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EFFECTS OF CHANGES IN CONVECTIVE EFFICIENCY ON THE SOLAR RADIUS AND LUMINOSITY

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ABSTRACT

A sequence of solar models has been constructed in order to investigate the sensitivity of the solar radius and luminosity to small changes in the ratio α of the mixing length ℓ to the pressure-scale height H_p throughout the solar convective envelope. The basic procedure for determining this sensitivity was to impose a perturbation in α within the convective envelope and then to follow the resulting changes in the solar radius ΔR and luminosity ΔL for the next 10^6 yrs. These calculations gave the following results. 1) A perturbation in α produces immediate changes in the solar radius and luminosity. Initially ΔL and $\Delta\alpha$ are related by $\Delta L/L = 0.30\Delta\alpha/\alpha$. 2) The value of the ratio $W = \Delta \log R/\Delta \log L$ is strongly time dependent. Its value just after the perturbation in α is 6.5×10^{-4}. 3) The ratio $H = (\Delta \log L)^{-1}$ $d\,\Delta \log R/dt$ is much less time dependent and is a more suitable means for relating the changes in the solar radius and luminosity. 4) Both of these ratios imply that for any reasonable change in the solar luminosity the corresponding change in the solar radius is negligible.

I. INTRODUCTION

During this workshop there has been much discussion about possible changes in the solar radius and luminosity over timescales ranging from a year or less to a few hundred years. Because of the keen interest in this topic and because of its obvious relevance to climatic conditions here on earth it is of considerable importance to determine the sensitivity of the solar radius and luminosity to changes in the interior structure of the Sun. Knowledge of this sensitivity together with observational data on any radius and luminosity changes would greatly help in understanding the characteristics of the physical processes operating within the solar interior and, as a result, in understanding the influence which these processes might have on the Sun's future behavior. In addition, it is of considerable importance to determine theoretically the relationship between changes in the solar radius and luminosity resulting from interior perturbations, since then observational data on one of these changes could be used to estimate the size of the other (ref. 1), provided, of course, that the physical process causing the perturbations has been properly identified.

There are many ways in which the interior structure of the Sun might be perturbed. In approximately the outer 2 per cent of the Sun's mass the outward energy flux is carried largely by convection. Since convection in these layers is a turbulent process, it is entirely plausible that there may be random fluctuations in the efficiency of energy transport due, for example, to statistical fluctuations in the number of convective cells or to changes in the flow pattern. The convective envelope therefore represents one part of the Sun where interior perturbations might be expected naturally to arise.

Interior perturbations might also occur within the radiative core. Such perturbations would alter both the thermal and hydrostatic structure of the Sun. The thermal readjustment induced by such perturbations would take place over a Kelvin timescale which for the entire Sun is about 10^7 yrs (ref. 2). The hydrostatic readjustment, however, would take place on a dynamical timescale which is on the order of minutes (ref. 2) and would therefore manifest itself almost instantaneously as a change in the solar radius.

Studies of other stars provide some evidence that observable changes in the radius can result from perturbations within the core. The pulsation period of a class of variable stars known as RR Lyrae stars, found both in globular clusters and in the field, can be accurately determined by using observations spanning several decades. It has been found that the pulsation periods of the RR Lyrae stars typically vary at the rate of a few parts in 10^5 per century. Such changes in the pulsation period can be readily interpreted as changes in the mean stellar radius from one pulsation period to the next. The observed rates of period change considerably exceed the values expected from the normal evolution of the RR Lyrae stars - a fact that has proved to be a long-standing problem. Recent theoretical studies of RR Lyrae models (ref. 3) have shown that perturbations within the core of these stars can reproduce the observed characteristics of the period changes and can thus offer a reasonable solution for this problem. This result suggests that the radiative core of the Sun may also be a likely site for the perturbations responsible for any changes in the solar radius and luminosity.

The objective of the present paper is to give the results of one way of perturbing the solar interior, namely, by changing the efficiency of energy transport by convection throughout the convective envelope. In computing the structure of the solar convective envelope it is necessary to know the value of the convective-temperature gradient, i.e., the actual temperature gradient, at each point. The value of this gradient is determined by the requirement that the total energy flux carried by both convection and radiation be equal to the actual outward energy flux. The convective gradient can range between two limiting values, namely, the adiabatic- and the radiative-temperature gradients, depending on the degree of convective efficiency. When convection is very efficient, the convective gradient approaches the adiabatic gradient. This is normally the situation at higher densities and temperatures when the thermal energy content of the convective cells is relatively large. At lower densities and temperatures, convection can become quite inefficient, and, as a result, the convective gradient becomes significantly superadiabatic and can in fact approach the radiative gradient, which is defined to be the temperature gradient that would exist if all of the outward energy flux were carried by radiation.

The calculation of the convective gradient as a function of the physical conditions at each point in the convective envelope is generally done according to the prescription of the mixing-length theory. In this theory the turbulent convective motions which actually cover a wide range of scale lengths are assumed to be represented by convective cells that travel a characteristic length ℓ before dissolving into the surrounding medium. The mixing length ℓ is the main parameter governing the convective efficiency. An increase in ℓ enhances the convective efficiency, thereby lowering the convective gradient. Conversely, a decrease in ℓ reduces the convective efficiency, since the convective cells then cannot transport their excess thermal energy as far before dissipation. Ordinarily the value of ℓ at each point is expressed in terms of some scale height such as the pressure-scale height H_p ($= dr/d\ln P$). In this paper we will study the consequences of changing the ratio α ($= \ell/H_p$) and hence the convective efficiency in the solar convective envelope. Two points should, however, be kept in mind when considering the following results. First, there are other ways in which the properties of the solar convection could be altered, and hence this paper examines only one type of convective perturbation. Secondly, the perturbation results assume that the mixing-length theory adequately determines the structure of the solar convective envelope at least as far as small perturbations away from the equilibrium structure are concerned.

In the next section we describe first the unperturbed structure of the solar convective envelope and then the effects which a perturbation in α has on this structure at various times following the perturbation. The changes in the solar radius and luminosity resulting from a perturbation in α and the relationship between these changes are discussed in sections III and IV, respectively. We emphasize in section IV the advantages of using the time rate of change of the radius perturbation rather than the radius perturbation itself when relating the radius and luminosity perturbations. Finally, a summary of the main points is provided in section V.

II. SOLAR CONVECTIVE ENVELOPE

UNPERTURBED STRUCTURE

In order to examine the unperturbed structure of the solar convective envelope, one must first obtain a solar model with the proper luminosity and radius at an age of 4.7×10^9 yrs following the zero-age main-sequence (ZAMS) phase. The properties of a solar model are dependent on the assumed composition, i.e., the helium abundance Y and the heavy-element abundance Z, and on α. For the present calculations Z was taken to be 0.02. The luminosity of a solar model is particularly sensitive to Y, since changes in Y affect the mean molecular weight and hence the hydrostatic structure, leading to a change in the central temperature. This in turn alters the rate of hydrogen burning due to the strong temperature dependence of the nuclear reaction rates. On the other hand, α primarily affects the convective envelope and thus the radius. Several trial sequences showed that the values

$$Y = 0.2317, \alpha = 1.70 \quad (1)$$

reproduce the present solar luminosity and radius with an error of about 0.2 per cent. Accordingly these values were adopted for the model computations, and a standard evolutionary sequence was then computed from the ZAMS phase to the present Sun. The ZAMS luminosity and radius of this solar sequence were 0.723 $L_\odot$ and 0.893 $R_\odot$.

