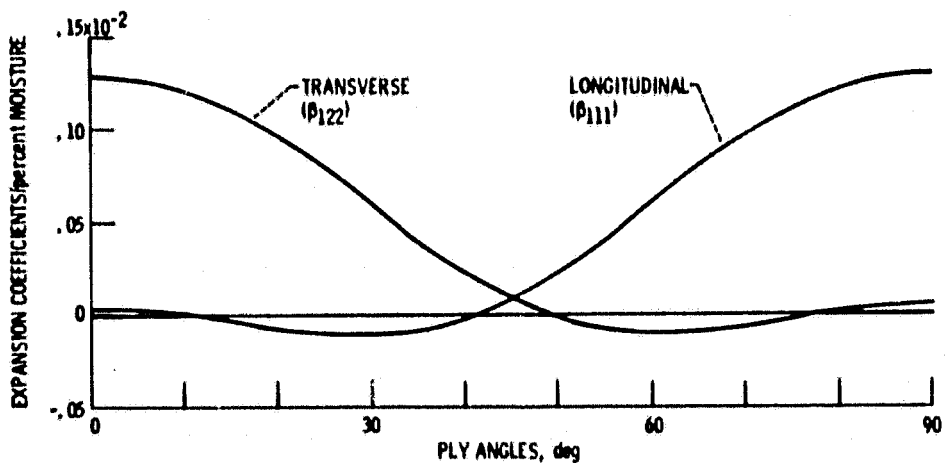


Figure 6. - Moisture expansion coefficients of high modulus graphite-fiber/epoxy (HMG/E) ±θ laminates.

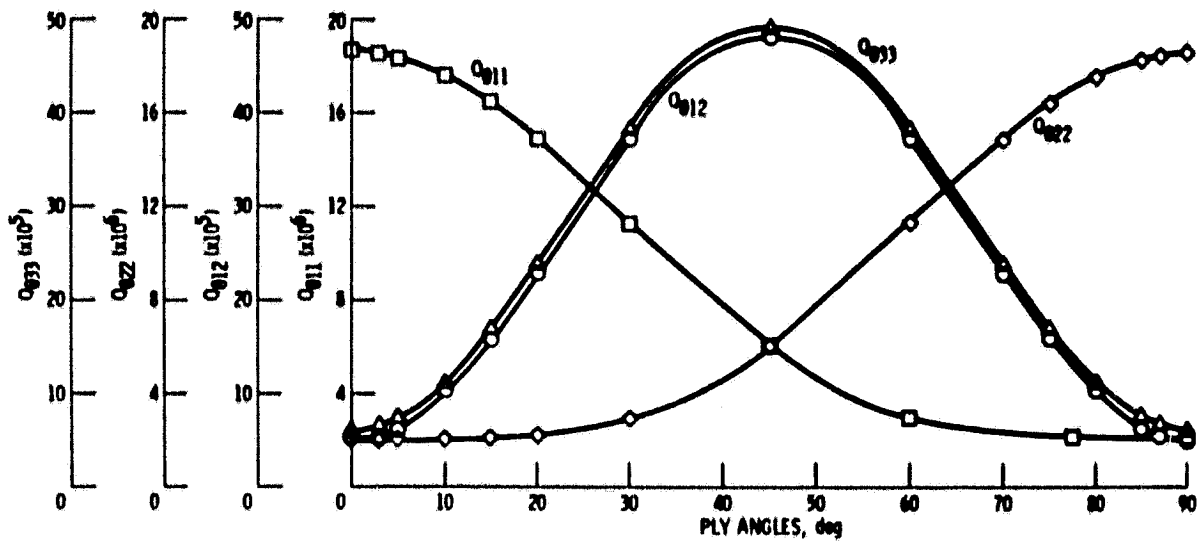


Figure 7. - Reduced stiffnesses of AS graphite-fiber/epoxy (AS/E) ±θ laminates.

