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## Some New Methods in Geomagnetic Field Modeling Applied to the 1960-1980 Epoch

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# SOME NEW METHODS IN GEOMAGNETIC FIELD MODELING APPLIED TO THE 1960-1 880 EPOC.H 

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[^0]As shown by Gauss in 1839 the potential of the geomagnetic field can be represented by a spherical harmonic series of the form:

$$
\begin{align*}
V & =a \sum_{n=1}^{N \operatorname{MAX1}} \sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\left[g_{n}^{m} \cos m \phi+h_{n}^{m} \sin m \phi\right] p_{n}^{m}(\cos \theta) \\
& +a \sum_{n=1}^{N M A X 2} \sum_{m=0}^{n}\left(\frac{r}{a}\right)^{n}\left[q_{n}^{m} \cos m \phi+s_{n}^{m} \sin m \phi\right] p_{n}^{m}(\cos \theta) \tag{1}
\end{align*}
$$

where: a is the mean radius of the earth,
$r, \theta, \phi$ are the standard spherical coordinates, and
$P_{n}{ }^{m}$ (in "modern" methodology) are the Schmidt quasi-normalized form of associated Legendre functions.

The magnetic field is then given by:

$$
\begin{equation*}
\bar{B}=-\nabla V=\left(R_{r}, R_{\theta}, R_{\phi}\right) . \tag{2}
\end{equation*}
$$

Theoretically, (1) applies exactly, at a given time, only when NMAX1 and NMAX2 go to infinity, under the assumption that the region under consideration, $a<r<b$, say, is source-free. The source-free assumption holds nearly exactly between the earth's surface and the ionosphere, but near-earth spacecraft pass through a region of "field aligned" currents in the auroral belt. The geometry is such that the field magnitude and vertical component are rela-ively unaffected (Langel, 1974) but the horizontal components may have several hundred $n T$ (nanotesla) contribution from these currents. This must be accounted fur in deriving (1). In practice, the values of NMAXI and NMA.․․ are limited by the data accuracy, by finite computer capabilities and, for NMAX2, the nature of its temporal variability. The data accuracy aspect will be
discussed in a later section. In (1) the terms in $(a / r)^{n+1}$ describe sources within $r<a$, or "internal" sources, and the terms in $(r / a)^{n}$ describr sources outside $r \geq b$, or "external" sources. Field measurements are used to derive the coefficients $g_{n}^{m}, h_{n}^{m}, q_{n}^{m}$, and $s_{n}^{m}$, usually by some form of least squares procedure.

Both the internal $\left(g_{n}^{m}\right.$ and $h_{n}^{m}$ ) and external ( $q_{n}^{m}$ and $s_{n}^{m}$ ) coefficients are known to vary with time. To date, temporal variations in the external terms have not been included in models. Variations in the internal field have been modeled by expanding the coefficients in Taylor series in time, e.g.:

$$
\begin{equation*}
g_{n}^{m}(t)=g_{n}^{m}\left(t_{0}\right)+\dot{g}_{n}^{m}\left(t_{0}\right) \star\left(t-t_{0}\right)+\ddot{g}_{n}^{m}\left(t_{0}\right)^{\star} \frac{\left(t-t_{0}\right)^{2}}{2!}+\cdots \tag{3}
\end{equation*}
$$

Most models include only the constant and first derivative (secular variation) terms, although some more recent models have incorporated the second derivative (secular acceleration) also (e.g., Cain et al, 1967; Barraclough and Malin, 1979). It should be noted that in some contexts the derivatives in equation (3) are combined with the factorials to produce the total coefficients which multiply the powers of time in the power series.

The principal sources of data for main field modeling have been (1) permanent magnetic observatories, (2) repeat measurements at selected sites with intervals between measurement of one to six years, (3) surveys from aircraft and ship, and (4) satellite measurements. Only the satellite surveys are truly global. Relevant surveys from which data are generally available were conducted by the Cosmos 49 spacecraft in nctober and November of 1964, by the $0 G 0-2,-4$, and -6 (POGO) spacecraft from nctober 1965 through July 1971, and most recently, by the MAGSAT
spacecraft from November 1979 tnrough June 1980. The Cosmos and POGO satellites measured only the field magnitude, which introduces an ambiguity in resulting spherical harmonic analyses (Backus, 1970; Hurwitz and Knapp, 1974; Stern and Bredekamp, 1975; Stern et al, 1980). The permanent magnetic observatories must still be regarded as the primary source of information regarding the temporal changes. Unfortunately, these data, and all data except the satellite data, are highly "contaminated" by fields originating in the crust of the earth. These anomaly fields can be tens to thousands of $n T$ in any of the components and represent a large nc: se source when attempting to model the bulk of the geomagnetic field which originates in the earth's core.

This paper describes an attempt to utilize the observatory data in a more optimal way by incorporating an estimation of individua; observatory anomaly fields into the solution and, in so doing, to allow thé derivation of a more accurate model of temporal variation. The results were presented as a possible contribution to the 1980 version of the IGRF (International Geomagnetic Reference Field) and the definitive Geomagnetic Reference Fields for 1965, 1970 and 1975 at the 1981 assembly of the International Association of Geomagnetism and Aeronomy (IAGA).

MODEL DEGREE AND ORDER

In (1) " $n$ " is the degree and " $m$ " the order of any given term. NMAXI is the maximum degree and order for internal terms and NMAX2 for external terms. Gauss' original model did not include external terms and used an NMAXI of four because he concluded that the available data did not warrant the inclusion of further terms. In the years since fiauss the available data base has improved considerably and, accordingly, the degree/order of published models has increased. Malin and Pocock (1969) analyzed the question of the appropriate degree/order for models hased on magnetic observatory data. Using data from 180 observatories they computed models from degree/order two to ten and compared the rms residuals. Their results are shown in Table 1.

| Degree/Order | rms residual ( nT ) |
| :---: | :---: |
| 2 | 3495.7 |
| 3 | 2218.2 |
| 4 | 889.0 |
| 5 | 538.7 |
| 6 | 388.5 |
| 7 | 364.5 |
| 8 | 359.5 |
| 9 | 354.3 |
| 10 | 351.9 |

They note a rapidly decreasing rms from 2 nd to 6 th order after which the decrease becomes very slow; this leveling off is attributed to the "crustal noise" in the data. They conclude that "at least six orders should be evaluated if the core field is to be fitted within 0.5 percent". Subsequent models not using satellite data are in reasonable agreement with their results and most of these models were of degree/ order eight, nine or ten.