The unperturbed structure of the convective envelope in the present Sun is perhaps best illustrated by the behavior of the adiabatic-, convective- and radiative-temperature gradients (= dlogT/dlogP), denoted by ∇_a, ∇_c and ∇_r, respectively. These three gradients are plotted in Figure 1 as functions of the logarithm of the amount of mass between the surface and the given point. Here M_r is the amount of mass within a distance r from the center of the Sun. The convective envelope in this model contains 0.016 $M_\odot$, corresponding to log $(M_\odot - M_r) = -1.796$. For a fully ionized nondegenerate gas with negligible radiation pressure ∇_a equals 0.40, and we note that ∇_a approaches this value throughout the inner part of the convective envelope, i.e., for log $(M_\odot - M_r) > \sim -4$. Between log $(M_\odot - M_r) = -11$ and -4, ∇_a is depressed due to the ionization of hydrogen and the first and second ionizations of helium. The difference between ∇_c and ∇_a is very small for log $(M_\odot - M_r) > \sim -7$. This "adiabatic" region contains the bulk of the mass within the convective envelope. In this region energy transport by convection is very efficient with radiation making only a negligible contribution to the outward flux, since $\nabla_r >> \nabla_c$. Because of this high convective efficiency ∇_c will not be very sensitive to α. Just the opposite is true in the layers near the surface (log $(M_\odot - M_r) < \sim -9$), where the convection becomes strongly superadiabatic. As one goes outward through this "superadiabatic" region, ∇_c begins to exceed ∇_a substantially with the maximum superadiabaticity being reached at log $(M_\odot - M_r) = -10.5$. The convective efficiency in the superadiabatic region is low due to the low density and thermal energy content of the convective cells. In the layers nearest the surface ∇_c approaches ∇_r and hence the energy transport there is largely by radiation. The structure of the superadiabatic region will be strongly dependent on α. The transition between the adiabatic and superadiabatic regions occurs around log $(M_\odot - M_r) = -8$.

PERTURBED STRUCTURE

Before discussing the quantitative results from detailed solar model computations it is worthwhile to mention first some further features of the solar convective envelope and to consider the physical reasons for the way in which the Sun responds to a change in α. The superadiabatic region contains little mass and has only a small thermal energy content. As a result, the thermal timescale of the region is quite short, on the order of 1 day (ref. 4), and consequently the superadiabatic region rapidly readjusts to any change in α. Within the adiabatic region ∇_c is nearly independent of α, and therefore changes in α that are confined to this region will not significantly affect the structure of the convective envelope. However, a change in α throughout the entire convective envelope will alter the boundary conditions at the top of the adiabatic region, and this in turn will force the adiabatic region to undergo both a dynamical and thermal readjustment. The dynamical response of the adiabatic region will restore hydrostatic equilibrium on a timescale of minutes and will thus be practically instantaneous. The contraction (or expansion) associated with this dynamical readjustment will release (or absorb) gravitational potential energy, thereby perturbing the outward energy flux L_r and causing the adiabatic region to depart from thermal equilibrium. The timescale for restoring thermal equilibrium is on the order of 10^5 yrs (ref. 5). The key point to remember is that the response of L_r and hence the solar luminosity is set, not by the thermal timescale of the adiabatic region, but by the much shorter thermal timescale of the superadiabatic region. Therefore one would expect a change in α to show up almost immediately as a perturbation in the solar luminosity.

Let us now outline the sequence of events to be expected if, for example, α increases. After about 1 day the superadiabatic region will have readjusted both thermally and hydrostatically. As is well-known from stellar model computations, an increase in α leads to a contraction of the adiabatic region and hence to the concomitant release of gravitational potential energy, resulting in an increase in the outward flux L_r and thus in the solar luminosity. The superadiabatic region will then expand in order to carry the additional outward flux, since this is the normal reaction of a region in which energy transport by radiation is important (ref. 4). Thus one has a situation in which the bulk of the convective envelope contracts on a timescale of $\sim 10^5$ yrs while the outermost layers initially expand on a timescale of days. Observationally this would appear as a sudden increase in the solar radius followed by a gradual decrease. A similar sequence of events would also occur if α decreases except that all of the perturbations would have opposite signs.

In order to verify the above predictions quantitatively, a sequence of solar models was constructed in which the time step between models, which is normally set by the nuclear timescale of the core, was gradually reduced to 1 yr. This choice for the minimum time step was made in order to follow the rapid changes expected in the solar radius and luminosity while avoiding the numerical difficulties sometimes encountered when even shorter time steps are used. At this point in the calculations the value of α was increased by $\Delta\alpha = 0.01$ throughout the convective envelope, and the subsequent evolution of the perturbed solar models was followed for about the next 10^6 yrs. After the change in α the time step was slowly increased but was always small compared with the timescale on which the perturbations were changing. The size of the perturbations resulting from this change in α are very small compared with the numerical accuracy of typical solar models. For this reason it was essential to maintain a high degree of numerical accuracy and especially to minimize the importance of numerical noise during the computations.

When constructing a stellar model one usually treats the outermost layers differently from the interior. In the outermost layers the stellar structure equations are integrated inward from the surface to some interior fitting point under the assumption of constant L_r. This is equivalent to ignoring any changes in the gravitational potential energy, i.e., to assuming thermal equilibrium. Given several of these integrations, one can then define the outer boundary conditions needed for the interior solution. Inside the fitting point the stellar structure equations are replaced by difference equations which are then solved by an iterative procedure. In the present solar models the fitting point was located at $\log (M_\odot - M_r) = -6$. Since according to the previous discussion the thermal timescale of the outermost 10^{-6} $M_\odot$ of the Sun is very short, our implicit assumption of thermal equilibrium in these layers should be justified. About 175 integration steps based on a high-order predictor-corrector procedure were used in computing the layers above the fitting point. Interior to the fitting point there were 247 mesh points of which 88 were in the convective envelope.

There are many sources of numerical noise which can enter into solar model computations. For example, stellar structure programs frequently contain iterative procedures for determining the density from the equation of state, the degree of ionization from the Saha equations and the superadiabaticity within convective regions. Tight convergence of these iterative procedures as well as the iterative procedure involved in the overall convergence of the models was required at all times. In addition, no changes were permitted in either the number or distribution of the mesh points. Such changes in the mesh points could introduce spurious perturbations by altering the truncation error with which the difference equations represent the basic differential equations of stellar structure. Special attention must therefore be paid to these as well as a number of other sources of numerical noise if reliable results are to be obtained. To insure that numerical noise was not important in the present calculations, we constructed an additional solar sequence in which the perturbation in α was a factor of 10 greater, i.e., $\Delta\alpha = 0.10$. The only difference was the expected scaling in the size of the perturbations by a factor of 10. In particular, the ratio of the perturbations in the solar radius and luminosity changed by less than 2 per cent.

Let us now consider some of the quantitative results for the readjustment of the solar convective envelope after the perturbation $\Delta\alpha = 0.01$. Figure 2 illustrates the difference in the radius $\Delta\log r$ between a perturbed model and the basic unperturbed model as a function of $M_\odot - M_r$ within the convective envelope. The four curves labelled a, b, c and d correspond to four perturbed models having ages of 1, 4900, 49,000 and 310,000 yrs, respectively, following the perturbation in α. The contraction is not yet noticeable in model a, because the time elapsed since the perturbation has been too short. Moreover, the increase in the surface radius $\Delta\log R$ in model a due to the expansion of the superadiabatic region amounted to only 5×10^{-7}. By model d the rate of contraction has slowed substantially so that this model is approaching the equilibrium structure for the new value of α. The amount of the contraction is considerably greater nearer the surface, and thus the convective envelope does not contract uniformly. This contraction increases the weight of the convective envelope on the radiative core, thereby causing the core also to contract, as indicated in Figure 2 for $M_\odot - M_r > 0.016\ M_\odot$.

The rate of release of gravitational potential energy ϵ_g in ergs/gm/sec within the convective envelope is shown in Figure 3 for each of the four perturbed models plotted in Figure 2. The maximum rate of contraction of the convective envelope occurs immediately after the perturbation in α, and thus the largest values of ϵ_g are produced at this time. However, the radiative core does not begin to contract until after there has been a decrease in the radius of the convective envelope and hence a change in the boundary conditions at the edge of the core. This explains why the release of gravitational energy in the core is negligible in model a while it becomes important in the later models. We note that ϵ_g is negative for $M_\odot - M_r < 6 \times 10^{-4}\ M_\odot$ in model a due to the expansion of the outer layers of the convective envelope. In the present calculations this expansion disappears 1 year after the perturbation in α; it might actually disappear sooner if shorter time steps are used. The slowing-down of the contraction with time, as indicated by the decrease in ϵ_g, is apparent in going from models a to d.

The release of gravitational potential energy perturbs the outward flux L_r at each point within the convective envelope. This flux perturbation ΔL_r is illustrated in Figure 4, where the difference in L_r between each of the four perturbed models in Figure 2 and the unperturbed model is shown over the same interval in $M_\odot - M_r$ as in Figure 3. The behavior of the flux perturbation in time is somewhat complicated in the inner half of the convective envelope due to two competing effects. Between models a and c the contribution to the flux perturbation from the contraction of the core increases, while at the same time the contribution of the convective envelope decreases. The drop in ΔL_r for $M_\odot - M_r < 6 \times 10^{-4}\ M_\odot$ in model a is again associated with the initial expansion of the outermost layers.