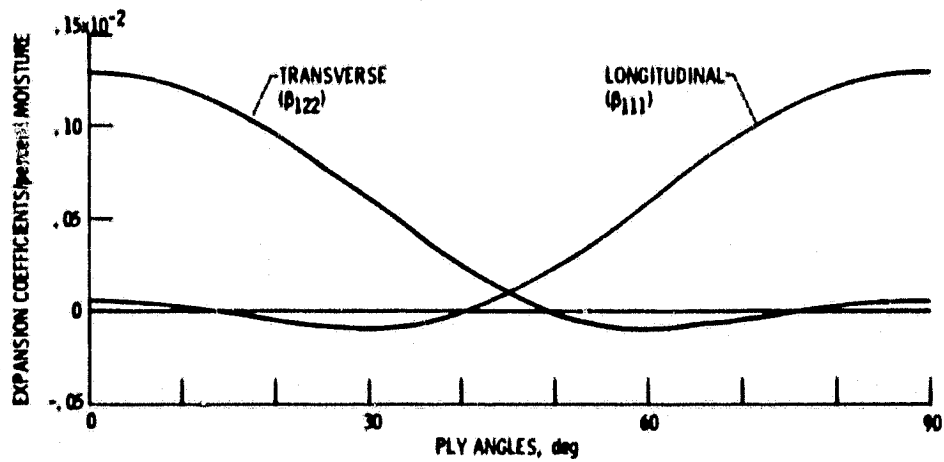


Figure 8. - Moisture expansion coefficients of AS graphite-fiber/epoxy (AS/E) ±θ laminates.

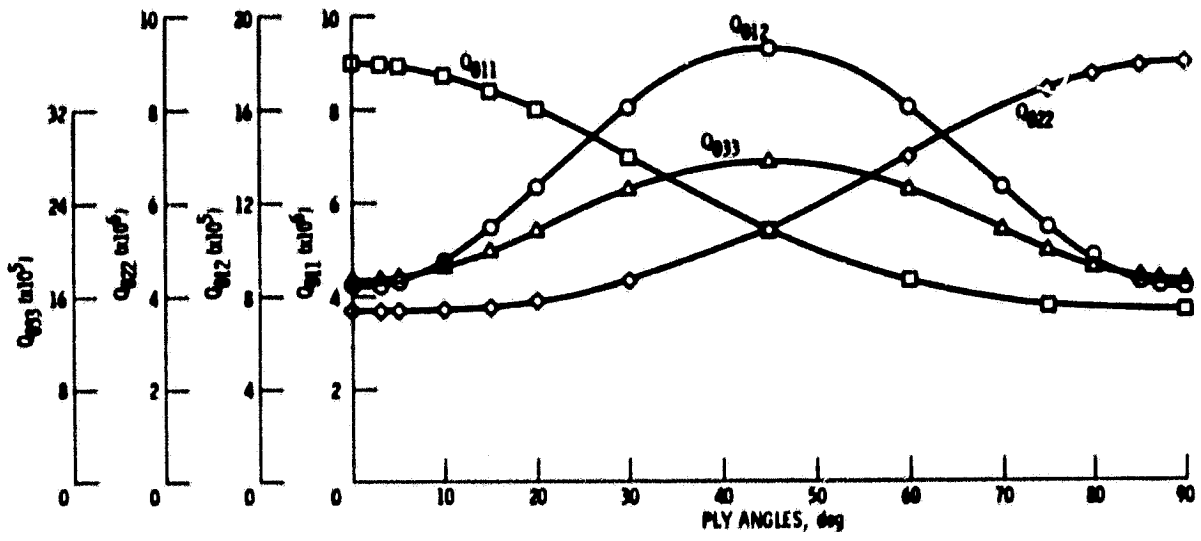


Figure 9. - Reduced stiffnesses of S-Glass-fiber/epoxy (S-G/E)  $\pm\theta$  laminates.

