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COMPUTER PREDICTION OF DUAL REFLECTOR ANTENNA RADIATION PROPERTIES

by

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ABSTRACT

A new program for calculating dual reflector antenna radiation patterns has been developed adding one more option to the original program developed jointly by NCSU and NASA. The previous program was capable of computing patterns for single reflector antennas with either smooth analytic surfaces or with surfaces composed of a number of panels.

Techniques based on the geometrical optics (GO) approach are used in tracing rays over the following regions:

1) From a feed antenna to the first reflector surface (subreflector).
2) From this reflector to a larger reflector surface (main reflector).
3) From the main reflector to a mathematical plane (aperture plane) in front of the main reflector.

The equations of GO are also used to calculate the reflected field components for each ray making use of the feed radiation pattern and the parameters defining the surfaces of the two reflectors. These resulting fields form an aperture distribution which is integrated numerically to compute the radiation pattern for a specified set of angles.

Spillover, diffraction and other factors [2] that affect the accuracy of the calculation of the far-out sidelobes, are neglected.

Examples and all test cases are mentioned to support the validity of the new algorithm.

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1. INTRODUCTION

The objective of the work reported herein was to develop an algorithm to calculate the radiation patterns of Cassegrain antennas, which belong to the general group of dual reflector antennas. (See Appendix A.) The approach taken is to adopt and extend an existing algorithm which was developed for single reflector antennas.

The original algorithm for single reflector antennas was published in 1976 [1]. Later on that year this program appeared as a NCSU report [2], but in a modified version. Between 1976 and 1978 this algorithm was extended to deal with new surfaces such as ellipsoids and spheres [3]. In 1980, Botula modified the algorithm giving it the capability to analyze antennas with either smooth analytic surfaces or with surfaces composed of a number of panels [5].

The method of the electric vector potential and the geometrical optics approach were used to compute the radiation field of the antenna in question.

This thesis includes:

1) All modifications and additions inserted into the program to increase the accuracy of the calculated results for multipanel single reflector antennas;

2) The equations written to describe hyperbolic surfaces; and

3) The equations used to describe all reflections of rays from both surfaces of a Cassegrain antenna and the

intersections of these rays with the two surfaces.

FORTRAN G level was the language used in writing the algorithm. The computing time was slightly increased due to the fact that more ray tracing is involved in a dual reflector antenna case.

2. ANALYSIS AND FORMULATION

2.1 Theoretical Development

The majority of operations in this algorithm are essentially the same as those in the single reflector algorithm. The GO approach is applied to calculate the reflected electric field using the feed radiation pattern and all parameters defining the surfaces comprising a reflector antenna. The electric field is computed over a planar aperture in front of the reflector surface. As a result, an integration over the aperture plane yields the radiation patterns of the antenna in question.

To understand the line of thought and development of the new algorithm it is necessary to review some aspects of the old program and see where the new additions appear. A more refined and detailed explanation of all equations in the old algorithm is given in references [1] to [5].

Figures 2.1 and 2.2 depict the coordinate systems used in the single and dual reflector algorithms.

The first difference is that the new algorithm has the capability of analyzing both dual reflector antennas and single reflector antennas, i.e., the old algorithm became part of the new one. The two reflector surfaces are described in terms of the reference coordinate system (x, y, and z) in which most of the mathematical operations are performed. The second difference between the old and new programs lies in the types of reflector surfaces that can be analyzed. Previously, five types were available: planes, spheres, ellipsoids, paraboloids, and parabolic cylinders, whereas now hyperboloids can also be treated as another type of surface.

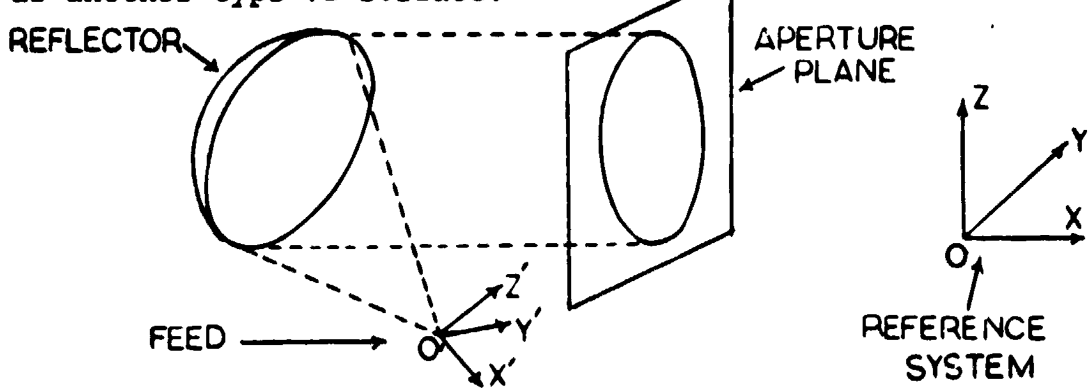

Fig. 2.1. Coordinate system for a single reflector antenna system

It should be stressed here that these six types of surfaces are available for each reflector for the case of dual reflector antennas.

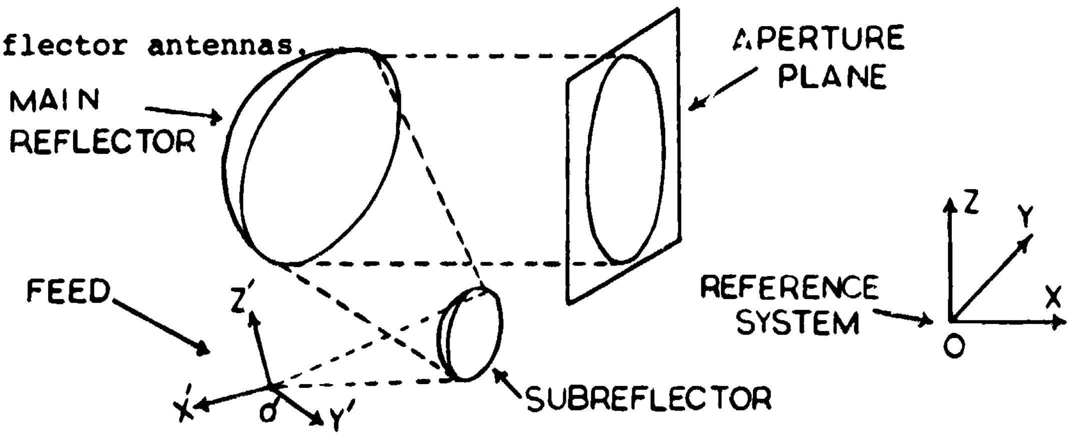

Fig. 2.2. Coordinate system for a dual reflector antenna system

Spherical coordinates are used for the radiation pattern calculations. The convention used concerning the angles θ and ϕ is shown in Figure 2.3.

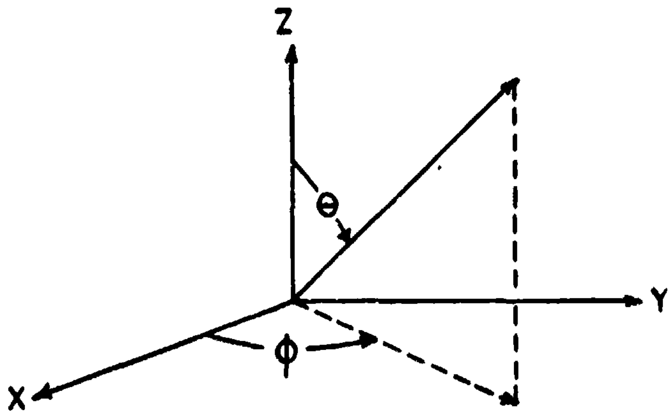

Fig. 2.3. Convention used for angles θ and φ

The feed position is expressed in terms of the primed coordinates x′, y′, and z′. The feed radiation pattern is expressed in spherical coordinates, based on the feed cartesian coordinate system using the same convention for the angles θ′ and φ′ as the reference spherical system. Here, θ′ and φ′ are referred to the feed coordinate system. The phase center of the feed antenna is the origin of its coordinate system.

The two coordinate systems are related to each other via a three-dimensional rotational matrix [A], whose derivation can be found in [2]. The rotational operation of this matrix is used to make the feed system parallel to the reference system, making use of the three angles ALPHA, BETA, and GAMMA as shown in Figure 2.4. All counterclockwise rotations are defined as positive when looking in the negative direction along the axis of rotation.

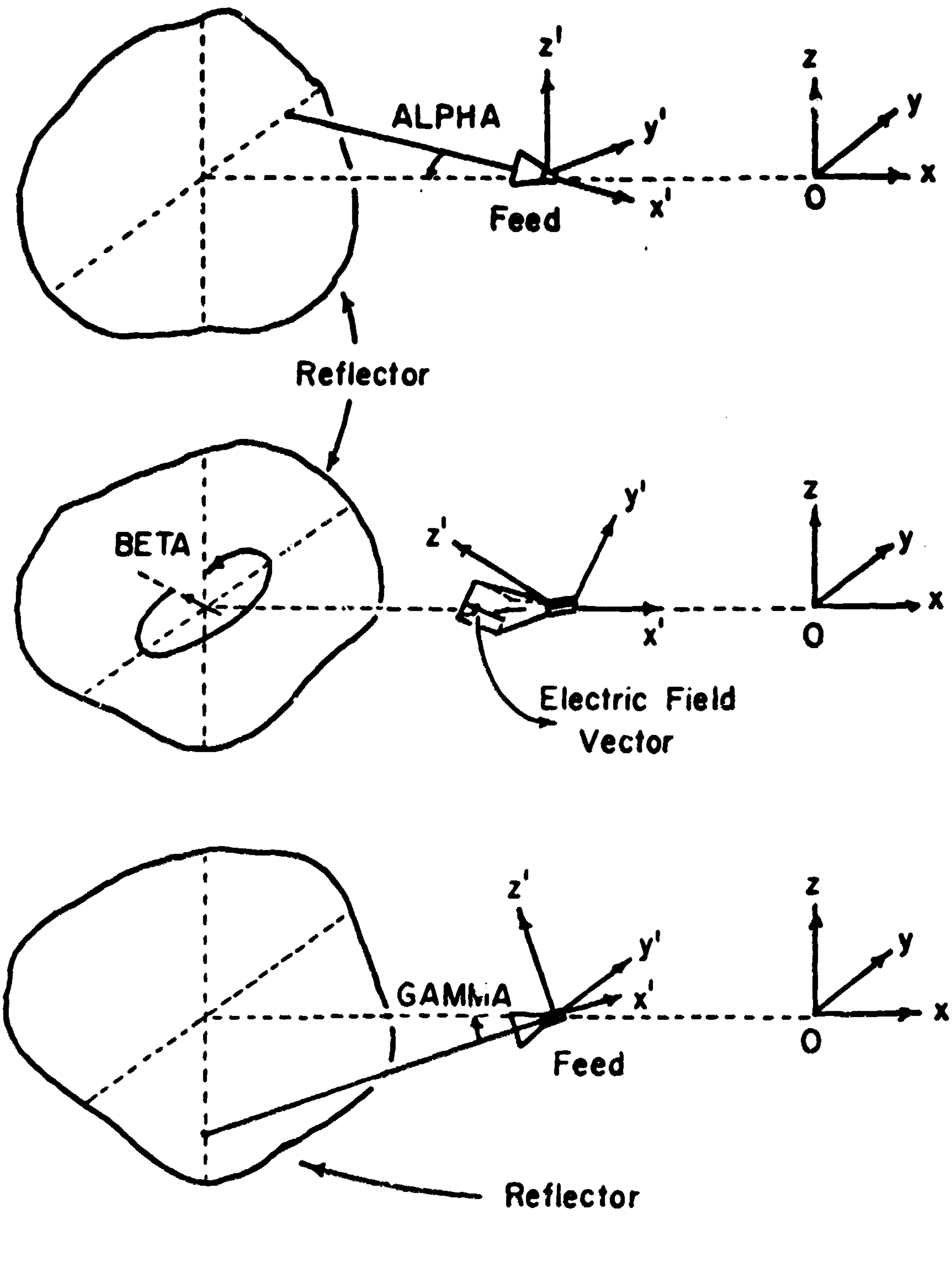

Fig. 2.4. Feed rotation angles

ALPHA is the rotation about the z´-axis, BETA is the rotation about the x´-axis and GAMMA is the rotation about the y´ -axis.

Each ray starts from the feed and is traced up to the aperture plane. Five pieces of information are associated with each ray: a set of angles θ´ and ϕ´, the appropriate θ´ and ϕ´ polarized electric field strengths and the initial phase, all taken from the feed antenna pattern. Figures 2.5 and 2.6 show all vector operations involved in ray tracing.

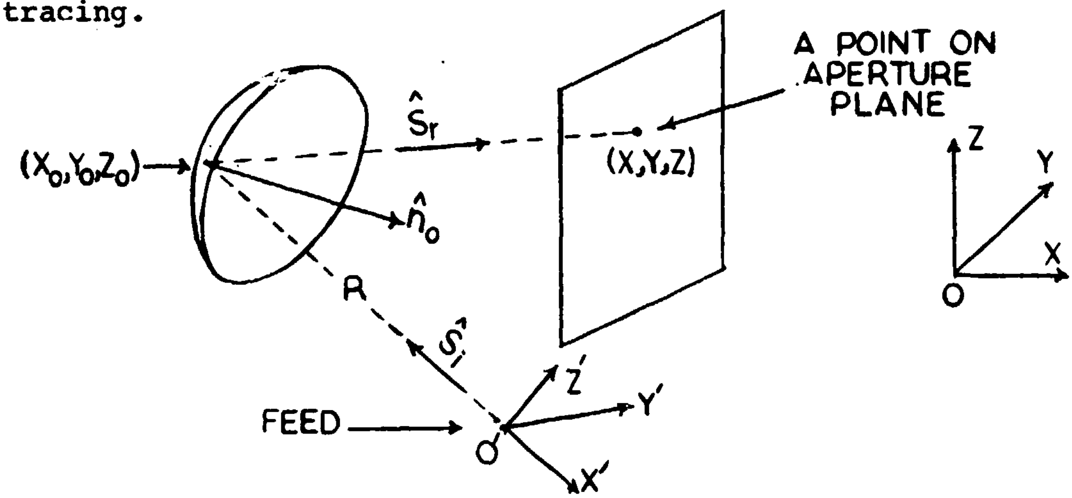

Fig. 2.5. Vector operations for a single reflector antenna

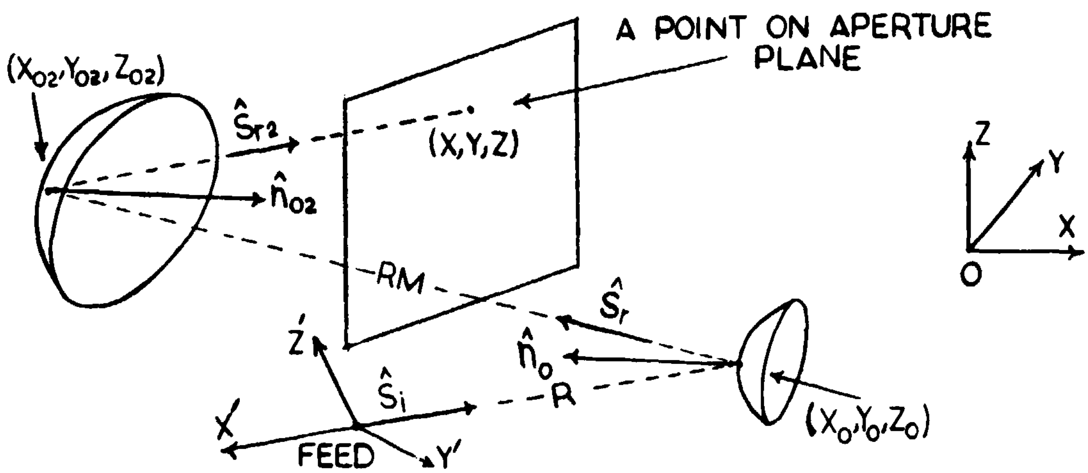

Fig. 2.6. Vector operations for a dual reflector antenna

The symbols in these figures are defined as follows:

1) $\hat{s}_i$ is a unit vector in the direction of an arbitrary ray incident on the reflector (or on the subreflector).

2) R is the distance from the phase center of the feed to the point at which the incident ray strikes the reflector (or the subreflector).

3) $\hat{n}_o$ is the unit normal vector to the reflector surface (or the subreflector).

4) $\hat{s}_r$ is a vector in the direction of the reflected ray, (or reflected from the subreflector) and incident on the main reflector in the case of a dual reflector antenna.

5) RM is the distance from (x_0, y_0, z_0) on the subreflector to (x_{02}, y_{02}, z_{02}) on the main reflector, i.e., the distance from a point on the subreflector to a point at which the reflected ray strikes the main reflector.

6) $\hat{s}_{r2}$ is a vector in the direction of the ray reflected by the main reflector.

7) D is the distance from the point of reflection (x_0, y_0, z_0) to the aperture plane for a single reflector or from the point (x_{02}, y_{02}, z_{02}) on the main reflector to the aperture plane for the dual reflector case.

The unit vector $\hat{s}_i$ which is expressed in spherical feed coordinates is written in its cartesian coordinate system as:

$$\hat{s}_i = s'_x \hat{x} + s'_y \hat{y} + s'_z \hat{z}$$

where

$$s'_x = \sin\theta' \cos\phi'$$

$$s'_y = \sin\theta' \sin\phi' \text{ and}$$

$$s'_z = \cos\theta'$$

θ' and ϕ' are also expressed in terms of the feed cartesian coordinates. The feed system is not only rotated but translated with respect to the reference system. That means that a rotation as well as a translation should be performed to express the vector $\hat{s}_i$ in the reference system. To achieve this task, the origin of the reference system must be known in the feed system.

The intersection of a ray having the unit vector $\hat{s}_i$, with the reflector or subreflector surface is defined by a vector $\vec{V}$ as shown in Figure 2.7.

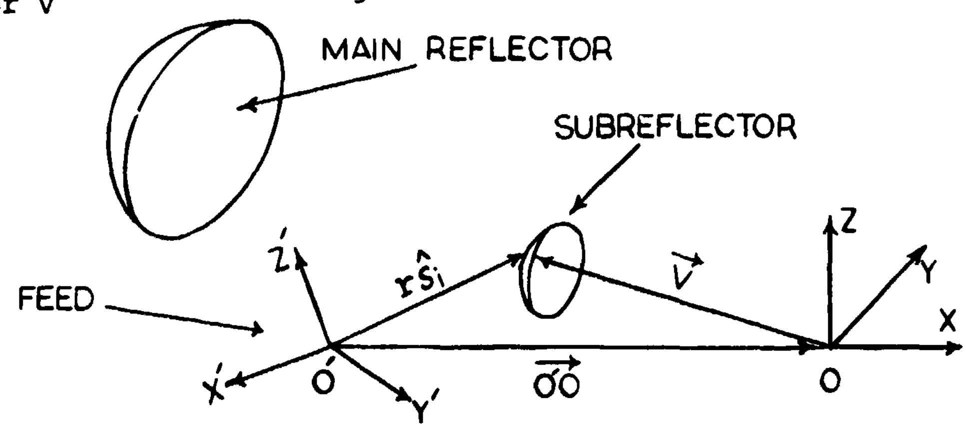

Fig. 2.7. Vector operation

Thus $\vec{V} = r\,\hat{s}_i - \overrightarrow{00}$ provided that $\hat{s}_i$ and $\overrightarrow{00}$ are expressed in the reference coordinate system. To accomplish the transformation a 3x2 matrix [BB] is formed. This matrix has the ray unit vector $(\hat{s}_i)$ and the translation vector as its columns. The rotational operation takes place by premultiplying [BB] by the rotation matrix [A].

$$[A]\ [BB] = [B]$$

Each ray is now described in the reference system by the parametric equations

$$x = B_{11}r - B_{12}$$

$$y = B_{21}r - B_{22}$$

$$z = B_{31}r - B_{32}$$

The point of intersection is found by solving simultaneously the equations mentioned above and the equation of the reflector surface. To find a vector $(\hat{s}_r)$ in the direction of the reflected ray, the unit normal to the reflector surface, at the incident point is evaluated and Snell's Law is used, i.e.

$$\hat{s}_r = \hat{s}_i - 2(\hat{n}_0 \cdot \hat{s}_i)\ \hat{n}_0$$

Similarly, the reflected field except for phase, is given by

$\vec{E}_r - 2(\hat{n}_0 \cdot \vec{E}_i)\ \hat{n}_0 - \vec{E}_i$ where $\vec{E}_i$ is the incident field, attenuated, of course, by a factor $\frac{1}{R}$, since we assume that the reflector is in the far field of the feed antenna.

All vector operations are the same for both the single and dual reflector antenna options.

The two options are now considered separately.

A) Dual Reflector System

The parametric equations for a ray along $\hat{s}_r$, which is treated now as the incident ray on the main reflector, are:

$$x = x_0 + h \cos\alpha x$$

$$y = y_0 + h \cos\alpha y$$

$$z = z_0 + h \cos\alpha z$$

where h is the distance travelled from the point (x_0, y_0, z_0) on the subreflector along the ray, and

$$\cos\alpha x = \frac{s_{rx}}{\vec{s}_r}$$

$$\cos\alpha y = \frac{s_{ry}}{\vec{s}_r} \qquad \text{direction cosines}$$

$$\cos\alpha z = \frac{s_{rz}}{\vec{s}_r}$$

and s_{rx}, s_{ry}, s_{rz} are the components of the reflected vector $\vec{s}_r$. To find the intersections of the ray and the main reflector, simultaneous solution of the above parametric equations with the equations of the surface of the main reflector is required.

The unit normal to the surface is evaluated at this point and used to compute a vector in the direction of the reflected ray, i.e.,

$$\vec{s}_{r2} = \vec{s}_{i2} - 2\,(\hat{n}_{02} \cdot \vec{s}_{i2})\,\hat{n}_{02}$$

where $\vec{s}_{i2} = \hat{s}_r$ is a unit vector incident on the main reflector, and $\hat{n}_{02}$ is the unit normal on the surface of the main reflector in cartesian components.

$$\vec{s}_{r2} = \hat{x}\left[s_{ix2} - 2n_{x02}(n_{x02} \cdot 2_{ix2} + n_{y02} \cdot 2_{iy2} + n_{z02} \cdot s_{iz2})\right]$$
$$+ \hat{y}\left[s_{iy2} - 2n_{y02}(n_{x02} \cdot 2_{ix2} + n_{y02} \cdot 2_{iy2} + n_{z02} \cdot s_{iz2})\right]$$
$$+ \hat{z}\left[s_{iz2} - 2n_{z02}(n_{x02} \cdot 2_{ix2} + n_{y02} \cdot 2_{iy2} - n_{z02} \cdot s_{iz2})\right]$$

where

$$s_{ix2} = s_{rx}$$

$$s_{iy2} = s_{ry}$$

$$s_{iz2} = s_{rz}$$

are the components of the ray vector reflected by the subreflector. Now if

$$s_{rx2} = s_{ix2} - 2n_{x02}\ (n_{x02} \cdot 2_{ix2} + n_{y02} \cdot s_{iy2} + n_{z02} \cdot s_{iz2})$$

$$s_{ry2} = s_{iy2} - 2n_{y02}\ (n_{x02} \cdot s_{ix2} + n_{y02} \cdot s_{iy2} + n_{z02} \cdot s_{iz2})$$

$$s_{rz2} = s_{iz2} - 2n_{z02}\ (n_{x02} \cdot s_{ix2} + n_{y02} \cdot s_{iy2} + n_{z02} \cdot s_{iz2})$$

then

$$\vec{s}_{r2} = \hat{x}\ s_{rx2} + \hat{y}\ s_{ry2} + \hat{z}\ s_{rz2}$$

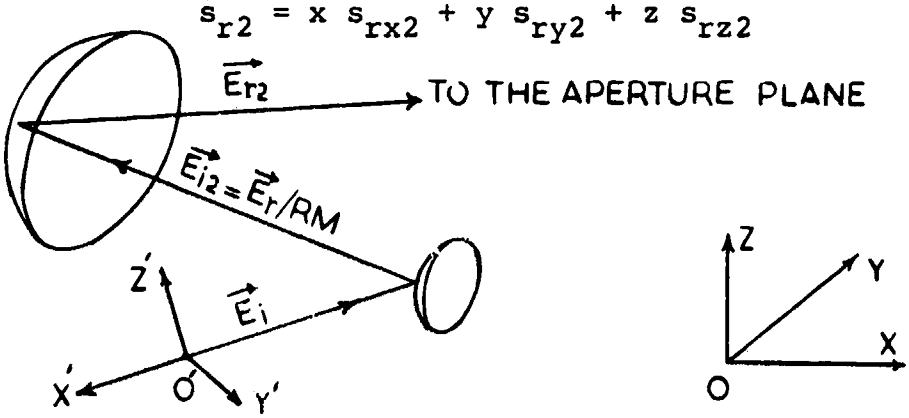

Fig. 2.8. Electric field vectors

Similarly, the reflected field (Figure 2.8), assuming a perfectly conducting reflector, is given by:

$$\vec{E}_{r2} = 2\,(\hat{n}_{02}\cdot\vec{E}_{i2})\,\hat{n}_{02} - \vec{E}_{i2}$$

$$\text{where } \vec{E}_{i2} = \frac{\vec{E}_r}{RM}\,.$$

$\vec{E}_{i2}$ is the incident electric field on the main reflector and $\vec{E}_r$ is the electric field reflected by the subreflector. It is seen here that $\vec{E}_r$ is multiplied by a factor 1/RM since the main reflector is assumed to be in the far field of the subreflector.

In component form,

$$\vec{E} = \hat{x}\,\frac{E_{rx}}{RM} + \hat{y}\,\frac{E_{ry}}{RM} + \hat{z}\,\frac{E_{rz}}{RM}$$

$$= \hat{x}\,E_{ix2} + \hat{y}\,E_{iy2} + \hat{z}\,E_{iz2}$$

and $\vec{E}_{r2}$ becomes

$$\begin{aligned}\vec{E}_{r2} = {} & \hat{x}\left[2n_{x02}(n_{x02}\cdot E_{ix2}+n_{y02}\cdot E_{iy2}+n_{z02}\cdot E_{iz2})-E_{ix2}\right] \\ & + \hat{y}\left[2n_{y02}(n_{x02}\cdot E_{ix2}+n_{y02}\cdot E_{iy2}+n_{z02}\cdot E_{iz2})-E_{iy2}\right] \\ & + \hat{z}\left[2n_{z02}(n_{x02}\cdot E_{ix2}+n_{y02}\cdot E_{iy2}+n_{z02}\cdot E_{iz2})-E_{iz2}\right]\end{aligned}$$

The procedure of finding the intersection of the reflected ray (by the main reflector) and the aperture plane is as follows:

Find the parametric equation for a line along $\vec{s}_{r2}$ given by:

$$x = x_{02} + h'\cos\alpha'x$$

$$y = y_{02} + h'\cos\alpha'y$$

$$z + z_{02} + h'\cos\alpha'z$$

where

$$\cos\alpha'x = \frac{\hat{x} \cdot \vec{s}_{r2}}{|\vec{s}_{r2}|} = \frac{s_{rx2}}{|\vec{s}_{r2}|}$$

$$\cos\alpha'y = \frac{\hat{y} \cdot \vec{s}_{r2}}{|\vec{s}_{r2}|} = \frac{s_{ry2}}{|\vec{s}_{r2}|}$$

$$\cos\alpha'z = \frac{\hat{z} \cdot \vec{s}_{r2}}{|\vec{s}_{r2}|} = \frac{s_{rz2}}{|\vec{s}_{r2}|}$$

and h' is the distance travelled along the ray. The aperture plane is at $x = x_c$, which defines $h' = \frac{x_c - x_{02}}{\cos\alpha'x}$. The (y, z) coordinates where this ray strikes the aperture plane are:

$$y = y_{02} + (x_c - x_{02}) \frac{\cos\alpha'y}{\cos\alpha'x} = y_{02} + (x_c - x_{02}) \frac{s_{ry2}}{s_{rx2}}$$

$$z = z_{02} + (x_c - x_{02}) \frac{\cos\alpha'z}{\cos\alpha'x} = z_{02} + (x_c - x_{02}) \frac{s_{rz2}}{s_{rx2}}$$

Then

$$D = \sqrt{(x_c - x_{02})^2 + (y - y_{02})^2 + (z - z_{02})^2}$$

and the phase of the field upon reaching the aperture plane is given as:

$$\psi_2 = \frac{2\pi}{\lambda}(R + RM + D) + \text{Initial Phase.}$$

Thus, five parameters are computed for each ray at a point on the aperture plane: the y and z coordinates, the y and z components of the electric field, and the phase of the field.

B) Single Reflector System

In this case each ray is traced from the feed to the reflector up to the aperture plane in the same way as before. It is clear that in this case a smaller number of equations have to be written and the phase is given by

$\psi = \frac{2\pi}{\lambda}$ (R+d) + Initial phase. A more detailed discussion of the above operation is provided by Kauffman [2].

2.2 Calculation of Radiation Patterns

In both cases, the tangent aperture field is given by:

$$\vec{E}_{AP} = (\hat{y}\ E_{ry} + \hat{z}\ E_{rz})\ e^{-j\psi} \text{ for a single reflector}$$

where E_{ry}, E_{rz} are the tangential components of the aperture electric field, or $\vec{E}_{AP} = (\hat{y}\ E_{ry2} + \hat{z}\ E_{rz2})\ e^{-j\psi_2}$ for a dual reflector.

In order to evaluate the secondary radiation pattern at a particular point in space, we integrate numerically over the aperture. The integrals to be evaluated are:

$$E_\theta = \iint_{\substack{\text{Aperture} \\ \text{Surface}}} E_{rz} \cos\phi\ e^{-j\psi} e^{jk\left[y \sin\theta \ \sin\phi + z \cos\theta\right]} d_y d_z$$

and

$$E_\phi = \iint_{\substack{\text{Aperture}\\\text{Surface}}} \left[E_{ry}\sin\theta + E_{rz}\cos\theta\ \sin\phi\right] e^{-j\psi}$$

$$e^{jk\left[y\ \sin\theta\ \sin\phi + z\ \cos\theta\right]} d_y d_z$$

where the aperture surface is the area of the reflector aperture projected on the aperture plane. It is necessary to integrate only those points which result from reflections from the actual surface and not from its mathematical extension. This is achieved by interpolating a series of edge points on the boundary, using information from points which exist outside the aperture. All points then existing outside the reflector surface are disregarded.

Before the integration takes place, all points on the aperture plane are quantized in their y-coordinate. All details on quantization and integration are fully provided by Kauffman [2], Agrawal [3], and Botula [5].

2.3 Transition from the Old Algorithm to the New One

The block diagram in Figure 2.9 shows the locations where changes, additions and modifications were applied to the old algorithm to obtain the new one.

These general additions and changes, which will be explained later in more detail, are the following:

1. NPUT: Was enlarged to read in and print out data for both reflectors for a dual reflector antenna system. This feature

did not exist before. NPUT also calls an additional subroutine, named SUBPNT.

2. SUBPNT: Was added to determine the four extreme points on the subreflector, given the four extreme points on the main reflector.

3. APRTUR: Was extended for the following reasons:

   A) to incorporate hyperboloidal surfaces, as an addition to the previous list of surfaces.

   B) To compute, automatically, the location of the aperture plane (x_c) in terms of parameters pertinent to the antenna under consideration. This is accomplished by calling the subroutine FINDXC.

4. FINDXC: FINDXC was added to provide APRTUR with an approximate value of x_c. x_c is evaluated for both reflector systems, following different approximations depending on whether the antenna is a dual or a single reflector system.

5. CASSA: A new subroutine was inserted in APRTUR to account for all the tracing from the subreflector to the main reflector, up to

the aperture plane for the case of a dual reflector system.

The rest of the program is unchanged.