The above discussion has focused on the structural readjustment that takes place within the solar convective envelope following a perturbation in α. We now turn our attention to the question of what potentially observable changes a perturbation in α might produce in the solar radius and luminosity.

III. CHANGES IN THE SOLAR RADIUS AND LUMINOSITY

The changes in the solar radius $\Delta \log R$ and luminosity $\Delta \log L$ during the first 8×10^5 yrs after the perturbation in α are presented in Figure 5. The zero-point of the time scale in Figure 5 as well as in all subsequent figures corresponds to the time t when the perturbation $\Delta\alpha = 0.01$ was imposed within the convective envelope. The response of the solar luminosity to this perturbation appears to be nearly instantaneous for the time resolution of this figure. Following the large initial response $\Delta \log L$ decays with an e-folding time on the order of a few times 10^5 yrs. By the latest times shown in Figure

5 the perturbed solar models are approaching their new equilibrium structure which is characterized by a decrease in log R and an increase in log L. The present results demonstrate that the initial response of the solar luminosity considerably exceeds the difference in log L between the unperturbed and new equilibrium states. Also plotted in Figure 5 is the change in the rate of hydrogen burning $\Delta \log L_H$. The contraction of the core, as indicated previously, raises the temperature in the layers near the center, thus increasing the rate of the nuclear reactions.

It is of some interest to examine the behavior of $\Delta \log L$ and $\Delta \log R$ immediately following the perturbation in α. Figure 6 shows this behavior for $\Delta \log L$. The time scale in this figure has been expanded by approximately a factor of 2000 compared with Figure 5 and consequently covers only the first 400 yrs after the perturbation in α. Even on this expanded timescale there is a sudden response of the solar luminosity at $t = 0$. This response would actually have been more abrupt if time steps less than 1 year had been used in the computations. This result confirms our previous conjecture that changes in convective efficiency of the type considered here will almost immediately affect the surface luminosity. We note from Figure 6 that $\Delta \log L$ is nearly constant over a timescale of several hundred years. From these results it follows that the change in the solar luminosity produced by a perturbation $\Delta\alpha$ is given by

$$\frac{\Delta L}{L} = 0.30 \frac{\Delta \alpha}{\alpha} \qquad (2)$$

for short times after the perturbation. A similar expression has been derived by Dearborn and Blake (ref. 4), who found a coefficient of 0.64 on the right-hand side of equation (2).

The more complicated behavior of $\Delta \log R$ is illustrated in Figure 7 for the same time interval as in Figure 6. The ordinate in Figure 7 has been expanded by roughly a factor of 1000 in comparison with Figure 5. The sudden increase of the solar radius due to the expansion of the superadiabatic region is readily apparent at $t = 0$. This initial expansion is followed by an overall contraction of the convective envelope and hence in the solar radius as the adiabatic region reacts to the change in α. At $t = 250$ yrs the radius again equals its unperturbed value. The maximum value of $\Delta \log R$ just after the perturbation in α was quite small, only 5×10^{-7}, which explains why the initial expansion was not evident in Figure 5. This maximum value of $\Delta \log R$ is related to the perturbation $\Delta\alpha$ by the equation

$$\frac{\Delta R}{R} = 2.0 \times 10^{-4} \frac{\Delta \alpha}{\alpha} . \qquad (3)$$

Two features of Figure 7 should be emphasized. First, the value of $\Delta \log R$ is strongly time dependent even over the short time interval covered by this figure. Second, the rate of change of $\Delta \log R$, $d \Delta \log R/dt$, is, in contrast, nearly constant.

In this section we have discussed each of the changes $\Delta \log L$ and $\Delta \log R$ separately. We now wish to consider how these changes are related to each other.

IV. RELATIONSHIP BETWEEN CHANGES IN THE SOLAR RADIUS AND LUMINOSITY

One of the objectives of previous studies (refs. 1, 4, 6, 7, 8) on the effects of perturbations in α was to determine the ratio

$$W = \frac{\Delta \log R}{\Delta \log L} . \tag{4}$$

This ratio can be straightforwardly obtained from the present calculations to give the results shown in Figure 8, where the time interval is the same as in Figures 6 and 7. The strong time dependence of W is immediately evident. In fact, the value of W changes sign at t = 250 yrs. Since $\Delta \log L$ is nearly constant over the time interval in Figure 8, this time dependence is actually a reflection of the strong time variation of $\Delta \log R$. The values of W in Figure 8 can be approximated by the equation

$$W(t) = 6.5 \times 10^{-4} - 2.6 \times 10^{-6}\, t, \tag{5}$$

where t is in years. The original estimates of W(t = 0) ranged from 0.075 to 5×10^{-3} (refs. 1, 4). More recent determinations have averaged from 5×10^{-4} to 10×10^{-4} (refs. 6, 7, 8) and are therefore in agreement with the present value.

One would like to use W to determine, for example, the change in the solar luminosity associated with observational estimates for changes in the solar radius. However, there are two major disadvantages with using W for this purpose. First, it is only appropriate to use W if the perturbation in α has occurred during the time interval spanned by the radius observations. Otherwise any observed change in log R would actually be the change between two perturbed states rather than between the unperturbed and perturbed states. From the last section we know that a perturbation in α gives rise to changes $\Delta \log R$ and $\Delta \log L$ that persist for several times 10^5 yrs. Thus, if an observed change in the solar radius is ascribed to a perturbation in α, the probability that this perturbation occurred during the interval of the observations is very small. Second, there is the problem caused by the strong time dependence of W. Even if the first disadvantage is ignored, one must still know how much time has elapsed since the perturbation in α in order to compute the proper value of W from equation (5).

The above difficulties can be overcome by using an alternative expression relating $\Delta \log R$ and $\Delta \log L$, namely, the ratio

$$H = \frac{1}{\Delta \log L} \frac{d\, \Delta \log R}{dt} = \frac{1}{\Delta \log L} \frac{d \log R}{dt} . \tag{6}$$

This ratio is plotted in Figure 9, where the time interval is again the same as in Figures 6 and 7. The average value of H in Figure 9 is

$$H = -2.6 \times 10^{-6}\ yr^{-1}. \tag{7}$$

The variation in the value of H is substantially less than was the case for W. This result is not surprising in view of our previous comments that $\Delta\log L$ and $d\,\Delta\log R/dt$ in Figures 6 and 7 are nearly constant. The fact that H is not strongly time dependent can also be given a straightforward physical explanation. The rate of contraction of the convective envelope $d\,\Delta\log R/dt$ determines the rate of release of gravitational potential energy which in turn determines the luminosity change $\Delta\log L$. Thus $d\,\Delta\log R/dt$ and $\Delta\log L$ actually represent different ways of measuring the same quantity, namely, the mean value of ε_g in the convective envelope, and consequently we would expect this ratio to be approximately constant at least for short times following the perturbation in α. Over much longer time intervals, however, the value of H will change significantly, as is illustrated by Figure 10, but even here the relative change is much less than that shown by W in Figure 8. For example, after 10^5 yrs the value of H differs by only a factor of 2 from its value at $t = 0$. When using H to relate $\Delta\log L$ to an observed radius change, one is implicitly assuming that the perturbation in α occurred prior to the time of the observations, but, as mentioned before, this is very likely to be the case. We conclude therefore that the inherent disadvantages of the ratio W can be circumvented to a large extent by using the ratio H.

Dunham et al. (ref. 9) have reported a decrease in the solar radius of 0.70 ± 0.12 between 1925 and 1980 from measurements of the size of the path of totality during a number of solar eclipses. The corresponding change in log R is thus -3×10^{-4}. Let us now see what this observational result implies for the change in the solar luminosity under the assumption that a perturbation in α is responsible for the radius change. There are two cases to consider. First, let us assume that the perturbation in α occurred sometime after 1925 so that W is the appropriate ratio to use. From equation (5) it follows that $5.1 \times 10^{-4} < W < 6.5 \times 10^{-4}$. The change in log L determined from these values of W lies in the range $-0.62 < \Delta\log L < -0.49$, implying that the solar luminosity in 1925 differed from the present luminosity by a factor of 3 or 4. As the second case, let us assume that the perturbation in α occurred before 1925 so that we must apply the ratio H. The radius measurements then give $-6 \times 10^{-6}\ yr^{-1}$ for the average value of $d\,\Delta\log R/dt$ since 1925. By combining this observational result with the value of H from equation (7), we find that $\Delta\log L = 2.2$, again implying an impossibly large change in the solar luminosity. The change in log L would have been even greater if a smaller value of $|H|$ had been used, as would be appropriate for later times according to Figure 10. We conclude therefore that the change in the solar radius since 1925 either has not been as large as reported by Dunham et al. or has been produced by some process other than the one studied in this paper.

