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## A Concept for a Fuel Efficient Flight Planning Aid for General Aviation

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B. P. Collins, A. L. Haines,  
and C. J. Wales

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# A Concept for a Fuel Efficient Flight Planning Aid for General Aviation

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and C. J. Wales

*The MITRE Corporation  
McLean, Virginia*

Prepared for  
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## 1. BACKGROUND

### 1.1 Problem Description

Energy costs have risen dramatically in the past several years. This has impacted all segments of aviation. Operations that have historically reflected time costs, must now be reoriented toward minimization of a cost function heavily dominated by fuel costs. Many accepted flight planning practices are being reevaluated and redesigned.

One useful tool in this effort to reflect the realities of fuel costs is a model to estimate fuel burn.

### 1.2 Fuel Model History

MITRE has developed for the Federal Aviation Administration (FAA) an approach to fuel burn modeling (References 1, 2). This model estimates fuel consumption as a function of path profile (geometric) data. The internal aircraft parameters such as thrust are not required for the model to estimate fuel consumption.

The resultant FAA Fuel Burn Model is a flexible, efficient tool for many applications. It can be used to evaluate proposed alternative routing structures, and for many other tasks (References 2, 3). It also has the capability to be adapted to a flight planning role.

### 1.3 Project Concept

MITRE was asked by the FAA and NASA's Langley Research Center to develop a concept for a General Aviation flight planning aid. This aid was to help provide the general aviation (or commuter) pilot with a first cut, fuel efficient, flight planning aid. The aid had to be relatively inexpensive in concept, and easy to use, to fit the general aviation budget and operating environment.

The basic concept revolved around the use of the fuel burn model on a hand held programmable calculator. This would permit the pilot to use it in an airborne update environment, as well as in a ground planning environment. In the ground environment, the calculator might be linked to other computational devices, to identify weather data, or for initialization of the data base for in-flight use.

#### 1.4 Solution Concept

The solution approach was to take the existing fuel burn model as a basis for calculation of fuel burns over specified segments. The optimal path would be established by a dynamic programming technique (see for example Reference 4). This technique iteratively, but systematically, constructs the set of feasible flight paths and iterates to the single optimal.

#### 1.5 Report Organization

Section 2 will review the salient facts on the basic fuel burn equation, as applied to general aviation aircraft. Detailed derivations are contained in Appendix A and B. Section 3 describes generically the input data necessary to the flight plan optimization. A more detailed description of a possible operational scenario is found in Appendix C. Section 4 presents the basic dynamic programming approach. Section 5 then reviews the interaction with the pilot, in terms of implementation of the optimal plan, and in terms of the process of in-flight update. Finally, the specialization of the dynamic programming concept for a hand held programmable calculator is addressed in Section 6.

## 2. DEFINITIONS AND ASSUMPTIONS

### 2.1 Assumptions

Several assumptions inherent in the development of the fuel burn model are continued here in the application to fuel efficient flight paths. The primary of these are:

- o Weight changes over increments will not materially affect fuel burn calculations.
- o Flight path angles are small:  $\cos$  (Flight path angle) approximates 1.
- o Velocity and altitude changes within an increment are linear.

### 2.2 Fuel Burn Equation

The fuel burn equation has been developed to measure fuel consumption as a function of path profile characteristics. The equation accepts as inputs (see Table 2-1):

- o Stored constants and functions
  - general non-specific functions (density, etc.)
  - specific constants related to aircraft and engine type
- o Path/profile increment data
  - altitude
  - velocity
  - time or distance
  - weight
  - configuration (flaps, gear)

**TABLE 2-1  
FUEL CONSUMPTION ESTIMATION ALGORITHM  
INPUTS/OUTPUTS**

## **Inputs**

- **Stored Constants**
  - General Nonspecific Constants (Density, Etc.)
  - Specific Constants Related To Aircraft and Engine Type
  
- **Path/Profile Increment Data**
  - Altitude
  - Velocity
  - Time
  - Weight
  - Configuration (Flaps, Etc.)

## **Outputs**

- **Calculated Fuel Burn Over The Increment**
- **Performance Limit Checks**

The fuel burn equation has been derived from basic principles of conservation of energy. Energy balance requires:

$$\begin{aligned} &(\text{Energy In}) - (\text{Energy Loss}) \\ &= (\text{Energy Change}) \end{aligned}$$

Specializing this to aircraft flight, we have:

$$\begin{aligned} &f(\text{Thrust}) - f(\text{Drag}) \\ &= f(\text{Kinetic Energy Change}) \\ &+ f(\text{Potential Energy Change}). \end{aligned}$$

The details of the derivation are presented in Appendix A. Additional material on jet aircraft derivations is found in Reference 1. The initial derivation is given in Figure 2-1 for ISA conditions. Figure 2-2 gives the construction of the  $R_1$ ,  $R_2$  multipliers reflecting flaps and gear in low altitude flight. Figure 2-3 shows the modified equation necessary to reflect actual wind data. It requires mainly a correct accounting of the two factors:

- o Aircraft flight through the airmass (leading to instantaneous fuel flow rate)
- o Aircraft progress along the ground track (leading to real forward progress along trip itinerary)

For simplification of the following discussions, we will simply talk of fuel burn (B) as calculable as a function of W, h, V, and T with aircraft constants being known and included. The effects of actual winds, flaps, and gear will be noted only when they have a significant impact on the fuel burn.

$$B = K_{15} TV_T F_n + K_{16} Te^{K_{17} h}$$

where

$$F_n = \frac{R_1 K_1 \dot{p} S_w V_T^2}{2} + \frac{2R_2 K_2 W^2}{\dot{p} S_w V_T^2} + \frac{W}{gT} (v_{T2} - v_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

$R_i$  defined in Figure 2-2

**FIGURE 2-1  
FUEL BURN EQUATION FOR TURBOPROP (ISA CONDITIONS)**

$$R_1 = \begin{cases} GU_1 F^3 + GU_2 F^2 + GU_3 F + 1 & \text{(gear up)} \\ GD_1 F^3 + GD_2 F^2 + GD_3 F + GD_4 & \text{(gear down)} \end{cases}$$

$$R_2 = FDM_1 F^3 + FDM_2 F^2 + FDM_3 F + 1$$

where  $F$  is flap angle in degrees

$GU_i$ ,  $GD_i$ , and  $FDM_i$  are aircraft constants

**FIGURE 2-2  
FLAP AND GEAR DRAG MULTIPLIERS**

∞

$$B = K_{15} TV_T F_n + K_{16} Te^{K_{17} h}$$

where

$$F_n = \frac{R_1 K_1}{2} \bar{\rho}_h S_w V_T^2 + \frac{2R_2 K_2 W^2}{\bar{\rho}_h S_w V_T^2} + \frac{W}{gT} (V_{T2} - V_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

$R_i$  previously defined

$\bar{\rho}_h$  is altitude dependent density

T is derived as  $\frac{\text{Ground Distance}}{\text{Ground Velocity}}$

**FIGURE 2-3  
FUEL BURN EQUATION FOR TURBOPROP  
(ACTUAL WINDS AND TEMPERATURE)**



### 3. INPUT DATA

The general solution approach will be developed in Section 4. This section first defines the content and source of the data that is needed as input to the dynamic program.

#### 3.1 Stored Data

Stored data will be specific to the individual aircraft type. Data will include:

- o Aircraft/engine type - (to identify data)
- o Aircraft specific constants for the fuel burn equation
- o Maximum service ceiling (feet)
- o Maximum velocity (knots)
- o Maximum takeoff weight (pounds)
- o Operating empty weight (pounds)

In addition, stored data for the individual aircraft or operator could include:

- o Effective service ceiling
- o Adaptations to the aircraft type generic constants

#### 3.2 Pilot Input Data

The flight specific data is supplied by the pilot. This would normally include the following:

- o Aircraft/Engine ID (to verify stored data is applicable)
- o Departure and Arrival Velocities (knots)
- o Departure and Arrival Altitudes (feet of density altitude)
- o Effective Service Ceiling (feet) (for this trip, reflecting availability of equipment, known operator, or ATC imposed constraints)
- o Landing Gross Weight (pounds) (defined as empty weight plus payload and reserve fuel and reflecting other operator imposed constraints)
- o Number of Waypoints (generally keyed to ATC defined navigational aids, fixes or airports)

For each of the trip waypoints, the pilot (or an automated link to a weather data base) must provide:

- o Course Heading (degrees)
- o Magnetic Variation (degrees)
- o Distance to Waypoint (or X, Y Ground Coordinates)(nmi)
- o Number of Altitudes (for which weather data is to be input)

For each waypoint, and for the selected number of altitudes, the following are input:

- o Density Altitude (feet)
- o True Wind Direction (degrees)
- o Wind Velocity (knots)
- o Temperature (°F)

### 3.3 Processing of Weather Data

The weather data as input must be processed to a form more suitable to the dynamic program. The following conversions are made:

- o Wind Direction from True to Magnetic:

Wind (Magnetic)

$$= \text{Wind (True)} + \text{Magnetic Variation}$$

- Wind into North-East Components:

$$\text{WN} = \text{Wind Velocity} [\text{Cos}(\text{Wind Magnetic})]$$

$$\text{WE} = \text{Wind Velocity} [\text{Sin}(\text{Wind Magnetic})]$$

where

WN = North Wind Component

WE = East Wind Component

Several interpolations are determined. Winds and temperatures at a waypoint are fit over the several altitudes by a least squares regression:

$$\text{WN}_h = C_1 + C_2h$$

$$\text{WE}_h = C_3 + C_4h$$

$$\text{Temp}_h = C_5 + C_6h$$

where

$WN_h$  = North wind component at altitude h

$WE_h$  = East wind component at altitude h

h = Density altitude

$Temp_h$  = Temperature at altitude h ( $^{\circ}F$ )

$C_1$  = Constants from regression

These values then may be linearly interpolated as necessary between the pilot input defined waypoints. Once the necessary interpolations are completed, the winds at each node point are converted to head wind components as follows:

$$WC_h = C_7 + C_8h$$

where

$$C_7 = C_1 \cos (\text{course}) + C_3 \sin (\text{course})$$

$$C_8 = C_2 \cos (\text{course}) + C_4 \sin (\text{course})$$

$WC_h$  = headwind at altitude h

## 4. MATHEMATICS OF DYNAMIC PROGRAMMING MATRIX

This section presents the basic dynamic programming approach to the derivation of a fuel efficient profile. This approach uses the ground track and wind/temperature data described in Section 3. It presumes the filed ground course is followed. The resultant is a fuel efficient altitude and velocity at each user defined waypoint.

### 4.1 Overview

For the purpose of illustrating the approach, we will assume for the moment that velocity is known at each waypoint. The basic dynamic program approach is illustrated for a simple four-step problem in Figures 4-1 through 4-6.

We initially define a set of potential density altitudes for each of the defined distance waypoints (Figure 4-1). The altitudes for the departure and arrival points are known, and thus some immediate simplification is possible (Figure 4-2). We start from the arrival point and iteratively, but selectively, calculate fuel burns. First we calculate the fuel burn from each candidate altitude at distance  $d_3$  to the arrival point  $d_4$  (Figure 4-3). We then consider distance  $d_2$ . For each candidate altitude at  $d_2$ , we calculate the fuel burn to each altitude at  $d_3$ . Since we already know the fuel burn from  $d_3$  to  $d_4$ , we can calculate the most fuel efficient route from each altitude point at  $d_2$  to  $d_4$ . The solution for one  $d_2$  altitude is shown in Figure 4-4. Iterating this for each altitude at  $d_2$  we have the most fuel efficient path for each, as shown in Figure 4-5. Note that one of the altitudes at  $d_3$  does not lie on any fuel efficient (partial) path, and thus is discarded from further consideration. We finally calculate the fuel burn from the single altitude at the depart point  $d_1$  to each altitude at  $d_2$ . We already know each (partial) fuel efficient path from altitudes at  $d_2$  to  $d_4$ , so we now can calculate the optimal from  $d_1$  to  $d_4$ , shown by the heavy line in Figure 4-6.

### 4.2 Definitions

In fact, we need to determine both altitude and velocity at each distance point. The same dynamic program approach holds, as shown in Figure 4-7. The primary change is in the number of fuel burn calculations that are required at each step. Thus for the example shown, at  $d_3$ , we have 4 altitudes x 5 velocities = 20 values (versus just 4 altitudes).