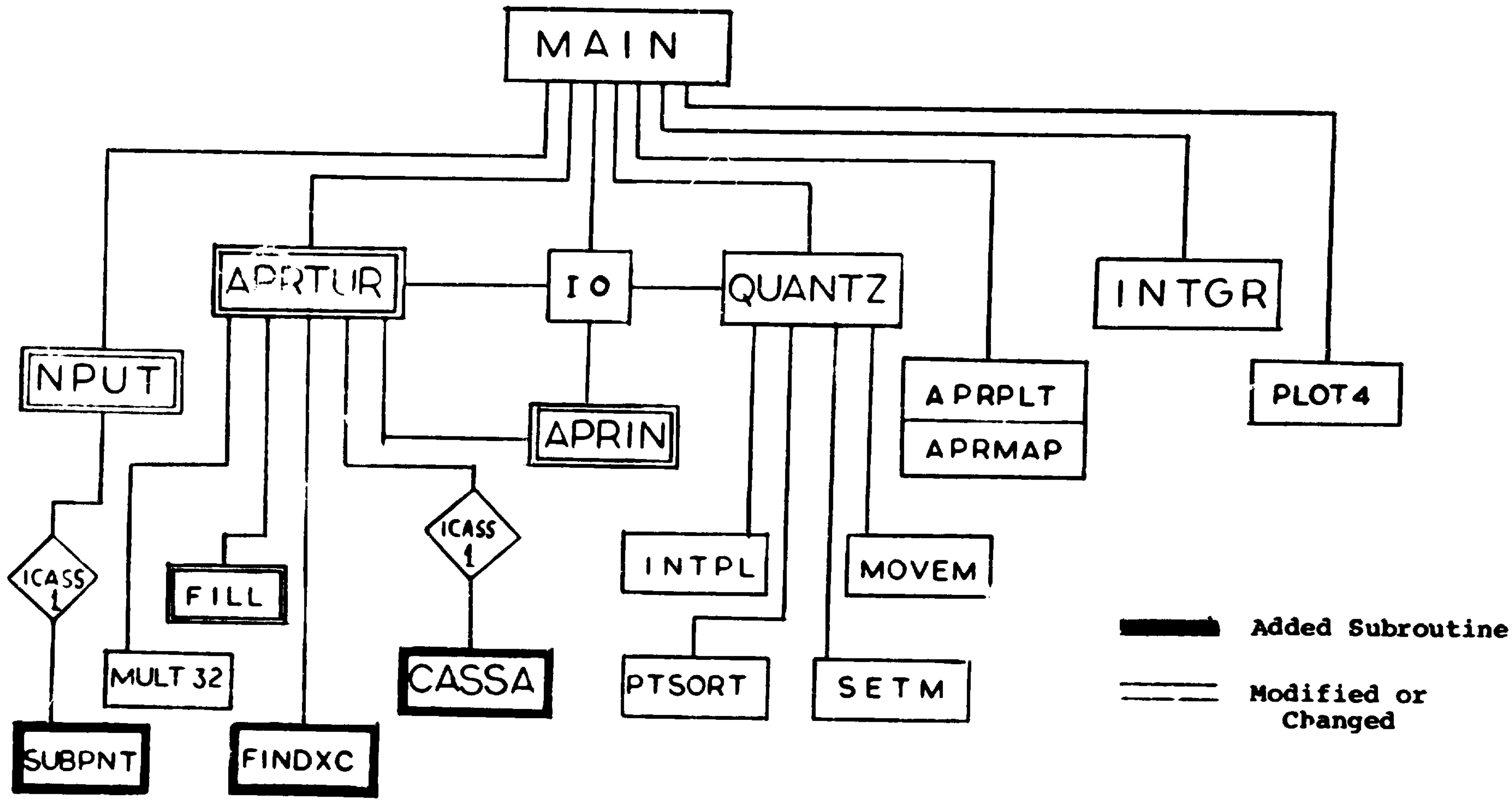

Fig. 2.9. Structure of new algorithm

3. STRUCTURE OF REFLECTR

3.1 New Variables

New variables were introduced to account for the increased complexity of the program. Some old variables and common storage blocks were changed to give the new algorithm a general character. Since the new variables come as a follow-up of the old ones, all common storage blocks and variables are introduced here.

1) BLOCKG/YCBL, ZCBL, HFMABL, HFMIBL (Aperture plane blockage information).

   YCBL, ZCBL: y and z center coordinates of the aperture plane blockage ellipse.

   HFMABL, HFMIBL:

   Half-major and half-minor axes of the aperture plane blockage ellipse.

2) CASS/SR(3), XO, YO, ZO, Y, Z, RM, D, XO2, YO2, ZO2, ER2(3), ER(3) (Only for Cassegrain antennas).

XO, YO, ZO,	A point where a ray emanating from the feed intersects the subreflector.
XO2, YO2, ZO2	A point of intersection of the main reflector and the ray.
Y, Z	The y and z coordinates of each ray on the aperture plane.
RM	Length of a ray from the subreflector to the main reflector.

D — Distance of aperture plane from main reflector.

SR(3) — A vector $\vec{s}_r$ in the direction of a ray reflected by the subreflector.

ER2(3) — The three components of the electric field reflected by the main reflector.

ER(3) — The three components of the electric field reflected by the subreflector.

3) COLOS/DELT, XC, ANGING, PM(3,4), RS, XMX, ZMX, ZMN, YMX (Parameters used for determining x_c.)

DELT — The θ' angle subtended by the subreflector. (See Figure 2.2.)

ANGING — Angular increment. (See Botula [5] for more details.

PM(3,4) — Four extreme points on the main reflector.

RS — Distance from an extreme point on the subreflector to the origin.

XMX, YMX, ZMX — A point on the subreflector which is the closest point to the origin.

ZMN — The minimum Z coordinate of the subreflector.

4) CONTRL/NOPT(3), NLIST, IOPT, ICASS, ILIST (100)

NOPT(3) — Three number specifying options regarding printer, plotter, and aperture plane, data output, respectively. (See [5] Section 6.)

NLIST — The number of panels for which the algorithm will print complete illumination and quantizing data.

IOPT — A variable which is zero when the program is to run normally, and one when the single-panel option is in effect.

ICASS — A variable which is one if a Cassegrain antenna is to be analyzed, and zero for a single reflector antenna.

ILIST(100) — The specific panels for which the algorithm is to provide complete illumination and quantizing data. (See Botula [5], Section 4.)

5) The common blocks: A) DIMENS, B) EXTENT, C) MATH and D) PATTRN, have remained the same as in [5].

6) FEED/EP(91), ET(91), NP, NT, XS, YS, ZS.

(Feed antenna parameters)

EP(91), ET(91)

Array containing the electric field strengths of the feed antenna in one-degree increments off-axis in the $\theta = 90^{\circ}$ and $\phi = 180^{\circ}$ planes, respectively.

NP, NT

The number of increments of phi and theta used in the illumination pattern, respectively.

XS, YS, ZS

A point on each panel which is the closest point to the origin of the reference coordinate system.

PARAMS/AORORF, BELLP, CELLP, DIST, PSI, PLNPNT (3), PLNORM (3), FEED (3), ALPHA, BETA, GAMMA, XLAM, AOROR2, BELLP2, CELLP2, PSI2, DIST2, POINT (3), NORM (3), SURFC1, NPNL, NPOINT, SURFC2. (Antenna system parameters.)

In the following, the variables that appear first are defined on the subreflector, and those that appear second are defined on the main reflector.

AORORF, AOROR2: The focal length of a paraboloidal reflector, the focal length of a parabolic cylindrical reflector, the radius of a spherical reflector, the semi-major axis of an ellipsoidal reflector along X, or half the transverse axis (x-direction) of a hyperboloidal reflector (Appendix B), depending on which surface is intended to represent the reflector.

BELLP, BELLP2: The semi-minor axes (along y and z, respectively) of an ellipsoidal reflector surface. Note that this does not define a completely arbitrary ellipsoid since the axes along y and z must be equal. For the case of a hyperboloidal reflector surface, this value represents the y semi-axis of the ellipse in the yz plane of the hyperboloid.

CELLP, CELLP2: Used only for a hyperboloid and stands for the z semi-axis of the ellipse in the yz plane of the hyperboloid.

DIST: A parameter used in translating the origin of the hyperbolic subreflector coordinate system so that it coincides with that of the main reflector. (See Appendix B.)

PLNPNT(3), POINT(3): The coordinates of a point on a planar reflector surface (x, y, z).

PLNORM(3), NORM(3): The components of a unit normal vector to a planar reflector surface (x, y, z).

FEED(3): The reference coordinate system origin as expressed in the feed coordinate system (x, y, z).

XLAM: Wavelength of the feed antenna radiation.

ALPHA, BETA, GAMMA: Rotation angles mentioned before (Figure 2.4.)

SURFC1, SURFC2: Integer variables which determine the type of reflector surface. (This code is applied to the subreflector as well as the main reflector.)

1) Surface is a plane.

2) Surface is an ellipsoid.

3) Surface is a sphere.

4) Surface is a paraboloid.

5) Surface is a parabolic cylinder.

6) Surface is a hyperboloid.

NPNL: Determines the number of panels the reflector is made of. The value of one means that a list of perimeter points and other surface parameters for each panel must be supplied. In this case, the aperture boundary is approximated by a polygon. The value of zero means that the single-panel option is in effect and hence an ellipse is used to represent the boundary of that panel.

NPOINT: The number of rays stored for processing in the P array at any given time.

3.2 NPUT

This is an input/output routine. If ICASS = 0, the program is to analyze a single reflector antenna system with two options:

1) With IOPT = 1 for a single-panel option.

2) With IOPT = 0 for a multipanel option.

In both cases, the four extreme points of the reflector surface are required. If ICASS = 1 a dual reflector antenna is to be analyzed. For this case, the four extreme points of the main reflector are read in and used to find the four extreme points of the subreflector by calling subroutine SUBPNT. (SUBPNT explained later in this Section.)

NPUT also reads other parameters concerning the feed. This is important since all pieces of information read here are used in conjunction with the FILL routine which is called later in the program. The connecting agent in this operation is the common storage block, named FEED.

Previously, the four extreme points on the reflector were read into the P array only when NPNL was zero, and the variable x_c was also provided by the user. In this algorithm the four extreme points are read regardless of the particular value of NPNL. The reason for this is that the above points are needed to compute the variable x_c later in the program. Furthermore, new printing statements were added to be used dd for dual reflector antennas.

3.3 SUBPNT

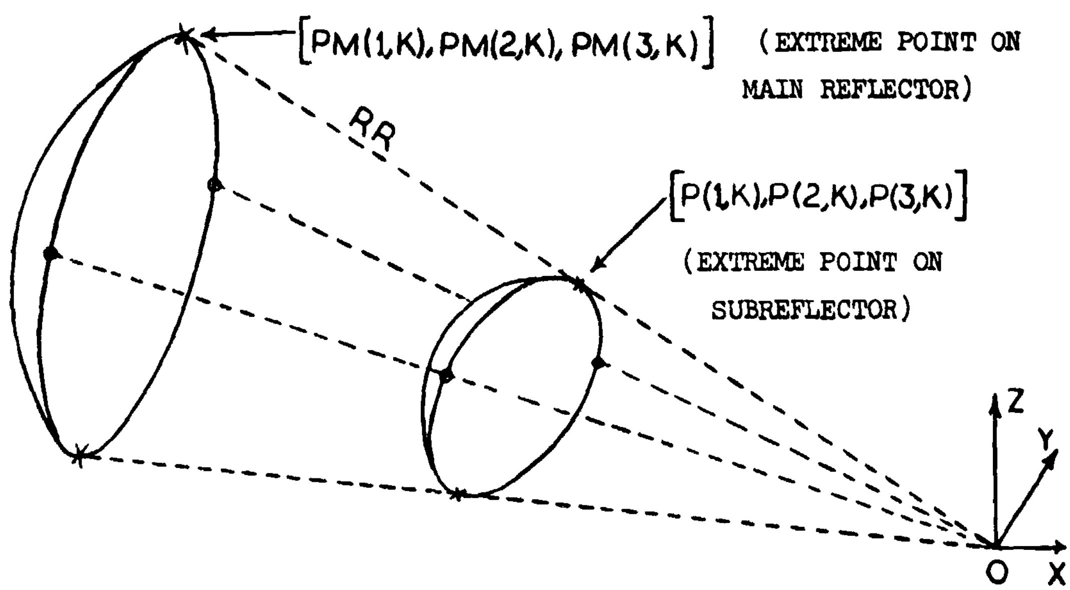

Fig. 3.1. Finding the four extreme points

SUBPNT is called only for a dual reflector antenna. There is a "Do" loop which computes the distance (RR) from the extreme point on the main reflector to the reference point.

$$RR = \left[(PM(1,K))^2+(PM(2,K))^2+(PM(3,K))^2\right]^{\frac{1}{2}}$$

where PM(1,K), PM(2,K), and PM(3,K) are the coordinates of each extreme point on the main reflector. Then, the direction cosines are found as:

DIR1 = PM(1,K)/RR (direction cosine in the x-direction)

DIR2 = PM(2,K)/RR (direction cosine in the y-direction)

DIR3 = PM(3,K)/RR (direction cosine in the z-direction)

The parametric equations of a line passing through the origin (reference point), and a point on the main reflector are given by:

P(1,K) = PM(1,K) - RR·DIR1

P(2,K) = PM(2,K) - RR·DIR2

P(3,K) = PM(3,K) - RR·DIR3

where P(1,K), P(2,K), P(3,K) is an extreme point on the reflector. To determine this point, the above parametric equations and the equation of the surface of the subreflector are solved simultaneously. (See Appendix C for details.)

This operation is repeated four times, i.e., once for each extreme point of the subreflector.

3.4 APRTUR, APRIN, AND FILL

APRTUR does all the ray tracing for the single reflector antenna and it calls a new subroutine named CASSA for additional tracing in the dual reflector case. Figure 11 shows the difference in approach between the old and new algorithms in determining the location of the aperture plane before integration for a multipanel, single reflector antenna.

This difference gives some increased accuracy in predicting the radiation pattern of a multipanel, single reflector antenna. (See results, Section 5.) In the case of a single reflector, a short "Do" loop is used to find XMX, YMX, and ZMX, a point of the reflector which is the closest one to the origin.

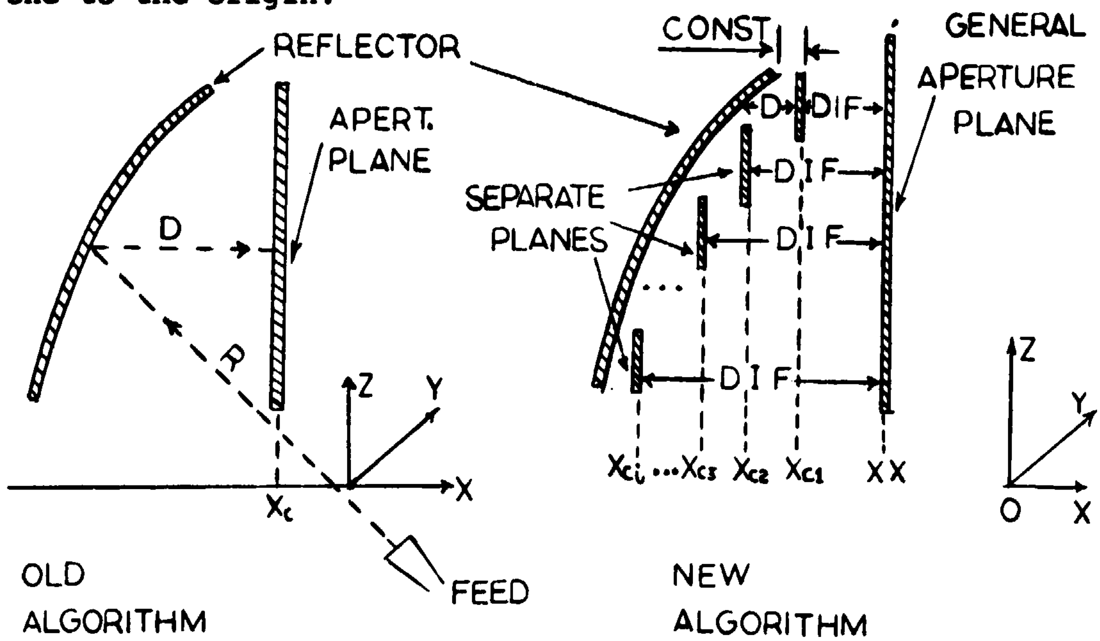

Fig. 3.2. Location of aperture plane

Then a rotation matrix A is computed from the rotation angles ALPHA, BETA, and GAMMA. The inverse of that matrix is also found. If the dual reflector option is in effect, the rotation matrix is calculated immediately skipping the above-mentioned "Do" loop. For single reflector antennas comprised of a number of panels, subroutine APRIN is called to provide data for each panel individually.

Two important additions have been made in APRIN: 1) For each plane reflector a normal is computed automatically using the principle of the CROSS product. (See Appendix D.) 2) Statements 20-28 make use of a "Do" loop to search for

(x_s, y_s, z_s), a point on each panel, which is also the closest point to the origin. It is an important point because it is used later, in APTRUR, to find the location of the aperture plane (x_{ci}) for each panel individually. (See Figure 3.2 for geometry). For a complete discussion of APRIN, see [5].

From statements 50 to 65, APRTUR finds the angles subtended by the reflector or the reflector panel. Notice that in the dual reflector case, the angles subtended by the subreflector are the ones to be measured and not those for the main reflector. All points, either the perimeter points for a panel, or the four extreme points for a single panel option, are expressed as angles in the feed system. Then, a search for the maximum and minimum θ' and ϕ' angles represented by the above-mentioned set of points is performed to determine the angles subtended by a panel or a subreflector. (See Appendix B in [5].)

ILLUMINATION ARRAY - Statements 65-95 generate the appropriate illumination array to insure a well-ordered illumination of the chosen reflector option. The previous method of illumination has been kept the same since it serves the purpose of the new algorithm in a rather convenient way. (See Section 2.3 in [5].)

For the dual reflector case, the angles subtended by the subreflector are the ones to be considered instead of those of the main reflector. The reason for this is the fact that an overillumination of the subreflector results in an overillumination of the main reflector. Overillumination is desired so that the projected boundary of the main reflector on the aperture plane can be defined before integration is performed. The rays corresponding to the upper and lower limits of θ' miss the real subreflector. They get reflected by its mathematical extension, and as a result, they miss the main reflector too.

If a Cassegrain antenna is to be studied, as soon as ANGINC is computed in APRTUR, subroutine FINDXC is called. (See Section 3.5.) This is the first time where FINDXC appears in the program to provide APRTUR with the location of the aperture plane (x_c). APRTUR, with a "Do" loop in statement 95, loads all illumination angles into the P array just after the angle pairs corresponding to the perimeter points. SUBROUTINE FILL is called to provide the angle pairs in the P array with the field strength and phase values.

FILL - This routine is changed and adjusted to each antenna whose radiation pattern is to be computed. A detailed description of this subroutine and its various forms appear in [2], [3] and [5]. A new subroutine has been written for a vertical polarization case. (See Appendix E.)

Furthermore, in APRTUR for single reflector antennas as the (x_0, y_0, z_0) point is found, the location of a separate plane are determined. This part of the algorithm is not carried out for dual reflectors. The procedure for determining xx and x_{ci} is as follows:

If the single panel option is in effect, then subroutine FINDXC is called. This is the second location in the program where FINDXC appears. (See Section 3.5.) If a multipanel option is in effect, then R_1 (Figure 12) is expressed as:

$$R_1 = \left[(x_s + B12)^2 + (y_s + B22)^2 + (z_s + B32)^2\right]^{\frac{1}{2}} - 1.0 = R' - 1.0$$

where $R' = \left[(x_s + B12)^2 + (y_s + B22)^2 + (z_s + B32)^2\right]^{\frac{1}{2}}$ is the distance between (x_s, y_s, z_s) and (B12, B22, B32).

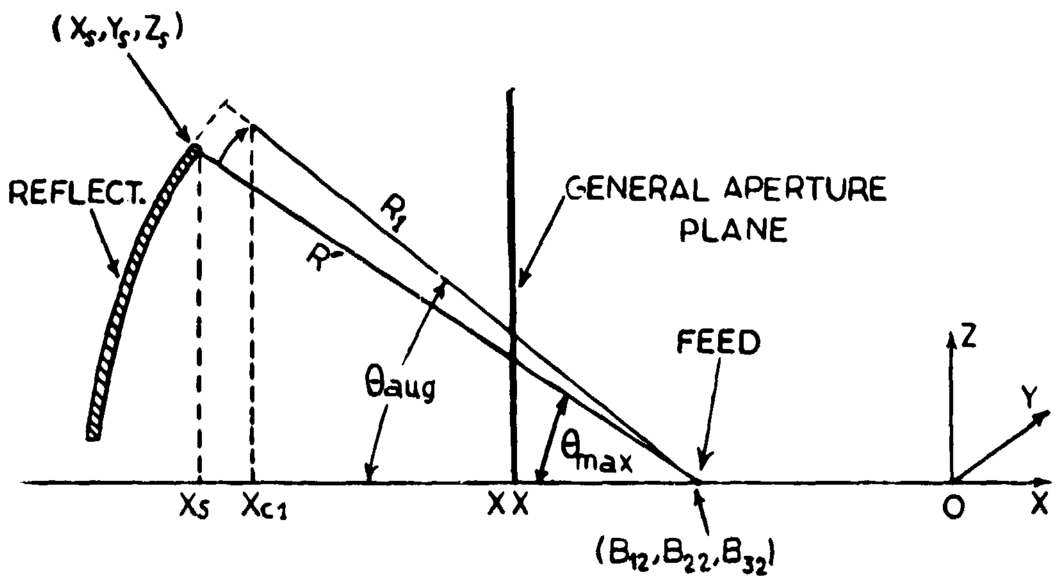

Fig. 3.3. Computation of x_c's and xx

Also, the angle θ_{max} subtended by the reflector is expressed as:

$$\theta_{max} = \tan^{-1}\left(- \frac{x_s + B32}{x_s + B12}\right)$$

, where $(x_s + B12)$ is a negative value and $(z_s + B12)$ a positive one. Hence, to obtain a positive θ_{max} angle, a negative sign is added. The angle θ_{max} is augmented by 2.5 ANGING, i.e., 2.5 times an angular increment. The reason that $R_1 = R' - 1.0$ and $\theta_{aug} = \theta_{max} + 2.5$ ANGING are used instead of R' and θ_{max}, is to make sure that the panel will be overilluminated. Thus x_c is found as:

$$x_c = -(R_1 \cos(\theta_{aug}) + B(1,2)).$$

The distance between x_c and x_s for the first panel is computed as CONST $= |x_c - x_s|$. This number becomes an important factor in locating the aperture plane for the rest of the panels. The idea is to put an aperture plane in front of every panel and with a distance equal to CONST away from it. This results in having an ordered arrangement of aperture planes in front of the reflector. So, the rest of the x_c's are given as:

$x_c = x_s +$ CONST where x_s is provided by APRIN, in advance. Once all x_c's have been found, the location of a general plane (xx) is determined, using FINDXC. (See Section 3.5.) Each panel is first projected onto its own individual aperture plane, and then phase-referenced to the general aperture plane. Thus, the general plane sums up all these projections that comprise the total projection of the antenna on the aperture plane. This method of preparation of the aperture plane before integration yields better results compared with the previous method.

The difference in phase is written as:

DIFF = $\left| x_c - x_s \right|$ and the PHASE = $\frac{2\pi}{\lambda}$ (R+D+DIF) + Initial Phase

where R = distance from the feed to reflector.

D = a distance from the reflector to the individual aperture plane.

DIF = distance from the individual aperture plane to the general one.

If the dual reflector antenna option is in effect, subroutine CASSA is called by APRTUR to continue the ray tracing operation over the region lying between the subreflector and the main reflector. (See Section 3.6.)

3.5 FINDXC

This subroutine is called, as mentioned before, at two different locations in APRTUR.

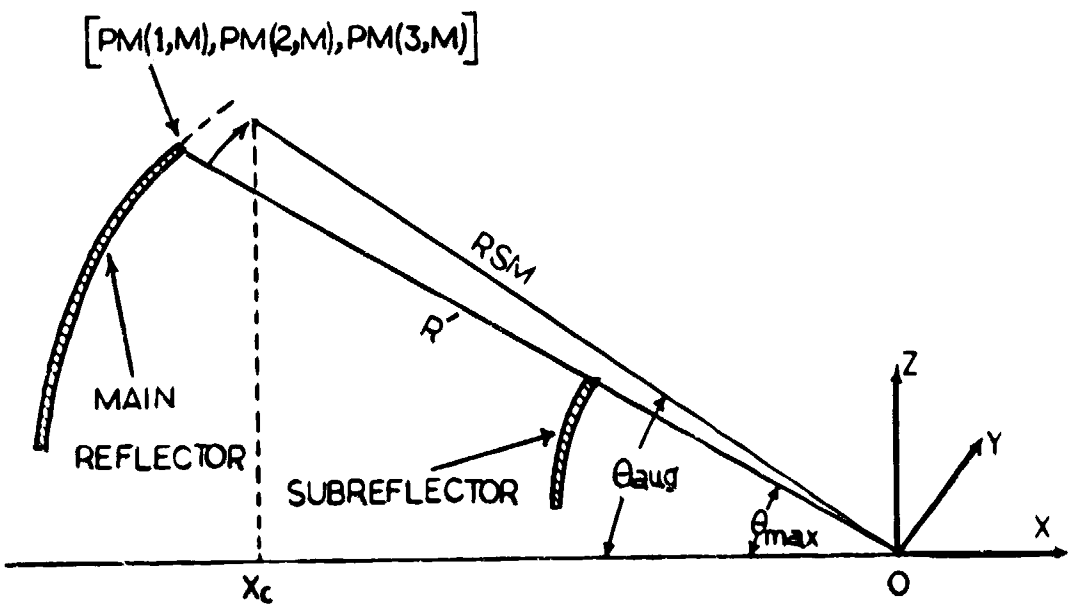

Fig. 3.4. Location of an aperture plane at x_c for a dual reflector

In the dual reflector antenna case, FINDXC is called immediately after ANGING is computed. In this case, x_c is evaluated directly from the geometry of the two reflectors. From Figure 3.4, a point with the largest z coordinate on the main reflector is determined and its distance (R') from the reference system is computed. Then, new parameter RSM is computed as:

$$RSM = \left[(PM(1,M))^2+(PM(2,M))^2+(PM(3,M))^2\right]^{1/2}-1.0 = R' - 1.0$$

where $R' = \left[(PM(1,M))^2+(PM(2,M))^2+(PM(3,M))^2\right]^{1/2}$

Also, θ_{max} the angle subtended by the main reflector is expressed :

$\theta_{max} = \tan^{-1}\left(-\frac{PM(3,M)}{PM(2,M)}\right)$ where the negative sign is provided here to obtain a positive θ_{max} angle, since PM(3,M) is positive and PM(1)M is negative. In the reference system another angle, called θ augmented is estimated as:

$\theta aug = \theta max + 3.0 \cdot ANGING$(in radians) and x_c is then calculated using the expression.

$$x_c = -RSM \cos(\theta aug.)$$

The fact that RSM is used instead of R' and θ_{aug} instead of θ_{max} is to insure overillumination and to make sure that this subroutine works for all sub and main reflector combinations, no matter what their geometrical relationships are. This subroutine could, if necessary, be changed to deal with each sub and main reflector combinations separately.

The second call of FINDXC by APRTUR is concerned with finding the location (xx) of the general aperture plane for the multipanel option reflector, or x_c for the single panel option. This task is accomplished as follows: (See Figure 3.5.)

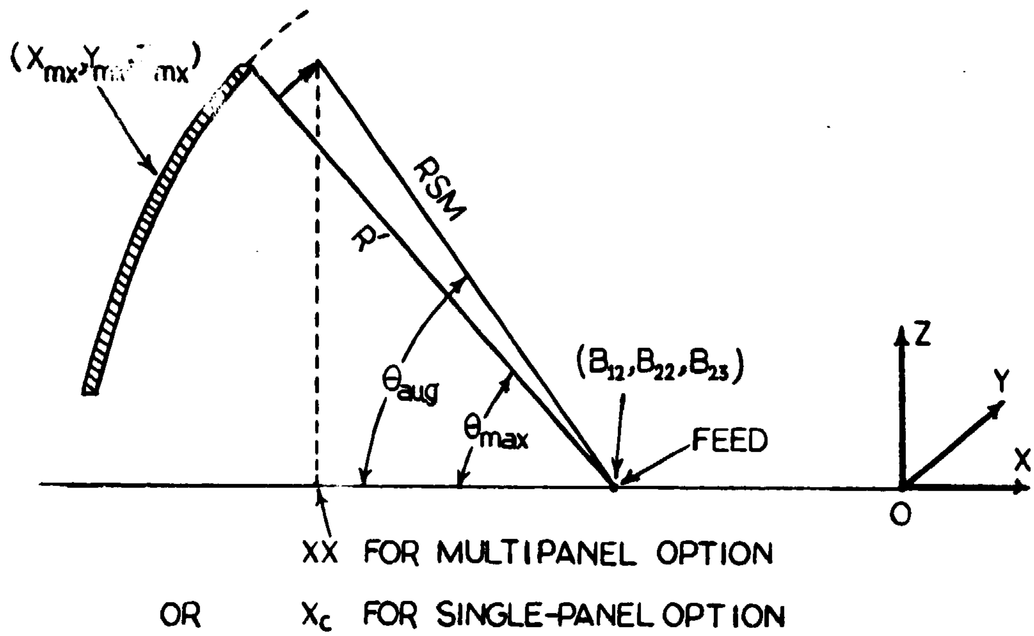

Fig. 3.5. Location of xx or x_c for multipanel or single panel antenna, respectively

First, find the distance R′ between the point (x_{mx}, y_{mx}, z_{mx}), and the feed, i.e.,

$$R' = \left[(x_{mx} + B12)^2 + (y_{mx} + B22)^2 + (z_{mx} + B32)^2\right]^{\frac{1}{2}}$$

where (x_{mx}, y_{mx}, z_{mx}) is the point on the reflector which is the closest to the origin of the reference system. It should be noted that this point is computed at the beginning of the APRTUR routine. Second,

$$\theta_{max} = \tan^{-1}\left(-\frac{(z_{mx} + B_{32})}{x_{mx} + B_{12})}\right)$$ gives the maximum angle subtended by the reflector. This angle is increased by a 2.5 ANGING to give θaug = θmax + 2.5 ANGING (in radians) and third, to find x_c, R′ is reduced by 1.5 to yield

$$RSM = \left[(x_{mx} + B_{12})^2 + (y_{mx} + B_{22})^2 + (z_{mx} + B_{32})^2\right]^{1/2} - 1.5$$

and hence

x_c or xx = - RSM cos(θaug) + B(1,2) for a single panel or a multipanel antenna, respectively.

It is noted here that the distance R′ is reduced by 1.5 instead of 1.0 (as was done in the case of individual panels) to insure that xx will be less than x_c, in the multipanel case. The whole arrangement of separate aperture places and a general one is shown in Figure 11, Part B.

It can be seen that xx has to be behind all individual aperture planes. If the multipanel option is not in effect, xx becomes x_c.

3.6 CASSA

This subroutine accomplishes all the ray tracing from the subreflector to the main reflector up to the aperture plane. It starts with finding the direction cosines of a vector along the ray reflected by the subreflector. Parametric equations of a line are expressed as:

$$x_{02} = x_0 + RM \cdot DC(1)$$
$$y_{02} = y_0 + RM \cdot DC(2)$$
$$z_{02} = z_0 + RM \cdot DC(3)$$

where (x_{02}, y_{02}, z_{02}) is a point on the main reflector, (x_0, y_0, z_0) is a point on subreflector, RM distance between these two points and DC(1) DC(2) DC(3) are the direction cosines with respect to x, y and z axes, respectively. The solution of simultaneous equations consisting of the above parametric equations and the equation of the reflector surface yield the point x_{02}, y_{02}, z_{02}. Although this subroutine has been written to deal with six analytical surfaces, it could be extended to incorporate any other number of types of surfaces, if desired. Surfaces expressed numerically could also be added to this algorithm, especially for the dual reflector antenna option, where shaping of one or both of the reflectors is now widely used in their actual design.

Once the point x_{02}, y_{02}, z_{02} is evaluated, the normal (NHAT2(1), NHAT2(2), NHAT2(3)) on the surface at that point is computed as follows:

Let the surface be represented as $g(x, y, z) = C$.

$$\text{Then } \hat{n}_{02} = \frac{\nabla g(x_{02}, y_{02}, z_{02})}{|\nabla g|}$$

A detailed explanation of computing normals and intersections of rays with surfaces is not given in this thesis, since a complete discussion can be found in all references from [1] to [5], in their description of subroutine APRTUR. The only difference lies in the fact that the parameters used in CASSA are pertinent to the surface of the main reflector and not the subreflector.

The normal on the main reflector is used to apply Snell's law of reflection to find a vector in the direction of the reflected ray (SR2(1), SR2(2), SR2(3)). This part of the algorithm is described in Section 2.1. A point, (y, z) on the aperture plane is then computed, and passed over to APRTUR where it is stored, to be retrieved later by QUANTZ.

The principles of geometrical optics are used to determine the electric field during these two phases of ray tracing. All equations in this part of the algorithm are mentioned in Section 2.1. In general, all operations taking place in CASSA are depicted in Figures 2.6 and 2.8.

3.7 Main Procedure and the Utility Routines

The main procedure and all the rest of the utility subroutines were kept the same as before with a minor change in their storage blocks. A complete development of these subroutines and the main procedure is provided by Botula in [5].

4. EXAMPLES AND TEST CASES

4.1 Introduction

Two test cases on the Cassegrain antennas are provided here to demonstrate the use of the program and support the validity of the algorithm. These cases are the following:

FIRST, a classical Cassegrain antenna which was used to check the algorithm in the case of uniform illumination, but with no blockage.

SECOND, a dual offset reflector antenna, used to check the results obtained by this algorithm against clculated data obtained from two other algorithms.

4.2 Example and First Test Case

The classical Cassegrain antenna, shown in Figure 4.1 employs a hyperboloid for the subreflector and a paraboloid for the main reflector. One of the two foci of the hyperboloid is the real focal point of the system, and is located at the origin of the feed coordinate system; the other is a virtual focal point which is located at the focus of the paraboloid which coincides with the origin of the reference system. As a result, all rays originating from the real focus and reflected from both surfaces travel equal distances to a plane in front of the antenna. (See Figure 4.1.)

