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Another Self-Similar Blast Wave: Early Time Asymptote with Shock Heated Electrons and High Thermal Conductivity

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Abstract

Accurate approximations are presented for the self-similar structures of nonradiating blast waves with adiabatic ions, isothermal electrons, and equal ion and electron temperatures at the shock. The cases considered evolve in cavities with power law ambient densities (including the uniform ambient density case) and have negligible external pressure. The results provide the early time asymptote for systems with shock heating of electrons and strong thermal conduction. In addition, they provide analytical results against which two fluid numerical hydrodynamic codes can be checked.

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Cowie (1977) and Cox and Anderson (1982, CA) have pointed out that when non-Coulomb processes heat the electrons in the shock front of a blast wave, strong thermal conduction at early times leads to a nearly isothermal electron temperature distribution, but that at these same early times [$t \ll t_{eq} \approx 5000$ yr$(E_o/10^{51}$ ergs$)^{3/14}(1$ cm$^{-3}/n_o)^{4/7}$ for uniform external density] Coulomb collisions are slow to exchange energy between electrons and ions and the ion fluid remains essentially adiabatic.

In the present paper, we apply methods developed in CA to this problem, utilizing an approximation scheme due to Kahn (1975). In so doing, we find the early self-similar blast wave structure with adiabatic ions, isothermal electrons and equal electron and ion temperatures at the shock. In subsequent papers, we shall present the time development of the system away from this early time form as Coulomb interactions become important and thermal conduction fades in significance (Cox and Edgar, 1982, Paper II). In addition, we shall complete the project begun in CA and Cox and Franco (1981, CF) presenting the X-ray appearance of a large set of possible explosions viewed from within, considering the possibility that the soft X-ray background derives from our being within such an explosion. (The appropriate parameters, from CA, appear to be $n_o \sim 0.004$ cm^{-3}, $R_s \sim 100$ pc, $t \sim 10^5$ years, for $E_o \sim 5 \times 10^{50}$ ergs.)

Apart from its role in furthering the project mentioned, the present results will help to erase a certain parochialism sometimes present in important discussions about the nature of shock fronts

in the presence of strong thermal conduction. In addition it provides analytic results against which two-fluid numerical hydrodynamic flow codes can be checked.

Note that CA considered the effects of non-zero ambient pressure on a blast wave. Those effects are neglected in the present treatment, although for the soft X-ray background model, the evolving remnant is in an extremely difficult regime for which the external pressure, thermal conduction, and both Coulomb and non-Coulomb heating of electrons are all potentially of comparable importance.

Turning to the problem at hand, the shock wave jump conditions for zero external pressure, generalized in CA to include thermal conduction flux F_s arriving at the front are

$$\rho_s = x_s \rho_o$$

$$u_s = \frac{x_s - 1}{x_s} v_s$$

$$p_s = \rho_o v_s^2 \left(\frac{x_s - 1}{x_s} \right) \qquad (1)$$

$$F_s = \rho_o v_s^3 \frac{(x_s - 1)(4 - x_s)}{2 x_s^2}$$

where p, ρ, and u are pressure, density, and mass velocity, respectively, and the subscripts s and o denote post-shock and pre-shock values.

In addition v_s is the shock velocity and x is the normalized density. Taking R_s as the shock radius and assuming power law density behavior, $\rho_o \propto R_s^{-\omega}$, the derivatives of the first three lines of equation (1) with respect to R_s can be written

$$\begin{aligned} \rho_s^* &= x_s^{*-\omega} \\ u_s^* &= x_s^*/(x_s-1) + v_s^* \\ p_s^* &= \rho_o^* + 2v_s^* + x_s^*/(x_s-1) \end{aligned} \tag{2}$$

where the asterisk is used to abbreviate the logarithmic derivative: $f_s^* = d\ell n f_s / d\ell n R_s$.

In the present context, with a self-similar evolution, x_s is constant and $x_s^* = 0$. In addition, $p_s R_s^3$ is constant so $p_s^* = -3$. We shall retain the x_s dependence of the equations in anticipation of Paper II but set $p_s^* = -3$, as this will assumed to remain nearly valid even as the system evolves away from the early time asymptote explored here. The second two lines of (2) can then be rewritten

$$\begin{aligned} u_s^* &= [\omega-3 + \frac{x_s^*}{x_s-1}]/2 \\ v_s^* &= [\omega-3 - \frac{x_s^*}{x_s-1}]/2 \end{aligned} \tag{3}$$

When the equations of motion are expanded about the shock position as in CA, the relations between the post-shock derivatives of the variables (still assuming $p_s^* = -3$) are found to be:

$$\alpha \equiv \frac{\partial \ell n \rho}{\partial \ell n R}$$

$$\sigma \equiv \frac{\partial \ell n u}{\partial \ell n R} = (\alpha - \rho_s^*) \frac{x_s}{x_s - 1} - (2 + \alpha) \tag{5}$$

$$\beta \equiv \frac{\partial \ell n p}{\partial \ell n R} = \sigma - x_s u_s^*$$

$$= \frac{\alpha}{x_s - 1} - \frac{3}{2} \frac{x_s x_s^*}{x_s - 1} + \frac{3}{2} x_s - 2 - \omega x_s \left[\frac{1}{2} - \frac{1}{x_s - 1}\right]$$

and

$$\frac{2}{3}\left(\frac{R_s \ell_s}{v_s p_s}\right) x_s = (\beta - \frac{5}{3}\alpha) + x_s(3 + \frac{5}{3}x_s^* - \frac{5}{3}\omega) \tag{6}$$

where $\ell = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 F)$ is the divergence of the thermal conduction flux. As can be seen by comparing equations (1) and (6), the jump conditions are directly related to the thermal conduction flux arriving at the shock, but the post-shock derivatives depend on the divergence of F just behind the shock. Using the jump conditions, the left hand side of (6) can be rewritten

$$\frac{2}{3}\left(\frac{R_s \ell_s}{v_s p_s}\right) x_s = \frac{4 - x_s}{3}\left(\frac{R_s \ell_s}{F_s}\right). \tag{7}$$

Knowledge of the parameter $(R_s \ell_s / F_s)$ is essential to knowledge of the post-shock derivatives, but cannot itself be known without evaluating $\int R^2 \ell dR$ over the entire structure.