- **Define Distance and Altitude Points**
  - Limited in Number by Computational Capacity (4 Is Example)
  - May Correspond (in Part) to User Defined Waypoints
  - Divisions Need Not Be Linearly Chosen
- **(Velocity Is Treated Analogously)**

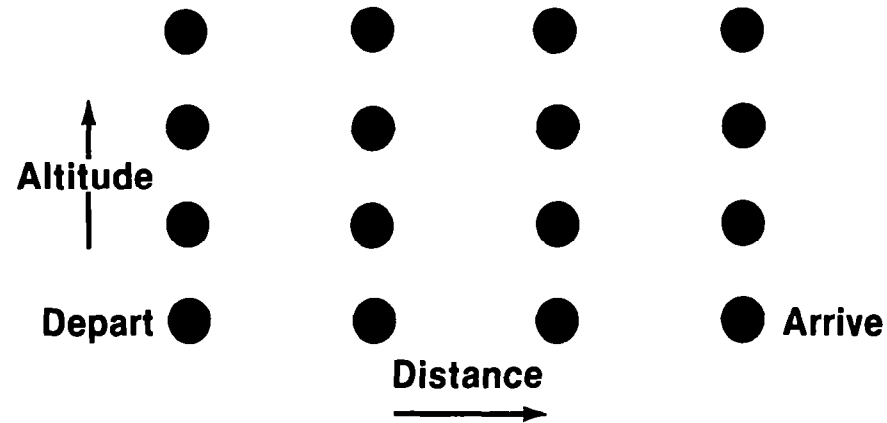
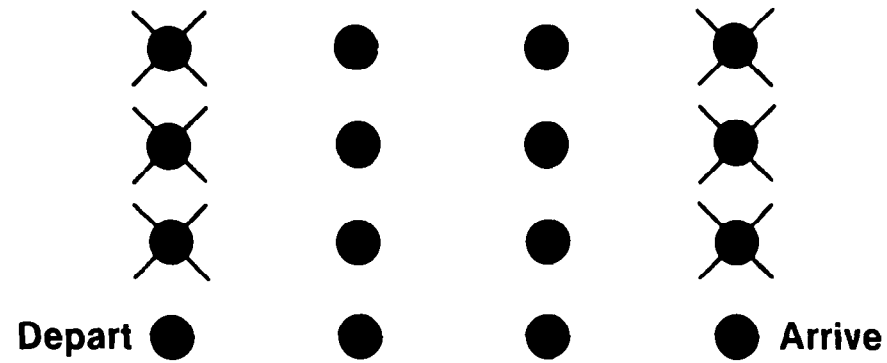


FIGURE 4-1  
MATRIX DEFINITION

- **Altitude at Depart/Arrive Points Is Known**



**FIGURE 4-2**  
**MATRIX SIMPLIFICATION**

- **Calculate Fuel Burn**

- From Each of Candidate Altitudes at Distance Point  $d_3$
- To Arrive Point

(Assuming for Moment that Velocity Is Known)

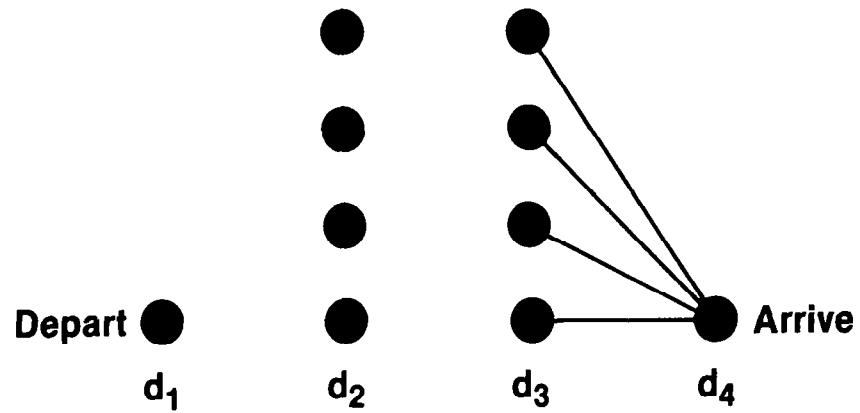


FIGURE 4-3  
DYNAMIC PROGRAM- $d_3$  STEP

- **Calculate Fuel Burn**
  - From Each Candidate Altitude at Distance Point  $d_2$
  - To Each Candidate Altitude at Distance Point  $d_3$
- **Calculate**
  - For Each Altitude at  $d_2$
  - Smallest Sum of Fuel Burn  $d_2$  to  $d_3$  to  $d_4$
- **Save Only Minimum Fuel Burn Path**
  - For Each Altitude at  $d_2$

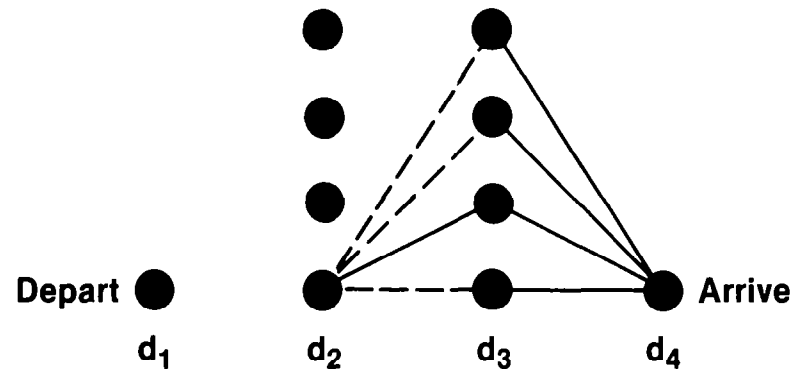


FIGURE 4-4  
DYNAMIC PROGRAM- $d_2$  STEP



- For Each Altitude at  $d_2$ , Have Fuel Optimal Path to  $d_4$
- Note Some  $d_3$  to  $d_4$  Path Segments Deleted From Further Consideration

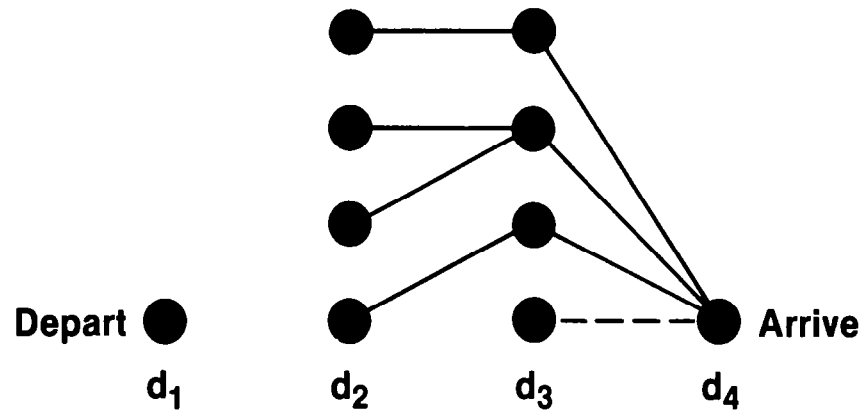


FIGURE 4-5  
DYNAMIC PROGRAM- $d_2$  STEP RESULT

- Repeat Previous Step for  $d_1$
- Note Only Single Candidate Altitude at  $d_1$
- Complete Optimal Path Is Defined

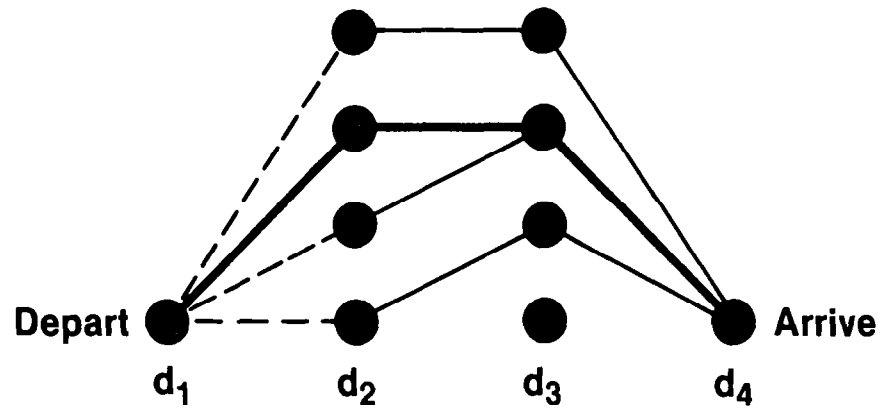


FIGURE 4-6  
DYNAMIC PROGRAM- $d_1$  STEP

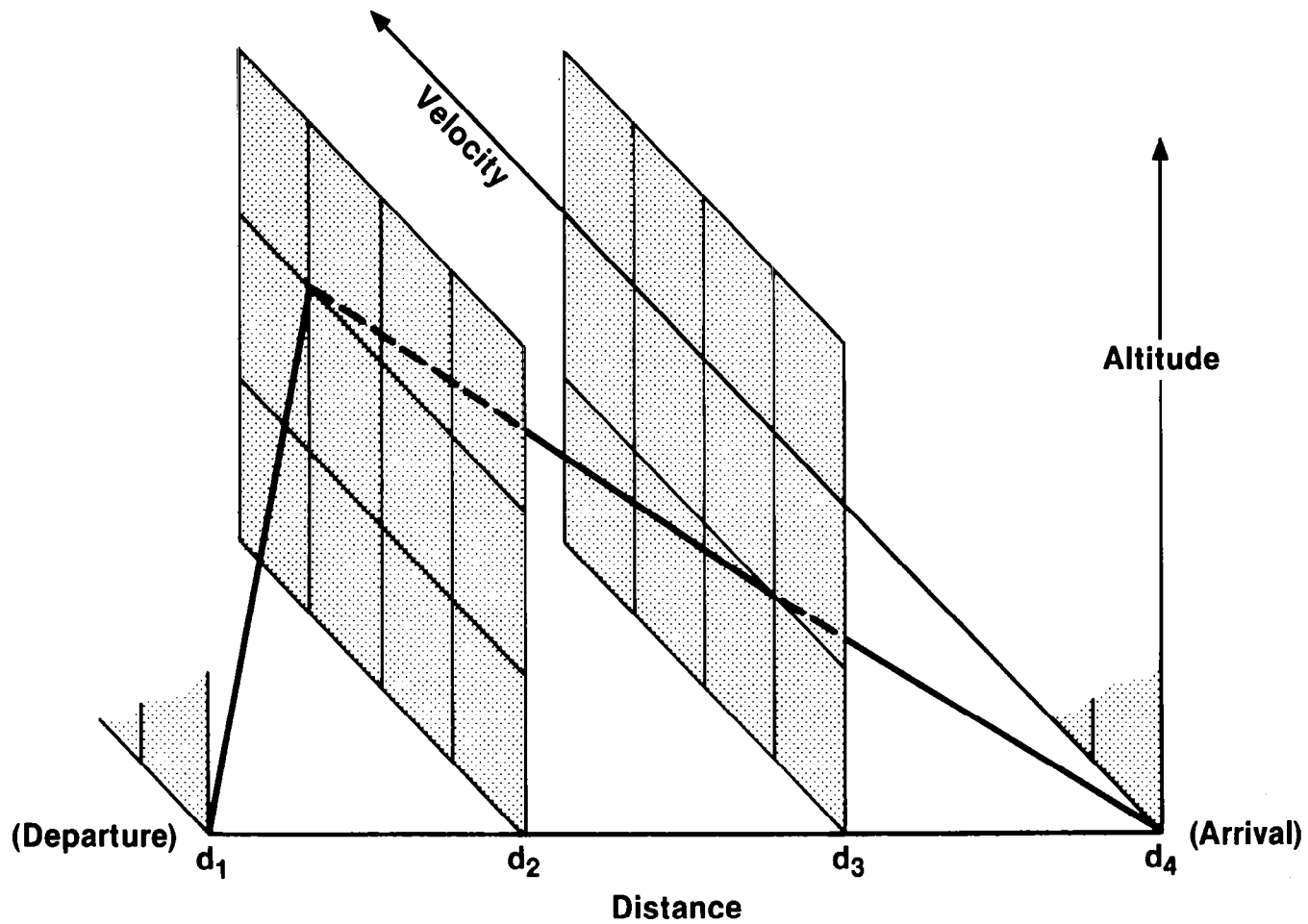


FIGURE 4-7  
OPTIMAL FLIGHT PLANNING RESTRICTED SEARCH TECHNIQUE

We now proceed to introduce the basic notation we will need. We define a three dimensional matrix in

- o Density altitude (index i)
- o Velocity (index j)
- o Distance (index k)

We define the limits of (i,j,k) as M, N, P respectively, not necessarily equal.

We will make the definition of altitude and velocity with ranges independent of distance. (This assumption can easily be changed, as noted later.)

Altitude elements are defined by the user or as

$$H_i = H_{\text{MIN}} + \frac{(i-1)}{(M-1)}(H_{\text{MAX}} - H_{\text{MIN}})$$

where

$H_{\text{MIN}}$  = minimum altitude  
(departure or arrival)

$H_{\text{MAX}}$  = effective service ceiling  
(if less than maximum service ceiling; otherwise  
maximum service ceiling.)

Similarly, velocity elements are defined by the user or as

$$V_j = V_{\text{MIN}} + \frac{(j-1)}{(N-1)}(V_{\text{NE}} - V_{\text{MIN}})$$

where

$V_{\text{MIN}}$  = minimum velocity  
(departure or arrival)

$V_{\text{NE}}$  = velocity not to be exceeded

We will define distance elements by the user waypoints. If the number of waypoints is less than P (limit of distance index) then additional distance elements will be added. This will be accomplished by adding a distance element  $D_k$  such that

$$D_k = \frac{D_{q+1} - D_q}{2} + D_q \quad \text{where } D_q \text{ is chosen}$$

such that  $D_{q+1} - D_q = \text{Max}_m (D_{m+1} - D_m)$

At this new distance element the weather data is linearly interpolated between distance elements  $D_q$  and  $D_{q+1}$  and the course heading is set equal to the course heading at  $D_q$ . This is repeated until the number of distance elements equals  $P$ .

Finally, the arrival weight is defined as the weight at point  $(1,1,P)$  in the matrix. This weight must be greater than the specified aircraft operating weight empty.

### 4.3 Matrix Steps

The number of calculations implied by the matrices defined as in Figure 4-7 will expand rapidly for relatively small numbers of distance, altitude, and velocity points. Thus it becomes desirable to define the points carefully so they have relevance to the final optimum flight path. It is also important to limit calculations when they are clearly not relevant to the final path. Table 4-1 lists some of these considerations. The limits of aircraft performance can be used to reject some paths. Similarly smoothness rules can be applied. We know, for instance, that the optimal path will not have a sequence of significant climb and descent combinations. These considerations are used in the logical definition of matrix steps below.

#### 1. Initial Segment

- a. Perform calculations between distance points  $D_p$  to  $D_{p-1}$ . Calculate fuel flow rate from each  $(i,j,P-1)$  to  $(1,1,P)$ . Insure this is no less than idle fuel flow.
- b. Set path flags to indicate path to the arrival point:  
$$\text{OH}(i,j,P-1) = 1$$
$$\text{OV}(i,j,P-1) = 1$$
- c. Set flags to indicate aircraft is descending towards arrival point and decelerating:  
$$\text{ALT}(i,j,P-1) = 1$$
$$\text{VEL}(i,j,P-1) = 1$$

**TABLE 4-1  
SIMPLIFICATION OF CALCULATIONS  
(RESTRICTION OF SEARCH)**

- **Performance Constraints**
  - **Maximum Altitude**
  - **Maximum Velocity**
  - **Maximum Rate of Climb**
  - **Maximum Rate of Descent**
  
- **Smoothness Rules**
  - **Climb After Descent**
  - **Velocity Increase After Decrease**

**(Limits Imposed by Wind and Temperature)**

These flags will be set to 0 when aircraft is ascending from departure point and accelerating respectively.

d. For the next step, set

$$D_k = D_{p-1}.$$

## 2. Generic Segment

Perform calculations between two distance points

$D_{k-1}$  to  $D_k$ .