V. SUMMARY

From the present results it is possible to draw the following conclusions:

1) Changes in the efficiency of convection throughout the solar convective envelope lead to sudden changes in both the solar radius and luminosity. The relationship between the change in the luminosity and the change in α is given by equation (2).

2) The value of the ratio $W = \Delta\log R/\Delta\log L$ is strongly time dependent. For this and other reasons W does not seem to be a very suitable means for relating changes in the solar radius and luminosity. Immediately after a perturbation in α the value of W is 6.5×10^{-4}.

3) A more satisfactory way to relate the radius and luminosity changes is represented by the ratio $H = (\Delta\log L)^{-1} d\,\Delta\log R/dt$. This ratio is much less time dependent, varying from -2.6×10^{-6} to $-1.3 \times 10^{-6}\ yr^{-1}$ during the first 10^5 yrs following a perturbation in α.

4) According to the present values of W and H, any observationally detectable change in the solar radius would imply an impossibly large change in the solar luminosity. Consequently changes in convective efficiency of the type considered here cannot be responsible for any observed radius changes in the Sun.

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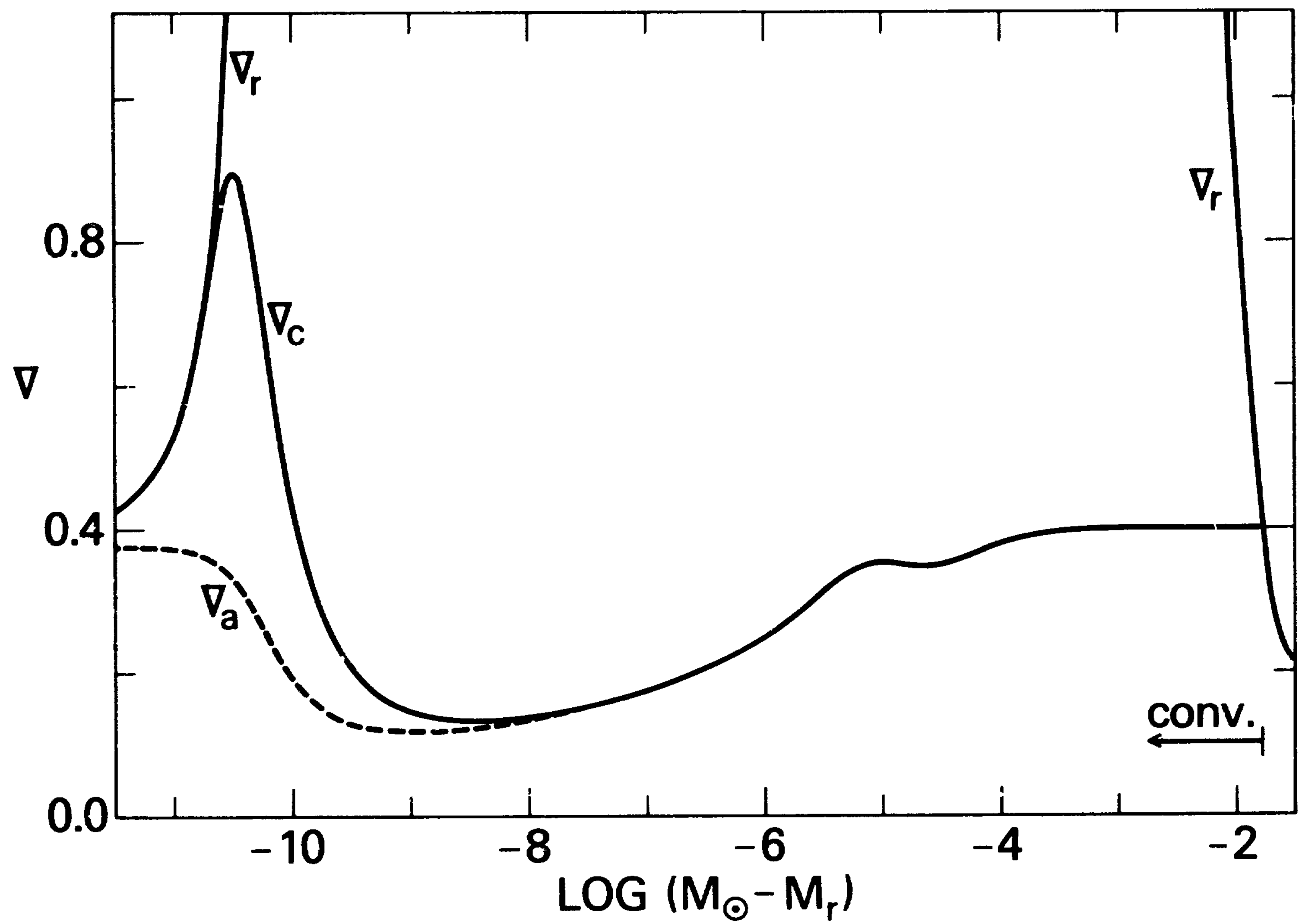

Figure 1. Variation of the adiabatic-, convective- and radiative-temperature gradients ∇_a, ∇_c and ∇_r, respectively, as functions of log ($M_\odot$ - M_r) within the solar convective envelope

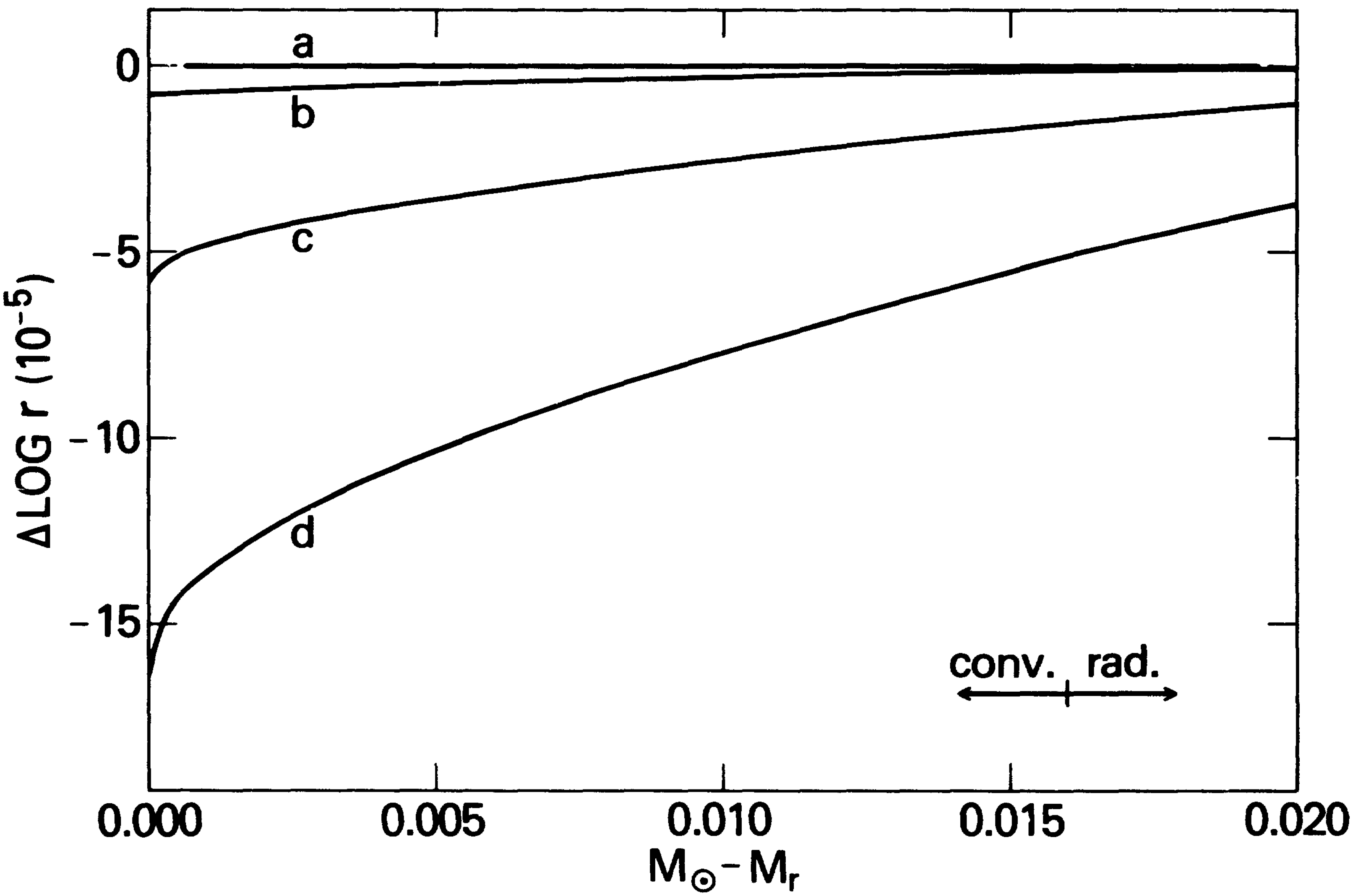

Figure 2. Perturbation in the radius $\Delta \log r$ as a function of $M_{\odot} - M_r$ within the solar convective envelope at four times following the perturbation $\Delta\alpha = 0.01$

