NOTICE

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State-Variable Analysis of Non-linear Circuits with a Desk Computer

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STATE-VARIABLE ANALYSIS OF NONLINEAR CIRCUITS WITH A DESK COMPUTER

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STATE-VARIABLE ANALYSIS OF NON-LINEAR CIRCUITS WITH A DESK COMPUTER

INTRODUCTION

This work was prompted by the need to analyze the transient performance of the power regulator unit for the multi-mission modular spacecraft, in particular under the condition of short-circuit failure of the switching transistor of a power module. Due to the non-linearities introduced by the filter inductors and by the solar array characteristics, the state-variable programs available for desk computers could not be used. The purpose of this work is to fill up this void and to be general enough to handle most nonlinear circuit or system analyses.

The nonlinearities considered here are not restricted to any particular circuit element. They may arise from any passive or active source. What the program needs is the fundamental relationship governing each nonlinearity in the form of points on a curve; for example, the flux linkage-current relationship of a nonlinear inductance or the voltage-current relationship of a source.

The starting point of the program is a set of first-order differential equations and algebraic equations describing the system. That is provided by the user. It was therefore deemed useful to include in this document the methodology of writing equations directly from a simple examination of a circuit. Examples have been included, where appropriate, in order to illustrate such methodology.

The program is interactive and offers many options to the user, among which is plotting of the results. It was used very successfully for the transient analysis of the power module mentioned above.

Circuits As Graphs

The graph representation of a circuit enables focusing on the manner in which the various elements of the circuit are interconnected. Each node of the circuit has its counterpart in the graph, a node being the point at which two or more circuit elements join. For the purposes of this analysis, all sources (voltage or current) are considered by themselves, dissociated from any other circuit elements; besides, their values are presumed to be variable. Any passive circuit element (resistance, inductance or capacitance) found connected in parallel with a voltage source or in series with a current source may be removed as it has no bearing on the analysis at hand. The graph of the circuit is then obtained by representing each circuit element, except those removed, by a line (edge) joining the nodes at the terminals of that element. This is illustrated in Fig. 1.

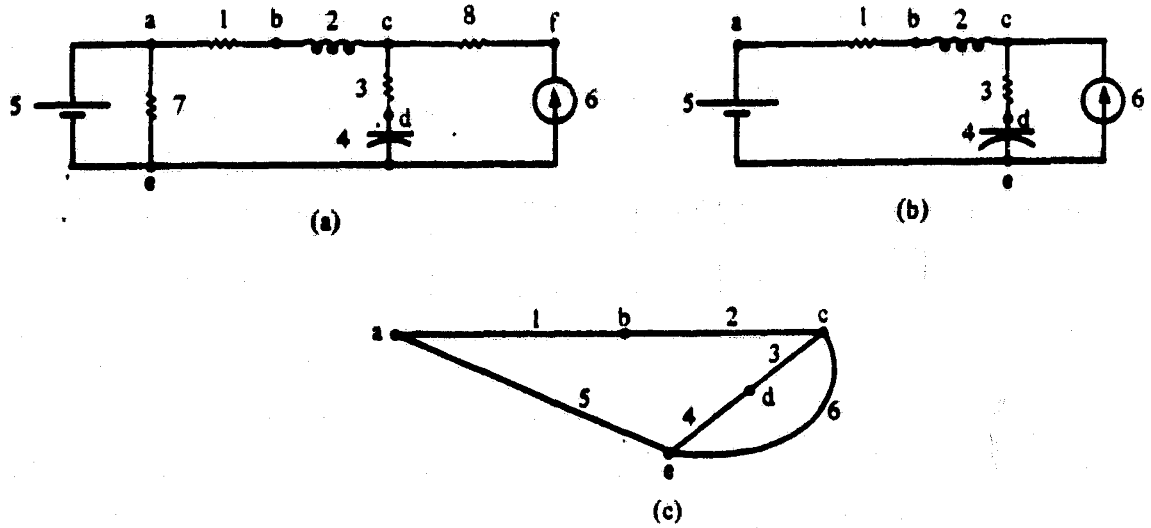


Figure 1. Circuit and its graph. (a) Original circuit. (b) Modified circuit where elements 7 and 8 are dropped because they were in parallel with a voltage source and series with a current source, respectively. (c) Circuit graph with 5 nodes and 6 edges.

The tree of a graph is obtained, using the following procedure. Starting out with the node configuration only, enough edges are subsequently added to interconnect the nodes without forming any closed paths. The edges forming a tree are called tree branches and the remaining edges are called links. It is, in general, possible to derive a large variety of trees from a graph as illustrated in Fig. 2. In (b) the tree branches are

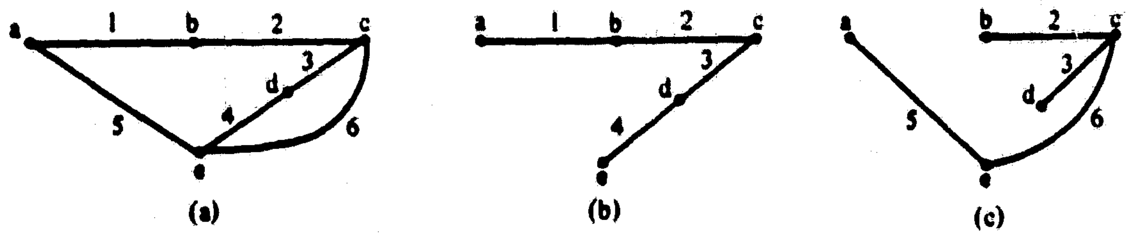


Figure 2. Graph and some of its trees. (a) Original graph. (b) & (c) Possible, but not all, trees.

1, 2, 3, and 4, while the links are 5 and 6. In (c) 2, 3, 5, and 6 are tree branches, 1 and 4 are links.

Due to Kirchhoff's voltage law (KVL), the tree branch voltages may be considered independent variables. The addition of any link to a tree produces a unique closed path. This enables the writing of a link voltage in terms of a unique combination of tree branch voltages. Thus, in the tree of Fig. 2(b), the voltage of link 5 can be expressed in terms of the voltages of branches 1, 2, 3, and 4, while the voltage of link 6 depends on branch voltages 3 and 4. In Fig. 2(c), link 1 needs branches 2, 5, and 6 for its voltage, and link 4 branches 3 and 6. Each link can thus be associated with a unique set of tree branches, namely, those branches which lie along the closed path defined by the link, and form a "tie set." See Fig. 3(a).

Likewise, each tree branch may be associated with a unique set of links to form a "cut set" as follows. A closed surface can be found, crossed by that tree branch alone, such that the tree nodes are split into two distinct groups. The links crossing that surface are those associated with the tree branch. For the tree of Fig. 2(b), the following associations hold, as illustrated in Fig. 3(b).

Tree branch	Cut set	Link	Tie set
1	1,5	5	1,2,3,4,5
2	2,5	6	3,4,6
3	3,5,6		
4	4,5,6		

For the tree of Fig. 2(c), the associations are:

Tree branch	Cut set	Link	Tie set
2	1,2	1	1,2,5,6
3	3,4	4	3,4,6
5	1,5		
6	1,4,6		

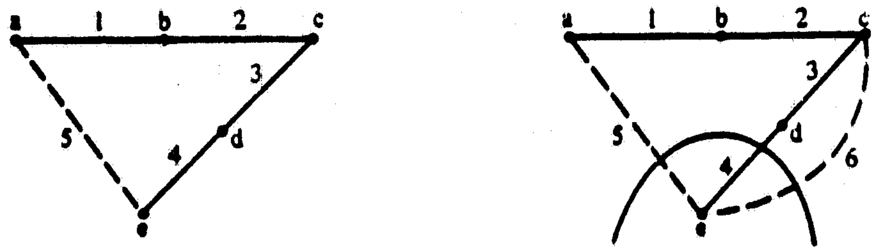


Figure 3. Use of a tree in defining: (a) link voltage, (b) tree branch current.

The Concept of State

At any instant of time, the amount of energy stored in an energy-storing element is an indication of the "state" of that element. In a capacitance the energy stored is $\frac{1}{2} C v^2$, in an inductance it is $\frac{1}{2} Li^2$. Thus, the voltage v may be considered the state of a capacitance, describing completely its present, independent of its past. Similarly, the current i may be considered the state of an inductance.