The specific pair of points under consideration will be denoted  $(i,j,k-1)$  and  $(m,n,k)$ .

a. Check to see if any of the performance constraints are violated. These are:

1) Maximum fuel flow for climb or cruise

2) Descent prior to ascent, if

$$H_i > H_m \text{ and } ALT(i,j,k-1) = 0$$

3) Deceleration prior to acceleration if  $V_j > V_n$   
and  $VEL(i,j,k-1) = 0$

4) Climb gradient  $((H_i - H_m) / (D_k - D_{k-1}))$   
greater than 10%. This can easily be changed to  
a function of altitude.

5)  $W(m,n,k) > 90,000$  (indicates performance  
constraints violated to reach point  $(m,n,k)$ )

b. If any of the above are violated:

$$B(m,n) = 99,999$$

c. Otherwise fuel burn from  $(i,j,k-1)$  to  $(m,n,k)$  is  
calculated and denoted  $B(m,n)$ .

d. Thence

$$W(i, j, k-1) = \min_{m, n} [B(m, n) + W(m, n, k)].$$

If the optimum burn  $B(m, n)$  is less than 2% different from fuel burn with constant altitude/velocity, then latter is retained ( $i = m, j = n$ ).

e. Flags are set:

$$OH(i, j, k-1) = \min(m) \text{ index}$$

$$OV(i, j, k-1) = \min(n) \text{ index}$$

$$ALT(i, j, k-1) = 0 \text{ if } H_m > H_i$$

$$VEL(i, j, k-1) = 0 \text{ if } V_n > V_j$$

f. Steps a to f are repeated for each  $(i, j, k-1)$ .

3. Decrement distance by one index and repeat step 2. If new  $D_{k-1} = 0$  (that is  $k-1 = 1$ ), step 2 is only done for point  $(1, 1, 1)$ .

4. The optimum path is determined by path flags, iteratively

a. Distance =  $D_2$

$$\text{Density Altitude} = H(OH(1, 1, 1))$$

$$\text{Velocity} = V(OV(1, 1, 1))$$

b. Distance =  $D_3$

$$\text{Density Altitude} = H[OH(OH(1, 1, 1), OV(1, 1, 1), 2)]$$

$$\text{Velocity} = V[OV(OH(1, 1, 1), OH(1, 1, 1), 2)]$$

c. etc.



5. The density altitude is converted to pressure altitude before output by

$$H_p = (\text{Temp}_H - 59.) / (-.003566)$$

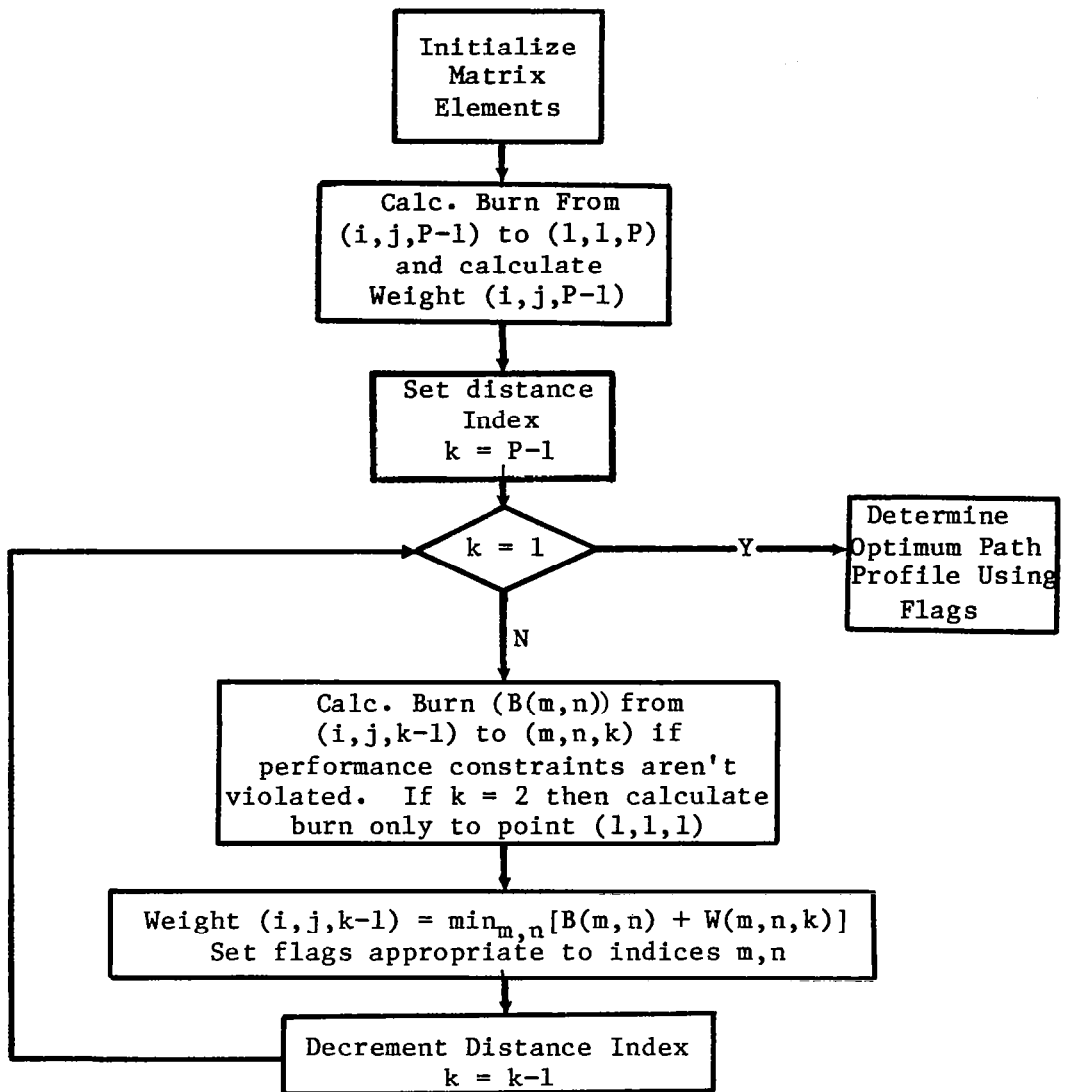
where:

$\text{Temp}_H$  = temperature at density altitude H

$$\text{Temp}_H = C_5 + C_6h$$

$H_p$  = pressure altitude in feet.

The calculation steps are summarized in Figure 4-8.



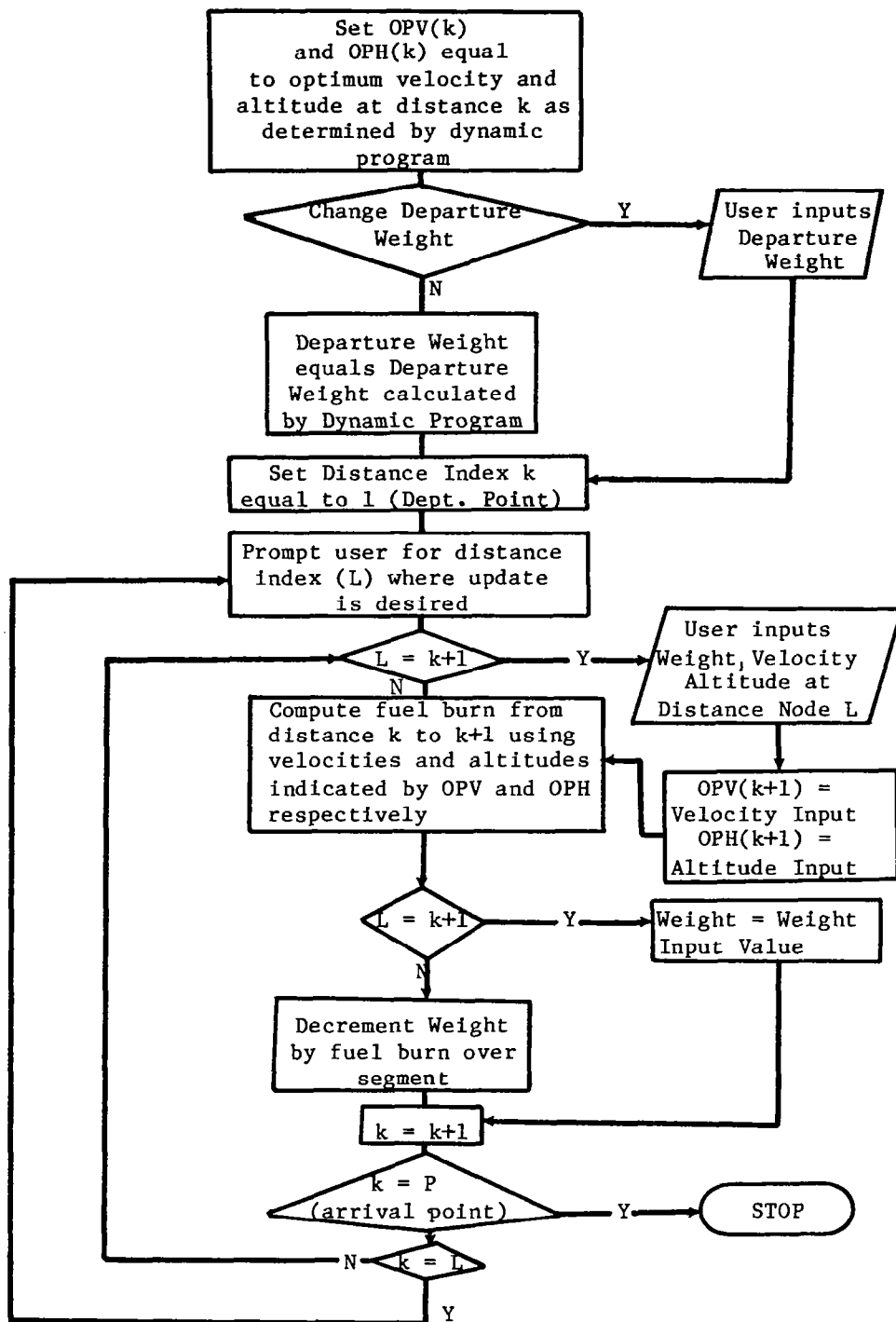
**FIGURE 4-8**  
**MATRIX CALCULATION STEPS**

## 5. INFLIGHT UPDATE

After the optimum flight path has been determined by the dynamic programming technique an option exists for a user inflight update. If this option is selected, the fuel burn along the flight path, either indicated by the user or as determined by the dynamic program, is calculated. Also available is an option to update departure weight or aircraft weight at a distance node point.

The steps through the inflight update are logically defined below (and in Figure 5-1) where  $OPH(k)$  and  $OPV(k)$  are the optimum altitude and velocity respectively at distance  $D(k)$  as determined by the dynamic program.

- 1) Allow the user the option to set departure weight. Otherwise the weight at the departure point ( $W(1,1,1)$ ) computed by the dynamic program will be used.
- 2) Set the distance index  $k$  equal to 1 (departure point).
- 3) Prompt user for distance node ( $L$ ) where update is desired. (Note - only the weight can be changed at the departure point not the altitude or velocity).
- 4) If the distance node for update ( $L$ ) equals the node at the end of the flight segment ( $k+1$ ) then the user will be prompted for the velocity and density altitude which will be set equal to  $OPV(k+1)$  and  $OPH(k+1)$  respectively. The user will also be offered the option to update the weight at the distance node ( $k+1$ ).
- 5) Compute the fuel burn from distance ( $k$ ) to distance ( $k+1$ ).
- 6) If the user has updated the weight at this distance node, set the weight equal to the user input; otherwise, decrement the weight by the fuel burn of the segment.
- 7) Increment distance index  $k = k+1$ .
- 8) Steps 4 to 7 are repeated until either  $k$  equals the arrival point node or  $k=L$ .
- 9) If  $k$  does not equal the arrival point node then repeat steps 3 to 8.



**FIGURE 5-1  
UPDATE CALCULATIONS**

## 6. CONCEPTS FOR EXPANSION

The restricted search version of the dynamic programming approach, as described in Section 4, can be expanded to provide greater accuracy.

Accuracy of the solution and speed of the computation can be enhanced by further consideration of factors such as those listed in Table 6-1. Depending on the description of the individual problem, the number of possible values for distance, altitude, and velocity can be different. An initial capability has been implemented in the current algorithm. The current algorithm can also be extended to consider unequal increments in the division of distance, altitude, and velocity. This permits more efficient usage of fixed computational resources in those areas of distance, altitude, and velocity that are closest to the likely range of the optimal flight path.

Perhaps the most critical improvement is in considering the limits of altitude and velocity as a function of distance. Clearly there are limits on climb and descent rates that can be reflected in the choice of altitude and velocity. Minimum values may be derived from pilot or file data on minimum en route altitudes and the like. Similarly, the number of candidate altitudes and velocities can be made a function of distance.

Several additional features or algorithm capabilities could be considered. These could include

- o Time constraints
- o In-flight change of segments
- o Alternate airport planning
- o Point of no return indication
- o Adaptive aircraft constants to reflect
  - errors in data input (especially weather)
  - aircraft specific characteristics
- o Automatic interfaces between ground computer and
  - weather sources
  - geographic data (to define location, minimum altitudes at standard navigational fixes)

**TABLE 6-1  
DESIGN VARIATIONS**

- **For Fixed Computational Resources, Accuracy and Speed Solution Can Be Enhanced by Careful Consideration of**
  - **Vector Limits Different on Distance, Altitude, Velocity**
  - **Unequal Distance, Altitude, Velocity Segments**
  - **Calculation of Min/Max Altitude and Velocity as Function of Distance**

The results of the current algorithm should be checked and verified. For this purpose, it may be useful to develop a form of the algorithm for a mainframe computer, to provide for enhanced computational power and accuracy. In any case, the algorithm needs to be flight checked against actual aircraft in typical flight regimes.

The current algorithm, based upon an Apple II computer, is not transportable to the cockpit. As such, the initial update capability described in Section 5 is not readily usable in a practical operational environment. To do this, another approach to a solution will be required. Section 7 describes an approach to a dynamic programming solution that restricts the moves that can be made in altitude and velocity between distance increments. This solution should be implementable on a hand-held programmable calculator.

## 7. CONCEPT FOR IMPLEMENTATION

Sections 4 and 5 defined the application of a restricted search dynamic programming technique to the problem of determining the unconstrained minimum fuel aircraft trajectory along a predetermined track or route. This section defines a modification to the technique that reduces the computational complexity and the computer memory requirements. The resulting technique then has the potential for implementation on a programmable calculator.

