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Formulation of Additional Observables for ENTREE

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ABSTRACT

S-band X and Y angles, SAMS, and TACAN range and bearing have been incorporated into the ENTREE software for use by experimenters at LaRC for entry trajectory reconstruction purposes. Background discussions in Section I present the need for this added capability. Formulations for the various observables are presented. Both north-south and east-west antenna mounts have been provided for in the S-band angle computations. Sub-vehicle terrain height variations are included in the SAMS model. Local magnetic variations have been incorporated for the TACAN bearing computations. Observable formulations are discussed in detail in Section II of this report. The partial computations are discussed in Section III.

I. Background discussion

The principal entry trajectory reconstruction software (Ref. 1) for LaRC experimenters provided for C-band range, azimuth and elevation observables but was structured to accomodate additional modelling. Though not completely formulated, Doppler, S-band ranging, and altimeter data were included. Instantaneous formulations for Doppler and S-band ranging were modelled. AMA, Inc., under the subject contract, has rectified these models.[1] It is felt that the documentations, References 1 and 2, are adequate for C-band data, Doppler and S-band ranging. The purpose of this report is to provide a third source to complete the documentation for altimeter modelling as well as describe the new observable formulations required to process the entire spectrum of anticipated data for the LaRC experiment. Thus, the remaining observable formulation included herein pertains to:

OBSERVABLE

(a) SAMS

(b) S-band X and Y angles

and   (c) TACAN bearing[2]

Section II presents the observable modelling. SAMS, S-band angular observations and TACAN bearing computations are presented as sub sections IIa, IIb, and IIc, respectively. The partials of these observables with respect to the state vector are presented as Section III in the same order.

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[1]Analysis has shown that the instantaneous formulations are inadequate. Reference 2 discusses the recommended modelling for the S-band ranging and Doppler observables. Partials, however, are based on the instantaneous formulations which are deemed sufficient.

[2]The computation of the TACAN range measurement and the associated partial vectors is identical to that of the previously documented (Ref. 1) C-band range measurement.

II. Observable Modelling

(a) SAMS data

The observable formulation used to process the recently proposed SAMS altimetric measurements computes the geocentric altitude of the spacecraft above the reference ellipsoid as follows:

$$h_c = K \{ R_{s/c} - R_L - h_T (\Phi_D , \lambda) \} + h_b \tag{1}$$

where $R_{s/c}$ is the geocentric radius vector of the spacecraft,

R_L is the local radius of the Earth ellipsoid at the sub-spacecraft point (geocentrically located,)

h_T is the local terrain height,[3]

K is the instrument scale factor,

and h_b is the instrument bias.

The local planet radius is computed as follows:

$$R_L = \frac{R_E}{\sqrt{1 + \left((\frac{R_E}{R_P})^2 - 1 \right) \sin^2 \Phi_c}} \tag{2}$$

in which R_E is the Earth equatorial radius

R_P is the Earth polar radius

and Φ_c is the geocentric latitude

Though the expression in Eq. (1) is not a rigorous computation of the true geodetic altitude above the Fischer ellipsoid it is considered sufficiently accurate in its simpler form ($\sim$ 2 ft).

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[3] The terrain height is modelled as a bi-variate table with geodetic latitude (Φ_D) and longitude (λ) as the independent variables.

(b) S-band Angle Data

(1) <u>X-Angle:</u> For east-west antenna mounts, the X-angle is defined to be the angle between the local vertical (up) direction and the projection of the range vector onto the east-west local vertical plane, measured positive to the east. For north-south mounts, the X-angle is the angle between local vertical (up) and the projection of the range vector onto the local N-S vertical plane, measured positive south. As shown in Figures 1 and 2, the X-angle can be given as a function of the azimuth and elevation angles. In particular, for east-west mounts:

$$X\text{-angle}_{E-W} = \tan^{-1}\left(\frac{Y_s}{X_s}\right) = \tan^{-1}\left(\frac{\sin Az}{\tan El}\right) \tag{3}$$

For north-south mounts:

$$X\text{-angle}_{N-S} = \left(\frac{-Z_s}{X_s}\right) = \tan^{-1}\left(\frac{-\cos Az}{\tan El}\right) \tag{4}$$