The transition of an element from one state to another requires the flow of energy into or out of the element, i.e. a certain behavior of the electrical quantity which is not descriptive of its state. In the case of a capacitance, the transition from a voltage v_1 to a voltage v_2 necessitates a flow of current whose behavior in time is responsible for taking the capacitance from the first state to the second. An inductance undergoes a transition from a state i_1, to a state i_2 on the heels of a voltage performance in time.

From a different standpoint, it can be said that a knowledge of the behavior of the current of a capacitance between two instants of time t_1 and t_2 is not enough to determine the voltage (state) of the capacitance at time t_2 ; it is essential to know also the state from which it started, i.e. the voltage at t_1. The same holds true for an inductance where the voltage variations across it between times t_1 and t_2 cannot determine its state i_2 unless its state i_1 is known. As for a resistance, no particular behavior in time is needed of either of its electrical quantities, i and v. It is a memoryless element with no need for a concept of transition. The idea of state is therefore foreign to it. Let us move on now to a more complex configuration, the circuit.

The state of a circuit may be conceived as the set of states of all of its energy-storing elements. The electrical quantities, which are capable of effecting a transition of the circuit from one state to another (the capacitance currents and the inductance voltages) are interrelated by the circuit topology and the source values. Those electrical quantities may be obtained at any time if the state of the circuit and the source values are known. As an example, the circuit of Fig. 4(a) is in the following state at t = 0.1 second:

Voltage across C = 3 V
Current through L = 0.2 A

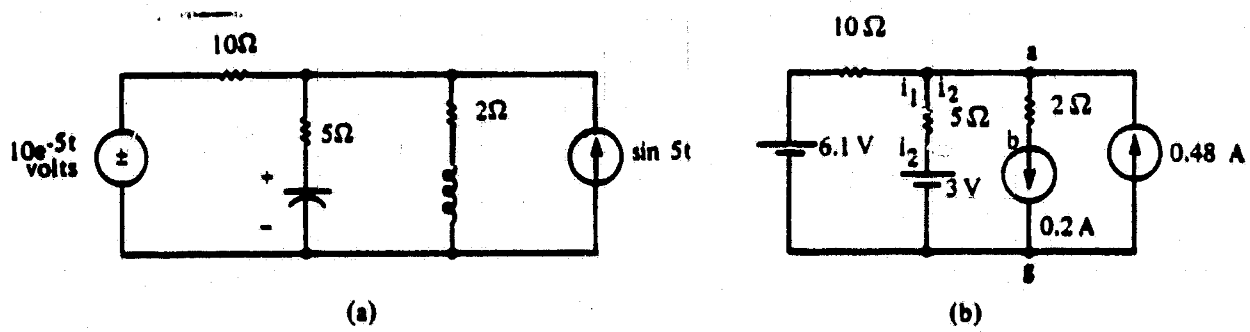


Fig. 4. An example: (a) the circuit at any time;
(b) the circuit at t = 0.1 s.

This state and the source values are shown in Fig. 4(b). Using simple dc circuit analysis techniques, the following results are obtained:

i_1 = 0.11 A
i_2 = 0.39 A
v_{ag} = 5.0 V
v_{bg} = 4.6 V

The capacitance current and the inductance voltage at $t = 0.1$ s are thus known, namely, 0.39 A and 4.6 V, respectively. The transition to the "next" circuit state can then presumably be determined.

In the equivalent circuit of Fig. 4(b), no distinction is made, or is necessary, between the 6.1 V of the source and the 3 V of the capacitance state, or between the 0.48 A of the source and the 0.2 A of the inductance state. It is therefore convenient to include the source variables in the set defining the circuit state, as the augmented set permits the determination of the voltage and current of every element in the circuit.

State-Variable Approach to Analysis

In the sequel two assumptions are implicitly made for any circuit:

(a) no tie set contains only capacitances; (b) no cut set contains only inductances.

As a result, it is possible to find a tree where all the circuit capacitances are included, and none of the circuit inductances. To simplify this first analysis, resistances are assumed to be all tree branches or all links. Voltage sources are to belong to the tree, while current sources have to be links. The state variables are then defined as the capacitance voltages, the inductance currents, and the source variables (voltage or current).

If the circuit is linear, the next step is to write a set of first-order differential equations for the state variables, obtained directly from KVL and Kirhoff's current law (KCL). For n state variables, denoted by $X_1, X_2, \ldots, X_n$, the rth equation is of the form:

$$\frac{dXr}{dt} = a_{r1} X_1 + a_{r2} X_2 + \ldots + a_{rn} X_n$$

There are n such equations, which in matrix form may be expressed as:

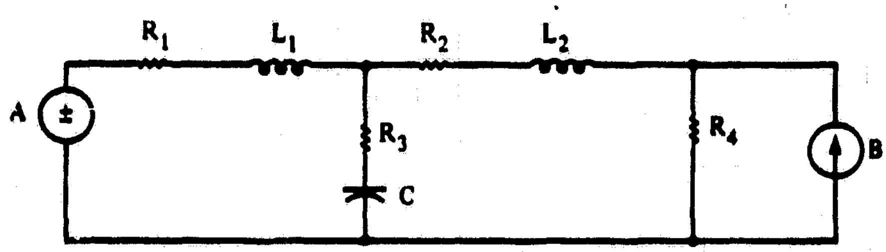
$$\dot{\underline{X}} = \underline{A}\,\underline{X}$$

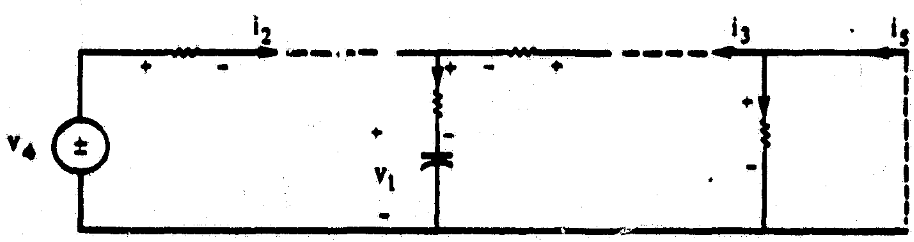
The detailed procedure now follows. First, if the resistances are tree branches, express their individual currents in terms of state-variable currents. The voltage is then the current expression multiplied by R. On the other hand, if the resistances are links, their individual voltages are expressed in terms of state-variable voltages. The current is then the voltage expression divided by R. In either case, both the voltage and current of each resistance may be expressed in terms of state variables only. And now the state equations.

Each capacitance is a tree branch and its voltage v is a state variable. The derivative dv/dt is equal to the capacitance current divided by C. Using the graph method described above, the capacitance current may be expressed in terms of link currents, i.e. state-variable currents only as sought. The same holds true for the inductances.

Each inductance is a link and its current i is a state variable. The derivative di/dt is equal to the inductance voltage divided by L. Using the graph method, the inductance voltage may be expressed in terms of tree-branch voltages, i.e. state-variable voltages only. Finally, a (set of) first-order differential equation(s) is obtained for each source, obviously in terms of its own state variable(s) only. For more details concerning sources, see Hewlett-Packard manual for model 30 calculators, entitled "State Variables PAC," pages 36-37.

Example 1.





tree (5 state variables)

The sources are dc with values A and B. A tree is formed including all capacitances, excluding all inductances, and including all resistances. The tree also includes the voltage source, but not the current source. All state variable polarities chosen arbitrarily.