The situation changed drastically with the advent of satellite data, particularly the surveys by the POGO satellites from 1965 tn 1971. These data are far enough above the earth that the crustal anomaly fields are less than $20 n T$ maximum, with rms below 5 nT . Further, the coverage is now truly global with no large gaps. The latest published models which inciude these data are mainly of degree/order 12 (Rarraclough et al, 1975; Peddie and Fabiano, 1976; Barker et al, 1981). Those of us who have been attempting to study crustal magnetic anomalies have been using a model of degree/order 13 (Langel et al, 1980a). This choice is confirmed by Langel and Estes (1982) who derived the spatial power spectruni (Lowes, 1966, 1974).

$$
\begin{equation*}
R_{n}=(n+1) \sum_{m=0}^{n}\left[\left(g_{n}^{m}\right)^{2}+\left(h_{n}^{m}\right)^{2}\right] \tag{4}
\end{equation*}
$$

for $n=1$ to 23 using Magsat data. The derived spectrum showed a clear break near $n=14$ which was interpreted to mean that the core field dominates for $n<13$ and the crustal field for $n>15$.

The situation for the temporal derivative terms is much more complicated. In the model to be presented, we have included first derivatives to degree/order thirteen, second derivatives to degree/order six, and third derivates to degree/order four. For each coefficient we have calculated the ratio of coefficient magnitude to the standard error of the coefficient. As a rule of thumb, we assume that if this ratio is $\geq 2$, the coefficient has statistical significance above the $95 \%$ confidence level (see, e.g. Rarraclough and Malin, 1979).

Table 2 shows the ratio distribution for these terms. On this hasis we are justified in including some terms from each of the 13 degrees of the first derivative and, perhaps, should have extended the second and third derivative terms to still higher degree/order. Investigation of this question is beyond the scope of the present paper.

Aside from the question of the accuracy to which individual coefficients are determined, one has to ask what the descriptive and predictive properties are for models with higher derivatives. We will address this question in a subsequent section.

There remains the question of external fields (NMAX2). As Malin and Pocock (1969 and references therein) point out, various attempts have been made to separate the internal and external parts of the field. However, the results have been widely different; in some cases unrealistically large external fields have been found, but have not been statistically significant. Again, a change has come with the ginhal vector survey by MAGSAT. Langel et al (1980b) used data from November $5-6,1979$ to derive the $\operatorname{MGST}(6 / 80)$ model. This model included statistically significant external terms of degree/order equal to one. Furthermore, plots of the MAGSAT data clearly demonstrated the need for such terms to adequately represent the data. However, it is known that

Table 2:
Summary of Distribution of Ratio of Coefficent Magnitude to Coefficent Standard Error.

|  | Number |  | RATIOS |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Degree | of Terms | $>100$ | 10-100 | 2-10 | <2 |

First Derivatives

| 1 | 3 | 1 | 2 | 0 | $n$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 2 | 5 | 3 | 0 | 2 | 0 |
| 3 | 7 | 0 | 6 | 1 | 0 |
| 4 | 9 | 0 | 6 | 3 | 0 |
| 5 | 11 | 0 | 8 | 3 | 0 |
| 6 | 13 | 0 | 10 | 3 | 0 |
| 7 | 15 | 1 | 9 | 4 | 1 |
| 8 | 17 | 0 | 12 | 4 | 1 |
| 9 | 19 | 0 | 12 | 6 | 1 |
| 10 | 21 | 0 | 8 | 8 | 5 |
| 11 | 23 | 0 | 6 | 15 | 2 |
| 12 | 25 | 0 | 2 | 17 | 6 |
| 13 | 27 | 0 | 4 | 11 | 12 |

Second Derivatives

| 1 | 3 | 0 | 3 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 2 | 5 | 0 | 1 | 4 | 0 |
| 3 | 7 | 0 | 6 | 1 | 0 |
| 4 | 9 | 0 | 5 | 3 | 1 |
| 5 | 11 | 0 | 7 | 3 | 1 |
| 6 | 13 | 0 | 12 | 0 | 1 |

Third nerivatives

| 1 | 3 | 0 | 3 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 5 | 1 | 3 | 1 | 0 |
| 3 | 7 | 0 | 7 | 0 | 0 |
| 4 | 9 | 0 | 5 | 3 | 1 |

the sources of the external fields, which are the magnetospheric ring current, magnetopause current and magnetotail current, vary widely with time in a fashion not yet amenable to this type of model and, moreover, vary more strongly as functions of local time than of longitude as set forth in equation (1). While external terms may be valuable as an indication of the average quitet level of external fields, we have no compelifing reason to include them in models describing extended periods of time.

## METHOD OF ANALYSIS

The method of determining the parameters of ( 1 ) is essentially the same as that described by Cain et al (1967) with revisions to include higher temporal derivatives and to incorporate magnetic observatory data in a new way. Because of the data types involved the problem is non-linear and must be solved iteratively. The Rayesian least squares estimation equations are as follows:

$$
\begin{equation*}
\left.\delta \hat{\rho}_{n+1}=\left(A^{\top} W A+\Omega_{0}\right)^{-1}\right)^{-1}\left[A^{\top} W \delta y_{n}+R_{0}^{-1}\left(\hat{\rho}_{0} \hat{\theta}_{n}\right)\right] \tag{5}
\end{equation*}
$$

where

A
$\hat{\rho} \quad$ is the vector of adjusted parameters
oy is the vector of residuals, i.e. measured data minus predicted value from previous iteration

W is the weight matrix for the measurements
$\Omega_{0} \quad$ is the a priori parameter covariance matrix
$\hat{0} 0$ is the a priori estimate of the parameters
and the estimeice at the $(n+1)$ st iteration is

$$
\begin{equation*}
\hat{\rho}_{n+1}=\hat{\rho}_{n}+\delta \hat{\rho}_{n+1} \cdot \tag{6}
\end{equation*}
$$

In the notation of Cain et al (1967), each measured quantity, say $C$, is a function of the coefficients $g_{n}^{m}, n_{n}^{m}, q_{n}^{m}$, and $s_{n}{ }^{m}$ from equation (1), denoted collectively by " $p$ ", and of the standard $r, \theta, \phi, t$ space-time coordinatus:

$$
\begin{equation*}
C=C(p ; r, \theta, \phi, t) . \tag{7}
\end{equation*}
$$

The partial derivatives in $A$ are then the set $\partial C / \partial p$. The weight matrix $W$ is diagonal and formed from the standard deviations of the various measurements, so that for the ith measurement with sigma $\sigma_{i}$

$$
\begin{equation*}
W_{11}=\frac{1}{\sigma_{j}^{2}} \tag{8}
\end{equation*}
$$

As already noted, the data from magnetic observatories represent the most useful data set for determining the temporal variation of the internal field. The incorporation of such data directly into a main internal core field model, however, suffers from the fact that the magnetic field measured at the observatory may have a significant contribution due to local crustal fields. The field at an observatory is represented as the vector sum

$$
\begin{equation*}
\bar{B}=\bar{B}_{i}+\bar{B}_{m} \tag{9}
\end{equation*}
$$

where $\bar{B}_{\mathfrak{i}}$ is the internal core field contribution from the scalar potential of equation (1) and $\bar{R}_{m}$ is the local anomaly field, which may
change appreciably over a distance of a few kilometers. The time derivative of $\bar{B}_{m}$ is assumed to be negligible so that

$$
\begin{equation*}
\dot{\bar{B}}=\dot{\bar{B}}_{i} . \tag{10}
\end{equation*}
$$