The conservation of energy equation

$$\frac{D}{Dt}\left(\frac{p}{\rho^{5/3}}\right) = -\frac{2}{3}\frac{\ell}{\rho^{5/3}} \tag{8}$$

can be separated into its two fluid components

$$\begin{aligned} \frac{D}{Dt}\left(\frac{p_i}{\rho^{5/3}}\right) &= -\frac{2}{3}\frac{h}{\rho^{5/3}} \qquad \text{(ions)} \\ \frac{D}{Dt}\left(\frac{p_e}{\rho^{5/3}}\right) &= +\frac{2}{3}\frac{h-\ell}{\rho^{5/3}} \qquad \text{(electrons)} \end{aligned} \tag{9}$$

where the energy exchange rate between ions and electrons is (Spitzer, 1962; Itoh, 1978; CA)

$$h = akn^2(T_i - T_e)/T_e^{3/2} = \frac{2.3}{1.1}akn^2\frac{T - T_e}{T_e^{3/2}} \tag{10}$$

Here k is Boltzmann's constant, $n = n_H + n_{He}$, $a = \ell n\Lambda/153$ cgs, $\ell n\Lambda = \ell n[1.2 \times 10^5(T^{1/2}T_e/n^{1/2})]$, $n_{He} = 0.1\ n_H$ was assumed, T_i is the ion temperature, T_e the electron temperature, and $T = (1.1\ T_i + 1.2\ T_e)/2.3$ is the average temperature. Similarly, $p_i = nkT_i$, $p_e = (1.2/1.1)nkT_e$, $p = (2.3/1.1)nkT$.

These separated equations can be manipulated into a variety of useful forms. For example, defining $g = T_e/T$,

$$\frac{Dg}{Dt} = -\frac{2}{3}\frac{\ell}{p}\left[\frac{2.3}{1.2} - g\right] + \frac{2}{3}\left(\frac{2.3}{1.2}\right)a\left(\frac{n}{T^{3/2}}\right)\frac{1-g}{g}. \tag{11}$$

The second term on the right side is the Coulomb heating and is negligible for times small compared to the equilibration time mentioned earlier (see CF for values when $\omega \neq 0$), unless the first term is also small. The first term is conduction cooling and will be small at early times either if the electron temperature is not equilibrated behind the shock or thermal conduction is thoroughly quenched by a magnetic field. Itoh's (1978) useful approximation to the electron temperature distributions for adiabatic models follows from setting $\ell = 0$ and recognizing that $n/T^{3/2}$ is then constant for a parcel. The g's from the RHS of (11) can be included in a total time derivative on the LHS which is then constant. In the present study, a more useful result is

$$\frac{D}{Dt}\left[\frac{\left\{\left(\frac{2.3}{1.2}-g\right)T\right\}^{3/2}}{n}\right] = -a\left(\frac{2.3}{1.2}\right)\left(\frac{2.3}{1.2}-g\right)^{1/2}\frac{(1-g)}{g^{3/2}} \qquad (12)$$

together with a slightly altered version of (8):

$$\frac{D}{Dt}\left[\frac{T^{3/2}}{n}\right] = -\left(\frac{T^{3/2}}{n}\right)\frac{\ell}{p}\,. \qquad (13)$$

In this form, two separate variables are identified, one of which is sensitive only to Coulomb heating, and one of which is sensitive only to ℓ.

Since we are assuming that at the shock front, $T_e = T_i = T$, due to non-Coulomb heating processes, and are interested in the similarity solution for small times, we will neglect the RHS of equation (12). Near the center of the structure, g will be very small so that it would appear that this neglect is not entirely justified. But in this case

equation (12) implies that $D/Dt[\ell n(T^{3/2}/n)] \propto n$, which vanishes for small R, so the approximation holds.

For $t \ll t_{eq}$, then, a particular parcel of gas has two constants of the motion which allow it to be located within the structure and partly constrain its properties. One of these constants is M(R), the mass enclosed by the radius through the parcel. The other is the differentiated quantity in (12).

A convenient normalized mass coordinate is $\mu = M(R)/M(R_s)$. When a parcel of current mass coordinate μ was first shocked, its temperature was $T_s(R_s)/\mu$ where $T_s(R_s)$ is the current value of the post-shock temperature (at $\mu = 1$). Since $M(R_s) \propto R_s^{3-\omega}$, the parcel was originally located and shocked at $R_i(\mu) = \mu^{\frac{1}{3-\omega}} R_s$. Its density at that time was $x_s(R_i)\, n_o(R_i) = x_s(R_i)(R_s/R_i)^{\omega}\, n_o(R_s)$. Since we are considering only the early self similar form, $x_s(R_i) = x_s(R_s) = x_s$, and thus $n_s(R_i) = x_s n_o(R_s)\mu^{-\omega/(3-\omega)}$. In addition, the value of g just inside the shock was 1, so that the second constant of the motion can be written in terms of μ and current values of T_s and n_o,

$$\left[\left(\frac{2.3}{1.2} - g\right)T/n^{2/3}\right]_{\mu} = \frac{1.1}{1.2}\frac{T_s}{\mu} \Big/ \left[x_s n_o(R_s)\mu^{-\omega/(3-\omega)}\right]^{2/3}$$

$$= \frac{1.1}{1.2}\frac{T_s}{(x_s n_o)^{2/3}}\,\mu^{-\frac{(9-5\omega)}{3(3-\omega)}}. \qquad (14)$$

We are furthermore exploring an example for which thermal conduction is sufficiently rapid that $T_e \approx T_s$ everywhere, making $g \approx T_s/T$. Substituting this into (14), recalling $x = n/n_o$, and solving for T,

$$\frac{T}{T_s} = \frac{1.2}{2.3} + \frac{1.1}{2.3}\left(\frac{x}{x_s}\right)^{2/3} \mu^{-\frac{(9-5\omega)}{3(3-\omega)}}. \qquad (15)$$

where x is the current normalized density at the location of the mass element with present mass coordinate μ.

By conservation of mass, $\partial\mu/\partial r = (3-\omega)r^2x$ where $r = R/R_s$. With this result, the local value of ℓ can be derived from equation (13). A number of steps are involved in the algebra, including expansion of $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r}$ and use of the result $u = [r - \mu/(r^2x)]\, v_s$ which follows from mass conservation. The net result after several fortuitous cancellations is (with $x' = \partial x/\partial r$)

$$\frac{R_s\ell}{F_s} = \frac{1.2}{2.3}\,\frac{1}{4-x_s}\left[(9-5\omega)x - \frac{2x'\mu}{r^2x}\right]. \qquad (16)$$

A self-consistent solution requires $\int_0^1 r^2\left[\frac{R_s\ell}{F_s}\right]dr = 1$ since F_s is the integral of the divergence ℓ. The first term in (16) is easily integrated since $\int_0^1 r^2x\,dr = (3-\omega)^{-1}$ but the second term still requires a complete knowledge of the structure.