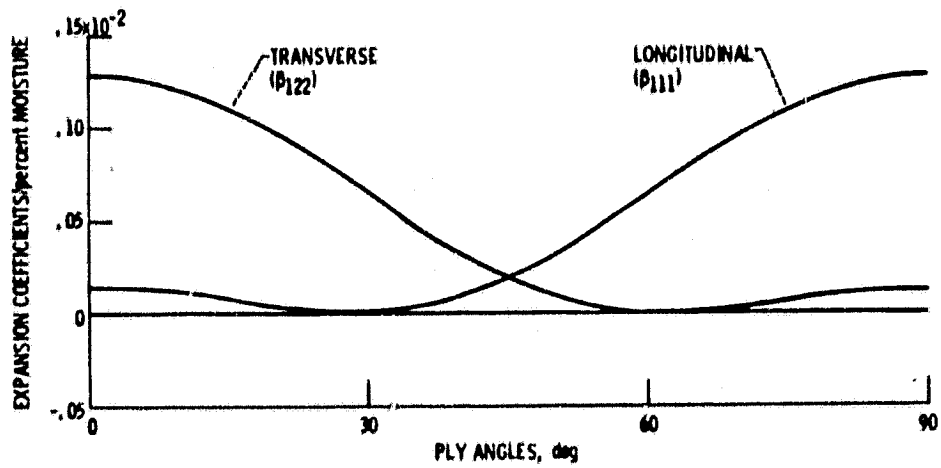


Figure 10. - Moisture expansion coefficients of S-glass-fiber/epoxy (S-G/E)  $\pm\theta$  laminates.

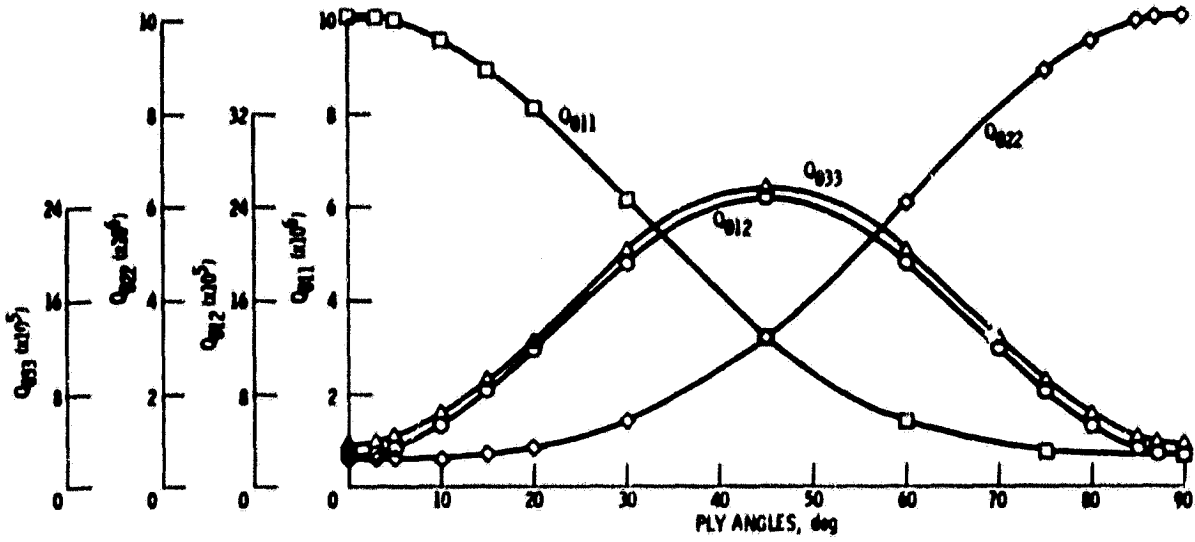


Figure 11. - Reduced stiffnesses of Kevlar 49-fiber/epoxy (K/E)  $\pm\theta$  laminates.