Resistance R_1 : current i_2

voltage $R_1 i_2$

Resistance R_2 : current i_3

voltage $R_2 i_3$

Resistance R_3 : current $i_2 + i_3$

voltage $R_3 (i_2 + i_3)$

Resistance R_4 : current $-i_3 + i_5$

voltage $R_4 (-i_3 + i_5)$

$$\frac{dv_1}{dt} = \frac{1}{C}(i_2 + i_3)$$

$$\frac{di_2}{dt} = \frac{1}{L_1}[-R_1 i_2 + v_4 - v_1 - R_3 (i_2 + i_3)]$$

$$\frac{di_3}{dt} = \frac{1}{L_2}[R_4 (-i_3 + i_5) - v_1 - R_3 (i_2 + i_3) - R_2 i_3]$$

$$\frac{dv_4}{dt} = 0$$

$$\frac{di_5}{dt} = 0$$

In matrix form,

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ i_2 \\ i_3 \\ v_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & 1/C & 1/C & 0 & 0 \\ -1/L_1 & -(R_1+R_3)/L_1 & -R_3/L_1 & 1/L_1 & 0 \\ -1/L_2 & -R_3/L_2 & -(R_4+R_3+R_2)/L_2 & 0 & R_4/L_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \\ i_3 \\ v_4 \\ i_5 \end{bmatrix}$$

coefficient matrix

The initial state vector is

$v_1(0)$

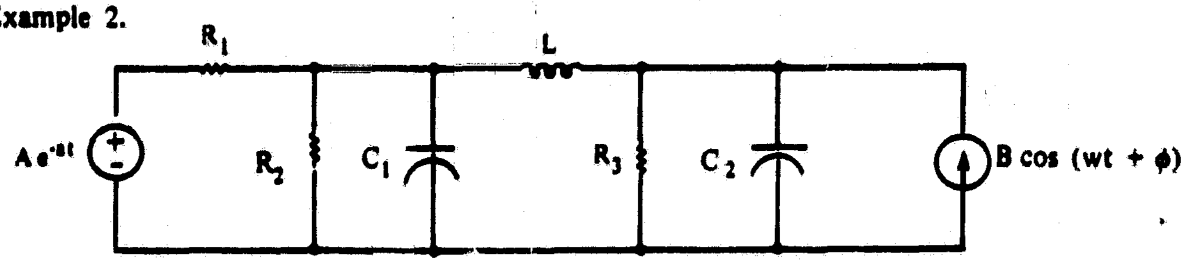
$i_2(0)$ where $v_1(0)$ is the initial voltage of C_1

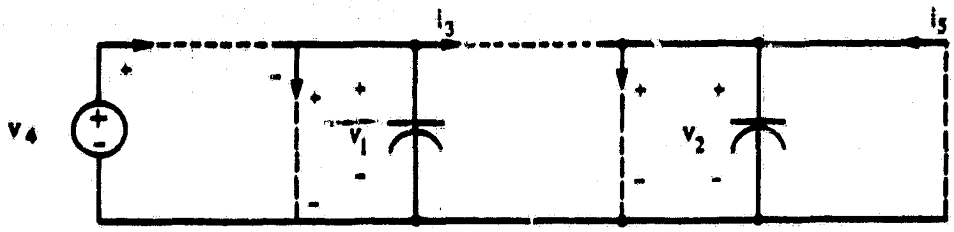
$i_3(0)$ $i_2(0)$ is the initial current of L_1

A $i_3(0)$ is the initial current of L_2

B

Example 2.





tree

Here all resistances are links.

Resistance R_1 : voltage = $v_4 - v_1$

current = $(v_4 - v_1)/R_1$

Resistance R_2 : voltage = v_1

current = v_1/R_2

Resistance R_3 : voltage = v_2

current = v_2/R_3

$$\frac{dv_1}{dt} = \frac{1}{C_1}[(v_4 - v_1)/R_1 - v_1/R_2 - i_3]$$

$$\frac{dv_2}{dt} = \frac{1}{C_2}[i_3 - v_2/R_3 + i_5]$$

$$\frac{di_3}{dt} = \frac{1}{L}[v_1 - v_2]$$

$$\frac{dv_4}{dt} = -av_4 \qquad \text{for voltage source}$$

$$\left.\begin{aligned}\frac{di_5}{dt} &= X_6 \\ \frac{dX_6}{dt} &= -\omega^2 i_5\end{aligned}\right\} \qquad \begin{aligned}&\text{for current source, since} \\ &di_5/dt = -\ \omega B \sin(\omega t + \phi) \\ &d^2 i_5/dt^2 = -\omega^2 B \cos(\omega t + \phi)\end{aligned}$$

In matrix form,

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ v_4 \\ i_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} -(1/R_1 + 1/R_2)/C_1 & 0 & -1/C_1 & 1/C_1 & 0 & 0 \\ 0 & -1/R_3C_2 & 1/C_2 & 0 & 1/C_2 & 0 \\ 1/L & -1/L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ v_4 \\ i_5 \\ X_6 \end{bmatrix}$$

The initial state vector is

$$\begin{bmatrix} v_1(0) \\ v_2(0) \\ i_3(0) \\ A \\ B\cos\phi \\ -\omega B \sin\phi \end{bmatrix}$$

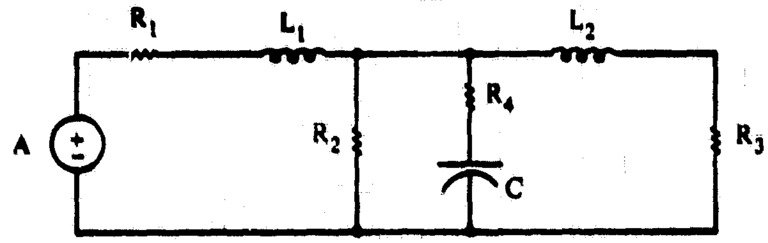
where $v_1(0)$ is the initial voltage of C_1

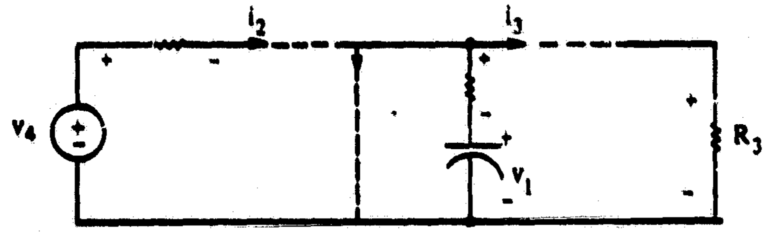
$v_2(0)$ is the initial voltage of C_2

$i_3(0)$ is the initial current of L

Concerning the status of resistances in a graph, it is not always possible to have them all either tree branches or links. A mix is the norm rather than the exception. In this case consider temporarily that the voltages of tree-branch resistances and the currents of link resistances are state variables. Write an equation for the current of each tree-branch resistance and the voltage of each link resistance, using state variables only. Solve for the resistance state variables in terms of the remaining state variables.

Example 3.





tree

Let the temporary state variables be v_5 for R_1, i_6 for R_2, v_7 for R_3, and v_8 for R_4.

$$v_5/R_1 = i_2 \qquad\qquad v_5 = R_1 i_2$$

$$i_6 R_2 = v_8 + v_1 \qquad\qquad R_2 i_6 - v_8 = v_1$$

$$v_7/R_3 = i_3 \qquad\qquad v_7 = R_3 i_3$$

$$v_8/R_4 = i_2 - i_6 - i_3 \qquad\qquad v_8/R_4 + i_6 = i_2 - i_3$$

Solve for v_5, i_6, v_7, and v_8.

$$v_5 = R_1 i_2$$

$$i_6 = \frac{1}{R_2 + R_4} v_1 + \frac{R_4}{R_2 + R_4} i_2 - \frac{R_4}{R_2 + R_4} i_3$$

$$v_7 = \frac{R_3 i_3}{R}$$

$$v_8 = -\frac{R_4}{R_2 + R_4} v_1 + \frac{R_2 R_4}{R_2 + R_4} i_2 - \frac{R_2 R_4}{R_2 + R_4} i_3$$

The previous procedure is now followed.