### 7.1 Background

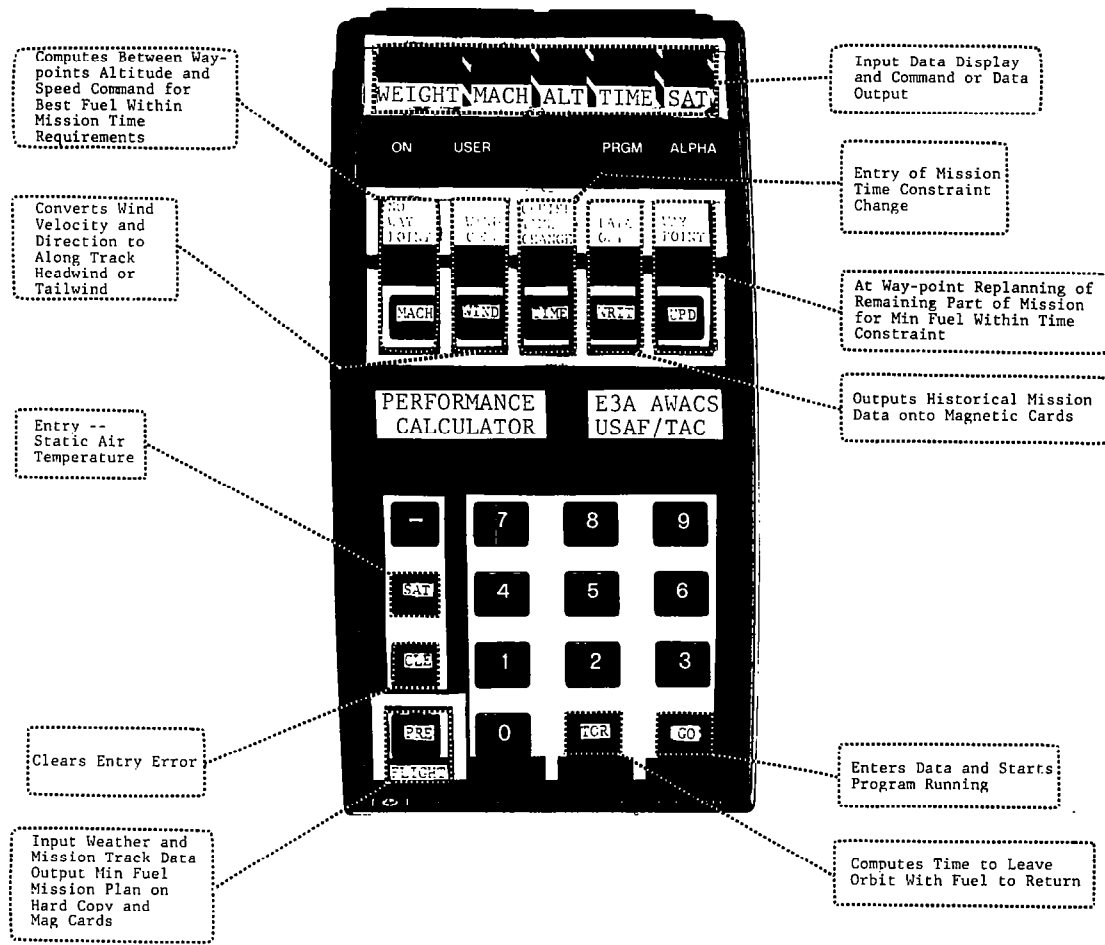
The concept of utilizing a programmable calculator to perform pre-flight planning, inflight update and post-flight performance evaluation has been utilized in one case as part of the E3A AWACS fuel conservation and mission enhancement program. Figure 7-1 depicts a HP-41C programmable calculator similar to that used in the application in conjunction with an appropriate overlay. The software represented mainly an attempt to minimize the amount of fuel burned on each trip segment in the presence of actual winds and temperatures and with a total trip time constant. This program incrementally calculated each maximum range increment time and then adjusted this increment time so that the total time was consistent with a time constraint. It was recognized that, at best, this concept represented a first generation attempt at constrained trajectory optimization. The concept was flight tested and judged to be a success because it caused the aircraft to arrive at a geographic point at a precise time and with a fuel saving in the range of 5 to 7 percent. The next logical step was to upgrade the software so that it truly calculated an unconstrained minimum fuel trajectory.

### 7.2 Algorithm Concept

The algorithm described in Sections 4 and 5 must undergo a transformation in order to be compatible with the capability of the HP-41CV programmable calculator. A major simplification could be effected by the determination of relationships between altitude/distance and velocity/distance such that all feasible values could be examined in terms of a planar matrix.

As given in Table 7-1, distance is used to determine the number, range, and value for the altitude and velocity points in the





**FIGURE 7-1  
PROGRAMMABLE CALCULATOR SAMPLE FORMAT**

**TABLE 7-1  
TOWARD A REVISED ALGORITHM  
(RESTRICTION ON MATRIX MOVES)**

- **Using Altitude and Velocity as Function of Distance, Can Develop Revised Algorithm**
- **Distance Determines**
  - **Range of Altitude, Velocity Points**
  - **Number of Different Values To Be Used**
  - **Selection of Specific Altitude, Velocity Points**
- **Dynamic Program Steps Restricted**
  - **Arrive at Each Permitted Altitude, Velocity Point by One of Two Paths Only**

dynamic programming matrix. The dynamic program is thus modified from the "restricted search" approach by the addition of restrictions on the steps or moves in the dynamic programming solution. This "restricted move" approach is then the basis for a potential technique, implementable on a programmable calculator.

This technique is illustrated with the aid of Figure 7-2. Part (c) shows a 10 X 10 matrix with the horizontal axis representing the variations in a kinetic energy index ( $I_k$ ), the vertical index representing the variation in a potential energy index ( $I_p$ ), and the diagonals representing the variation in a distance index ( $I_d$ ). The lower left vertex represents the departure point and the upper right the arrival point in a total path profile. Figure 7-2(a) depicts an altitude ( $h$ ) surface that is a function of a potential energy index ( $I_p$ ) and a distance index ( $I_d$ ). This relationship can be expressed as,

$$h_{p,d} = f(I_p, I_d)$$

The ridge of the surface represents the altitude that can be achieved by the aircraft operating at maximum performance limits from departure to arrival. Thus an aircraft, at any distance index, can never reach an altitude greater than that which is located on the ridge of the surface. An empirical relationship that can be utilized to generate the surface is:

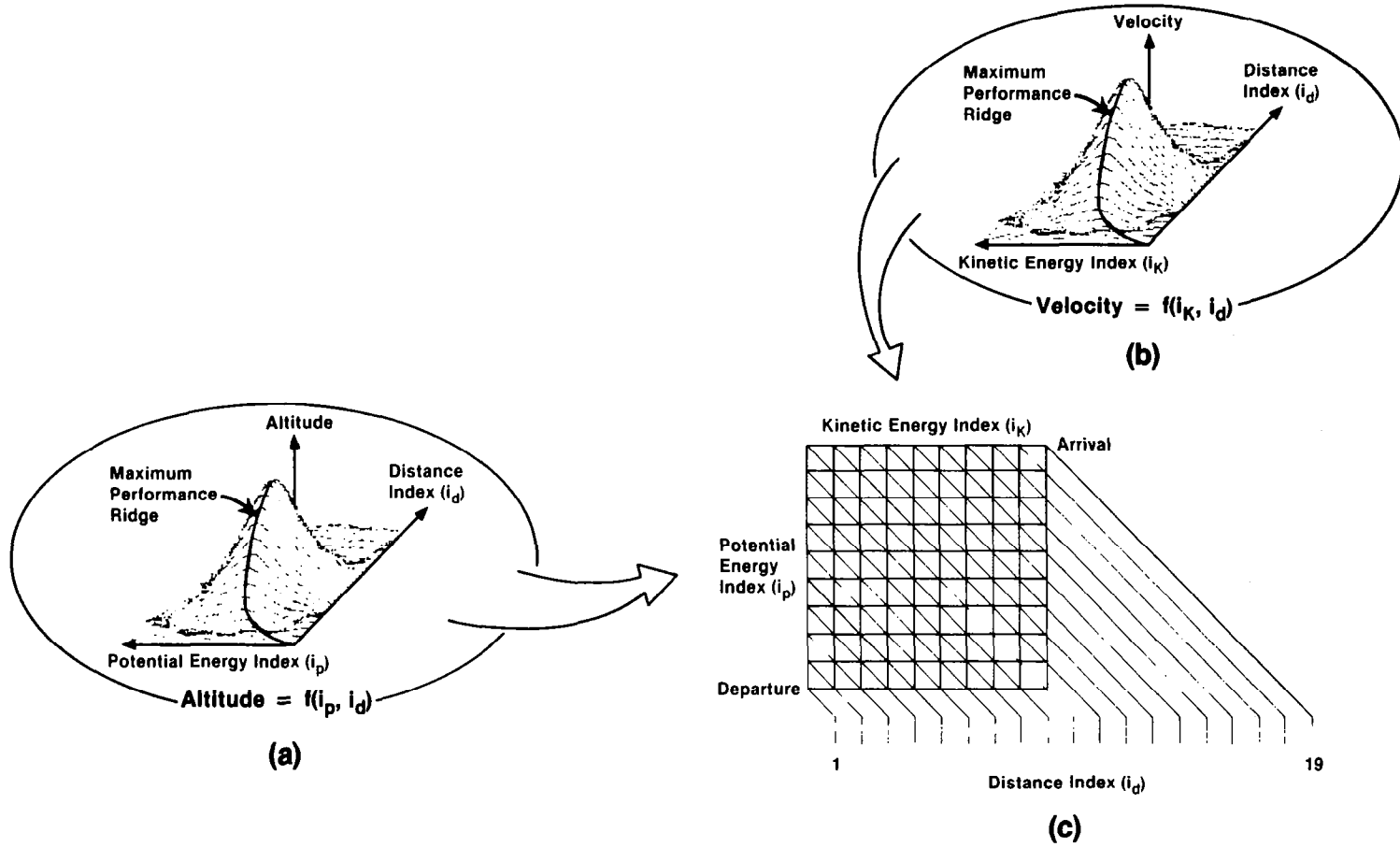
$$h_{p,d} = C_1 e^{C_2(I_p - C_3)} \left[ 1 - e^{C_5 I_d} - e^{C_6 |I_d - C_4|^{C_7}} \right]$$

In the above equation  $C_n$  are constants and  $I_d$  and  $I_p$  as defined. Additionally,

$$C_1 = f(\text{trip maximum altitudes ceiling})$$

$$(C_2 - C_4, C_7) = f(\text{aircraft performance constraints as derived from manufacturer's data})$$

$$C_5 \text{ and } C_6 = f(\text{total trip distance})$$



**FIGURE 7-2**  
**OPTIMAL FLIGHT PLANNING RESTRICTED MOVE TECHNIQUE**

In the example of Figure 7-2,  $I_d$  ranges from 1 to 19. Given an  $I_d$  then  $I_p$  ranges from:

$$\max (1, I_d - 9) \text{ to } \min (10, I_d)$$

As an example, this equation means that at a distance from the departure point represented by  $I_d = 6$ , there are 6 values of  $I_p$  that can be selected, or there are 6 nodal points on the  $I_d = 6$  diagonal matrix line. The selection of an  $I_p$  value defines a nodal point in the matrix and the equation then defines a unique altitude. This altitude relationship determines a family of ridges on which exists a possible optimal solution. An additional observation is that the maximum ridge level that can be reached in a segment is determined by the preceding part of the optimal solution. An explicit form for the  $h_{p,d}$  function is given in Appendix E.

Figure 7-2(b) shows a similar relationship to Figure 7-2(a) except that it concerns velocity ( $V$ ), distance index ( $I_d$ ) and a kinetic energy index ( $I_k$ ). The relationship is,

$$V = f(I_d, I_k).$$

However, the maximum ridge generated does not represent a maximum velocity ridge; but, rather represents a maximum efficiency velocity ridge. The basic assumption is that the most fuel efficient velocity at an altitude is the best velocity to be selected if that velocity and altitude is a part of a total optimal solution.

The equation has a similar form as that for the altitude case and is as follows:

$$v_{d,k} = c_{10} e^{c_{11}(I_k - c_{12})} \left[ 1 - e^{c_{14} I_d} - e^{c_{15} |I_d - c_{13}|^{c_{16}}} \right]$$

In the above equation  $C_n$  are constants, and  $I_d$  and  $I_k$  as defined. Additionally,

$$C_{10} = f(\text{trip maximum velocity})$$

$$(C_{11} - C_{13}, C_{16}) = f(\text{aircraft performance constants as derived from manufacturer's data})$$

$$C_{14} \text{ and } C_{15} = f(\text{total trip distance})$$

Thus, by means of the velocity and altitude functions it is possible to compute an altitude and velocity at each nodal point after selecting an  $I_p$ ,  $I_k$  and  $I_d$ . This matrix is pictured in Figure 7-2(c).

### 7.3 Dynamic Program Calculations

The first step is to systematically compute a velocity and altitude at each nodal point in the matrix. The velocity at each nodal point is modified to ground velocity by the use of upper winds and temperatures data that have been entered. The altitude is with respect to air density and will be modified to pressure altitude only at time of output.

The next step is to compute the total minimum fuel consumption required to go from any nodal point in the matrix to the departure point. Although the matrix presentation in Figure 7-2(c) seems different than that of Section 4, many similarities exist.

First fuel burns are calculated from the two points of  $I_d = 18$  to the single arrival point ( $I_d = 19$ ). This is a function as well of  $H_{p,d}$ ,  $V_{k,d}$ , weight and aircraft constants. We then calculate the fuel burn (based on updated weight) from each of the three points at  $I_d = 17$  to the one or two points at  $I_d = 18$  to which they are connected. For each point in the matrix there are only one or two points at the previous (smaller)  $I_d$  leading to the reference point. Similarly there are only one or two points leading from the reference point to the next (larger)  $I_d$  value. The observation is critical to the concept of a restricted move approach to the dynamic program.

The process outlined above is repeated "backwards" through the matrix. At each step the flags to indicate the partial optimal path are stored as in the concept in Section 4. When the process concludes at  $I_d = 1$ , the optimal path is retrievable from the flags.

Some simplification in the calculations can be achieved by recognition of the various performance constraints as outlined in Section 4.