A global satellite data set, on the other hand, is comparatively free fror the effects of crustal anomalies and is certainly free from the effects of the higher-amplitude, more localized anomalies. Assuming that for a main ffeld model the crustal influence on satellite data may he treated as random noise, these data may then be used in conjunction with observatory data to isolate the non-core fields at the observatory. In terms of the algorithm of equations (5), (7) and (8) this is accomplished by writing, for cich measured component at each observatory

$$
\begin{equation*}
C=C\left(p, C_{a} ; r, \theta, \phi, t\right), \tag{12}
\end{equation*}
$$

where $C_{d}$ is the anomaly in that component at that ohscrvatory. Terms such as $\partial C / \partial C_{d}$ are then added to the matrix, $A$, of portial derivatives. The vector, $D$, of adjusted parameters from equation (5) then includes $p$ and all anomaly components. This procedure allows the data to properiy distribute their influence among the temporal and constant parameters in a least square sense. Such a solution is well determined only when the satellite data, not strongly sensitive to the crustal fields, are included in the solution. A forward elimination technique is used for the anomaly components in accumulating the normal matrix $A^{\top} W A$, so the cost of obtaining the solution for the model coefficients is greatly roduced. The values of the anomaly components, if desired, are then obtained by back substitution.

DATA SET

The data utilized in the present analysis include data from (1) magnetic observatories, (2) repeat stations, (3) marine surveys, (4) the

POGO satellite surveys and (5) the MGSAT satellite survey. The PNGO data consisted of the 47000 scaiar observations used for the POEn(8/71) model (Langel, 1974) augmented with 24000 quiet nGn-6 observations from mid-1969 through early 1971. The MAGSAT data set, consisting of scalar and vector measurements from November 5 and 6, 1979, is identical to that used for the $\operatorname{MGST}(6 / 80)$ model (Langel et al, 1980b). The maasurement standard deviation used to weight the POGO data was 7 nT and for MAGSAT data was 10 nT , based on fits to these data alone.

Annual means data were taken from 148 ohservatories, selected on the basis of geographical distribition, longevity of measurement availability, and data quality. This resulted in our not using some high quality observatories in regions of higher observatory density. Moreover, only those annual means with three vector components we a accepted. The observatories utilized are listed in Table 3 together with the anomaly vector bias from the solution, the time of cata avallability and the $\sigma_{i}$ used in equation ( 8 ). The spellings used are those of the NOAA worldwide Magnetic Survey of the National Geophysical and Solar-Terrestrial Data C.enter. In cases where a significant change at the observatory (e.g., shift of loration, change of instrument) was known to have occurred, the data from that observatory were broken into subsets whtcil were treated independently with respect to the anomaly blas vecturs. In all cases the measurcd elements were converted to $X, Y$ and $Z$ components where $X$ is north, $Y$ is east and $Z$ is down in a geodetic coordinate system assuming an equatorial radius of 6378.165 km and a reciprocal flattening of 298.25. To determine the $\sigma_{i}$ in Table 3 a least squares quadratic curve was fit to each component at each observatory and the standard deviation of the data to the solution adopted as $\sigma_{i}$.

To fill in surface areas vold of data, seiected marine survey and repeat station data were utilized. In order to accommodate the non-

Table 3


observatory surface data in the solution in a consistent manner, techniques were used to remove the major parts of the crustal contributions. From the avallable scalar marine data over the years 1970-1974, 39 long, straight tracks of length greater than 1200 km were selected. A low pass filter was applied to each track, removing anomaly wavelengths shorter than 500 km . Approximately 300 measurements were then taken along the filtered tracks. A measurement standard deviation of 10 nT was used in weighting the data in the soilition, although analysis of crossing points for the 39 tracks indicated differences on the order of 50 nT when corrected for secular variation. The higher weighting reflects our regard for the relative importance of these data in an area devoid of other surface observations. Approximately 600 measurements from 150 repeat stations were utilized to fill sparse data regions in Central and South America, Africa and Australia. Only stations with three or more occupations and good data quality were accepted. As there were generally insufficient data available for the repeat stations to solve for independent anomaly bias vectors, quadratic polynomial fits to these data were time differentiated to remove the assumed constant crustal influence. Measurement standard deviations for the "differentiated" repeat data were arrived at by utilizing the "differentiated" value, together with observatory and marine data, to derive a degree/order eight model. The standard deviations to that model were 0.2 deg/year for $\dot{\rho}$ and $I$ and $5 \mathrm{nT} /$ year for $\mathrm{H}, \mathcal{Z}$ and $B$, taken collectively. These were adopted as measurement standard deviations for the repeat station data in the present analysis.

RESIILTS

In order to test the usefulness of solving for anomalies at the observatories and of including second and third time derivatives, a model not incorporating MAGSAT data was developed and its prediction capahility tested by comparing it to the $\operatorname{MGST}(6 / 80)$ model based only on MAGSAT
data. The test model, denoted $\operatorname{PMAG}(7 / 80)$, as well as the final model, denoted GSFC(9/80), was of degree/order 13 in its constant and first derivative terms, of degree/order six in its second derivative terms and of degree/order four in its third derivative ternis. Table 4a gives the coefficient values and their first derivatives (secular variation), and Table $4 b$ the second and third derivatives, for the $\operatorname{GSFC}(9 / 80)$ model. The standard error is given in parenthesis beside each coefficient. This is to be used as an error estimate only witin caution because its accuracy depends upon (1) the accuracy of the $\sigma_{1}$ used to weight the data (and the validity of the assumptions that the data are uncorrelated and have gaussian error) ?nd (2) the validity (accuracy and completeness) of our model. In practice the standard errors tend to underestimate the actual error but are nonetheless useful as an indication of the magnitude of error in each coefficient and of the relative accuracies between coefficients.