Evaluating equation (16) at $r = \mu = 1$, $x'/x_s = \alpha$, however, provides another relation between the post shock derivatives

$$\frac{R_s\ell_s}{F_s} = \frac{1.2}{2.3}\,\frac{1}{4-x_s}\left[(9-5\omega)x_s - 2\alpha\right] \qquad (17)$$

which, when combined with (5), (6), and (7) with $x_s^* = 0$ can be solved for α, β, σ and $R_s\ell_s/F_s$ as functions of x_s alone. (There are four parameters and four equations, one each from conservation of mass and momentum, and two from the separated energy equations.) There is then a one parameter family of edge conditions only one of which will correspond to a complete solution satisfying the integrated flux condition.

Thus far no approximations regarding the structure have been made. Three possible approaches can be taken at this point. One is to put the exact equations of motion and boundary conditions on a machine and ask it for the answer. A second is to develop a method for finding a reasonable approximation to $(R_s\ell_s/F_s)$ from which all of the post shock parameters and their first derivatives would follow automatically. A third is to apply Kahn's (1975) technique to approximate the entire structure, given x_s and the parameters it determines, and then to perform the flux integral as a function of x_s, choosing the value of x_s which effects the flux normalization.

We have carried out the last two of these approaches, the second because the estimate helps to understand the results, the third because (i) Kahn's (1975) approximation technique has been shown to produce extremely accurate results in the adiabatic problem, (see appendix to CF); (ii) because the structure can be described analytically; and (iii) because this scheme is used in Paper II to find the later behavior so what we actually require for Paper II is the asymptotic behavior of the approximation. The truth, of course, is that we have done the problem

we felt competent to solve, confident that the answers would be close to correct and could be presented conveniently.

In attempting a first approximation to $(R_s \ell_s / F_s)$, we note that all electrons have the same temperature at any given time, implying that they all lose energy at an equal rate. If all losses were via thermal conduction, ℓ/ρ would be a constant, ℓ_s/ρ_s. The flux normalization integral can be written

$$\int_0^1 r^2 \ell dr = \frac{F_s}{R_s} = \int_0^1 r^2 \left(\frac{\ell}{x}\right) x dr = \left\langle \frac{\ell}{x} \right\rangle \int_0^1 r^2 x dr = \frac{1}{3-\omega} \left\langle \frac{\ell}{x} \right\rangle$$

or

$$\frac{R_s \ell_s}{F_s} = x_s (3-\omega) \left[\frac{\ell_s}{x_s} / \left\langle \frac{\ell}{x} \right\rangle \right], \tag{18}$$

so we expect the quantity in square brackets to be of order unity, and we have found a reasonable estimate for $R_s \ell_s / F_s$. Electrons also lose energy by expansion, however, and these losses are most severe near the edge. Thus the edge electrons necessarily have ℓ/ρ less than the mass average in order to have the same total loss rate.

In order to obtain an estimate of the size of this effect, an analysis similar in spirit to the one presented here was carried out for a totally isothermal blast wave, the solution to which has been given by Sollinger, Rappaport, and Buff (1975, SRB). Assuming that $\frac{\ell_s}{x_s} = \left\langle \frac{\ell}{x} \right\rangle$ led to $x_s = 2.457$ whereas SRB quote $x_s = 2.378$, a difference of only 3.3%. Using $x_s = 2.378$ to solve for the ratio of edge to mass-averaged ℓ/ρ, $R_s \ell_s / [F_s x_s (3-\omega)]$,

the result was 0.862. Thus the greater importance of expansion losses near the edge leads to an ℓ_s/ρ_s only 14% lower than average.

Returning to the present investigations, when all the substitutions are made to utilize the mass averaged ℓ/ρ, the results can be expressed as

$$\frac{\left(\frac{R_s \ell_s}{F_s}\right)}{x_s(3-\omega)} = \frac{\left(\frac{\ell_s}{x_s}\right)}{\underset{x}{\ell}} = \frac{36[(6x_s^2-11x_s-4) - \omega x_s(4x_s-7)]}{(3-\omega)(4-x_s)x_s(91x_s-160)} \qquad (19)$$

where some of the peculiar numbers result from the inclusion of 10% helium by number. For $\omega = 0$, the ratio equals 1 for $x_s = 3.3456$ or 1.5176. As $\omega \to -\infty$ (explosions in cavities have negative ω), these values approach $x_s = 2.3972$ and $x_s = 1.779$ respectively. For the totally isothermal case, the equation corresponding to (19) differs only in that $(91x_s-160)$ in the denominator is replaced by $36(x_s-2)$ in which case the two solutions for $\omega = 0$ and ratio 1 are $x_s = 2.457$ and 1. We have not explored the significance, if any, of the lower compression possibilities, for example, whether or not they could correspond to thermal waves. It is clear, however, that the higher compression solutions of each pair are the analogs of the $x_s = 2.378$ solution of SRB.

Notice that there is no single value of x_s, u_s, or other post shock parameter which is always appropriate to shocks with infinite thermal conductivity behind them. The shock compression depends on

the complete structure, here summarized by the ω dependence. Notice too that as pointed out by Cowie (1977) the SRB result exaggerates the deviations expected from the usual adiabatic structure if the ions remain adiabatic.

As previously intimated, the results can be improved upon considerably and the complete structure given by resorting to Kahn's (1975) approximation scheme for the structure, generalized as in CA and CF. We first solve for $\alpha(x_s)$ using equations (17), (5), (6), and (7) with $x_s^* = 0$ (again with $n_{He}/n_H = 0.1$);

$$\alpha = \frac{\{(x_s-1)[(20.25)x_s - 13.8] - \frac{\omega x_s}{2}[(17.9)x_s-31.7]\}}{9.1x_s-16} \tag{20}$$

A second parameter required is

$$q = \frac{x_s(3+\alpha) - (3-\omega)x_s^2}{x_s-5/2} \tag{21}$$

which was derived by matching the form of the structure to the post shock density slope, α. The resulting structure is given by

$$\mu(r) = [r^{5/2}\exp\{(x_s-\tfrac{5}{2})\frac{r^q-1}{q}\}]^{3-\omega}$$

$$x(r) = [\tfrac{5}{2} + (x_s-\tfrac{5}{2})r^q]\frac{1}{r^3}\mu(r) \tag{22}$$

with $T(r)$ provided by (15). This form automatically conserves mass, provides the correct post shock values of all parameters (when supplemented by the jump conditions) and gives the correct post shock

derivatives. It also correctly matches logarithmic derivatives in the limit of small r. All that is required is to find the correct value of x_s.