Additionally, an X-angle bias term can be modelled, considered, or solved for in ENTREE for each S-band station if so desired.
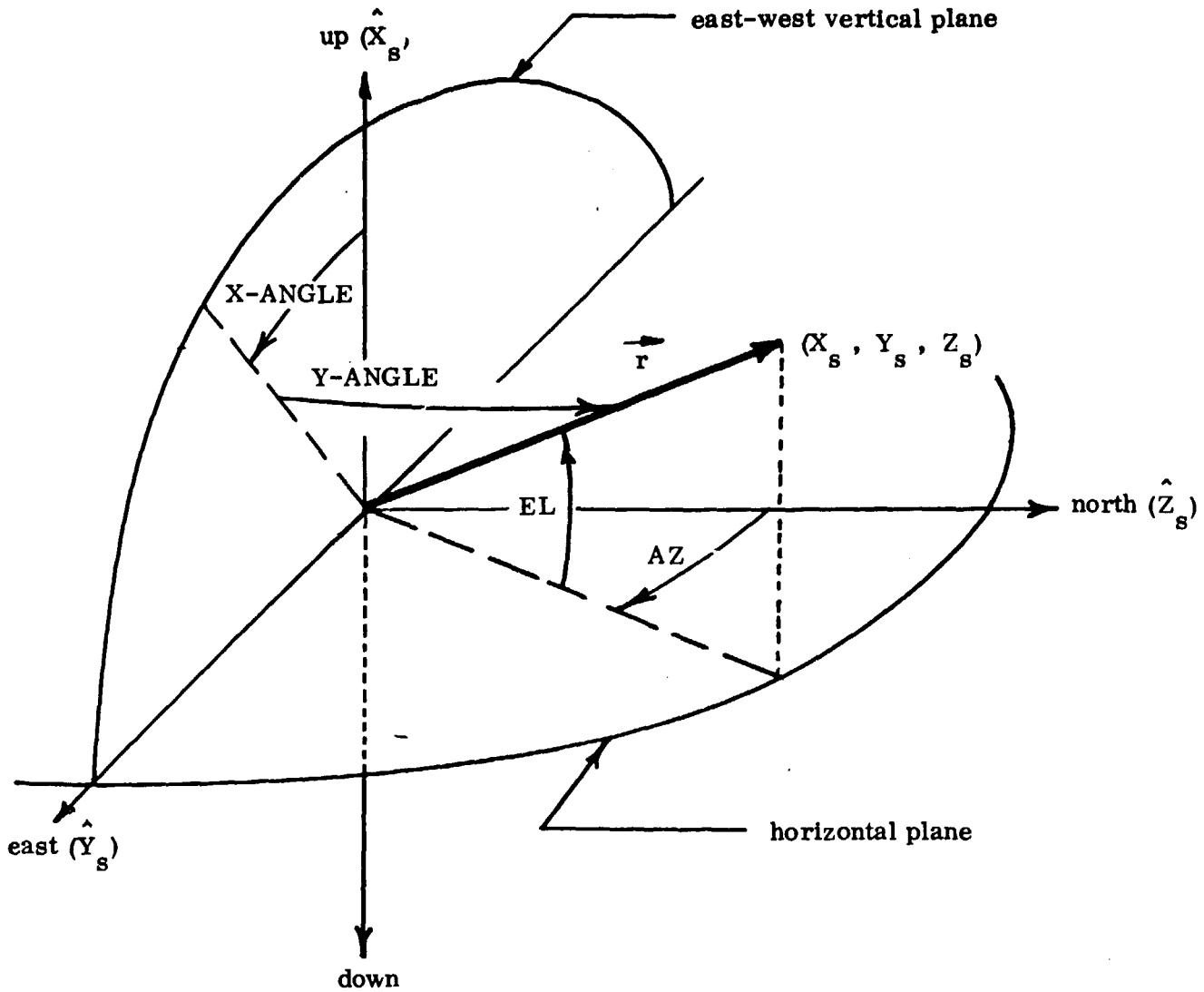
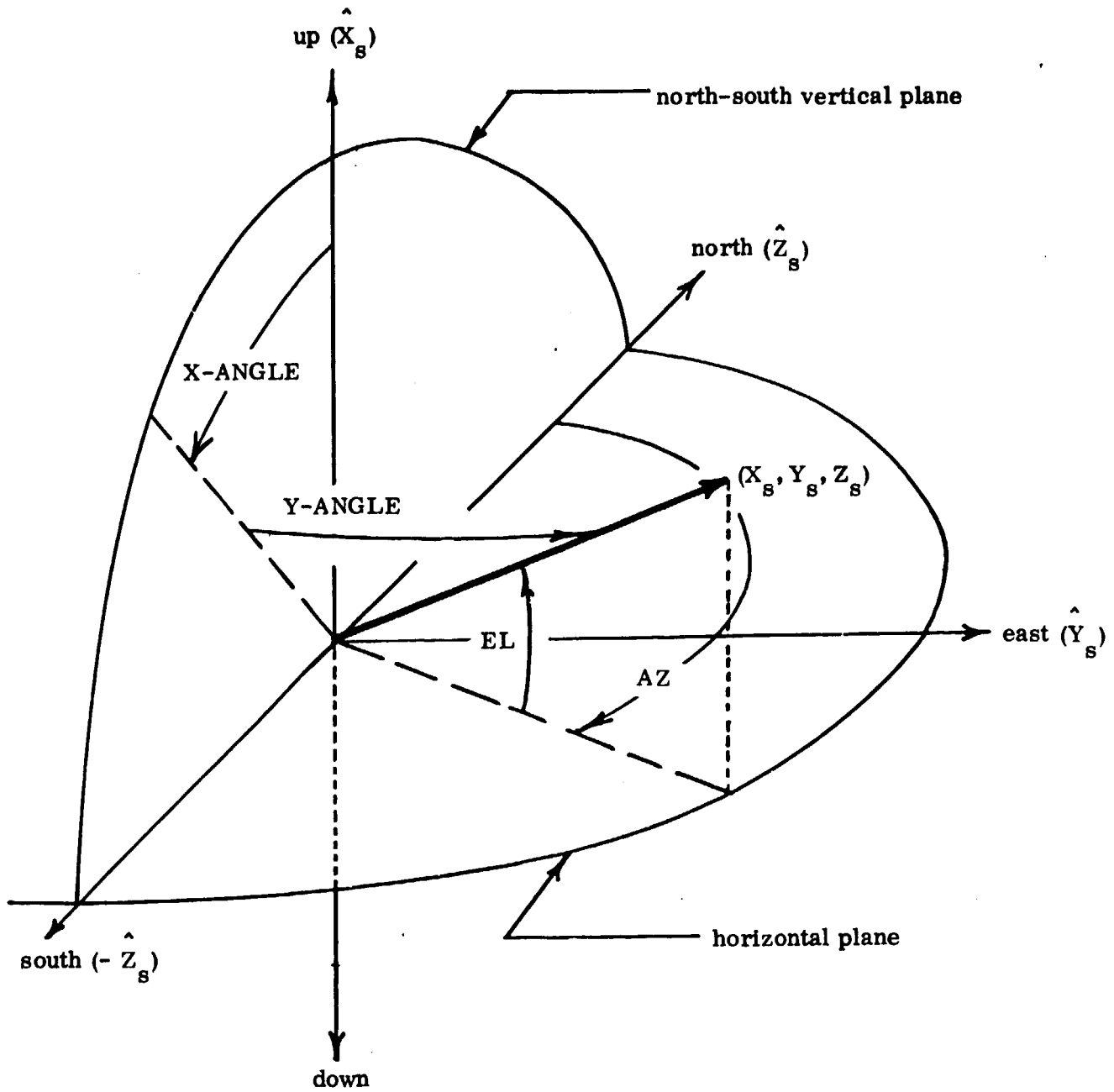Figure 1.   East-West Mount X-ANGLE and Y-ANGLE Definition

