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TIME DOMAIN IDENTIFICATION OF AN OPTIMAL CONTROL PILOT MODEL WITH EMPHASIS ON THE OBJECTIVE FUNCTION

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Introduction

In this paper, we will propose a method for the identification of the pilot's control compensation using time domain techniques. From this information we hope to infer a quadratic cost function, supported by the data, that represents a reasonable expression for the pilot's control objective in the task being performed, or an inferred piloting "strategy". (Note here that we are using the term strategy as synonomous with control objective, and not with control law.)

The ultimate goals of this research topic include a better understanding of the fundamental piloting techniques in complex tasks, such as landing approach; the development of a metric measurable in simulations and flight test that correlate with subjective pilot opinion; and to further validate pilot models and pilot-vehicle analysis methods. At this time we will present the methodology and some preliminary numerical results.

The Pilot Model and Objective Function

The analyses relies on the well-known [1] optimal-control theoretic technique for modeling the human pilot's manual control function. The hypothesis upon which it is based is that the well trained, well motivated pilot chooses his control inputs (e.g. stick force) to meet the pilot's (internal) objective in the task, subject to his human limitations. His objective is further assumed to be expressible in terms of a quadratic "cost" function

$$J_p = E\left\{\lim_{T\to\infty} \frac{1}{T}\int_0^T \left(Y_p^T QY_p + u_p^T Ru_p + \dot{u}_p^T G\dot{u}_p\right) dt\right\} \qquad (0)$$

where Y_p = vector of pilot's observed variables (e.g., attitude, acceleration)

u_p = vector of pilot's control inputs

Q,R,G = Pilot-Selected (internal) weightings

The human limitations modeled include information-acquisition and processing time delay, observation and control input errors, and neuromuscular dynamics. A block diagram of the resulting model structure is shown in Figure 1.

The components of this model may be grouped into two parts, one dealing with the information acquisition and state estimation, and one related to the control law or control policy operating on the estimated state. As has been shown in the references on this modeling approach, the "solution" for the pilot's control inputs, as predicted by the model, is expressed as

$$\dot{u}_p = g_x^T \hat{x} + g_u^T u_p + \nu_u$$

where $\hat{x}$ = internal estimate of the system state

g_x, g_u = control gains

ν_u = motor noise, or control input errors

(Readers unfamiliar with the further details of the model are referred to the reference.)

The key points germain to this analysis are that the above equation is a mathematical expression representing the pilot's overt control actions (u_p), and these control actions are measurable experimentally. Furthermore, the gains g_x and g_u are functions of the plant (vehicle) dynamics and his objective function, and thereby represent his control "techniques", level of skill, and familiarity with the vehicle dynamics.

Another factor of importance is that not only is the objective function, from which the gains are determined, a mathematical part of a pilot control model, but it's resulting magnitude obtained from exercising the <u>model</u> has been found to correlate with the subjective pilot opinion obtained from <u>simulation</u> and <u>flight test</u>. Such a correlation is shown in Figure 2, as an example, taken from Refs. 2 and 3. This of course assumes one has been able