Resistance R_1 : current $= i_2$

voltage $= R_1 i_2$

Resistance R_2 : current $= \frac{1}{R_2+R_4} v_1 + \frac{R_4}{R_2+R_4} i_2 - \frac{R_4}{R_2+R_4} i_3$

voltage $= \frac{R_2}{R_2+R_4} v_1 + \frac{R_2R_4}{R_2+R_4} i_2 - \frac{R_2R_4}{R_2+R_4} i_3$

Resistance R_3 : current $= i_3$

voltage $= R_3 i_3$

Resistance R_4 : current $= -\frac{1}{R_2+R_4} v_1 + \frac{R_2}{R_2+R_4} i_2 - \frac{R_2}{R_2+R_4} i_3$

voltage $= -\frac{R_4}{R_2+R_4} v_1 + \frac{R_2R_4}{R_2+R_4} i_2 - \frac{R_2R_4}{R_2+R_4} i_3$

$$\frac{dv_1}{dt} = \frac{1}{C}\left[i_2 - \frac{1}{R_2+R_4} v_1 - \frac{R_4}{R_2+R_4} i_2 + \frac{R_4}{R_2+R_4} i_3 - i_3\right]$$

$$\frac{di_2}{dt} = \frac{1}{L_1}\left[-R_1 i_2 + v_4 - v_1 + \frac{R_4}{R_2+R_4} v_1 - \frac{R_2R_4}{R_2+R_4} i_2 + \frac{R_2R_4}{R_2+R_4} i_3\right]$$

$$\frac{di_3}{dt} = \frac{1}{L_2}\left[-\frac{R_4}{R_2+R_4} v_1 + \frac{R_2R_4}{R_2+R_4} i_2 - \frac{R_2R_4}{R_2+R_4} i_3 + v_1 - R_3 i_3\right]$$

$$\frac{dv_4}{dt} = 0$$

Notice that in all the state equations written so far the differentiated variables were the state variables. This is always true of linear systems, not necessarily true of nonlinear systems. For the latter a distinction is made between the differentiated variables and the state variables. The state equations take the form $\underline{\dot{X}}' = \underline{A}\underline{X}$ instead of $\underline{\dot{X}} = \underline{A}\underline{X}$. The basic equation characterizing an inductance is $v = d\lambda/dt$, where λ is the coil flux linkage. If λ is produced by a coil current i, then the inductance $L = \lambda/i$. L is not constant unless λ is directly proportional to i, in which case $v = d\lambda/dt = d(Li)dt = L\, di/dt$. For a magnetic core whose B-H curve is available, the $\lambda - i$ relationship is derived in the following manner.

$\lambda = N\phi = (NA)\ B$.

$i = (\ell/N)\ H$

where N is the number of turns of the coil,
A is the cross-section of the magnetic core in m^2
ℓ is the mean length of the magnetic core in m.

In setting up the state equation of an inductance, write $d\lambda/dt$ in terms of the state variables using KVL. For a nonlinear inductance, the differentiated variable is thus λ, and the state variable i.

The above expressions for λ and i are based on SI units, i.e. B is in tesla (weber/m^2) and H in ampere/meter. The B-H curve is often available in cgs units. In that case,

$\lambda = (NA \times 10^{-8})\ B$

$i = [\ell/(0.4\pi N)]\ H$

where A is in square centimeter,
ℓ is in centimeter,
B is in gauss,
H is in oersted.

As for a capacitance, the basic equation ~~is~~ i = dq/dt, where q is the charge in coulomb on the plates of the capacitance. The charge is related to voltage across the capacitance by q = Cv, where C is the capacitance in farad. If C is constant (linear capacitance), then i = C dv/dt; otherwise, the relationship between q and v must be known. In setting up the state equation, write dq/dt in terms of the state variables using KCL. For a nonlinear capacitance, the differentiated variable is thus q and the state variable v.

For a resistance v = Ri. If R is not constant (nonlinear resistance), one of its electrical quantities is taken as a state variable. If the resistance is a tree branch, its state variable is the voltage. If the resistance is a link, its state variable is the current. In either case, the other electrical quantity is expressed in terms of the state variables of the whole problem, using KVL in case of a link, KCL in case of a tree branch. This other quantity is obviously not differentiated.

Example 4.

Consider the circuit of example 1 in this section. Let C, L_i, and R_4 be nonlinear. Using the same tree, assign voltage v_6 to R_4. v_6 is a state variable since R_4 is a tree branch. The state equations become:

$$\frac{dq_1}{dt} = i_2 + i_3$$

$$\frac{d\lambda_2}{dt} = - R_1 i_2 + v_4 - v_1 - R_3 (i_2 + i_3)$$

$$\frac{di_3}{dt} = \frac{1}{L_2} [v_6 - v_1 - R_3 (i_2 + i_3) - R_2 i_3]$$

$$\frac{dv_4}{dt} = 0$$

$$\frac{di_5}{dt} = 0$$

$$i_6 = -i_3 + i_5$$

In matrix form,

$$\frac{d}{dt}\begin{bmatrix} q_1 \\ \lambda_2 \\ i_3 \\ v_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ -1 & -(R_1 + R_3) & -R_3 & 1 & 0 & 0 \\ -1/L_2 & -R_3/L_2 & -(R_2 + R_3)/L_2 & 0 & 0 & 1/L_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \\ i_3 \\ v_4 \\ i_5 \\ v_6 \end{bmatrix}$$

or $\underline{\dot{X}}' = \underline{A}\underline{X}$

$$\begin{bmatrix} \dot{i}_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \\ i_3 \\ v_4 \\ i_5 \\ i_6 \end{bmatrix}$$

or $\underline{X}'' = \underline{R}\underline{X}$

NOTE: It is recommended that a tree be chosen such that a nonlinear resistance which is a link be not associated with a loop (tie set) containing another nonlinear resistance. Likewise, a nonlinear resistance which is a tree branch should preferably not be associated with a cut set containing another nonlinear resistance.

Method of Solution

From the initial state vector $\underline{X}(0)$ and the curves of the nonlinear elements, the initial values of the differentiated variables $\underline{X}'(0)$ are obtained. The time interval of interest is now divided into equal increments Δt, small enough not to impair the accuracy of the solution, yet large enough not to cause the computer to take too long in reaching that solution. Increments of the differentiated variables are obtained:

$$\Delta\underline{X}'(\Delta t) = \underline{A}\ \underline{X}(0)\ \Delta t$$

The values of the differentiated variables at time Δt are:

$$\underline{X}'(\Delta t) = \underline{X}'(0) + \Delta\underline{X}'(\Delta t)$$

Using the curves of the nonlinear elements, one can now obtain the state variable values at time Δt, namely, $\underline{\tilde{X}}(\Delta t)$. If no nonlinear resistances exist,

then $\underline{X}(\Delta t) = \underline{\tilde{X}}(\Delta t)$. Otherwise, obtain

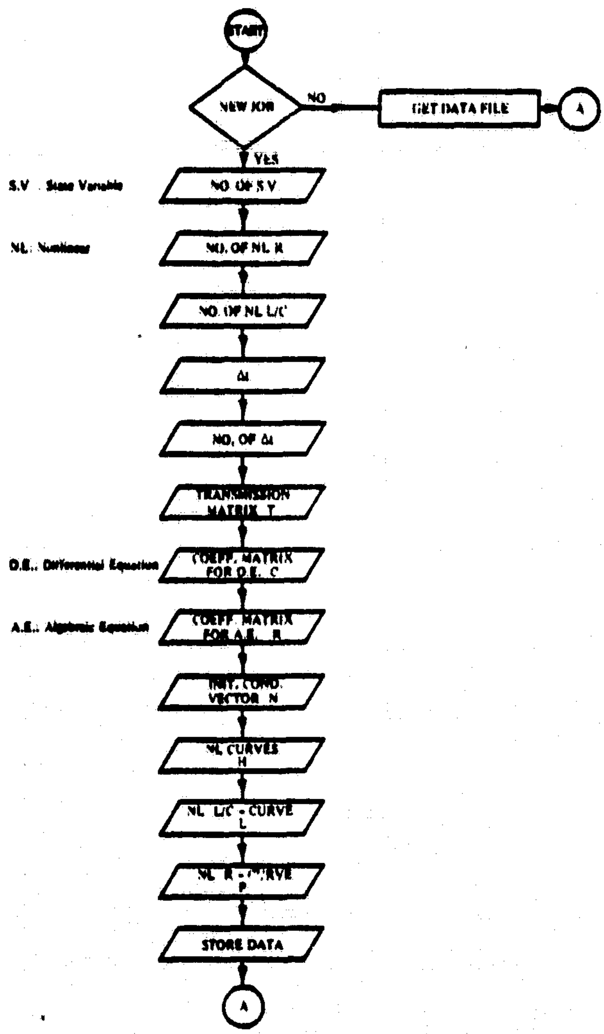
$$\underline{X}''(\Delta t) = \underline{R}\ \underline{\tilde{X}}(\Delta t)$$

