Recommended ENTREE S-band Range and Doppler Models

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SUMMARY

Based on comparisons between instantaneous and light-time-delay (LTD) formulations for S-band range and Doppler observables, the following expanded models for instantaneous S-band observables are recommended for inclusion in the ENTREE program (Ref. 1):

$$R_s = \rho - \frac{\rho}{c}\dot{\rho}$$

$$f_2 = \frac{2K}{c}\left(\dot{\rho} - \frac{\dot{\rho}^2}{c} - \frac{\rho\ddot{\rho}}{c} + \frac{T_c^2}{24}\dddot{\rho}\right)$$

where

R_s, f_2	=	S-band range and Doppler, respectively
ρ, $\dot{\rho}$	=	Instantaneous slant range and slant range rate, respectively
c	=	Speed of light
K	=	Doppler conversion factor
$\ddot{\rho}$, $\dddot{\rho}$	=	Higher order derivatives of instantaneous slant range
T_c	=	Doppler count time

In a worst case situation using Guam tracking for a representative Shuttle entry trajectory, the proposed ENTREE model differs from the LTD model by approximately ± 0.2 ft in range and ± 0.02 Hz in Doppler. In contrast, the maximum differences between the existing ENTREE model and the LTD model are approximately (-80, + 60) ft in range and (-4, -1) Hz in Doppler.

INTRODUCTION

Instantaneous formulations for S-band range and Doppler observables are reasonable approximations when applied to near-Earth spacecraft. Nevertheless, the modeling errors resulting from instantaneous assumptions can be orders of magnitude larger than the accuracies of the measurements. In order to provide the best mathematical models for use in ENTREE, an effort was undertaken (pursuant to Task I of this contract) for the purposes of: (1) Determining limiting accuracies of existing formulations in ENTREE; (2) Recommending improvements which do not dramatically alter the current software (e.g., an iterative light time solution algorithm is not considered viable for inclusion in ENTREE).

The S-band range and Doppler formulations given in Ref. 2 were adopted as the real world for this study. Comparisons were performed against instantaneous models with various levels of improvements. Simulated tracking data from Guam for a representative Shuttle entry trajectory (obtained from Dick Powell, VAB/SSD, in Feb., 1980) were utilized. The Guam tracking pass is a worst case illustration of the limitations of the instantaneous formulations because the slant range and slant range rate are very large near the ends of the data arc.

MATHEMATICAL MODELS

Light-Time Delay (LTD) Model

Very rigorous formulations for S-band Doppler and range observables are given in Sections VIII and IX, respectively, of Ref. 2. The derivations take into account: (1) Relativistic effects; (2) Station, S/C, Earth motion over the signal transit time (light time); (3) Differing time references (ET, UTC, UT1) and their interrelationships. Corrections which must be applied to the computed observables to account for the effects of: (1) Offset of tracking point from station location; (2) Earth's troposphere; (3) Earth's ionosphere are discussed in Section XII of Ref. 2.

The observation equations can be summarized as:

$$R_s = \frac{c}{2}(\tau_u + \tau_d + \ldots)$$

$$f_2 = \frac{K}{c}(\dot{r}_u + \dot{r}_d + \ldots)$$

where

τ_u, τ_d = converged light travel times for the up and down-legs, respectively

$\dot{r}_u$, $\dot{r}_d$ = rate of change of up, down-leg slant ranges, respectively (Note: r_u, r_d are not instantaneous slant ranges)

Existing ENTREE Model

The S-band model currently implemented uses the simplest instantaneous formulations:

$$R_s = \rho$$

$$f_2 = \frac{2K}{c}\dot{\rho}$$

Expanded Instantaneous Model

The inherent inaccuracies of the simplest instantaneous formulations are due to neglect of: (1) One-way light time in range, Doppler computations; (2) Count (averaging) time in Doppler computations. Improvements can be gained by using the simple notion that both range and Doppler are measurements of where the S/C <u>was</u> τ_d seconds earlier, not where it <u>is</u> at station receive time. As a result, the error in range is approximately equal to τ_d times the slant range rate. Therefore, a better approximation for S-band range is:

$$R_s = \rho - \tau_d \dot{\rho}$$

or

$$R_s = \rho - \frac{\rho}{c} \dot{\rho}$$

Neglecting the count time, a better approximation for Doppler may be obtained by taking the time derivative of R_s and converting to Hz. The resulting equation is

$$f_2 = \frac{2K}{c} \left(\dot{\rho} - \frac{\dot{\rho}^2}{c} - \frac{\rho \ddot{\rho}}{c} \right)$$