Figure 2. North-South Mount X-ANGLE and Y-ANGLE Definition

(2) <u>Y-Angle:</u> For east-west antenna mounts, the Y-angle is defined to be the angle between the projection of the vector onto the local vertical east-west plane and the position vector itself, measured positive to the north. For north-south mounts, the Y-angle is measured between the projection of the position vector onto the north-south plane and the position vector itself, positive to the east. Again, referring to Figures 1 and 2, the Y-angle can be shown to be given as a function of the previously computed azimuth, elevation, and X-angles.

For east-west mounts:

$$\text{Y-angle}_{E-W} = \tan^{-1}\left(\frac{Z_s}{X_s \cos(\text{X-Angle})_{E-W} + Y_s \sin(\text{X-Angle})_{E-W}}\right) \quad (5a)$$

$$= \tan^{-1}\left(\frac{\cos Az}{\tan El \cos(\text{X-Angle})_{E-W} + \sin Az \sin(\text{X-Angle})_{E-W}}\right) \quad (5b)$$

For north-south mounts:

$$\text{Y-angle}_{N-S} = \tan^{-1}\left(\frac{\sin Az}{\tan El \cos(\text{X-Angle})_{N-S} - \cos Az \sin(\text{X-Angle})_{N-S}}\right) \quad (6)$$

The Y-angle bias term is modelled in ENTREE as a constant, solve-for or consider parameter for each S-band station if desired.

(c) TACAN bearing

TACAN bearing is very similar to C-band azimuth except the measured angle is a "fly-to" angle for the benefit of pilots. In other words, if a vehicle was due southwest of a tracking station, the C-band azimuth angle measurement would be 225° (or -135°) from north, whereas the TACAN bearing angle measurement would be +45 degrees with respect to north. In addition, the

reference north direction for TACAN is <u>magnetic</u> north (again in deference to pilots flying with magnetic compasses) whereas it is true north for C-band azimuth. Hence, TACAN bearing is given by

$$\text{TAC-B} = \text{Az} + \pi - \mu \quad (-\pi, \pi) \tag{7}$$

where μ is the local true north to magnetic north variation (positive to the east).