Table 5 summarizes a statistical evaluation of these models and two other recent (pre-MAGSAT) models. Model AWC75 (secular variation part) was derived by Peddie and Fabiano (1976) using data from 1967 through 1974 and model WC80 was derived by Rarker et al (1981) using data from 1950 through 1980, but their secular variation model is thought to be applicable mainly from 1974-1977. AWC75 and WC8O include constant terms through degree/order 12 and first time derivatives through degree/order eight. In Table 5 MGST( $6 / 80$ ) is included as a standard of comparison for the 1980 epoch. Of the pre-MAGSAT models it is seen that $\operatorname{PMAG}(7 / 80)$ is the best predictive model. This indicates that, at least over a three-year interval, its temporal derivatives are not wildly varying beyond the data span. It should be noted that a similar model which did not include the observatory anomaly solution performed hadly when used as a predictor, i.e. the presence of the anomaly solution affects the sciction for the temporal terms in a positive way. The $\operatorname{GSFC}(9 / 80)$ mocit represents the 1980 field well because it incorporates MAGSAT data. It is, however, slightly deteriorated from MGST(6/80). This

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Table 4a
onec furs macNeTic FiELD model


|  | m | 8 |  | $\cdots$ |  | \%rin |  | $n_{n}^{m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | - | . | 48 |  |  | 201 |  |  |  |
|  | - |  |  | 40.7 | (1.0) | 90 | 10.10 | -9,4 | (10.21) |
|  | - |  | 4-4 |  |  | -18909 | 16.14 |  |  |
|  | 1 |  | 108 | -24008 | 1as | 4.8 | 40.13 | -1t. 54 | (0.100 |
|  | 2 |  | 112) |  | (19.1) | 204 | \%.* | -4.08 | 0.\% |
|  | - |  | 104 |  |  | 481 | 12.121 |  |  |
| , | , |  | 108 | 418 | Rea | -2 | 10.12) | -6.39 | *.88 |
|  | 2 |  | 108) | 2 me | 6县 | -2.19 | 60.121 | 1.4 | 4.13 |
| , | 3 |  | 18.0.0. | -913 | 11.4 | 387 | 10.14 | 4.42 | 10.4 |
|  | - |  | (1) |  |  | -2.29 | 60.10 |  |  |
|  | 1 | 1 | 1 m | 2198 | 10.4 | -294 | racm | 20 | (0.te) |
| , | 1 |  | 45 |  | 1094 | -1000 | 6.19\% | 10\% | 10.10 |
| , | 3 |  | - | ma | ten | -204 | 10.111 | 6.7 | 40.20) |
| , | 4 |  |  | -2000 | 19,4 | -184 | (6.12) | -2.73 | 10.13) |
|  | - | -217.1 | 10.78 |  |  | -1.04 | 10.04 |  |  |
|  | 1 | 374 | Hen | $4{ }^{4}$ | 10.3: | 4.80 | 10.04 | 3.20 | 10.04 |
| , | 2 | 2048 | \%as | 190 | E.24 | -0.73 | (1904) | $0 \cdot 0$ | (10.04 |
|  | 2 | -ma | (1)4 | -1mat | 10.4 | 4.41 | 1603 | +142 | rosed |
|  | 4 | - 618 | 120N | .77.7 | 10.9 | 0.11 | (1006) | 1.60 | 10.00 |
| , | - | 47.1 | $\omega . n$ | 118 | 10.2) | 1.48 | 16.07 | 0.4 | (0.07) |
| - | - | 1. | 4 |  |  | 0.75 | (0.03) |  |  |
| , | 1 | 0 | 022 | -148 | 10.2) | -0.e4 | 10.031 | 0.98 | 10.00 |
| , | 2 | 48. | 4 | 4 | 10.2) | 3.48 | 10.041 | -120 | 10.031 |
| , | 1 | . 701.4 | 10.31 | 20. | 10.31 | 2.80 | 19.04) | -.. 0 | 10.04 |
| - | 4 | 20 | 03) | -40 | 10.a) | 0.4 | 10.04 | 0.11 | 10.04 |
|  | ! | 4.1 | 10.4 | -1.8 | 10.21 | 1.74 | 10.041 | 0.80 | 10.04 |
| , | - | -rep. 1 | 0.a) | 17.5 | 10.9 | 1.34 | 10.04 | 2.72 | 10.071 |
| 1 | - | 71.0 | 6.11 |  |  | 0.00 | 10.011 |  |  |
| 7 | 1 | -0.1 | 6.11 | 43.2 | 10.11 | -0.3) | 10.011 | -1.44 | 10.011 |
| \% | 2 | 13 | 6.11 | -27.1 | 10.11 | 0.6 | 10.011 | -0.4 | (0.041 |
| 7 | 3 | 20.1 | 10.8 | -5.8 | 10.21 | 0.48 | 10.011 | 0.15 | 10.07) |
|  | 4 | -138 | 19.2 | 18.8 | 10.21 | 0.4 | 10.01) | 0.58 | (0.01) |
| 1 | 5 | 0.1 | 10.31 | 17.8 | 10.31 | 0.17 | 10.031 | -0.17 | 10.02) |
| 1 | 8 | 10. | 6.2) | -236 | 10.31 | -0.20 | 10.011 | 0.84 | 10.0.) |
| 1 | 1 | -2.7 | (0.2) | - 0.8 | 10.71 | 0.00 | (0.04) | 0.54 | 10.04 |
| - | 0 | 14.8 | 10.11 |  |  | 0.60 | 10.01) |  |  |
| - | 1 | 72 | 10.11 | 18 | 10.11 | 0.00 | 10.01) | -0.00 | (0.01) |
| , | 2 | 0.9 | 10.1) | -17.7 | 10.11 | 0.22 | 10.01) | -0.30 | 10.011 |
| - | 3 | - 4.4 | 10.11 | 3.2 | 10.11 | 0.04 | 10.011 | -0.00 | 10.011 |
| , | * | 7.1 | 10.2) | -23.4 | 10.21 | -0.24 | 10.011 | -0.34 | (0.01) |
| - | 5 | 4.0 | 10.21 | 2.4 | 10.2) | -0.3\% | 10.011 | 0.31 | 10.019 |
| , | + | 3.7 | 10.3) | 16.3 | 10.31 | 0.63 | 10.021 | -0.84 | 10.02) |
| - | 7 | 7.1 | 10.3) | -13.4 | 10.21 | -0.48 | 10.021 | -0.67 | 10.01) |
| : | B | -13 | 10.7) | - 18.2 | 10.71 | -0.28 | 10.031 | 0.06 | 10.04 |