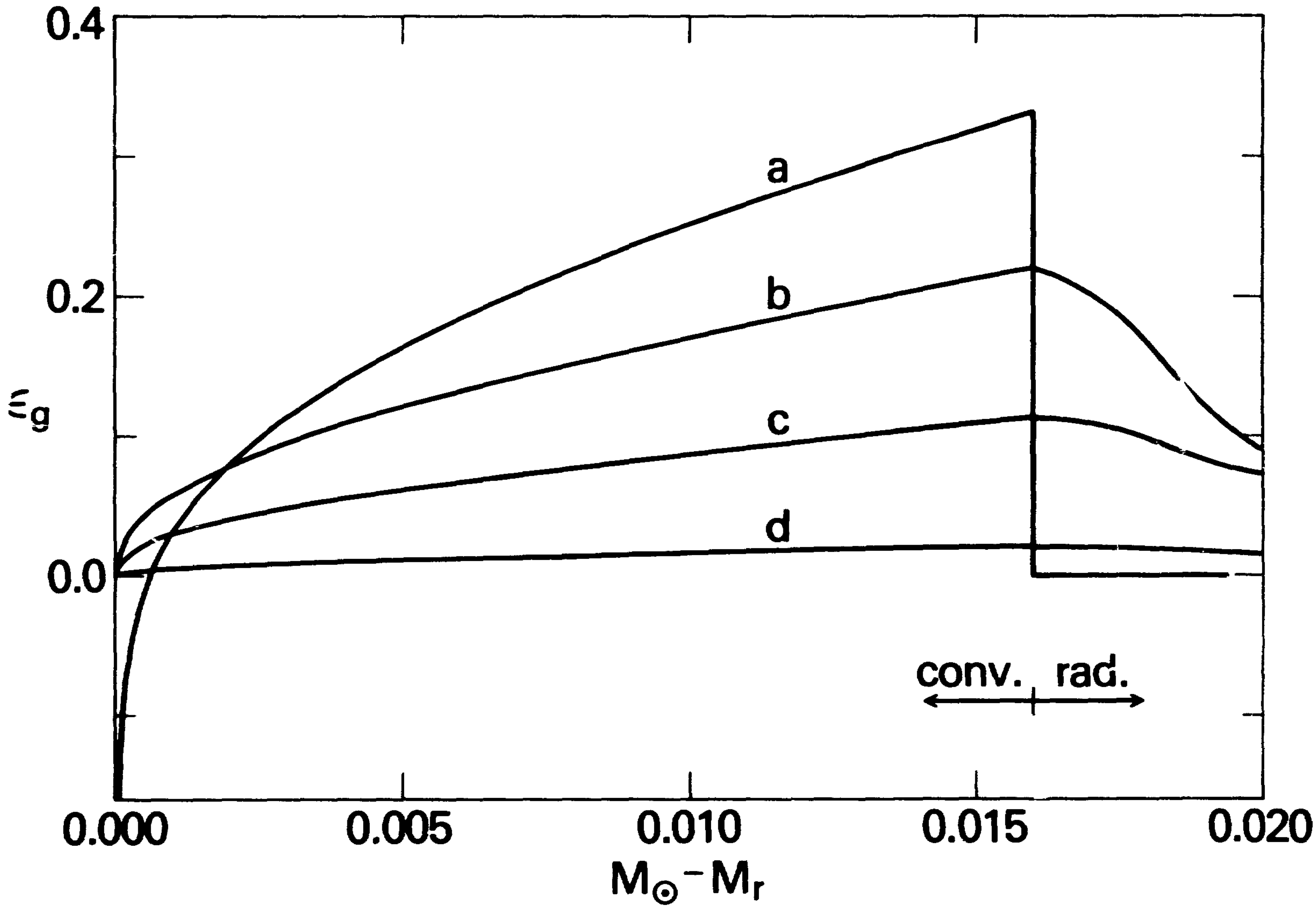

Figure 3. Rate of release of gravitational potential energy ϵ_g as a function of $M_\odot - M_r$ within the solar convective envelope at four times following the perturbation $\Delta\alpha = 0.01$

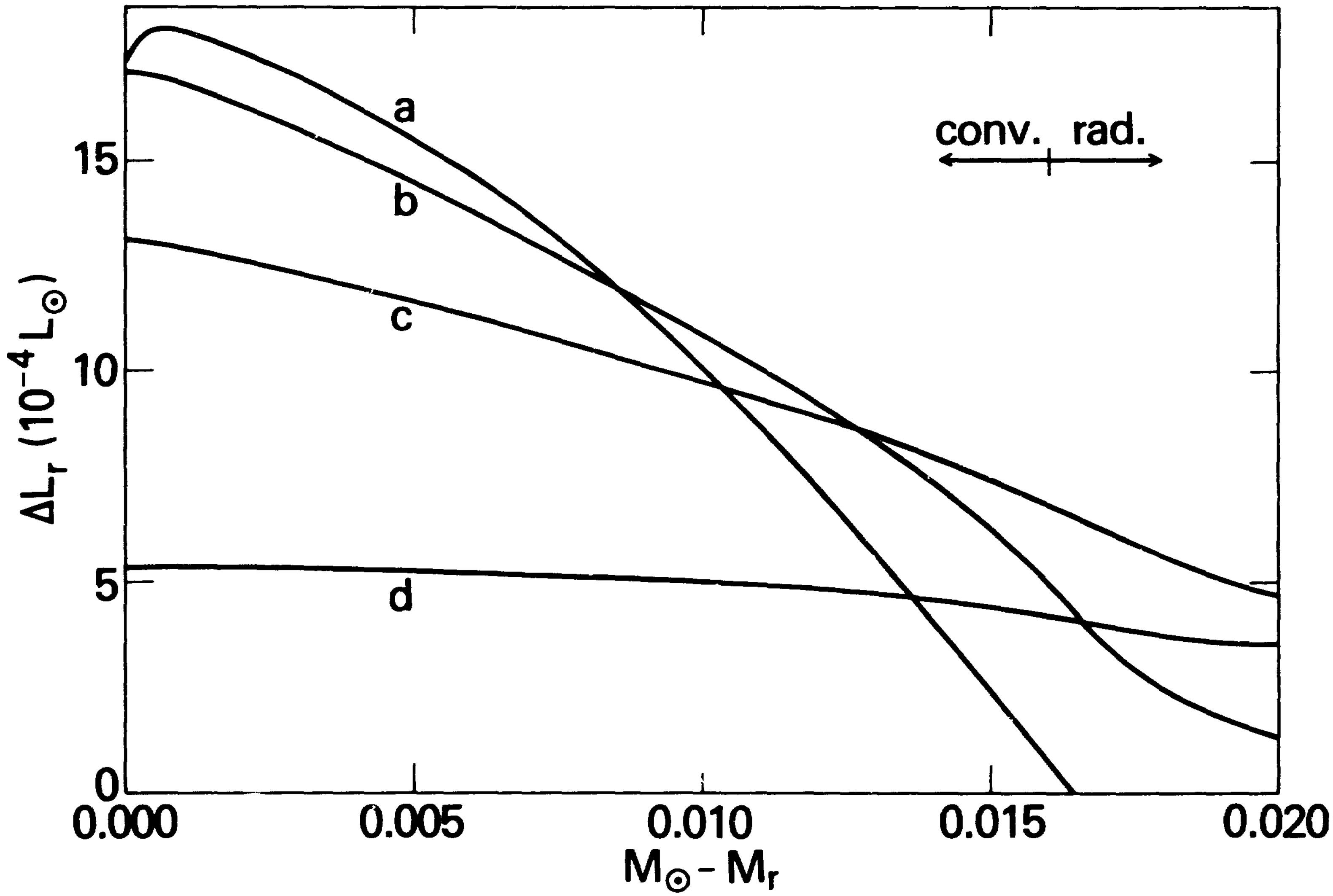

Figure 4. Perturbation in the outward flux ΔL_r as a function of $M_\odot - M_r$ within the solar convective envelope at four times following the perturbation $\Delta\alpha = 0.01$