In this approximation, the normalization integral for the flux becomes equivalent to $Q(x_s, \omega) = 1$ where

$$Q = \frac{1.1}{2.3}\frac{1}{4-x_s}\left\{\frac{3(1-\omega)}{3-\omega} + 6I_o - 2qI_1\right\}, \tag{23}$$

$$I_o = \int_0^1 \frac{\mu(r)}{r}\,dr,$$

and

$$I_1 = \int_0^1 \left\{\frac{\left(x_s-\frac{5}{2}\right)r^q}{\frac{5}{2}+\left(x_s-\frac{5}{2}\right)r^q}\right\}\frac{\mu(r)}{r}\,dr. \tag{24}$$

These integrals have been evaluated numerically for $\omega = 0$, -2, and -4, varying x_s until the $Q = 1$ condition was satisfied. The resulting values of x_s, α, q, and $R_s\ell_s/F_s$ are given in Table I.

Several comparisons between the two levels of approximation are in order before considering the complete structures associated with these results. First we note that values of x_s obtained by setting equation (19) equal to 1 are 3.3, 3.1, and 2.6 per cent larger (for $\omega = 0$, -2, and -4 respectively) than those in Table I, the same degree of accuracy obtained for the isothermal case. Secondly, the values of $R_s\ell_s/F_s$ in Table I are very well approximated by $x_s(2.53-\omega)$ while the mass average is $x_s(3-\omega)$. Therefore, cases which we have not presented should be closely approximated by setting equation (19) equal to $(2.53-\omega)/(3-\omega)$.

Finally, the density, pressure, temperature, and $R_s \ell/[F_s x]$ (normalized ℓ/ρ) structures are shown graphically for $\omega = 0$, -2, and -4 in Figures 1 through 3. They bear a strong resemblance to the adiabatic structures. The central pressure plateaus are slightly lower ($p/p_s = 0.26$, 0.35, 0.39 for $\omega = 0$, -2, -4 respectively) than when the same approximation is applied to the adiabatic case ($p/p_s = 0.31$. 0.38, 0.40). The edge densities are lower ($x_s = 3.24$, 2.91, 2.77) than the $x_s = 4$ for the adiabatic case, but the density gradients are less so that the interior densities are higher by factors of 1 34, 1.41, 1.44 for $\omega = 0$, -2, -4. For $\omega = 0$, the crossover occurs near $R/R_s = 0.9$, $x = 1.5$. Dividing the pressures by the densities implies that the edge temperatures are higher by factors of 1.23, 1.37, and 1.44 for the same p_s and ρ_o, but the central temperatures (local mean, T) are lower by factors of 0.62, 0.66, and 0.68 relative to the adiabatic structures. For given total energy, R_s, and ρ_o, we shall see that p_s differs by at most a few per cent from the adiabatic case so these ratios are altered very little.

In order to know the explosion energy represented by a particular solution, it is necessary to integrate the thermal and kinetic energies over the structure and sum them. The thermal energy is

$$E_T = \int_0^{R_s} 4\pi R^2 \left(\frac{3}{2}p\right) dR$$

$$= \frac{M(R_s)v_s^2}{2}\left[\frac{(x_s-1)(3-\omega)}{x_s} I_T\right] \qquad (25)$$

$$= \frac{3}{2} V_s p_s [I_T]$$

where $I_T = 3\int_0^1 r^2(p/p_s)dr$, and $V_s = 4\pi R_s^3/3$.

Similarly, the kinetic energy is

$$E_K = \int_0^{R_s} 4\pi R^2 \rho(u^2/2)dR$$

$$= \frac{M(R_s)v_s^2}{2}\left[\left(\frac{x_s-1}{x_s}\right)^2 I_K\right] \qquad (26)$$

$$= \frac{3}{2} V_s p_s \left[\frac{x_s-1}{x_s(3-\omega)} I_K\right]$$

where $I_K = (3-\omega)\int_0^1 r^2 x(u/u_s)^2 dr$.

The results of numerical integrations of I_T and I_K for $\omega = 0$, -2, and -4 are provided in Table II. Table II also lists $E_o/(3V_s p_s/2)$ for these same value of ω (where $E_o = E_T + E_K$), as well as the corresponding values from CF for the completely adiabatic structures.

From these results and the jump condition for p_s, the time development can be found. For example with $\omega = 0$, ρ_o = constant, $x_s = 3.2383$, $p_s = \rho_o v_s^2 (x_s-1)/x_s = 0.6912\ \rho_o v_s^2$, $3V_s p_s/2 = 4.343\ R_s^3 \rho_o v_s^2$ $= E_o/0.6430$ (from Table II), or $E_o = 2.793\ R_s^3 \rho_o v_s^2 = 0.447\ R_s^5 \rho_o/t^2$ (since for $\omega = 0$, $v_s = 0.4\ R_s/t$). Thus, $R_s = (2.238\ E_o t^2/\rho_o)^{1/5}$. The adiabatic result is similar, but with 2.238 replaced by 2.03. Thus strong electron thermal conduction in the $\omega = 0$ structure increases R_s and v_s by only about 2% over an adiabatic system.

In the approach taken in Paper II, it is necessary to make an a priori choice for the behavior of p_s^*, after which a self-consistent solution is found for the equations for motion (again assuming the Kahnian density distribution). Any fault of the choice for p_s^* then appears as a time variation of the systemic total energy.

We note that as the system evolves away from the early time asymptote, it is evolving toward equilibrated electron and ion temperatures and toward negligibility of the thermal conductivity (e.g. CA). Hence (until radiative cooling becomes important), the late time asymptote is just the adiabatic structure discussed in CF. And the fact that the total energy integrals of Table II show very similar values of $E_o/(3V_s p_s/2)$ for the two asymptotes guarantees that on average, $p_s^* = \partial \ln p_s / \partial \ln R_s = -3$ during the transition. In addition, major fluctuations in p_s^* seem unlikely since so little readjustment is required in the density distribution. (Fluctuations in p_s^* would provide transient alterations in the pressure gradient, which in turn cause transient accelerations which rearrange the mass distribution.) The only possible difficulty may arise from the suddenness with which the transition is made. Thus taking $p_s^* = -3$ throughout the transition is a reasonable first step, although the transition structure must then be examined for nonconservation of energy.