| , | 0 | 5.2 | 10.11 |  |  | -0.34 | 10.011 |  |  |
| , | 1 | 10.7 | 10.11 | -21.8 | 10.11 | 0.17 | 10091 | 0.06 | 10.011 |
| , | 2 | 1.0 | 10.11 | 16.0 | 10.11 | 0.04 | (0.011 | 0.06 | 10.011 |
| , | 3 | -12.9 | $10.1)$ | e. | 10.11 | 0.02 | 10.011 | 0.24 | 10.911 |
| - | 4 | 1.2 | ¢0.1) | -4.8 | 10.11 | -0.12 | 10.011 | -0.19 | 10.011 |
| , | 5 | -2. | 10.11 | -13 | 10.11 | 0.27 | 10.011 | -0.27 | (10.01) |
| - | * | - 1.1 | 10.2) | 0.3 | 10.21 | -0.04 | 10.011 | -0.08 | 10.011 |
| , | 7 | 7.1 | 10.2) | 108 | 10.2) | 0.27 | 10.011 | -0.00 | 10.011 |
| - | \% | 1.5 | 10.2) | - 6.3 | 10.21 | -0.00 | 10.01 | -0.23 | 10.011 |
| , | - | -E. | 10.51 | 2.1 | 10.81 | 0.48 | 10.093 | 0.15 | 10.03 |
| 10 | - | -33 | 10.11 |  |  | -12 | 10.011 |  |  |
| 1 | 1 | - 4.1 | 10.11 | 1.0 | 10.11 | -0.et | 18.071 | -0.07 | 10.011 |
| 10 | 2 | 2.7 | 10.11 | 4.1 | 10.11 | 0.06 | 10.011 | -0.0.0. | 10.011 |
| * | 8 | -6 | 10.11 | 2.8 | 10.11 | -0.00 | 10.81 | 0.06 | 10.011 |
| \% | 4 | -1. | $10.1)$ | 55 | 10.11 | 0.01 | 10.011 | -0.00 | 10.011 |
| 1 | 5 | 52 | 6.1) | 4.3 | 10.11 | 0.01 | 10.011 | -0.01 | 10.011 |
| 10 | c | 2.8 | $0.1)$ | -1.3 | 10.11 | -0.19 | 10.011 | -0. 14 | 10.011 |
| 1 | 1 | 1.3 | 10.2 | -1.1 | 10.2) | 0.18 | 10.011 | 0.12 | 10.019 |
| 10 | - | 2.4 | 10.21 | 4.4 | 10.21 | 0.07 | 10.011 | 0.14 | 10.011 |
| 10 | * | 3.3 | 6.3 | -0.8 | 10.21 | 0.00 | 10.011 | -0.00 | 10.011 |
| 1 | 10 | $\rightarrow 2$ | 10.sy | -4 | 10.5) | 0.03 | 10.03) | -0.05 | 10.031 |
| 1 | , | 23 | 16.11 |  |  | -0.02 | 10.081 |  |  |
| 1 | 1 | -16 | 10.11 | 1.1 | 10.11 | 0.93 | 10.01) | -0.03 | 10.04) |
| 1 | 2 | -23 | 10.11 | 2.4 | 6.11 | -0.03 | 10.011 | -0.0. | 10.011 |
| 11 | 3 | 2.1 | 10.11 | -2.1 | 10.11 | -0.13 | 10.01) | -0.02 | 19.911 |
| 1 | 4 | 0.3 | 10.11 | -2.3 | 10.11 | 0.00 | 10.011 | 0.05 | 10.011 |
| 1 |  | -4 | 10.11 | 0.4 | 10.1) | -0.02 | 10.011 | -0.02 | 10.011 |
| 1 | - | $\rightarrow 3$ | 6.11 | -0. 1 | 10.11 | -0.01 | 10.011 | 0.02 | 10.011 |
| 11 | 1 | 1.4 | 10.1) | -3.15 | 10.11 | -0.01 | (0.01) | 0.10 | 10.011 |
| 1 | 1 | 13 | te. 11 | -0.2 | 10.2) | -0.11 | 18.091 | 0.04 | 10.011 |
| 1 | * | 48 | 421 | -0.7 | 10.21 | 0.60 | 10.011 | 0.15 | 10.011 |
| 11 | 0 | 1. | 10.21 | -1.7 | 10.2) | -0. 10 | 10.011 | -0.00 | 10.011 |
| 11 | 11 | 14 | 10.4 | -0. 1 | 10.41 | 0.06 | 10.09 | -0.39 | 10.081 |
| 18 | - | . 20 | H.11 |  |  | -0.en | (0.091 |  |  |
| 1 | 1 | 4.2 | m.11 | 0.8 | 10.1) | -0.08 | 10.011 | 0.00 | 10.011 |
| 11 | 3 | 0.8 | 10.11 | 0.5 | 10.11 | 0.11 | 10.011 | -0.00 | 10.011 |
| 12 | 1 | 43 | n.11 | 2.0 | 10.11 | -0.08 | 10.011 | 0.00 | 18.011 |
| 18 | 4 | 4.2 | 6.11 | -1.0 | 16.11 | 4.04 | 10.011 | -0. 13 | 10.011 |
| 12 | - | 1.1 | ve.11 | 0.5 | 16.11 | 0.04 | 10.011 | 0.00 | 10.011 |
| 12 | - | 42 | 10.11 | 0.2 | 10.11 | 0.09 | (0.01) | -0.00 | 10.011 |
| 12 | 7 | 6.1 | 10.11 | 0.2 | 10.11 | 0.4 | 10.011 | 0.63 | 10.011 |
| 12 | - | 0.3 | *0.1) | -0.3 | 10.11 | 0.01 | 10.011 | -0.10 | 10.011 |
| 12 | $\bullet$ | -18 | 10.11 | -0.6 | 10.11 | -0.05 | 10.011 | -0.00 | 10.011 |
| 12 | ${ }^{*}$ | 0. | 10.2 | -0.8 | 10.31 | 0.01 | 10.911 | 0.00 | 10.011 |
| 12 | 11 | 0.4 | ¢. 21 | 0.8 | 60.2) | 0.00 | 10.091 | -0.07 | 10.011 |
| 12 | 12 | 0.2 | 10.21 | -. 1 | 10.31 | -0.e4 | (0.02) | -0.01 | (0.02) |
| 13 | - | 0. | +.11 |  |  | 0.02 | 10.011 |  |  |
| 12 | 1 | 4.2 | 10.11 | 0.3 | 10.11 | 0.03 | $10.01)$ | 0.07 | 10.011 |
| 13 | 2 | 0.2 | *.1) | 0.2 | 10.11 | -0.06 | 10.011 | -0.01 | 10.011 |
| 13 | 3 | -0. | 6.11 | 18 | 10.11 | -0.00 | 10.011 | 0.03 | 10.011 |
| 13 | 4 | 02 | 10.11 | 0.8 | 10.11 | -0.00 | 10.011 | 0.09 | 10.011 |
| 13 | 6 | 0.8 | 0.1 | -1.0 | 10.1 | 0.01 | 10.011 | -0.00 | 10.01 |
| 12 | $\cdots$ | 0.2 | 10.11 | 0.2 | 10.11 | 0.03 | 10.011 | 0.00 | 10.011 |
| ${ }^{3}$ |  | 0.1 | 10.11 | 12 | 10.11 | 0.03 | 10.011 | 000 | 10.011 |
| 13 | 8 | -3 | 10.11 | -0.2 | (0.1) | 0.00 | 10.011 | -0.01 | 10.071 |
| 13 | - | 0.2 | 10.11 | 0.7 | 10.11 | -0.01 | 10.011 | 0.01 | 10.011 |
| 12 | 10 | -0.4 | te.11 | -0.1 | 10.11 | 0.00 | 10.011 | -0 02 | 10.011 |
| 13 | 11 | 0.2 | 0.2) | -9.5 | 10.81 | 0.02 | 10.011 | -0.04 | 10.011 |
| 13 | 12 | -0. | 10.21 | 0.4 | 10.23 | 0.00 | 10.011 | 000 | 10011 |
| 13 | 12 | 0.8 | 10.3 | -0.4 | 10.31 | 0.07 | $10.08)$ | 0.01 | 10.027 |