A considerable savings in calculation is possible. For a sample of  $I_{dmax} = 19$  and  $I_{pmax} = I_{kmax} = 10$ , as shown in Figure 7-2, a maximum of 180 fuel burn calculations are required. For the approach of Section 4, a maximum of 160,200 fuel burn calculations could be required. (This number is drastically reduced with the consideration of performance constraints.)

It is emphasized that the technique has not been programmed and flight tested and thus should be considered preliminary until this work has been completed.

#### 7.4 Functional Description

Section 4 describes the optimizing technique that computes the minimum fuel consumption path profile. Additional programs provide input and output data handling in terms of data storage organization. Since these programs are described in Sections 3 and 5, they will not be repeated. Some details of the input/output process for the programmable calculator are contained in Appendix E.





## APPENDIX A

### FUEL BURN MODEL

#### A.1 Turbojet Equation

The basic equation for turbojet aircraft was derived in Reference 1. Much of this remains the same, and will not be repeated here in detail.

##### A.1.1 General

We will use the following general definitions:

$W_f$  = fuel flow rate

$F_n$  = thrust

$E_T$  = thrust work

$D$  = drag

$E_D$  = drag work

KE = kinetic energy

PE = potential energy

$T$  = time in segment

$d$  = ground distance in segment

$V_T$  = true velocity (average in segment)

$B$  = fuel burn

The general energy balance equation is

$$E_T = E_D + \Delta PE + \Delta KE \quad (A-1)$$

We note also the following relationships for the case of no wind:

$$E_D = \int_0^d D dx = \int_0^T DV_T dt \quad (A-2)$$

$$E_T = \int_0^d F_n dx \cong dF_n = TV_T F_n \quad (A-3)$$

$$B \cong W_f T \quad (A-4)$$

#### A.1.2 Original Derivation

From Reference 1, the basic expression for  $E_D$  has been derived, and is

$$E_D = \frac{R_1 K_1 \bar{\rho} S_w TV_T^3}{2} + \frac{2R_2 K_2 W^2 T}{\bar{\rho} S_w V_T} \quad (A-5)$$

where

$R_i$  are flap/gear configuration drag multipliers

$K_i$  are aircraft constants

$\bar{\rho}$  is atmospheric density

$S_w$  is wing area

$W$  is aircraft weight.

A satisfactory form is then found to estimate  $F_n/W_f$ . This function is used since data is typically arrayed as:

$$\frac{F_n}{W_f} = f(V_T, h).$$

The empirical relation that defines this ratio is

$$\frac{F_n}{W_f} = K_{10} e^{K_{11} V_T} + K_{12} h^2 + K_{13} h + K_{14} \quad (A-6)$$

where

$K_i$  are constants

$h$  is altitude.

This yields then

$$E_T = W_f TV_T \left[ K_{10} e^{K_{11} V_T} + K_{12} \left( \frac{h_1^2 + h_1 h_2 + h_2^2}{3} \right) + K_{13} \left( \frac{h_1 + h_2}{2} \right) + K_{14} \right]. \quad (A-7)$$

Finally, we recognize

$$KE = \frac{W}{2g} (v_{T2}^2 - v_{T1}^2) \quad (A-8)$$

where  $g$  is gravitational acceleration.

$$PE = W(h_2 - h_1). \quad (A-9)$$

We thence have

$$B = W_f T = \frac{E_D + \Delta KE + \Delta PE}{E_T / W_f T} \quad (A-10)$$

This, with the values of the preceding equations is the turbojet fuel burn equation.

### A.1.3 Low Altitude Enhancements

The original solution was developed for climb/cruise/descent, for above 2000 feet and 250 Kn. Use of the fuel burn equation outside of these limits required a modification to reflect the higher thrust values for the low altitude/low speed regime. Thus, for thrust values in excess of  $K_7$ , we add to the fuel burn equation

$$LAMI = K_8 F_n + K_9 \quad (A-11)$$

where  $F_n$  is derived from original fuel burn equation, noting it is of the form

$$E_T = TV_T F_n = E_D + \Delta KE + \Delta PE. \quad (A-12)$$

The detailed form of the equation is given in Figures A-1 and A-2.

### A.1.4 Actual Winds and Temperature

In the event actual winds and temperature are input into the fuel burn equation, account must be taken of the differences between ground and true velocity.

True velocity ( $V_T$ ) is used in describing the motion of the aircraft in the air mass, and determines the instantaneous fuel flow. Ground velocity ( $V_G$ ) reflects the motion of the aircraft with respect to the ground, and determines aircraft progress along the route of flight. Ground velocity is defined as the magnitude of the ground velocity vector ( $\vec{V}_G$ ) defined in equation A-13.

$$\vec{V}_G = \vec{V}_T - \vec{WV} \quad (A-13)$$

where  $\vec{V}_G$  = vector with magnitude ground velocity and direction of true heading

$\vec{V}_T$  = vector with magnitude true velocity and direction of true course

$\vec{WV}$  = vector with magnitude wind velocity and direction that the wind is blowing from.

$$B = \frac{TV_T F_n}{V_T A} + T(LAM1)$$

where

$$F_n = \frac{R_1 K_1 \bar{\rho} S_w V_T^2}{2} + \frac{2R_2 K_2 W^2}{\bar{\rho} S_w V_T^2} + \frac{W}{gT} (v_{T2} - v_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

$$A = K_{10} e^{K_{11} V_T} + K_{12} \left( \frac{h_1^2 + h_1 h_2 + h_2^2}{3} \right) + K_{13} \left( \frac{h_1 + h_2}{2} \right) + K_{14}$$

$$LAM1 = \begin{cases} 0 & \text{if } F_n < K_7 \\ K_8 F_n + K_9 & \text{otherwise} \end{cases}$$

$R_1, R_2$  contained in Figure A-2

**FIGURE A-1**  
**FUEL BURN EQUATION FOR TURBOJET (ISA CONDITIONS)**

$$R_1 = \begin{cases} GU_1 F^3 + GU_2 F^2 + GU_3 F + 1 & \text{(gear up)} \\ GD_1 F^3 + GD_2 F^2 + GD_3 F + GD_4 & \text{(gear down)} \end{cases}$$

$$R_2 = FDM_1 F^3 + FDM_2 F^2 + FDM_3 F + 1$$

where

F is flap angle in degrees

$GU_i$ ,  $GD_i$ , and  $FDM_i$  are aircraft constants

**FIGURE A-2  
FLAP AND GEAR DRAG MULTIPLIERS**

In addition, actual conditions should be reflected in calculating density as a function of average temperature (T, in degrees Fahrenheit) and flight level (hp).

Density below flight level 36089 is given by

$$\rho_h = \frac{1.233}{459.67 + T} (1 - .00000687hp)^{5.2563} \quad (A-14)$$

and above flight level 36089 it is given by

$$\rho_h = \frac{.27544}{459.67 + T} [.2235 \exp ((36089 - hp)/53.35)] \quad (A-15)$$

where

T = degrees Fahrenheit.

The resultant fuel burn equation is given in Figure A-3.

## A.2 Turboprop Equation

### A.2.1 General

Section A.1 presented the equation form that can be utilized to compute the estimated fuel burn of a turbojet aircraft. Before developing a similar equation that will apply to propeller aircraft, it is relevant to discuss those differences in powerplants that account for equational differences.

In the case of turbojet powered aircraft, the thrust force is generated reactively by the mass flow velocity of the jet nozzle gases in combination with the intake compressor by-pass airflow. The thrust is a function of the difference between the aircraft's velocity through the air and the mass flow velocity through the engine nozzle. In turboprop powered aircraft the thrust force is derived mainly through the propeller which the turbine operates in addition to the compressor. The power generated is essentially constant with speed and the thrust decreases with speed. Thus the equational form of the fuel burn equations for the propeller aircraft and turbojet aircraft are different.

$$B = \frac{TV_T F_n}{V_T A} + T(LAM1)$$

where

$$F_n = \frac{R_1 K_1 \bar{\rho}_h S_w V_T^2}{2} + \frac{2R_2 K_2 W^2}{\bar{\rho}_h S_w V_T^2} + \frac{W}{gT} (V_{T2} - V_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

A; LAM1;  $R_i$  previously defined in Figures A-1 and A-2.

$\bar{\rho}_h$  is altitude dependent density

T is derived as  $\frac{\text{Ground Distance}}{\text{Ground Velocity}}$

**FIGURE A-3**  
**FUEL BURN EQUATION FOR TURBOJET**  
**(ACTUAL WINDS AND TEMPERATURES)**



### A.2.2 General Derivation

It is convenient to concentrate on the relationships of power, since power relates to the shaft output of the engine. Data is readily available to establish a curve fit between P and  $W_f$  of the form:

$$W_f = aP + be^{ch} \quad (A-16)$$

where

a, b, c are constants

P is output shaft horsepower.

Thrust and shaft horsepower are related by

$$P = \frac{V_T F_n}{\eta_P 550} \quad (A-17)$$

where  $\eta_P$  is the propeller efficiency, and is approximated by (.82).

Substituting and letting

$$K_{15} = a/(\eta_P 550)$$

$$K_{16} = b$$

$$K_{17} = c$$

results in

$$W_f = K_{15} V_T F_n + K_{16} e^{K_{17} h}. \quad (A-18)$$

We take the derived value of  $F_n$  from equation (A-12)

$$F_n = \frac{E_D + \Delta KE + \Delta PE}{TV_T}. \quad (A-19)$$

Finally,

$$B = W_f T = K_{15} TV_T F_n + K_{16} T e^{K_{17} h} \quad (\text{A-20})$$

The resultant fuel burn equation is given in Figure A-4.

### A.2.3 Actual Winds and Temperatures

Actual winds and temperatures are reflected in the fuel burn equation analogously to the turbojet case. The resultant is given in Figure A-5.

## A.3 Turbocharged Piston Equation

### A.3.1 General Derivation

As in the turboprop aircraft, the power is converted into thrust by the propeller. In a turbocharged engine, constant power output can be maintained to a certain altitude.

Turbocharged piston engine aircraft normally use a lean fuel mixture during cruise and descent and richer fuel mixtures during takeoff and climb conditions. This can result in different fuel flows at the same altitude and velocity. When the lean fuel mixture is used, an empirical curve fit of fuel flow ( $W_{fL}$ ) is a linear function of brake horsepower  $P$ :

$$W_{fL} = aP + b \quad (\text{A-21})$$

From operating manuals it is possible to derive an empirical curve fit for fuel flow at richer fuel mixtures ( $W_{fR}$ ), assuming standard operating procedures. This function is quadratic instead of linear in  $P$ , to accommodate different fuel mixtures at different power settings:

$$W_{fR} = cP^2 + sP + r \quad (\text{A-22})$$

making the assumption that the richer fuel mixtures are only used when the aircraft is in climb. Equations (A-21) and (A-22)

$$B = K_{15} TV_T F_n + K_{16} Te^{K_{17} h}$$

where

$$F_n = \frac{R_1 K_1 \bar{\rho} S_w V_T^2}{2} + \frac{2R_2 K_2 W^2}{\bar{\rho} S_w V_T^2} + \frac{W}{gT} (V_{T2} - V_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

$R_1$  previously defined

**FIGURE A-4  
FUEL BURN EQUATION FOR TURBOPROP (ISA CONDITIONS)**

$$B = K_{15} TV_T F_n + K_{16} Te^{K_{17} h}$$

where

$$F_n = \frac{R_1 K_1}{2} \bar{\rho}_h S_w V_T^2 + \frac{2R_2 K_2 W^2}{\bar{\rho}_h S_w V_T^2} + \frac{W}{gT} (V_{T2} - V_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

$R_1$  previously defined

$\bar{\rho}_h$  is altitude dependent density

T is derived as  $\frac{\text{Ground Distance}}{\text{Ground Velocity}}$

**FIGURE A-5  
FUEL BURN EQUATION FOR TURBOPROP  
(ACTUAL WINDS AND TEMPERATURE)**

can be combined using an exponential function, in the rate of climb (R/C), as a step function:

$$W_f = \left( e^{K(|R/C| + R/C)} \right) (W_{fL}) + \left( 1 - e^{K(|R/C| + R/C)} \right) (W_{fR}) \quad (A-23)$$

With the constant K properly chosen, this exponential will be equal to 1 for climb, and 0 at all other times.

Substituting

$$R/C = V_T(h_2 - h_1)/d \quad (A-24)$$

$$P = V_T F_n / (\eta_p 550) \quad (A-25)$$

into equation (A-23) we have:

$$W_f = \delta \left( \frac{a V_T F_n}{\eta_p 550} \right) + b + (1 - \delta) \left( \frac{c V_T^2 F_n^2}{(\eta_p 550)^2} + \frac{s V_T F_n}{\eta_p 550} + r \right) \quad (A-26)$$

where

$$\delta = e^{KV_T/d \left[ |h_2 - h_1| + (h_2 - h_1) \right]}$$

Letting

$$K_{15} = K$$

$$K_{16} = a/(\eta_p 550)$$

$$K_{17} = b$$

$$K_{18} = c/(\eta_p 550)^2$$

$$K_{19} = s/(\eta_p 550)$$

$$K_{20} = r$$

and noting

$$B = W_f T \quad (A-27)$$

we have fuel burn as:

$$B = \delta (K_{16} V_{T_n} F_n + K_{17}) + T(1-\delta)(K_{18} V_{T_n}^2 F_n^2 + K_{19} V_{T_n} F_n + K_{20}) \quad (A-28)$$

where  $F_n$  is defined in the turbojet derivation.

The results are summarized in Figure A-6.