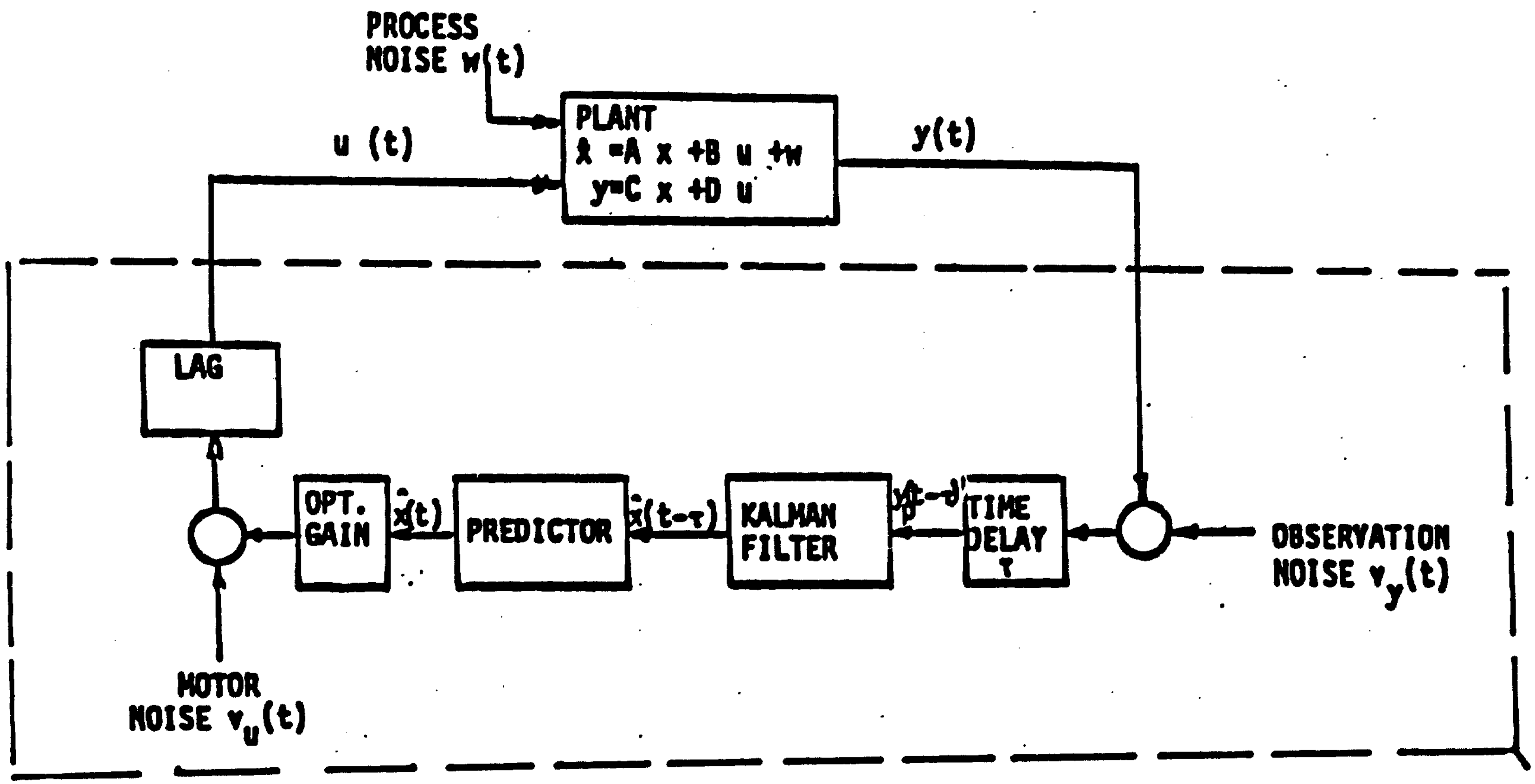


Figure 1, Model Structure

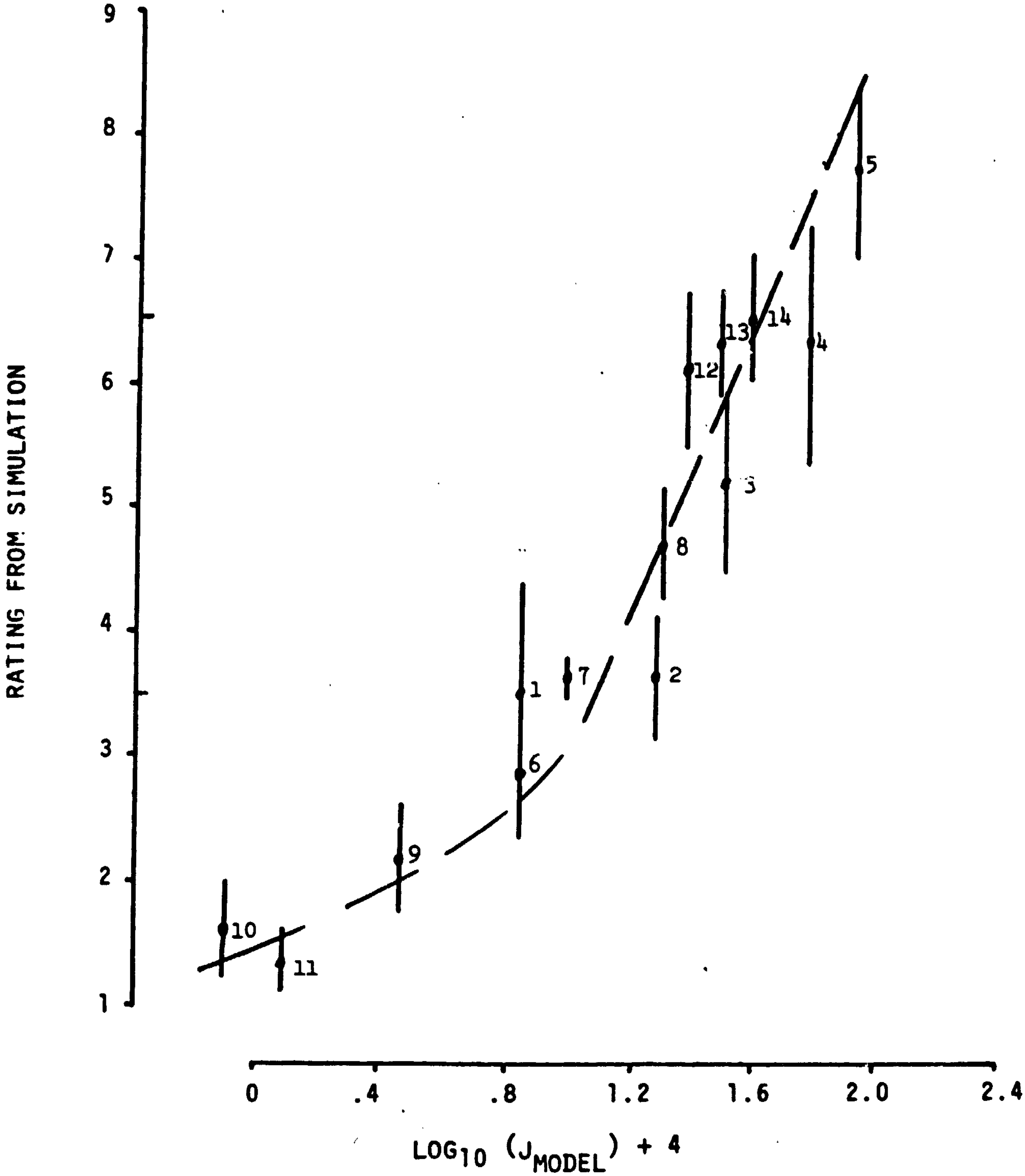


Figure 2, Rating Correlation

to correctly express the pilot's (internal) cost function, which is in fact his strategy that depends on his perception of the task. Now this is easy to do in simple laboratory tasks in which the subject has been instructed to minimize some displayed error, for example. But it is not at all clear just what flight parameters are being "regulated" or "tracked", other than ILS glide slope and localizer error in the case of landing approach. This is but one example, other complex piloting tasks might be considered equally as well.