Table 4b

GSFC (9/E0) MACNETIC FIELD MODEL
(MEAN RADIUS OF THE EARTH I8 6371.2 KM; MEAN EPOCH IS 1980.0) (CONTINUED)

| n | m | \% |  | hn |  | ${ }^{* 0.0}$ |  | $\because \ddot{n}_{n}^{m}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | -0.818 | (0052) |  |  | -0.0098 | (0.0027) |  |  |
| 1 | 1 | -0.440 | (0.0.41) | 1.188 | (0.048) | -0.04 | (0.0041) | 0.1914 | (0.0046) |
| 2 | 0 | 0.388 | (0.026) |  |  | 0.0384 | (0.0023) |  |  |
| 2 | 1 | 0.497 | (0.023) | 2830 | (0.030) | 0.024 | (0.0023) | -0.2974 | (0.0028) |
| 2 | 2 | 0.004 | (0.037) | -1218 | (0.041) | 0.014 | (0.003) | -0.0804 | (0.0042) |
| 3 | 0 | 1.304 | (0.023) |  |  | 0.1140 | (0.0020) |  |  |
| 3 | 1 | 1.337 | (0.024) | -2.218 | (0.023) | 0.1134 | (0.0022) | -0.1836 | (0.0024) |
| 3 | 2 | 0.362 | (0.024) | -0.166 | (0.028) | 0.0334 | (0.0022) | -0.0168 | (0.0023) |
| 3 | 3 | 1.147 | (0.034) | -0.408 | (0.033) | 0.0940 | (0.0094) | -0.0060 | (0.0033) |
| 4 | 0 | -0.248 | (0.019) |  |  | -0.0240 | (0.0017) |  |  |
| 4 | 1 | -0.086 | (0.018) | -0.240 | (0.021) | -0.0012 | (0.0016) | -0.0366 | (0.0018) |
| 4 | 2 | -0.806 | (0.020) | 0.087 | (0.021) | -0.0604 | (0.0019) | 0.0090 | (0.0020) |
| 4 | 3 | -0.284 | (0.021) | 1.108 | (0.021) | -0.0064 | (0.0019) | 0.1092 | (0.0019) |
| 4 | 4 | -0.448 | (0.028) | -0.192 | (0.029) | -0.0364 | (0.0028) | -0.0186 | (0.0030) |
| 5 | 0 | -0.180 | (0.003) |  |  |  |  |  |  |
| 5 | 1 | -0.098 | (0.003) | 0.173 | (0.003) |  |  |  |  |
| 5 | 2 | -0.188 | (0.003) | -0.136 | (0.003) |  |  |  |  |
| 5 | 3 | -0.272 | (0.004) | 0.107 | 10.004; |  |  |  |  |
| 5 | 4 | 0.031 | (0.004) | -0.056 | (0.004) |  |  |  |  |
| 5 | 5 | 0.019 | (0.006) | 0.018 | (0.006) |  |  |  |  |
| 6 | 0 | 0.077 | (0.002) |  |  |  |  |  |  |
| 6 | 1 | -0.083 | (0.002) | 0.048 | (0.003) |  |  |  |  |
| 6 | 2 | 0.168 | (0.003) | -0.074 | (0.003) |  |  |  |  |
| 6 | 3 | -0.026 | (0.003) | -0.136 | (0.003) |  |  |  |  |
| 6 | 4 | 0.038 | (0.003) | 0.096 | (0.003) |  |  |  |  |
| 6 | 5 | 0.078 | (0.003) | 0.006 | (0.003) |  |  |  |  |
| 6 | 6 | 0.188 | (0.006) | 0.081 | (0.006) |  |  |  |  |

Table 5

MAEL

| DATA TYPE | stantistic TrPE | Mest $6 / 600$ | AMc75 | Wemon | Pmars(7/8n) | GSFC.(9/80) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Magsat |  |  |  |  |  |  |
| scalar | r.m.s | -8.2 | 138.9 | 112.7 | 82.2 | 10.1* |
|  | meen | 0.1 | 6n, ${ }^{\text {a }}$ | -20.6 | -24.0 | -1.6 |
|  | stc. dev. | 8.2 | 125.2 | 116.9 | 78.8 | 1 n .0 |
| $x$-component*** | P.m.s. | 7.6 | 100.1 | 91.7 | 65.5 | 9.0 |
|  | mean | -0.2 | 24.7 | -33.4 | -27.9 | -1.1 |
|  | sta. dev. | 7.6 | 97.0 | R5. 5 | 59.3 | 8.0 |
| Y-component *** | r.m.s. | 7.4 | 79.6 | 59.7 | 62.7 | 8.5 |
|  | mean | 3.1 | -1.3 | -1.1 | -1.6 | -n.1 |
|  | ste. dev. | 7.4 | 79.5 | 59.7 | 62.7 | 9.5 |
| Z-component******) | r.m.s. |  | 157.4 | 113.2 | 98.3 | 0.2 |
|  | mean | -2.5 | 24.9 | 2 n .0 | 2.1 | -3.5 |
|  | stc. dev. |  |  | 111.3 | 9.7 | 3.3 |
| pogn |  |  |  |  |  |  |
| scaiar | r.m.s. | - | 69.8 | 121.6 | 5.7 | $6 . ?$ |
|  | nuan | - | 43.8 | 21.3 | 0.0 | -7. ${ }^{5}$ |
|  | std. dev. | $\because$ | 54.4 | 119.8 | 6.7 | 6.7 |
| obsarvatory $x$-component |  |  | [1967-1974] | [1974-1977] | [1900-19771 | :196n-1977 |
|  | P.m.s. | -- | 2 Ca .5 | 302.7 | $37.1 *$ | 39.900 |
|  | man | - | -24.7 | 41.5 | 3.5 | 5.6 |
|  | std. dev. | - | 283.4 | 299.9 | 37.6 | 39.6 |
| Y-component | P.m.s. | -- | 352.8 | 240.9 | 18.7 | 19.3 |
|  | meen | - | -48.3 | -15.月 | - 0.06 | 0.4 |
|  | std. dev. | -- | 349.5 | 240.5 | 18.7 | 19.3 |
| 2-component |  | - | 515.6 |  |  |  |
|  | meen | - | -23.7 | -71.0 | -0.05 | -n. 11 |
|  | std. dev. | - | 515.0 | 579.9 | 16.5 | 15.2 |