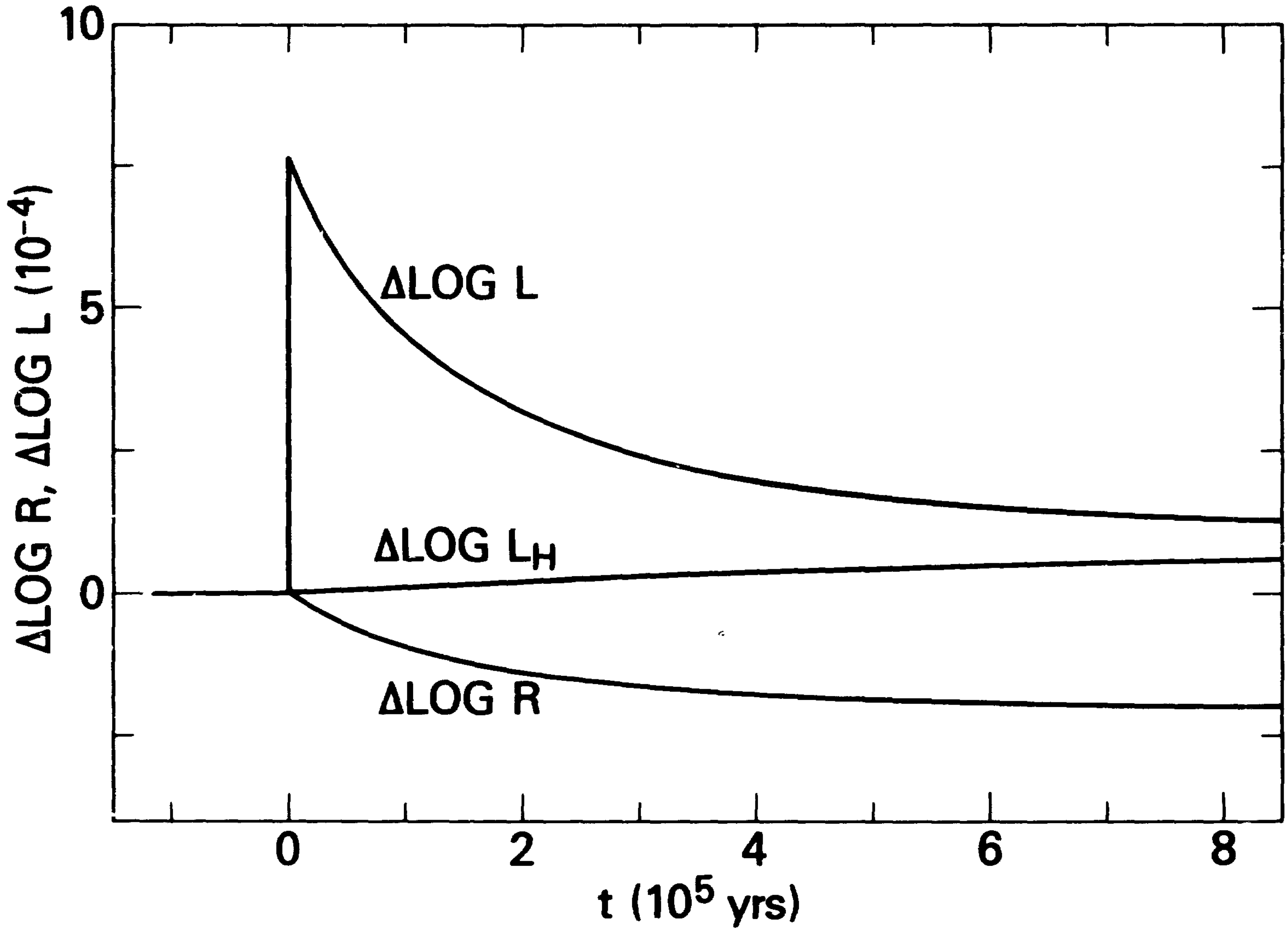

Figure 5. Time dependence of the change in the luminosity $\Delta \log L$, the surface radius $\Delta \log R$ and the rate of hydrogen burning $\Delta \log L_H$ following the perturbation $\Delta\alpha = 0.01$ within the solar convective envelope

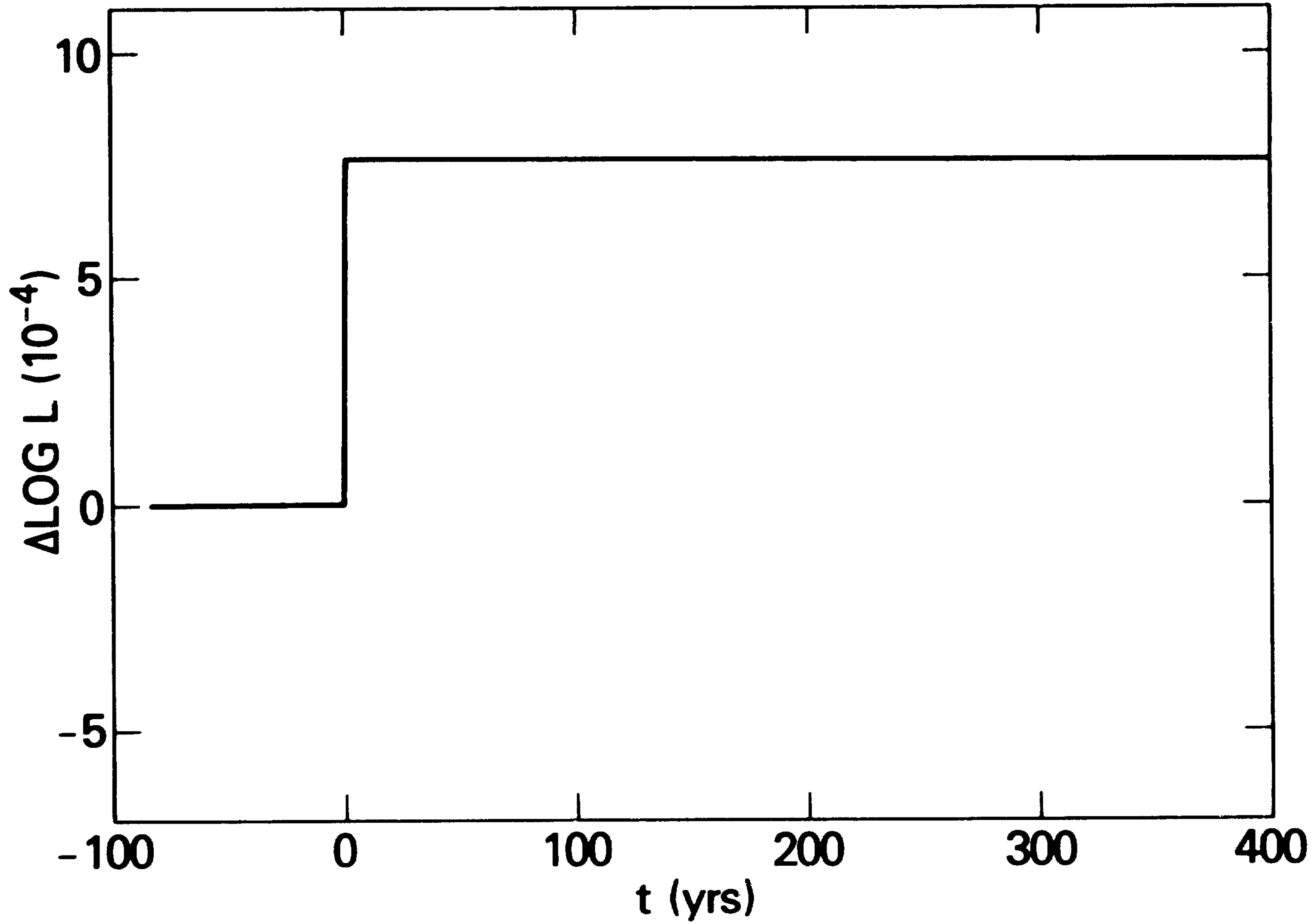

Figure 6. Time dependence of the change in the luminosity Δlog L shortly after the perturbation $\Delta\alpha = 0.01$ within the solar convective envelope