### A.3.2 Actual Winds and Temperature

Accounting for actual winds and temperatures is analogous to the previous derivations. The resultant is presented in Figure A-7.

$$B = \delta T(K_{16} V_T F_n + K_{17}) + T(1 - \delta)(K_{18} V_T^2 F_n^2 + K_{19} V_T F_n + K_{20})$$

where

$$\delta = e^{K_{15} (1/T)} \left[ |h_2 - h_1| + (h_2 - h_1) \right]$$

$$F_n = \frac{R_1 K_1}{2} \bar{\rho}_w S_w V_T^2 + \frac{2R_2 K_2 W^2}{\bar{\rho}_w S_w V_T^2} + \frac{W}{gT} (V_{T2} - V_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

**FIGURE A-6**  
**FUEL BURN EQUATION FOR TURBOCHARGED PISTON**  
**(ISA CONDITIONS)**

$$B = \delta T(K_{16} V_T F_n + K_{17}) + T(1 - \delta)(K_{18} V_T^2 F_n^2 + K_{19} V_T F_n + K_{20})$$

where

$$F_n = \frac{R_1 K_1}{2} \bar{\rho}_h S_w V_T^2 + \frac{2R_2 K_2^2 W}{\bar{\rho}_h S_w V_T^2} + \frac{W}{gT} (V_{T2} - V_{T1}) + \frac{W}{TV_T} (h_2 - h_1)$$

$\delta$  previously defined

$R_1$  previously defined

$\bar{\rho}_h$  is altitude dependent density

$T$  is derived as  $\frac{\text{Ground Distance}}{\text{Ground Velocity}}$

**FIGURE A-7**  
**FUEL BURN EQUATION FOR TURBOCHARGED PISTON**  
**(ACTUAL WINDS AND TEMPERATURE)**



## APPENDIX B

### AIRCRAFT CONSTANTS

#### B.1 General

This appendix contains aircraft constants for the fuel burn equation for both a turboprop aircraft (King Air 200) and a turbocharged piston aircraft (Cessna 421C). The use of these aircraft constants with the fuel burn equation and the techniques used to derive them are described in detail in References 1 and 2, and Appendix A. Accuracy checks using the aircraft constants and cruise conditions are also given for both of these aircraft.

#### B.2 Notation

H = altitude (feet)

V = velocity (true airspeed in knots)

FB<sub>REF</sub> = fuel burn, reference value (pounds/hour)

FB<sub>CAL</sub> = fuel burn, calculated value (pounds/hour)

$$\Delta = \text{FB}_{\text{REF}} - \text{FB}_{\text{CAL}}$$

$$\% = \frac{\Delta}{\text{FB}_{\text{REF}}} \times 100$$

#### B.3 Beechcraft Super King Air 200

Tables B-1 and B-2 contain the aircraft constants and an accuracy check respectively for the Beechcraft Super King Air 200 powered by two Pratt and Whitney PT6A-41 engines. The reference fuel burn values used in the accuracy check (Table B-2) were taken from the Super King Air 200 Pilot's Operating Handbook (Reference 5).

#### B.4 Cessna 421C

Tables B-3 and B-4 contain the aircraft constants and an accuracy check, respectively, for the Cessna 421C (Golden Eagle) powered by two Teledyne Continental Motors GTSIO-520-L engines. The reference fuel burn values used in the accuracy check (Table B-4) were taken from the Cessna 421 Golden Eagle Information Manual (Reference 6).

TABLE B-1  
BEEHCRAFT SUPER KING AIR 200 CONSTANTS

$S_w = 303$	$FDM_1 = 0$
$K_1 = .0256014$	$FDM_2 = 0$
$K_2 = .04241259$	$FDM_3 = -.0057$
$K_{15} = .0000002692$	$WFIDLE = .067$
$K_{16} = .080443$	$L_1 = 0$
$K_{17} = -.000034$	$L_2 = 0$
$GD_1 = 0$	$L_3 = 0$
$GD_2 = 0$	$L_4 = 0$
$GD_3 = 0$	$VNE = 289$
$GD_4 = 2.3573$	$VS = 75$
$GU_1 = 0$	$MTOW = 12500$
$GU_2 = 0$	$OEW = 7755$
$GU_3 = .01547$	

	<u>Takeoff</u>	<u>Climb</u>	<u>Cruise</u>
$A_1 =$	0	0	0
$A_2 =$	0	0	0
$A_3 =$	.10E-09	-.44E-10	-.44E-10
$A_4 =$	-.42238E-05	-.39419E-05	-.39419E-05
$A_5 =$	.28228	.29681	.29681

Information from Beechcraft Super King Air 200 Pilot's Operating Handbook and Airplane Flight Manual (Ref. 5) and Jane's All the World's Aircraft (Ref. 7) was used in deriving this set of constants.

TABLE B-2  
BEECHCRAFT SUPER KING AIR 200 ACCURACY CHECK

Time = 3600 sec.  
Weight = 11,000 lbs.

<u>RPM</u>	<u>H</u>	<u>V</u>	<u>FBREF</u>	<u>FBAL</u>	<u>Δ</u>	<u>%</u>
1700	0	240	946	919	-27	-2.8
1700	2000	245	918	903	-15	-1.6
1700	4000	250	892	887	-5	-0.6
1700	6000	254	866	864	-2	-0.2
1700	8000	259	846	849	3	0.3
1700	10000	264	828	833	5	0.6
1700	12000	270	812	823	11	1.3
1700	14000	275	800	807	7	0.9
1700	16000	280	792	791	-1	-0.1
1700	18000	280	746	747	1	0.1
1700	20000	277	690	690	0	0.0
1700	22000	275	644	642	-2	-0.3
1700	24000	273	598	599	1	0.2
1700	26000	269	554	552	-2	-0.4
1700	28000	265	514	511	-3	-0.6
1700	29000	263	494	492	-2	-0.4
1700	31000	257	458	453	-5	-1.1
1700	33000	250	422	417	-5	-1.1
1800	0	245	982	956	-26	-2.6
1800	2000	250	954	939	-15	-1.6
1800	4000	255	926	923	-3	-0.3
1800	6000	260	902	906	4	0.4
1800	8000	265	882	889	7	0.8
1800	10000	270	866	873	7	0.8
1800	12000	276	850	863	13	1.5
1800	14000	281	838	845	7	0.8
1800	16000	283	802	810	8	1.0
1800	18000	281	746	752	8	1.1
1800	20000	279	692	700	8	1.1
1800	22000	277	646	652	6	0.9
1800	24000	274	598	603	5	0.8
1800	26000	271	554	560	6	1.1
1800	28000	267	514	518	4	0.8
1800	29000	264	494	495	1	0.2
1800	31000	258	458	455	-3	-0.6
1800	33000	250	422	417	5	1.2

TABLE B-2  
(Concluded)

<u>RPM</u>	<u>H</u>	<u>V</u>	<u>FB<sub>REF</sub></u>	<u>FB<sub>CAL</sub></u>	<u>Δ</u>	<u>%</u>
1900	0	250	1016	995	-21	-2.1
1900	2000	255	988	977	-11	-1.1
1900	4000	260	962	960	-2	-0.2
1900	6000	265	940	943	3	0.3
1900	8000	270	920	925	5	0.5
1900	10000	276	904	915	9	1.0
1900	12000	281	890	897	7	0.8
1900	14000	287	876	885	9	1.0
1900	16000	286	822	828	6	0.7
1900	18000	285	770	775	5	0.6
1900	20000	283	716	721	5	0.7
1900	22000	281	668	671	3	0.4
1900	24000	279	620	625	5	0.8
1900	26000	275	574	575	1	0.2
1900	28000	271	532	531	-1	-0.2
1900	29000	269	512	511	-1	-0.2
1900	31000	263	474	469	-5	-1.0
1900	33000	256	434	431	-3	-0.7

Mean = .06%  
Standard Deviation = .98%  
Variance = .95%

TABLE B-3  
CESSNA 421C (GOLDEN EAGLE) CONSTANTS

S <sub>w</sub> = 215	GU <sub>1</sub> = 0
K <sub>1</sub> = .0274935	GU <sub>2</sub> = .00041563
K <sub>2</sub> = .0415015	GU <sub>3</sub> = .030365
K <sub>15</sub> = -.57564	FDM <sub>1</sub> = 0
K <sub>16</sub> = 2.636E-07	FDM <sub>2</sub> = 0
K <sub>17</sub> = .0057675	FDM <sub>3</sub> = 0
K <sub>18</sub> = 2.12978E-12	WFIDLE = .01111
K <sub>19</sub> = 5.3845E-07	L <sub>1</sub> = 0
K <sub>20</sub> = .0798467	L <sub>2</sub> = 0
GD <sub>1</sub> = 0	L <sub>3</sub> = 0
GD <sub>2</sub> = .0004145	L <sub>4</sub> = 0
GD <sub>3</sub> = .03042	VNE = 258
GD <sub>4</sub> = 1.9641	VS = 74
MTOW = 7450	OEW = 4426

	TAKEOFF	CLIMB	CRUISE
A <sub>1</sub> =	0	0	0
A <sub>2</sub> =	0	0	0
A <sub>3</sub> =	-1.68E-10	-1.68E-10	-1.68E-10
A <sub>4</sub> =	2.410128E-06	2.410128E-06	2.410128E-06
A <sub>5</sub> =	.16033551	.16033551	.16033551

Information from the Cessna 421 Golden Eagle Information Manual (Ref. 6) and Jane's All the World's Aircraft (Ref. 7) was used in deriving this set of constants.

TABLE B-4  
CESSNA 421C (GOLDEN EAGLE) ACCURACY CHECK

<u>RPM</u>	<u>MP</u>	<u>H</u>	<u>V</u>	<u>FBREF</u>	<u>FBAL</u>	<u>Δ</u>	<u>%</u>
1900	32.5	0	186	257	254.3	-2.7	-1.0
1900	31.0	0	183	246	244.9	-1.1	-0.4
1900	29.0	0	177	228	227.1	-0.9	-0.4
1900	27.0	0	171	211	210.7	-0.3	-0.1
1900	25.0	0	164	194	193.1	-0.9	-0.5
1900	32.5	5000	195	257	255.7	-1.3	-0.5
1900	31.0	5000	191	246	244.0	-2.0	-0.8
1900	29.0	5000	185	228	227.4	-0.6	-0.02
1900	27.0	5000	178	211	209.6	-1.4	-0.7
1900	25.0	5000	171	194	193.3	-0.7	-0.03
1900	32.5	10000	204	257	255.4	-1.6	-0.6
1900	31.0	10000	200	246	244.5	-1.5	-0.6
1900	29.0	10000	194	228	229.0	1.0	0.4
1900	27.0	10000	187	211	212.3	1.3	0.6
1900	25.0	10000	179	194	194.9	0.9	0.5
1900	32.5	15000	214	257	256.1	-0.9	-0.3
1900	31.0	15000	210	246	245.9	-0.1	0.4
1900	29.0	15000	203	228	229.2	1.2	0.5
1900	27.0	15000	195	211	211.6	0.6	0.3
1900	25.0	15000	186	194	193.9	-0.1	-.05
1900	32.5	20000	224	257	255.2	-1.8	-0.7
1900	31.0	20000	220	246	245.9	-0.1	-.04
1900	29.0	20000	212	228	228.4	0.4	0.2
1900	27.0	20000	204	211	212.4	1.4	0.7
1900	25.0	20000	194	194	194.5	0.5	0.2
1900	32.5	25000	236	257	257.5	0.5	0.2
1900	31.0	25000	231	246	246.8	0.8	0.3
1900	29.0	25000	223	228	230.8	2.8	1.2
1900	27.0	25000	212	211	211	0	0
1900	25.0	25000	200	194	192.2	-1.8	-0.9
1700	32.5	0	177	230	227.1	-2.9	-1.2
1700	31.0	0	174	219	218.8	-0.2	-0.1
1700	29.0	0	168	204	202.9	-1.1	-.5
1700	27.0	0	161	187	186.0	1.0	.5
1700	25.0	0	154	171	170.7	-0.3	-.2
1700	23.0	0	146	155	155.2	0.2	.1
1700	32.5	5000	186	230	230.1	0.1	.04
1700	31.0	5000	181	219	217.0	-2.0	-0.9

TABLE B-4  
(Concluded)

<u>RPM</u>	<u>MP</u>	<u>H</u>	<u>V</u>	<u>FBREF</u>	<u>FBAL</u>	<u>Δ</u>	<u>%</u>
1700	29.0	5000	175	204	202.4	-1.6	-0.8
1700	27.0	5000	168	187	186.7	-0.3	-0.2
1700	25.0	5000	161	171	172.6	1.6	0.9
1700	23.0	5000	151	155	154.8	-0.2	0.1
1700	32.5	10000	194	230	229	-1.0	-0.4
1700	31.0	10000	190	219	219.3	0.3	0.1
1700	29.0	10000	183	204	203.3	-0.7	-0.3
1700	27.0	10000	175	187	186.9	-0.1	-0.05
1700	25.0	10000	167	171	172.1	1.1	0.6
1700	23.0	10000	155	155	153.2	-1.8	-1.2
1700	32.5	15000	203	230	229.2	-0.8	-0.3
1700	31.0	15000	198	219	218	-1.0	-0.4
1700	29.0	15000	191	204	203.5	-0.5	-0.2
1700	27.0	15000	183	187	188.4	1.4	0.7
1700	25.0	15000	172	171	170.2	-0.8	-0.5
1700	32.5	20000	213	230	230.4	0.4	0.2
1700	31.0	20000	208	219	220.2	1.2	0.5
1700	29.0	20000	200	204	205	1.0	0.5
1700	27.0	20000	189	187	186.4	-0.6	-0.3
1700	25.0	20000	177	171	166.7	-4.3	-2.5
1700	27.0	25000	194	187	183.9	-3.1	-1.6

Percent Differences:

Mean = 0.16  
Standard Deviation = 0.64  
Variance = 0.40

## APPENDIX C

### OPERATIONAL SCENARIO

This appendix presents the concept of the operational scenario for the use of the flight planning aid.

#### C.1 Introduction

The conservation of aircraft fuel starts with the initial trip planning, and ends with a post flight evaluation of actual fuel usage. During preflight planning, the optimum fuel usage performance characteristics of an aircraft must be combined with route alternatives and upper air mass conditions to arrive at the flight path and operating parameters to achieve the maximum fuel economy. This involves the evaluation of "trade off" situations such as operating at a less fuel efficient altitude in order to take advantage of an upper wind condition.

This first level of fuel burn optimization is based upon pretakeoff route conditions. After becoming airborne, the actual weather conditions must be evaluated at reasonable time intervals and the required adjustments made in order to optimize the fuel conservation process.

During the post flight review the actual versus predicted fuel usage can be evaluated. This is an important step because, during the flight, judgments were made based upon use of the technique. The review can be utilized to modify or adjust the individual technique so that future flights will benefit.

Examination of these three steps indicates that the solutions, if performed manually, would be cumbersome, time consuming, complex and error prone. Thus, the need for a flight planning aid points to some type of computer capability. The state-of-the-art of modestly-priced programmable calculators has reached the level that adequate computational capacity is readily available.