Magsat residuals to GSFC(9/80) mere taken including the masi(6/8n) external terms with GFSC(0/8n) because the Magsat data used to derfve GSFC(9/80) was cormeted for that external field.
mapma(7/80) and GSFC(9/40) inelude astimates of observatary "anomalies" which are taken into account whent computing residuals.
manoer not include dati at latitudes greater than $50^{\circ}$ or lass than $-50^{\circ}$.
is not unexpected since it applies to 20-year time period whereas $\operatorname{MGST}(6 / 80)$ is direct fit to the two days of data involved.

Comparison with observatories shows typlcally high residuals for the models which do not estimate observatory anomalies (AWC75 and wC.8n). On the other hand, the observatory residuals for $\operatorname{PMAG}(7 / 80)$ and $\operatorname{GSFC}(9 / 80)$ are of the range one would expect from the published accuracy estimates of the observations, the internal consistency of the data from individual observatories, and the characteristics of unmodeled temporal variations. Note that no measurements with extreme residuals have been eliminated from this calculation, as is often done. This is because such extreme values are often due to the anomaly values solved for in $\operatorname{PMAG}(7 / 80)$ and $\operatorname{GSFC}(9 / 80)$ and so to compare the models all measurements should be utilized.

For further comparison, and to get a quantitative measure of the predictive capability of the models, statistics were computed of each model versus observatory data on a year-by-year basis. In doing so, we adopted the statistical measure used by Mead (1979) in a similar analysis, namely, half the width of the median $68 \%$ of the residual values. This is designated $\bar{\sigma}$, and would he equivalent to one standard error, $\sigma$, if the distribution were normal. The reason for adopting this measure is to avoid the situation where a few very large residuals dominate the statistics. Note that this is a different statistic than used in Table 5, and so the numbers are not directly comparable. In the plots to follow, some of the year-to-year changes in $\bar{\sigma}$ are due to a changing distribution of magnetic observatories. Figures $1-3$ show the variation of $\bar{\sigma}$ with time for the $X, Y$, and $Z$ components of observatory data relative to five field models: WCBO, IGS75, AWC75, $\operatorname{PMAG}(7 / 80)$, and $\operatorname{GSFC}(9 / 80)$. IGS75 is the designation for the model derived by Rarraclough et al (1975) from all available data from 1955 through about 1974. $\bar{\sigma}$ is rounded to the nearest 5 nT . The observatorins used in this
evaluation are those listed in Table 3 (i.e. those used in deriving GSFC(9/80) and those listed in the Appendix. The statistic for GSFC(9/80) is computed without taking into account the lncal anomaly solution.

Examination of these plots shows that over its "lifetime" the GSFC( $9 / 80$ ) model describes the observatory data as well as or better than the other models. The deterioration of its representation prior to 1960 is also apparent. Of the models plotted, IGS75 has the longest useful "lifetime", presumably because the span of the data used to derive the model extended to the 1950's.

It is very apparent that these models, considered collectively, suffer from a data limitation such that a spherical harmonic analysis of reasonable degree/order cannot represent the data with a $\bar{\sigma}$ of better than about $100 n T$ in $X, 90 n T$ in $Y$ and $150 n T$ in $Z$. We attribute this to the presence of "crustal noise" in the data and belfeve that it is the fundamental limiting factor both on the accuracy of models based on such data and on our ability to evaluate model accuracy using such data.

A clearer picture of the model degradation is found in Figure 4. Here the statistics for $\operatorname{GSFC}(9 / 80)$ versus three components of observatory data are plotted taking into account the local anomaly solutions. The $\bar{\sigma}$ for each component is now in the 5 - 20 nT range from 1962.5 on. These values are commensurate with the accuracy of measurement at observatories; i.e., we believe we have very nearly eliminated the effect of crustal anomalies and that the statistic is now dominated by the actual measurement noise. neterioration of the model hegins at about 1961.5, within the dota interval used by the model. This is attributed to the lack of satellite data, with their global coverage, prior to 1965. The amount of deterioration then increases rapidly outside the data interval. It is roughly doubled at 1958.5. Comparison of Figure 4
with Figures 1 - 3 indicates that the model deterioration becomes comparable to the "crustal noise" at about 1956. For many purposes this might be taken to be the useful limit of extrapolation of the model.

Figures 5-8 show the yearly averages at a series of observatories together with the field predicted by GSFC(9/80). Examination of these plots shows the need for the third time derivative, particularly for the $X$ component at Alibag, Boulder, Gornotayezhnaya and Guam, and the $Y$ and $Z$ components at Gornotayezhnaya and Guam.

Figures 9 - 10 show data from two observatories not utilized in obtaining the solution. We have not, then, calculated the crustal component for these stations and none is included in the plots. This is most evident at Kodaikanal. Examination of Figures 9-1 $\mathbf{9}$ shows that although the magnitude of the model differs from the data, the temporal change of the data is well represented throughout the 1960-1980 time period. Furthermore, the third time derivative is important for the $Y$ components at Hurbanovo.