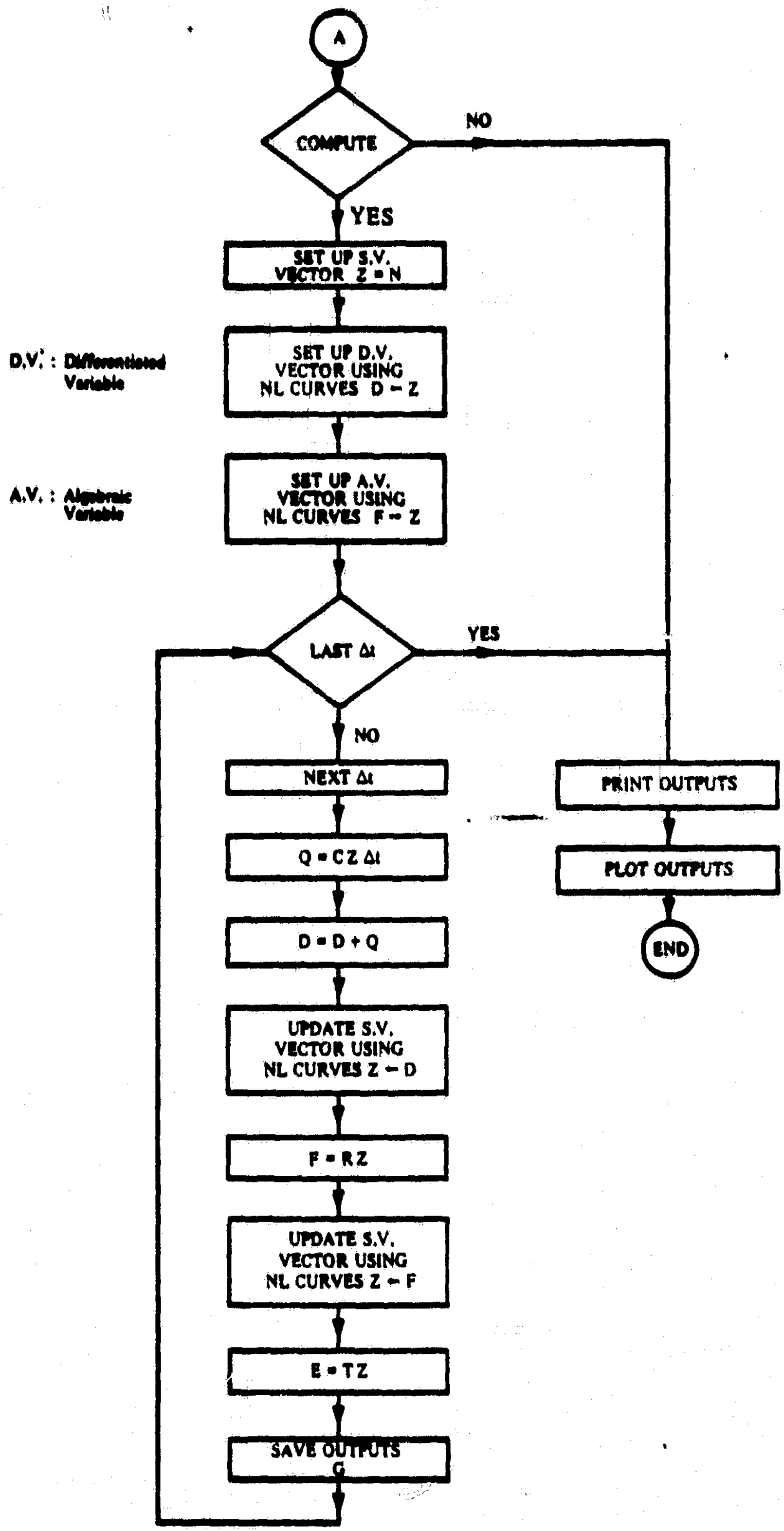
Using the curves of the nonlinear resistances, a further update of the state variables $\tilde{\underline{X}}\ (\Delta t)$ is obtained, namely, $\underline{X}\ (\Delta t)$.

$\underline{X}\ (\Delta t)$ is now taken as the initial state vector and the same procedure is repeated to obtain $\underline{X}\ (2\Delta t)$, and so on.

The outputs at any time $\kappa\Delta t$ are given by $\underline{Y}\ (\kappa\Delta t) = \underline{B}\ \underline{X}\ (\kappa\Delta t)$ where $\underline{Y}$ is the output vector and $\underline{B}$ the transmission matrix. Each output is a linear combination of the state variables. Thus, each row of $\underline{B}$, say row i, contains the coefficients relating output i to the state variables.

Computer Program – Flow Chart




A
COMPUTE
NO
YES
SET UP S.V. VECTOR Z = N
D.V. : Differentiated Variable
SET UP D.V. VECTOR USING NL CURVES D ~ Z
A.V. : Algebraic Variable
SET UP A.V. VECTOR USING NL CURVES F ~ Z
LAST Δt
YES
NO
NEXT Δt
Q = C Z Δt
D = D + Q
UPDATE S.V. VECTOR USING NL CURVES Z ~ D
F = R Z
UPDATE S.V. VECTOR USING NL CURVES Z ~ F
E = T Z
SAVE OUTPUTS G
PRINT OUTPUTS
PLOT OUTPUTS
END

The program was implemented on a Hewlett-Packard 9830 computer, provided with a 9866 printer and a 9862 plotter. A listing of the program may be found in Appendix A.

The COMMON statement permits modification of individual pieces of data once the program terminates. The modified data may be stored on tape with STORE DATA file# command.

Following is a list of the main variables used in the program, together with a description of their role.

C Coefficient matrix for the differential equations, 10 x 10. Each row of C multiplied by state-variable vector Z produces the derivative of a differentiated variable.

D Vector of differentiated variables, 10 x 1.

E Output vector at time considered, 4 x 1.

F Vector of non-state variables of dissipative elements, 3 x 1.

G Output matrix, 4 x 181. Each row gives the consecutive values of an output in time, starting with t = 0.

H Nonlinear characteristics matrix, 10 x 10. Each pair of columns represents a curve; hence, up to 5 curves. The first column of a pair contains the x-coordinate values, the second column the y-coordinates: for an inductance i and λ, for a capacitance v and q, for a resistance i and v.

L Matrix of nonlinear energy-storing element position in Z and corresponding curve number in H, 10 x 2.

N(1) Number of state variables, max. 10.

N(2) Number of basic time intervals.

N(3) Print interval.

N(4) Number of outputs, max. 4.

N(5) Number of output values to be printed out.

N(6) Number of different curves of nonlinear characteristics, 0-5.

N(7) Number of nonlinear energy-storing elements, 1-10.

N(8) Basic time interval.

N(9)
through Initial condition values.
N(18)

N(19) Number of nonlinear dissipative elements, 0-3.

P Matrix for nonlinear dissipative elements, 1 x 9. To each element corresponds a triplet: state variable position in Z, curve number, and kind of state variable (1 for current, 2 for voltage).

Q Vector of change in differentiated variables, 10 x 1.

Q3 Data file number.

Q4 Number of energy-storing elements, i.e. number of differential equations.

R Coefficient matrix for the linear equations of the nonlinear dissipative elements, 3 x 10.

T Transmissions matrix. 4 x 10. Each row of T multiplied by state-variable vector Z produces an output value.

Z State-variable vector at any time, 10 x 1.

Computer Program – Instructions for Use

1. NEW JOB (Y OR N)?

   Enter N if you already have a data file for the job; Y if you wish to produce new data, and then proceed to question 3.

2. FILE # ?

   Enter number of existing data file. Proceed to question 20.

3. NO. OF STATE VARIABLES (1 TO 10)?

   Enter number of state variables. This number may be larger than the order of the system, the latter corresponding to the number of first-order differential equations. Each nonlinear dissipative element (resistance, friction) requires a state variable, but gives rise to an algebraic equation only.

4. NO. OF NONLINEAR DISSIPATIVE ELEMENTS (0, 1, 2, 3)?

   Enter number of nonlinear dissipative elements (resistances), up to three.

5. NO. OF NONLINEAR ENERGY-STORING ELEMENTS (0 TO 10)?

   Enter number of nonlinear energy-storing elements (inductances, capacitances), up to 10.

6. BASIC TIME INTERVAL (SECONDS)?

   Enter the elementary time interval of integration. For example, if the period of interest is 1 millisecond and this period is divided into 500 basic time intervals, the length of each basic time interval is 2 microseconds. Too large a basic time interval speeds up the solution, but impairs its accuracy.

7. NO. OF TIME INTERVALS?

   Enter number of basic intervals into which the total time of interest is divided.