The Identification Procedure

We seek then a method by which we may identify those pilot parameters that reflect his control techniques, or control strategy. Referring back to the pilot model control law, or

$$\dot{u}_p = g_x^T \hat{x} + g_u^T u_p + \nu_u$$

we note that the gains g_x operate on the estimated state $\hat{x}$. Now the separation principle of optimal estimation and control theory states that the control gains (g_x, g_u) are independent of the state estimation process. Further, the optimal state estimator, in general and in the pilot model, is independent of the overall objective function being minimized by the controller (estimator and control) law. Therefore, if we are mainly after the pilot's control strategy as expressed by, or at least a function of, his objective function, we need only to focus on the gains (g_x, g_u) and not on those variables related only to the state estimator. These latter variables include the time delay, and observation and motor noise covariance matrices, parameters of interest in the identification technique of Levison [4], for example. If our approach is successful, fewer parameters must be identified from the data, which is always an advantage, but the parameters affecting the estimation process are assumed.

The identification method proposed is as follows. The control law expressed previously, may be rewritten as

$$\dot{u}_p = g_x^T x - g_x^T \varepsilon + g_u^T u_p + \nu_u$$

where ε = error in estimating the true (actual) state x. Note that along with the pilot's control u_p, these true states, such as angle of attack or pitch attitude are <u>measureable</u>, but the state estimate, $\hat{x}$, is a quantity internal in to pilot, as modeled. <u>Hence $\hat{x}$ is not measurable</u> ---nor are ε or ν_u. Transposing the above, multiplying by x = col $[x, u_p]$, and taking expected values yields

$$\begin{bmatrix} E(x\dot{u}_p^T) \\ \hline E(u_p\dot{u}_p^T) \end{bmatrix} = \left\{ \begin{bmatrix} E(xx^T) & E(xu_p^T) \\ \hline E(u_px^T) & E(u_pu_p^T) \end{bmatrix} - \begin{bmatrix} E(x\varepsilon^T) & 0 \\ \hline E(u_p\varepsilon^T) & 0 \end{bmatrix} \right\} \begin{bmatrix} g_x \\ \hline g_u \end{bmatrix}$$

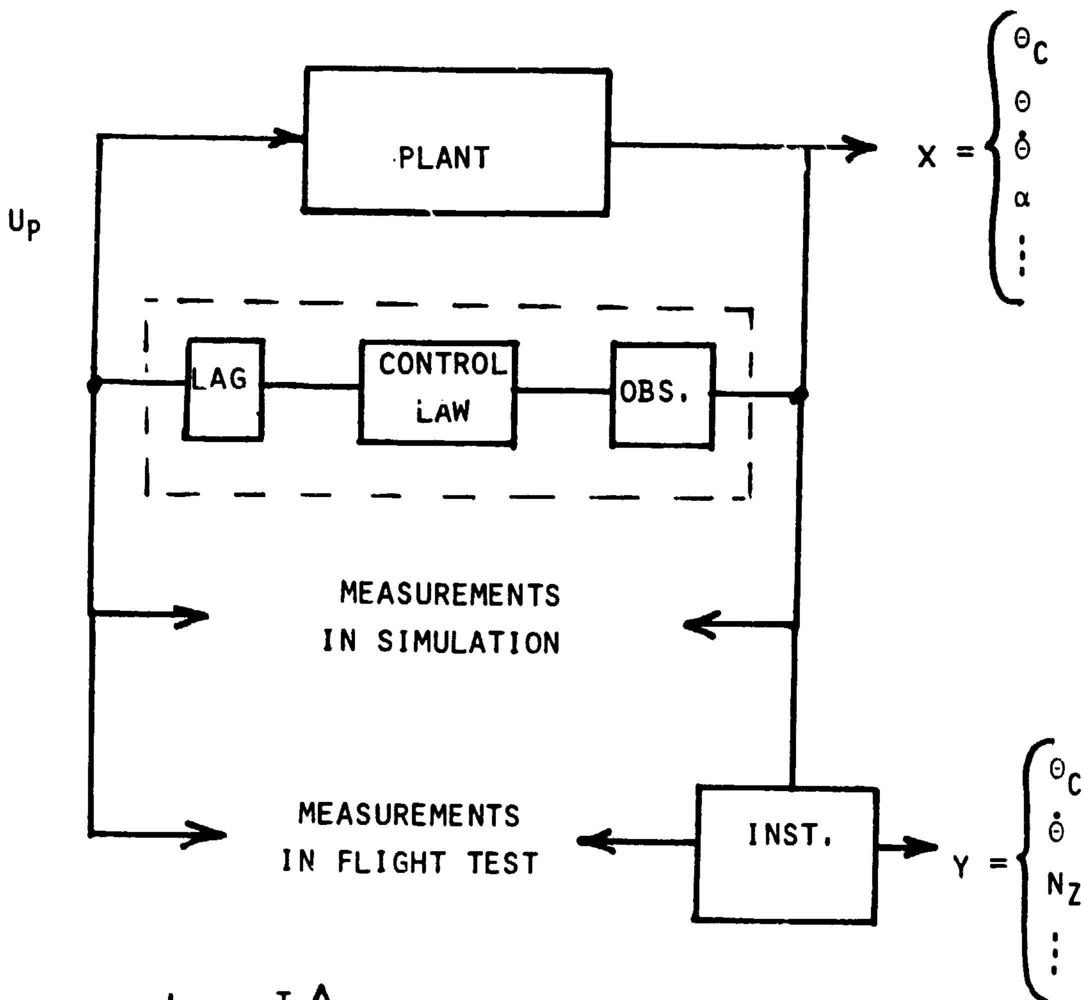
$$+ \begin{bmatrix} E(x\nu_u^T) \\ \hline E(u_p\nu_u^T) \end{bmatrix}$$

or $N_{\dot{u}} = M \begin{bmatrix} g_x \\ \hline g_u \end{bmatrix} + N_{\nu_u}$ (I)

Now to evaluate these matrices we note first that, in a simulation at least, the vectors x(t) and $u_p(t)$ are measurable, so estimates of their covariance matrices (e.g., $E(xx^T)$) may be obtained from measurements of sampled time histories. (Also, in this paper we assume that good estimates of $\dot{u}_p$ are available from filtered measurements of u_p. The details of accomplishing this filtering are under current investigation, but digital techniques as well as analog methods are still available.) For reference, refer to Figure 3.