If the speed of light were infinite, Doppler would still not be range rate because it is averaged over the count time, T_c. Taylor series expansions about the mid-point of the count interval yield

$$\rho_L = \rho_M - \dot{\rho}_M \frac{T_c}{2} + \ddot{\rho}_M \frac{T_c^2}{8} - \dddot{\rho}_M \frac{T_c^3}{24} + \ldots$$

$$\rho_R = \rho_M + \dot{\rho}_M \frac{T_c}{2} + \ddot{\rho}_M \frac{T_c^2}{8} + \dddot{\rho}_M \frac{T_c^3}{24} + \ldots$$

The average range rate over the count interval is then

$$\dot{\rho}_{Avg} = \frac{\rho_R - \rho_L}{T_c} \approx \dot{\rho}_M + \dddot{\rho}_M \frac{T_c^2}{24}$$

where L, R, M represent the left, right end-points and the mid-point of the count interval, respectively. Addition of this count time correction term gives

$$f_2 = \frac{2K}{c} \left(\dot{\rho} - \frac{\dot{\rho}^2}{c} - \frac{\rho \ddot{\rho}}{c} + \frac{T_c^2}{24} \dddot{\rho} \right)$$

SOFTWARE DESCRIPTION

Comparative data between the LTD and instantaneous formulations were generated using the author's program SESIMS (Shuttle Entry Simulated Observables). SESIMS has been utilized a number of times since March, 1980 to generate simulated instantaneous observations for ENTREE checkout. Its primary purpose, however, is to perform model comparisons.

The complexity of both the LTD and instantaneous formulations can be controlled on input. The LTD model can be simplified by "turning off" in any combination: (1) Relativistic effects on light time solution; (2) Higher order speed of light terms in Doppler calculation; (3) Earth motion over the light time. This model can also be regulated by the input values assigned to the speed of light and the Doppler count time. The default instantaneous model is the existing ENTREE model. It can be expanded by "turning on" in any combination the correction terms containing: (1) $\dot{\rho}$; (2) $\ddot{\rho}$; (3) $\dddot{\rho}$.

RESULTS

Figs. 1-4 show the range, Doppler differences between the LTD and instantaneous models for various levels of "expansion" in the instantaneous formulations. A Doppler count time of 1^s and the most complete LTD model were used throughout. The "stray" points in several of the plots are due to acceleration discontinuities (reaction jet firings) and have no bearing on the overall comparisons.

Fig. 1 illustrates the limitations of the existing ENTREE models. Fig. 2 shows the improvement obtained by adding the $\dot{\rho}$ correction term, which results in a complete model for range, but not for Doppler. The range comparisons are excellent; the only noticeable change in Doppler is in the signature, although some of the bias has been removed. Fig. 3 contains comparative data based on the addition of $\dot{\rho}$ and $\ddot{\rho}$ correction terms. (The range comparisons in Figs. 3a, 4a are identical to those shown in Fig. 2a, since no further expansion beyond $\dot{\rho}$ correction is required). The Doppler differences in Fig. 3b are unbiased, which represents an improvement. The spread, however, is approximately ± 1.5 Hz and is due to neglect of the 1^s Doppler count time. The differences will vary directly with the square of the count time and could be as much as ± 150 Hz for a 10^s count interval. The importance of the count time correction term (which requires $\dddot{\rho}$) is thus established. Fig. 4b shows the excellent Doppler comparisons resulting from the incorporation of the $\dddot{\rho}$ term.

A numerical example showing the magnitude of the range, Doppler correction terms at 0^s into the tracking arc is given in Table I.