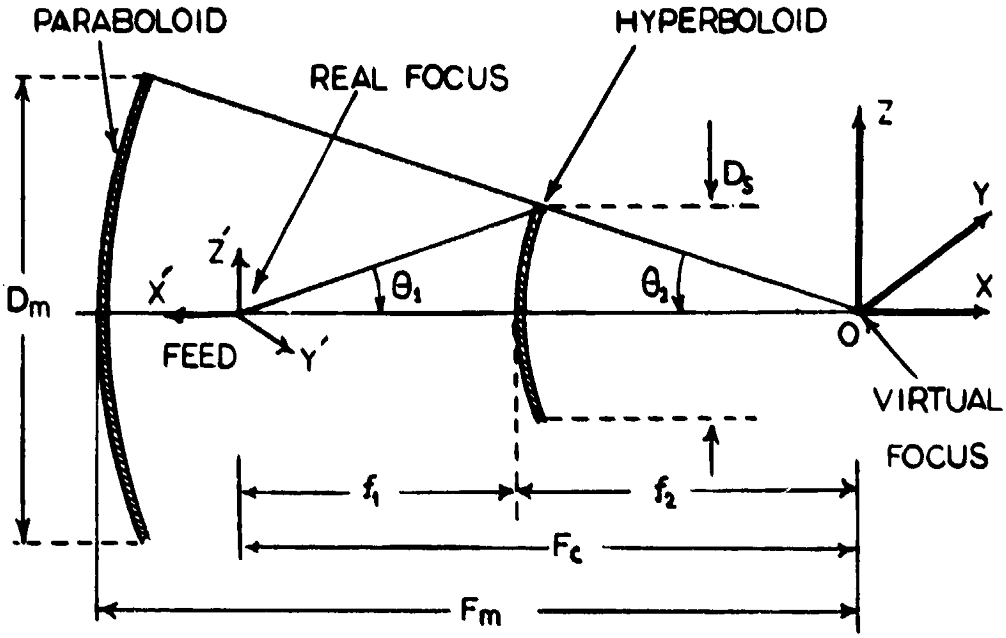

Fig. 4.1. Classical Cassegrain antenna system

Table 4.1 gives a number of parameters that define completely the geometry of the antenna system. All parameters required by NPUT will now be evaluated from this Table.

TABLE 4.1.

Main reflector focal length	(F_m)	100.0 in.
Main reflector illumination angle	(Θ_2)	60.785
Eccentricity of subreflector	(ϵ)	1.50177
Distance between two foci	(F_c)	91.0 in.
Wavelength	(λ)	4.734 in.

FEED PARAMETERS

Since the origin of the feed coordinate system is located at the real focus of the hyperboloid and at distance x = -91.0 from the origin of the reference system, the feed parameters can be given as:

1) Feed (1) = -91.0 in., Feed (2) = 0.0, Feed (3) = 0.0

2) ALPHA = 0.0, BETA = 0.0, GAMMA = -180.0

MAIN REFLECTOR PARAMETERS

SURFC2 is set equal to 4, since a paraboloidal reflector is to be used as a main reflector.

F_m = 100.0, as was given in the Table.

The four extreme points of that reflector to be read in are:

Upper point

$$r = \frac{2\ F_m}{1 + \cos\theta_{max}} = \frac{2\ F_m}{1 + \cos\theta_2} = \frac{2.(100.0)}{1 + \cos(60.785^o)} = 134.4$$

$$x = r\cos\theta_{max} = -r\cos(60.785^o) = -65.599$$

$$y = 0.0$$

$$z = r\sin\theta_{max} = r\sin(60.785^o) = 117.304$$

Lower point

$$r = \frac{2F_m}{1 + \cos\theta_{min}} = \frac{2.100.0}{1 + \cos(-60.785^o)} = 134.4$$

$$x = -r\cos\theta_{min} = -65.599$$

$$y = 0.0$$

$$z = r\sin\theta_{min} = r\sin(-60.785^o) = -117.304$$

These two points correspond to the θ' extrema in the feed system. Also, the two points representing the y - extrema are almost exactly the ϕ' extrema as well. The z coordinates of these points are identical.

$$z = \frac{z_{min} + z_{max}}{2} = \frac{117.304 - 117.304}{2} = 0.0$$

The reflector, as seen from the geometry of the antenna system, is 234.608 inches wide and symmetric with respect to the xz plane, hence $y = \pm\frac{234.608}{2} = \pm 117.304$ in. Finally, the paraboloid equation provides the x coordinates

$$x = \frac{y^2 + z^2}{4F_m} - F_m = 65.599$$

Thus, the four aperture points become:

Upper point:	(-65.599,0.0,117.304 = PM(1,1),PM(2,1),PM(3.1))
Lower point:	(-65.699,0.0,-117.304 = PM(1,2),PM(2,2),PM(3,2))
Leftmost point:	(-65.599,-117.304,0.0) = PM(1,3),PM(2,3),PM(3,3))
Rightmost point:	(-65.599,117.304,0.0) = PM(1,4),PM(2,4),PM(3,4))

It should be noted here that the diameter of the main reflector can also be found from the relationship given in Appendix A as follows:

$$\tan\frac{\theta_2}{2} = \frac{1}{4}\frac{D_m}{F_m} \to D_m = 4F_m \tan\frac{60.785^\circ}{2} = 234.608$$

SUBREFLECTOR PARAMETERS

SURFC1 is set equal to 6, since a hyperboloidal surface is to be used for a subreflector. NPNL takes the value of zero, since neither the subreflector nor the main reflector is composed of panels.

The parameters a (semi-transverse axis along x = AORORF),
b (semi-axis along the y direction = BELLP), and
c (semi-axis along z direction = CELLP)

are computed as follows: (See Appendix B for details.)

$$a = \frac{F_c}{2} = \frac{91.0}{2\ 1.50177} = 30.2976$$

$$c = b = a\sqrt{\varepsilon^2-1} = 30.2976 \quad \sqrt{(1.50177)^2-1} = 33.95$$

Also, DIST $= \frac{F_c}{2} = \frac{91.00}{2} = 45.0$ which is a parameter used in translating the origin of the subreflector coordinate system so that it coincides with that of the main reflector. (See Appendix B for details.)

There is no need to read in x_c, since this value is computed in the program as a function of the antenna system parameters. In this case, the FILL routine was not used, and no data for the E and H plane patterns of the feed were used in the input file.

4.3 General Input File

For format information, refer to the program listing, Appendix F.

A) Dual Reflector Cases

Cards	Information
1-4	Title Cards
5	Feed (1-3), ALPHA, BETA, GAMMA, XLAM
6	SURFC2, AOROR2, BELLP2, CELLP2, DIST2, PSI2
7	POINT(1-3), NORM(1-3)
8	SURFC1, NPNL, AORORF, BELLP, CELLP, DIST, PSI
9	PLNPNT(1-3), PLNORM(1-3)
10,11,12,13	Four extreme points (x_{02}, y_{02}, z_{02}), on the edge of the main reflector. One point goes on each card.
14	YCBL, ZCBL, HFMABL, HMIBL (Blockage of main reflector by subreflector)
15-N	Any data required by the FILL routine
N+1	NOPT, NLIST
N+2	MAJOR, AMAJOR, MINOR, AMINOR(1-3)(Pattern request cards)
N+3	DONE typed in the first four columns of the card

B) Single Reflector Cases

Cards	Information
1-4	Title Cards
5	Feed (1-3), ALPHA, BETA, GAMMA, XLAM
6	SURFC1, NPNL, AORORF, BELLP, CELLP, DIST, PSI
7	PLNPNT(1-3), PLNORM(1-3)
8,9,10,11	Four extreme points (x, y, z) on the reflector. One point goes on each card (single panel option only)
12	YCBL, ZCBL, HFMABL, HMIBL (Blockage of reflector fy feed)
12-N	Any data required by FILL ROUTINE
N+1	NOPT, NLIST(if NOPT specifies that only certain panels are to be printed or plotted, cards containing the list of these panels follow this card)
N+2	MAJOR, AMAJOR, MINOR, AMINOR(1-3)
N+3	DONE is typed in the first four columns of the card

These cards are followed by the panel data. The organization of the panel data is as follows:

Cards	Information
1	NPERIM, SURFC1, NPTPPL
2-(NPERIM+1)	(x, y, z) perimeter point (one perimeter point per card)
NPERIM+2	AORORF, or AORORF and BELLP, or AORORF and PSI, or PLNPNT(3), or AORORF, BELLP, CELLP, and DIST, depending on which parameters are needed to describe the surface specified by SURFC1.

All cards carrying information for individual panels appear in the main input file after the DONE card.

4.4 Development of a Uniformly Illuminated, Classical Cassegrain Antenna

All parameters needed for this case were computed in Section 4.2. None of the available FILL subroutines was used and the H and E plane patterns (for the feed) were not read in as data in this particular case. The reason for that was to insure uniform illumination over the main reflector. The procedure adopted to achieve this task was as follows:

1. FILL is not called in APERTUR.
2. All lines in APERTUR related to the amplitude and phase of the E field were moved to subroutine CASSA.
3. In subroutine CASSA the following modifications took place:

   P1 = R·RM and P2 = 0.0

$E_{Ti} = \frac{P1}{R}$ (i.e., equal to RM) and $E_{Pi} = \frac{P2}{R} = 0.0$

where E_{Ti} and E_{Pi} are the θ and ϕ electric field components of the incident (on the subreflector) ray, respectively.

From E_{Ti}, and applying Snell's law to rays reflected by the two surfaces, E_r and E_{i2} were evaluated, where E_r is the electric field vector along a ray reflected by the subreflector, and E_{i2} is the electric field vector along a ray incident on the main reflector.

It is obvious that in the far field, $E_{i2} = \frac{E_r}{RM}$

$$\text{Also, } E_r/RM = \frac{E_{Ti}/R}{RM} = \frac{P1/R}{RM} = \frac{P1}{R \cdot RM} = \frac{R \cdot RM}{R \cdot RM} = 1.0$$

which means that the E field was kept constant at the value of one along every ray. Thus, the constant amplitude requirement for uniform illumination was met.

4. The constant phase requirement was also satisfied by the above arrangement, since the phase was set equal to:

$$\text{PHASE} = \frac{2\pi}{\lambda}(R+RM+D).$$

Notice that R+RM+D is always constant for a focused Cassegrain antenna. (See Appendix A.)

Table 4.2 shows the input file for Case A. The first four cards contain title information which is also reproduced at the printout. Information about the feed coordinate system (FEED, ALPHA, BETA, GAMMA, and XLAM) appear on Card 5.

Cards 6 and 7 contain information about the surface of the main reflector. Card 6 is for SURFC2, AOROR2, BELLP2, CELLP2, DIST2, PSI2 and Card 7 is for POINT, NORM. For this main reflector, SURFC2 = 4 and AOROR2 = 100.0. None of the other parameters is required for this surface, so all are given the value of zero. Cards 8 and 9 contain information for the subreflector surface. Card 8 is for SURFC1, NPNL, AORORF, BELLP, CELLP, DIST, PSI and Card 9 for PLNPNT and PLNORM. For that type of subreflector surface SURFC1 = 6, NPNL - 0.0, AORORF = 30.2976, BELLP = 33.95, CELLP = 33.95, and DIST = 45.500. The rest of the other parameters are given the value zero, since none of them is required for this surface. Cards 10, 11, 12 and 13 contain the four extreme points (x02, Y02, Z02) of the main reflector. Card 14 carries the required blockage information, i.e., YCBL, ZCBL, HFMABL, and HFMIBL. In this case, aperture blockage is not considered and so all the above parameters are set equal to zero. Since the FILL routine is not used in this case and no data for the feed radiation patterns are needed, Card 15 is used to determine the output option code. Here the computer is instructed to print and plot information about the two surfaces, as follows:

NOPT(1) = 2 (print all results)

NOPT(2) = 2 (plot aperture after quantizing)

NOPT(3) = 1 (print aperture array onto a disc file at the end of QUANTZ).

NLIST is equal to zero since the antenna in question is not divided into panels. Cards 16 and 17 are the radiation pattern requests. One pattern is required in $\phi = 0^{o}$ plane for θ from 85.0^{o} to 95.0^{o} by increments of 0.5^{o}, and another one in the $\theta = 90.0^{o}$ plane for $\phi = -4.0^{o}$ to 4.0^{o} by 0.5^{o}. The next and last card (No. 18) has DONE typed in the first four columns, which signifies the end of the pattern requests and the end of the input file. The result of this check case are shown in Appendix G.

Figure 4.2 shows a comparison of the results obtained by this algorithm with those results reported by Silver for a uniformly illuminated circular aperture [6].

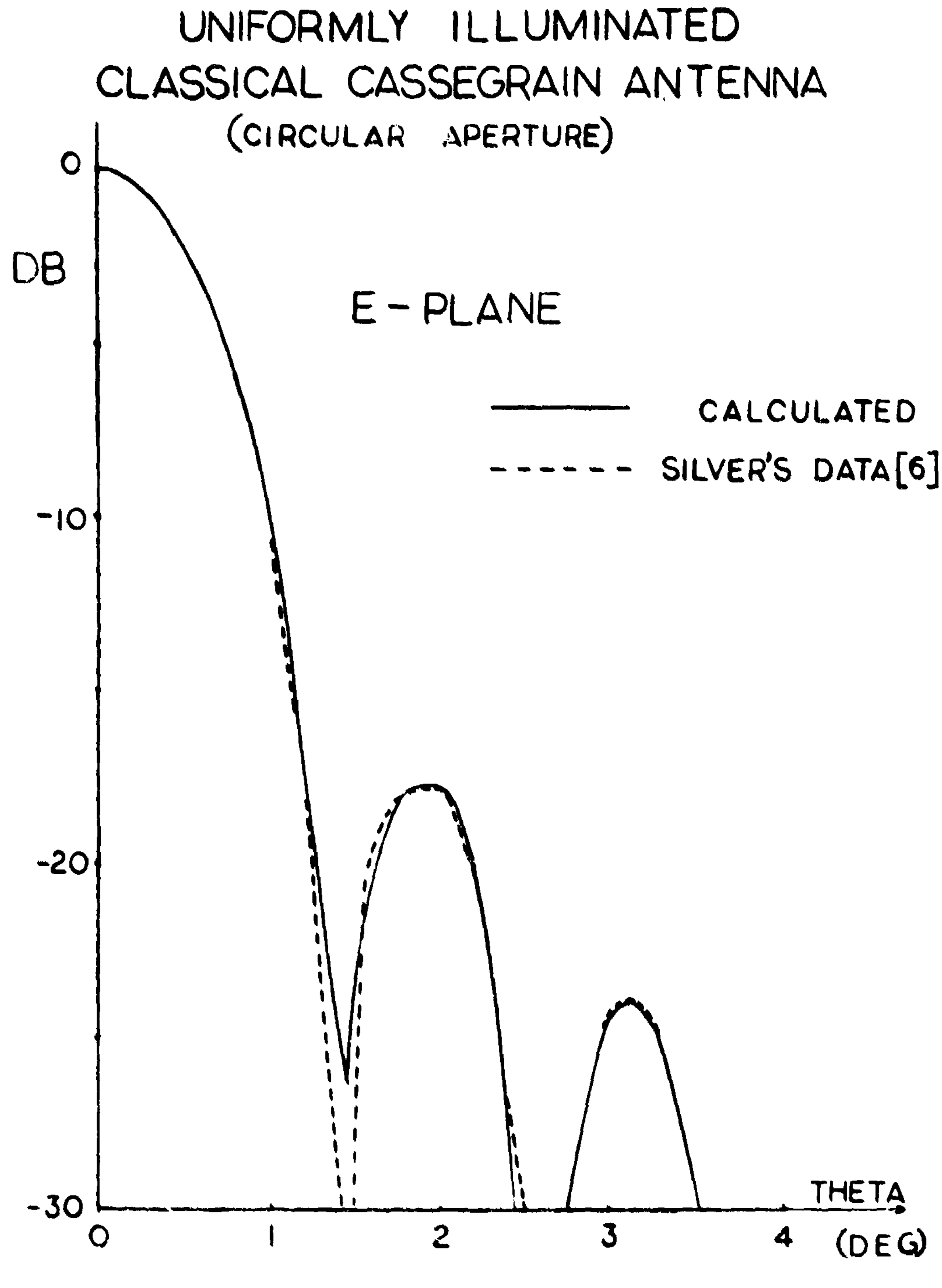

Fig. 4.2. Classical Cassegrain antenna radiation pattern (Due to the symmetry, only one-half of the pattern is shown)

TABLE 4.2

CASE A INPUT FILE

1	CASSEGRAIN ANTENNA EXAMPLE						
2	A PARABOLOID-HYPERBOLOID COMBINATION						
3	FEBRUARY 13, 1981, NCSU			PGMR:CHRISTOS FCLTY:RD-DAN PRT:HILLSBORO			
4	(A BLANK CARD)						
5	-91.005	0.0	0.0	0.0	0.0	-180.0	4.734
6	4	100.0	0.0	0.0	0.0	0.0	
7	0.0	0.0	0.0	0.0	0.0	0.0	
8	6	030.2976	33.95	33.95	45.50	0.0	
9	0.0	0.0	0.0	0.0	0.0	0.0	
10	-65.5997	-117.304	0.0				
11	-65.5997	117.304	0.0				
12	-65.5997	0.0	-117.304				
13	-65.5997	0.0	117.304				
14	0.0	0.0	0.0	0.0			
15	221						
16	PHI	0.0	THETA	85.0	95.0	0.5	
17	THETA	90.0	PHI	-4.0	4.0	0.5	
18	DONE						

4.5 Second Test Case

Dual Offset Reflector Antenna

Here, the algorithm is tested with calculated data reported by TICRA A/S [8], and C. C. Chen [9]. The reason for choosing an offset case as a second test case is the fact that offset geometry does not have the symmetry of the first test case, which can sometimes mask errors.

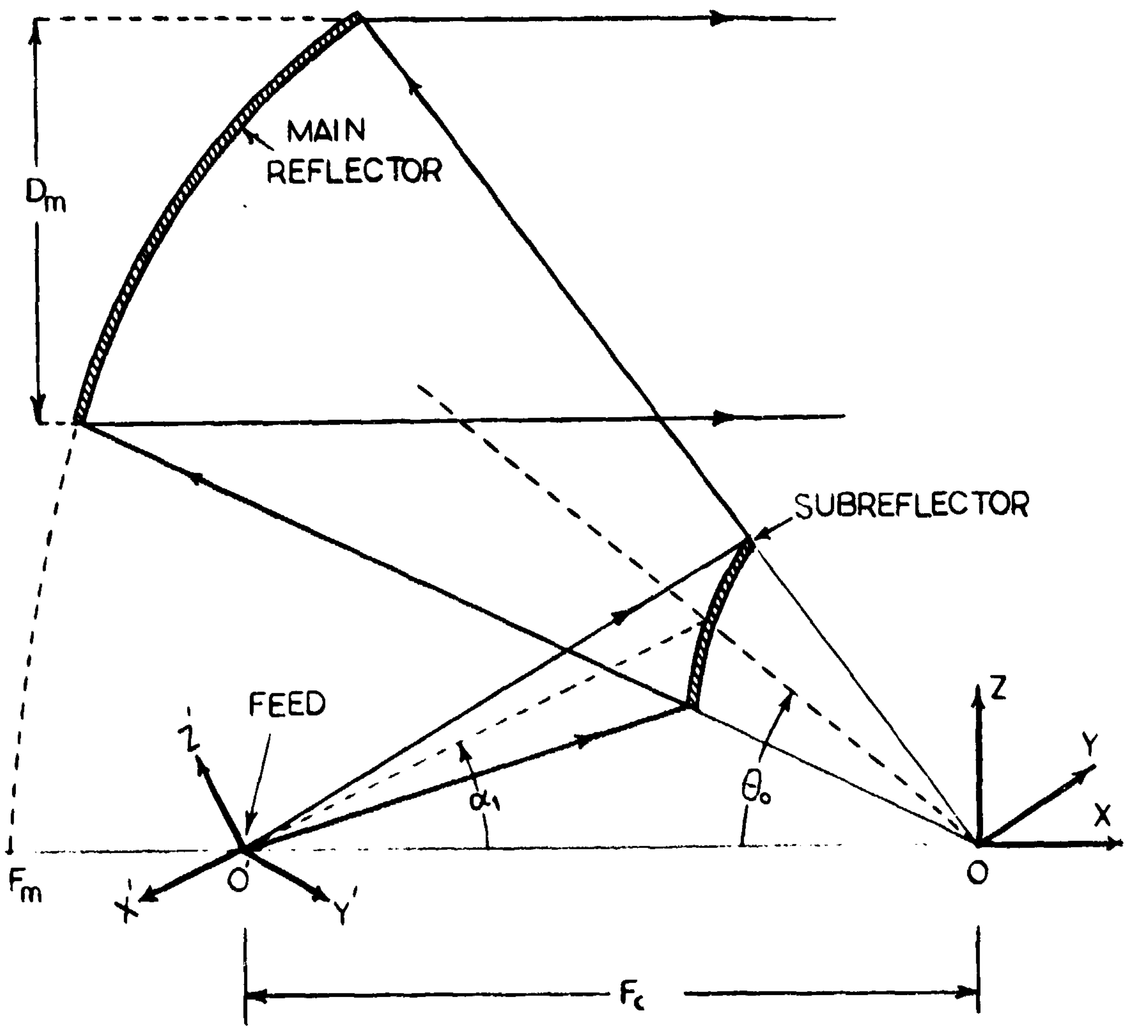

Fig.4.3. Dual offset antenna geometry

TABLE 4.3

ε	2.47
Fc	33.07 in
λ	0.98425 in
Fm	69.685 in
Offset angle (θ^o)	37.6^o
Aperture diameter (Dm)	64.8 λ
Tilted angle of feed axis (α_1)	16.4^o

-11 db taper was used.

Using the relationships between the hyperboloid and paraboloid from Appendix B, and using the given data in Table 4.3, one can estimate AORORF, BELLP, CELLP and DIST. Furthermore, in this case, ALPHA = 0.0, BETA = 0.0 and GAMMA = -163.6, since the axis of the feed makes an angle (α_1) of 14.6^0 with the x axis of the reference system, as shown in Figure 4.3. Feed (1), Feed (2), and Feed (3), as well as the four extreme points of the main reflector are easily calculated. The input file is shown in Table 4.4. In this case, the input file is arranged in the same way as before up to the fourteenth card. Cards 15 to 52 contain information about the feed radiation pattern. Card 53 contains NOPT, NLIST, and Cards 54 and 55 are used for the pattern requests. Finally, DONE is typed on Card 56. The secondary radiation pattern is shown in Figure 4.4, and compared with data obtained from the other two algorithms.

TABLE 4.4

CASE B INPUT FILE

1	OFFSET CASSEGRAIN ANTENNA EXAMPLE						
2	A PARABOLOID - HYPERBOLOID COMBINATION						
3	FEBRUARY 19, 1981 NCSU PGMR-CHRISTOS FCLTY:RD-DAN						
4	TICRA AP/S						
5	-31.725344	0.0	9.337276	0.0	0.0	-163.6	.98425
6	4	69.685	0.0	0.0	0.0	0.0	
7	0.0	0.0	0.0	0.0	0.0	0.0	
8	6	6.694507	15.119643	15.119643	16.535433	0.0	
9	0.0	0.0	0.0	0.0	0.0	0.0	
10	-57.18974	-31.88699	49.66035				
11	-57.18974	31.88699	49.66035				
12	-45.82776	0.0	81.54734				
13	-68.55171	0.0	17.77336				
14	0.0	0.0	0.0	0.0			
15	1.000000	.99053	.96266	.91793	.85878		
16	.78830	.70997	.62737	.54392	.46269		
17	.38617	.31623	.25407	.20029	.15491		
18	.11756	.08755	.06394	.04563	.03223		
19-32	.00000	.00000	.00000	.00000	.00000		
33	.00000						
34	1.0000	.99053	.96266	.91793	.85878		
35	.78830	.70997	.62737	.54392	.46269		
36	.38617	.31623	.25407	.20029	.15491		
37	.11756	.08755	.06394	.04563	.03223		

Table 4.4 (continued)

38-51	.00000	.00000	.00000	.00000	.00000	
52	.00000					
53	221					
54	PHI	0.0	THETA	87.0	93.0	0.25
55	THETA	90.0	PHI	-3.0	3.0	0.25
56	DONE					

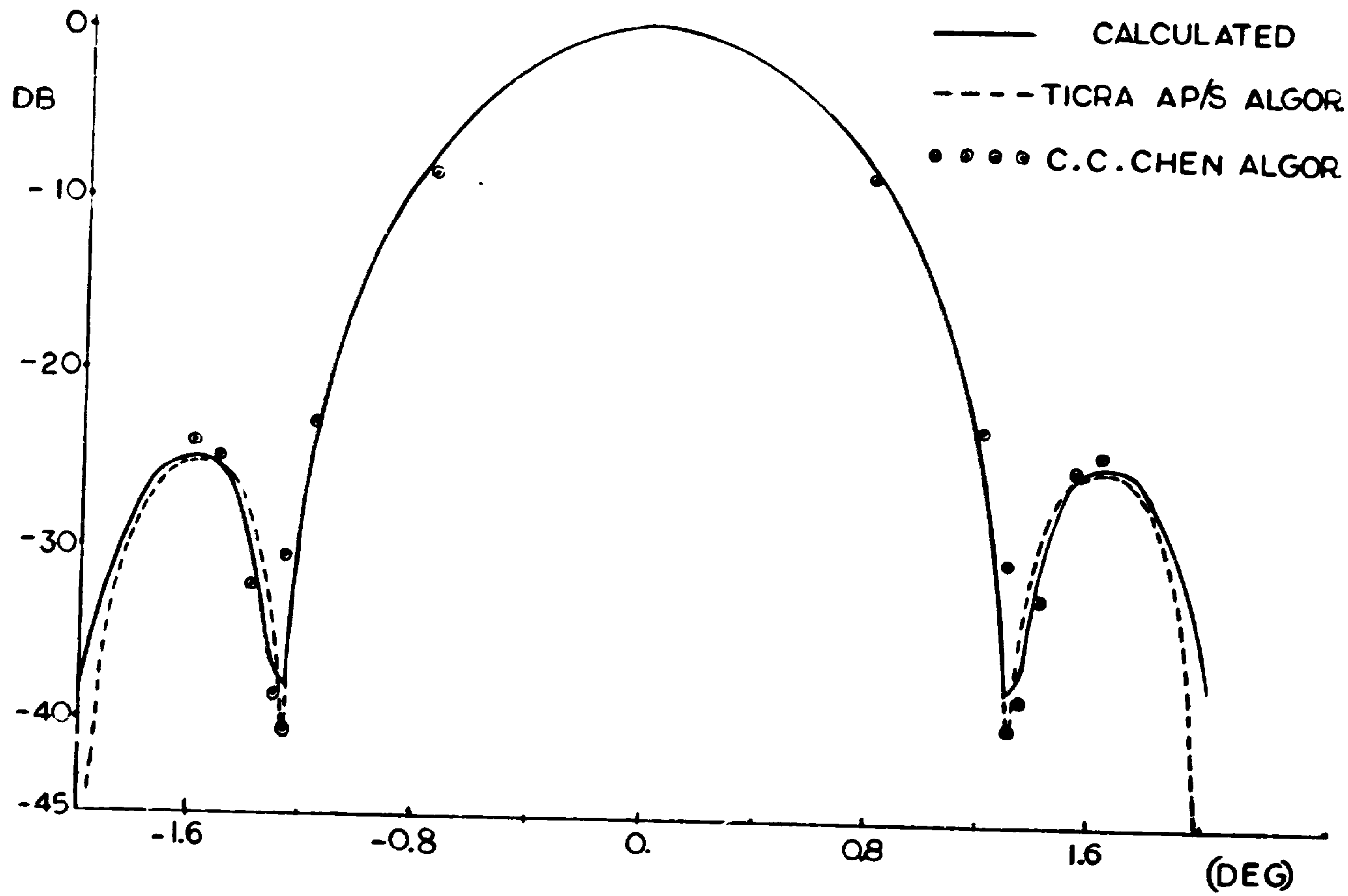

Fig. 4.4. Offset Cassegrain E-plane pattern

5. A SINGLE REFLECTOR ANTENNA EXAMPLE (A SEGMENTED SPHERICAL REFLECTOR)

5.1 Description of the Problem

A single reflector composed of 54 panels was constructed and tested by NASA at the Langley Research Center. Its measured radiation patterns were compared twice: First, with calculated results obtained by using the old version [5]; and second, with calculated results obtained via the modified version incorporated in the new algorithm. A complete description of the antenna and its parameters is provided by Botula in [5]. Here, the input file and the results only are given.

5.2 Results and Comments

Figures 5.1 and 5.2 depict the projections on all panels on the aperture plane. The result obtained by the old version is shown in Figure 5.1, whereas the result from the revised algorithm is shown in Figure 5.2.

Figures 5.3-5.6, inclusively, show the secondary radiation pattern for both versions. The reason for this discrepancy in the above results lies in the amount of overlapping between the projected panels on the aperture plane. The more the overlapping, the less accurate results are obtained compared to measured data.

The reason for this overlapping is due to the fact that the rays reflected by the perimeter points of each panel tend to diverge on their way to the aperture plane. To reduce their divergence, the aperture plane is brought closer to each panel so that the rays travel over shorter distances before they strike the aperture plane. Once this occurs, the projected panel is then phase referenced to the general aperture plane.

This procedure, which is summarized in Figure 3.2, yields less overlapping and better results than the old version.

5.3 Input File

TABLE 5.1

INPUT FILE FOR A SINGLE REFLECTOR ANTENNA

1	Faceted Spherical Reflector Test Case (LSST)						
2	Surface composed of 54 panels, three perimeter points per panel,						
3	no blockage, Feed phase center 0.5 lamda inside horn aperture, E-plane only						
4	(Blank Card)						
5	9.441	0.00	8.026	0.0	0.0	-40.0	0.3335
6	3	5424.0	0.0	0.0	0.0		
7	0.0	0.0	0.0	0.0	0.0	0.0	
8	-19.6617	0.0	13.7628				
9	-20.7015	5.0252	11.0542				
10	-22.2326	-5.0581	7.4916				
11	-23.8206	0.0	2.9285				
12	0.0	0.0	0.0	0.0			
13-50	Illumination data for FILL routine						
51	101	3					

Table 5.1 (continued)

52	1	15	51				
53	THETA	91.0	PHI	0.0	15.0	0.5	
54	PHI	0.0	THETA	81.0	100.0	0.5	
55	DONE						
56	3	1	150				
57	X1	Y1	Z1	Three points for the first panel on reflector			
58	X2	Y2	Z2				
59	X3	Y3	Z3				
60	3	1	150				
61	X1	Y1	Z1	Three points for the second panel on reflector			
62	X2	Y2	Z2				
63	X3	Y3	Z3				

64-273 This process is repeated for all 54 panels.