EXPERIMENTAL CONSIDERATIONS

TRACKING TASK - ATTITUDE, ACCEL. ...



$$\dot{U}_P = G_X^T \hat{X}_P + G_U U_P + V_U$$

Figure 3

With regard to the remaining terms involving ε and ν_u, both are not measurable and need attention. To resolve this consider the complete system dynamics model by the relation

$$\dot{x} = Ax + Bu_p + w$$

and

$$\dot{u}_p = g_x^T x - g_x^T \varepsilon + g_u^T u_p + \nu_u$$

where the relation between state and estimate, or $\hat{x} = x - \varepsilon$ has been employed. The pilot's internal state estimation error, ε, is treated as follows. Define $\varepsilon_u = u_p - \hat{u}_p$ to be the error in estimation of the pilot's own control input, and then let

$$\bar{\varepsilon} = \text{col}\left[\varepsilon,\ \varepsilon_u\right]$$

Now the covariance of $\bar{\varepsilon}$ may be shown to be governed by the relation

$$\text{cov}\ (\bar{\varepsilon}) = E(\bar{\varepsilon}\ \bar{\varepsilon}^T) \triangleq P$$

$$\dot{P} = A_1\ P + PA_1^T + W_1$$

Also we have

$$A_1\ \Sigma + \Sigma\ A_1^T + W_1 - \Sigma C^T\ V_y^{-1}\ C\Sigma = 0;\ \Sigma = \text{cov}\ (e_{KF})$$

and

$$A_1 = \left[\begin{array}{c|c} A & B \\ \hline 0 & g_u \end{array}\right] \qquad C = \text{pilot's observation matrix}$$

$$Y_p = C\begin{bmatrix} x(t-\tau) \\ u(t-\tau) \end{bmatrix} + \nu_y$$

These relations are all obtained from Ref. (5) and from the pilot model equations given in Ref. (1). Here e_{KF} is the Kalman filter estimation error for the delayed state, Σ the covariance of e_{KF}, and

$$W_1 = \begin{bmatrix} W & 0 \\ 0 & V_u \end{bmatrix}$$

Also W is the covariance of the plant disturbance w, and V_u and V_y are motor noise and measurement noise covariance, respectively, all assumed known. Now the $\dot{P}$ equation may be integrated over the time delay τ, with the initial condition on P from P(0) = Σ, the Kalman filter error covariance. Now, since the predictor has the property that $E(\hat{x}\, \bar{\varepsilon}^T) = 0$, we have

$$E\left\{\begin{bmatrix} x \\ u_p \end{bmatrix} \bar{\varepsilon}^T\right\} = E(\bar{\varepsilon}\, \bar{\varepsilon}^T) = P$$

So then the terms $E(x\, \varepsilon^T)$ and $E(u_p \varepsilon^T)$ are available from P, and these are required to form M.

Finally, it can be shown (Ref. (5)), pg. 331) that with the processes w and ν_u uncorrelated we have in this case

$$E(x \nu_u^T) = 0$$
$$E(u_p \nu_u^T) = \frac{1}{2} V_u$$

Returning then to the estimation of the gains (equation I), we see that all the terms in the matrices $N_{\dot{u}}$, N_{ν_u} and M may be calculated, either analytically or from the measurements of x, u_p (and $\dot{u}_p$). The estimate for the gain vector is then

$$\begin{bmatrix} g_x \\ g_u \end{bmatrix}_{est} = M^{-1} \left[N_{\dot{u}} - N_{\nu_u}\right] \qquad \text{(II)}$$

Note finally that the matrix M is formed from two matrices

$$M = M_x - M_{cor}$$

where the M_{cor} and N_{ν_u} matrices may be thought of as corrections added to a basic least-squares technique. The potential importance of these terms (M_{cor} and N_{ν_u}) will be demonstrated in an example later.

The algorithm is as follows:

1) Select noise covariance matrices, W, V_u, and V_y

2) Select a time delay τ, neuromuscular time constant τ_N (or matrix $T_N = g_u^{-1}$).

3) Form A_1 and solve for Kalman Filter error covariance Σ.

4) Solve for covariance matrix $P(\tau)$ and then the $E(\bar{\epsilon}\,\bar{\epsilon}^T)$ is available. (Note, all these steps may be accomplished before or after the experimental data is obtained.)

5) Perform experiment to obtain state and control (and control rate) time histories.

6) From the time histories, obtain estimates for $E(xx^T)$, $E(xu_p^T)$, $E(u_p u_p^T)$, $E(x\dot{u}_p^T)$ and $E(u_p \dot{u}_p^T)$, or the matrices M_x and $N_{\dot{u}}$.

7) Identify M_{cor} and N_{ν_u} in $E(\bar{\epsilon}\,\bar{\epsilon}^T)$ found in step 4.

8) Form $M = M_x - M_{cor}$ and determine $\begin{bmatrix} g_x \\ g_u \end{bmatrix}_{est}$ from Equation II.

9) Check g_u vs T_N^{-1} (selected in 2 above) and iterate (steps 2-8) again as necessary. Note now that selecting τ_N affects the solution for Σ and $P(\tau)$, along with the effective V_u or

$$V_{u_{eff}} = T_N^{-1} V_u \left(T_N^{-1}\right)^T$$

while selecting τ only affects $P(\tau)$ in the procedure.