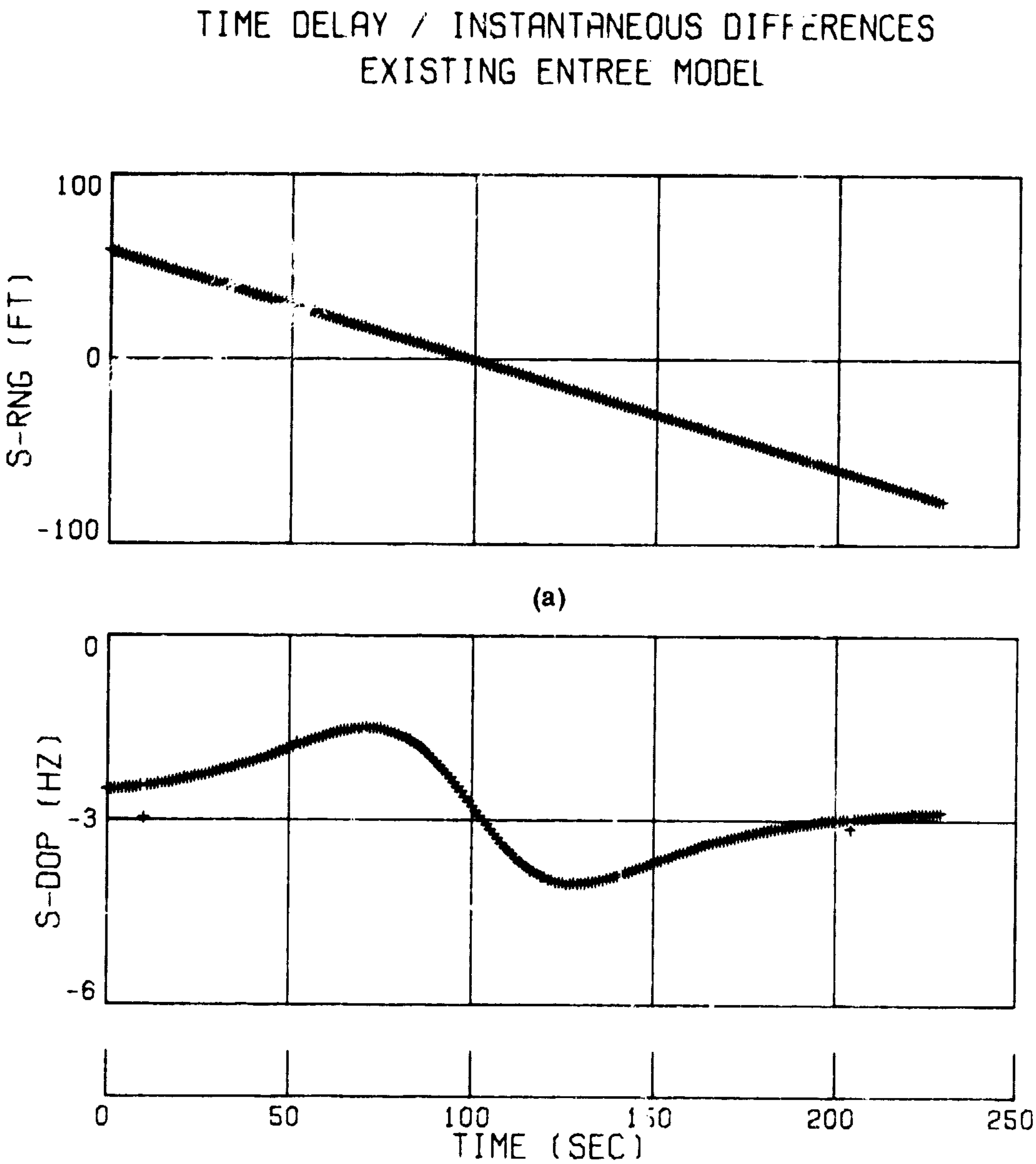
ALTERNATE METHODS FOR IMPROVING INSTANTANEOUS MODELS

Time-shifting the range observable by the one-way light time yields the same level of improvement as the addition of the correction term $\left(-\frac{\rho\dot{\rho}}{c}\right)$. However, time-shifts of the Doppler observable are equivalent to the $\ddot{\rho}$ correction term only. If the Doppler count time were zero, then a time-shift with the addition of the $\dot{\rho}$ correction term would produce comparisons as good as those shown in Fig. 4b. Unfortunately, as shown in Fig. 3b, Doppler count time may be a very significant error source, so time-shifting is no help for Doppler.

A range-difference formulation for Doppler would eliminate the need for incorporating $\ddot{\rho}$, $\dddot{\rho}$ computations in ENTREE. By definition, the count time problem would disappear. Time shifts to accomodate the light time effects could probably be used when evaluating range at the end points of the count interval. A range-difference Doppler formulation is costly, however, since effectively twice as many observations are computed. The number of file positioning steps for state interpolation also goes up dramatically, although the positioning may not be too costly in ENTREE because of the indexed sequential PQR file.