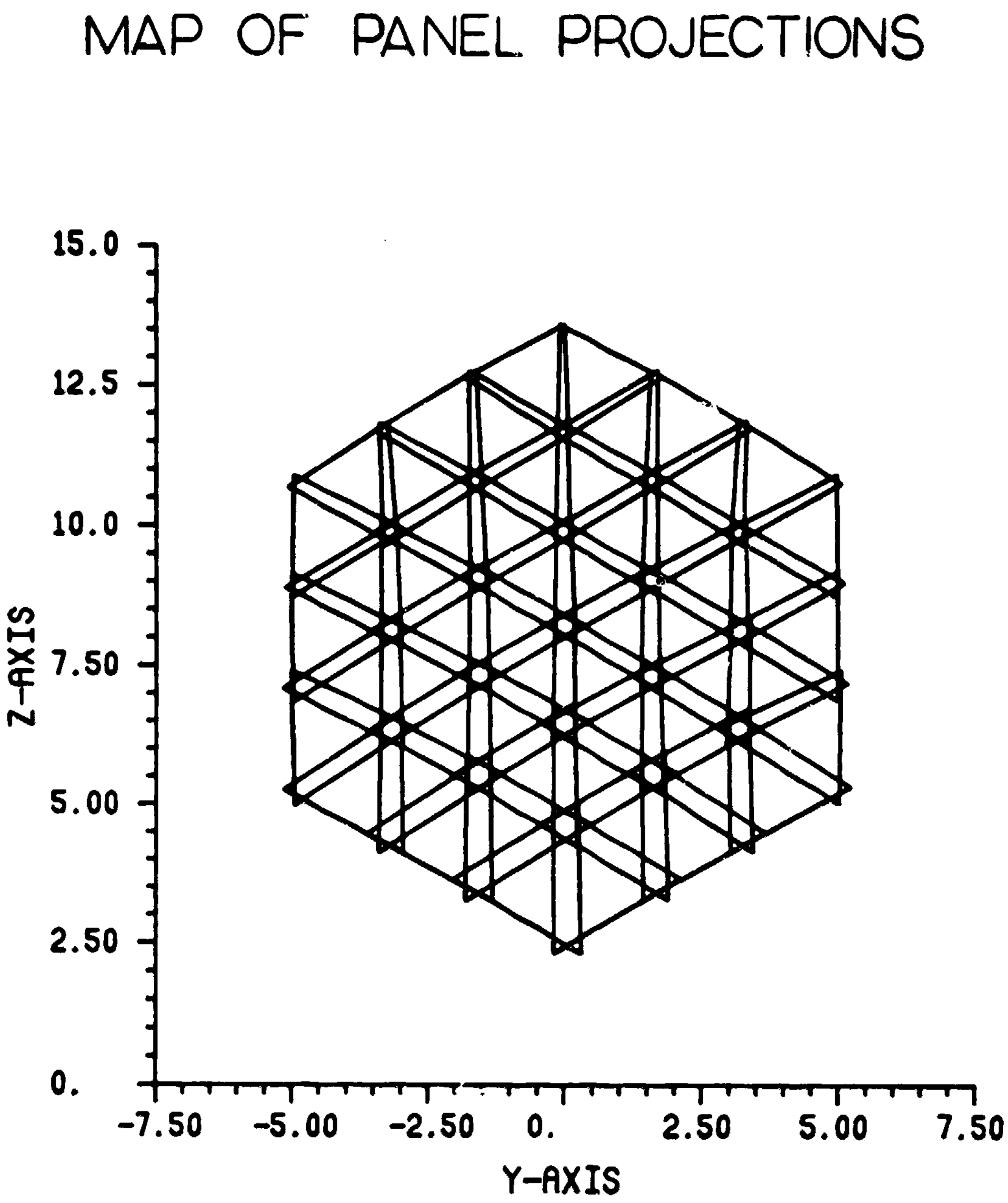

Fig. 5.1. Old algorithm

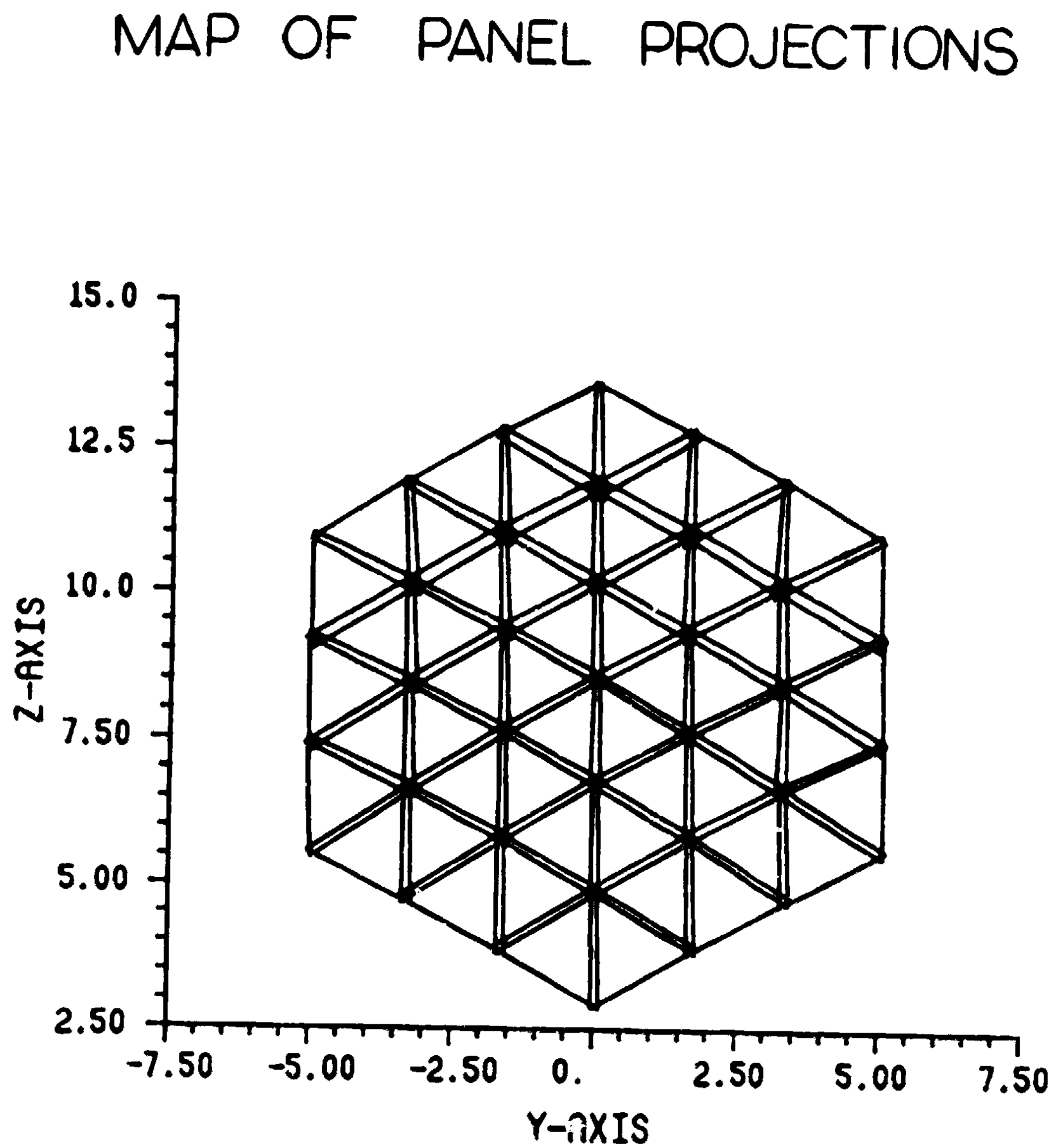

Fig. 5.2. New algorithm

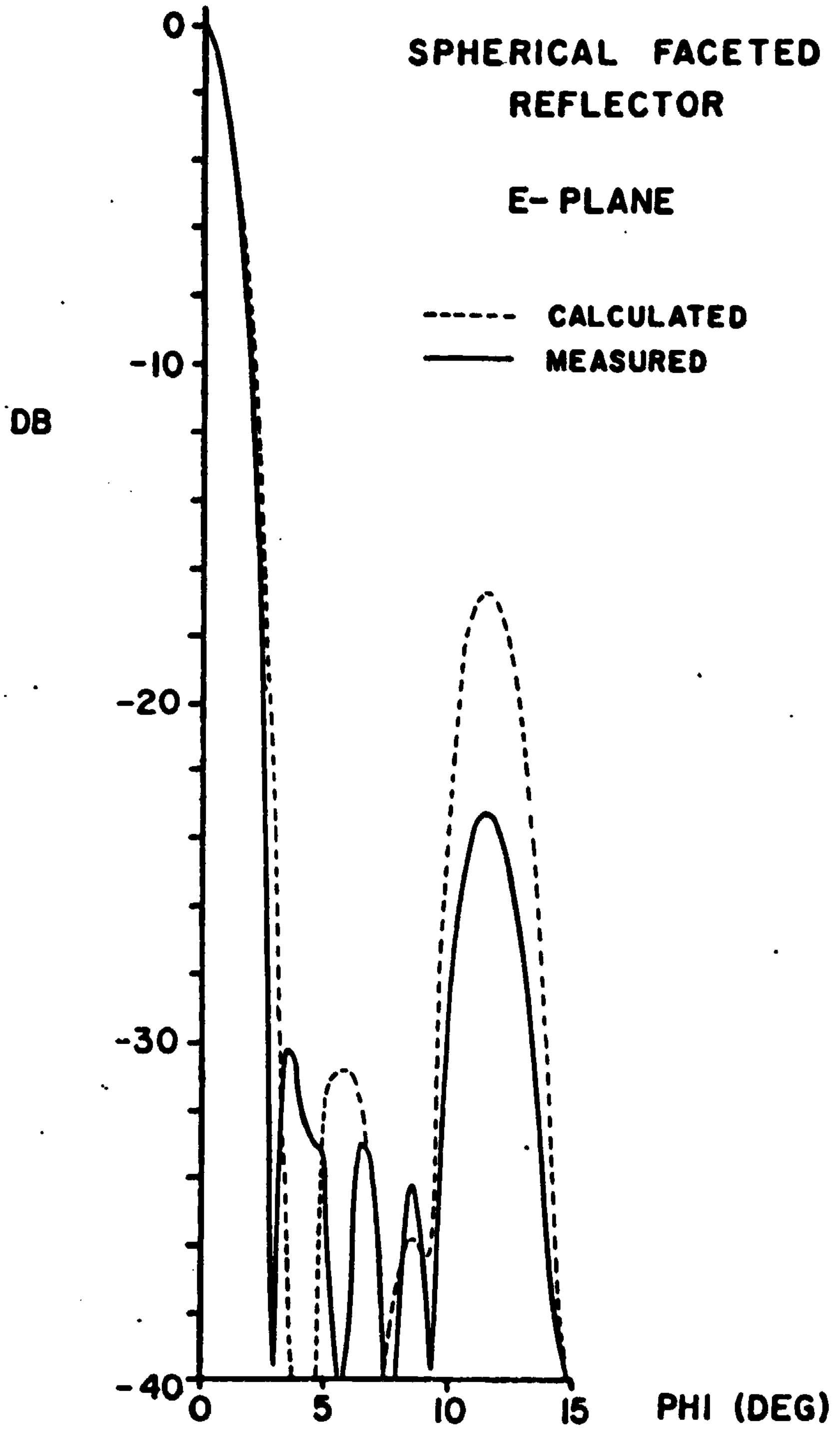

Fig. 5.3. Sphere E-plane pattern (old algorithm)

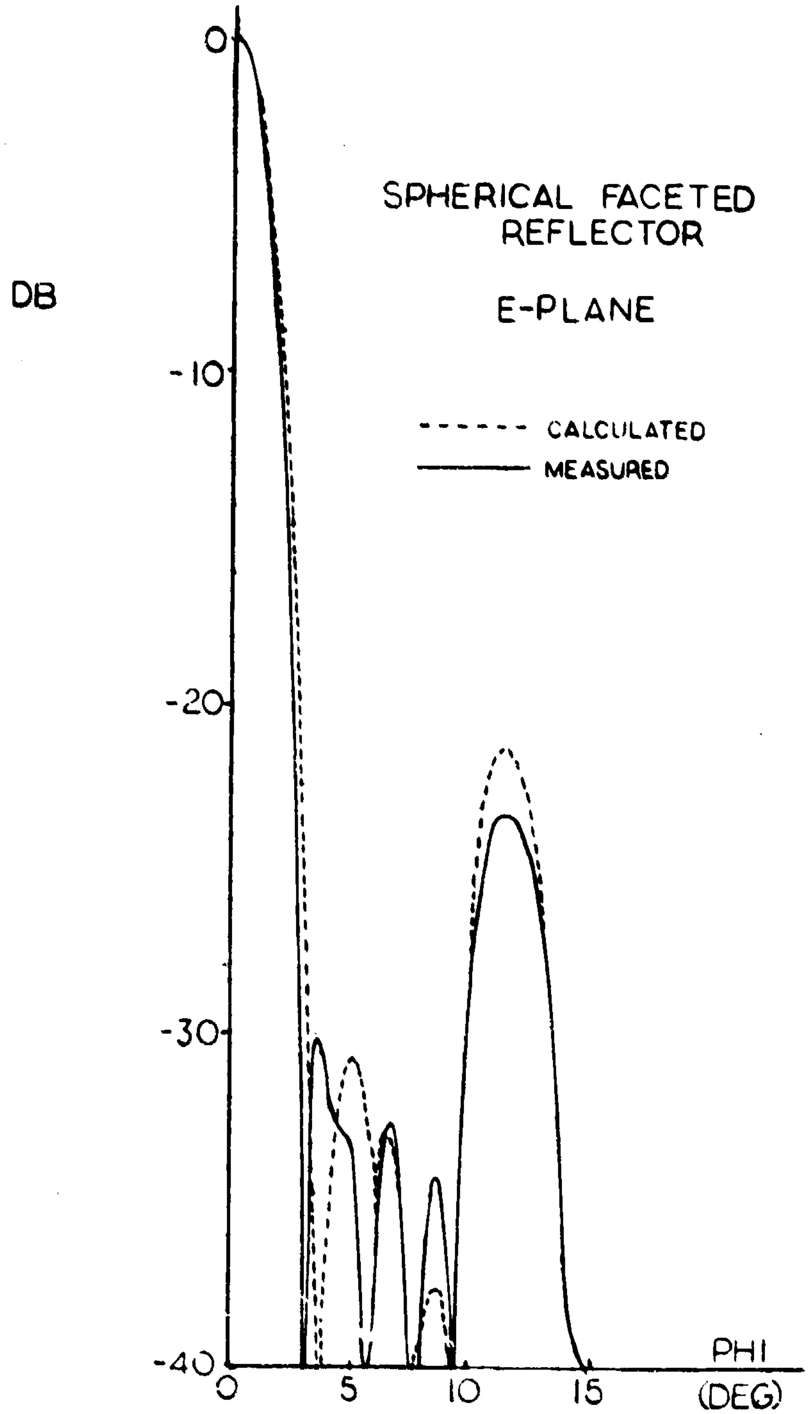

Fig. 5.4. New algorithm

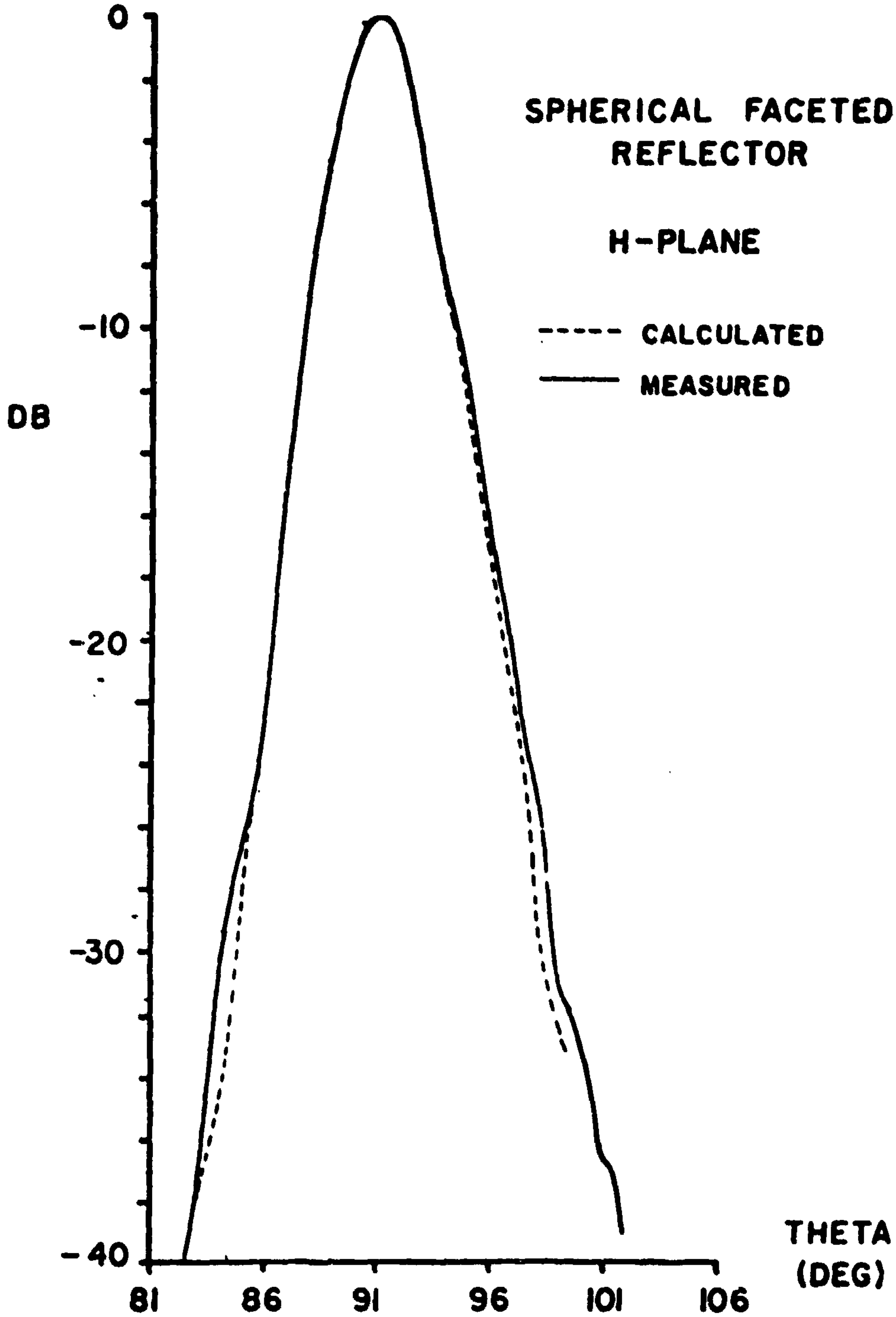

Fig. 5.5. Sphere H-plane (old algorithm)

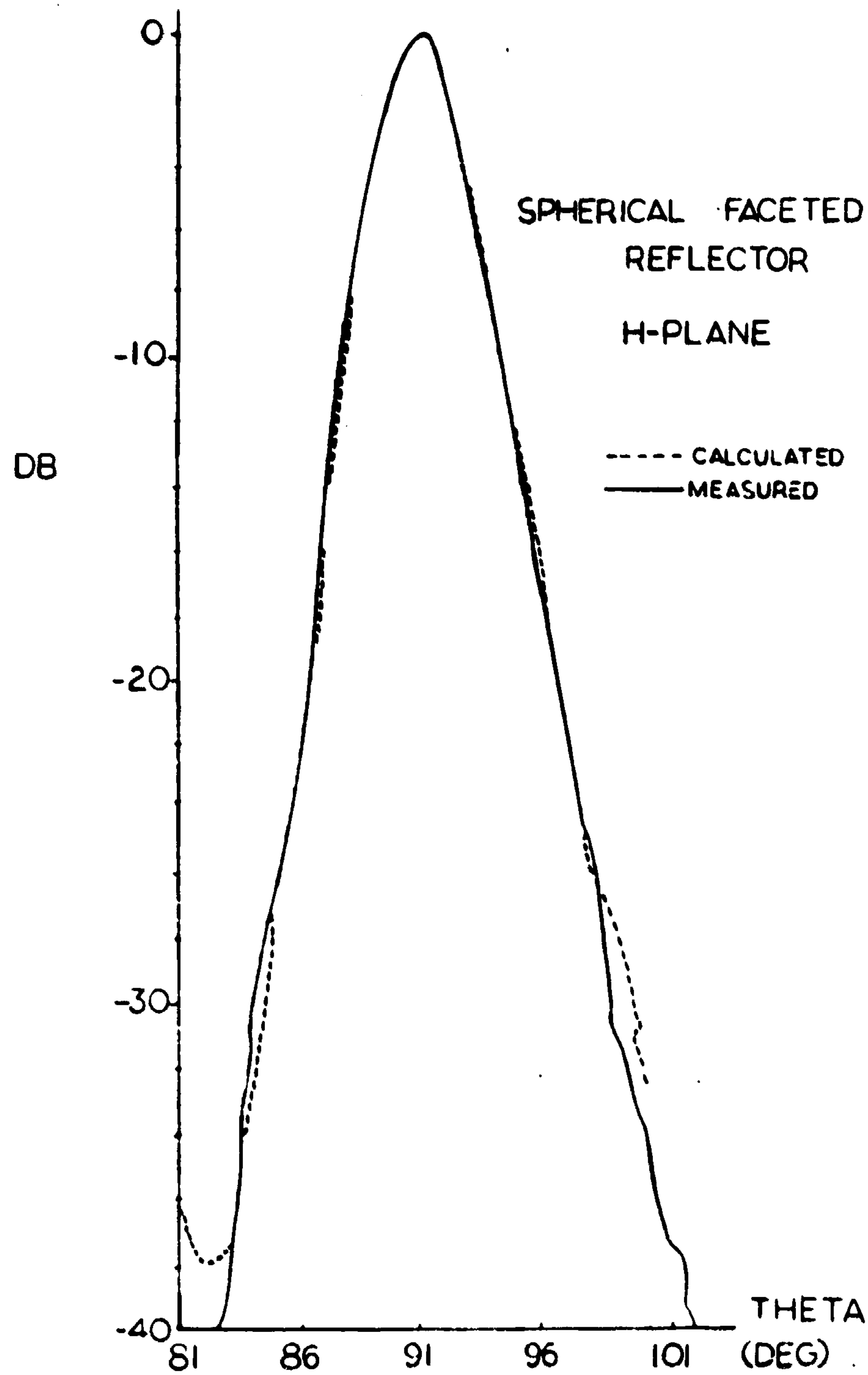

Fig. 5.6. New algorithm

6. CONCLUSIONS

An algorithm capable of computing radiation patterns of single reflector antennas has been modified and extended to analyze dual reflector antennas. A new technique for determining the aperture plane for multipanel single reflector antennas has been incorporated into the new program. The location of any aperture plane and the normals on each plane panel are computed automatically. Furthermore, equations for hyperbolic surfaces have been added.

The capability of expressing any non-analytic surface numerically will render the present algorithm very versatile. This fact will make the analysis of dual reflector antennas with shaped surfaces possible.

Presently, the algorithm requires that the feed center coincide with the real focus of the hyperboloid for a Cassegrain antenna, but modifications could be inserted to deal with any off-focus applications.

The results for the dual reflector antennas obtained by this algorithm show good agreement with those obtained by other algorithms. It is believed that a direct comparison with measured patterns will give a better estimate of the accuracy of the present algorithm.

7. LIST OF REFERENCES

1. Kauffman, J. R., W. F. Croswell and L. J. Jowers. 1976. Analysis of the radiation patterns of reflector antennas. IEEE Transactions on Antennas and propagation, Vol. AP-24, No. 1.

2. Kauffman, J. F. 1976. Calculation of the radiation patterns of reflector antennas. North Carolina State University, Department of Electrical Engineering (Report).

3. Agrawal, Praedeep K. 1978. A computer program to calculate radiation properties of reflector antennas. NASA Technical Memorandum 78721, Langley Res. Ctr., Hampton, Virginia.

4. Agrawal, Pradeep K. 1979. A preliminary study of a very large space radiometric antenna. NASA Technical Memorandum 80047, Langley Res. Ctr., Hampton, Virginia.

5. Botula, Alan. 1980. Computer Prediction of Large Reflector Antenna Radiation Properties. North Carolina State University, Department of Electrical Engineering (Report).

6. Silver, S. Microwave antenna theory and design. M.I.T. Rad. Lab. Series, McGraw-Hill Book Co., Inc. New York. Vol. 12, pp. 192-195, 1949.

7. Hannan, P. W. Microwave antennas derived from the Cassegrain telescope, IRE Trans. Antennas Propagation, pp. 140-153, Mar. 1961.

8. Albertsen, N.C. March 1977. Dual offset reflector antenna shaped for low cross polarization. TICRA Aps. Copenhagen, Denmark.

9. Chen, C. C. (Private communication).

8. APPENDICES

8.1. APPENDIX A

CASSEGRAIN ANTENNA GEOMETRY

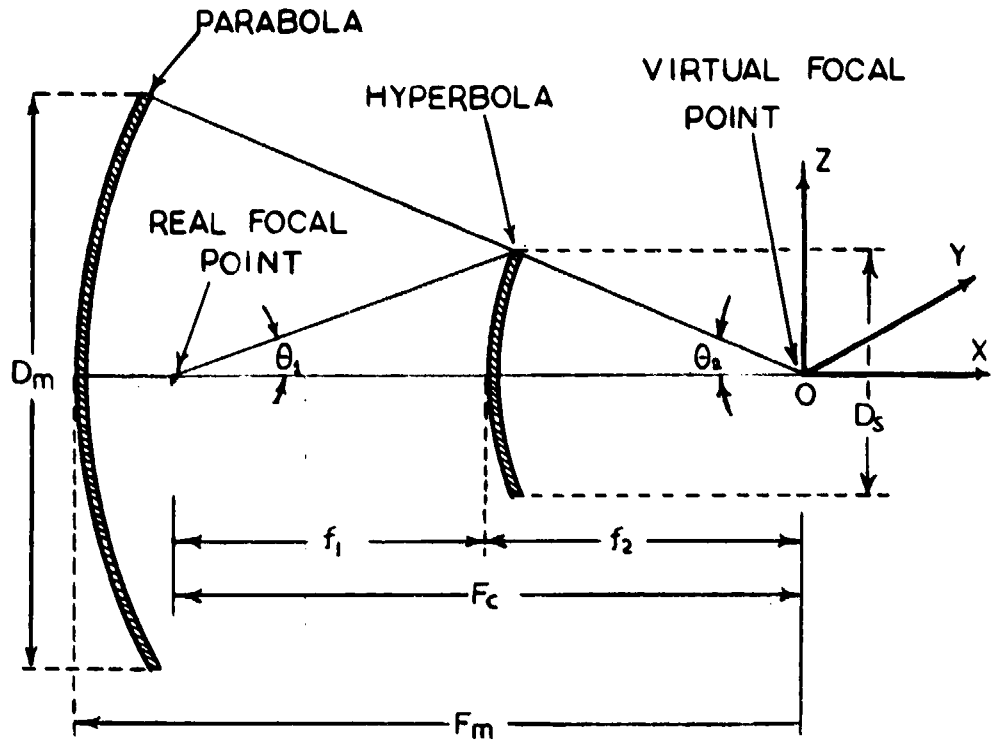

Fig. 8.1. Geometry of classical Cassegrain antenna

The classical Cassegrain geometry shown above employs a parabolic contour for the main reflector and a hyperbolic contour for the subreflector. One of the foci of the hyperboloid is the real focal point of the system and is located at the origin of the feed coordinate system; the other is a virtual focal point which is located at thc focus of the paraboloid. As a result, all parts of a wave emanating from the real focal point and then reflected from both reflector surfaces, travel equal distances to a plane in front of the antenna.

Four fixed parameters are adequate to completely describe a Cassegrain system, two for each reflector. In Figure 8.1, seven parameters are shown. If four are known, the other three can be derived from the mathematical relationships between the two reflector surfaces. For the main reflector,

$$\tan \frac{1}{2}\,\theta_2 = \frac{1}{4}\,\frac{Dm}{Fm}\ , \text{ and}$$

for the subreflector:

$$\frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2} = \frac{2\ F_c}{D_s}\ , \text{ and}$$

$$1 - \frac{\sin\frac{1}{2}(\theta_2-\theta_1)}{\sin\frac{1}{2}(\theta_2+\theta_1)} = 2\,\frac{f_1}{f_2}$$

where: F_c - distance between two foci,

f_1, f_2 = focal lengths of hyperboloid,

Dm = diameter of main reflector,

D_s = diameter of subreflector,

Fm = focal length of paraboloid

θ_2 = one-half of the angle subtended by the main reflector

θ_1 = one-half of the angle subtended by the subreflector.

For example, if Dm, Fm, F_c and θ_1 are determined by considerations of antenna performance and space limitations, then θ_2, D_s, and f_2 can be derived.

Note θ, which determines the beamwidth required of the feed radiation pattern, may be determined independe.:tly of the ratio Fm/Dm which specified the sape of the main reflector.

The surface of the main reflector is given by: $y^2 + z^2 = 4\ Fm\ (Fm + x)$, and the surface of the subreflector is expressed as:

$$\frac{(x + DIST)^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

where DIST $= \frac{F_c}{2} = a + \left|-x_0\right|$ (See Figure A.2) is the distance used to translate the origin of the hyperbola coordinate system so that it coincides with the origin of the referenced system.

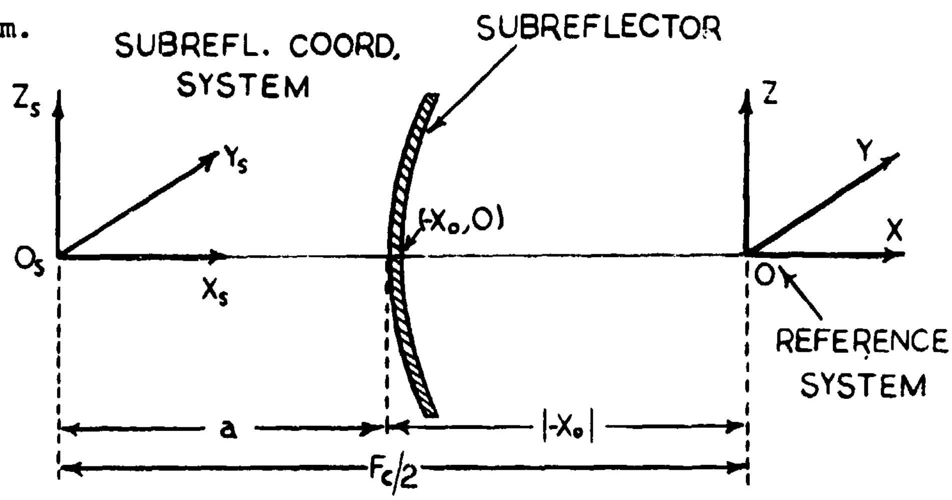

Fig. 8.2. Subreflector coordinate system

a = half the transverse axis (along x-axis)

b = semi-axis along the y direction in the ellipse lying in the yz plane.

c = semi-axis along the z direction in the ellipse lying in the yz plane.

If ε (eccentricity) of the hyperboloid is known, the following equations can be used:

$$\varepsilon = \frac{\sin\frac{1}{2}(\theta_2+\theta_1)}{\sin\frac{1}{2}(\theta_2-\theta_1)}$$

$$a = \frac{F_c}{2\varepsilon}\ ,\ b = a\sqrt{\varepsilon^2-1},\ \text{and}\ \frac{f_2}{f_1} = \frac{\varepsilon+1}{\varepsilon-1} = M$$

where M is the magnification factor of the hyperboloid.

8.2. APPENDIX B

ADDITION OF HYPERBOLOID

The equation of the hyperboloid, depicted in Figure 8.3, in the cartesian system is given as:

$$\frac{x_s^2}{a^2} - \frac{y_s^2}{b^2} - \frac{z_s^2}{c^2} = 1,$$ where a = half the transverse axis along x

b = semi-axis of the ellipse in the yz plane

c = semi-axis of the ellipse in the yz plane

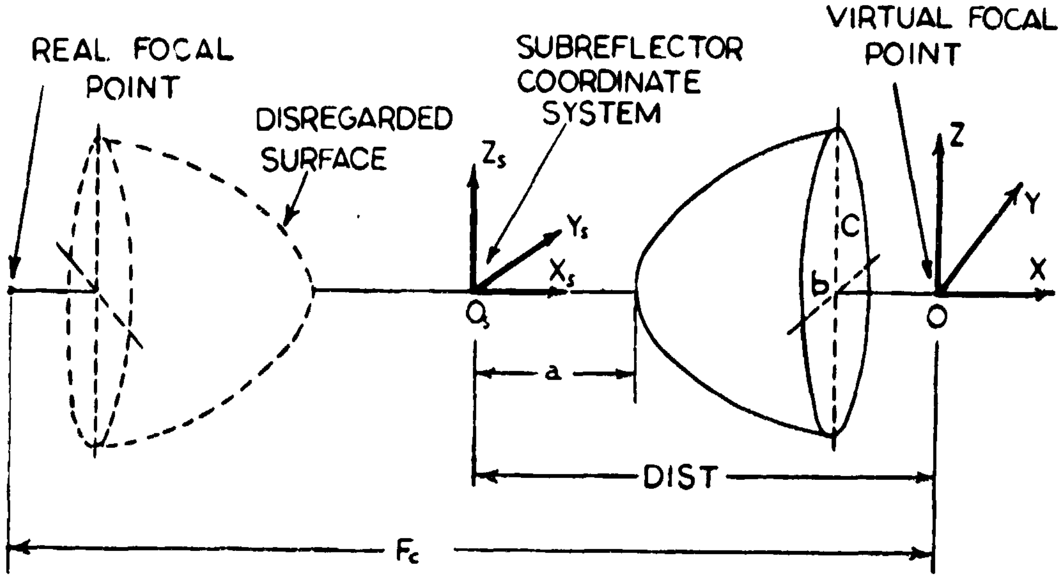

Fig. 8.3. The hyperboloid

In this equation, the hyperboloid is expressed in the x_s, y_s and z_s coordinate system. To express the same surface in the x, y and z system, a translation has to take place along the x axis, so that the origins of the two systems O_s and O coincide. It is clear that $y_s = y$ and $z_s = z$, and hence no change is needed to be made in the y and z directions.

If DIST is the distance between O_s and O, then x can be expressed as $x = x_s - DIST$, or $x_s = x + DIST$, and hence the hyperboloid equation in the x, y, z system becomes:

$$\frac{(x + DIST)^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \text{ where } DIST = \frac{Fc}{2} \qquad (8.1)$$

and Fc = distance between the two foci of the hyperboloid.

The parametric equations for a ray are:

$$\begin{aligned} x &= RB_{11} - B_{12} \\ y &= RB_{21} - B_{22} \\ Z &= RB_{31} - B_{32} \end{aligned} \qquad (8.2)$$

Substitute Equation (8.1) back into the equation of the hyperboloid to obtain:

$$\frac{(RB_{11} - B_{12} + DIST)^2}{a^2} - \frac{(RB_{21} - B_{22})^2}{b^2} - \frac{(RB_{31} - B_{32})^2}{c^2} - 1 = 0 \qquad (8.3)$$

or

$$\frac{R^2 B_{11}^2}{a^2} + \frac{B_{12}^2}{a^2} + \frac{DIST^2}{a^2} - \frac{2R\, B_{11}B_{12}}{a^2} + \frac{2R\, B_{11}\, DIST}{a^2} - \frac{2B_{12}DIST}{a^2}$$

$$- \frac{R^2B_{21}^2}{b^2} - \frac{B_{22}^2}{b^2} + \frac{2R\, B_{21}B_{22}}{b^2} - \frac{R^2B_{31}^2}{c^2} - \frac{B_{32}^2}{c^2} + \frac{2B_{31}B_{32}}{c^2} - 1 = 0$$

Equation (8.3) is of the form

$$AR^2 + BR + C = 0 \tag{8.4}$$

where

$$A = \frac{B_{11}^2}{a^2} - \frac{B_{21}^2}{b^2} - \frac{B_{31}^2}{c^2} \tag{8.5}$$

$$B = -2\left(\frac{B_{11}\,B_{12}}{a^2} - \frac{B_{11}\,\mathrm{DIST}}{a^2} - \frac{B_{21}\,B_{22}}{b^2} - \frac{B_{31}\,B_{32}}{c^2}\right) \tag{8.6}$$

$$C = \frac{B_{12}^2}{a^2} + \frac{(\mathrm{DIST})^2}{a^2} - \frac{B_{12}^2\,\mathrm{DIST}}{a^2} - \frac{B_{22}^2}{b^2} - \frac{B_{32}^2}{c^2} - 1 \tag{8.7}$$

Equations (8.5), (8.6), and (8.7) are evaluated by the program and (8.4) is solved to find the intersection point of the ray with the surface.