We summarize our results by way of example. For a particular application, ϵ_o, ω, and ρ_o for a particular value of R will be known. If ω is 0, -2, or -4, $E_o/[3V_s p_s/2] = E_o/[2\pi R_s^3/p_s]$ is found from Table II, which then is solved for $p_s(R_s)$. For other negative values of ω, the

results can be interpolated to reasonable accuracy. Similarly, values of x_s, α, and q can be taken from Table I and substituted into equations (22) and (15) to find the normalized density, temperature, pressure [from $p = (2.3/1.1)nkT$], and mass coordinate distributions. The jump conditions, (1), together with $p_s(R_s)$, $\rho_o(R_s)$, and x_s, provide $v_s(R_s)$ which can be integrated easily to find $R_s(t)$. Finally, the electron temperature is everywhere $T_s(R_s)$.

Again, for values of ω other than those in Table I, x_s can be found by setting equation (19) equal to $(2.53-\omega)/(3-\omega)$, after which x and q follow from (20) and (21). Thereafter solution proceeds as before.

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TABLE I

ω	x_s	α	q	$R_s\ell/F_s$
0	3.2383	8.6044	8.2879	8.176
-2	2.9121	13.8907	16.4663	13.212
-4	2.7721	18.9004	25.4258	18.097

TABLE II

ω	Present Case			Adiabatic (CF)		
	I_T	I_K	Energy[a]	I_T	I_K	Energy[a]
0	0.474	0.735	0.643	0.470	0.740	0.655
-2	0.473	0.800	0.578	0.465	0.773	0.581
-4	0.434	0.844	0.511	0.464	0.790	0.549

[a] $E_o/[3V_s p_s/2] = I_T + (x_s-1)I_K/[x_s(3-\omega)]$

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FIGURE CAPTIONS

Fig. 1. -- Self-similar structure of a blastwave with ρ_o = constant, adiabatic ions, isothermal electrons, and $T_e = T_i$ at the shock. $x_s = 3.2383$.

a) p/p_s ———— and x/x_s — — — — versus R/R_s

b) $R_s \ell/[F_s x_s]$ —— —— ——

T/T_s ————————

T_e/T_s — — — — —

p/p_s — — — —

x/x_s — — — —

as functions of the relative mass coordinate μ.

Fig. 2. -- Same as Fig. 1 but with $\rho_o \propto R_s^{-2}$. $x_s = 2.9121$.

Fig. 3. -- Same as Fig. 1 but with $\rho_o \propto R_s^{-4}$. $x_s = 2.7721$.

