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THE EXTENSION OF THE THERMAL-VACUUM TEST OPTIMIZATION PROGRAM TO MULTIPLE FLIGHTS

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JULY 1981

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INTRODUCTION

The TVTO Model of Kruger and Norris[1] was developed to provide an approach to the optimization of a test program based on prediction of flight performance with a single flight option in mind, and only minimal consideration of reflight was made. In this paper we extend the process to more than one flight as in Space Shuttle missions. We utilize the concept of "utility" which was first developed under the name of "availability" by Bloomquist,[2,3] and further developed by Kruger and Norris. In addition to the concept of utility, a model developed by Williams and Kruger[4] is used to follow performance through the various options that one encounters when one has the reflight and retrievability capabilities of Space Shuttle.

The "Lost Value" model proposed by Kruger and Norris is modified to produce a measure of the probability of a mission's success, achieving a desired utility using a minimal cost test strategy. The resulting matrix of probabilities and their associated costs provides a means for project management to evaluate various test and reflight strategies.

Finally, recommendations for future study are provided.

THE CONCEPT OF UTILITY

One may ask, why use a concept such as "utility" instead of a more tried and standard concept such as "reliability". The answer lies, in part, in the mathematical theoretical development of the field of reliability and the ever developing complexity of space payloads (satellites, experiments on Space Shuttle, etc.).

As mentioned in Gnedenko, et. al. [5], "one of the most intriguing problems in reliability theory is the development of principles of design of a complex apparatus that will function even when some of its elements will not. Biological systems possess this valuable property to a high degree. The study of biological systems from the point of view of the principles of their design and high reliability will provide many tools that will be used for technological accomplishment. We are convinced that nature has taken a course not only along the lines of extravagant standby redundancy but primarily through selection of optimum system solutions, i.e., a careful choice of elements capable of maintaining an extraordinary stability in performance. No doubt, study of a peculiar feature of biological systems from the standpoint of reliability theory will enable researchers to discover principles not yet conceived, since we tend to approach technical problems exclusively from the point of view of "traditional technology."

As spacecraft become more complex, their nature approaches that of a biological system. It is with this view in mind that utility models the behavior of complex spacecraft payloads better than reliability. Even though utility can be classified as a reliability concept, it is not the same as the classical concept of reliability. Classically, reliability is thought of as the probability of failure-free operation of a unit, component, or system during some time t. Utility is a measure of the overall usefulness of a spacecraft or a payload and is thought of as the successfulness of a mission as compared to a perfect mission. Thus it allows for multiple failures to occur during the flight. Utility does not deal with the probability of the failure free operation of a system. As a sequence of failures occur, these cause payloads to perform to a lesser degree than perfection. This is very similar to how a biological system performs. As failures or aging occurs, biological systems still function, but to a lesser degree than before. With this in mind, it seems like the transitional step to the study of biological system is to study spacecraft operation. Although the previous statement is somewhat contrary to Gnedenko's quote, we believe the line of discovery is from simple systems to complex systems.

In spacecraft operation or payload performance, each anomaly can be classified as to its seriousness or the amount of degradation that it causes to the particular mission. Utility can then be calculated from the observed occurrence of a random sequence of anomalies or types of failures by assigning a certain criticality to each of the failures in the sequence and then considering utility of the payload or spacecraft after the occurrence of any failure as the product of one minus the criticality term at the particular failure multiplied by the previous remaining utility.

The instantaneous utility U, is defined as follows:

$$U = \prod_{i \in \Omega} (1 - D_i)^{n_i} \tag{1}$$

where D_i denotes the criticality of a type i failure, Ω is an index set for the various criticalities of failures that occur during space flight, and n_i is the total number of failures for any particular type of criticality during any space flight.

The average utility, $\bar{U}$, is defined in Kruger and Norris as

$$\bar{U} = \frac{1}{t} \int U \, dt \tag{2}$$

where t is the duration of the mission. It should be noted that this definition is a slight modification of the definition of utility given by Williams[6]; this modification arises in the discussion of the calculation of criticality. For a complete discussion of the concept of the criticality of failures and their classification, see Bloomquist[2,3], Timmins[7], and Williams[6].

To extend equation (1) to multiple flights, one must consider various options such as repair and refurbishment; repair, refurbishment, and retest; and repair or refurbishment. These various options and their effect on the calculation of utility will generally fall into one of three decision-making options. If one repairs perfectly, then the initial (or instantaneous) utility in that particular flight reverts back to the maximum value of 1.0. If one does not repair or retest, then the initial instantaneous utility of the subsequent mission is related to the instantaneous utility at the end of the last mission. If there is testing in the intermediate stage, then the average utility of the proceeding mission is influenced by this amount of testing, and the average utility of the mission is changed accordingly. The mathematics of these options and their effect on utility during multiple flights will be discussed in the section on Failure Flow Process.

Calculation Of Utility For Multiple Flights

To calculate utility for multiple flights, we have a finite sequence of utilities, $\{U_i\}_{i=1}^{n}$, which corresponds to some particular managerial decision-making process. For example, $\bar{U}_1$ would be the average utility obtained after subjecting the spacecraft to the thermal-vacuum test environment at the component and system levels. The utility thus obtained is a result of some testing; the more one tests, the more utility one obtains up to a fixed point less than 1.0.

Depending upon some desired average utility, $\bar{U}_d$, one either reflys a spacecraft without repair, or repairs and then reflys, etc., thus exercising a managerial option. If, for instance, one selects the refly option, then a second average utility, $\bar{U}_2$, is obtained. As this decision process continues, one obtains a sequence of utilities. The argument for the calculation of $\bar{U}$ as affected by multiple flight utility can be generalized even though it is speculated that most experiments will only experience one or two flights beyond the original flight.

Since utility can be thought of as the amount of information collected as compared to the amount of information that can be collected were there no anomalies, it is a class property. By this we mean that it is a measurement of the overall utility of all of the components. As before, utility is an integral over time where the number of failures over that time interval is represented by a functional failure mode form. If we want to calculate average utility, then we divide the integral of the instantaneous utility by the length of time interval. We thus have

$$\bar{U} = \frac{1}{t_f} \int_0^{t_f} (1 - D^*)^{F(t)} \, dt \qquad (3)$$

where t_f is the final time, D^* is the particular average measure of criticality of a failure, and $F(t)$ is the cumulative failure function.

If we consider continuous failure modes over several flight and reflight options, then we may consider the case of repair and non-repair options. To help visualize this scheme, we let t_1 be the total time in flight one, t_2 be the total time in flight two, and t_3 be the total time in flight three. If there were no repair before flight two, then the average utility for flight two, is

$$\bar{U}_2 = \frac{1}{t_2} \int_{t_1}^{(t_1+t_2)} (1 - D^*)^{F(t)} \, dt \, . \qquad (4)$$

If there were repair before flight two, then

$$\bar{U}_2 = \frac{1}{t_2} \int_0^{t_2} (1 - D^*)^{F_R(t)} \qquad (5)$$

where $F_R(t)$ is the cumulative failure function when repairs are made. If there were no repairs after flights one and two, then the average utility over flight three would be

$$\bar{U}_3 = \frac{1}{t_3} \int_{(t_1+t_2)}^{(t_1+t_2+t_3)} (1 - D^*)^{F(t)} \, dt \qquad (6)$$

and if there was repair after flight two,

$$\bar{U}_3 = \frac{1}{t_3} \int_0^{t_3} (1 - D^*)^{F_R(t)} dt \tag{7}$$

The exact failure modes $F(t)$ and $F_R(t)$ will be calculated as in the Failure Flow Process section. We can see that the effect on the limits of integration of a repair is to start the time clock at the beginning or to integrate from zero to a final time.