Now, to find the inside normal of the surface, the gradient of Equation (8.1) is taken as:

$$\nabla g(x,y,z) = \vec{n}(x,y,z) \tag{8.8}$$

where

$$g(x,y,z) = \frac{(x + \mathrm{DIST})^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 \tag{8.9}$$

it follows that:

$$\nabla g = \hat{x}\,\frac{\partial g}{\partial x} + \hat{y}\,\frac{\partial g}{\partial y} + \hat{z}\,\frac{\partial g}{\partial z} = \hat{x}\,\frac{2(x + \mathrm{DIST})}{a^2} - \hat{y}\,\frac{2y}{b^2} - \hat{z}\,\frac{2z}{c^2} \tag{8.10}$$

or

$$\frac{\partial g}{\partial x} = \frac{2(x + \mathrm{DIST})}{a^2}$$

$$\frac{\partial g}{\partial y} = -\frac{2y}{b^2} \tag{8.11}$$

$$\frac{\partial g}{\partial z} = -\frac{2z}{c^2}$$

Normalization results in obtaining the unit vector $\hat{n}$ as:

$$\hat{n} = \frac{\nabla g(x,y,z)}{\nabla g} = \frac{\hat{x}\ 2(x + DIST)/a^2 - \hat{y}(2/b^2) - \hat{z}(2z/a^2)}{\left(\frac{4(x + DIST)^2}{a^4} + \frac{4y^2}{b^4} + \frac{4z^2}{c^4}\right)^{\frac{1}{2}}} \qquad (8.12)$$

The factor 2 cancels out from both numerator and denominator. Let the denominator be expressed as:

$$DEN = \left[\frac{(x + DIST)^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right]^{\frac{1}{2}} \qquad (8.13)$$

then

$$n_x = \frac{(x + DIST)/a^2}{DEN}$$

$$n_y = \frac{-y/b^2}{DEN}$$

$$n_z = \frac{-z/c^2}{DEN}$$

8.3. APPENDIX C

SUBROUTINE SUBPNT

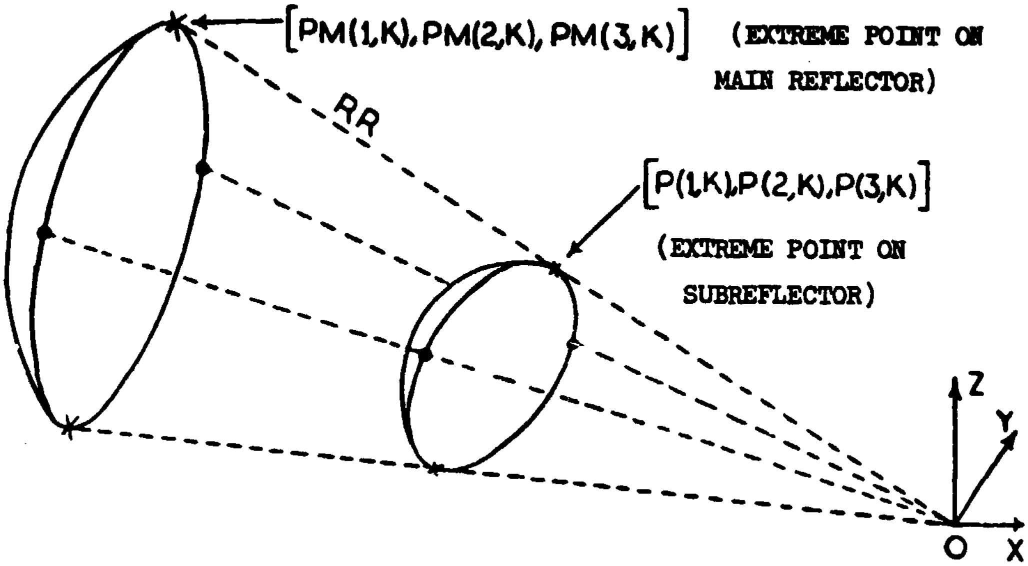

Fig. 8.4. Determination of subreflector four outermost perimeter points

In this subroutine, the four extreme points of the main reflector are used to find the four extreme points on the subreflector. This task is accomplished as follows:

Take a given extreme point on the main reflector and write the parametric equations of the line (RR) connecting that point to the origin of the reference system (0).

Express the direction cosines as:

DIR1 = COSA = PM(1,K)/RR (8.14)

DIR2 = COSB = PM(2,K)/RR (8.15)

DIR3 = COSC = PM(3,K)/RR (8.16)

Hence, the parametric equation of that line is given by:

$$x_0 = P(1,K) = PM(1,K) - RR\cdot DIR1 \qquad (8.17)$$

$$y_0 = P(2,K) = PM(2,K) - RR\cdot DIR2 \qquad (8.18)$$

$$z_0 = P(3,K) = PM(3,K) - RR\cdot DIR3 \qquad (8.19$$

where (P(1,K), (⁊(2,K) and P(3,K)) is a point on the sub-reflector which is to be found.

Now, substitute Equations (8.17), (8.18), and (8.19) in the equation for the surface of the hyperboloid, that is in

$$\frac{(x+DIST)^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \qquad (8.20)$$

to obtain:

$$\frac{[PM(1,K) - RR\cdot DIR1 + DIST]^2}{a^2} - \frac{[PM(2,K) - RR\cdot DIR2]^2}{b^2} - \frac{[PM(3,K) - RR\cdot DIR3]^2}{c^2} = 1 \qquad (8.21)$$

or

$$\frac{(PM(1,K)^2}{a^2} + \frac{(DIST)^2}{a^2} + \frac{(RR)^2(DIR1)^2}{a^2} - \frac{2RR\cdot DIR1\cdot PM(1,K)}{a^2}$$

$$- \frac{2RR\cdot DIR1\cdot DIST}{a^2} + \frac{2PM(1,K)\cdot DIST}{a^2} - \frac{(PM(2,K))^2}{b^2} - \frac{(RR)^2(DIR2)^2}{b^2}$$

$$+ \frac{2PM(2,K)\cdot RR\cdot DIR2}{b^2} - \frac{(PM(3,K))^2}{c^2} - \frac{(RR)^2(DIR3)^2}{c^2}$$

$$+ \frac{2\cdot RR\cdot PM(3,K)\cdot DIR3}{c^2} - 1 = 0 \qquad (8.22)$$

This equation is of the firm $(ARR)\ (RR)^2 + BRR \cdot RR + CRR = 0$

(8.23)

$$\text{where } ARR = \frac{(DIR1)^2}{a^2} - \frac{(DIR2)^2}{b^2} - \frac{(DIR3)^3}{c^2} \qquad (8.24)$$

$$BRR = 2\left[(-PM(1,K)-DIST)\cdot DIR1/a^2+PM(3,K)\cdot DIR2/b^2 + PM(3,K)\cdot DIR3/c^2\right] \qquad (8.25)$$

$$CRR = \left[(PM(1,K))^2+(DIST)^2+2.0\cdot PM(1,K)\cdot DIST\right]/a^2 - (PM(2,K))^2/b^2 - (PM(3,K))^2/c^2 - 1 \qquad (8.26)$$

Equations (8.24), (8.25), and (8.26) are evaluated by the program and (8.23) is solved to find RR. Substituting for the value of RR in Equations (8.14), 8.15), and (8.16), a point on the subreflector is obtained.

8.4. APPENDIX D

DEVELOPMENT OF NORMALS ON A PLANE PANEL

In the APRIN routine a certain number of perimeter points for each panel are read in. To determine a unit normal on each panel, the following procedure is applied:

1) Any three perimeter points are used to form two vectors, as shown in Figure 8.2.

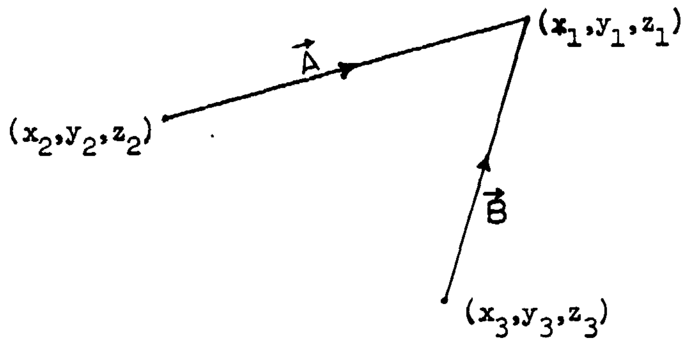

Fig. 8.5. Formation of two vectors from three perimeter points

where $\vec{A} = (x_1-x_2)\ \hat{i} + (y_1-y_2)\ \hat{j} + (z_1-z_2)\ \hat{k}$, and

$$\vec{B} = (x_1-x_3)\ \hat{i} + (y_1-y_3)\ \hat{j} + (z_1-z_3)\ \hat{k}$$

2) The cross product operation is used to find a vector normal ($\vec{N}$) to the plane defined by the vectors $\vec{A}$ and $\vec{B}$:

$$\vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1-x_2 & y_1-y_2 & z_1-z_2 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \end{vmatrix} =$$

$$\begin{aligned} \vec{N} = \ & (y_1-y_2)\cdot(z_1-z_3)-(y_1-y_3)\cdot(z_1-z_2) \quad \hat{i} \\ + \ & (x_1-x_3)\cdot(z_1-z_2)-(x_1-x_2)\cdot(z_1-z_3) \quad \hat{j} \\ + \ & (x_1-x_2)\cdot(y_1-y_3)-(x_1-x_3)(y_1-y_2) \quad \hat{k} \end{aligned}$$

3) The unit normal $\hat{N}$ is computed by: $N = \frac{\vec{N}}{|\vec{N}|}$

4) If this normal on the surface of the panel has a negative x component, then the vector is inverted to yield a positive x component, since any normal vector on the surface of the reflector should be directed toward the origin of the reference system, i.e., along the positive x axis. (See Figure 2.5).

8.5. APPENDIX E

FILL ROUTINE FOR A VERTICALLY POLARIZED FEED

The basis of this subroutine can be found in (5). To use it, the E- and H-plane patterns of the feed must be provided by the programmer in increments of 1°.

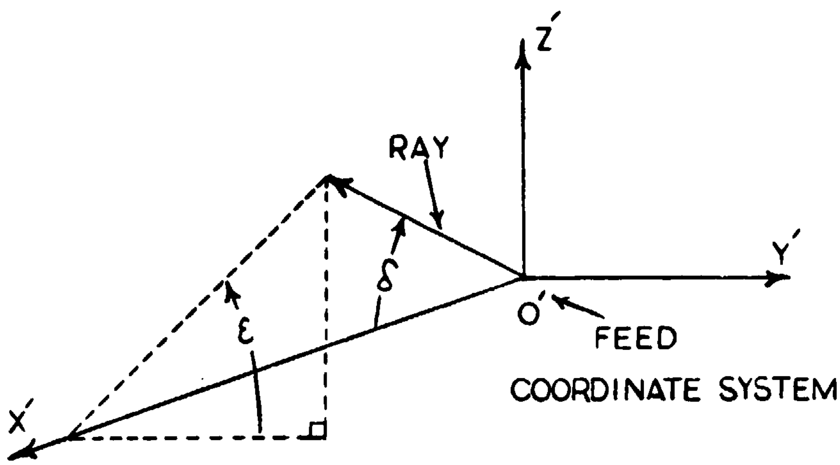

Fig. 8.6. Definition of angles δ and ε

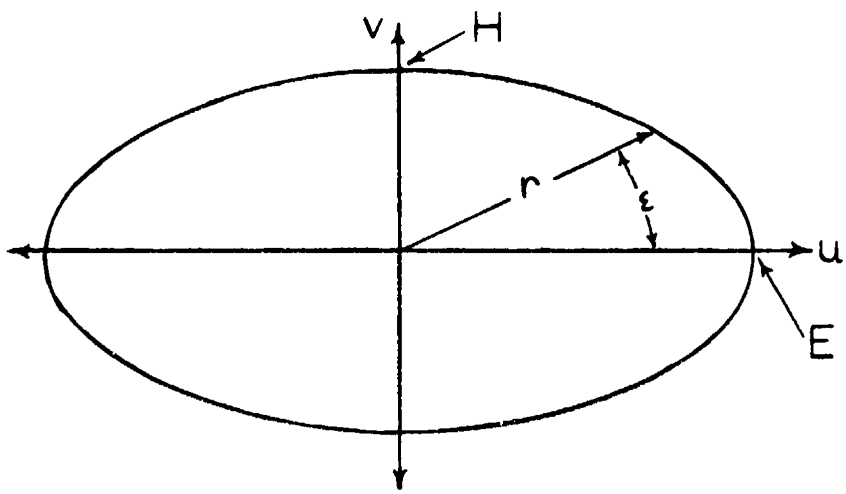

Fig. 8.7. Ellipse used for interpolation

In Figure 8.3, the angles used in FILL are shown. (From (5).)

$$\delta = \cos^{-1}(\sin\theta' \cos\phi') \tag{8.27}$$

where θ' and ϕ' are angles in the feed system.

$$\text{and } \varepsilon = \tan^{-1}\frac{\cos\theta'}{\sin\theta' \sin\phi'} \tag{8.28}$$

Figure 8.4 depicts the interpolation ellipse which is given by:

$$\frac{U^2}{E^2} + \frac{V^2}{H^2} = 1 \tag{8.29}$$

where

$$u = r\cos\varepsilon,\ v = r\sin\varepsilon, \tag{8.30}$$

$$E = E_{\theta' = 90^o},\ \text{and } H = E_{\phi' = 180} \tag{8.31}$$

Hence:

$$r = Etot = \frac{E_{\theta'=90} \cdot E_{\phi'=180}}{(E^2_{\theta'=90} \sin^2\varepsilon + E^2_{\phi'=180}\cos^2\varepsilon)^{1/2}} \tag{8.32}$$

The code of this subroutine is shown in Appendix F. In that code, PROJX = $\cos\delta$ and PROJEX = $\sin\varepsilon$. To insure vertical (i.e.,θ') polarization, PROJEX is set equal to zero. That means $u = r\cos\varepsilon$ and $v=0$. Substitution for u and v is Equation (8.32).

$$Etot = \frac{E_{\theta'= 90}\ E_{\phi'=180}}{E^2_{\phi'= 180} \cdot \cos^2\varepsilon^{1/2}} = \frac{E_{\phi'=90}}{\cos^2\varepsilon}$$

where $\cos^2\varepsilon = 1 - \sin^2\varepsilon$. Since a θ' polarized feed is associated with the z component of a cartesian system P(3,I), and P(4,I) are given as:

P(3,I) = Etot (along z)
P(4,I) = 0.0 (along y)

8.6. APPENDIX F

LISTING OF THE CODE FOR REFLECTR

MAIN

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 MAJOR(5),MINOR(5),NORM
      COMPLEX*16 ETOT(2,400),FIELDY(400),FIELDZ(400)
      INTEGER SURFC1,SURFC2
      COMMON/PARAMS/AORORF,BELLP,CELLP,DIST,PS1,PLNPNT(3),PLNORM(3),
     .        FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AUROR2,BELLP2,CELLP2,
     .        PS12,D'ST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      COMMON/APRPRM/NPTPPL,NPERIM
      COMMON/COLOS/DELT,XC,ANGINC,PM(3,4),RS,XMX,ZMX,ZMN,YMX
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      COMMON/DIMENS/YDIM,ZDIM,YCT,ZCT
      COMMON/EXTENT/YMIN,YMAX,ZMIN,ZMAX
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      COMMON/PATTRN/ETOT,AMINOR(3,5),AMAJOR(5),MINOR,MAJOR,NANGLE(5)
      DIMENSION P(5,2750),YFLD(75),ZFLD(75),PWER(75),PR(2,500)
      DATA DONE/5HDONE /,MLVL,NPARTS/0,7/
      DATA YLO,YHI,ZLO,ZHI/1.0D+10,-1.0D+10,1.0D+10,-1.0D+10/
      FDB(X)=20.0*DLOG10(X)
      PDB(X)=10.0*DLOG10(X)
      MAXPTS=2750
      CALL NPUT(P,NPAT)
      DO 400  I=1,NPNL
      CALL APRTUR(P,I)
      PRINT 777
      CALL QUANTZ(P,NPERIM,I)
      PRINT 778
      IF(IO1(3,I).EQ.0) GO TO 80
      ISW=1
      IF (IOPT.EQ.1)  ISW=-1
      CALL APRPLT(P,NPOINT,ISW)
      PRINT 780
80    CONTINUE
      IF (YMIN.LT.YLO) YLO=YMIN
      IF (YMAX.GT.YHI) YHI=YMAX
      IF (ZMIN.LT.ZLO) ZLO=ZMIN
      IF (ZMAX.GT.ZHI) ZHI=ZMAX
      IF(ICASS.EQ.1) NPERIM=4
      DO 95  L=1,NPERIM
      PR(1,L+MLVL)=P(1,L+NPOINT)
95    PR(2,L+MLVL)=P(2,L+NPOINT)
      MLVL=MLVL+NPERIM+1
      PR(1,MLVL)=1.0D+40
      ISUM=0
      DO 200  K=1,NPAT
      CALL INTGR(P,MAJOR(K),AMAJOR(K),AMINOR(1,K),FIELDY,FIELDZ)
      PRINT 779
      NANG=NANGLE(K)
      DO 150  L=1,NANG
      ETOT(1,L+ISUM)=ETOT(1,L+ISUM)+FIELDY(L)
150   ETOT(2,L+ISUM)=ETOT(2,L+ISUM)+FIELDZ(L)
200   ISUM=ISUM+NANG
400   CONTINUE
      PRINT 781
      IF (IOPT.EQ.1)  GO TO 420
      IF (IO1(2,1).EQ.0) GO TO 420
      YDIM=YHI-YLO
```

```
      ZDIM=ZHI-ZLO
      YCT=(YHI+YLO)/2.0
      ZCT=(ZHI+ZLO)/2.0
      CALL APRMAP(CR,NPNL,-1)
      PRINT 782
420   ISUM=0
      DO 770  I=1,NPAT
      NANG=NANGLE(I)
      FMAXY=-1.0D+40
      FMAXZ=-1.0D+40
      DO 450  J=1,NANG
      YFLD(J)=CDABS(ETOT(1,J+ISUM))
      ZFLD(J)=CDABS(ETOT(2,J+ISUM))
      FMAXY=DMAX1(FMAXY,YFLD(J))
450   FMAXZ=DMAX1(FMAXZ,ZFLD(J))
      ISUM=ISUM+NANG
      D=AMINOR(1,I)
      FMYDB= -60.0D0
      FMZDB= -60.0D0
      PWRMDB=-60.0D0
      PWR=FMAXZ*FMAXZ+FMAXY*FMAXY
      IF (FMAXY.GT.1.0D-10) FMYDB=FDB(FMAXY)
      IF (FMAXZ.GT.1.0D-10) FMZDB=FDB(FMAXZ)
      IF ( PWR .GT.1.0D-10) PWRMDB=PDB( PWR )
      PRINT 600,MAJOR(I),AMAJOR(I),MINOR(I),(AMINOR(J,I),J=1,3)
600   FORMAT(1H1,///24X,
     .       'TABLE OF ELECTRIC FIELD STRENGTHS (DB)',/'+',23X,
     .       '_____ __ ________ _____ _________ ____',
     .     ///19X,'PRINCIPAL PLANE OF CUT IS ',A5,' = ',F8.3,' DEG'
     .       //19X,'ANGLE ',A5,' FROM',F8.3,' TO',F8.3,' BY',F6.3,' DEG')
      PRINT 666, MINOR(I)
666   FORMAT(//13X,A5,4X,'DB(Z/Z)',4X,'DB(Y/Z)',4X,'DB(Z/Y)',5X,
     .       'DB(Y/Y)',5X,'PWRDB',/)
      DO 700  K=1,NANG
      PWER(K)=PWRMDB-100.0D0
      DBY    =FMYDB -100.0D0
      DBZ    =FMZDB -100.0D0
      PWR=ZFLD(K)*ZFLD(K)+YFLD(K)*YFLD(K)
      IF (YFLD(K).GT.1.0D-15) DBY=FDB(YFLD(K))
      IF (ZFLD(K).GT.1.0D-15) DBZ=FDB(ZFLD(K))
      IF (PWR.GT.1.0D-20) PWER(K)=PDB(PWR)
      IF (FMYDB.EQ.-60.0D0) DBY=-60.0D0
      IF (FMZDB.EQ.-60.0D0) DBZ=-60.0D0
      DBZZ=DBZ-FMZDB
      DBYY=DBY-FMYDB
      DBZY=DBZ-FMYDB
      DBYZ=DBY-FMZDB
      PWRDB=PWER(K)-PWRMDB
      PRINT 690, D,DBZZ,DBYZ,DBZY,DBYY,PWRDB
690   FORMAT(10X,F9.3,5F11.5)
      D=D+AMINOR(3,I)
      YFLD(K)=DBY
      ZFLD(K)=DBZ
700   CONTINUE
      PRINT 750, FMAXZ,FMZDB,FMAXY,FMYDB
750   FORMAT(//15X,'MAXIMUM FIELD VALUES-'//15X,
```

```
     .    .      20LOG(MAX(FIELD-Z))=20LOG(',1PE15.7,')=',0PF12.7//15X,
     .    .      20LOG(MAX(FIELD-Y))=20LOG(',1PE15.7,')=',0PF12.7)
      PRINT 755, NPARTS
755   FORMAT(//14X,
     .      ' INTERPOLATION NUMBER USED FOR INTEGRATION IS.........',I5)
      PRINT 765,MAJOR(I),AMAJOR(I)
765   FORMAT(1H1,///20X,'PRINCIPAL PLANE = ',A5,F7.3,' DEGREES')
      CALL PLOT4(64H NORMALIZED Z-COMPONENT OF SECONDARY PATTERN (DB)
     .          ,FMZDB,ZFLD,NANG,MINOR(I),AMINOR(1,I))
      PRINT 765,MAJOR(I),AMAJOR(I)
      CALL PLOT4(64H NORMALIZED Y-COMPONENT OF SECONDARY PATTERN (DB)
     .          ,FMYDB,YFLD,NANG,MINOR(I),AMINOR(1,I))
      PRINT 765,MAJOR(I),AMAJOR(I)
      CALL PLOT4(64H              NORMALIZED POWER PATTERN (DB)
     .          ,PWRMDB,PWER,NANG,MINOR(I),AMINOR(1,I))
770   CONTINUE
      IF (IO1(4,1).EQ.1) WRITE(7,775)
      IF (IO1(5,1).EQ.0) STOP
      REWIND 7
      CALL PTLIST
775   FORMAT('    -1234')
776   FORMAT(/'      ------------------ FINISHED INPUT  ----------------')
777   FORMAT(/'      ----------------- FINISHED APERTUR -----------------')
778   FORMAT(/'      ----------------- FINISHED QUANTIZ -----------------')
779   FORMAT( '      ----------------- FINISHED INTGRT -----------------')
780   FORMAT( '                    *** EXECUTED  APRPLT ***             ')
781   FORMAT(/'      ---------- PATTERN COMPUTATIONS COMPLETE -----------')
782   FORMAT(/'                    *** EXECUTED  APRMAP ***             ')
      STOP
      END
```

NPUT

```
      SUBROUTINE NPUT(P,NPAT)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 MAJOR(5),MINOR(5),NORM
      COMPLEX*16 ETOT(2,400)
      INTEGER SURFC1,SURFC2
      COMMON/BLOCKG/YCBL,ZCBL,HFMABL,HFMIBL
      COMMON/FEED/EP(91),ET(91),NP,NT,XS,YS,ZS
      COMMON/COLOS/DELT,XC,ANGINC,PM(3,4),RS,XMX,ZMX,ZMN,YMX
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      COMMON/PARAMS/AORORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .        FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AOROR2,BELLP2,CELLP2,
     .        PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      COMMON/PATTRN/ETOT,AMINOR(3,5),AMAJOR(5),MINOR,MAJOR,NANGLE(5)
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      DIMENSION P(5,2750),TITLE(40)
      DATA DONE/5HDONE /
      ICASS=0
       IOPT=0
      READ 5,TITLE
 5    FORMAT (10A8)
      READ(1,10) FEED,ALPHA,BETA,GAMMA,XLAM
 10   FORMAT(7F10.4)
      IF (ICASS.NE.1) GO TO 35
      READ(1,20) SURFC2,AOROR2,BELLP2,CELLP2,DIST2,PSI2,POINT,NORM
 20   FORMAT(I1,9X,5F10.4/6F10.4)
 35   READ(1,37) SURFC1,NPNL,AORORF,BELLP,CELLP,DIST,PSI,PLNPNT,PLNORM
 37   FORMAT(I1,7X,I2,5F10.4/6F10.4)
      IF(ICASS.NE.1) GO TO 40
      READ (1,39) ((PM(I,J),I=1,3),J=1,4)
 39   FORMAT (3F10.4)
C     END OF MAIN REFLECTOR INPUT DATA
      CALL SUBPNT(P)
      GO TO 43
 40   READ (1,41) ((P(I,J),I=1,3),J=1,4)
 41   FORMAT (3F10.4)
C     END OF SUB  REFLECTOR INPUT DATA
 43   READ(1,50) XX,YCBL,ZCBL,HFMABL,HMIBL
 50   FORMAT(5F10.4)
C     FEED RADIATION PATTERN
      READ(1,55) EP
      READ(1,55) ET
 55   FORMAT(5F15.5)
      READ (1,60) NOPT,NLIST
 60   FORMAT (3I1,2X,I5)
      IF (NOPT(1).EQ.1.OR.NOPT(2).EQ.1) READ(1,70) (ILIST(I),I=1,NLIST)
 70   FORMAT (16I5)
      ISUM=0
      NPAT=1
 77   READ(1,80) MAJOR(NPAT),AMAJOR(NPAT),MINOR(NPAT),(AMINOR(I,NPAT),
     .           I=1,3)
 80   FORMAT(A5,5X,F10.4,A5,5X,3F10.4)
      IF (MAJOR(NPAT).EQ.DONE) GO TO 88
      NANGLE(NPAT)=(AMINOR(2,NPAT)-AMINOR(1,NPAT))/AMINOR(3,NPAT)+1.5
      IF (NANGLE(NPAT).GT.75)  GO TO 85
      ISUM=ISUM+NANGLE(NPAT)
      NPAT=NPAT+1
```

```
      IF (NPAT.LT.6) GO TO 77
      PRINT 330
      STOP
85    PRINT 335
      STOP
88    IF (ISUM.LE.400)  GO TO 95
      PRINT 340,ISUM
      STOP
95    NPAT=NPAT-1
      DO 98  L=1,ISUM
      ETOT(1,L)=(0.0D0,0.0D0)
98    ETOT(2,L)=(0.0D0,0.0D0)
      PRINT 576,TITLE,XLAM,FEED,ALPHA,BETA,GAMMA
      PRINT 577,XC,YCBL,ZCBL,HFMABL,HFMIBL,NPNL
      IF (ICASS.NE.1) GO TO 180
      PRINT 578
      GO TO (120,130,140,150,160,161),SURFC2
120   PRINT 579,POINT,NORM
      GO TO 179
130   PRINT 580,AOROR2,BELLP2
      GO TO 179
140   PRINT 581,AOROR2
      GO TO 179
150   PRINT 582,AOROR2
      GO TO 179
160   PRINT 583,AOROR2,PSI2
      GO TO 179
161   PRINT 584,AOROR2,BELLP2,CELLP2,DIST2
179   PRINT 585,((PM(I,J),I=1,3),J=1,4)
      PRINT 586
180   GO TO (220,230,240,250,260,270),SURFC1
220   PRINT 579,PLNPNT,PLNORM
      GO TO 300
230   PRINT 580,AORORF,BELLP
      GO TO 300
240   PRINT 581,AORORF
      GO TO 300
250   PRINT 582,AORORF
      GO TO 300
260   PRINT 583,AORORF,PSI
      GO TO 300
270   PRINT 584,AORORF,BELLP,CELLP,DIST
300   IF (NPNL.GE.1) GO TO 310
      IOPT=1
      NPNL=1
310   PRINT 585,((P(I,J),I=1,3),J=1,4)
      PRINT 587
      PRINT 589
      PRINT 600,EP
      PRINT 587
      PRINT 588
      PRINT 600,ET
      PRINT 400,NPAT
      DO 320 M=1,NPAT
320   PRINT 500,MAJOR(M),AMAJOR(M),MINOR(M),(AMINOR(KK,M),KK=1,3)
330   FORMAT('********** ERROR-MORE THAN 5 PATTERN ',
```

```
     .          'CALCULATIONS REQUESTED  **********   ')
335   FORMAT('********** ERROR-MORE THAN 75 ANGLES IN',
     .          ' ONE PATTERN REQUEST **********')
340   FORMAT(' **********  ERROR - REQUESTED',I5,' ANGLES TO BE ',
     .          'CALCULATED EXCEEDS AVAIL. STORAGE  **********')
400   FORMAT(//
     .          ' NUMBER OF PATTERN GROUPS REQUESTED.................',I5/)
500   FORMAT(5X,A5,' =',F10.4,10X,A5,' FROM',F10.4,' TO',F10.4,' BY',
     .          F10.4)
576   FORMAT(1H1,////,15X,' FAR FIELD RADIATION PATTERN CALCULATION   '/
     .    //' ',10A8/' ',10A8/' ',10A8/' ',10A8//
     .     ' INPUT PARAMETERS-                                     ',//
     .     '    WAVELENGTH OF ELECTRIC FIELD.......................',F9.4/
     .     '    LOCATION OF COORDINATE ORIGIN WRT FEED (X,Y,Z).....',3F8.3
     .    /'    FEED ROTATION ANGLES(ALPHA,BETA,GAMMA)............',3F8.3)
577   FORMAT(
     .     '    APERTURE PLANE LOCATION(XC)........................',F7.2/
     .     '    SUB DISH SHADOW CENTER COORDINATES IN APERT. PL....',2F7.2
     .    /'    HALF MAJOR AXIS OF SUB DISH SHADOW ...............',F7.2/
     .     '    HALF MINOR AXIS OF SUB DISH SHADOW................',F7.2/
     .     '    NUMBER OF PANELS IN REFLECTOR......................',I6/)
578   FORMAT (//' MAIN DISH DESCRIPTION AND ITS PARAMETERS-          '/)
579   FORMAT ('   IT IS A PLANAR REFLECTOR                          ',/
     .     '    A POINT ON THE REFLECTR SURFACE(X,Y,Z)............',3F8.3/
     .     '    COMPONENTS OF UNIT NORMAL TO SURFACE(X,Y,X).......',3F8.3)
580   FORMAT ('   IT IS AN ELLIPTICAL REFLECTOR                     ',
     .    /'    MAJOR AXIS OF THE ELLIPTICAL REFLECTOR.............',F78.3
     .    /'    MINOR AXIS OF THE ELLIPTICAL REFLECTOR.............',F8.3)
581   FORMAT('   IT IS A SHERICAL REFLECTR                          ',
     .    /'    RADIUS OF REFLECTR SHERE...........................',F8.3)
582   FORMAT('   IT IS A PARABOLIC REFLECTOR                        ',
     .    /'    FOCAL LENGTH OF THE REFLECTOR......................',F8.3)
583   FORMAT ('IT IS A PARABOLIC CYLINDRICAL REFLECTOR               ',
     .    /'    FOCAL LENGTH OF PARABOLIC CYLINDER.................',F8.3/
     .     '    ANGLE OF ROTATION ABOUT X-AXIS (PSI)...............',F8.3)
584   FORMAT('   IT IS A HYBERBOLIC REFLECTOR                       ',
     .    /'    MAJOR AXIS OF REFL. IN X DIRECTION.................',F8.3
     .    /'    AXIS OF REFLECTOR IN Y DIRECTION...................',F8.3
     .    /'    AXIS OF REFLECTOR IN Z DIRECTION...................',F8.3
     .    /'    DISTANCE USED FOR TRANSLATION OF ORIG. OF AXES.....',F8.3)
585   FORMAT(
     .     '    *****  PROGRAM IN SINGLE PANEL MODE  *****'
     .    /'    MINIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)...........',3F8.3
     .    /'    MAXIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)...........',3F8.3
     .    /'    MAXIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)...........',3F8.3
     .    /'    MINIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)...........',3F8.3
     .     )
586   FORMAT (//' SUBDISH DESCRIPTION AND ITS PARAMETERS-          '/)
587   FORMAT(//' PATTERN OF FEED IN ONE DEG INCREMENTS OFF-AXIS-'/)
588   FORMAT(' E-PLANE '/)
589   FORMAT(' H-PLANE '/)
600   FORMAT(2X,5F16.10)
      PI=DARCOS(-1.0D0)
      PI2=PI+PI
      PID2=0.5*PI
      DTOR=PI/180.
      RTOD= 180./PI
      RETURN
      END
```

APRTUR