Comparison to Classical Results

It is interesting to note that the "corrections" performed by including M_{cor} and N_{ν_u} are qualitatively related to an identification technique (discussed in Ref. 6, and elsewhere) used to determine the human describing function in a compensatory task, which goes back to the development of the "crossover model" of McGruer et al. Shown in Figure 4 is a schematic of this situation, showing the closed-loop tracking of some commanded θ_c. Measurements may be taken of $\theta_c(t)$, $\varepsilon(t)$, $u_p(t)$, and $\theta(t)$ and manipulated in the frequency domain to obtain frequency spectra

$$G_1(j\omega) = U_p(j\omega)/\theta_c(j\omega)$$
$$G_2(j\omega) = \varepsilon(j\omega)/\theta_c(j\omega)$$
$$G_3(j\omega) = U_p(j\omega)/\varepsilon(j\omega)$$

Now, in this model the pilot's control is considered to consist of two parts, one correlated with the input θ_c, the other uncorrelated with the input. The latter component was defined to be "remnant." Mathematically,

$$u_p(j\omega) = Y_p(j\omega)\ \varepsilon(j\omega) + r(j\omega) \text{ and } r(j\omega)/\theta_c(j\omega) \to 0 \text{ in effect.}$$

Block diagram manipulation leads then to the desired relation

$$Y_p(j\omega) = G_1(j\omega)/G_2(j\omega)$$

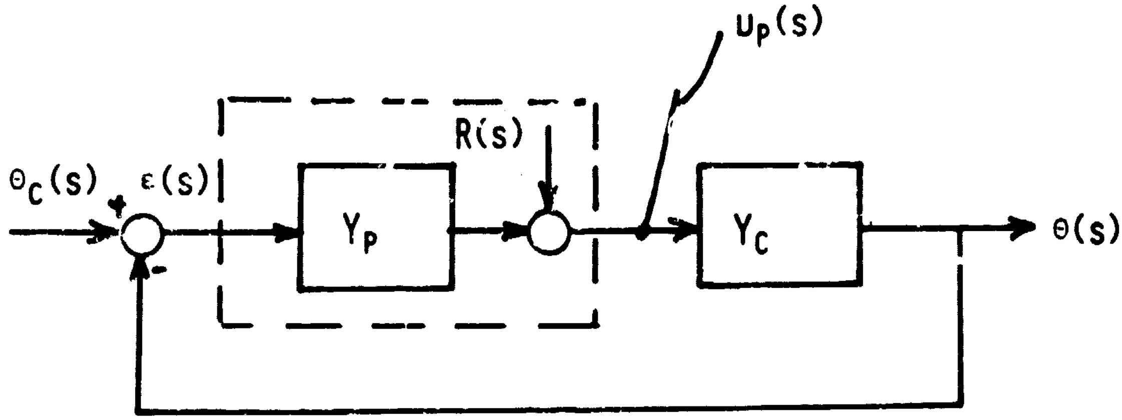
rather than the simpler, and incorrect, expression $Y_p(j\omega) = G_3(j\omega)$. This was due to the presence of remnant $r(j\omega)$ in the <u>measured</u> control input, and the necessity to eliminate it's effect by defining it as the uncorrelated component of u_p, and using this property. Comparing to our control law, transformed just for discussion purposes, we have

$$\dot{u}_p(j\omega) = \underbrace{g_x^T\, x(j\omega) + g_u^T\, u_p(j\omega)}\ -\ \underbrace{g_x^T\, \bar{\varepsilon}(j\omega) + \nu_u(j\omega)}_{\text{unmeasurable separately}}$$

compared to

$$u_p(j\omega) = Y_p(j\omega)\ \varepsilon(j\omega) + \underbrace{r(j\omega)}_{\text{unmeasurable separately}}$$

CLASSICAL RESULTS



MEASURE $U_p(s) = Y_p\,\varepsilon(s) + R(s)$

$$Y_p = G_1(j\omega)/G_2(j\omega) \neq G_3(j\omega)$$

WHERE $G_1(s) = U_p(s)/\Theta_c(s)$

$$G_2(s) = \varepsilon(s)/\Theta_c(s)$$

$$G_3(s) = U_p(s)/\varepsilon(s)$$

Figure 4

The significant difference is that $r(j\omega)$ was, in effect, discarded in finding Y_p, but $g_x^T\varepsilon$ is not uncorrelated with x or u_p and must be accounted fo in the identification problem.

A Numerical Example

To evaluate the numerical properties and the sensitivity to the a priori selected parameters (V_y, W, V_u, τ) a fast time simulation of the pilot model equations has been assembled, and the simulated control task is shown in Figure 5 . As shown, the task is that of pursuit tracking with $11.7/s^2$ controlled element dynamics, and the displayed command signal is filtered white noise with the filter transfer function given ($\theta_c(s)/w(s)$). The state vector is shown, the known gain vector to be identified is listed, and the weights in the objective function used are given. A sample time history of the state and simulated pilot's control input is depicted in Figure 6. Such time histories were sampled at 10 msec intervals and the gains estimated from time windows of data 5, 10, 15, 20, 25, 30 and 35 seconds wide. The root-sum-squred percent error of the five estimated gains is shown in Figure 7. Where

$$E_{RSS} = \left[\sum_{i=1}^{5} \left[\frac{g_i - \hat{g}_i}{g_i} \right]^2 \right]^{1/2}$$

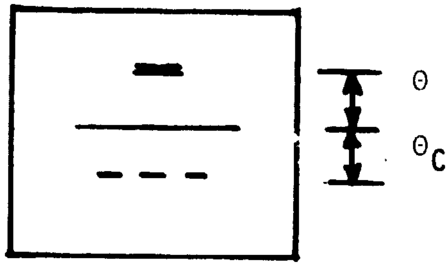
As shown, about 30 seconds of data is required to obtain less than 10% rss error in this example. Other dynamics of higher order, and therefore more gains, will be evaluated in the near future and the convergence will not be as rapid.

The importance of using the proper corrections (e.g., M_{cor} and N_{ν_u}) is shown in Figure 8 , in which the five exact gains, $g_1 \rightarrow g_5$ are shown, along with two sets of gain estimates. The set labeled "uncorrected" was obtained via straight-forward least squares (i.e., M_{cor} and N_{ν_u} not included).