Certainty of Obtaining a Specified $\bar{U}$

One of the basic assumptions made in the TVTO model of Kruger and Norris is that a certain average utility (termed availability in that report) results from a series of tests. If a project manager specifies a certain desired level of utility, then a decision-making process that involves cost and uncertainty would be necessary.

Uncertainty means some inability to predict accurately. In the case of testing, this means that we are not 100 percent certain that we will obtain the utility that we specified. Therefore it is necessary to calculate the risk involved in obtaining a specified average utility.

Once the manager has specified the degree of certainty desired, the objective of a cost strategy is to obtain the maximum performance for the minimum cost. Following this line of reasoning, we establish minimum cost curves for a given utility. These minimum cost curves are unique to the particular project and have to be derived under specific conditions.

We now turn to the derivation of the probability of obtaining a given $\bar{U}$. As mentioned in Williams[6], $\bar{U}$ is a function of D^* (some average measure of criticality) and n_i (some number of failures over a specified time). For any particular flight, the time period is specified. For example, various missions were investigated in Kruger and Norris.

It is important to note that n_i is represented by a function of the form

$$F_o(t) = A_o t^{B_o}$$

and that D^* is replaced by some constant value as given in Williams[6]. This value is, for a specified 90 percent certainty, the upper confidence limit for D^* which we denote by D^*_U . Once D^* is replaced by D^*_U at a particular confidence level, $\bar{U}$ then becomes a function of the failures in any particular flight.

To specify the probability calculation of $\bar{U}$ or the risk of $\bar{U} \geq \bar{U}_d$, where $\bar{U}_d$ is given, we must answer the basic question, "Does a given $\bar{U}_d$ give or specify a unique failure mode function $F_o(t)$"? Looking at printouts from the various options in the calculation of utility versus total number of failures, the answer appears to be affirmative. To formalize the above idea, we state the following:

Theorem (1.0). A given $\bar{U}_d$ specifies or gives a unique failure mode function $F_o(t)$.

Proof: Recall that

$$\bar{U} = \frac{1}{t_f} \int_o^{t_f} (1 - D_U^*)^{A_o t^{B_o}} dt \tag{8}$$

In the above equation, t_f, D_U^*, and B_o are all given. B_o is given with the assumption that it is process dependent. This assumption will be elaborated upon in the section on Failure Flow Process.

Assume that $\bar{U}$ is given. This implies

$$C = \int_o^A (K)^{A_o t^{B_o}} = \int_o^A (K^{t^{B_o}})^{A_o} dt \tag{9}$$

where $C = \bar{U} \cdot t_f$, $A = t_f$, and $K = 1 - D_U^*$.

Letting $x = A_o$ and using these substitutions, we have

$$F(x) = \int_o^A f(t)^x \, dt \tag{10}$$

where

$$f(t) = K^{t^{B_o}} .$$

Using difference quotients and applying the limit, we have

$$F'(x) = \lim_{h \to 0} \int_o^A \frac{f(t)^{(x+h)} - f(t)^x}{h} dt$$

$$F'(x) = \int_o^A f^x(t) \lim_{h \to 0} \left[\frac{f^h(t) - 1}{h} \right] dt = \int_o^A f^x(t) \ln f(t) \, dt . \tag{11}$$

Since $0 < f(t) < 1$, then $F'(x) < 0$; this implies that $\bar{U}$ is decreasing continuously over time. Since $\bar{U}$ or $F(x)$ is decreasing over time, this implies that if $x_1 \neq x_2$, then $F(x_1) \neq F(x_2)$. This being the case, there are no two different A_o's which give rise to the same $F(x)$. This in turn implies that $F_o(t) = A_o t^{B_o}$ is specified for a given $\bar{U}$.

The importance of Theorem 1.0 is that it enables one to make probabilistic or risk statements about $\bar{U}$ from the probabilistic structure of the Product Limit estimation procedure for $\hat{F}(t)$ found in

Williams and Kruger[4]. Before we develop a method for calculating the probability of obtaining a specified $\bar{U}$, we give a brief review of the product limit estimation procedure found in Williams and Kruger.

The Product Limit estimate of the distribution of failures during the component and system level tests and the orbital flight of a payload measures the performance of a spacecraft in terms of its ability to survive in that particular mode of operation. As in reference (4) we make the following designations:

(1) $\hat{F}(t) \equiv n_i$, the number of failures

(2) $\hat{F}^*(t) \equiv n_i/N \equiv \hat{F}(t)/N$ where N is the number of components in the payload

(3) $\hat{P}(t) \equiv 1 - \hat{F}^*(t)$.

The next question we explore is how to ascertain the probability of obtaining a certain utility. Suppose from Theorem 1.0, we have a unique failure mode function which may be obtained from the failure flow process. Thus, $\bar{U} = C$, which corresponds to $\hat{F}(t) = n$ or to $\hat{F}^*(t) = n/N$ or to $\hat{P}(t) = (N - n)/N$.

If we now calculate probability based on the above statements, we have not expressed the variability of the system and are operating at a 50 percent risk level. Another way to visualize this is that we are operating on the curve that describes cumulative failures, $\hat{F}^*(t)$, without any variability, or we are estimating with point estimates. To deal with this situation, we have one of two alternatives:

1) we test and accept less utility at a given level of certainty

or

2) we test more to achieve a higher level of utility at the given level of certainty.

Note, a test-fix process is assumed.

In order to develop a scheme to accomplish either of these two above objectives and to observe the variability of $\bar{U}$, we refer to Williams and Kruger and derive the appropriate probability statements. An example will help visualize the approaches.

EXAMPLE I

Suppose $\bar{U} = 0.40$ corresponds to or is equivalent to $\hat{F}(t) = 15$. This implies that $\hat{F}^*(t) = 15/16$, where $N = 60$; this in turn implies that $\hat{P}(t) = 45/60$.

Using equation (12), reference (4), we find that

$$F^*(t) = 1 + (\hat{F}^*(t) - 1) \, [1 \pm Z_{\alpha/2} \sqrt{\hat{V}(t)}\,] \qquad (12)$$

or equivalently:

$$F^*(t) = \hat{F}^*(t) \pm (F^*(t) - 1)\,(Z_{\alpha/2})\,\sqrt{\hat{V}(t)} \qquad (13)$$

Suppose that $\sqrt{\hat{V}(t)} = 1/32$ for sake of this example. Inserting the appropriate values into the above equation at the 95 percent confidence level, we have

$$F^*(t) = \hat{F}^*(t) \pm \left(\frac{45}{60}\right)(1.96)\left(\frac{1}{32}\right)$$

$$= \left(\frac{15}{60}\right) \pm 0.045 = 0.25 \pm 0.045 = [0.205, 0.295]$$

$$\hat{F}(t) = 60\,[0.205, 0.295] = [12.3, 17.7] \ .$$

This means that we are 95 percent confident that we do not have any more than 17.7 failures. This also translates into a statement about utility. If we want a 95 percent bound on utility, we have to relate 17.7 failures to utility. This may be done by solving for A_0 in the general failure process model in the orbital case or by searching for the appropriate number of corresponding failures for a given utility on the failure flow process program. After this is done, we see that the corresponding utility is less than the utility we achieved.