Figure 7





Observatory and other surface data are inadequate in space and time for determining accurate field models. Hints of the limitations of these data for defining the main field have repeatedly surfaced, as for example in the study already cited by Malin and Pocock (1969). Comparing Figure 1-3 with Figure 4 brings this limitation into clearer focus. The fact is that there is a "noise" with an "rms" ( $\sigma$ ) of about 100-150 nT in the observatory data set. We attribute this to the existence of crustal anomalies, although it is conceivable that error sources such as poor knowledge of data location, instrument inaccuracy or local magnetic contamination are contributing factors at some locations. Crustal anomalies are worldwide in distribution, have amplitude as high as several thousands of $n T$, and have a broad spectrum of spatial wavelengths. Because of the poor spatial distribution of surface data, any attempt to even partially model the anomaly field with such data will be plagued with aliasing problems. We conclude that these data alone provide sufficient accuracy only for models up to a degree and order of about eight. Satellite data are not immune to these limitations, although they are not as severe. Langel and Estes (1981) showed that crustal anomaly fields dominate the Magsat data for wavelengths shorter than those corresponding to degree and order fifteen. This means that with present methods of modeling, we cannot determine the main field representation beyond degree and order thirteen or fourteen, regardless of the quality of our data.

What must be appreciated is that the limitations of the data are limitations not only on modeling but also on the evaluation of models. For example, in considering Figure 1 , the differences between models $A$, I, P and G between 1960 and 1975 are at the "noise" (anomaly) level of the observatory data and are not likely to be truly significant. To generalize this conclusion, a model derived from satellite data with accuracies of, say, $10-20 n T$, cannot be accurately evaluated using observatory data unless the model is in error by several hundred $n$ T.

The method of solving for observatory biases or anomalies presented ham offers a partial selution to this data limitation. Further work is required to determine the dependance of the bias determination on other model parameters such as the time span involved, the degree and order of the constant and temporal terms, etc. One step in this direction has been taken in that we have determined the biases for several models of differing degree and order with only small changes in the values of the biases so determined.

Further such tests need to be performed. Moreover, it needs to be determined if the calculated biases are in reasonable accord with, say, aeromagnetic anomaly data. This is complicated by the fact that the untangling of the aeromagnetic anomalies from the main field model used to reduce the anomaly data is not straightforward.

Adequate representation and prediction of the temporal change of the earth's main field has, in our minds, not been achieved. In this paper, we have taken two steps which we believe improve the situation. First, we have included solution for the localized observatory anomaly fields and, second, we have utilized third time derivatives. The success of the second step depends upon inclusion of the first.

The use of second and third derivatives has certainly resulted in a model of good accuracy for a twenty year period. The advantages of this over the use of several individual models for a shorter period of time are continuity and that the highly accurate satellite data are allowed to provide some constraint at times remote from the data epoch and so, hopefully, increase the overall accuracy. Tests indicated that PMAG(7/80) predicted two to three years forward in time more accurately than other Pre-Magsat models. Comparison with observatory data (Figures 1-3, 5-10) indicates that $\operatorname{GSFC}(9 / 80)$ "predicts" (backward in time from 1960) within the 100-200 nT level for about four years. However, for
longer prediction, such models are clearly not suitable because the higher derivative terms begin to dominate and the resulting error increases at a more rapid rate than for models with, say, only first derivatives. A solution might be to taper the higher temporal derivatives to zero a few years beyond the existing data interval. This, however, is quite ad hoc and has little real justification. Another approach would be to continue to add temporal terms, including higher order derivatives, until. hopefully, some sort of convergence is achieved. From the discussion of Table 2, it is likely that higher degree and order second and third derivatives are significant. It is not clear, however, that temporal convergence will ever be achieved within any reasonable computer limitations or even that it is possihle. The problem, of course, is that model constraints based on the physics of the core dynamo are not built into these models. In our view, until this is done it will not be possible to derive truly adequate "forecast" models.

## NOKMOLLEDGMENTS

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Appendix: Observatories used for the statistics of Figures $1-3$ but not included in the model solution are: Abinger, Abisko, Acacias, Aso, Averroes, Bagulo, Bouzareah, Budakeszi, Budkov, Cambridge Bay, Castellaccio, Centro Geofisico, Cheltenham, navao, nehra Nun, nruzhnaya, Ebro, Eights, El Abiod Sidi, Elisabethville, Eskdalemuir, Gibilmanna, Gonzalez Videla, Hallett Station, Heard Island, Heiss Island, Helwan, Hollandia, Hurbanovo, liadan, Isla da Pascua, Jassy, Julianehaab II, Kanoya, Kanozan, Karavia, Katuura, Kiruna, Kodaikanal, Kuyper, L. America III, L. America V, Lazarevo, Luanda Golf, Manhay, Maputo, Marion Island, Mizusawa, Monte Capellino, Murmansk, Nagycenk, Nantes, Nitzanim, Norway Station, Novo-Kazalinsk, Orcadas del Sur, Patrick, Pendeli, Port-Alfred, Regensberg, Roi Baudouif, San Fernando, Simferopol, Srednikan, Stonyhurst, Surlari, Swider, Taipei, Tangerang, Thule I, Tikhaya Ray, Toledo, Tulsa, Voroshilov, Vykhodnoy, Watheroo, Wien Auhof, Wien Kobenzl, Witteveen, Yellowknife.

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## FIOME CAPTIOMS

Figure 1: Variation of $\bar{\sigma}_{X}$ with time for observatory date relative to five field models, not including observatory anomalles.

Figure 2: Variation of $\bar{\sigma}_{\gamma}$ with time for observatory data relative to five field models, not including observatory anomalies.

Figure 3: Variation of $\bar{\sigma}_{2}$ with time for observatory date relative to five field models, not including observatory anomalies.

Figure 4: Variation of $\bar{\sigma}$ with time for $\operatorname{GSFC}(9 / 80)$, inciuding observatory anomalies.

Figure 5: Comparison of Alibag annual means ( $x, y, z$ ) to values computed from $\operatorname{GSFC}(9 / 80)$, including the observatory anomaly. nata from this observatory was used in the solution for GSFP( $9 / 80$ ).

Figure 6: Comparison of Boulder annual means $(X, Y, Z)$ to values computed from GSFC( $9 / 80$ ), including the observatory anomily. Data from this objervatory was used in the solution for GSFC.(9/80).

Figure 7: Comparison of Gornotayezhnaya annual means ( $X, Y, Z$ ) to values computed from GSFC(9/80), including the observatory anomaly. Data from this observatory was used in the solution for $\operatorname{GSFC}(9 / 80)$.

Figure 8: Comparison of Guam annual means ( $X, Y, Z$ ) ter values computed from GSFC(9/80), including the observatory anomaly, nata from this observatory was used in the solution for $\operatorname{GSFC}(9 / 80)$.

Figure 9: Comparison of Hurbanovo annual means ( $X, Y, Z$ ) to values computed from GSFC(9/80). No observatory anomaly was computed because data from this observatory was not used in the solution for GSFC(9/80).

Figure 10: Comparison of Kodaikanal annual means $(X, Y, Z)$ to values computed from $\operatorname{GSFC}(9 / 80)$. No observatory anomaly was computed because data from this observatory was not used in the solution for GSFC(9/80).


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