RECOMMENDATIONS FOR FURTHER STUDY

The improvements in the instantaneous formulations for S-band range and Doppler observables discussed in this report are considered an important first step in upgrading ENTREE modeling accuracy. Additional refinements will necessarily include: (1) Algorithms for correcting observables for the effects of Earth's troposphere and perhaps antenna offsets; (2) Provision for time system differences at S/C and tracker (Station clock measures UTC, station rotates in UT1 and S/C moves in ET); (3) Thorough review of models for all other data types (TACAN, C-Band, Altimeter) to determine the adequacy of existing formulations.


TIME DELAY / INSTANTANEOUS DIFFERENCES
EXISTING ENTREE MODEL
100
0
-100
S-RNG (FT)
0
-3
-6
S-DOP (HZ)
0
50
100
150
200
250
TIME (SEC)

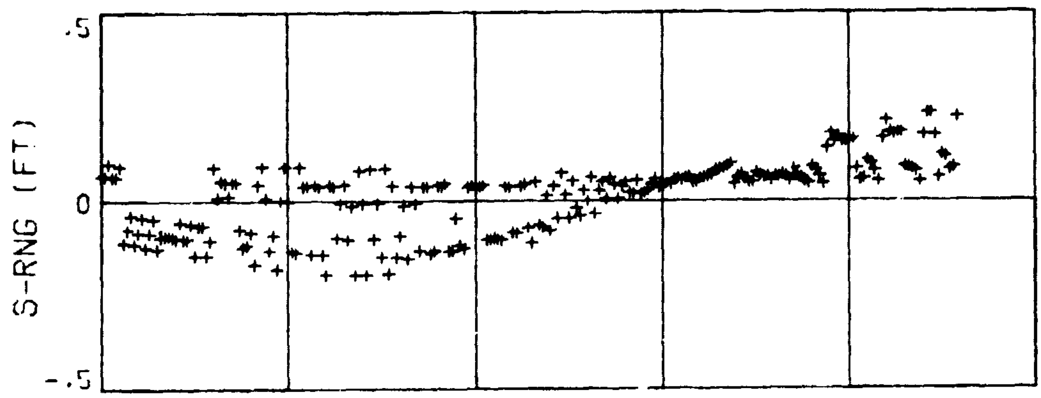
(a)

(b)

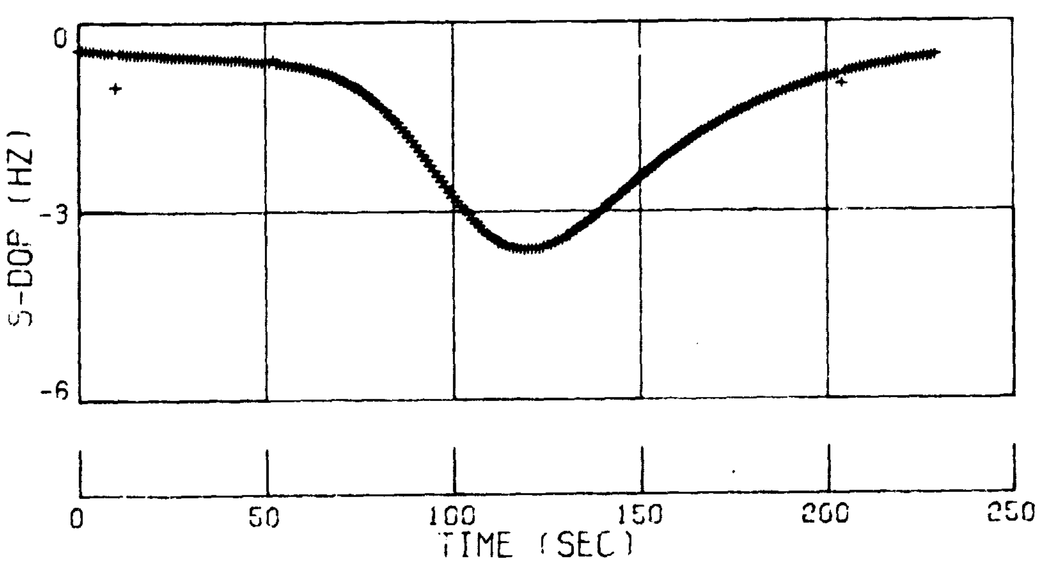
Fig. 1

TIME DELAY / INSTANTANEOUS DIFFERENCES

$\dot{\rho}$ CORRECTION TO ENTREE MODEL



(a)