8. PRINT INTERVAL (1, 2, 3, ...)?

   If the basic time interval has been chosen judiciously, the variation of the solution from one basic time interval to the next is, in general, too small to be relevant. When the solution is eventually printed out, you may want to get the solution every κ basic time intervals. This κ is called "print interval." For example, let the number of basic time intervals be 500. If the print interval is selected as 5 only solution values # 1, 6, 11, 16, ... will eventually be printed out, that is 101 values: the initial condition and 100 intervals. Value # 1 is the initial one, before any integration takes place. Incidentally, only those 101 values are kept in memory, to be later stored with the data. Up to 181 output values may thus be stored, i.e. NO. OF TIME INTERVALS/PRINT INTERVAL $\leq$ 180.

9. NO. OF OUTPUTS (1, 2, 3, OR 4)?

Enter number of output functions, up to 4. An output function is a linear combination of the state variables.

10. TRANSMISSION VECTOR J

J takes the values of 1 to the number of outputs designated in instruction # 9. For output function J, enter the coefficients of its linear combination, one at a time as requested on display.

11. COEFFICIENT MATRIX FOR DIFFERENTIAL EQUATIONS

Enter the coefficients, one at a time, as requested on display. This matrix is not square if nonlinear dissipative elements exist.

12. COEFFICIENT MATRIX FOR ALGEBRAIC EQS. OF N.L. DISSIPATIVE ELEMENTS

Enter the coefficients, one at a time, as requested on display. This information is not requested if the answer to instruction 4 is 0.

13. INIT. COND. VECTOR

Enter the initial values of all state variables, one at a time, as requested. The initial values of the state variables corresponding to nonlinear dissipative elements may have to be estimated. Once the computation starts, the program will try to improve the accuracy of these values to within 0.1% in 10 iterations. If it does not succeed, it will request improved initial conditions and terminate.

14. NO. OF NONLINEAR CHARACTERISTICS (0 TO 5) ?

Enter the number of curves describing the characteristics of nonlinear elements. This is less than, or equal to, the number of nonlinear elements as two or more such elements may possess the same characteristics.

15. ENTER CURVE # J (BY PAIR OF VALUES)

J takes the value 1 to the number of curves designated in instruction # 14. For curve J, enter two values at a time, as requested. The first value is the x-coordinate: The current I for a resistance and an inductance, the voltage V for a capacitance. The second value is the y-coordinate: the voltage V for a resistance, the flux linkage λ (volt-sec) for an inductance, and the charge Q for a capacitance. Ten such pairs must be entered for each curve. This information is not requested if the answer to question # 14 is zero.

16. FOR EACH NONLINEAR ENERGY-STORING ELEMENT, ENTER STATE-VARIABLE POSITION AND CURVE #

For each of the nonlinear energy-storing elements numbering the answer to question # 5, enter a pair of values at a time: the state-variable position number in the state-variable vector as entered, for example, under # 13 above, and the associated curve number as entered under 15 above. More than one element may be associated with the same curve.

17. FOR EACH NONLINEAR DISSIPATIVE ELEMENT, ENTER STATE-VARIABLE POSITION, CURVE # , AND KIND OF STATE VARIABLE (1 for I, 2 for V)

For each of the nonlinear dissipative elements numbering the answer to question # 4 above, enter a triplet of values at a time: the state variable position number in the state variable vector, the associated curve number, and a designation of 1 if the element is a link or 2 if the element is a tree branch.

18. STORE DATA (Y OR N) ?

Enter Y if you wish all the above data stored on tape, N if you don't. In the latter case skip to instruction # 20.

19. FILE # ?

Enter the number of the file where the data is to be stored. NOTE: The file must have been previously marked (see HP manual tor FIND and MARK commands).

20. COMPUTE (Y OR N) ?

Enter Y if you wish the computation to be carried out in order to obtain solution, N if you don't. The purpose of this instruction is to separate the data input from the data processing. Thus, the whole job need not be done in one sitting. The instruction is also found useful in separating the computation job from the output job.

21. INTERVAL # J. BE PATIENT ! ! !

This message occurs every 50 integration intervals. Nothing to answer.

22. FINAL STATUS OF STATE-VARIABLE VECTOR IS:

The solution has ended. The last values of the state variables are printed out. They may later be used as initial values to a subsequent time interval. The solution and data are stored in the file specified under instruction # 19 if the answer to # 18 is yes. This information may later be retrieved for recomputation, and/or printout, and/or plot. Nothing to answer.

23. TOTAL TIME INTERVAL = — SECONDS

Nothing to answer

24. PRINTOUT NEEDED (Y OR N) ?

Enter Y if you wish to have the outputs printed out, N if you don't.

25. PLOT NEEDED (Y OR N) ?

Enter Y if you wish to have the outputs plotted, and get plotter ready. Enter N if you do not need plot, and skip to # 28.

26. SET PLOTTER, PRESS CONT

Raise pen, place paper on plotter board, and set all margins. Then press CONT and EXECUTE

27. MIN AND MAX VALUES ?

Enter pair. It establishes the range of the y-axis. Choose it by referring to the values printed out under # 24 above. (MAX-MIN)/5 should be a convenient value for a marked interval on the y-axis. If the output just plotted is not the last one, the program goes back to # 26.

28. END OF JOB

Nothing to answer. The cassette tape is rewound.

Possible messages which are followed by program termination:

1) OUT OF CHARACTERISTICS BOUNDS, PROGRAM ABORTED.

Variable of nonlinear element is outside range of curve.

2) PROGRAM ABORTED. IMPROVE INITIAL CONDITIONS.

Initial values entered for state variables of dissipative elements are not correct.

3) NO. OF N.L. ELEMENTS = - - -, NO CURVE.

Answers to instructions 4 and 5 are inconsistent with answer to # 14.

It is recommended that the data entered be checked for accuracy before computation. After entering all data for a new job, answer N to question #20 and all succeeding questions. This ends the program. Next, execute the following commands to obtain a printout of the data entered:

```
MAT PRINT N      [EXECUTE]
MAT PRINT T      [EXECUTE]
MAT PRINT C      [EXECUTE]
MAT PRINT R      [EXECUTE]
MAT PRINT H      [EXECUTE]
MAT PRINT L      [EXECUTE]
MAT PRINT P      [EXECUTE]
```

If corrections are to be made, the program does not have to be rerun. Simply enter the corrections, one at a time, as follows. If element C (3,5), say, is to be corrected then:

```
C (3,5) = new value      [EXECUTE]
```

and so on. After all corrections are made, store the data. Thus, if the data had been stored by the program in file # 4, then:

STORE DATA 4 [EXECUTE]

Now, in order to compute, run the program again considering that you are dealing with an old job.

1. NEW JOB (Y OR N) ?

   N

2 FILE # ?

   4

3. COMPUTE (Y OR N)

   Y

This feature is particularly convenient when the circuit model changes due to the presence of diodes, SCRs, ... The program would first be run for, say, 50 time intervals with a diode current as one of the outputs. Check the output values printed out. Suppose that at interval # 38 the diode current reverses direction. The solution is therefore valid only to interval # 37. Then,

N(2) = 37 [EXECUTE]

N(5) = 38 [EXECUTE]

STORE DATA same file no. [EXECUTE]

Rerun the program. At the end, the final values of the state variables are to be used as initial values for the next run with different equations for a new model and, say, for 80 time intervals. Then,

N(2) = 80

N(5) = 81

N(9) = Z (1,1) and likewise for the rest of the initial conditions.

Changes in the model

STORE DATA new file no.

The solution over the whole time interval of interest may thus be formed of a number of solutions over consecutive subintervals. In order to obtain a single plot for the overall solution, the next task is to splice all subsolutions together, with the proper print interval, and store the result in a single file. This can be achieved with the program listed in Appendix B and whose details follow.

1. NO. OF OUTPUTS?

   Enter the number of output functions stored in the files to be spliced. This number must be the same for both files.

2. TOTAL NO. OF INTERVALS (BOTH FILES)?

   Enter a + b, where a and b are the number of stored data intervals in files 1 and 2, respectively.

3. SUPPLEMENTARY PRINT INTERVAL?

   Both files 1 and 2 to be spliced must also be based on the same print interval, say c. If a value d is entered here, it will be considered as a print interval on the output data of the pre-spliced files. The resulting file will have a print interval of c times d.

4. FIRST FILE #?

   Enter number of front file.

5. SECOND FILE #?

   Enter number of rear file.