The availability of relatively inexpensive minicomputers that are capable of interfacing with larger machines via modems provide a potential for performing complex track flight planning at virtually all airport locations.



## C.2 Utilization Alternatives

The operational utilization of the minimum peak consumption trajectory technique can be separated into four categories with each having different capabilities and data requirements. They are as follows:

<u>ALTERNATIVES</u>	<u>PREFLIGHT</u>	<u>INFLIGHT</u>
I	Programmable Calculator	Programmable Calculator
II	Minicomputer/ Time Sharing	Programmable Calculator
III	Minicomputer/ Time Sharing	Performance Computer
IV	Performance Computer	Performance Computer

### C.2.1 Alternative I

The use of the programmable calculator for both preflight planning and inflight update and control represents the least cost implementation of the minimum fuel trajectory technique. During preflight planning (Table C-1) the pilot examines the winds and temperatures along various routes and chooses the route that, in his judgment, has the best conditions. The appropriate programs are loaded into the calculator by utilizing either bar code or magnetic cards. The total trip is divided into segments and the course, magnetic variation, and distance of each segment determined (Table C-2). Next, the upper winds and temperatures at various altitudes for each segment are determined. The maximum acceptable operating altitude along with the landing weight (including alternate fuel reserves) is also determined. This data is entered interactively into the calculator. The output is a printed listing for each segment of distance, fuel consumption, speed, altitude, wind component, and temperature. Additionally, the complete matrix data is stored within the calculator and on magnetic cards for inflight use.

On entering the cockpit for departure, the pilot loads his inflight programs and trip data cards. At departure the time and actual gross weight are entered. During climb the pilot can receive climb speed and power setting by entering altitude and temperature. At the first waypoint the pilot enters time, altitude, velocity,

**TABLE C-1  
PRELIMINARY FUNCTION CONCEPT  
STEP I: PILOT SUPPLIED DATA**

### **Route Of Flight Data**

- **Determine Route Of Flight**
- **Divide Into Approximately 20 Minute Flight Segments**
- **Define Distance And Course For Each Segment**

### **En Route Weather**

- **Define Wind Velocity, Heading And Temperature**
  - **Maximum Operating Altitude**  
(Based Upon Aircraft Limitations,  
Icing, Turbulence, Oxygen Availability)

### **Calculator Programs**

- **Have Available Program Modules/Magnetic Tape With Particular Aircraft Constants Stored**
- Or
- **General Program Module/Magnetic Tape/Paper Copy With A List Of Particular Aircraft Constants**

**TABLE C-2  
PRELIMINARY FUNCTIONAL CONCEPT  
STEP II: PRE-FLIGHT PLANNING**

**Initial Data**

- Aircraft ID, Engines
- Empty Weight
- Altitude Constraint

**Segment Data Input**

- Distance
- Altitude
- Temperature
- Wind
- Course

**Output Best And Next Best**

- Climb, Cruise, Descent
  - IAS/Mach
  - Power Settings
  - Altitudes

temperature, and gross weight or fuel on board (Table C-3). Also, at this point a projected minimum fuel arrival time will be displayed. The pilot can either accept or modify the value. The program then reprojects, incrementally, the remaining portion of the flight and displays a speed, altitude, and power setting command.

The entry is reported at each way point throughout the flight. The time is continually updated and can be displayed upon command to the pilot. Additionally, a start descent time is calculated and displayed. At start of descent the pilot enters the time, altitude, temperature, and fuel on board at which the descent speed is displayed with the appropriate power setting. As in climb, the speed and power setting is displayed at entry of a time, altitude, and temperature. Upon landing, the time and fuel on board is entered. The data contained in the calculator is recorded on magnetic cards.

The post flight analysis (Table C-4) is performed by the pilot entering the appropriate programs in combination with the flight data that was recorded on the data cards. A printout records the actual versus projected plans and allows the pilot to evaluate the trip fuel efficiency.

#### C.2.2 Alternative II

This alternative assumes that the ground or preflight planning is performed by a minicomputer that is linked to a VOR and a winds and temperatures data base (Table C-5). It also assumes that the computer has the capability to evaluate multiple tracks either iteratively or by utilizing a dynamic programming data base. The pilot enters landing gross weight, arrival point (VOR), maximum acceptable altitude, departure point, and aircraft/engine types. The system then provides a printed output as described in Alternative I, including a bar code for loading into a programmable calculator.

The inflight and postflight operations are the same as described in Alternative I.

#### C.2.3 Alternatives III and IV

These alternatives are listed for the purpose of highlighting the point that the techniques described can be implemented on any computer, providing that the required interface capability has been provided.

**TABLE C-3  
PRELIMINARY FUNCTIONAL CONCEPT  
STEP III: IN-FLIGHT UPDATE AND CONTROL**

**Updated Inputs For Way Points**

- Time
- Altitude
- Fuel Burn/Flow
- Temperature

**Revised Output**

- IAS/Mach
- Power Settings
- Altitude

**TABLE C-4  
PRELIMINARY FUNCTIONAL CONCEPT  
STEP IV: POST-FLIGHT REVIEW INPUT AND OUTPUT**

**To Be Determined Based On Pilot Needs**

**Objectives**

- **Compare Best To Actual Fuel Conservation Performance**
  - Stored In Calculator
  - Reflects Encountered Conditions
- **Present Results So That Future Performance Will Be Improved**

**TABLE C-5  
OPERATIONAL SCENARIO**

- **Preflight**

- **Time Share or Separate Computer at Fixed Base Operator**
- **(Later) Possible Automated Weather Input**
- **Initial Optimal Solution**

- **Inflight**

- **Programmable Calculator, Initialized With Ground Solution**
- **Inflight Updated Inputs and Solution**

APPENDIX D  
SAMPLE PROBLEM

This appendix contains a sample optimization problem done on the Apple II computer. This problem consisted of determining the optimum path profile for a Super King Air 200 aircraft during a 360 nmi flight.

D.1 Inputs

The actual inputs for this problem are given in Table D-1 (as echoed out to the printer as output). The altitude modes were calculated by the program while the velocity modes were designated by the user.

D.2 Matrix Definition

The matrix nodal points in altitude, velocity, and distance are defined by either the user or the program. In the sample problem the velocity nodes are defined by the user while the altitude nodes are defined by the program. Since the number of distance nodes is greater than the number of waypoints, the program embeds distance nodes between waypoints. The altitude, velocity, and distance nodes defined for the sample problem were printed out and are contained among the output figures in Table D-1.

D.3 Optimal Path Profile

Once an optimum path profile has been determined, the path is printed in the form shown in Table D-1, along with the departure and arrival weights. The altitudes printed out differ from the altitude nodal points since the nodal points are density altitudes while pressure altitude is given. The program also prints a list of flight segments using the optimum path containing start and end distances, velocities (TAS), and pressure altitudes for each flight segment, and the fuel burn over the segment as shown for the sample problem in Table D-1.

D.4 Outputs

The outputs for this sample problem are shown in Table D-1 and have been described previously.



TABLE D-1  
SAMPLE OUTPUT

FLIGHT TITLE  
 ? SAMPLE FLIGHT  
 INPUT SER.CEILING,DEP.ALT.,DEP.VEL,ARR.ALT,ARR.VEL  
 ? 33000.,5000.,135.,5000.,135.  
 NO. OF ALT.NODES,VEL.NODES,DIST.NODES,WAYPOINTS?  
 ? 10,10,10,5  
 INPUT ALTITUDE NODES(Y/N)  
 ? N  
 INPUT VELOCITY NODES(Y/N)  
 ? Y  
 INPUT VELOCITY NODES  
 ? 135.,155.,175.,190.,200.,210.,220.,230.,250.,270.  
 LANDING GROSS WEIGHT  
 ? 11250.

WAYPOINT NUMBER            1  
 DISTANCE= 0.000000  
 HEADING= 40.0000  
 VARIATION= 3.0000  
 ALT=15000.0    WD= 90.0    WV=30.000    TEMP= 5.50  
 ALT=25000.0    WD= 70.0    WV=40.000    TEMP= -30.20  
 ALT=35000.0    WD= 50.0    WV=70.000    TEMP= -65.80

WAYPOINT NUMBER            2  
 DISTANCE= 54.000000  
 HEADING= 120.0000  
 VARIATION= 3.0000  
 ALT=15000.0    WD=160.0    WV=55.000    TEMP= 2.00  
 ALT=25000.0    WD=180.0    WV=60.000    TEMP= -22.50  
 ALT=35000.0    WD=230.0    WV=70.000    TEMP= -58.00

WAYPOINT NUMBER            3  
 DISTANCE=113.000000  
 HEADING= 80.0000  
 VARIATION= 5.0000  
 ALT=15000.0    WD=130.0    WV=40.000    TEMP= -1.00  
 ALT=25000.0    WD=180.0    WV=50.000    TEMP= -25.00  
 ALT=35000.0    WD=210.0    WV=60.000    TEMP= -59.00

TABLE D-1  
(Continued)

WAYPOINT NUMBER	4		
DISTANCE=	265.000000		
HEADING=	90.0000		
VARIATION=	8.0000		
ALT=15000.0	WD=270.0	WV=30.000	TEMP= -3.00
ALT=25000.0	WD=260.0	WV=40.000	TEMP= -26.00
ALT=35000.0	WD=220.0	WV=50.000	TEMP= -62.00

WAYPOINT NUMBER	5		
DISTANCE=	315.000000		
HEADING=	90.0000		
VARIATION=	8.0000		
ALT=15000.0	WD=300.0	WV=10.000	TEMP= 5.50
ALT=25000.0	WD=290.0	WV=20.000	TEMP= -30.20
ALT=35000.0	WD=270.0	WV=40.000	TEMP= -65.80

ALTITUDE NODE	1=	5000.0000
ALTITUDE NODE	2=	8111.1111
ALTITUDE NODE	3=	11222.2222
ALTITUDE NODE	4=	14333.3333
ALTITUDE NODE	5=	17444.4444
ALTITUDE NODE	6=	20555.5556
ALTITUDE NODE	7=	23666.6667
ALTITUDE NODE	8=	26777.7778
ALTITUDE NODE	9=	29888.8889
ALTITUDE NODE	10=	33000.0000

VELOCITY NODE	1=	135.0000
VELOCITY NODE	2=	155.0000
VELOCITY NODE	3=	175.0000
VELOCITY NODE	4=	190.0000
VELOCITY NODE	5=	200.0000
VELOCITY NODE	6=	210.0000
VELOCITY NODE	7=	220.0000
VELOCITY NODE	8=	230.0000
VELOCITY NODE	9=	250.0000
VELOCITY NODE	10=	270.0000

TABLE D-1  
(Continued)

DISTANCE NODE	1=	0.0000
DISTANCE NODE	2=	27.0000
DISTANCE NODE	3=	54.0000
DISTANCE NODE	4=	83.5000
DISTANCE NODE	5=	113.0000
DISTANCE NODE	6=	151.0000
DISTANCE NODE	7=	189.0000
DISTANCE NODE	8=	227.0000
DISTANCE NODE	9=	265.0000
DISTANCE NODE	10=	315.0000

ARRIVAL WEIGHT= 11250.0000  
DEPARTURE WEIGHT= 11784.3645

OPTIMUM PATH PROFILE

NODE	DIS.	ALT.	V(KTAS)
1	0.00	5010.28	135.00
2	27.00	20329.35	190.00
3	54.00	27949.15	190.00
4	83.50	30781.45	200.00
5	113.00	30996.45	200.00
6	151.00	31164.70	200.00
7	189.00	31332.96	200.00
8	227.00	31501.22	200.00
9	265.00	23948.40	270.00
10	315.00	5010.28	135.00

TABLE D-1  
(Concluded)

OPTIMUM PATH FUEL BURN

D1	D2	V1	V2	H1	H2	B
0.00	27.00	135.00	190.00	5010.28	20329.35	109.62
27.00	54.00	190.00	190.00	20329.35	27949.15	72.91
54.00	83.50	190.00	200.00	27949.15	30781.45	51.76
83.50	113.00	200.00	200.00	30781.45	30996.45	40.52
113.00	151.00	200.00	200.00	30996.45	31164.70	52.56
151.00	189.00	200.00	200.00	31164.70	31332.96	53.77
189.00	227.00	200.00	200.00	31332.96	31501.22	55.04
227.00	265.00	200.00	270.00	31501.22	23948.40	40.02
265.00	315.00	270.00	135.00	23948.40	5010.28	60.01

## APPENDIX E

### PROGRAMMING DETAILS

This appendix describes some of the programming detail necessary to implement the concepts presented in the paper.

#### E.1 Apple II Algorithm

This section describes programming details to implement on an Apple II computer (in FORTRAN) the dynamic programming technique described previously.

##### E.1.1 Storage

Since the Apple II computer has limited storage, it is necessary to combine several arrays together to permit maximum usage of storage. In the Apple II program the OH and OV arrays described in Section 4.3 and the ALT and VEL arrays also described in Section 4.3 have been combined respectively. The OH and OV arrays are combined into an array IOP such that

$$IOP(i,j,k) = OH(i,j,k)*100 + OV(i,j,k).$$

The ALT and VEL array have been combined such that

$$IFLAG(i,j,k) = ALT(i,j,k)*10 + VEL(i,j,k)$$

##### E.1.2 Input Data

The following inputs to the Apple II will be supplied by the user.

1. Flight Plan Title (25 characters or less)
2. Effective Service Ceiling (feet) (Program will default to Aircraft Service Ceiling if exceeded)
3. Departure Altitude (feet) (density altitude)
4. Departure Velocity (knots)
5. Arrival Altitude (feet) (density altitude)
6. Arrival Velocity (knots)

7. Number of Altitude Nodes (If the user wishes to input the altitude node points, then he inputs a "Y" when prompted and then inputs the density altitude node points. These node points must be such that  $H_i < H_{i+1}$ ).
8. Number of velocity nodes (User can input velocity nodes in similar fashion to inputted altitude nodes.)
9. Number of Distance Nodes
10. Number of Waypoints (Must be equal to or less than number of distance nodes.)
11. Landing Gross Weight (Must be greater than the aircraft operating empty weight else program will reprompt for Landing Gross Weight.)