EXAMPLE

PLANT: $\frac{\Theta(s)}{\delta(s)} = \frac{11.7}{s}$

$$\frac{\Theta_C(s)}{W(s)} = \frac{3.67}{s^2 + 3s + 2.25}$$



$\varepsilon = \Theta_C - \Theta$

$$X^T = \left[\Theta_C \;\; \dot{\Theta}_C \;\; \Theta \;\; \dot{\Theta}\right]$$

$$\dot{X} = AX + BU + N$$

$$\dot{U}_P = G_X^T \hat{X}_P + G_U U_P + V_U$$

$$\left[G_X^T \;\; G_U\right] = \left[5.53,\ 1.86,\ -6.76,\ -3.69,\ -9.28\right]$$

$Q_\varepsilon = 16/.35$, $Q_{\dot{\varepsilon}} = 1/.35$, $Q_{\dot{\delta}} = 1$

Figure 5

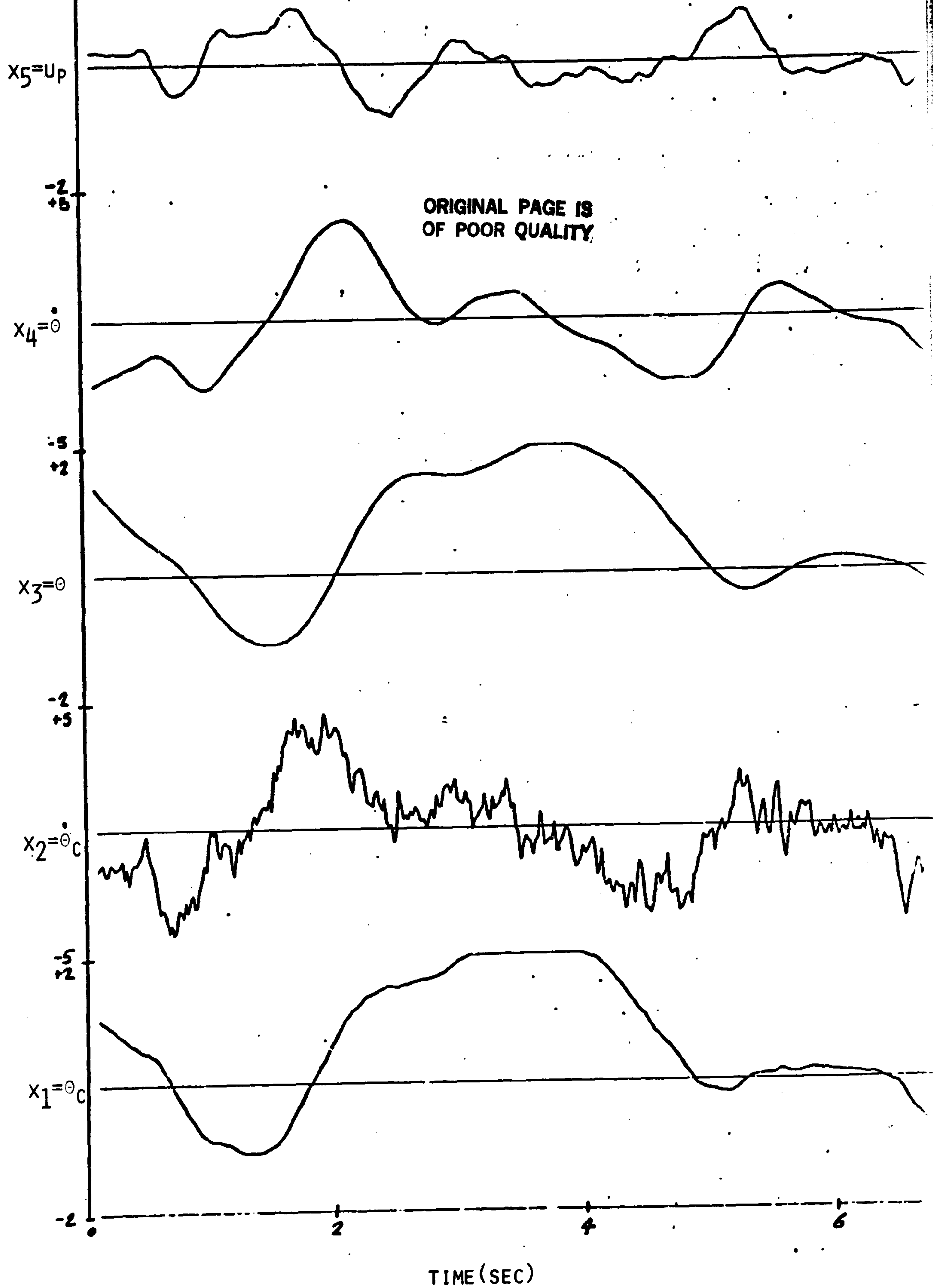


Figure 6 Time Histories

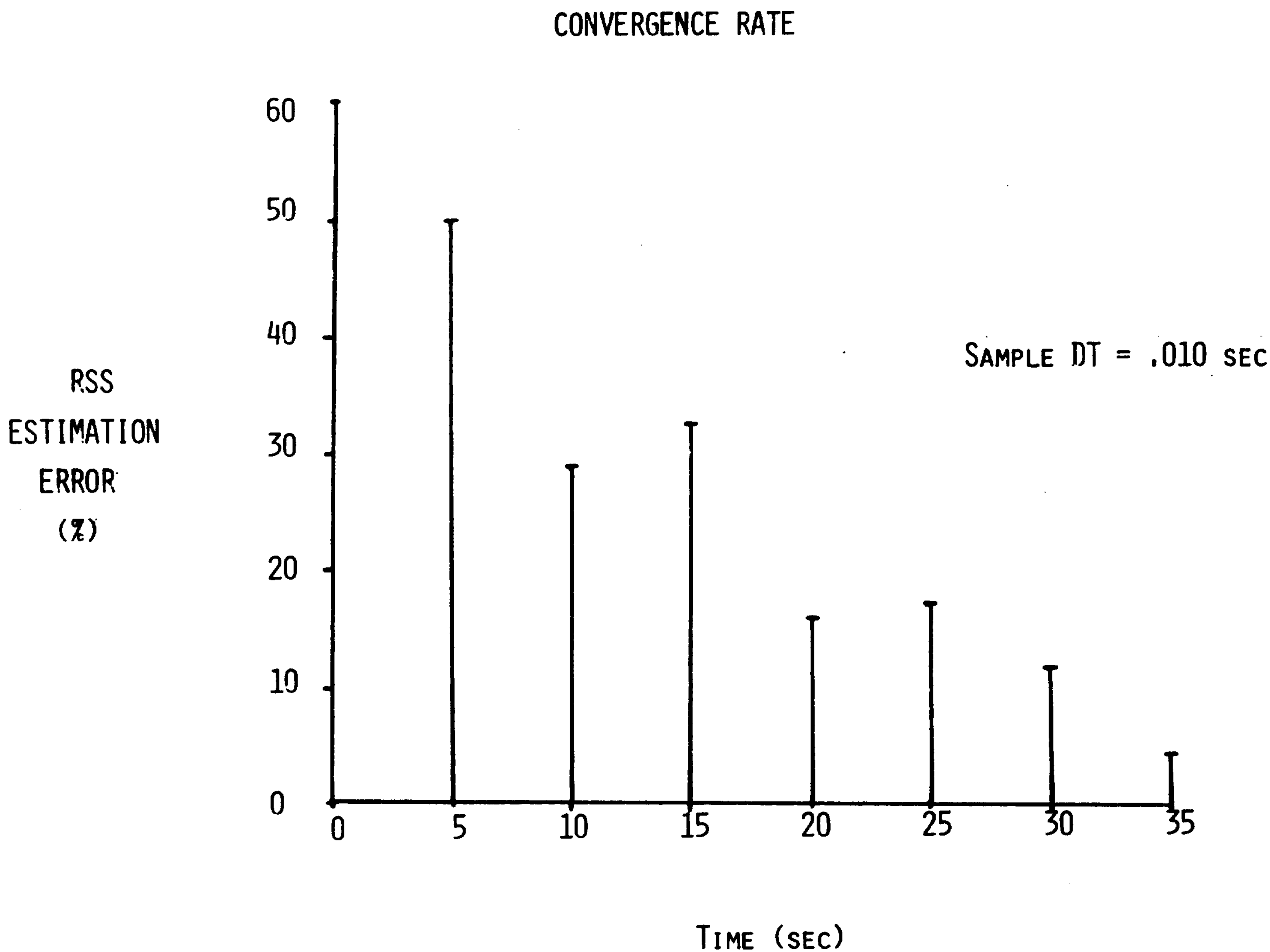

CONVERGENCE RATE
RSS
ESTIMATION
ERROR
(%)
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
SAMPLE DT = .010 SEC
TIME (SEC)