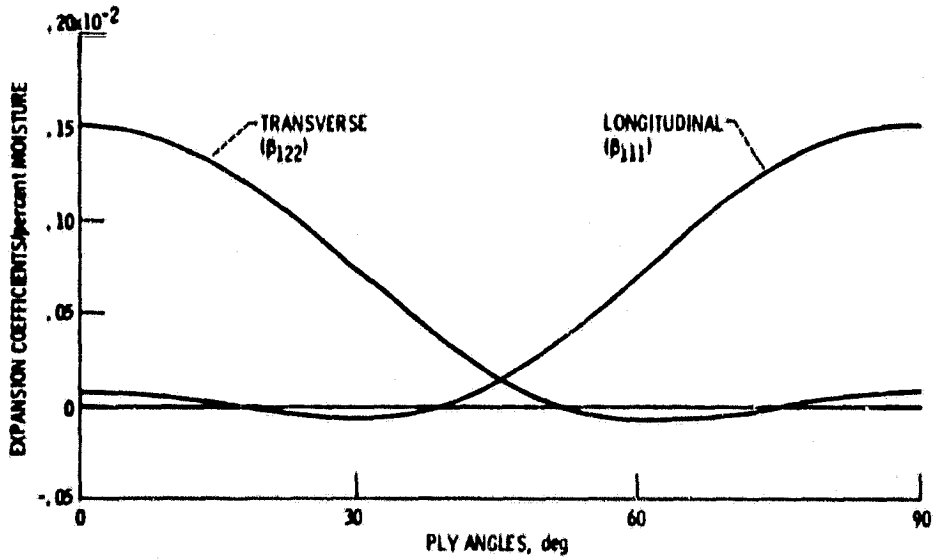


Figure 12. - Moisture expansion coefficients of Kevlar 49-fiber/epoxy (K/E)  $\pm\theta$  laminates.

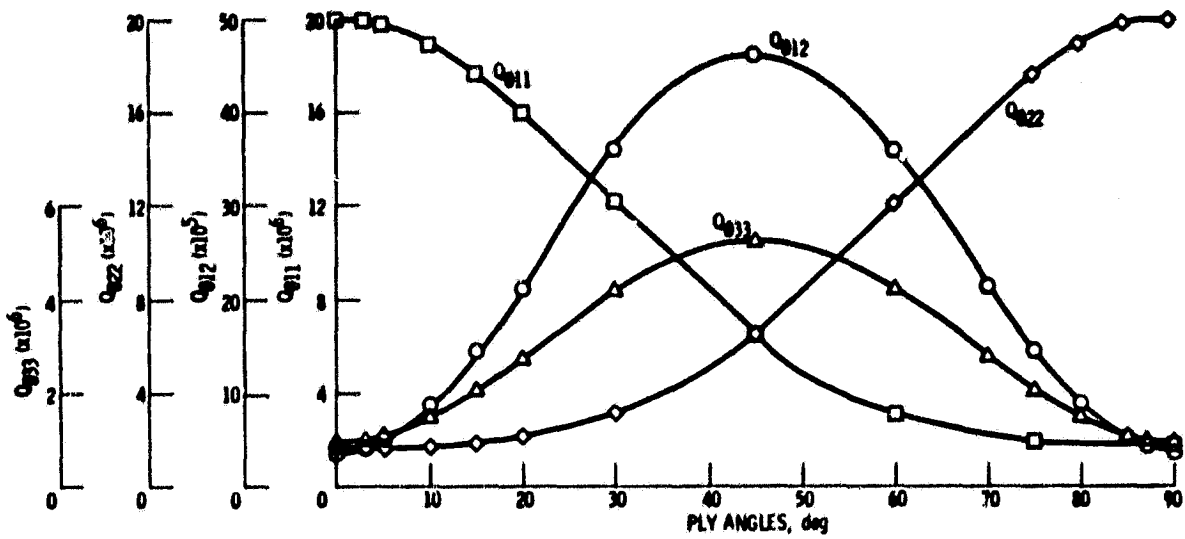


Figure 13. - Reduced stiffnesses of intraply hybrid (80% HMG/E//20% S-G/E) ±θ laminates.