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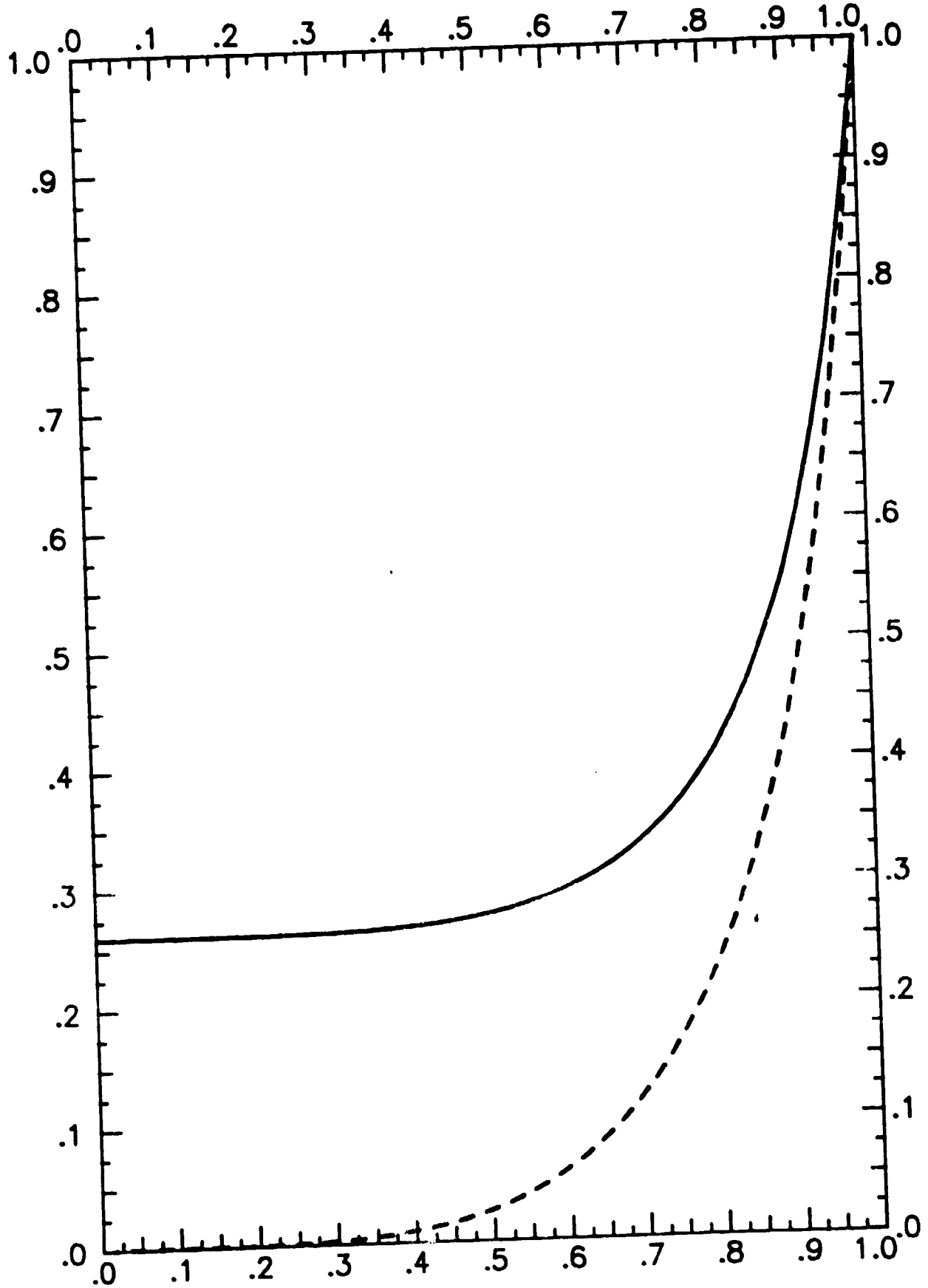
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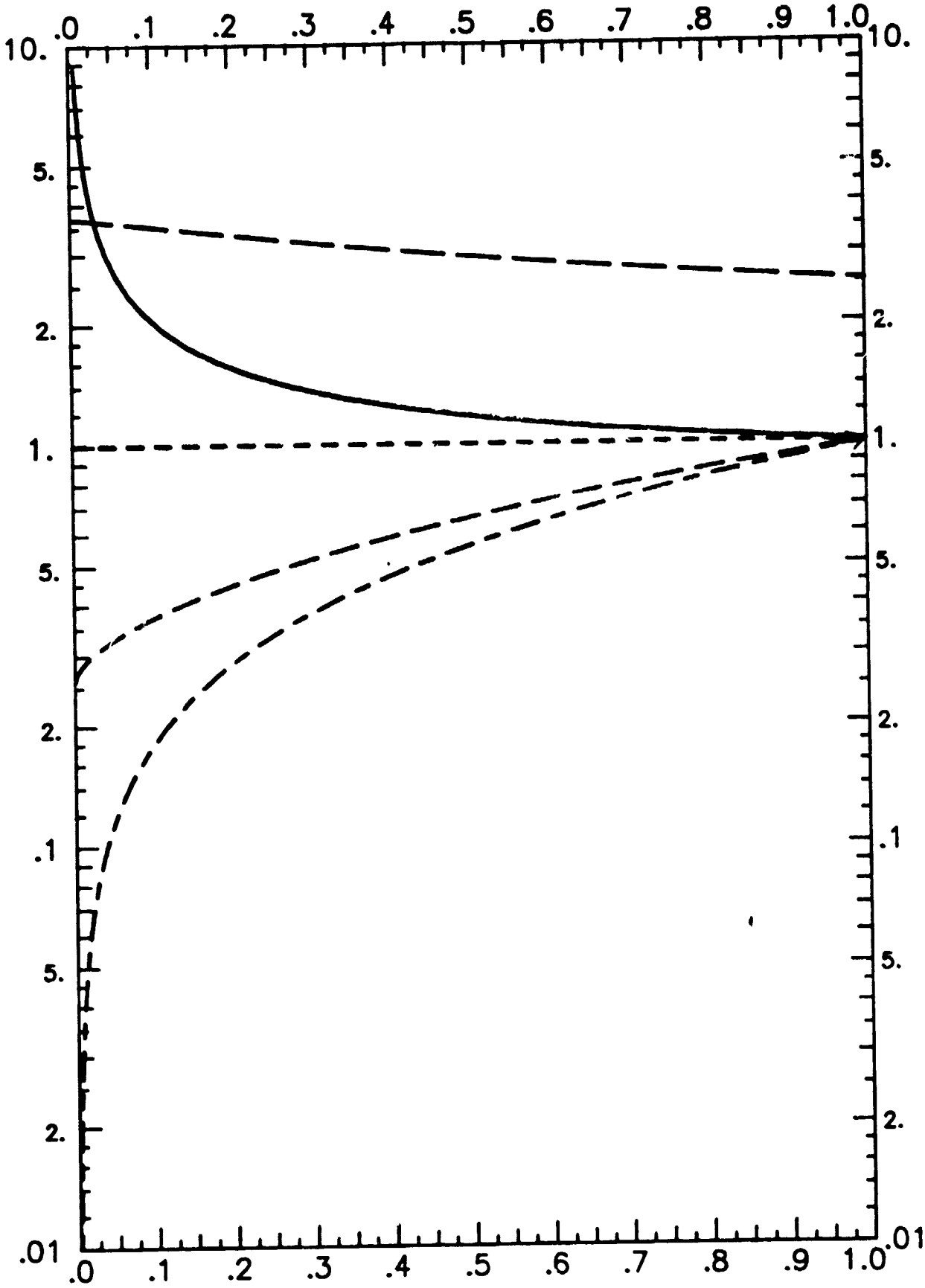