6. NO. OF TIME INTERVALS LEFT OUT = - - -

   Nothing to answer. It indicates the number of data pieces lost from the end of the second file due to the supplementary print interval. For example, let the number of intervals of the first and second file be 37 and 80, respectively. If the supplementary print interval chosen is 2, the 80th output value of the second file is lost from the spliced combination.

7. FILE # FOR NEW DATA ?

   Enter number of file where resulting spliced data are to be stored.

APPENDIX A

```
10 COM N[19],T[4,10],C[10,10],Z[10,1],G[4,101],H[10,10],L[10,2],R[3,10],P[1,9]
20 DIM D[10,1],E[4,1],Q[10,1],F[3,1]
30 PRINT "NEW JOB  (Y OR N)"
40 INPUT Q$
50 IF Q$="Y" THEN 100
60 PRINT "FILE #"
70 INPUT Q3
80 LOAD  DATA Q3
90 GOTO 1120
100 PRINT "NO. OF STATE VARIABLES (1 TO 10)"
110 INPUT N[1]
120 PRINT "NO. OF NONLINEAR DISSIPATIVE ELEMENTS (0,1,2,3)"
130 INPUT N[19]
140 PRINT "NO. OF NONLINEAR ENERGY-STORING ELEMENTS (0 TO 10)"
150 INPUT N[7]
160 PRINT "BASIC TIME INTERVAL (SECONDS)"
170 INPUT N[8]
180 PRINT "NO. OF TIME INTERVALS"
190 INPUT N[2]
200 PRINT "PRINT INTERVAL (1,2,3,...)"
210 INPUT N[3]
220 PRINT "NO. OF OUTPUTS (1,2,3,OR 4)"
230 INPUT N[4]
240 MAT T=ZER[N[4],N[1]]
250 Q4=N[1]-N[19]
260 MAT C=ZER[Q4,N[1]]
270 MAT Z=ZER[N[1],1]
280 IF N[19]=0 THEN 310
290 MAT R=ZER[N[19],N[1]]
300 MAT P=ZER[1,3*N[19]]
310 N[5]=INT(N[2]/N[3])+1
320 MAT G=ZER[N[4],N[5]]
330 FOR J=1 TO N[4]
340 PRINT "TRANSMISSION VECTOR"J
350 FOR I=1 TO N[1]
360 DISP "(";I;")";
370 INPUT T[J,I]
380 NEXT I
390 NEXT J
400 PRINT "COEFFICIENT MATRIX FOR DIFFERENTIAL EQUATIONS"
410 FOR I=1 TO Q4
420 FOR J=1 TO N[1]
430 DISP "(";I;",";J;")";
440 INPUT C[I,J]
450 NEXT J
460 NEXT I
470 IF N[19]=0 THEN 550
480 PRINT "COEFFICIENT MATRIX FOR ALGEBRAIC EQS. OF N.L. DISSIPATIVE ELEMENTS"
490 FOR I=1 TO N[19]
500 FOR J=1 TO N[1]
510 DISP "(";I;",";J;")";
520 INPUT R[I,J]
530 NEXT J
540 NEXT I
550 PRINT "INIT. COND. VECTOR"
560 FOR I=1 TO N[1]
570 DISP "(";I;")";
580 INPUT Z[I,1]
590 N[8+I]=Z[I,1]
600 NEXT I
```

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```
610 FOR I=N[1]+1 TO 10
620 N[8+I]=0
630 NEXT I
640 PRINT "NO. OF NONLINEAR CHARACTERISTICS (0 TO 5)"
650 INPUT N[6]
660 IF N[6]=0 THEN 970
670 REDIM H[10,2*N[6]]
680 FOR J=1 TO N[6]
690 PRINT "ENTER CURVE #"J;"(BY PAIR OF VALUES)"
700 FOR I=1 TO 10
710 DISP "PAIR #";I;
720 INPUT H[I,2*J-1],H[I,2*J]
730 NEXT I
740 NEXT J
750 IF N[7]=0 THEN 850
760 REDIM L[N[7],2]
770 PRINT
780 PRINT "FOR EACH NONLINEAR ENERGY-STORING ELEMENT,"
790 PRINT "ENTER STATE-VARIABLE POSITION AND CURVE #"
800 FOR I=1 TO N[7]
810 DISP "(";I;")";
820 INPUT L[I,1],L[I,2]
830 NEXT I
840 GOTO 860
850 MAT L=ZER[1,1]
860 IF N[19]=0 THEN 940
870 PRINT
880 PRINT "FOR EACH NONLINEAR DISSIPATIVE ELEMENT, ENTER STATE-VARIABLE"
890 PRINT "POSITION, CURVE #, AND KIND OF STATE VARIABLE (1 FOR I, 2 FOR V)"
900 FOR I=1 TO N[19]
910 INPUT P[1,3*I-2],P[1,3*I-1],P[1,3*I]
920 NEXT I
930 GOTO 1050
940 MAT P=ZER[1,1]
950 MAT R=ZER[1,1]
960 GOTO 1050
970 IF N[7]+N[19]=0 THEN 1010
980 PRINT "NO. OF N.L. ELEMENTS=";N[7]+N[19];", NO CURVE."
990 DISP "ERROR! PROGRAM ABORTED."
1000 STOP
1010 MAT P=ZER[1,1]
1020 MAT R=ZER[1,1]
1030 MAT H=ZER[1,1]
1040 MAT L=ZER[1,1]
1050 PRINT "STORE DATA  (Y OR N)";
1060 INPUT R$
1070 IF R$="N" THEN 1270
1080 PRINT "FILE #";
1090 INPUT Q3
1100 STORE  DATA Q3
1110 GOTO 1270
1120 Q4=N[1]-N[19]
1130 R$="Y"
1140 REDIM T[N[4],N[1]],C[Q4,N[1]],Z[N[1],1],G[N[4],N[5]]
1150 IF N[6]=0 THEN 1260
1160 REDIM H[10,2*N[6]]
1170 IF N[7]=0 THEN 1200
1180 REDIM L[N[7],2]
1190 GOTO 1210
1200 REDIM L[1,1]
```