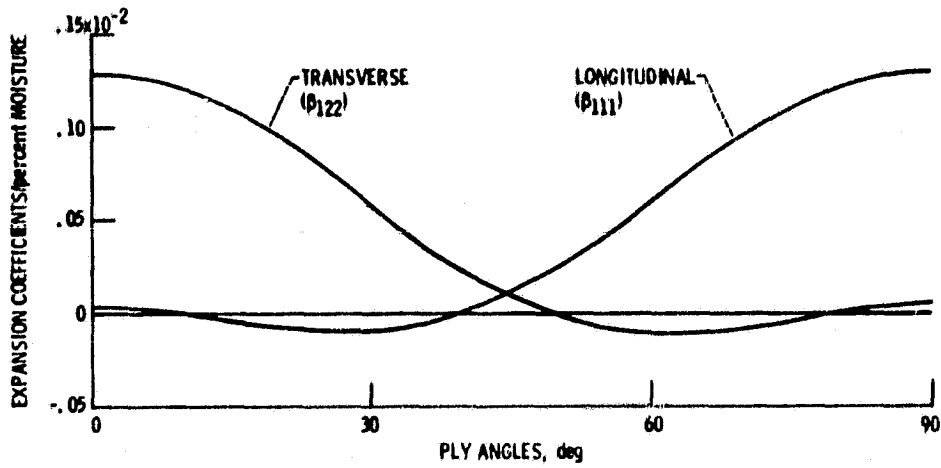


Figure 14. - Moisture expansion coefficients of intraply hybrid (80 percent HMS/E//20 percent S-G/E) ±θ laminates.

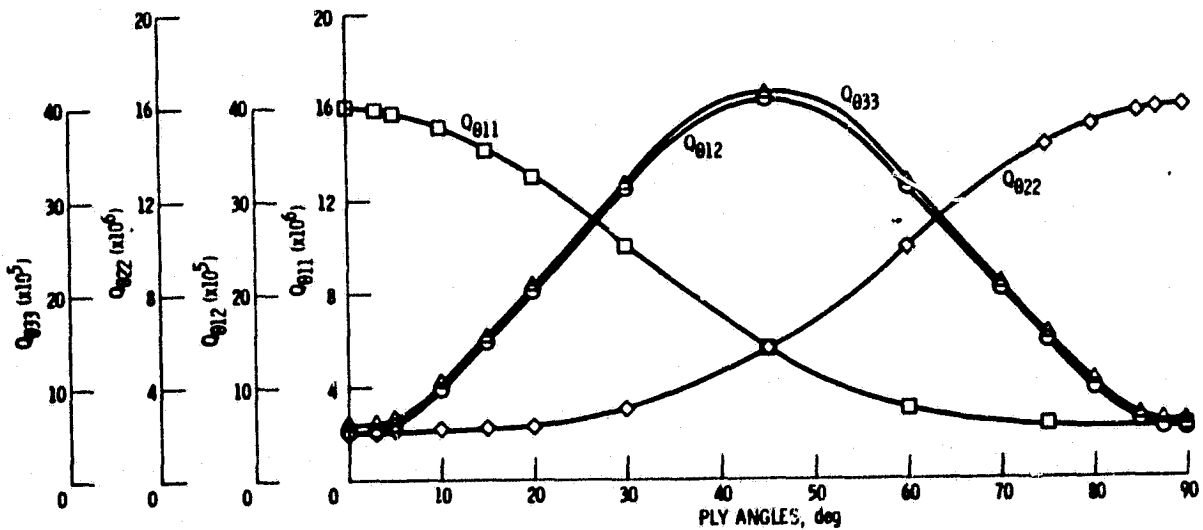


Figure 15. - Reduced stiffnesses of intraply hybrid (80% AS/E//20% S-G/E) ±θ laminates.

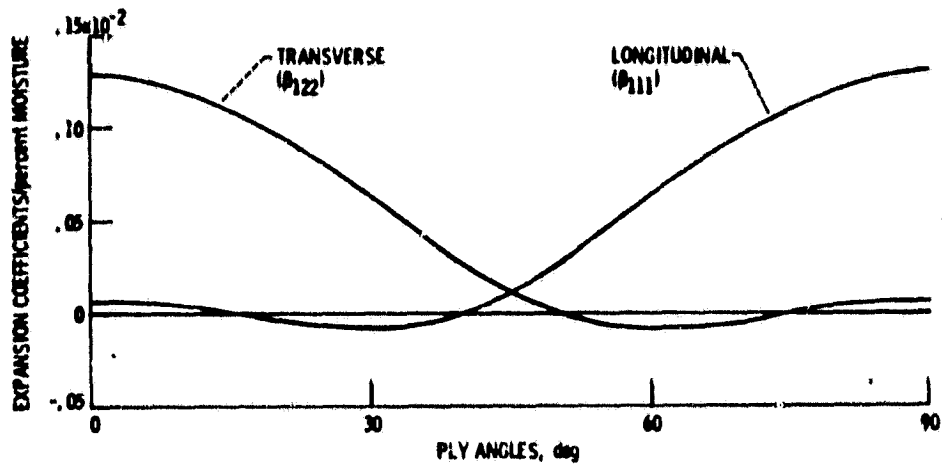


Figure 16. - Moisture expansion coefficients of intraply hybrid (80 percent AS/E//20 percent S-G/E)  $\pm$  laminates.