EXAMPLE II

The other approach is to specify a desired utility, and then test-fix beyond that level to obtain the desired utility at the degree of confidence that the project manager specifies. This may be illustrated as follows: Suppose $\bar{U}_d = 0.40$ and one tests beyond this level to obtain some utility say $\bar{U}_2 = 0.50$.

As in the previous example, we obtain a confidence interval for the number of failures that correspond to the particular bounds on $\bar{U}$, say (6, 12), with the interval centered at 9. Now suppose that $\bar{U} = 0.40$ corresponds to 11 failures. To find $Pr(\bar{U} < 0.40 \mid \bar{U}_a = 0.50)$, the probability that $\bar{U}$ falls below 0.40 given that one has achieved a utility of 0.50, one has to calculate $Pr(F(t) \geq 11 \mid \hat{F}(t) = 9)$. This is a measure of the risk of not obtaining $\bar{U} = 0.40$, given that one has obtained $\bar{U} = 0.50$ through a prior testing program. Thus, the probability of obtaining at least $\bar{U} = 0.40$ is $Pr(F(t) \leq 11 \mid \hat{F}(t) = 9)$.

To formalize the examples in terms of $\bar{U}$'s, we consider equation (13). Let this confidence interval correspond to, say, $\bar{U}_a = C_1$. To make a probability statement about $\bar{U}_d = C_2$ given that $\bar{U}_a = C_1$ where $\bar{U}_d < \bar{U}_a$, we have to relate $\bar{U}_a = C_1$ to a given number of failures and then to $F_a^*(t) = n_1/N$ by dividing the number of failures by the number of components. Once this has been done, we must relate $\bar{U}_d = C_2$ to $F_d^*(t) = n_2/N$.

After these relationships are made, then

$$\Pr(\bar{U}_d > C_2 \mid \bar{U}_a = C_1) = \Pr(F_d^*(t) < n_2/N \mid F_a^*(t) = n_1/N)$$

$$= \Pr\left[Z_{\alpha/2} < \frac{\hat{F}_a^*(t) - F_d^*(t)}{[\hat{F}_a^*(t) - 1]\sqrt{\hat{V}(t)}}\right] \tag{14}$$

We have a pseudo-normality situation; that is, a situation where one can make probability statements about a given variable through the normality of another related variable. For purposes of discussion and diagramatic arguments, we visualize this symbolically as

$$\bar{U}_a \sim n(\bar{U}_d, \sigma(\bar{U}_a)) \tag{15}$$

$$Z_{p.s.n.} = (\bar{U}_a - \bar{U}_d)/(\sigma(\bar{U}_a)) \tag{16}$$

where $Z_{p.s.n.}$ stands for the pseudo-normal random variable and is calculated by the formal given by equation (16). $\hat{F}_a^*(t)$ is calculated from the Product Limit estimation procedure as described in Williams and Kruger[4]. As seen by equation (14), it is necessary to calculate or estimate the variance $\hat{V}(t)$. To do this, we make use of a random number generator and use a Monte Carlo technique which we describe in the following section.

Monte Carlo Estimation of $\hat{V}(t)$

Using methods found in Kruger and Norris, we may determine the unique number of failures for a given utility. This gives one $F_0(t) = \hat{A}t^{\hat{B}_0}$. To illustrate how we use this model to place failures in time, we consider the following example.

Suppose that a given $\bar{U}$ corresponds to the situation where one has five failures over a specified orbital time. One has then to place these failures in time over the time interval for the orbital case. Essentially, we equate

$$\hat{A}(t_1^{\hat{B}_0}) = n_1 \rightarrow t_1 = \left(\frac{n_1}{\hat{A}}\right)^{1/\hat{B}_0}$$

$$\vdots$$

$$\hat{A}(t_n^{\hat{B}_0}) = n_n \rightarrow t_n = \left(\frac{n_n}{\hat{A}}\right)^{1/\hat{B}_0}.$$

Thus, we have produced a sequence of times which we can use in the Product Limit estimation process to calculate $\hat{V}(t)$ (see equation (10) in Williams and Kruger).

The above method illustrates what we do in the Monte Carlo estimation procedure. In the case where we have five failures, we use a uniform random number generator and obtain five numbers on the interval (0, 1). Suppose that we obtained, after ordering, the sequence, (0.2, 0.34, 0.52, 0.6, 0.82). We next multiply these numbers by five and obtain (1, 1.7, 2.6, 3, 4.1). Letting $n_1 = 1$, $n_2 = 1.7$, $n_3 = 2.6$, $n_4 = 3$, and $n_5 = 4.1$, we calculate $t_1, ..., t_5$ with the above formula for $n_1, ..., n_5$, $\hat{B}_0$, and $\hat{A}$.

After obtaining the failure times, we use the Product Limit estimation procedure to define the parameters necessary to calculate $\hat{V}_1(t)$. We repeat the process and calculate $\hat{V}_2(t)$. After n steps,

$$\hat{V}(t) = \sum_{i=1}^{n} \hat{V}_i(t) \quad (17)$$

By the central limit theorem and Monte Carlo techniques, $\hat{V}(t)$ converges to $V(t)$. Since the generalization of this example is apparent, we do not describe it here.

To conclude this section, we may note that the mathematical procedures for the option "we test and accept less utility at a given level of certainty" is very similar to the derivation described above and will be omitted.

APPROACH TO CHOOSING A COST

To adapt the preceding discussion of utility to a cost model, it is beneficial to examine the environment of the project manager. Increasingly, budget consideration will constrain testing and refurbish/reflight decisions. Furthermore, there very likely will be competition among a variety of projects for these limited resources (see Sayles and Chandler[8]). The allocation of resources – dollars, equipment or personnel – among the available alternatives will be a growing problem for the manager.

The fiscal constraints play a large role in project management (see Lloyd and Lipow[9]). To date, most spacecraft test cost optimization models have relied on expected value concepts, e.g. Campbell[10], Donelson[11], and Naegele and Sellinschegg[12]. While incorporating the range of possible costs, the single expected value – which in some models is supplemented with a measure of variability – may not be of great help to a manager facing budget limitations. At times the optimal test strategy may not fit into the available budget, or other needs are competing for funds making a non-optimal solution necessary. The suggested format of the modified cost model clearly displays the marginal changes in performance which may be purchased for additional test dollars. The format provides probabilities of success for various levels of investment in testing.

Given the few repetitions of an experiment available to most project managers and their need to know close-to-actual cost, we have modified the expected value model in this report. While continuing to rely on the probabilistic mathematical approach of expected value, the output of the modified cost model provides probabilities of success for various levels of costs. The model computes the least cost test and refurbish/reflight strategy that will attain a particular probability of achieving desired utility. An example of an output matrix for a specified, desired utility is shown in Table I.

Table I

Probability of Achieving $\bar{U} = 0.53$ Versus Least Cost Strategy

Probability of achieving $\bar{U} = 0.53$	Cost of least cost strategy
60%	50,000
70%	65,000
75%	73,000
80%	85,000
85%	97,000
90%	112,000
95%	153,000

Referring to Table I, the project manager or research team has determined that a utility of 0.53 will provide sufficient data for mission success (for a discussion of determining $\bar{U}_d$ from past spacecraft performance, see Williams and Kruger[4]). The least cost strategies for obtaining the probabilities in the left column for the particular spacecraft's characteristics – complexity of design, weight, volume, refurbishment costs, flight parameters, and extra STS services – are calculated and listed in the right column. With this information the project manager is able to see:

1. how much a certain probability of success will cost,

2. the highest degree of certainty affordable within the project budget, and

3. how additional investments of specified dollar amounts will enchance the probability of success, or, conversely, how specific dollar amounts debited will reduce the probability of success.