```
1210 IF N[19]=0 THEN 1240
1220 REDIM R[N[19],N[1]],P[1,3*N[19]]
1230 GOTO 1270
1240 REDIM R[1,1],P[1,1],F[1,1]
1250 GOTO 1270
1260 REDIM H[1,1],L[1,1],R[1,1],P[1,1]
1270 REDIM D[Q4,1],E[N[4],1],Q[Q4,1]
1280 IF N[19]=0 THEN 1300
1290 REDIM F[N[19],1]
1300 PRINT "COMPUTE (Y OR N)"
1310 INPUT Q$
1320 IF Q$="N" THEN 2040
1330 FOR I=1 TO N[1]
1340 Z[I,1]=N[8+I]
1350 IF I>Q4 THEN 1370
1360 D[I,1]=N[8+I]
1370 NEXT I
1380 IF N[6]=0 THEN 1640
1390 IF N[7]=0 THEN 1460
1400 FOR J1=1 TO N[7]
1410 P1=D[L[J1,1],1]
1420 P3=L[J1,2]
1430 GOSUB 2730
1440 D[L[J1,1],1]=P2
1450 NEXT J1
1460 IF N[19]=0 THEN 1640
1470 FOR Q1=1 TO 10
1480 MAT F=R*Z
1490 Q2=0
1500 FOR J1=1 TO N[19]
1510 P1=F[J1,1]
1520 P3=P[1,3*J1-1]
1530 GOSUB P[1,3*J1] OF 2900,2730
1540 IF Z[Q4+J1,1]=0 THEN 1560
1550 Q2=Q2+ABS((Z[Q4+J1,1]-P2)/Z[Q4+J1,1])
1560 Z[Q4+J1,1]=P2
1570 NEXT J1
1580 IF Q2<0.001*N[19] THEN 1620
1590 IF Q1#10 THEN 1630
1600 PRINT "PROGRAM ABORTED. IMPROVE INITIAL CONDITIONS."
1610 STOP
1620 Q1=10
1630 NEXT Q1
1640 MAT E=T*Z
1650 FOR I=1 TO N[4]
1660 G[I,1]=E[I,1]
1670 NEXT I
1680 FOR K1=1 TO N[2]
1690 MAT Q=C*Z
1700 MAT Q=(N[8])*Q
1710 MAT D=D+Q
1720 REDIM Z[Q4,1]
1730 MAT Z=D
1740 REDIM Z[N[1],1]
1750 IF N[6]=0 THEN 1950
1760 IF N[7]=0 THEN 1830
1770 FOR J1=1 TO N[7]
1780 P1=Z[L[J1,1],1]
1790 P3=L[J1,2]
1800 GOSUB 2900
```

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```
1810 Z[L[J1,1],1]=P2
1820 NEXT J1
1830 IF N[19]=0 THEN 1930
1840 FOR Q1=1 TO 2
1850 MAT F=R*Z
1860 FOR J1=1 TO N[19]
1870 P1=F[J1,1]
1880 P3=P[1,3*J1-1]
1890 GOSUB P[1,3*J1] OF 2900,2730
1900 Z[Q4+J1,1]=P2
1910 NEXT J1
1920 NEXT Q1
1930 IF K1#(INT(K1/50))*50 THEN 1950
1940 PRINT "INTERVAL #"K1"BE PATIENT!!!"
1950 IF K1#(INT(K1/N[3]))*N[3] THEN 2010
1960 MAT E=T*Z
1970 K2=K1/N[3]+1
1980 FOR I=1 TO N[4]
1990 G[I,K2]=E[I,1]
2000 NEXT I
2010 NEXT K1
2020 IF R$="N" THEN 2040
2030 STORE  DATA Q3
2040 PRINT
2050 PRINT "FINAL STATUS OF STATE-VARIABLE VECTOR IS:"
2060 REDIM Z[1,N[1]]
2070 MAT PRINT Z
2080 PRINT "TOTAL TIME INTERVAL="N[2]*N[8]"SECONDS"
2090 PRINT
2100 DISP "PRINTOUT NEEDED (Y OR N)";
2110 INPUT Q$
2120 IF Q$="N" THEN 2210
2130 PRINT
2140 FOR I=1 TO N[4]
2150 PRINT "OUTPUT"I
2160 FOR J=1 TO N[5]
2170 PRINT G[I,J];
2180 NEXT J
2190 PRINT
2200 NEXT I
2210 DISP "PLOT NEEDED (Y OR N)";
2220 INPUT Q$
2230 IF Q$="N" THEN 2670
2240 FOR I=1 TO N[4]
2250 DISP "SET PLOTTER, PRESS CONT"
2260 STOP
2270 PEN
2280 DISP "MIN AND MAX VALUES";
2290 INPUT E1,E2
2300 SCALE 0,1.1*N[2]*N[8],E1,E2
2310 E3=INT(N[2]/100)*10*N[8]
2320 E4=ABS(E2-E1)
2330 E5=LGT(E4)
2340 E4=E4/10↑E5
2350 E6=INT(E4)*10↑(E5-1)
2360 IF E2*E1>0 THEN 2390
2370 E4=0
2380 GOTO 2430
2390 IF E1<0 THEN 2420
2400 E4=E1
```

```
2410 GOTO 2430
2420 E4=E2
2430 E5=1.1*H[2]*H[8]
2440 XAXIS E4,E3,0,E5
2450 YAXIS 0,E6,E1,E2
2460 LABEL (*,1.5,1.7,0,8.5/11)
2470 FOR Y=0 TO 10 STEP 2
2480 IF ABS(E1+Y*E6-E4)<E6 THEN 2520
2490 PLOT 0,Y*E6+E1,1
2500 CPLOT 2,-0.3
2510 LABEL (2600)(Y*E6+E1)
2520 NEXT Y
2530 LABEL (*,1.5,1.7,PI/2,8.5/11)
2540 FOR X=0 TO 10 STEP 2
2550 IF X=0 THEN 2590
2560 PLOT X*E3,0,1
2570 CPLOT 2,-0.3
2580 LABEL (2600)(X*E3)
2590 NEXT X
2600 FORMAT E8.1
2610 PLOT 0,G[I,1],-2
2620 FOR J=2 TO H[5]
2630 PLOT (J-1)*H[3]*H[8],G[I,J],2
2640 NEXT J
2650 PEN
2660 NEXT I
2670 REWIND
2680 PRINT
2690 PRINT "END OF JOB"
2700 END
2710 REM: FROM STATE VAR. TO PROBLEM VAR.
2720 STOP
2730 K3=2*P3-1
2740 K4=2*P3
2750 FOR I=1 TO 10
2760 IF P1=H[I,K3] THEN 2820
2770 IF P1>H[I,K3] THEN 2840
2780 IF I=1 THEN 2850
2790 J=I-1
2800 P2=H[J,K4]+(H[I,K4]-H[J,K4])*(P1-H[J,K3])/(H[I,K3]-H[J,K3])
2810 GOTO 2870
2820 P2=H[I,K4]
2830 GOTO 2870
2840 NEXT I
2850 PRINT "OUT OF CHARACTERISTICS BOUNDS. PROGRAM ABORTED."
2860 STOP
2870 RETURN
2880 REM: FROM PROBLEN VAR. TO STATE VAR.
2890 STOP
2900 K3=2*P3-1
2910 K4=2*P3
2920 FOR I=1 TO 10
2930 IF P1=H[I,K4] THEN 2990
2940 IF P1>H[I,K4] THEN 3010
2950 IF I=1 THEN 3020
2960 J=I-1
2970 P2=H[J,K3]+(H[I,K3]-H[J,K3])*(P1-H[J,K4])/(H[I,K4]-H[J,K4])
2980 GOTO 3040
2990 P2=H[I,K3]
3000 GOTO 3040
3010 NEXT I
3020 PRINT "OUT OF CHARACTERISTICS BOUNDS. PROGRAM ABORTED."
3030 STOP
3040 RETURN
```

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APPENDIX B

```
10 COM N[19],T[4,10],C[10,10],Z[10,1],G[4,181],H[10,10],L[10,2],R[3,10],P[1,9]
20 DIM A[4,181]
30 DISP "NO. OF OUTPUTS";
40 INPUT E4
50 PRINT "TOTAL NO. OF INTERVALS (BOTH FILES)"
60 INPUT E9
70 PRINT "SUPPLEMENTARY PRINT INTERVAL"
80 INPUT E6
90 E8=INT(E9/E6)+1
100 REDIM A[E4,E8]
110 DISP "FIRST FILE #";
120 INPUT Q3
130 LOAD  DATA Q3
140 IF E4=N[4] THEN 180
150 PRINT "ERROR! DIFFERENT NO. OF OUTPUTS."
160 DISP "PROGRAM TERMINATED"
170 STOP
180 E3=N[3]
190 E2=N[2]
200 E5=N[5]
210 REDIM G[E4,E5]
220 E8=1
230 FOR J=1 TO E5 STEP E6
240 FOR I=1 TO E4
250 A[I,E8]=G[I,J]
260 NEXT I
270 E8=E8+1
280 NEXT J
290 E7=E5-((E8-2)*E6+1)
300 E7=E6-E7+1
310 REDIM G[4,181]
320 DISP "SECOND FILE #";
330 INPUT Q3
340 LOAD  DATA Q3
350 IF E4=N[4] THEN 370
360 GOTO 150
370 IF E3=N[3] THEN 400
380 PRINT "ERROR! FILES DIFFER IN PRINT INTERVAL."
390 GOTO 160
400 IF E5+N[5]-2=E9 THEN 430
410 PRINT "ERROR! TOTAL NO. OF INTERVALS ="(E5+N[5]-2)
420 GOTO 160
430 REDIM G[E4,N[5]]
440 FOR J=E7 TO N[5] STEP E6
450 FOR I=1 TO E4
460 A[I,E8]=G[I,J]
470 NEXT I
480 E8=E8+1
490 NEXT J
500 E8=E8-1
510 E7=(E5+N[5]-2)-(E8-1)*E6
520 PRINT "NO. OF TIME INTERVALS LEFT OUT=";E7
530 REDIM G[E4,E8]
540 MAT G=A
550 N[5]=E8
560 N[2]=N[2]+E2
570 N[3]=N[3]*E6
580 DISP "FILE # FOR NEW DATA";
590 INPUT Q3
600 DISP "IF TAPE READY, PRESS CONT"
610 STOP
620 STORE  DATA Q3
630 DISP "PRINTOUT NEEDED (Y OR N)";
640 INPUT Q$
650 IF Q$="N" THEN 670
660 MAT PRINT G
670 DISP "END OF JOB"
680 END
```