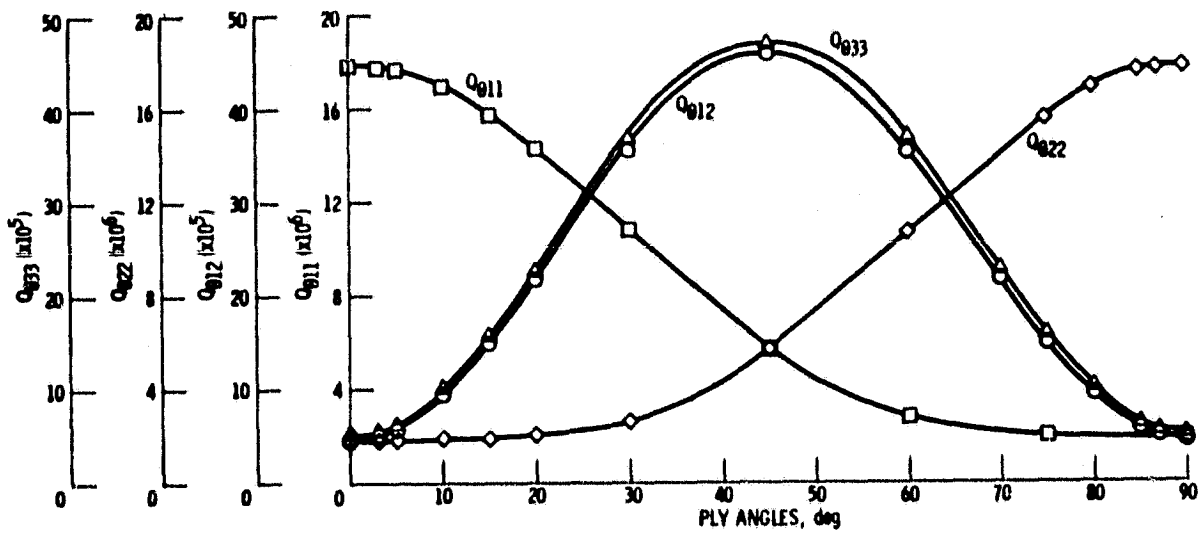


Figure 17. - Reduced stiffnesses of intraply hybrid (80% AS/E//20% K/E)  $\pm$  laminates.

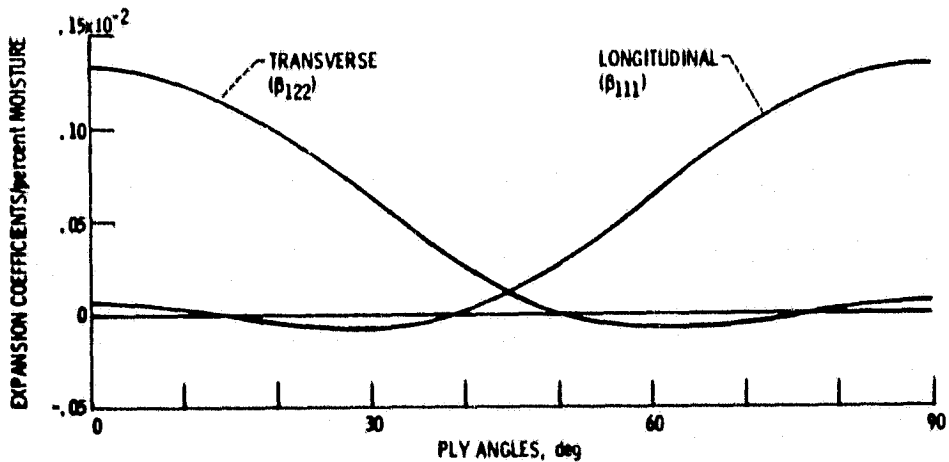


Figure 18. - Moisture expansion coefficients of intraply hybrid (80 percent AS/E//20 percent K/E)  $\pm$  laminates.