A further benefit of the information format of Table 1 is that a manager may make cost comparisons between experiments on a given program. If a program has a number of experiments competing for resources, the format presents information that can be used for allocating funds among them. For instance, a manager may decide to reduce test expenditures on one experiment, thereby accepting a slightly lower probability of success; the funds freed by this reduction in spending may then be applied to another experiment enhancing its likelihood of success.

Certainty of Achieving a Desired Average Utility

The design of the modified cost model relies on the assumption of a near normal distribution of utility and on a distinction between $\bar{U}_d$, desired average utility, and $\bar{U}_a$, achievable average utility. As the utility concept has emerged, it has become common to speak of $\bar{U}_a$ as if there was a 50% probability of actually attaining that level of utility or one higher. Through testing, the $\bar{U}_a$ level could be increased to a higher utility, again with a 50% probability of that utility or a higher one being reached. Given the roughly normal character of the distributions of utility, this means that

$\bar{U}_{\bar{a}}$ is the center or mean value of such a distribution. It also suggests that through more testing the entire distribution shifts to center on a new $\bar{U}_a$ value. Figure 1 illustrates this idea.

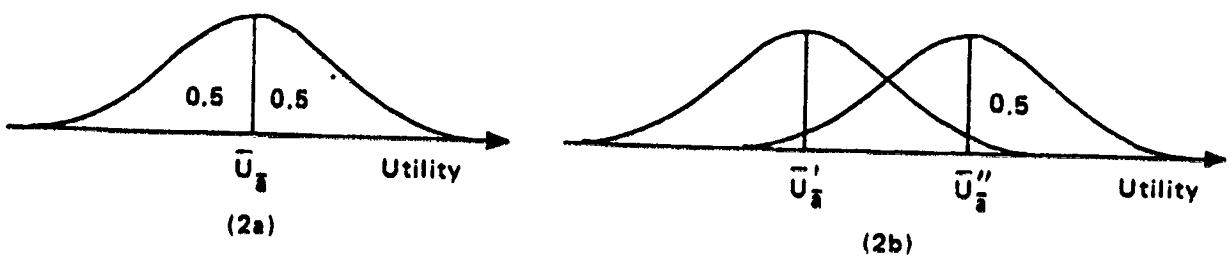

Figure 1

The Effect of Testing on Utility
After Testing, $\bar{U}_{\bar{a}}'' > \bar{U}_{\bar{a}}'$

In figure 2a, there is a 50% probability of attaining $\bar{U}_a'$. After testing and repairs, there is a 50% probability of attaining $\bar{U}_{\bar{a}}''$ or higher, where $\bar{U}_{\bar{a}}' < \bar{U}_{\bar{a}}''$.

Rather than speaking of attaining the new post-test utility with a probability of 50%, it is possible to think of attaining the original utility, $\bar{U}_{\bar{a}}'$, or higher with a probability greater than 50%. The post-test probability of attaining at least $\bar{U}_{\bar{a}}'$ is shown in figure 2 as the crosshatched area. Note that this area is based on the distribution centered at $\bar{U}_{\bar{a}}''$, the post-test utility with 50% probability of being achieved.

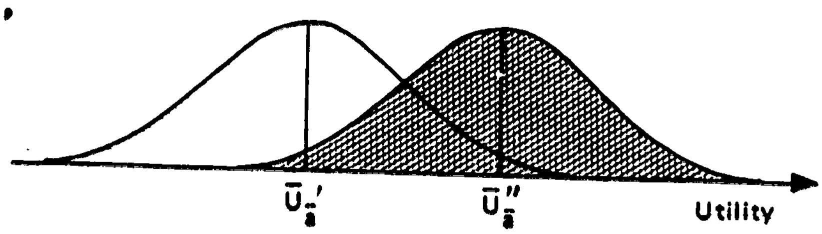

Figure 2

Post-Test Probability of Attaining at Least $\bar{U}_{\bar{a}}'$.

If a desired average utility, $\bar{U}_d$, is specified as necessary for a mission's success, then through testing it is possible to increase the probability that $\bar{U}_a > \bar{U}_d$. With testing or payload performance improvement we have increases in $\bar{U}_a$ at 50% probability, or, correspondingly, increase in $Pr(\bar{U}_a > \bar{U}_d)$ for reflight missions. As the test program progresses, the utility distribution curve shifts with means $\bar{U}_{\bar{a}}' < \bar{U}_{\bar{a}}'' < \bar{U}_{\bar{a}}''' < \ldots$. Each step signifies improved utility or payload performance through more thorough testing and correction of identified failures.

Assume that testing has brought a payload to the point where $Pr(\bar{U}_d) = S$ or $\bar{U}_a = \bar{U}_d$. With further testing and corrections, there will be increases in $\bar{U}_a$. As $\bar{U}_a$ and $\bar{U}_d$ diverge, the accompanying increases in $Pr(\bar{U}_a > \bar{U}_d)$ may be estimated as follows:

Given: $\bar{U}_a > \bar{U}_d$, $Pr(U_a > U_d) = 0.50 + Pr(Z_L < Z < 0)$ where $Pr(Z_L < Z < 0)$ is the probability associated with the Z-score of:

$$Z_L = \frac{\bar{U}_d - \bar{U}_a}{\sigma_{\bar{U}_a}}$$

where U_d results from the number of failures that give rise to the desired utility, $\bar{U}_a$ results from the number of failures that give rise to the average achieved utility, and $\sigma_{\bar{U}_a}$ is the standard deviation of the number of failures that give rise to $\bar{U}_a$. (Note: see section on Certainty of Obtaining a Specified U.)

As testing proceeds uniformly and $\bar{U}_a$ increases beyond $\bar{U}_d$ and approaches $\bar{U}_{max}$, the incremental improvements in $\bar{U}_a$ get smaller. These decreasing incremental gains are due to:

1. the existence of a $\bar{U}_{max}$ term for various flights and

2. the fact that one has to test longer at both the component and system level to detect failures as testing progresses through time.

Since the cost of testing is a function of time, and the time increments necessary to detect failures increase as $\bar{U}_a$ increases, the test cost increases as we try to improve $\bar{U}_a$. To help visualize these concepts, we consider figure 3.