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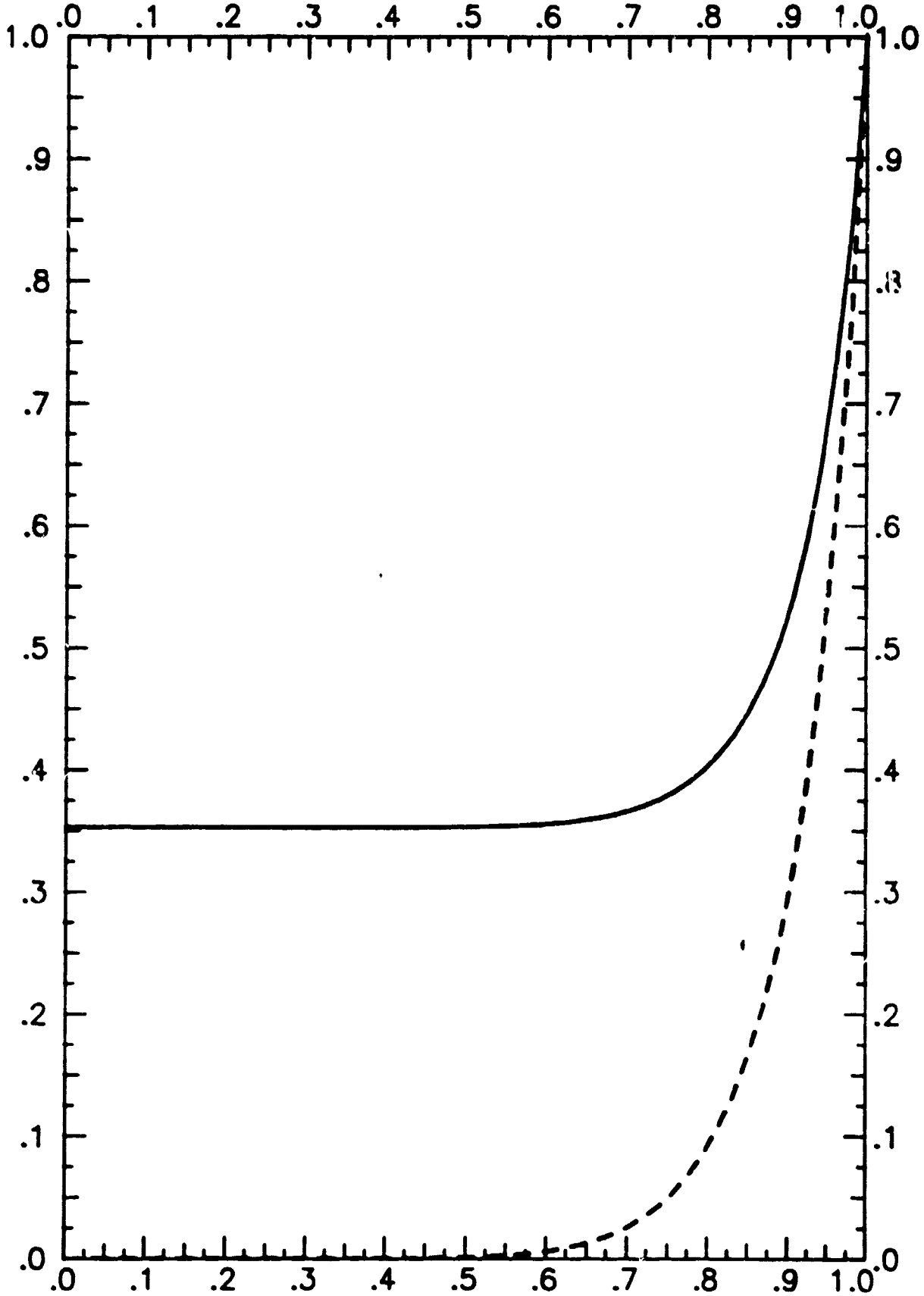
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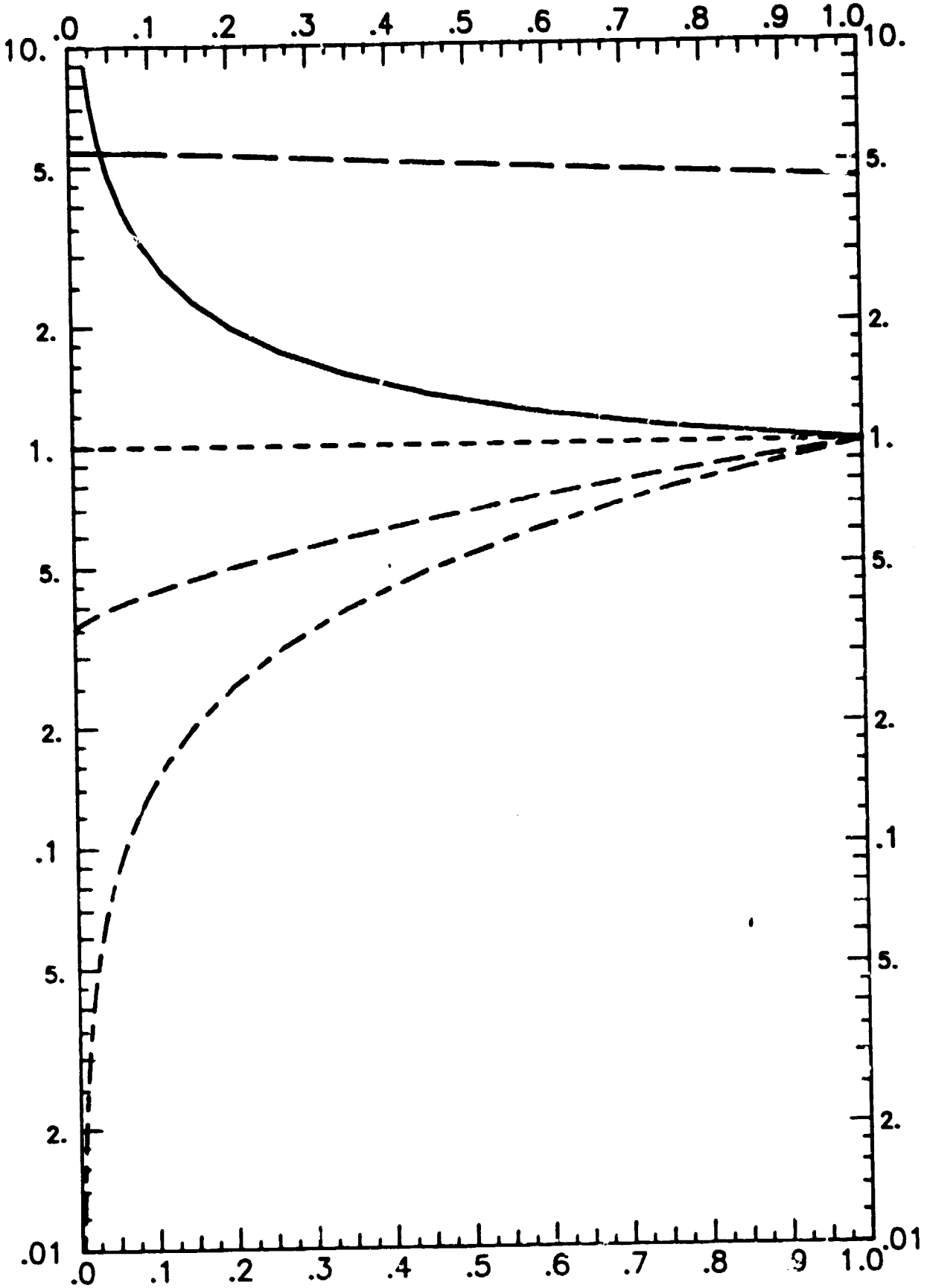
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