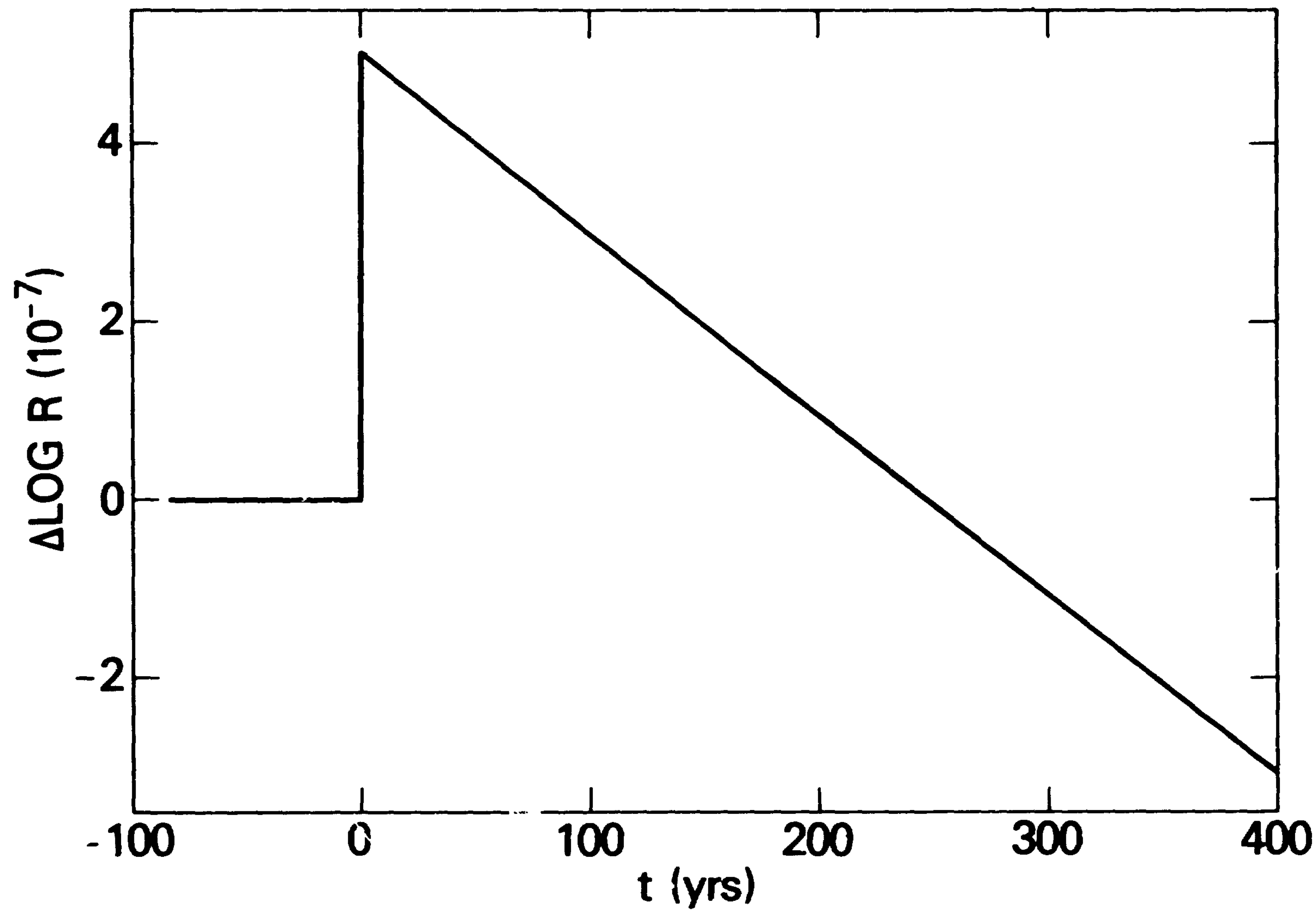

Figure 7. Time dependence of the change in the surface radius $\Delta \log R$ shortly after the perturbation $\Delta\alpha = 0.01$ within the solar convective envelope

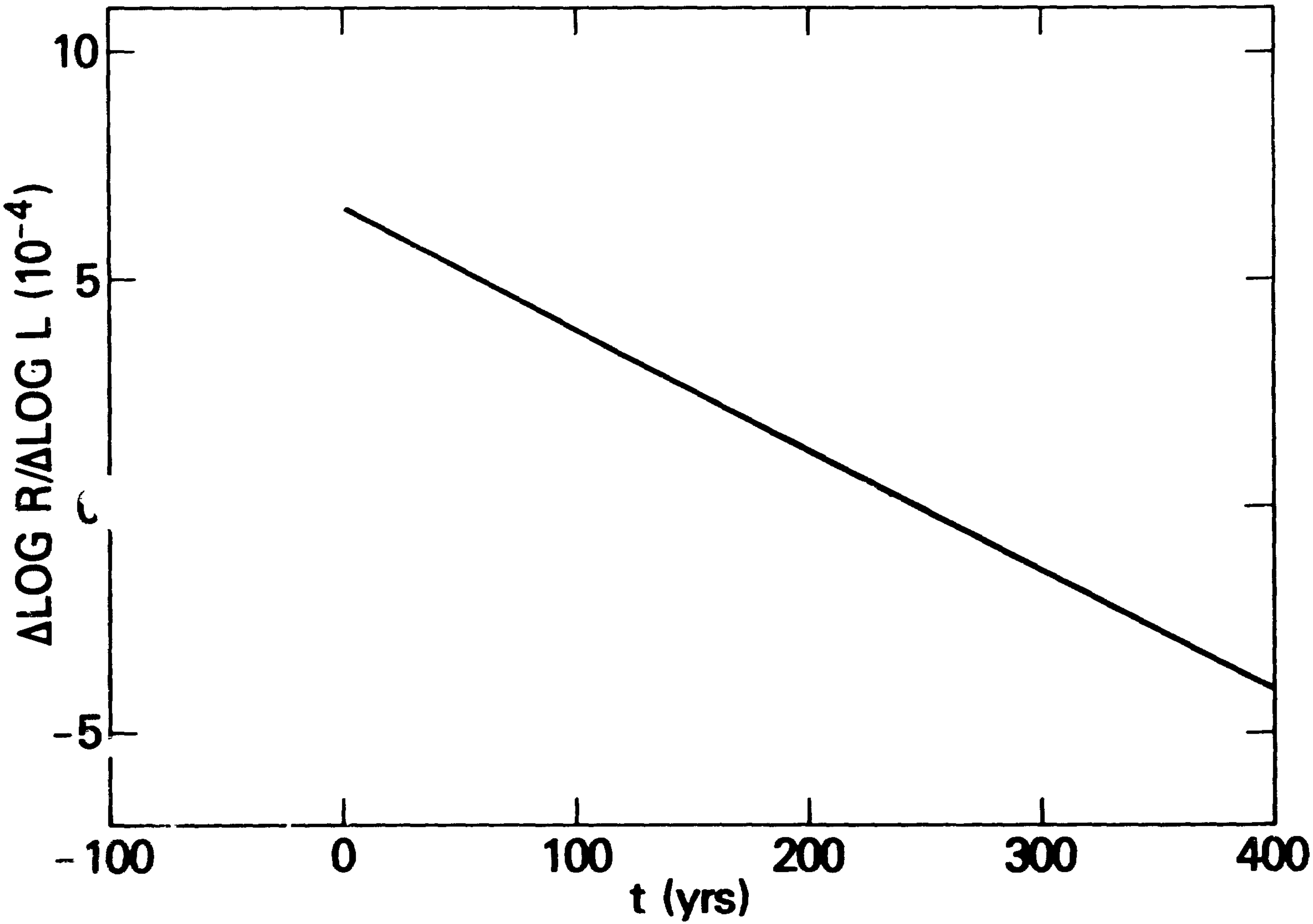

Figure 8. Time dependence of the ratio W = $\Delta\log R/\Delta\log L$ shortly after the perturbation $\Delta\alpha = 0.01$ within the solar convective envelope