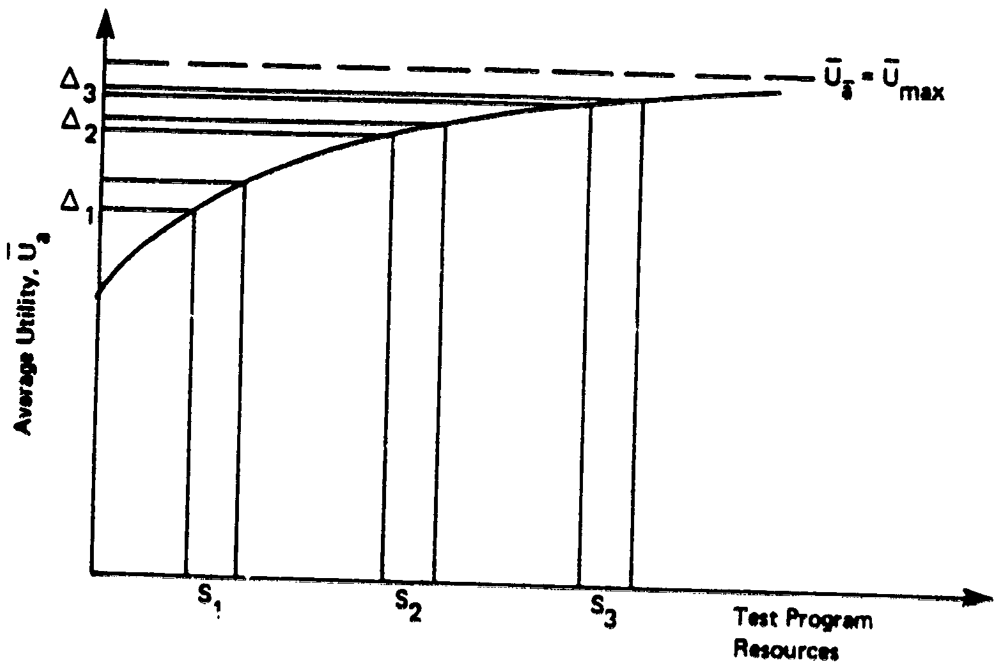

Figure 3

The Effect upon Test Program Resources of Incremental Changes in $\bar{U}_a$

From figure 3 we see that additional investments, S_1, S_2, and S_3, in testing to increase $\bar{U}_a$ produce decreasing gains in $\bar{U}_a$, Δ_1. Δ_2, and Δ_3. Eventually these gains approach an asymptote where $\bar{U}_{\bar{a}} = \bar{U}_{max}$ and set an upper bound to the improvement in $\bar{U}_{\bar{a}}$ that is possible through testing. The concept of $\bar{U}_{max}$ arises from the fact that there are those failures for which this test is inapplicable, and therefore, even if this test were perfect, a certain number of failures remain and give rise to an average utility of less than 1.0.

Increases in $\bar{U}_a$ are purchased through testing and corrections. Therefore, we may assume a relationship between $\bar{U}_a$ and cost. Assuming a uniformly increasing function with time, where cost is the ordinate and average utility, $\bar{U}_a$, is the abcissa, we may visualize the minimal cost curve as in figure 4.

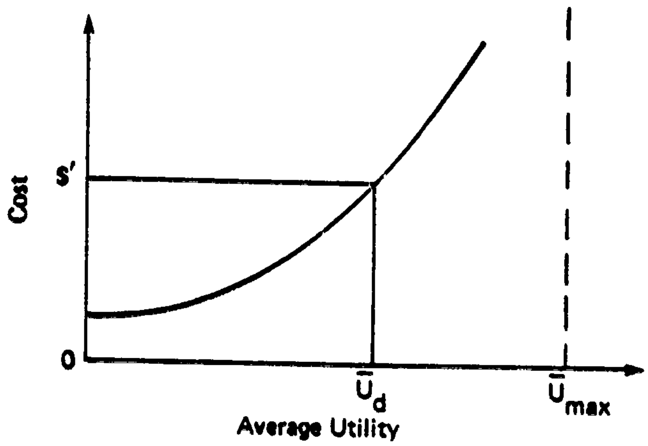

Figure 4

Hypothetical Minimum Cost Curve

We take the cost curve in figure 4 as the minimum cost curve for attaining the various average utilities. S' is the minimum cost test strategy to achieve $\bar{U}_a = \bar{U}_d$. The minimum cost curve is derived empirically from the TVTO computer model output. The output provides mixes of component and system level tests and the cost and improvement in $\bar{U}_a$ for each mix. The model takes into account payload complexity when making cost estimates.

Extending the Model to Multiple Flights

To this point, the cost model has included only ground-based testing – mixes of component and systems level testing – and a single flight. With the advent of the Space Shuttle, reflight becomes a viable alternative. There are three situations in which the refurbish/refly strategy must be considered:

1. if the desired $\bar{U}_d$ is greater than $\bar{U}_{max}$,

2. if the cost of obtaining $\bar{U}_d$ via ground based testing is beyond the project's budget limit, and

3. if the cost of attaining $\bar{U}_d$ with ground base testing alone is greater than the cost of refurbishing and reflying the payload.

Examining these situations separately will help to disclose some of the features of the refurbish/refly option.

In the first situation, when $\bar{U}_d > \bar{U}_{max}$, unlimited resources devoted to testing will not yield the necessary probability of achieving $\bar{U}_d$. In large part this is due to modes of failure unrelated to the test program.

After the first flight and succeeding flights, these modes of failure may be detected and corrected. Identifying and correcting flight failures will increase the payload's performance capability upon reflight.

It is unlikely that payloads will be flown solely for engineering performance information. Rather, they will perform their data collection or transmission duties as well as have their engineering performance monitored. The collection of data on the first flight may reduce the data collection needs of the second flight. The second flight may have less stringent performance requirements and yet result in an overall successful mission. The $\bar{U}_d$ per flight therefore, may be reduced through a flight/refurbish/refly strategy. A reduced $\bar{U}_d$ on the first flight will make it easier (possible) to attain a given $\bar{U}_d$ on the second flight.

The second situation is the case of budget limitation preventing the desired $\bar{U}_d$ from being reached. If the refurbish/refly option lies within the budget limits, it should be examined. This case could exist when the payload has low launch and refurbish cost (low volume and weight and easily reproducible parts) but high test costs (many components or systems requiring individual testing). As in the first situation, the combination of a more thorough "test" situation and the partial collection of data may provide an increase to the necessary $\bar{U}_d$.

In the first case it was suggested that the refurbish/refly strategy, by providing the "ultimate test" situation, may increase the $\bar{U}_{max}$ available by reducing the number of non-thermal-vacuum related failures that a payload will encounter. Estimates from a model have the potential to show how much ground-based testing simulates the actual conditions of space flight (in terms of the number of failures detected). The refurbish/refly strategy, by using the space environment, reduces the area of uncertainty; the potential for successful performance is thereby increased. Figure 5 illustrates this increase in $\bar{U}_{max}$ through by the use of a second flight option.

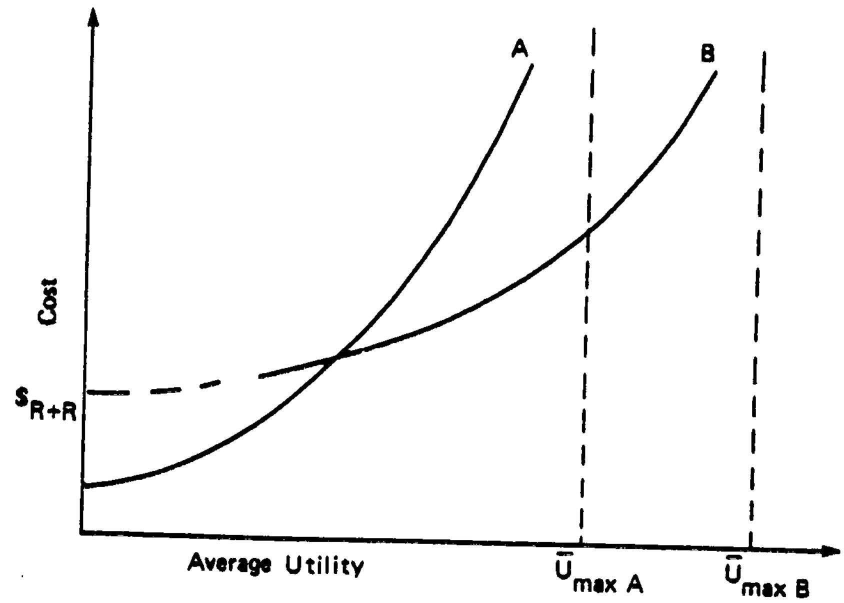

Figure 5

The Effect of Refurbishment and Reflight

Curve A in figure 5 depicts the improvements in average utility derived solely from ground-based testing. It is a testing, not a testing and reflight, strategy.