```
      SUBROUTINE APRTUR(P,ICALL)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 NHAT(3),NMAG,NORM
      INTEGER SURFC1,SURFC2
      COMMON/APRPRM/NPTPPL,NPERIM
      COMMON/CASS/SR(3),ER(3),X0,Y0,Z0,Y,Z,RM,D,X02,Y02,Z02,ER2(3)
      COMMON/FEED/EP(91),ET(91),NP,NT,XS,YS,ZS
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      COMMON/COLOS/DELT,XC,ANGINC,PM(3,4),RS,XMX,ZMX,ZMN,YMX
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      COMMON/PARAMS/AORORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .        FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AOROR2,BELLP2,CELLP2,
     .        PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
       DIMENSION AINV(3,3),B(3,2),BB(3,2),C(3),X(3),A(3,3),EI(3),
     .P(5,2750)
      IF(ICASS.EQ.1) GO TO 10
      M=1
      DO 2 I=2,4
      IF(P(3,M)-P(3,I))3,2,2
3     M=I
2     CONTINUE
      XMX=P(1,M)
      YMX=P(2,M)
      ZMX=P(3,M)
10    IF (ICALL.GT.1) GO TO 50
      ALPHAR=ALPHA*DTOR
      BETAR=BETA*DTOR
      GAMMAR=GAMMA*DTOR
      A(1,1)=DCOS(ALPHAR)*DCOS(GAMMAR)-DSIN(ALPHAR)*DSIN(BETAR)*
     . DSIN(GAMMAR)
      A(1,2)=DSIN(ALPHAR)*DCOS(GAMMAR)+DCOS(ALPHAR)*DSIN(BETAR)*
     . DSIN(GAMMAR)
      A(1,3)=-DCOS(BETAR)*DSIN(GAMMAR)
      A(2,1)=-DSIN(ALPHAR)*DCOS(BETAR)
      A(2,2)= DCOS(ALPHAR)*DCOS(BETAR)
      A(2,3)= DSIN(BETAR)
      A(3,1)=DCOS(ALPHAR)*DSIN(GAMMAR)+DSIN(ALPHAR)*DSIN(BETAR)*
     . DCOS(GAMMAR)
      A(3,2)=DSIN(ALPHAR)*DSIN(GAMMAR)-DCOS(ALPHAR)*DSIN(BETAR)*
     . DCOS(GAMMAR)
      A(3,3)= DCOS(BETAR)*DCOS(GAMMAR)
      DO 40  I=1,3
      DO 40  J=1,3
40    AINV(I,J)=A(J,I)
      NPERIM=4
      NPTPPL=2000
50    IF (IOPT.EQ.0)  CALL APRIN(P,ICALL)
      TMAX=0.0D0
      TMIN=PI
55    PMIN=PI+PID2
      PMAX=PID2
58    DO 65  I=1,NPERIM
      DO 60  J=1,3
60    X(J)=AINV(J,1)*P(1,I)+AINV(J,2)*P(2,I)+AINV(J,3)*P(3,I)
      R=DSQRT((X(1)+FEED(1))**2+(X(2)+FEED(2))**2+(X(3)+FEED(3))**2)
      P(1,I)=DARCOS((X(3)+FEED(3))/R)
```

```
      SINTHT=DSIN(P(1,I))
      IF (SINTHT.LT.1.D-10) SINTHT=1.D-10
      P(2,I)=PI-DARSIN((X(2)+FEED(2))/(R*SINTHT))
61    IF (P(1,I).GT.TMAX) TMAX=P(1,I)
      IF (P(1,I).LT.TMIN) TMIN=P(1,I)
      IF (P(2,I).GT.PMAX) PMAX=P(2,I)
      IF (P(2,I).LT.PMIN) PMIN=P(2,I)
65    CONTINUE
      DELP=PMAX-PMIN
      DELT=TMAX-TMIN
      NP=DSQRT(DELP*DFLOAT(NPTPPL)/DELT)+1.0
      NP=((NP-1)/2)*2+1
      ANGINC=DELP/(DFLOA.(NP)-2.6)
      IF(ICASS.EQ.1) CALL FINDXC(P,B)
      NTD2=DELT/(2.0*ANGINC)+1.0
      NT=2*NTD2+1
      PMIN=PMIN-0.8*ANGINC
      PMAX=PMAX+0.8*ANGINC
      TCT=(TMAX+TMIN)/2.0
      TMIN=TCT-DFLOAT(NTD2)*ANGINC
      TMAX=TCT+DFLOAT(NTD2)*ANGINC
      DO 95  J=1,NT
      DO 95  K=1,NP
      P(1,NPERIM+(J-1)*NP+K)=TMIN+(J-1)*ANGINC
95    P(2,NPERIM+(J-1)*NP+K)=PMIN+(K-1)*ANGINC
      NTNP=NT*NP
      NPOINT=NPERIM+NTNP
      TMIN=TMIN*RTOD
      TMAX=TMAX*RTOD
      PMIN=PMIN*RTOD
      PMAX=PMAX*RTOD
      ANGINC=ANGINC*RTOD
      IF(IOI(1,ICALL).EQ.1) PRINT 107,TMIN,TMAX,PMIN,PMAX,
     .                                ANGINC,NTNP,NPOINT
107   FORMAT(//' ILLUMINATION DATA-'//
     .   '   THETA ILLUMINATION FROM.......................',F9.3,'  TO
     .',     F9.3/
     .   '   PHI ILLUMINATION FROM.........................',F9.3,'  TO
     .',     F9.3/
     .   '   INCREMENTAL ANGLE (DEG)...........................',F7.4/
     .   '   THEREFORE TOTAL NUMBER OF GENERATED RAYS............',I7 /
     .   '   TOTAL NUMBER OF APERTURE PLANE POINTS...............',I7)
      IF (SURFC1.NE.5) GO TO 114
      CSPSI=DCOS(PSI*DTOR)
      SNPSI=DSIN(PSI*DTOR)
114   CALL FILLP(P,NPOINT)
      DO 600  I=1,NPOINT
      SINP=DSIN(P(2,I))
      COSP=DCOS(P(2,I))
      SINT=DSIN(P(1,I))
      COST=DCOS(P(1,I))
      BB(1,1)=SINT*COSP
      BB(2,1)=SINT*SINP
      BB(3,1)=COST
      BB(1,2)=+FEED(1)
      BB(2,2)=+FEED(2)
```

```
      BB(3,2)=+FEED(3)
      CALL MULT32(B,A,BB)
      GO TO (120,130,140,150,160,161),SURFC1
120   AR=0.0
      BR=B(1,1)*PLNORM(1)+B(2,1)*PLNORM(2)+B(3,1)*PLNORM(3)
      CR=-(B(1,2)+PLNPNT(1))*PLNORM(1)
     .   -(B(2,2)+PLNPNT(2))*PLNORM(2)
     .   -(B(3,2)+PLNPNT(3))*PLNORM(3)
      GO TO 180
130   AR=B(1,1)**2/AORORF**2+(B(2,1)**2+B(3,1)**2)/BELLP**2
      BR=-2.0*(B(1,1)*B(1,2)/AORORF**2+(B(2,1)*B(2,2)+B(3,1)*B(3,2))/
     .                          BELLP**2)
      CR=B(1,2)**2/AORORF**2+(B(2,2)**2+B(3,2)**2)/BELLP**2-1.0
      GO TO 180
140   AR=B(1,1)*B(1,1)+B(2,1)*B(2,1)+B(3,1)*B(3,1)
      BR=-2.*(B(1,1)*B(1,2)+B(2,1)*B(2,2)+B(3,1)*B(3,2))
      CR=B(1,2)*B(1,2)+B(2,2)*B(2,2)+B(3,2)*B(3,2)-AORORF*AORORF
      GO TO 180
150   AR=B(2,1)*B(2,1)+B(3,1)*B(3,1)
      BR=-2.0*(B(2,1)*B(2,2)+B(3,1)*B(3,2)+2.0*AORORF*B(1,1))
      CR=B(2,2)*B(2,2)+B(3,2)*B(3,2)+4.0*AORORF*B(1,2)-4.0*AORORF**2
      GO TO 180
160   AR=B(3,1)*B(3,1)*CSPSI*CSPSI-2.0*B(2,1)*B(3,1)*CSPSI*SNPSI
     .   +B(2,1)*B(2,1)*SNPSI*SNPSI
      BR=-2.0*(B(3,1)*B(3,2)*CSPSI*CSPSI
     .   -(B(3,1)*B(2,2)+B(2,1)*B(3,2))*CSPSI*SNPSI
     .   +B(2,1)*B(2,2)*SNPSI*SNPSI+2.0*AORORF*B(1,1))
      CR=B(3,2)*B(3,2)*CSPSI*CSPSI-2.0*B(2,2)*B(3,2)*CSPSI*SNPSI
     .   +B(2,2)*B(2,2)*SNPSI*SNPSI+4.0*AORORF*(B(1,2)-AORORF)
      GO TO 180
161   AR=(B(1,1)**2/AORORF**2)-(B(2,1)**2/BELLP**2)-(B(3,1)**2/CELLP**2)
      BR=-2.0*((B(1,1)*B(1,2)/AORORF**2)-(B(1,1)*DIST/AORORF**2)-(B(2,1)
     . *B(2,2)/BELLP**2)-(B(3,1)*B(3,2)/CELLP**2))
      CR=((B(1,2)*B(1,2)+DIST*DIST-2.0*B(1,2)*DIST)/AORORF**2)-(B(2,2)*
     .   B(2,2)/BELLP**2)-(B(3,2)*B(3,2)/CELLP**2)-1.0
      GO TO 181
180   IF (ICASS.NE.1) GO TO 181
      IF(DABS(AR).LT.1.0D-10) R=CR/BR
      IF (DABS(AR).LT.1.0D-10) GO TO 185
      R=(-BR-DSQRT(BR*BR-4.0*AR*CR))/(AR+AR)
      GO TO 185
181   IF(DABS(AR).LT.1.0D-5) R=-CR/BR
      IF(DABS(AR).LT.1.0D-5) GO TO 185
      V=BR*BR-4.0*AR*CR
      R=(-BR+DSQRT(V))/(AR+AR)
185   CONTINUE
      X0=B(1,1)*R-B(1,2)
      Y0=B(2,1)*R-B(2,2)
      Z0=B(3,1)*R-B(3,2)
      IF(I.GT.1) GO TO 219
      IF(ICASS.EQ.1) GO TO 219
      IF(ICALL.GT.1) GO TO 189
      IF(IOPT.EQ.1) GO TO 190
      R1=DSQRT((XS+B(1,2))**2+(YS+B(2,2))**2+(ZS+B(3,2))**2)-1.0
      THTMAX=DATAN(-(ZS+B(3,2))/(XS+B(1,2)))
      THTAUG=THTMAX+2.5*ANGINC*DTOR
```

```
      XC=-(R1*DCOS(THTAUG)+B(1,2))
      CONST =DABS(XC-XS)
      GO TO 190
189   XC=XS+CONST
      XX=ZMN
      IF(ICALL.GT.1) GO TO 219
190   CALL FINDXC(P,B)
      XX=ZMN
      IF(IOPT.EQ.1) XC=XX
219   GO TO (220,230,240,250,260,261),SURFCI
220   NHAT(1)=PLNORM(1)
      NHAT(2)=PLNORM(2)
      NHAT(3)=PLNORM(3)
      GO TO 288
230   NHAT(1)=-X0*BELLP**2/DSQRT(X0**2*BELLP**4+(Y0**2+Z0**2)*AORORF**4)
      NHAT(2)=-Y0*AORORF**2/DSQRT(X0**2*BELLP**4+(Y0**2+Z0**2)*
     . AORORF**4)
      NHAT(3)=-Z0*AORORF**2/DSQRT(X0**2*BELLP**4+(Y0**2+Z0**2)*
     . AORORF**4)
      GO TO 288
240   NHAT(1)=-X0/AORORF
      NHAT(2)=-Y0/AORORF
      NHAT(3)=-Z0/AORORF
      GO TO 288
250   NHAT(1)=2.0*AORORF/DSQRT(4.0*AORORF**2+Y0**2+Z0**2)
      NHAT(2)=        -Y0/DSQRT(4.0*AORORF**2+Y0**2+Z0**2)
      NHAT(3)=        -Z0/DSQRT(4.0*AORORF**2+Y0**2+Z0**2)
      GO TO 288
260   NMAG=DSQRT(4.0*AORORF*AORORF+(Z0*CSPSI*SNPSI-Y0*SNPSI*SNPSI)**2
     .              +(Y0*SNPSI*CSPSI-Z0*CSPSI*CSPSI)**2)
      NHAT(1)=2.0*AORORF/NMAG
      NHAT(2)=SNPSI*(Z0*CSPSI-Y0*SNPSI)/NMAG
      NHAT(3)=CSPSI*(Y0*SNPSI-Z0*CSPSI)/NMAG
      GO TO 288
261   DEN=DSQRT(((X0+DIST)**2/AORORF**4)+(Y0*Y0/BELLP**4)+(Z0*Z0/
     .     CELLP**4))
      NHAT(1)=(X0+DIST)/((AORORF**2)*DEN)
      NHAT(2)=-Y0/((BELLP**2)*DEN)
      NHAT(3)=-Z0/((CELLP**2)*DEN)
288   IF (ICASS.NE.1) GO TO 289
      NHAT(1)=-NHAT(1)
      NHAT(2)=-NHAT(2)
      NHAT(3)=-NHAT(3)
289   SCALAR=2.0*(B(1,1)*NHAT(1)+B(2,1)*NHAT(2)+B(3,1)*NHAT(3))
      DO 295  L=1,3
295   SR(L)= (B(L,1)-SCALAR*NHAT(L) )
      ETI=P(3,1)/R
      EPI=P(4,1)/R
      C(1)=COST*COSP*ETI-SINP*EPI
      C(2)=COST*SINP*ETI+COSP*EPI
      C(3)=-SINT*ETI
      DO 400  N=1,3
      EI(N)=0.0
      DO 400  M=1,3
400   EI(N)=EI(N)+A(N,M)*C(M)
      SCALAR=2.0*(EI(1)*NHAT(1)+EI(2)*NHAT(2)+EI(3)*NHAT(3))
```

```
      DO 500  K=1,3
500   ER(K)=SCALAR*NHAT(K)-EI(K)
      IF (ICASS.NE.1) GO TO 550
      CALL CASSA(P)
      PHASE=PI2*(R+RM+D)/XLAM
      P(1,I)=Y
      P(2,I)=Z
      P(3,I)=ER2(2)
      P(4,I)=ER2(3)
      P(5,I)=PHASE
      GO TO 600
550   Y=Y0+(XC-X0)*SR(2)/SR(1)
      Z=Z0+(XC-X0)*SR(3)/SR(1)
      D=DSQRT((XC-X0)*(XC-X0)+(Y-Y0)*(Y-Y0)+(Z-Z0)*(Z-Z0))
      DIF=DABS(XC-XX)
      PHASE=PI2*(R+D+DIF)/XLAM+P(5,I)
      P(1,I)=Y
      P(2,I)=Z
      P(3,I)=ER(2)
      P(4,I)=ER(3)
      P(5,I)=PHASE
600   CONTINUE
      RETURN
      END
```

SUBPNT

```
      SUBROUTINE SUBPNT(P)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 NORM
      INTEGER SURFC1,SURFC2
      COMMON/COLOS/DELT,XC,ANGINC,PM(3,4),RS,XMX,ZMX,ZMN,YMX
      COMMON/PARAMS/AURORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .        FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AOROR2,BELLP2,CELLP2,
     .        PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      DIMENSION P(5,2750)
      DO 33 K=1,4
      RR=DSQRT(PM(1,K)*PM(1,K)+PM(2,K)*PM(2,K)+PM(3,K)*PM(3,K))
      DIR1=PM(1,K)/RR
      DIR2=PM(2,K)/RR
      DIR3=PM(3,K)/RR
      ARR=DIR1**2/(AORORF**2)-(DIR2**2/(BELLP**2))-(DIR3**2/(CELLP**2))
      BRR=2.0*((-(PM(1,K)+DIST)*DIR1/(AORORF**2))+(PM(2,K)*DIR2/
     .(BELLP**2))+(PM(3,K)*DIR3/(CELLP**2)))
      CRR=((PM(1,K)**2+DIST**2+2.0*PM(1,K)*DIST)/(AORORF**2))-
     .(PM(2,K)**2)/(BELLP**2)-((PM(3,K)**2)/(CELLP**2))-1.0
      RR=(-BRR+DSQRT(BRR**2-4.0*ARR*CRR))/(ARR+ARR)
      P(1,K)=PM(1,K)-RR*DIR1
      P(2,K)=PM(2,K)-RR*DIR2
      P(3,K)=PM(3,K)-RR*DIR3
33    CONTINUE
      RETURN
      END
```

CASSA

```
      SUBROUTINE CASSA(P)
      IMPLICIT REAL *8 (A-H,O-Z)
      REAL*8 NHAT2(3),MAGSR,NMAG2,NORM,NHAT(3)
      INTEGER SURFC1,SURFC2
      COMMON/PARAMS/AORORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .        FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AOROR2,BELLP2,CELLP2,
     .        PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      COMMON/COLOS/DELT,XC,ANGINC,PM(3,4),RS,XMX,ZMX,ZKN,YMX
      COMMON/CASS/SR(3),ER(3),X0,Y0,Z0,Y,Z,RM,D,X02,Y02,Z02,ER2(3)
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      DIMENSION DC(3),EI2(3),P(5,2750),SR2(3),C(3)
      MAGSR=DSQRT(SR(1)*SR(1)+SR(2)*SR(2)+SR(3)*SR(3))
      DO 5 N=1,3
C     FIND DIRECTION COSINES
5     DC(N)=SR(N)/MAGSR
      GO TO (10,20,30,40,50,60),SURFC2
10    AA=0.0
      BB=NORM(1)*DC(1)+NORM(2)*DC(2)+NORM(3)*DC(3)
      CC=(X0-POINT(1))*NORM(1)+(Y0-POINT(2))*NORM(2)+
     .(Z0-POINT(3))*NORM(3)
      GO TO 100
20    AA=(DC(1)**2/AOROR2**2)+(DC(2)**2/BELLP2**2)+(DC(3)**2/CELLP2**2)
      BB=2.0*((X0*DC(1)/AOROR2**2)+(Y0*DC(2)/BELLP2**2)+
     .(Z0*DC(3)/BELLP2**2))
      CC=(X0**2/AOROR2**2)+(Y0**2/BELLP2**2)+(Z0**2/BELLP2**2)
      GO TO 100
30    AA=DC(1)*DC(1)+DC(2)*DC(2)+DC(3)*DC(3)
      BB=2.0*(X0*DC(1)+Y0*DC(2)+Z0*DC(3))
      CC=X0*X0+Y0*Y0+Z0*Z0-(AOROR2)**2
      GO TO 100
40    AA=DC(2)**2+DC(3)**2
      BB=2.0*(Y0*DC(2)+Z0*DC(3)-2.0*AOROR2*DC(1))
      CC=Y0*Y0+Z0*Z0-(4.0*AOROR2**2 )-(4.0*AOROR2*X0)
      GO TO 100
50    SNPSI2=DSIN(PSI2*DTOR)
      CSPSI2=DCOS(PSI2*DTOR)
      AA=(DC(3)*DC(3)*CSPSI2*CSPSI2)+(DC(2)*DC(2)*SNPSI2*SNPSI2)-
     .(2.0*DC(2)*DC(3)*CSPSI2*SNPSI2)
      BB=2.0*Z0*DC(3)*CSPSI2*CSPSI2+2.0*Y0*DC(2)*SNPSI2*SNPSI2-
     .2.0*(Y0*DC(3)+Z0*DC(2))*CSPSI2*SNPSI2-(4.0*AOROR2*DC(1))
      CC=(Z0**2)*(CSPSI2**2)+(Y0**2)*(SNPSI2**2)-2.0*(Y0*Z0*CSPSI2*
     .SNPSI2)-(4.0*AOROR2*AOROR2)-(4.0*AOROR2*X0)
      GO TO 100
60    AA=(DC(1)**2/AOROR2**2)+(DC(2)**2/BELLP2**2)+(DC(3)**2/CELLP2**2)
      BB=2.0*((X0*DC(1)/AOROR2**2)+(DIST2*DC(1)/AOROR2**2)-
     .(Y0*DC(2)/BELLP2**2)-(Z0*DC(3)/CELLP2**2))
      CC=(X0*X0/AOROR2**2)+(DIST2**2/AOROR2**2)-(Y0**2/BELLP2**2)
     .-(Z0**2/CELLP2**2)-1.0
100   IF(DABS(AA).LT.1.0D-10) RM=-CC/BB
      IF(DABS(AA).LT.1.0D-10) GO TO 110
      V2=BB*BB-4.0*AA*CC
      IF(V2.LT.0.0) V2=0.0
      RM=(-BB+DSQRT(V2))/(AA+AA)
110    CONTINUE
      X02=X0+RM*DC(1)
      Y02=Y0+RM*DC(2)
```

```
      Z02=Z0+RM*DC(3)
      GO TO (120,130,140,150,160,170),SURFC2
120   NHAT2(1)=NORM(1)
      NHAT2(2)=NORM(2)
      NHAT2(3)=NORM(3)
      GO TO 200
130   NHAT2(1)=-X02*BELLP2**2/DSQRT(X02**2*BELLP2**4+(Y02**2+Z02**2)*
     .ADROR2**4)
      NHAT2(2)=-Y02*ADROR2**2/DSQRT(X02**2*BELLP2**4+(Y02**2+Z02**2)*
     .ADROR2**4)
      NHAT2(3)=-Z02*ADROR2**2/DSQRT(X02**2*BELLP2**4+(Y02**2+Z02**2)*
     .ADROR2**4)
      GO TO 200
140   NHAT2(1)=-X02/ADROR2
      NHAT2(2)=-Y02/ADROR2
      NHAT2(3)=-Z02/ADROR2
      GO TO 200
150   NHAT2(1)=2.0*ADROR2/DSQRT(4.0*ADROR2**2+Y02**2+Z02**2)
      NHAT2(2)=        -Y02/DSQRT(4.0*ADROR2**2+Y02**2+Z02**2)
      NHAT2(3)=        -Z02/DSQRT(4.0*ADROR2**2+Y02**2+Z02**2)
      GO TO 200
160   NMAG2=DSQRT(4.0*ADROR2*ADROR2+(Z02*CSPSI2*SNPSI2-
     .Y02*SNPSI2*SNPSI2)**2+(Y02*SNPSI2*CSPSI2-Z02*CSPSI2*CSPSI2)**2)
      NHAT2(1)=2.0*ADROR2/NMAG2
      NHAT2(2)=SNPSI2*(Z02*CSPSI2-Y02*SNPSI2)/NMAG2
      NHAT2(3)=CSPSI2*(Y02*SNPSI2-Z02*CSPSI2)/NMAG2
      GO TO 200
170   DEN2=DSQRT(((X02+DIST2)**2/ADROR2**4)+(Y02*Y02/BELLP2**4)+
     .(Z02*Z02/CELLP2**4))
      NHAT2(1)=(X02+DIST2)/((ADROR2**2)*DEN2)
      NHAT2(2)=-Y02/((BELLP2**2)*DEN2)
      NHAT2(3)=-Z02/((CELLP2**2)*DEN2)
200   SCALA2=2.0*(DC(1)*NHAT2(1)+DC(2)*NHAT2(2)+DC(3)*NHAT2(3))
      DO 250 L=1,3
250   SR2(L)=DC(L)-SCALA2*NHAT2(L)
      EI2(N)=0.0
      DO 300 N=1,3
300   EI2(N)=ER(N)/RM
      DO 350 K=1,3
      SCALA3=2.0*(EI2(1)*NHAT2(1)+EI2(2)*NHAT2(2)+EI2(3)*NHAT2(3))
350   ER2(K)=SCALA3*NHAT2(K)-EI2(K)
      IF(DABS(SR2(1)).LT.1.0D-5) SR2(1)=1.0D-5
      Y=Y02+(XC-X02)*SR2(2)/SR2(1)
      Z=Z02+(XC-X02)*SR2(3)/SR2(1)
      D=DSQRT((XC-X02)**2+(Y-Y02)**2+(Z-Z02)**2)
      RETURN
      END
```

APRIN

```
      SUBROUTINE APRIN(P,ICALL)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 NORM
      INTEGER SURFC1,SURFC2
      COMMON/APRPRM/NPTPPL,NPERIM
      COMMON/PARAMS/AORORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .        FEED(3),ALPHA,BETA,GAMMA,XLAM,XC,AOROR2,BELLP2,CELLP2,
     .        PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      COMMON/FEED/EP(91),ET(91),NP,NT,XS,YS,ZS
      COMMON/CONTRL/NOPT(3),NLIST,ICPT,ICASS,ILIST(100)
      DIMENSION  P(5,2750)
      READ(1,10) NPERIM,SURFC1,NPTPPL
10    FORMAT(3I5)
      IF (NPERIM.LE.2)           GO TO 250
      IF (NPERIM.GT.40)          GO TO 260
      IF (SURFC1.GT.6)   GO TO 270
      IF (NPTPPL.GT.2500)        GO TO 270
      IF ((NPERIM*SURFC1).LE.0) GO TO 250
      READ(1,20) ((P(I,J),I=1,3),J=1,NPERIM)
20    FORMAT(3F10.6)
      M=1
      DO 2 I=2,NPERIM
      IF(P(3,M)-P(3,I))3,2,2
3     M=I
2     CONTINUE
      XS=P(1,M)
      YS=P(2,M)
      ZS=P(3,M)
28    GO TO (30,40,50,50,60,61),SURFC1
30    PLNORM(1)=(P(2,1)-P(2,2))*(P(3,1)-P(3,3))-
     .    (P(2,1)-P(2,3))*(P(3,1)-P(3,2))
      PLNORM(2)=(P(3,1)-P(3,2))*(P(1,1)-P(1,3))-
     .    (P(3,1)-P(3,3))*(P(1,1)-P(1,2))
      PLNORM(3)=(P(1,1)-P(1,2))*(P(2,1)-P(2,3))-
     .    (P(1,1)-P(1,3))*(P(2,1)-P(2,2))
      VMAG=DSQRT(PLNORM(1)**2+PLNORM(2)**2+PLNORM(3)**2)
      DO 35 K=1,3
      PLNORM(4-K)=PLNORM(4-K)/VMAG
      IF(PLNORM(1).LT.0.0) PLNORM(4-K)=-PLNORM(4-K)
35    CONTINUE
      PLNPNT(1)=P(1,1)
      PLNPNT(2)=P(2,1)
      PLNPNT(3)=P(3,1)
      GO TO 100
40    READ(1,45) AORORF,BELLP
45    FORMAT(2F10.3)
      GO TO 100
50    READ(1,55) AORORF
55    FORMAT(F10.3)
      GO TO 100
60    READ(1,65) AORORF,PSI
65    FORMAT(2F10.3)
      GO TO 100
61    READ(1,70) AORORF,BELLP,CELLP,DIST
70    FORMAT(4F10.3)
100   CONTINUE
```

```
199   IF(IDI(1,ICALL).EQ.0)  RETURN
      PRINT 200, ICALL
200   FORMAT('1',35X,'REFLECTOR PANEL NUMBER',I4)
      GO TO (320,330,340,350,360,370),SURFCI
250   PRINT 252,ICALL
252   FORMAT(///' ********** INPUT ERROR ON CARD ONE FOR PANEL NUMBER',
     .       I4,' EXECUTION TERMINATING **********')
      STOP
260   PRINT 262,ICALL
262   FORMAT(///' ********** STORAGE DOES NOT EXIST FOR NUMBER OF',
     .       ' PERIMETER POINTS SPECIFIED - PANEL',I4,' **********')
      STOP
270   PRINT 272,ICALL
272   FORMAT(///' ********** MAXIMUM ILLUMINATION REQUEST IS 2500',
     .       ' RAYS - PANEL',I4,' **********'////)
      NPTPPL=2500
      GO TO 28
320   PRINT 401,PLNPNT,PLNORM,NPERIM
      RETURN
330   PRINT 402,AORORF,BELLP,NPERIM
      RETURN
340   PRINT 403,AORORF,NPERIM
      RETURN
350   PRINT 404,AURORF,NPERIM
      RETURN
360   PRINT 405,AORORF,PSI,NPERIM
      RETURN
370   PRINT406,AORORF,BELLP,CELLP,DIST,NPERIM
      RETURN
401   FORMAT(///10X,'PANEL IS A PLANAR SURFACE',///
     .    '     A POINT ON THE REFLECTOR SURFACE (X,Y,Z)...........',3F7.2
     .   /'     COMPONENTS OF UNIT NORMAL TO SURFACE (X,Y,Z).......',3F7.2
     .   /'     NUMBER OF USER-SUPPLIED EDGE POINTS................',I7)
402   FORMAT(///10X,'PANEL IS AN ELLIPTICAL SECTION',///
     .    '     MAJOR AXIS OF ELLIPTICAL REFLECTOR.................',F7.2/
     .    '     MINOR AXIS OF ELLIPTICAL REFLECTOR.................',F7.2/
     .    '     NUMBER OF USER-SUPPLIED EDGE POINTS................',I7)
403   FORMAT(///10X,'PANEL IS A SPHERICAL SECTION'///
     .    '     RADIUS OF REFLECTOR SPHERE.........................',F7.2/
     .    '     NUMBER OF USER-SUPPLIED EDGE POINTS................',I7)
404   FORMAT(///10X,'PANEL IS A PARABOLIC SECTION',///
     .    '     FOCAL LENGTH OF THE PARABOLA.......................',F7.2/
     .    '     NUMBER OF USER-SUPPLIED EDGE POINTS................',I7)
405   FORMAT(///10X,'PANEL IS SECTION OF A PARABOLIC CYLINDER'///
     .    '     FOCAL LENGTH OF THE PARABOLA.......................',F8.3/
     .    '     FOCAL LINE ROTATION FROM Y-AXIS (PSI)..............',F8.3/
     .    '     NUMBER OF USER-SUPPLIED EDGE POINTS................',I7)
406   FORMAT(///10X,'PANEL IS A HYPERBOLIC SECTION'///
     .    '    MAJOR AXIS OF REFL.IN X DIRECTION...................',F8.3/
     .    '    AXIS OF REFLECTOR IN Y DIRECTION....................',F8.3/
     .    '     AXIS OF REFLECTOR IN Z DIRECTION...................',F8.3/
     .    '     NUMBER OF USER-SUPPLIED EDGE POINTS................',I7)
      END
```

FINDXC

```
      SUBROUTINE FINDXC(P,B)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/COLOS/DELT,XC,ANGINC,PM(3,4),RS,XMX,ZMX,ZMN,YMX
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      DIMENSION P(5,2750),B(3,2)
      IF(ICASS.NE.1) GO TO 15
      M=1
      DO 2 I=2,4
      IF(PM(3,M)-PM(3,I)) 3,2,2
3     M=I
2     CONTINUE
      RSM=DSQRT(PM(1,M)**2+PM(2,M)**2+PM(3,M)**2)-1.0
      THTMAX=DATAN(-PM(3,M)/PM(1,M))
      THTAUG=THTMAX+3.0*ANGINC
      XC=-RSM*DCOS(THTAUG)
      RETURN
15    RSM=DSQRT((XMX+B(1,2))**2+(YMX+B(2,2))**2+(ZMX+B(3,2))**2)-1.5
      THTMAX=DATAN(-(ZMX+B(3,2))/(XMX+B(1,2)))
      THTAUG=THTMAX+2.5*ANGINC*DTOR
      ZMN=-(RSM*DCOS(THTAUG)+B(1,2))
      RETURN
      END
```

IO1

```
      FUNCTION IO1 (INTENT,ITER)
      IMPLICIT REAL*8 (A-H,O-Z)
      INTEGER SURFC1,SURFC2
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      GO TO (20,30,40,50,60),INTENT
20    IF (NOPT(1).EQ.0) GO TO 90
      IF (NOPT(1).EQ.2) GO TO 91
22    DO 25 I=1,NLIST
      IF (ILIST(I).EQ.ITER) GO TO 91
25    CONTINUE
      GO TO 90
30    IF (NOPT(2).GT.0) GO TO 91
      GO TO 90
40    IF (NOPT(2).EQ.0) GO TO 90
      IF (NOPT(2).EQ.2) GO TO 91
      GO TO 22
50    IF (NOPT(3).GE.1) GO TO 91
      GO TO 90
60    IF (NOPT(3).EQ.2) GO TO 91
90    IO1=0
      RETURN
91    IO1=1
      RETURN
      END
```

INTGR

```
      SUBROUTINE INTGR(P,MAJOR,AMAJOR,AMINOR,FIELDY,FIELDZ)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 MAJOR,MINOR
      REAL*8 NORM
      COMPLEX*16 CTEMP,CZ1,CZ2,CY1,CY2,TSZ,TSY,DZI,DYI,ZIOLD,YIOLD,
     .          ZI,YI,FLDZ,FLDY,FIELDZ(200),FIELDY(200)
      INTEGER SURFC1,SURFC2
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      COMMON/PARAMS/AURORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .       FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AOROR2,BELLP2,CELLP2,
     .       PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      DIMENSION AMINOR(3),P(5,2750)
      DATA HPHI,HTHTA/5HPHI    ,5HTHETA/
      SEN=999.0
      NPARTS=7
      RPART=1.0/NPARTS
      ZLAM=PI2/XLAM
      CALL SETM(SEN,P(1,NPOINT+1),5)
      DEG=AMAJOR
      DEGR=DEG*DTOR
      DLOR=AMINOR(1)*DTOR
      DICR=AMINOR(3)*DTOR
      DSTOPR=AMINOR(2)*DTOR+DICR*0.5
      NTH=0
      D=DLOR
      IF (MAJOR.NE.HPHI) GO TO 3400
400   COSP=DCOS(DEGR)
      SINP=DSIN(DEGR)
      COST=DCOS(D)
      SINT=DSIN(D)
      GO TO 3425
3400  COSP=DCOS(D)
      SINP=DSIN(D)
      COST=DCOS(DEGR)
      SINT=DSIN(DEGR)
3425  NTH=NTH+1
      CTSP=COST*SINP
      ZK=ZLAM*COST
      YK=ZLAM*SINP*SINT
      IOLD=1
      INEW=2
      FLDY=(0.0,0.0)
      FLDZ=(0.0,0.0)
      YOLD=SEN
      YI=(0.0,0.0)
      ZI=(0.0,0.0)
3450  CONTINUE
      IF (P(1,IOLD).NE.P(1,INEW)) GO TO 4000
      Z=P(2,IOLD)
      ERY=P(3,IOLD)
      ERZ=P(4,IOLD)
      PH=P(5,IOLD)
      DZ=(P(2,INEW)-Z)*RPART
      DERY=(P(3,INEW)-ERY)*RPART
      DERZ=(P(4,INEW)-ERZ)*RPART
```