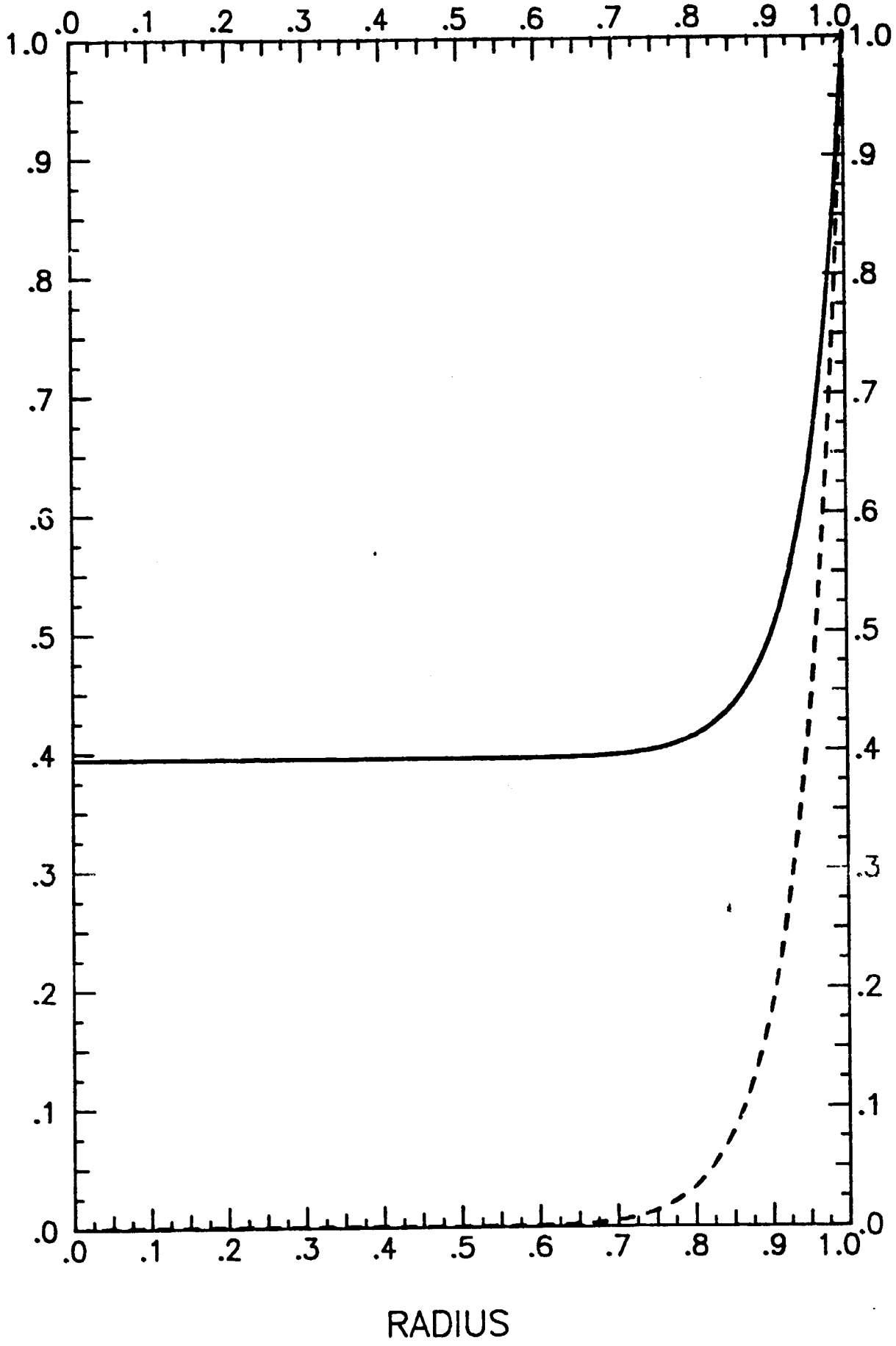
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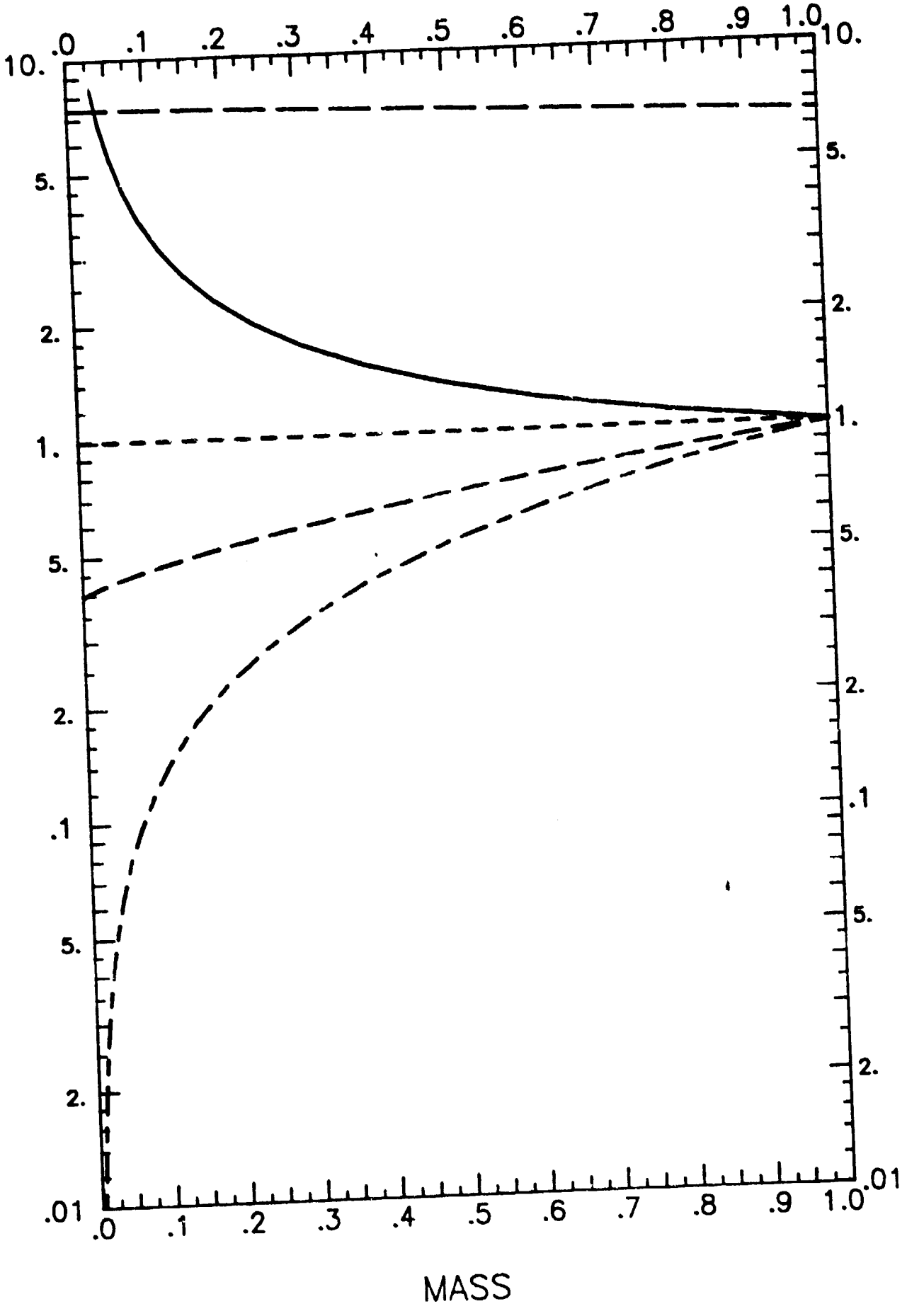
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