Figure 7

IMPORTANCE OF CORRECTIONS

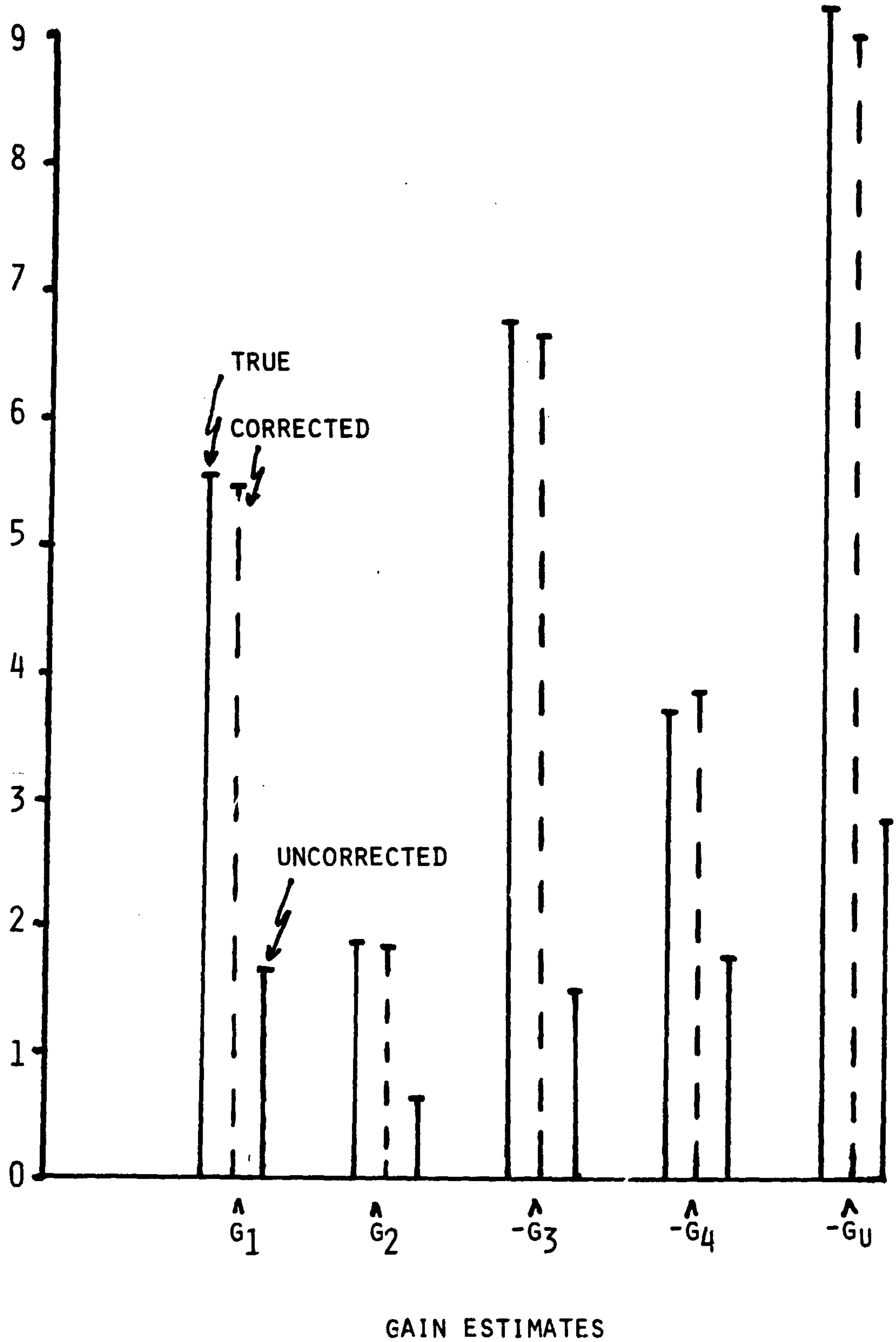


Figure 8

Conversely, the "corrected" set used perfectly corrected data, or the actual $\hat{X}$'s in the identification. Both sets of gain estimates are based of 50 seconds of data. Clearly, in this case again, the corrections are important. Further verification of the method is in process.

Inference of the Objective Function

Attention is now turned to estimation of the objective function weightings from the gain estimates just discussed. (Note, this is referred to in the control literature as the "inverse problem".) These weights are related to the gains via the Riccati matrix K, the solution of

$$\tilde{A}^T K + K\tilde{A} + \tilde{Q} - K\tilde{B}G^{-1}\tilde{B}^T K = 0$$

and

$$\left[g_x^T \,\vdots\, g_u^T \right] = -G^{-1}\tilde{B}^T K$$

where

$$\tilde{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \qquad \tilde{Q} = \begin{bmatrix} C^T Q_y C & 0 \\ 0 & R \end{bmatrix}$$

$$\tilde{B}^T = \left[0 \,\vdots\, I_u \right]$$ I_u = identity of dimension equal to u_p control vector

And recall that Q_y, R, and G are the weightings defined in Eqn (0). Now due to the structure of the OCM, we are able to reduce the above into some simpler relations. First, noting that letting $G = I_u$, without loss of generality (at least in the case of scalar control input u_p), we obtain

$$K = \begin{bmatrix} L & -g_x \\ -g_x^T & -g_u \end{bmatrix} , \quad g_u = g_u^T$$

and

$$R = g_u g_u^T + g_x^T B + B^T g_x \qquad \text{(III.a)}$$

$$0 = g_x g_u - LB + A^T g_x \qquad \text{(III.b)}$$

$$C^T Q_y C = g_x g_x^T - LA - A^T L \qquad \text{(III.c)}$$

Now L can be eliminated in the last two relations if desired, and have

$$B^T C^T Q_y CB = B^T g_x g_x^T B - H - H^T \qquad \text{(IV)}$$

where

$$H = (g_u g_x^T + g_x^T A)AB$$

By observing Equation III (and IV) we can see that if estimates of g_x and g_u are available, and plant and observation matrices A, B, and C are known, the R weighting can be obtained directly, but Q_y requires special attention. From Eqns. III.b and .c we see that if L can be obtained by solving III.b, an n x n matrix equation with only $C^T QC$ unknown results from III.c. But this is only possible if B^{-1} exists, which is only true if the number of independent control inputs (in u_p) equals the number of states (in x)-an unlikely situation.

An alternate attack using Eqn. IV leads to similar results. One could conceivably solve for a diagonal Q_y via a numerical method like Newton-Raphson, but that requires the matrix $CBB^T C^T$ to be invertable. This is possible if the number of control inputs (in u_p) equals the number of outputs (or y), (or the system transfer function matrix is square). Although this is less restrictive than the previous situation, it is also untrue in many applications of interest to us here. So the following conclusions may be stated, that in general a <u>unique</u> set of objective function weights may not be obtainable from gain estimates alone. This result is not new, we've just looked at it in the context of our specific problem.

Although improved methods are currently under investigation in this regard, we may always test assumed objective function weights to determine if they're feasible. This is considered a reasonable alternative since in an actual experiment, the analyst knows several reasonable statements for the objective function, and he may at least test them to see which one is best supported by the data. To pursue this approach, the accuracy of the gain estimate will also be developed such that statistical hypothesis tests may be performed. But for now, this is an important consideration.

In the case of our numerical example, Equation III.a leads to R = 0, and Equation IV yields

$$(11.7)^2(q_{\dot{\varepsilon}} + q_{\dot{\theta}}) = -2(11.7)g_u g_{x_3} + (11.7\ g_{x_r})^2$$

where

$$Q_y = \begin{bmatrix} q_\varepsilon & 0 & 0 & 0 \\ 0 & q_{\dot{\varepsilon}} & 0 & 0 \\ 0 & 0 & q_\theta & 0 \\ 0 & 0 & 0 & q_{\dot{\theta}} \end{bmatrix}$$

Using the estimated gains we obtain

$$(q_{\dot{\varepsilon}} + q_{\dot{\theta}}) \doteq 2.89 \qquad \text{(actually } q_{\dot{\varepsilon}} \text{ was 1.35 and } q_{\dot{\theta}} \text{ was 0)}$$

Now "guessing" that q_θ and $q_{\dot{\theta}}$ were zero, at first, we may iterate on q_ε and solve III.c, then check with III.b. If no $q_\varepsilon > 0$ led to a solution, then the assumption of q_θ and $q_{\dot{\theta}}$ equal to zero would need revision. Finally, note that from Equation III.b, we can actually solve for as many columns (and rows) of L as the number of control inputs (or rank B), and this part of the L matrix may be used to check results from III.c.

Acknowledgement

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