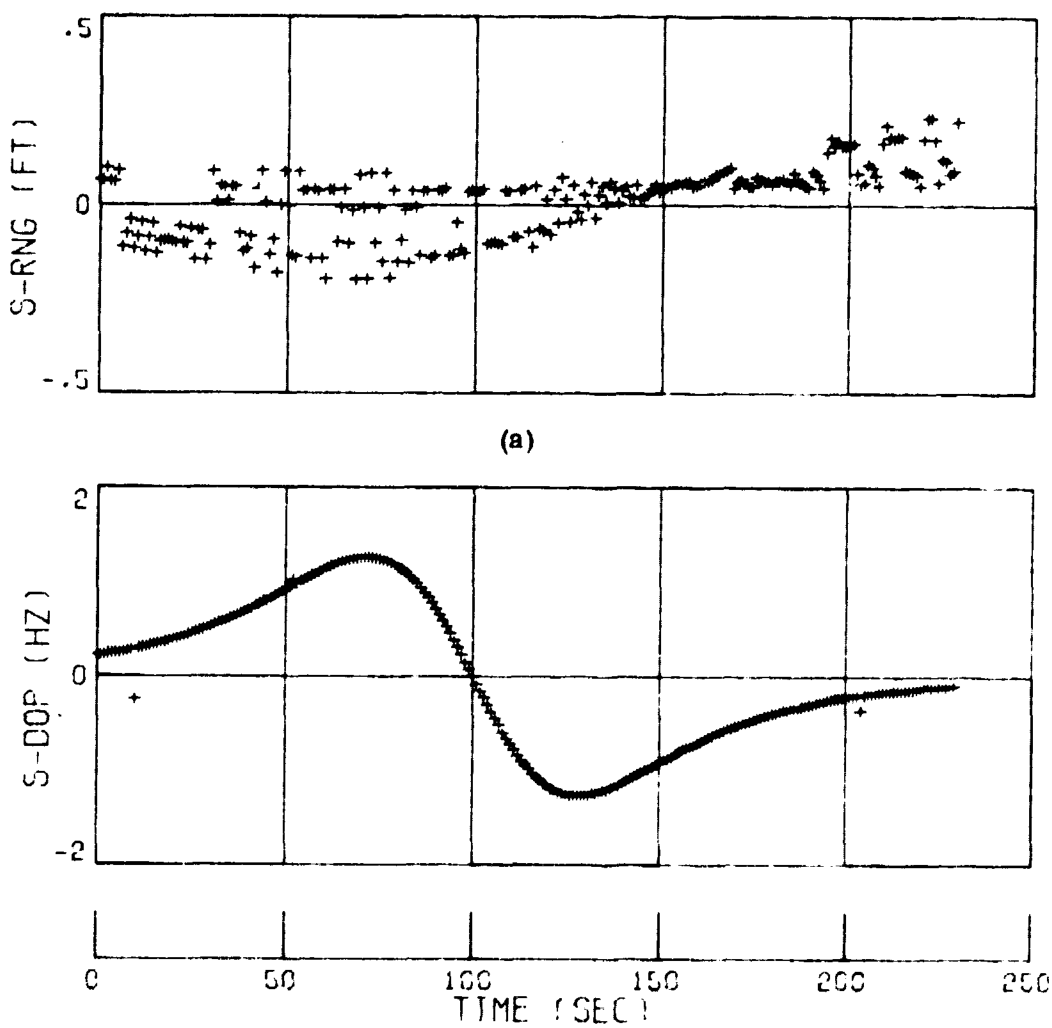


(b)

Fig. 2

TIME DELAY / INSTANTANEOUS DIFFERENCES

$\dot{\rho}$, $\ddot{\rho}$ CORRECTIONS TO ENTREE MODEL

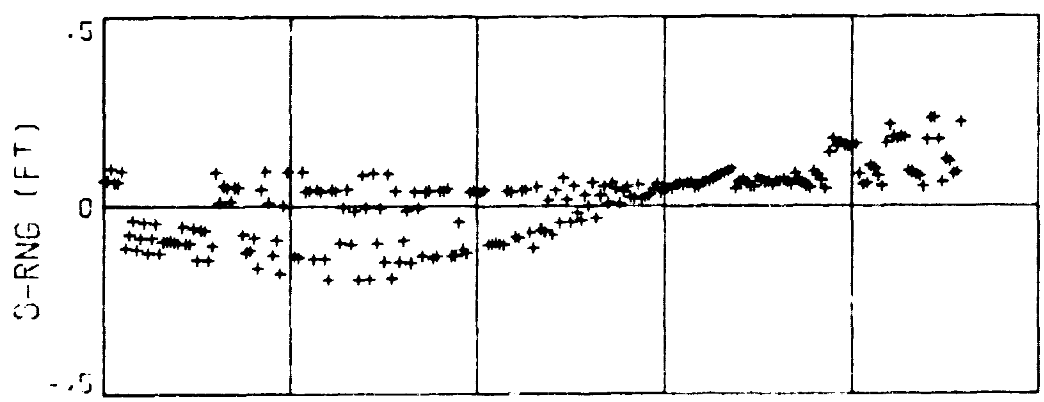


(b)