```
      DPH=(P(5,INEW)-PH)*RPART
      CTEMP=CDEXP(DCMPLX(0.0D0,ZK*Z-PH))
      CZ1=ERZ*COSP*CTEMP
      CY1=(ERY*SINT+ERZ*CTSP)*CTEMP
      TSY=(0.0,0.0)
      TSZ=(0.0,0.0)
      DO 3700 N=1,NPARTS
      Z=Z+DZ
      ERY=ERY+DERY
      ERZ=ERZ+DERZ
      PH=PH+DPH
      CTEMP=CDEXP(DCMPLX(0.0D0,ZK*Z-PH))
      CZ2=ERZ*COSP*CTEMP
      CY2=(ERY*SINT+ERZ*CTSP)*CTEMP
      TSZ=TSZ+CZ1+CZ2
      TSY=TSY+CY1+CY2
      CZ1=CZ2
      CY1=CY2
3700  CONTINUE
      ZI=ZI+TSZ*(0.5*DZ)
      YI=YI+TSY*(0.5*DZ)
3900  IOLD=IOLD+1
      INEW=INEW+1
      GO TO 3450
4000  CONTINUE
      YNEW=P(1,IOLD)
      IF (YOLD.EQ.SEN) GO TO 4400
4200  DZI=(ZI-ZIOLD)*RPART
      DYI=(YI-YIOLD)*RPART
      DY=(YNEW-YOLD)*RPART
      CTEMP=CDEXP(DCMPLX(0.0D0,YK*YOLD))
      CZ1=ZIOLD*CTEMP
      CY1=YIOLD*CTEMP
      TSY=(0.0,0.0)
      TSZ=(0.0,0.0)
      DO 4300 N=1,NPARTS
      YOLD=YOLD+DY
      ZIOLD=ZIOLD+DZI
      YIOLD=YIOLD+DYI
      CTEMP=CDEXP(DCMPLX(0.0D0,YK*YOLD))
      CZ2=ZIOLD*CTEMP
      CY2=YIOLD*CTEMP
      TSZ=TSZ+CZ1+CZ2
      TSY=TSY+CY1+CY2
      CZ1=CZ2
      CY1=CY2
4300  CONTINUE
      FLDZ=FLDZ+TSZ*(.5*DY)
      FLDY=FLDY+TSY*(.5*DY)
4400  CONTINUE
      YOLD=YNEW
      ZIOLD=ZI
      YIOLD=YI
      YI=(0.0,0.0)
      ZI=(0.0,0.0)
      IF (P(3,INEW).NE.SEN) GO TO 3900
```

```
      FIELDY(NTH)=FLDY
      FIELDZ(NTH)=FLDZ
      D=D+DICR
      IF (D.GT.DSTOPR) GO TO 5000
      IF (MAJOR.EQ.MPHI) GO TO 400
      GO TO 3400
5000  CONTINUE
      RETURN
      END
```

FILLP

```
      SUBROUTINE FILLP(P,NPT)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/FEED/EP(91),ET(91),NP,NT,XS,YS,ZS
      COMMON/MATH/PI,PI2,PID2,DTOR,RTOD
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      DIMENSION P(5,NPT)
      DO 100  I=1,NPT
      PROJX=DSIN(P(1,I))*DCOS(P(2,I))
      PROJEX=0.0D0
5     IF (DABS(P(2,I)-PI).GT.1.0D-5)
     .PROJEX=DSIN(DATAN(DCOS(P(1,I))/DSIN(P(1,I))/DSIN(P(2,I))))
10    ANGLX=DARCOS(DABS(PROJX))*RTOD
      LO=ANGLX+1.0D0
      IHI=LO+1
      PPFLD=(ANGLX-DFLOAT(LO-1))*(EP(IHI)-EP(LO))+EP(LO)
      TPFLD=(ANGLX-DFLOAT(LO-1))*(ET(IHI)-ET(LO))+ET(LO)
      SINE2=PROJEX*PROJEX
      COSE2=1.0D0-SINE2
      P(3,I)=PPFLD*TPFLD/(DSQRT(TPFLD*TPFLD*COSE2+
     .        PPFLD*PPFLD*SINE2))
      P(4,I)=0.0D0
      P(5,I)=0.0D0
100   CONTINUE
      RETURN
      END
```

QUANTZ

```
      SUBROUTINE QUANTZ(P,NPERIM,ICALL)
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 NORM
      INTEGER SURFC1,SURFC2
      COMMON/BLOCKG/YCBL,ZCBL,HFMABL,HFMIBL
      COMMON/DIMENS/YDIM,ZDIM,YCT,ZCT
      COMMON/EXTENT/YMIN,YMAX,ZMIN,ZMAX
      COMMON/CONTRL/NOPT(3),NLIST,IOPT,ICASS,ILIST(100)
      COMMON/FEED/EP(91),ET(91),NP,NT,XS,YS,ZS
      COMMON/PARAMS/AORORF,BELLP,CELLP,DIST,PSI,PLNPNT(3),PLNORM(3),
     .       FEED(3),ALPHA,BETA,GAMMA,XLAM,XX,AOROR2,BELLP2,CELLP2,
     .       PSI2,DIST2,POINT(3),NORM(3),SURFC1,NPNL,NPOINT,SURFC2
      DIMENSION P(5,NPOINT),PINT(5),POLD(5),PBLK(5),PR5(5,41),Z(2,101)
      IF(ICASS.EQ.1) NPERIM=4
      NBARS=NP-2
      YMIN=1.0D+10
      YMAX=-1.0D+10
      ZMIN=1.0D+10
      ZMAX=-1.0D+10
      NOS=2*NBARS
      CALL SETM(1.0D+20,Z,NOS)
      DO 20  I=1,NPERIM
      IF (P(1,I).GT.YMAX) YMAX=P(1,I)
      IF (P(1,I).LT.YMIN) YMIN=P(1,I)
      IF (P(2,I).GT.ZMAX) ZMAX=P(2,I)
      IF (P(2,I).LT.ZMIN) ZMIN=P(2,I)
20    CALL MOVEM(P(1,I),PR5(1,I),5)
      YDIM=YMAX-YMIN
      YCT=(YMAX+YMIN)/2.
      ZDIM=ZMAX-ZMIN
      ZCT=(ZMAX+ZMIN)/2.
      GRID=(YMAX-YMIN)/(DFLOAT(NBARS)-0.6)
      GRIDLO=YMIN+GRID/5.0D0
      GRIDHI=YMAX-GRID/5.0D0
31    IBGN=NPERIM+1
      NDEX=NPERIM
      DO 100  I=IBGN,NPOINT
      IF (P(1,I).GT.YMAX)  GO TO 98
      IF (P(1,I).LT.YMIN)  GO TO 98
      NGRID=(P(1,I)-GRIDLO)/GRID+0.5
      P(1,I)=GRIDLO+DFLOAT(NGRID)*GRID
      CALL MOVEM(P(1,I),P(1,I-NDEX),5)
      GO TO 100
98    NDEX=NDEX+1
100   CONTINUE
      NPOINT=NPOINT-NDEX
      CALL PTSORT(P,5,NPOINT)
      IF (IOPT.EQ.1)  GO TO 422
      CALL MOVEM(PR5(1,1),PR5(1,NPERIM+1),5)
      KDEX=2
      Y2=PR5(1,1)
      Z2=PR5(2,1)
200   Y1=PR5(1,KDEX)
      Z1=PR5(2,KDEX)
      IF (DABS(Y1-Y2).LT.1.0D-5)  GO TO 400
      SLOPE=(Z1-Z2)/(Y1-Y2)
```

```
      B=Z2-SLOPE*Y2
      IF (Y1-Y2) 220,230,230
220   YHI=Y2
      YLO=Y1
      GO TO 240
230   YHI=Y1
      YLO=Y2
240   INDEX=(YLO-GRIDLO)/GRID+1.0
250   INDEX=INDEX+1
      YQ=GRIDLO+DFLOAT(INDEX-1)*GRID
      IF (YQ.GT.YHI) GO TO 400
      ILOAD=1
      ZEE=SLOPE*YQ+B
      IF (Z(1,INDEX).LT.1.0D+10) ILOAD=2
      Z(ILOAD,INDEX)=ZEE
      GO TO 250
400   Y2=Y1
      Z2=Z1
      KDEX=KDEX+1
      IF (KDEX.LE.NPERIM+1)  GO TO 200
      DO 420  I=1,NBARS
      IF ((Z(1,I)+Z(2,I)).GT.1.0D+10) GO TO 1005
      IF (Z(2,I)-Z(1,I)) 410,420,420
410   Z2=Z(2,I)
      Z(2,I)=Z(1,I)
      Z(1,I)=Z2
420   CONTINUE
      GO TO 444
422   HFMAEX=YDIM/2.0D0
      HFMIEX=ZDIM/2.0D0
      DO 430  I=1,NBARS
      Y=GRIDLO+DFLOAT(I-1)*GRID
      V3=1.0D0-((Y-YCT)/HFMAEX)**2
      IF(V3.LT.0.0) V3=0.0
      ZZ=HFMIEX*DSQRT(V3)
      Z(1,I)=-ZZ+ZCT
      Z(2,I)= ZZ+ZCT
430   CONTINUE
444   L=0
      N=1
      CALL SETN(0.0,PBLK,5)
      YQ=P(1,1)
      IDEX=DINT((YQ-GRIDLO)/GRID+1.001)
      DO 900  I=1,NPOINT
      IF (P(1,I).EQ.YQ) GO TO 480
      IF (L.GT.2)       GO TO 470
      N=N-L
470   L=0
      YQ=P(1,I)
      IDEX=DINT((YQ-GRIDLO)/GRID+1.001)
480   PBLK(1)=P(1,I)
      PBLK(2)=P(2,I)
      TEST=-1.0
      IF (P(2,I).EQ.Z(1,IDEX).OR.P(2,I).EQ.Z(2,IDEX))  TEST= 0.0
      IF (P(2,I).GT.Z(1,IDEX).AND.P(2,I).LT.Z(2,IDEX)) TEST=1.0
      TESTBL=HFMABL*HFMABL*HFMIBL*HFMIBL
```

```
     .                        -HFMABL*HFMABL*(P(2,I)-ZCBL)*(P(2,I)-ZCBL)
     .                        -HFMIBL*HFMIBL*(P(1,I)-YCBL)*(P(1,I)-YCBL)
      IF (TEST) 701,501,501
501   IF (TESTBL.LE.0.0) GO TO 510
      CALL MOVEM(PBLK,P(1,N),5)
      GO TO 515
510   CALL MOVEM(P(1,I),P(1,N),5)
515   N=N+1
      L=L+1
      IF (TEST.EQ.0.0) GO TO 800
701   IF (L.EQ.0)       GO TO 800
      IF (TEST*TESTO) 704,800,800
704   CALL INTPL(POLD,P(1,I),PINT,Z(1,IDEX))
      NCHG=0
      IF (TEST.LT.0.0) GO TO 711
      CALL MOVEM(P(1,N-1),P(1,N),5)
      NCHG=1
711   CALL MOVEM(PINT,PBLK,2)
      TESTBL=HFMABL*HFMABL*HFMIBL*HFMIBL
     .                        -HFMABL*HFMABL*(PINT(2)-ZCBL)*(PINT(2)-ZCBL)
     .                        -HFMIBL*HFMIBL*(PINT(1)-YCBL)*(PINT(1)-YCBL)
      IF (TESTBL.LE.0.0)  GO TO 720
      CALL MOVEM(PBLK,P(1,N-NCHG),5)
      GO TO 725
720   CALL MOVEM(PINT,P(1,N-NCHG),5)
725   N=N+1
      L=L+1
800   CALL MOVEM(P(1,I),POLD,5)
      TESTO=TEST
900   CONTINUE
      NPOINT=N-1
      NLOC=5*NPERIM
      CALL MOVEM(PR5,P(1,N),NLOC)
      IF(IOI(4,1).EQ.1) WRITE(7,951) NPOINT,ICALL
      IF(IOI(4,1).EQ.1) WRITE(7,952) ((P(I,J),I=1,5),J=1,NPOINT)
      IF(IOI(1,ICALL).EQ.0)  RETURN
      PRINT 950, YMIN,YMAX,ZMIN,ZMAX,GRIDLO,GRIDHI,GRID,NBARS,NPOINT
950   FORMAT(//' QUANTIZING DATA-'//
     .   '  POINT PATTERN EXTENTS ON APERTURE PLANE.......YMIN=',F7.2/
     .   '                                            .......YMAX=',F7.2/
     .   '                                            .......ZMIN=',F7.2/
     .   '                                            .......ZMAX=',F7.2/
     .   '  GRID RANGES FROM...........................',F8.3,'  TO
     .',  F8.3/
     .   '  SPACING BETWEEN GRID BARS IS........................',F8.4/
     .   '  THEREFORE NUMBER OF GRID BARS.......................',I6/
     .   '  NUMBER OF POINTS SUPPLIED TO RADPAT.................',I7  )
951   FORMAT(2I8)
952   FORMAT(5D16.10)
      RETURN
1005  PRINT 1006
1006  FORMAT(///
     .  ' **********  Z ARRAY FOR POINT RESIDENCE NOT FILLED CORRECTLY
     . **********',//10X,'- STOP EXECUTION -')
      STOP
      END
```

8.7. APPENDIX G

OUTPUT FOR TEST CASES A AND B

```
FAR FIELD RADIATION PATTERN CALCULATION
-----------------------------------------------------------------------------------------

     CASSEGRAIN ANTENNA EXAMPLE....
  A PARABOLOID- HYPERBOLOID COMBINATION
  FEBRUARY 17,1981 NCSU        PGMR:CHRISTOS          FCLTY: RO-DAN BRT-HILLSBURG

INPUT PARAMETERS-

  WAVELENGTH OF ELECTRIC FIELD.........................   4.7340
  LOCATION OF COORDINATE ORIGIN WRT FEED (X,Y,Z).......  -91.005      0.0        0.0
  FEED ROTATION ANGLES(ALPHA,BETA,GAMMA)...............   0.0        0.0    -180.000
  APERTURE PLANE LOCATION(XC)..........................   0.0
  SUB DISH SHADOW CENTER COORDINATES IN APERT. PL......   0.0        0.0
  HALF MAJOR AXIS OF SUB DISH SHADOW...................   0.0
  HALF MINOR AXIS OF SUB DISH SHADOW...................   0.0
  NUMBER OF PANELS IN REFLECTOR........................    0


MAIN DISH DESCRIPTION AND ITS PARAMETERS-

  IT IS A PARABOLIC REFLECTOR
  FOCAL LENGTH OF THE REFLECTOR........................ 100.000
  *** PROGRAM IN SINGLE PANEL MODE ***
  MINIMUM-Y POINT ON THE REFLECTOR (X,Y,Z).............  -65.600-117.304      0.0
  MAXIMUM-Y POINT ON THE REFLECTOR (X,Y,Z).............  -65.600 117.304      0.0
  MAXIMUM-Z POINT ON THE REFLECTOR (X,Y,Z).............  -65.600    0.0   -117.304
  MINIMUM-Z POINT ON THE REFLECTOR (X,Y,Z).............  -65.600    0.0    117.304
```

```
SUBDISH DESCRIPTION AND ITS PARAMETERS-

   IT IS A HYPERBOLIC REFLECTOR
   MAJOR AXIS OF REFL. IN X DIRECTION..................   70.298
   AXIS OF REFLECTOR IN Y DIRECTION....................   33.950
   AXIS OF REFLECTOR IN Z DIRECTION....................   73.950
   DISTANCE USED FOR TRANSLATION OF ORIG. OF AXES......   45.500
   *****  PROGRAM IN SINGLE PANEL MODE  *****
   MINIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)............ -10.712  -19.156    0.0
   MAXIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)............ -10.712   19.156    0.0
   MAXIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)............ -10.712    0.0    -19.156
   MINIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)............ -10.712    0.0     19.156

NUMBER OF PATTERN GROUPS REQUESTED.....................   2

   THETA =   90.0000          PHI    FROM   -4.0000 TO    4.0000 BY    0.2500
   PHI   =    0.0             THETA  FROM   86.0000 TO   94.0000 BY    0.2500

ILLUMINATION DATA-

   THETA ILLUMINATION FROM.......................   76.075   TO   103.925
   PHI ILLUMINATION FROM.........................  166.075   TO   193.925
   INCREMENTAL ANGLE (DEG).......................   0.6329
   THEREFORE TOTAL NUMBER OF GENERATED RAYS......    2029
   TOTAL NUMBER OF APERTURE PLANE POINTS.........    2029

   --------------- FINISHED APERTUR ---------------
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POINT PATTERN EXTENTS ON APERTURE PLANE........YMIN=-117.29
                                      .......YMAX= 117.29
                                      .......ZMIN=-117.29
                                      .......ZMAX= 117.29
GRID RANGES FROM....................................-116.182  TO  116.182
SPACING BETWEEN GRID BARS IS...........................  5.5323
THEREFORE NUMBER OF GRID BARS..........................  43
NUMBER OF POINTS SUPPLIED TO RADPAT....................  1491

    ---------------- FINISHED QUANTIZ ----------------
    ---------------- FINISHED INTGRT  ----------------
    ---------------- FINISHED INTGRT  ----------------

    ---------- PATTERN COMPUTATIONS COMPLETE ----------
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TABLE OF ELECTRIC FIELD STRENGTHS (DB)

PRINCIPAL PLANE OF CUT IS THETA = 90.000 DEG

ANGLE PHI FROM -4.000 TO 4.000 BY 0.250 DEG

PHI	DB(Z/Z)	DB(Y/Z)	DB(Z/Y)	DB(Y/Y)	PWRDB
-4.000	-31.27215	-152.69030	121.41815	0.0	-31.27215
-3.750	-56.65126	-152.69030	96.03904	0.0	-56.65126
-3.500	-28.91043	-152.69030	123.77967	0.0	-28.91043
-3.250	-24.31747	-152.69030	128.37283	0.0	-24.31747
-3.000	-24.12432	-152.69030	128.56598	0.0	-24.12432
-2.750	-29.65977	-152.69030	123.03053	0.0	-29.65977
-2.500	-33.07042	-152.69030	119.61988	0.0	-33.07042
-2.250	-21.40500	-152.69030	131.28530	0.0	-21.40500
-2.000	-17.84811	-152.69030	134.84219	0.0	-17.84811
-1.750	-18.28170	-152.69030	134.40860	0.0	-18.28170
-1.500	-27.01067	-152.69030	125.67963	0.0	-27.01067
-1.250	-19.24178	-152.69030	133.44852	0.0	-19.24178

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-1.000	-9.78822	-152.69030	142.90208	0.0	-9.78822
-0.750	-4.94977	-152.69030	147.74053	0.0	-4.94977
-0.500	-2.07367	-152.69030	150.61662	0.0	-2.07367
-0.250	-0.50262	-152.69030	152.18768	0.0	-0.50262
0.0	0.0	-152.69030	152.69030	0.0	0.0
0.250	-0.50262	-152.69030	152.18768	0.0	-0.50262
0.500	-2.07367	-152.69030	150.61662	0.0	-2.07367
0.750	-4.94977	-152.69030	147.74053	0.0	-4.94977
1.000	-9.78822	-152.69030	142.90208	0.0	-9.78822
1.250	-19.24179	-152.69030	133.44852	0.0	-19.24178
1.500	-27.01067	-152.69030	125.67963	0.0	-27.01067
1.750	-18.28170	-152.69030	134.40860	0.0	-18.28170
2.000	-17.84811	-152.69030	134.84219	0.0	-17.84811
2.250	-21.40500	-152.69030	131.28530	0.0	-21.40500
2.500	-33.07042	-152.69030	119.61988	0.0	-33.07042
2.750	-29.65977	-152.69030	123.03053	0.0	-29.65977
3.000	-24.12432	-152.69030	128.56598	0.0	-24.12432
3.250	-24.31747	-152.69030	128.37283	0.0	-24.31747
3.500	-28.91043	-152.69030	123.77987	0.0	-28.91043
3.750	-56.65126	-152.69030	96.03904	0.0	-56.65126
4.000	-31.27215	-152.69030	121.41815	0.0	-31.27215

MAXIMUM FIELD VALUES-

20LOG(MAX(FIELD-Z))=20LOG(4.3103737D 04)= 92.6902984

20LOG(MAX(FIELD-Y))=20LOG(1.1131411D-13)= -60.0000000

INTERPOLATION NUMBER USED FOR INTEGRATION IS.......... 7

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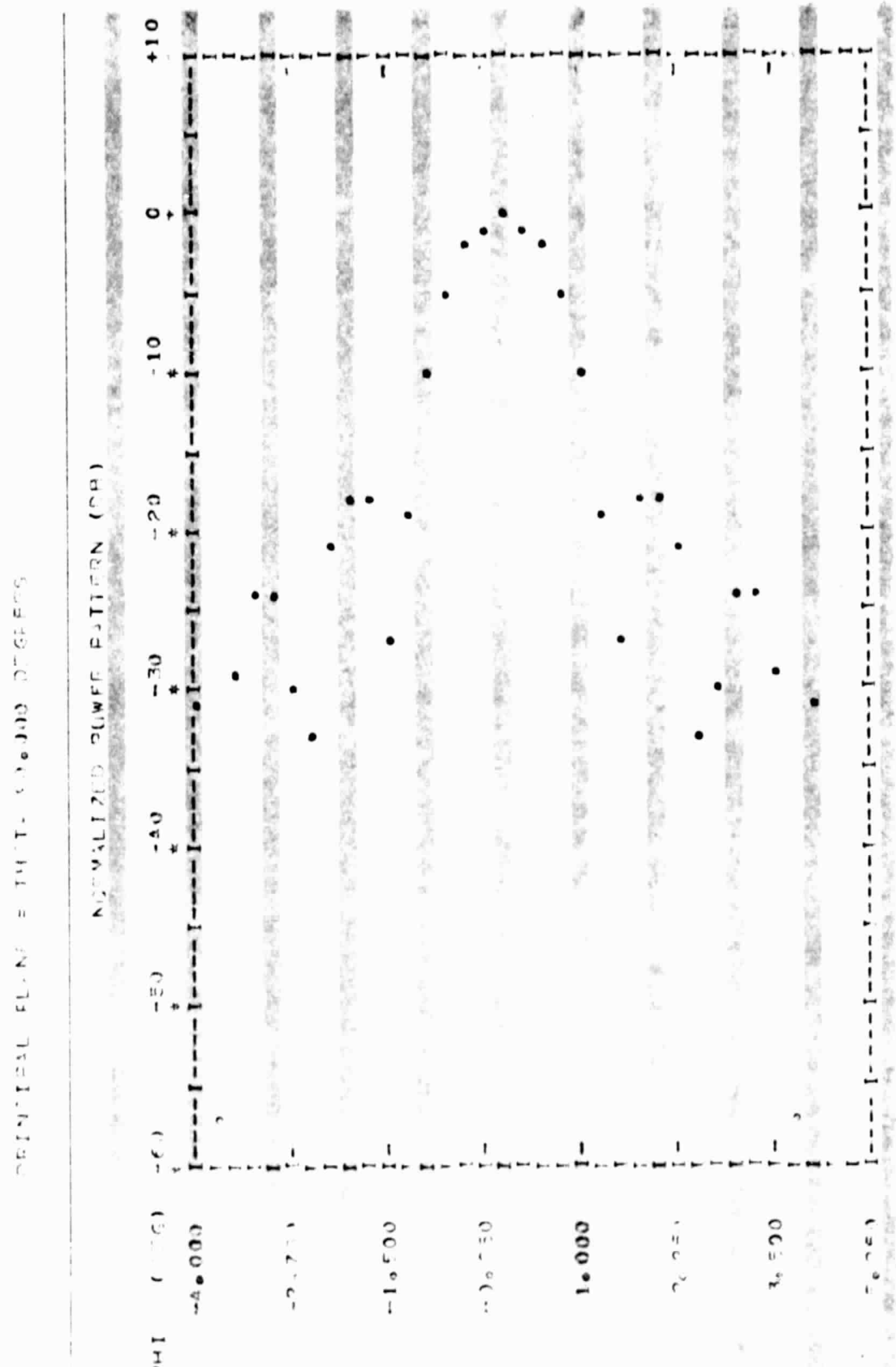

TABLE OF ELECTRIC FIELD STRENGTHS (DB)

PRINCIPAL PLANE OF CUT IS PHI = 0.0 DEG

ANGLE THETA FROM 86.000 TO 94.000 BY 0.250 DEG

THETA	DB(Z/Z)	DB(Y/Z)	DB(Z/Y)	DB(Y/Y)	PWRDB
86.000	-30.77263	-152.69030	121.91767	0.0	-30.77263
86.250	-57.96943	-152.69030	94.72086	0.0	-57.96943
86.500	-29.68609	-152.69030	123.00421	0.0	-29.68609
86.750	-24.69667	-152.69030	128.00363	0.0	-24.68667
87.000	-24.32675	-152.69030	128.36355	0.0	-24.32675
87.250	-29.74056	-152.69030	122.94974	0.0	-29.74056
87.500	-33.19706	-152.69030	119.50326	0.0	-33.19706
87.750	-21.40816	-152.69030	131.28214	0.0	-21.40816
88.000	-17.77584	-152.69030	134.91446	0.0	-17.77584
88.250	-18.09728	-152.69030	134.59302	0.0	-18.09728
88.500	-26.26741	-152.69030	126.42689	0.0	-26.26741
88.750	-19.62402	-152.69030	133.06628	0.0	-19.62402
89.000	-9.90730	-152.69030	142.78300	0.0	-9.90730
89.250	-5.00019	-152.69030	147.69011	0.0	-5.00019
89.500	-2.09307	-152.69030	150.59723	0.0	-2.09307
89.750	-0.50713	-152.69030	152.18317	0.0	-0.50713
90.000	0.0	-152.69030	152.69030	0.0	0.0

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90.250	-0.50713	-152.69030	152.18317	0.0	-0.50713
90.500	-2.09307	-152.69030	150.59723	0.0	-2.09307
90.750	-5.00019	-152.69030	147.69011	0.0	-5.00019
91.000	-9.90730	-152.69030	142.78300	0.0	-9.90730
91.250	-19.62402	-152.69030	133.06628	0.0	-19.62402
91.500	-26.26341	-152.69030	126.42689	0.0	-26.26341
91.750	-18.09728	-152.69030	134.59302	0.0	-18.09728
92.000	-17.77584	-152.69030	134.91446	0.0	-17.77584
92.250	-21.40816	-152.69030	131.28214	0.0	-21.40816
92.500	-33.18706	-152.69030	119.50324	0.0	-33.18706
92.750	-29.74056	-152.69030	122.94974	0.0	-29.74056
93.000	-24.32675	-152.69030	128.36355	0.0	-24.32675
93.250	-24.68667	-152.69030	128.00363	0.0	-24.68667
93.500	-29.68609	-152.69030	123.00421	0.0	-29.68609
93.750	-57.96943	-152.69030	94.72086	0.0	-57.96943
94.000	-30.77263	-152.69030	121.91767	0.0	-30.77263

MAXIMUM FIELD VALUES-

20LOG(MAX(FIELD-Z))=20LOG(4.31037370 04)= 92.6902984

20LOG(MAX(FIELD-Y))=20LOG(0.0)= -60.0000000

INTERPOLATION NUMBER USED FOR INTEGRATION IS.......... 7

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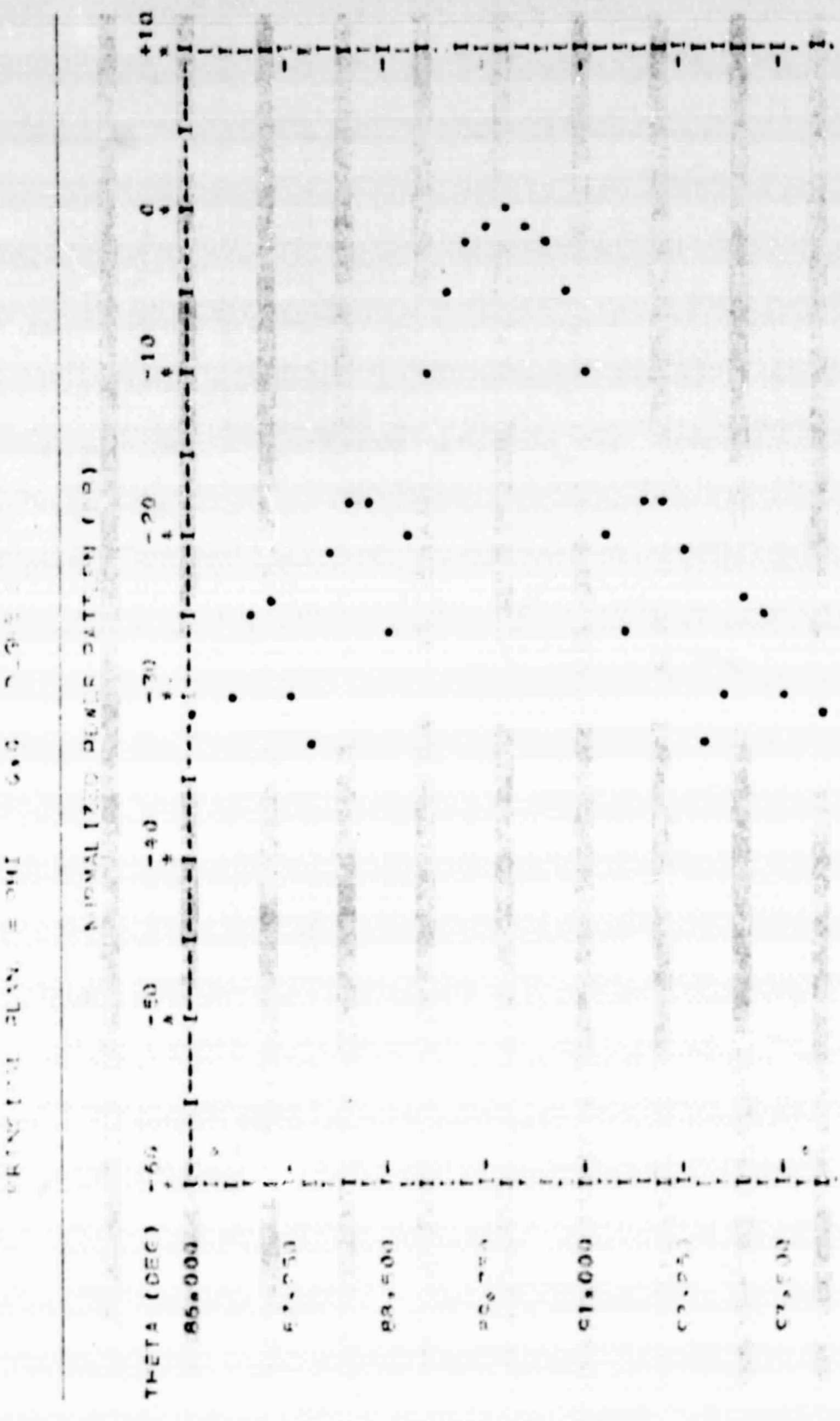
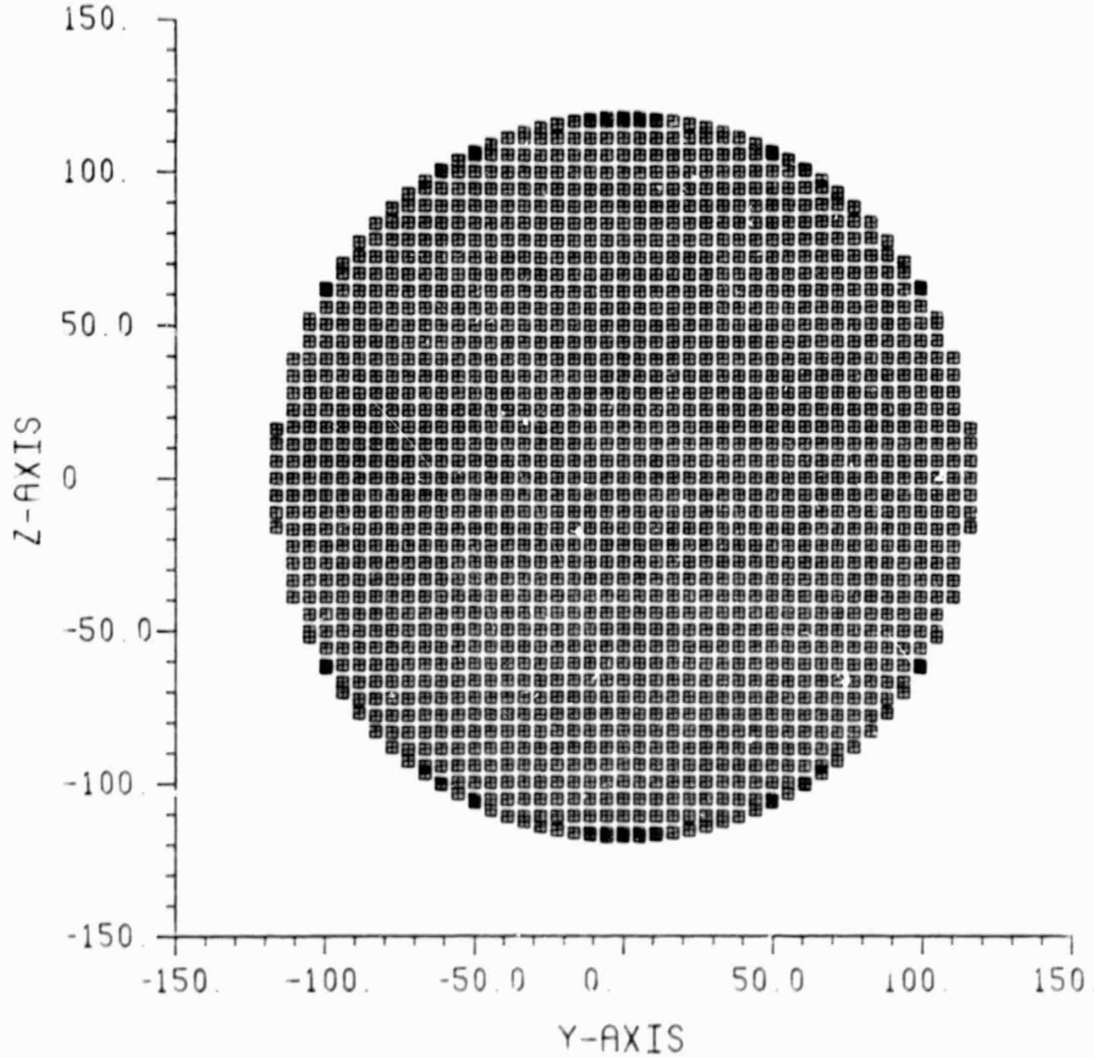