Curve B represents the cost curve for a test strategy that combines ground-based testing with refurbish/reflight. As curve B rises, the additional costs are for increases in refurbishment and ground-based testing. Curve B represents the costs to refurbish and refly a spacecraft once plus various levels of ground-based testing. As ground-based testing is increased, the costs increase, as does the attainable average utility. Strategy B provides more opportunities to detect failures than strategy A, thereby increasing $\bar{U}_{max}$, the maximum average utility achievable for a particular option.

The relation of curve A to curve B will be determined largely by payload size and complexity. Launch costs are a function of payload volume and weight, the orbit inclination, and the flight services required. Refurbishment costs appear to depend upon the complexity and sensitivity of the payload. Another cost which may be significant is the cost of delaying the project through a second flight cycle. If a group must be held in reserve while the refurbish/refly cycle is going on, these costs could be considerable. Since such delay costs are highly possible, they are dealt with in our general model.

There may be some concern that the cost curve for strategy B, the single reflight strategy, includes only the cost of one launch although the payload is actually flown twice. Since all missions include at least one flight, the expense of one launch is considered common to all missions and is not a part of the test costs. Flight costs are ignored as a portion of the test costs until the test strategy calls for additional flights; then these reflight costs are included as test costs. The inclusion of the reflight costs allows comparison of the various reflight and ground-based testing combinations to find the lowest cost strategy for obtaining a desired probability of success. Notice that in Curve A no flight costs whatsoever are included, though the vehicle will certainly be flown.

An important cost concern is the timing of the flights in the two strategies. STS costs are increased by the Bureau of Labor Statistics (BLS) cost escalator[14]. The timing of flights will affect costs due to this inflation factor. Figure 6 depicts a time line of possible series of events for strategies A and B, just testing or testing and reflying.

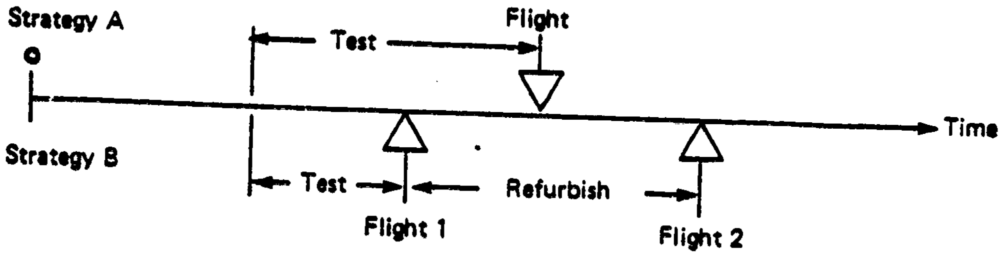

Figure 6

A Comparison of Strategies A and B

In the case shown, strategy A uses extensive ground-based testing and a single flight. Strategy B uses some ground-based testing and two flights. The first flight in B occurs before the flight in A. If the flights include identical services and the payload weight and volume is not different, then flight 1 or B will be less expensive than the flight in A. The flight in A will be less expensive than the flight 2 in B. An earlier flight will be less costly (assuming inflation) than a later one. The savings accruing to earlier flights may be estimated as

$$\text{Savings} = L(1+E)^{J_A} - L(1+E)^{J_B} \qquad (18)$$

Where L = present flight costs

E = monthly cost escalator

J_A = number of months until flight in A

J_B = number of months until flight in B

$L(1 - E)^J$ gives the cost of a flight J months for today assuming a constant rate of inflation E. Using this equation, the costs for all future flights may be computed and the total savings (or loss) of each strategy calculated.

Inflation is only one way that costs may fluctuate. There may be pricing increases independent of inflation. Such increases occur when it becomes apparent that the price charged for a service does not fully cover the cost of providing the service. Increases may also be pass-throughs. For instance, fuel prices may rise at a rate greater than the inflation rate. The proportion of increase not reflected in the BLS escalator must be passed through as a price of service increase. Since such increases are difficult to foresee, the costs incorporated in the model will have to be updated regularly.

Estimating refurbishment costs is difficult. The research done to date provides only rough estimates with which to model[13]. The cost estimates for refurbishment of LANDSAT vary from 30% to 75% of the original instrument costs[14]. The variations in costs appear to depend on the complexity of the instrument, the disassembly time, and whether parts are standardized. A re-examination of the LANDSAT-refurbishment raw data might reveal a functional relationship between complexity and refurbishment costs.

Returning to figure 5, curves A and B are assumed to be minimum test cost curves which incorporate the proper flight costs, escalators, and refurbishment costs. Figure 5 may be modified further by adding a curve C which depicts a strategy for two reflights with refurbishment as well as some ground-based testing.

Figure 7 illustrates the three strategies; A, B, and C.

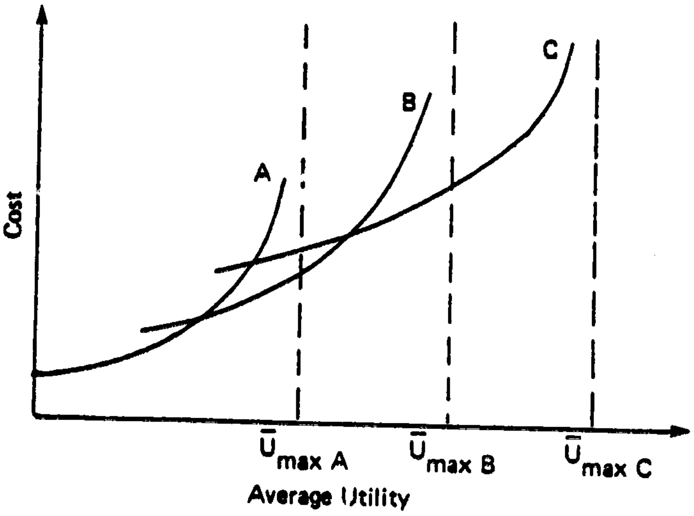

Figure 7

A Comparison of Three Strategies

Notice that $\bar{U}_{max\ c}$ is greater than either $\bar{U}_{max\ a}$ or $\bar{U}_{max\ b}$.

Curve C begins at a cost equivalent to the cost of ground-based testing and two refurbish/refly cycles. Strategy C, it should be mentioned, assumes three flights total.

Strategies B and C may provide lower costs methods for obtaining a given $\bar{U}_d$. For some $\bar{U}_d$ values, reflight strategies such as B and C may be the only method available. This is particularly likely as the desired $\bar{U}_d$ approaches $\bar{U}_{max}$. As the costs increase rapidly for additional performance certainty, it may become more economical to switch to a new strategy. Similarly, as the desired $Pr(\bar{U}_d)$ approaches the maximum available through strategy B, it may be less costly to use strategy C.

The model may be extended by adding strategies using increasing numbers of flights. Such extension, however, yields smaller and smaller increases in the maximum average utility; that is, the gains per dollar of shifting to "higher" strategies decrease. Figure 8 portrays the model extension and the convergence of the $\bar{U}_{max}$ levels for many strategies.