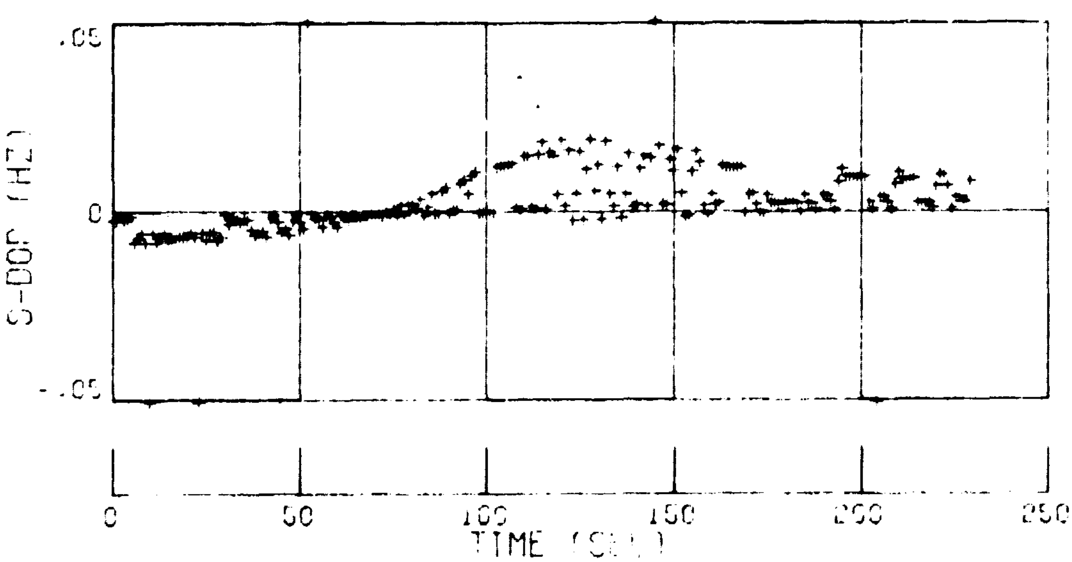
Fig. 3

TIME DELAY / INSTANTANEOUS DIFFERENCES

$\dot{\rho}$, $\ddot{\rho}$, $\dddot{\rho}$ CORRECTIONS TO ENTREE MODEL



(a)



(b)

Fig. 4

TERMS IN EXPANDED INSTANTANEOUS MODEL

(ELAPSED ARC TIME = 0^s)

ρ	=	2765477.4	ft	
$\dot{\rho}$	=	-20904.582	ft/sec	
$\ddot{\rho}$	=	49.541	ft/sec^2	
$\dddot{\rho}$	=	1.168	ft/sec^3	
c	=	9.8357106 E8	ft/sec	
$\frac{2K}{c}$	=	4.6637701	Hz/(ft/sec)	
$\frac{\rho}{c}$	=	2.812 E-3	sec	
$\frac{-\rho\dot{\rho}}{c}$	=	+58.8	ft	(Range correction term)
$\frac{2K}{c}(\dot{\rho})$	=	-97494.165	Hz	
$\frac{2K}{c}(\frac{-\dot{\rho}^2}{c})$	=	-2.072	Hz	(Doppler correction terms)
$\frac{2K}{c}(\frac{-\rho\ddot{\rho}}{c})$	=	-0.650	Hz	(Doppler correction terms)
$\frac{2K}{c}(\frac{T_c^2}{24}\dddot{\rho})$	=	+0.227	Hz	(Doppler correction terms)

Table I

REFERENCES:

1. Waligora, S. R. and Mittelman, J. P., "Entry Trajectory Estimation (ENTREE) Program System Description and User's Guide, CSC/SD-79/6145, Computer Sciences Corporation, November, 1979.

2. Moyer, T. D., "Mathematical Formulation of the Double Precision Orbit Determination Program (DPODP)", JPL TR 32-1527, May 15, 1971.