APERTURE PLANE AFTER QUANTIZING



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FAR FIELD RADIATION PATTERN CALCULATION

 OFFSET CASSEGRAIN ANTENNA EXAMPLE....
 A PARABOLOID- HYPERBOLOID COMBINATION
 MARCH 29 1981    NCSU     PGMR:CHRISTUS       FCLTY: RD-DAN PRT-HILLSBRO
               TICRA AP/S

INPUT PARAMETERS-

 WAVELENGTH OF ELECTRIC FIELD.........................   0.9843
 LOCATION OF COORDINATE ORIGIN WRT FEED (X,Y,Z)...... -31.725    0..      9.337
 FEED ROTATION ANGLES(ALPHA,BETA,GAMMA)...........    0.0     0.0   -163.600
 APERTURE PLANE LOCATION(XC)........................   0.0
 SUB DISH SHADOW CENTER COORDINATES IN APERT. PL.....   0.0     0.0
 HALF MAJOR AXIS OF SUB DISH SHADOW ................   0.0
 HALF MINOR AXIS OF SUB DISH SHADOW.................   0.0
 NUMBER OF PANELS IN REFLECTOR......................     0
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MAIN DISH DESCRIPTION AND ITS PARAMETERS-

  IT IS A PARABOLIC REFLECTOR
  FOCAL LENGTH OF THE REFLECTOR......................   69.685
  ..... PROGRAM IN SINGLE PANEL MODE .....
  MINIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)........... -57.190  -31.587   49.660
  MAXIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)........... -57.190   31.587   49.660
  MAXIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)........... -45.828    0.0     81.547
  MINIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)........... -63.552    0.0     17.773

SUBDISH DESCRIPTION AND ITS PARAMETERS-

  IT IS A HYBERBOLIC REFLECTOR
  MAJOR AXIS OF REFL. IN X DIRECTION.................    6.690
  AXIS OF REFLECTOR IN Y DIRECTION...................   15.120
  AXIS OF REFLECTOR IN Z DIRECTION...................   15.120
  DISTANCE USED FOR TRANSLATION OF ORIG. OF AXES.....   16.535
  ..... PROGRAM IN SINGLE PANEL MODE .....
  MINIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)...........  -8.740   -4.873    7.590
  MAXIMUM-Y POINT ON THE REFLECTOR (X,Y,Z)...........  -8.740    4.873    7.590
  MAXIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)...........  -7.570    0.0     13.470
  MINIMUM-Z POINT ON THE REFLECTOR (X,Y,Z)...........  -9.748    0.0      2.527
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PATTERN OF FEED IN ONE DEG INCREMENTS OFF-AXIS-

E-PLANE

1.0000000000	0.9903500000	0.9626800000	0.9179300000	0.8587800000
0.7883000000	0.7099700000	0.6273700000	0.5439200000	0.4626900000
0.3861700000	0.3162500000	0.2540700000	0.2002900000	0.1549100000
0.1175600000	0.0875500000	0.0639400000	0.0456300000	0.0322300000
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0				

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PATTERN OF FEED IN ONE DEG INCREMENTS OFF-AXIS-

E-PLANE

1.0000000000	0.9905000000	0.9823800000	0.9179300000	0.8587800000
0.7883000000	0.7099400000	0.6273700000	0.5439200000	0.4626900000
0.3881700000	0.3162000000	0.2540700000	0.2002900000	0.1549100000
0.1175600000	0.0875000000	0.0539400000	0.0456300000	0.0322300000
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
.0				

NUMBER OF PATTERN GROUPS REQUESTED................ 2

PHI = .0 THETA FROM 88.0000 TO 92.0000 BY 0.0800
THETA = 90.0000 PHI FROM -2.0000 TO 2.0000 BY 0.0800

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ILLUMINATION DATA-

  THETA ILLUMINATION FROM.......................   79.379  TO   101.848
  PHI ILLUMINATION FROM.........................  168.765  TO   191.235
  INCREMENTAL ANGLE (DEG)........................ 0.5107
  THEREFORE TOTAL NUMBER OF GENERATED RAYS.............   2025
  TOTAL NUMBER OF APERTURE PLANE POINTS................   2029

    ---------------- FINISHED APERTUR ----------------

QUANTIZING DATA-

  POINT PATTERN EXTENTS ON APERTURE PLANE........YMIN= -31.89
                                          .......YMAX=  31.89
                                          .......ZMIN=  17.77
                                          .......ZMAX=  81.55
  GRID RANGES FROM............................ -31.586  TO    31.586
  SPACING BETWEEN GRID BARS IS.......................   1.5041
  THEREFORE NUMBER OF GRID BARS......................    43
  NUMBER OF POINTS SUPPLIED TO RADPAT................  1499

    ---------------- FINISHED QUANTIZ ----------------
    ---------------- FINISHED INTGRT ----------------
    ---------------- FINISHED INTGRT ----------------

  ---------- PATTERN COMPUTATIONS COMPLETE ----------
```

TABLE OF ELECTRIC FIELD STRENGTHS (DB)

PRINCIPAL PLANE OF CUT IS PHI = 0.0 DEG

ANGLE THETA FROM 88.000 TO 92.000 BY 0.080 DEG

THETA	DB(Z/Z)	DB(Y/Z)	DB(Z/Y)	DB(Y/Y)	PWRDB
88.000	-25.97300	154.09250	-218.43254	-33.41645	-33.41545
88.080	-23.78100	157.02290	-218.29140	-29.88584	-29.88584
88.160	-20.23700	170.33051	-217.74536	-27.14724	-27.14724
88.240	-19.45170	172.25243	-216.95350	-25.25631	-25.25631
88.320	-18.64490	173.43997	-216.11368	-24.09678	-24.09378
88.400	-17.84000	173.66197	-215.34874	-23.62678	-23.62678
88.480	-17.23009	173.64017	-214.73883	-23.86557	-23.86557
88.560	-16.78000	172.64493	-214.26893	-24.86377	-24.86377
88.640	-16.42400	171.20337	-213.93343	-25.29937	-25.29937
88.720	-16.01644	171.49967	-213.52519	-26.00688	-26.00888
88.800	-15.34700	174.99031	-212.85388	-22.51823	-22.51823
88.880	-14.25673	179.00046	-211.75547	-18.50029	-18.50029
88.960	-12.75500	182.42007	-210.27460	-15.08007	-15.08007
89.040	-11.05670	185.25200	-208.55950	-12.25770	-12.25770
89.120	-9.30120	187.80030	-206.81001	-9.90723	-9.90723
89.200	-7.64117	189.57995	-205.14991	-7.92879	-7.92879
89.280	-6.13090	191.25503	-203.63965	-6.25306	-6.25306
89.360	-4.79299	192.67570	-202.30174	-4.83204	-4.83204
89.440	-3.63147	193.87592	-201.14021	-3.63183	-3.63183
89.520	-2.64255	194.88077	-200.15128	-2.62797	-2.62797
89.600	-1.81975	195.70519	-199.32855	-1.80256	-1.80256
89.680	-1.15630	196.36835	-198.66532	-1.14240	-1.14240
89.760	-0.64700	196.87090	-198.15577	-0.63764	-0.63764
89.840	-0.28644	197.22673	-197.79518	-0.28201	-0.28201
89.920	-0.07144	197.43546	-197.58018	-0.07028	-0.07028
90.000	0.0	197.50674	-197.50874	0.0	0.0

90.080	-0.07140	197.43344	-197.50020	-0.07030	-0.07030
90.160	-0.26640	197.22570	-197.79523	-0.28205	-0.28205
90.240	-0.64709	196.67084	-198.15504	-0.63790	-0.63790
90.320	-1.15661	195.36027	-198.50541	-1.14240	-1.14248
90.400	-1.81990	195.70609	-199.32865	-1.80206	-1.80236
90.480	-2.64200	194.88065	-200.15142	-2.62810	-2.62810
90.560	-3.63104	193.67376	-201.14033	-3.63198	-3.63196
90.640	-4.79319	192.67352	-202.30193	-4.83222	-4.83222
90.720	-6.13135	191.25347	-203.63967	-6.25327	-6.25327
90.800	-7.64142	189.57370	-205.15015	-7.92934	-7.92934
90.880	-9.30104	187.60122	-206.81120	-9.90753	-9.90753
90.960	-11.05405	185.25570	-208.55977	-12.25805	-12.25805
91.040	-12.70001	182.42326	-210.27485	-15.08049	-15.08049
91.120	-14.25694	179.00796	-211.75565	-18.50079	-18.50079
91.200	-15.34722	174.98995	-212.85595	-22.51877	-22.51877
91.280	-16.01644	171.49064	-213.52516	-26.00911	-26.00911
91.360	-16.42433	171.20961	-213.95535	-26.29913	-25.29913
91.440	-16.78042	172.64322	-214.28886	-24.86353	-24.86353
91.520	-17.23330	173.04329	-214.73576	-23.80545	-23.80545
91.600	-17.63990	173.08193	-215.34871	-23.62676	-23.62676
91.680	-18.60492	173.40339	-216.11367	-24.09886	-24.09886
91.760	-19.45173	172.25125	-216.95049	-25.25649	-25.25649
91.840	-20.23711	170.30122	-217.74556	-27.14752	-27.14752
91.920	-20.78104	167.60259	-218.29136	-29.88625	-29.88625
92.000	-20.97373	164.09204	-218.43250	-33.41699	-33.41599

MAXIMUM FIELD VALUES-

20LOG(MAX(FIELD-Z))=20LOG(1.4968921D-1)=-196.4961899

20LOG(MAX(FIELD-Y))=20LOG(1.1236414D 00)= 1.0125547

INTERPOLATION NUMBER USED FOR INTEGRATION IS.......... 7

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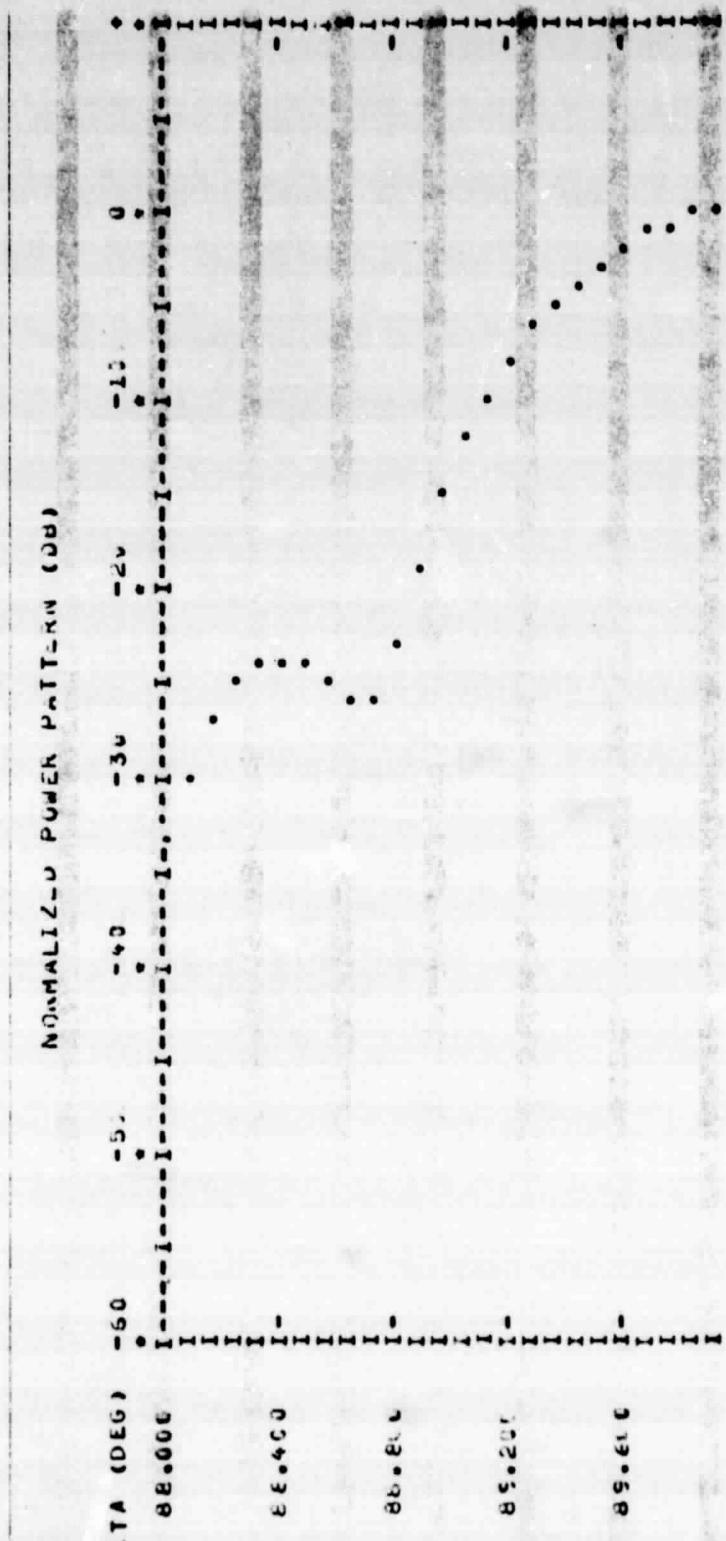

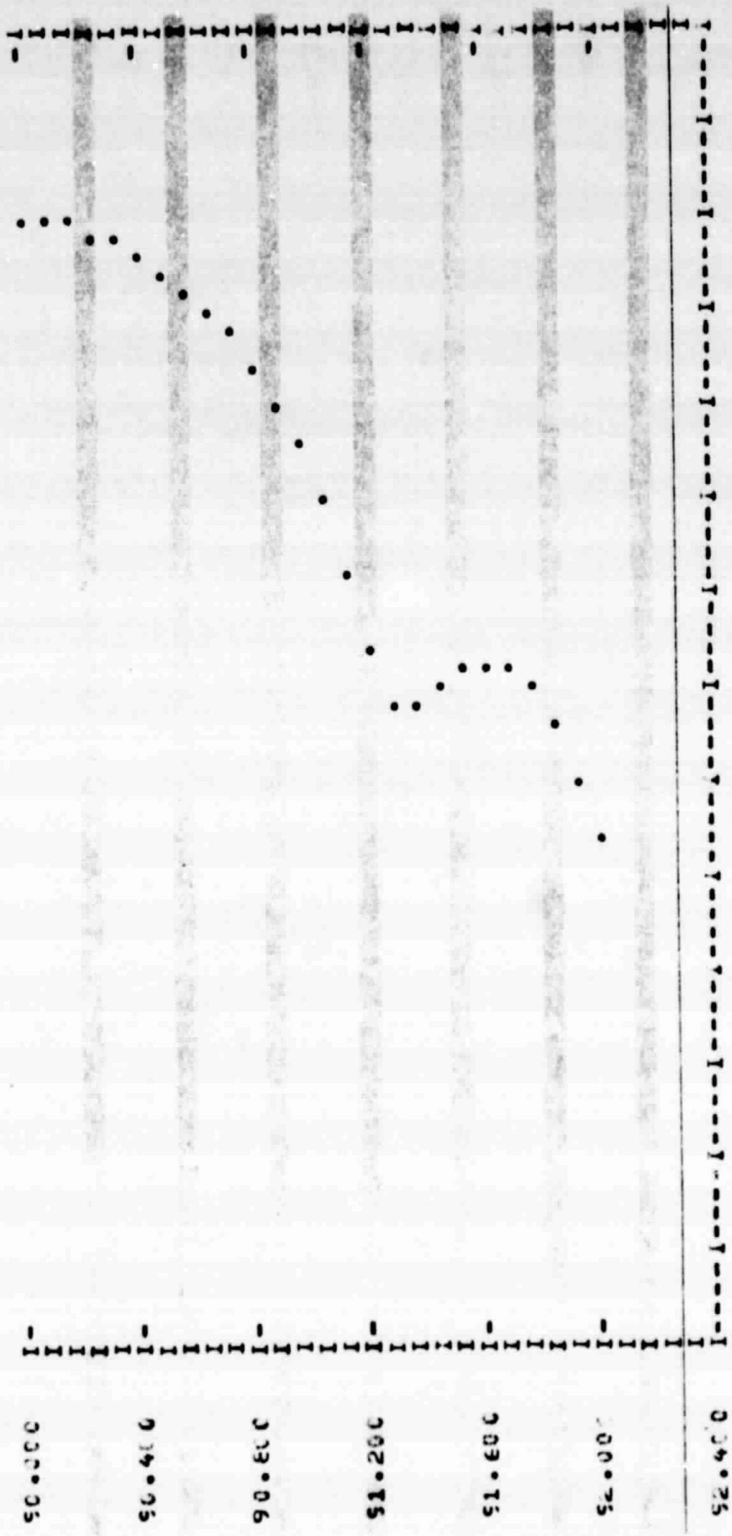

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OF POOR QUALITY

TABLE OF ELECTRIC FIELD STRENGTHS (DB)

PRINCIPAL PLANE OF CUT IS THETA = 90.000 DEG

ANGLE PHI FROM -2.000 TO 2.000 BY 0.080 DEG

PHI	DB(Z/Z)	DB(Y/Z)	DB(Z/Y)	DB(Y/Y)	PWRDB
-2.000	-18.46223	4.25798	-58.03587	-37.46564	-37.42845
-1.920	-16.66601	10.23614	-58.29023	-31.38746	-31.37895
-1.840	-17.83408	13.5[illegible]	-59.45770	-28.01826	-28.01344
-1.760	-20.58769	15.65806	-62.21151	-25.96554	-25.96481
-1.680	-27.62976	16.77747	-59.25358	-24.84615	-24.84629
-1.600	-31.32320	17.03763	-72.94678	-24.58599	-24.58622
-1.520	-19.06043	16.28565	-55.68498	-25.33799	-25.33702
-1.440	-13.59374	13.93403	-55.21736	-27.68959	-27.68222
-1.360	-9.97800	7.30766	-51.60170	-34.23896	-34.15732
-1.280	-7.30354	3.35513	-48.92696	-38.27052	-37.91264
-1.200	-5.23244	16.27104	-46.85606	-25.35256	-25.32227
-1.120	-3.60359	22.12418	-45.22721	-19.49944	-19.48814
-1.040	-2.32601	25.1[illegible]	-43.95163	-15.51666	-15.51273
-0.960	-1.35366	29.14613	-42.97729	-12.47549	-12.47193
-0.880	-0.64990	3[illegible]	-42.27352	-10.02308	-10.02079
-0.800	-0.20639	33.63693	-41.82431	-7.99264	-7.99114
-0.720	0.0	35.33491	-41.62362	-6.28871	-6.28774
-0.640	-0.03540	36.77177	-41.67771	-4.85185	-4.85125
-0.560	-0.37933	37.96[illegible]	-42.00335	-3.64261	-3.64228
-0.480	-1.01063	38.98999	-42.63445	-2.63363	-2.63349
-0.400	-2.00644	39.81326	-43.629[illegible]	-1.80536	-1.80538
-0.320	-3.46825	40.47193	-45.09187	-1.14369	-1.14381
-0.240	-5.60121	40.98325	-47.22452	-0.63857	-0.63858
-0.160	-8.86375	41.34143	-50.48738	-0.28219	-0.28245
-0.080	-14.72959	41.55530	-56.35321	-0.07032	-0.07061

0.000	-155.88522	41.62562	-197.50874	0.00	-0.00530
0.080	-14.72955	41.55555	-56.55321	-0.07052	-0.07061
0.160	-8.86375	41.34145	-50.48738	-0.28219	-0.28245
0.240	-5.60122	40.96525	-47.22482	-0.63857	-0.63558
0.320	-3.46825	40.47993	-45.09187	-1.14369	-1.14381
0.400	-2.00544	39.81526	-43.52926	-1.80556	-1.80558
0.480	-1.01065	38.98999	-42.53445	-2.63363	-2.63349
0.560	-0.37955	37.96551	-42.00355	-3.64251	-3.64228
0.640	-0.05449	36.77177	-41.67771	-4.85105	-4.85125
0.720	-0.00056	35.33491	-41.52362	-6.28871	-6.28774
0.800	-0.20059	33.63593	-41.82401	-7.99264	-7.99114
0.880	-0.64999	31.60554	-42.27332	-10.02308	-10.02079
0.960	-1.35366	29.14613	-42.97729	-12.47549	-12.47193
1.040	-2.32061	26.10495	-43.95163	-15.51666	-15.51273
1.120	-3.60359	22.12418	-45.22721	-19.49944	-19.48814
1.200	-5.23244	16.27104	-46.85506	-25.35258	-25.32227
1.280	-7.30354	3.35510	-48.92696	-38.27052	-37.91264
1.360	-9.97865	7.36755	-51.60170	-34.23596	-34.15732
1.440	-13.59374	13.93403	-55.21736	-27.68939	-27.68222
1.520	-19.06645	16.28565	-60.58406	-25.33799	-25.33712
1.600	-31.32316	17.03753	-72.94675	-24.58599	-24.58622
1.680	-27.52975	16.77747	-69.25335	-24.84615	-24.84529
1.760	-20.58705	15.65606	-62.21151	-25.96534	-25.96481
1.840	-17.83408	13.60555	-59.45775	-28.01826	-28.01544
1.920	-16.66661	10.23614	-58.29023	-31.38748	-31.37893
2.000	-16.46225	4.15795	-58.08587	-37.46534	-37.42845

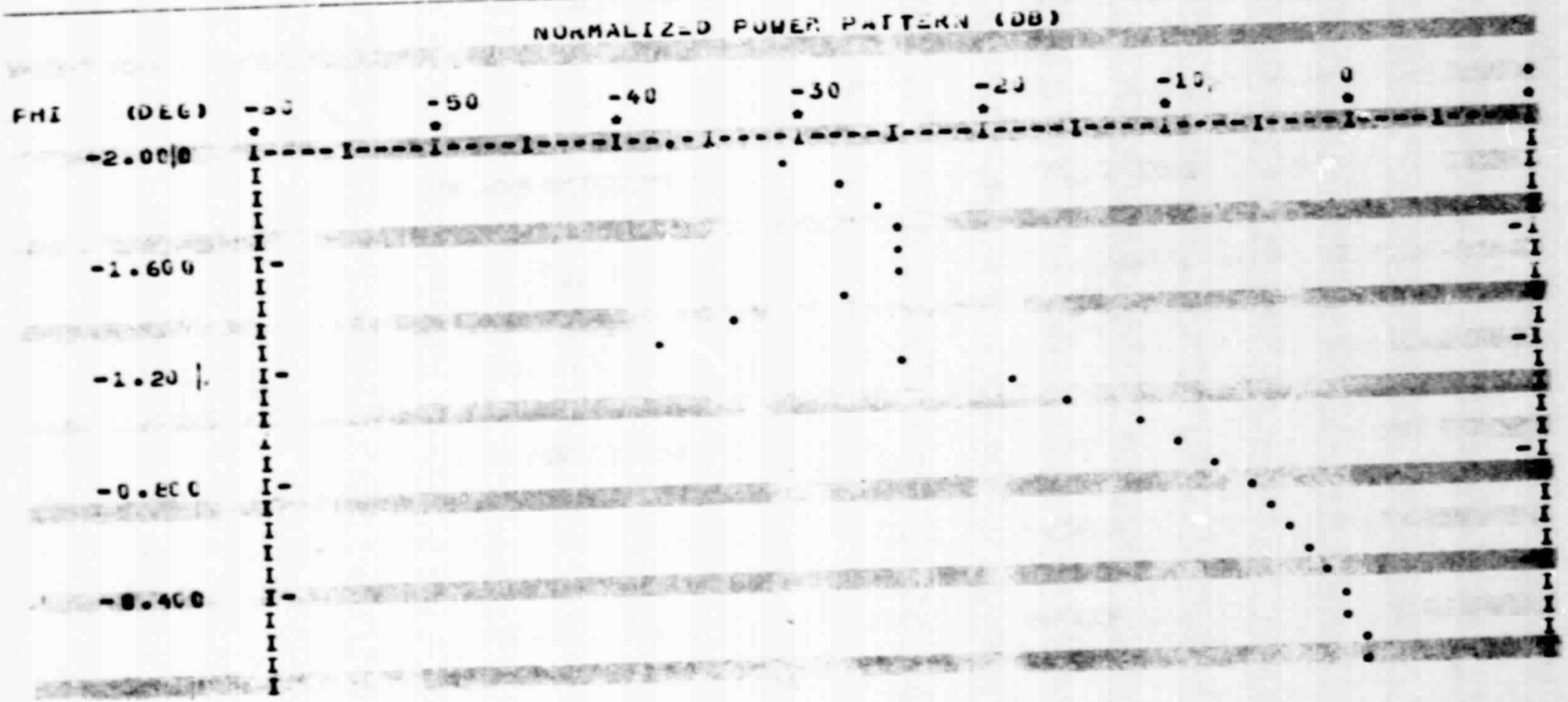
MAXIMUM FIELD VALUES-

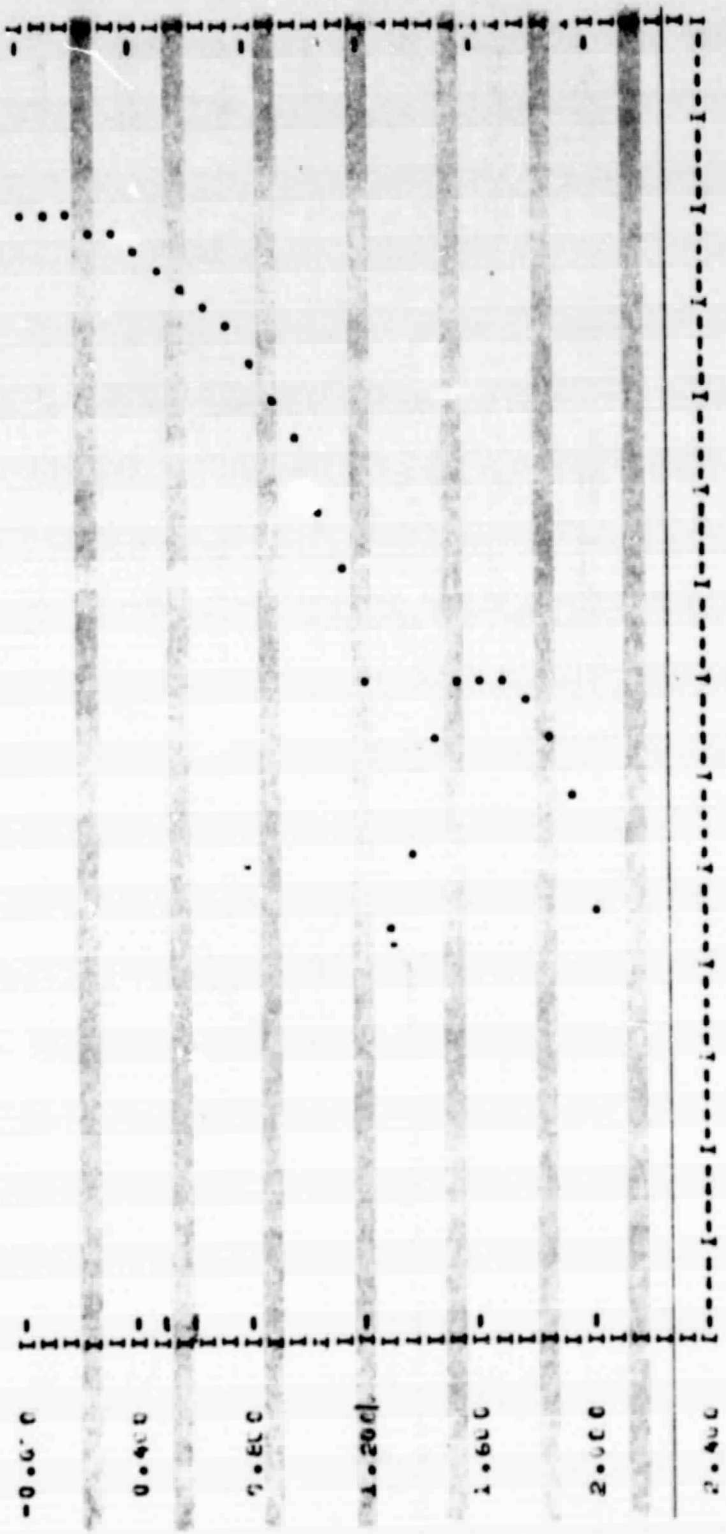
20LOG(MAX(FIELD-Z))-20LOG(9.3206600D-03)= -40.6110667

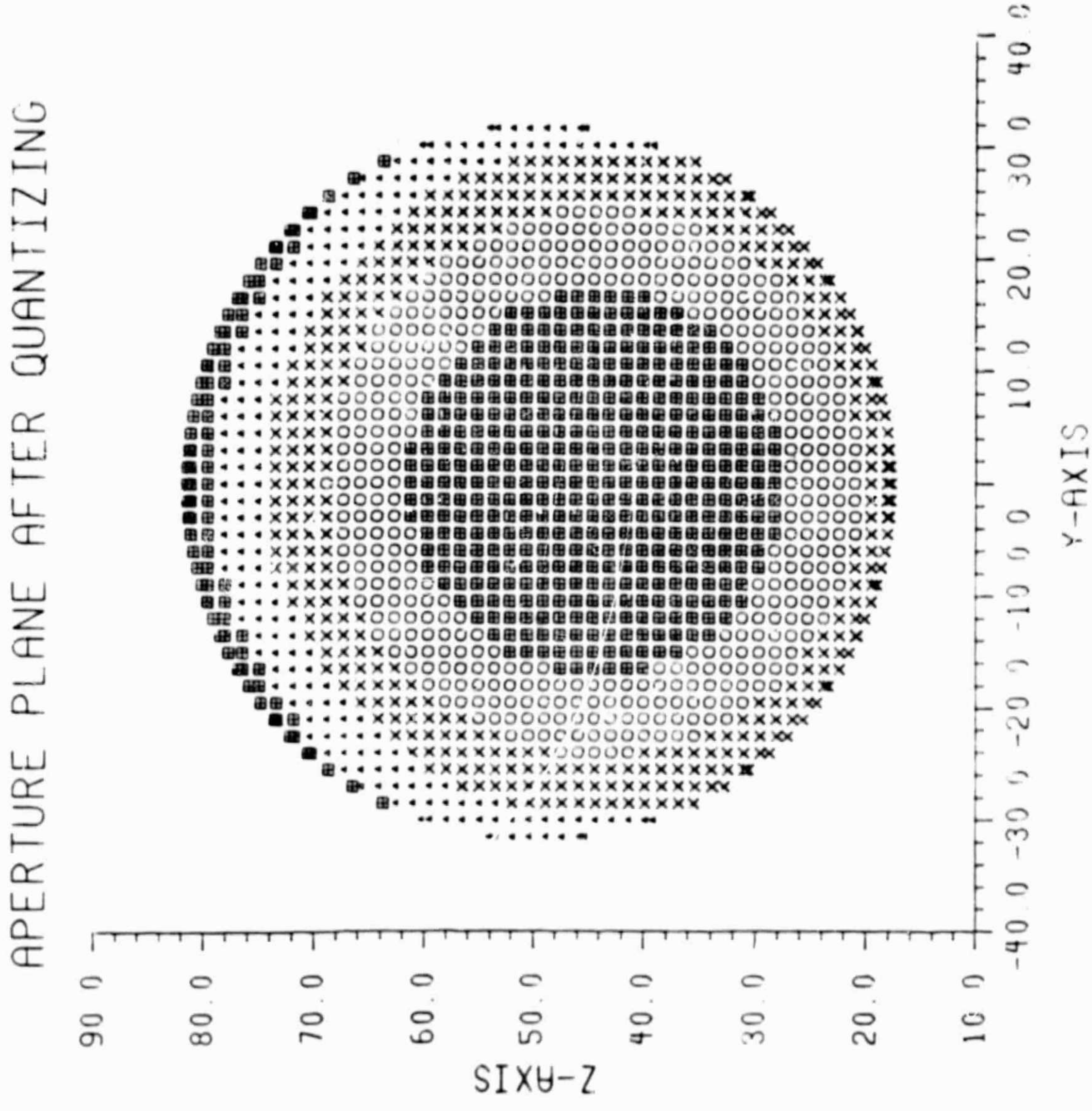
20LOG(MAX(FIELD-Y))-20LOG(1.1236414D 00)= 1.0125547

INTERPOLATION NUMBER USED FOR INTEGRATION IS.......... 7

PRINCIPAL PLANE = THETA 90.0 DEGREES






APERTURE PLANE AFTER QUANTIZING
Y-AXIS
Z-AXIS
-40.0
-30.0
-20.0
-10.0
0.0
10.0
20.0
30.0
40.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0