III. Partials

(a) SAMS

A very simple definition was adopted for the partial of altimetric data with respect to state, viz:

$$\frac{\partial h_c^T}{\partial S.V.} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \tag{8}$$

In Eqn. (8) S. V. represents the ENTREE internal state vector (Ref. 1) which consists of:

u	North component of the inertial velocity
v	East component of the inertial velocity
w	Vertical component of the inertial velocity
H	geocentric altitude of S/C above reference sphere
Φ_c	geocentric latitude
λ	longitude

$\left.\begin{array}{l} e_0 \\ e_1 \\ e_2 \\ e_3 \end{array}\right\}$ quaternions which define orientation of Shuttle body axes with respect to geocentric local vertical

A very weak sensitivity of h_c to latitude could be included but Viking experience showed no need for such treatment.

(b) X and Y-Angles

The X-Angle and Y-Angle partials were coded into ENTREE making use of the existing Az and El partials as follows:

$$\frac{\partial \text{ X-Angle}}{\partial \text{ S. V.}} = \frac{\partial \text{ X-Angle}}{\partial \text{ Az}} \frac{\partial \text{ Az}}{\partial \text{ S. V.}} + \frac{\partial \text{ X-Angle}}{\partial \text{ El}} \frac{\partial \text{ El}}{\partial \text{ S. V.}} \tag{9}$$

$$\frac{\partial \text{ Y-Angle}}{\partial \text{S. V.}} = \frac{\partial \text{Y-Angle}}{\partial \text{ Az}} \frac{\partial \text{ Az}}{\partial \text{ S. V.}} + \frac{\partial \text{ Y-Angle}}{\partial \text{ El}} \frac{\partial \text{ El}}{\partial \text{ S. V.}} \tag{10}$$

By using the X and Y-Angle computations shown earlier, the following equations can be derived:

$$\frac{\partial \text{X-Angle}}{\partial \text{ Az}} = \frac{\tan \text{El} \cos \text{Az}}{\tan^2 \text{El} + \sin^2 \text{Az}} \tag{11}$$

$$\frac{\partial \text{X-Angle}}{\partial \text{ El}} = \frac{- \sin \text{Az}}{\sin^2 \text{El} + \sin^2 \text{Az} \cos^2 \text{El}} \tag{12}$$

$$\frac{\partial \text{ Y-Angle}}{\partial \text{ Az}} = \frac{D^2}{D^2 + \cos^2 \text{Az}} \left[\frac{-\sin \text{Az}}{D} - \frac{\cos \text{Az}}{D^2} \left(\frac{\partial D}{\partial \text{ Az}} \right) \right] \tag{13}$$

where $D = \tan \text{El} \cos (\text{X-Angle}) + \sin \text{Az} \sin (\text{X-Angle})$

and $\dfrac{\partial D}{\partial \text{ Az}} = -\tan \text{El} \sin (\text{X-Angle}) \dfrac{\partial \text{ X-Angle}}{\partial \text{ Az}}$ (13a)

$\qquad\qquad + \cos \text{Az} \sin (\text{X-Angle}) + \sin \text{Az} \cos (\text{X-Angle}) \dfrac{\partial \text{ (X-Angle)}}{\partial \text{ Az}}$ (13b)

$$\frac{\partial \text{ Y-Angle}}{\partial \text{ El}} = \frac{- \cos \text{Az}}{D^2 + \cos^2 \text{Az}} \frac{\partial D}{\partial \text{ El}} \tag{14}$$

where $\dfrac{\partial D}{\partial \text{ El}} = \dfrac{\cos (\text{X-Angle})}{\cos^2 \text{El}} - \tan \text{El} \sin (\text{X-Angle}) \dfrac{\partial \text{ (X-Angle)}}{\partial \text{ El}} +$

$\qquad\qquad\qquad \sin \text{Az} \cos (\text{X-Angle}) \dfrac{\partial \text{ (X-Angle)}}{\partial \text{ El}}$ (14a)

The above equations apply for the east-west mounted antennas. For the north-south mount antennas, substitute sin Az everywhere that cos Az appears, and - cos Az everywhere that sin Az appears.

(c) TACAN Bearing

Since the TACAN bearing measurement differs from the azimuth measurement by a constant amount, the partial of TACAN bearing with respect to the state vector is identical to the azimuth partial documented in Ref. 1, i.e.,

$$\frac{\partial \text{ TAC-B}}{\partial \text{ S. V.}} = \frac{\partial \text{ Az}}{\partial \text{ S. V.}} \tag{15}$$

REFERENCES:

1. Waligora, S. R. and Mittelman, J. P., "Entry Trajectory Estimation (ENTREE) Program System Description and User's Guide,"CSC/SD-79/6145, Computer Sciences Corporation, November, 1979.

2. Kelly, G. M.,"Recommended ENTREE S-band Range and Doppler Models", Report No. 80-15, Analytical Mechanics Associates, Inc., September, 1980, NAS1-16087.