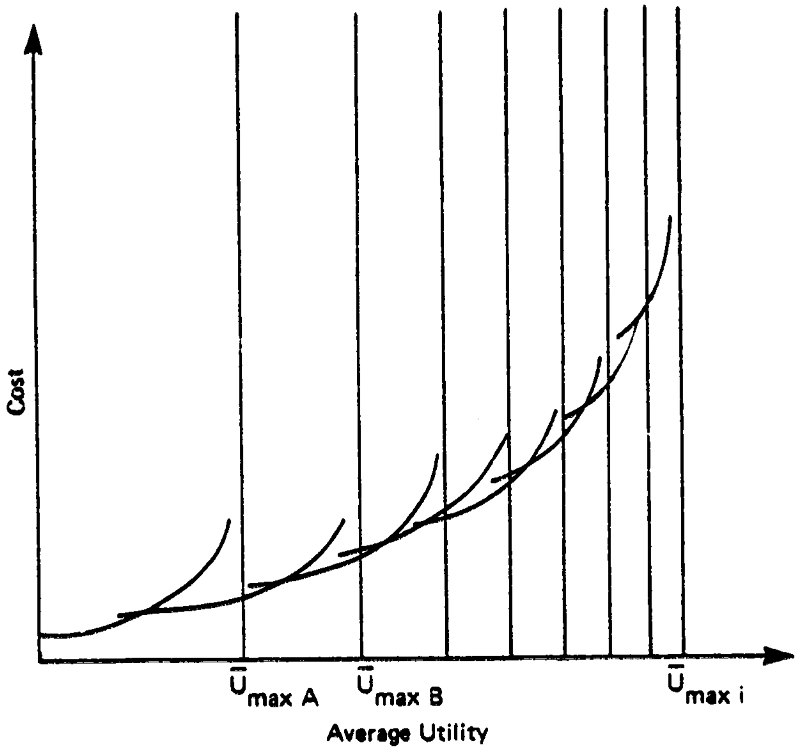

Figure 8

The Convergence of Attainable
Average Utility Towards a Maximum

The decreasing nature of the gain in $\bar{U}_{max}$ values results in a limit which is the ultimate $\bar{U}_{max}$ attainable after many flights. This value, max($\bar{U}_{max}$), is assumed to be less than 1.0. That the most certain probability of attaining success is less than 1.0 reflects the possibility of a random failure occurring at any time. Max ($\bar{U}_{max\ i}$) may be encountered in Space Transportation System by the Orbiter itself, its payload bay mechanisms, etc. after several years of operation. This value might also be approached by payloads that are flown repeatedly.

Given a payload, its volume, weight, flight needs, complexity, and the utility desired to have the mission perform successfully, a unique set of minimum test cost curves can be produced. These curves will relate gains in the average utility from ground-based testing as well as through refurbishment/reflight strategies. From the lowest cost frontier of the curve set, the scallop shaped line of figure 9, cost and levels of average utility may be matched. The output format with which this section began may thereby be derived.

The merits of using a "Lost Value" model as opposed to the "Utility vs. Cost" approach should be discussed for two different kinds of payloads.

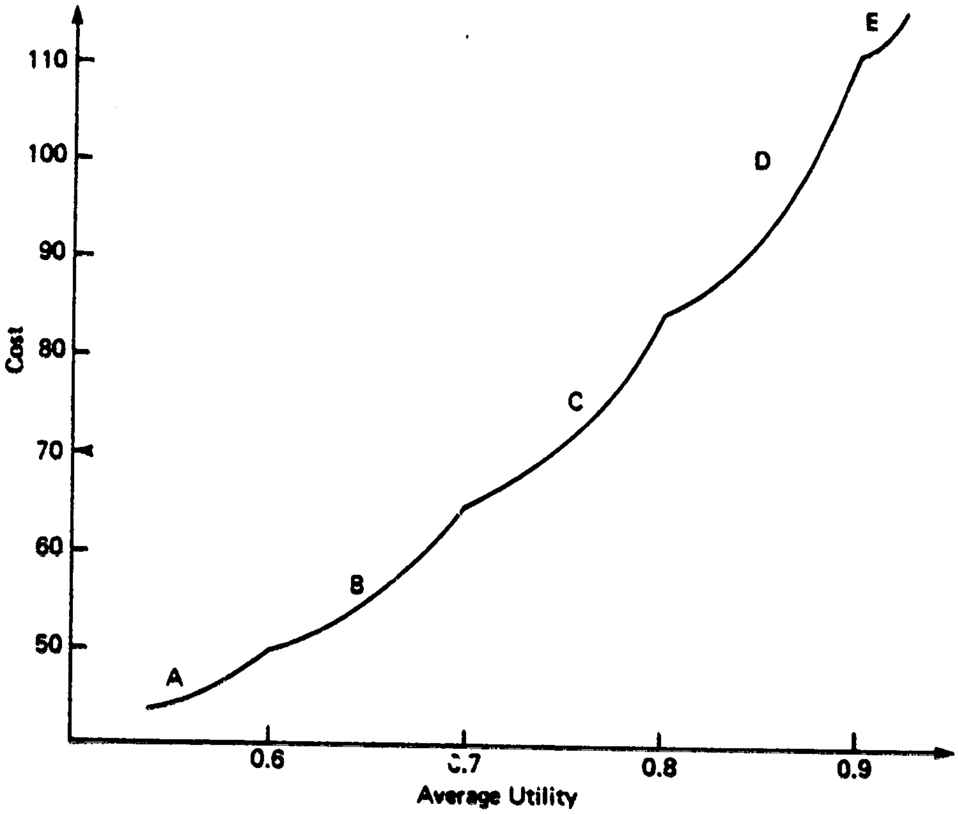

Figure 9

Variation of Cost with Strategy

If one launches a communications satellite in which the transmission of data can be measured as having value in terms of cost of data obtained and there is a one-time flight situation, then the "Lost Value" model or approach developed in Kruger-Norris[1] is adaptable as well as the approach developed in this section. The difference in the two approaches is that the Kruger and Norris Lost Value function takes the decision out of the projects manager's hands. It chooses the optimal test procedure for the manager.

If one launches scientific payloads where data has a tremendous value and there is a one-time flight, then the above comments are applicable to this type of payload.

On the other hand, if one has reflight capabilities for communication satellites or scientific payloads where data is highly valued, then the approach where one considers "cost vs. utility" applies. This approach also applies to the situation where one has scientific payloads where the value of the data obtained is not tremendous or unknown regardless of whether one reflys the payload or not.

To conclude this section, we remark that the mathematical procedures for the option, "we test to where we are and accept less utility at a given level of certainty" is very similar to the derivation described above and will be omitted.

THE FAILURE FLOW PROCESS

In order to develop an approach that would extend to multiple flights, we use the logic as suggested in Kruger and Norris[1]. A few simplifications were made in the general model, and before discussing the new approach, we discuss the basis for simplifying the model.

Rationale for Simplifying the Kruger and Norris TVTO Model

We now turn to the derivation of failure flow analysis and the modifications to utility under various decision criterion. The model developed by Kruger and Norris describes failure occurrences during the thermal-vacuum test procedure (accounting for effects of temperature cycling and temperature on the overall distribution of failures and normalized for the number of components) as

$$\frac{F}{N} = \frac{F_o}{N} + K\left\{(E_H + E_C)\left(\frac{y}{2}\right)^B + A(1-y)^B\left[\frac{T_H - T_C}{p(1-y)}\right]^Z\right\}(T+\gamma)^B \qquad (19)$$

To determine the various constants in equation (19), data from 109 component level tests were normalized, and iteration and fitting procedures were used to determine the parameters. The program that normalized the data and give the fitting procedures are given in reference (1).