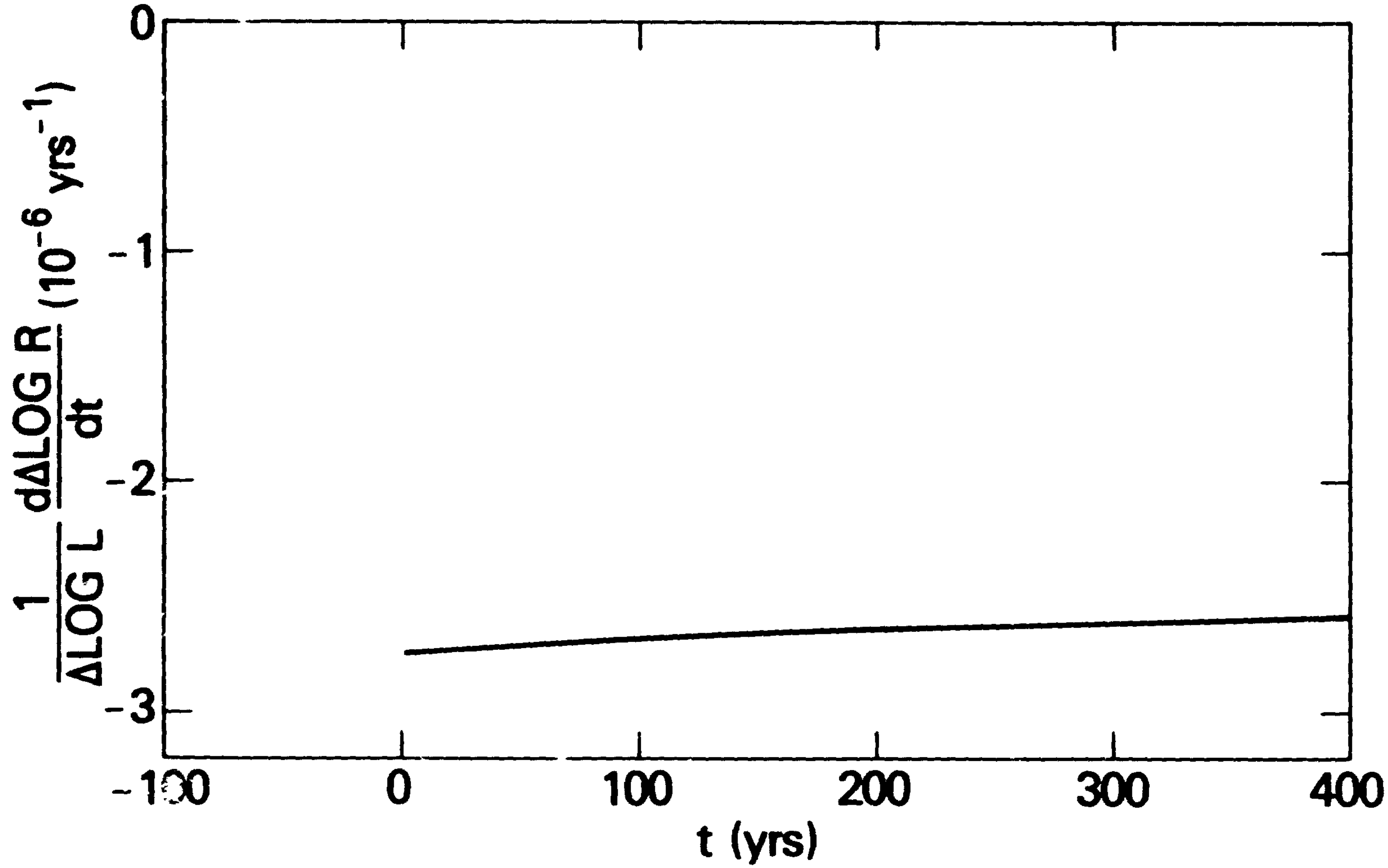

Figure 9. Time dependence of the ratio $H = (\Delta \log L)^{-1} d\, \Delta \log R/dt$ shortly after the perturbation $\Delta\alpha = 0.01$ within the solar convective envelope

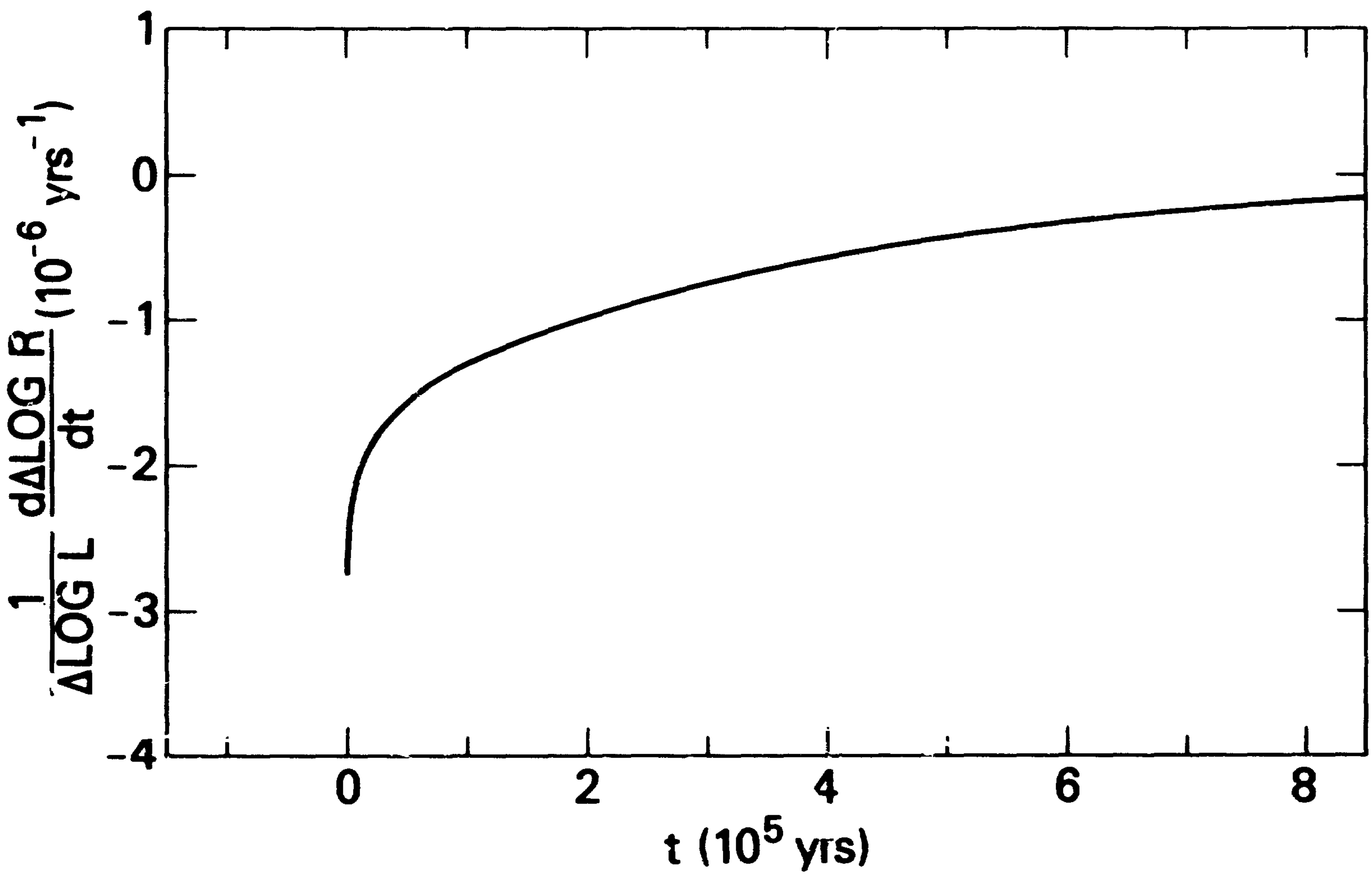

Figure 10. Time dependence of the ratio $H = (\Delta \log L)^{-1} d\ \Delta \log R/dt$ following the perturbation $\Delta\alpha = 0.01$ within the solar convective envelope