The program will then indicate the waypoint number and the user will then supply the following for that waypoint:

12. Course Heading (degrees from North)
13. Magnetic Variation (degrees from North)
14. Distance to Next Waypoint (nautical miles)
15. Number of altitudes at this waypoint for which weather data will be supplied
16. Altitude (density altitude in feet)
17. Wind Direction at Altitude (degrees from North)
18. Wind Velocity at Altitude (knots)
19. Temperature at Altitude (in degrees Fahrenheit)

Once the optimization has been completed and the optimal path profile has been printed out the user will be asked if he wishes to update. If the user responds with a "Y" then the following inputs will be required:

1. Departure Weight if the user responds in the affirmative to the prompt concerning updating departure weight.

2. Node Point where weight altitude, or velocity are to be updated. (This node point must be greater than previous node point updated or else the program will assume no additional updates will occur.)
3. Updated Weight at node point if the user responds in the affirmative
4. Density Altitude (feet) at node point
5. Velocity (knots) at node point

### E.1.3 Outputs

The following outputs from the Apple II computer to the user will normally include:

1. Flight Plan Title
2. Effective Service Ceiling (feet in density altitude)
3. Departure Altitude (feet in density altitude)
4. Departure Velocity (knots)
5. Arrival Altitude (feet in density altitude)
6. Arrival Velocity (knots)
7. For each user waypoint
  - a. Waypoint Number
  - b. Distance from Departure Point (nautical miles)
  - c. Course Heading (degrees from North)
  - d. Magnetic Variation (degrees from North)
  - e. For each Altitude
    - a) Altitude (feet in density altitude)
    - b) Wind Direction (degrees from North)
    - c) Wind Velocity (knots)
8. Arrival Weight (W(1,1,P)) (pounds)

9. Departure Weight ( $W(1,1,1)$ ) (pounds)  
(If the aircraft's Maximum Takeoff Weight is exceeded a warning will be printed.)
10. Optimum Path Profile  
This will include for each node point
  - a. Node point (number to be used for updating)
  - b. Distance from departure point (nautical miles)
  - c. Pressure altitude (feet)
  - d. Velocity (true airspeed in knots)
11. Optimum Path Fuel Burn  
Using the Departure Weight ( $W(1,1,1)$ ) the fuel burn will be computed along the optimum path. Outputs will include for each segment the start and end distance ( $D_1, D_2$ ), the start and end velocities ( $V_1, V_2$ ), the start and end pressure altitude ( $H_1, H_2$ ), and the fuel burn ( $B$ ) over the segment.
12. Updated Path Fuel Burn  
If Update has occurred the fuel burn along the updated profile will be computed using either the optimum departure weight ( $W(1,1,1)$ ) or the user's departure weight. Outputs will include the same as with the Optimum Path Fuel Burn. In addition, both the departure weight and updated arrival weight will be printed.

A sample output was included in Appendix D.

## E.2 Programmable Calculator Input Processing

The preflight planning program for the programmable calculator version has been divided into three subprograms or modules. These are:

- o Data Entry and Initialization
- o Flight Path Optimization
- o Data Output and Write Storage



Each of these modules accepts data, performs computations, and outputs data either internally or externally. The Flight Path Optimization module is common to the pre-flight; in-flight update; and, the post-flight evaluation. Of the three modules only the Data Entry and Initialization programming has been completed. Therefore, this section will deal only with this module.

The purpose of the Data Entry and Initialization subprogram is to load all programs, aircraft specific constants, general constants, optimization algorithm specific constants, and the initialization program. Figure E-1 depicts a flow chart that defines the program flow along with explanatory comments.

### E.3 Programmable Calculator Optimization Constants

The purpose of this section is to define the data that is used from the pilot's handbook and to show how the  $h = f(I_d, I_p)$  and  $V = f(I_d, I_k)$  equations' constants are determined.

#### E.3.1 Pilot's Handbook Data

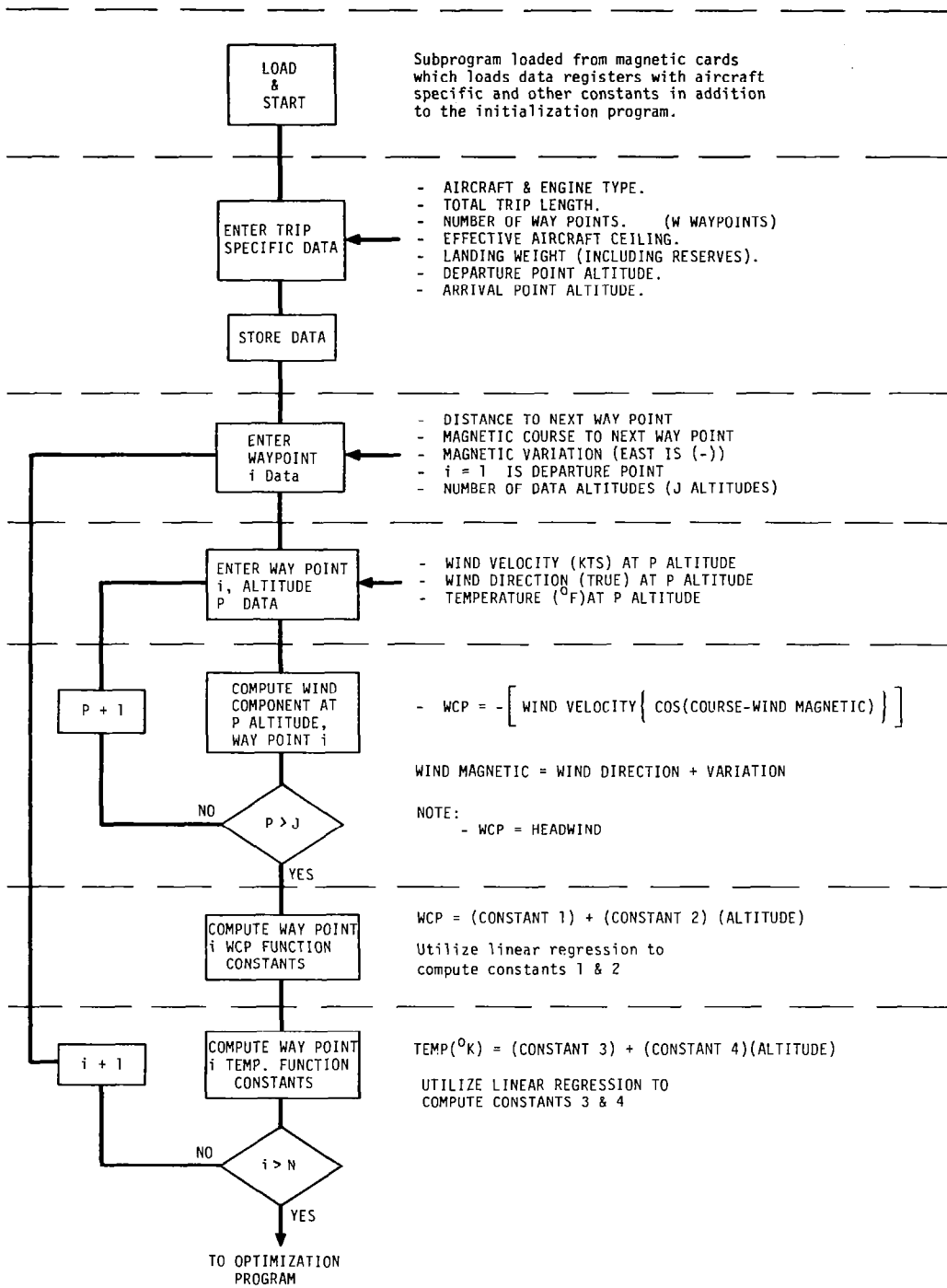
The following table depicts the data that was derived from the Beechcraft Super King Air 200 Pilot's Handbook: (A 300 mile trip is assumed as an example; however, the non-trip specific constants will remain the same.)

<u>Distance</u> <u>(nmi)</u>	<u>Altitude</u> <u>(ft)</u>	<u>Velocity</u> <u>(knots)</u>	<u><math>I_d</math></u>
2.3	1,000	125	0.146
7.3	10,000	143	0.462
22.4	22,000	166	1.42
106.5	35,000	230	6.74
151.2	35,000	230	9.57
222.79	22,000	169	14.10
276.28	10,000	138	17.49
300.00	1,000	119	19.00

The above data describes a maximum performance climb, cruise, and descent based on the handbook data.

#### E.3.2 Constant Determination

The constants contained in the altitude (h) and velocity (v) relationships are both aircraft type and trip dependent. In this typical example, a trip distance of 300 nautical miles will be used.



**FIGURE E-1  
DATA ENTRY AND INITIALIZATION**

I<sub>d</sub> Determination

The relationship of the distance index (I<sub>d</sub>) to the ground trip distance (d<sub>g</sub>) is:

$$I_d = K_t d_g$$

where, K<sub>t</sub> is a constant for a specific trip,

$$I_d = 0.0633d_g,$$

for 19 I<sub>d</sub> points, and a d<sub>g</sub>, total = 300 nmi.

Altitude Function

As described in Section 7, the altitude (h) function is,

$$h_{pd} = f(I_p)f(I_d) \text{ (assuming separability)}$$

An equation form that will generate a ridge line in altitude as a function of I<sub>d</sub> is,

$$h_d = f(I_d) = \text{ceiling altitude} \left[ 1 - e^{-C_n I_d} - e^{-C_n |I_d - 19|^n} \right]$$

In the above equation the  $1 - e^{-C_n I_d}$  term accounts for the maximum performance climb to the effective service ceiling

altitude. The term,  $e^{-C_n |I_d - 19|^n}$ , represents a maximum range descent at minimum power condition. The other part of

the h<sub>pd</sub> function is,  $h_p = f(I_p) = e^{C_n(I_p - C_n)}$ . This equation provides the cross section shape or variation in altitude with I<sub>p</sub>.

For the Super King Air 200, the constants have been determined and are as follows:

$$h = 35,000^{-0.1755(I_p - 5.5)} \left[ 1 - e^{-0.6975 I_d} - e^{-0.0175 |I_d - 19|^{2.5}} \right]$$

This example serves to demonstrate this method that is utilized to determine the h and v functions. Since the v function is similar, an example will not be presented.

APPENDIX F

NOMENCLATURE

<u>Symbol</u>	<u>Meaning</u>
a	Temporary constant
ALT	Altitude (program variable)
b	Temporary constant
B	Fuel burn
c	Temporary constant
$C_i$	Constants
d	Ground distance
$d_i$	Distance steps
D	Drag
$E_D$	Drag work
$E_T$	Thrust work
F	Degrees of flap
$FDM_i$	Flap/gear constants
$F_n$	Thrust
g	Gravitational acceleration
$GD_i$	Flap/gear constants
$GU_i$	Flap/gear constants
h	Altitude
$h_p$	Flight level
H	Altitude (program variable)
$H_p$	Pressure altitude

$h_{p,d}$	Altitude (indexed by potential energy and distance)
$i$	Density altitude index
$I_d$	Index of distance
$I_k$	Index of kinetic energy
$I_p$	Index of potential energy
IOP	Optimum index (program variable)
IFLAG	Optimum flag (program variable)
$j$	Velocity, index
$k$	Distance index
K	Constant
$K_i$	Aircraft specific constants
KE	Kinetic energy
L	Iteration counter
LAM1	Low altitude term
M	Limit of density altitude index
N	Limit of velocity index
OH	Altitude index
OPH	Update altitude index
OPV	Update velocity index
OV	Velocity index
P	Output shaft horsepower (or limit of distance index)
PE	Potential energy
r	Temporary constant
$R_f$	Flap/gear configuration drag multipliers
R/C	Rate of climb

s	Temporary constant
$S_w$	Wing area
T	Time (or Temperature)
Temp <sub>h</sub>	Altitude dependent temperature
V	Velocity
$V_G$	Ground velocity
$\overrightarrow{V_G}$	Vector with direction of true heading and magnitude ground velocity
$V_T$	True velocity
$\overrightarrow{V_T}$	Vector with direction of true course and magnitude true velocity
VEL	Velocity (program variable)
$V_{d,k}$	Velocity (indexed by distance and kinetic energy)
W	Aircraft weight
$W_f$	Fuel flow rate
$W_{fL}$	Fuel flow rate (lean)
$W_{fR}$	Fuel flow rate (rich)
WC	Head wind component
WE	East wind component
WN	North wind component
$\overrightarrow{WV}$	Vector with direction that the wind is blowing from and magnitude of wind velocity
$\delta$	Ratio constant (in fuel flow)
$\eta_p$	Propeller efficiency
$\bar{\rho}$	Atmospheric density
$\bar{\rho}_h$	Altitude dependent density

## APPENDIX G

### REFERENCES AND SOURCES

1. Collins, B. P., Haines, A. L., and Pool, D. A., "Derivation and Current Capabilities of the Path Profile Fuel Consumption Algorithm," The MITRE Corporation, MTR-80W195, September 1980.
2. Collins, B. P., "Energy Modeling for Aviation Fuel Efficiency," The MITRE Corporation, MP-81W12, May 1981.
3. Winer, D. E., and Hoch, C. J., "Energy Conservation in Terminal Airspace Through Fuel Consumption Modeling," Federal Aviation Administration, Office of Environment and Energy, 1980.
4. Bellman, R. E., and Dreyfus, S. E., Applied Dynamic Programming, Princeton University Press, 1962.
5. Beechcraft Aircraft Corporation, "Super King Air 200 Pilot's Operating Handbook and Airplane Flight Manual," October 1979.
6. Cessna Aircraft Company, "Cessna 421C Golden Eagle Information Manual," November 1979.
7. Jane's All the World's Aircraft, 1975-1976, edited by John W. R. Taylor; Franklin Watts Inc., NY, 1975.

1. Report No. NASA CR-3533	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle A CONCEPT FOR A FUEL EFFICIENT FLIGHT PLANNING AID FOR GENERAL AVIATION		5. Report Date March 1982	6. Performing Organization Code
		8. Performing Organization Report No. MTR-81W233	
7. Author(s) B. P. Collins, A. L. Haines, and C. J. Wales		10. Work Unit No. (TRAIS)	
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16. Abstract  MITRE has developed a core equation for estimation of fuel burn from path profile data. This equation has been used in this report as a necessary ingredient in a dynamic program to define a fuel efficient flight path.  The resultant algorithm is oriented toward use by general aviation. The pilot provides a description of the desired ground track, standard aircraft parameters, and weather at selected waypoints. The algorithm then derives the fuel efficient altitudes and velocities at the waypoints.			
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