A method of estimation of reliability growth was developed by Williams and Kruger[4] using the Product Limit estimation procedure of Kaplan and Meier[19]. This method has the statistical property of consistency. This property is a convergence property; by using the Product Limit estimation procedure, one has a theoretical, statistical representation of the distribution that describes the frequency of failures over time allowing for the adjustments, such as components entering and leaving the flow of tests, that were made to the data in the Kruger and Norris study. When one chooses six-hour increments and estimates the growth parameter B, using the Product Limit estimation procedure, one obtains $\hat{B} = 0.478$, and $\hat{\lambda} = 0.017$ for the model $F(t) = \lambda t^B$.

When one uses the normalized data from Kruger and Norris and does a fit to the above model, one obtains $\hat{B} = 0.485$, and $\hat{\lambda} = 0.018$. The multiple correlation coefficient for this data is $R^2 = 0.978$, which means that the model accounts for 97.8% of the variation in the system. From this result, one concludes that the normalization process in Kruger and Norris and a simple curve fit yield the same results for the estimation of B as the Product Limit estimates. In this sense, one may conclude that the normalization process for F/N found in Kruger and Norris yields the same estimates for B and λ as the process for Product Limit estimation or vice versa.

The Test Process

An approach to conceptualizing how failures are uncovered may be seen in Heuser[15] where failures are visualized as being detected or escaping tests. In this same way, it is helpful to look at and examine various assumptions and procedures that influence the flow of failures in thermal-vacuum tests.

The objective of any test program, whether it be environmental or reliability test screening, is to discover and eliminate failures that would occur in operational use. To accomplish this goal, one tries to conceive all possible environments to which the product would be subjected and then to create tests that simulate these operational environments. Decisions are also made as to some optimized test procedures in terms of cost, the quality of performance, the risk involved, the priority of the spacecraft, etc.

These kinds of decisions are made for thermal-vacuum tests recognizing that there have been previous tests at the parts level and several other levels (see Heuser[15]). Assuming the testing procedure has passed through a sequential stage like Heuser's description of Failure Flow Analysis, one encounters questions such as how much testing and what combination of testing needs to be done at the thermal-vacuum test stage.

To help guide these decisions, one needs to know the relationships between the various test stages. We assume, as in Kruger and Norris[1], that the procedure can be viewed as sequential in nature; that is, failures "flow" (i.e., if not uncovered proceed) from the component level tests to the systems level tests and on to the orbital flight. Further, the Kruger and Norris model assumes that by increasing test duration, one increases the likelihood of uncovering an incipient failure. With these assumptions, one would like to know how long to test at the component level to eliminate a reasonable amount of those failures which can be uncovered at that particular stage of testing. This decision must be based on cost and time considerations and on the future quality or performance of the spacecraft. To establish the cost-quality relationships in the thermal-vacuum area, it is necessary to model the failure mode, the cost mode, and the quality as testing takes place in this environment. It should be noted that quality encompasses the concept of utility.

We begin by discussing failure modes. To envision the process we consider the diagram of figure 10.

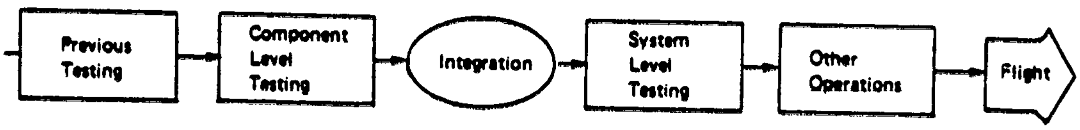

Figure 10

The Flow of Equipment

From previous testing and quality control of parts, we have screened out a certain number of failures (as appropriate references, we mention Heuser[15] and A. Krausz[16]).

If one has a procedure where failures are repaired when they occur, then it is conceivable (disregarding wearout modes and random failures) that by testing over a large enough time interval, one could eliminate those component failures that occur and are detectable during thermal-vacuum component level testing. One, however, has to consider the amount of testing in the light of cost and time constraints and in terms of the reliability or, in this paper, utility requirements. Since one cannot achieve perfect utility, then one must specify some level of utility that is necessary to meet the desired performance goal of the finished product.

It is in this framework of cost-benefit analysis that models play an essential role.

The Role of Models

Referring to Oxenfeldt[17]; a model is a simplified replication of reality that identifies its main components and indicates how they are interrelated. The following are some of the key elements of a model:

1. It is a simplified version of a more complex reality; the degree simplification varies according to the use for which it is intended.
2. Its purpose is to illuminate a real-life phenomenon; some simplification is required for ease and clarity of understanding.
3. Although simplified, the view of reality presented by a model does include its main elements and their interrelationships; simplification occurs by omitting non-essentials.
4. The model depicts reality for a particular purpose and a particular audience.
5. A model is an intellectual tool, a device that assists the thought process. Its value therefore is to be assessed primarily by the validity of the conclusions or decisions to which it leads.

Our model is used as a mathematical tool to predict performance as it progresses from one stage of the process to the next (as illustrated in figure 10) and to show what final utility is achieved at a particular cost after a certain amount of testing is done.

The TVTO Model Equations

Table II is provided as a guide to the equations that follow in this section.

Table II

List of Definitions

Symbol	Value or Definitions
A_c	0.0205
A_s, A_1, A_5	Variable coefficient
B_c	0.442
B_s	0.757
B_1	0.396
C	General coefficient

Table II (Con't)

Symbol	Value or Definitions
$F_c(t_f)$	Cumulative failures, component test
$F_s(t_f)$	Cumulative failures, system test
$F_1(t_f)$	Cumulative failures, in orbit
$f_c(t_f)$	$d(F_c(t_f))/dt$
$f_s(t_f)$	$d(F_s(t_f))/dt$
$f_1(t_f)$	$d(F_1(t_f))/dt$
N	Number of components
K_5	0.00512 (average failure rate for first 12h of component testing)
K_6	$N \times 10^{-7}$
K_7	$N \times 7.5 \times 10^{-6}$
t	time in test or in orbit
Subscripts c, s, 1, 2, 3	Refer to component or system level tests or first, second, or third flights, respectively
Subscript f	Final value
Subscript i	Interconnect
Subscript o	Other causes
Subscript t-v	Thermal-vacuum causes
Subscript (x,x)	Decisions D2 and D3 apply; D2 only if only one value is noted
Superscript bar, (as in $\bar{f}$)	Average value

Based on 150 component level tests (taken from the data used in NASA TM 80297 plus MMS data less that for the EU's and RIU's) with an average test duration of 187h, one can, using a 24h time increment with the Kaplan and Meier method, form an equation to describe the cumulative number of failures during component test as

$$F_c(t) = N A_c t^{B_c} . \tag{20}$$

On a per component basis, this can be differentiated to give a cumulative failure rate of

$$f_c(t) = A_c\, B_c\, t^{(B_c-1)} \quad . \tag{21}$$

failures per component per hour.

At the end of 187h, the failure rate is then

$$f_c(187) = 0.000489 \quad . \tag{22}$$

Based on 39 system level tests which had an average duration of 17.3 days (415.2h) and an average complement of 65 components, we can – using a 24h step as before – define the cumulative number of failures during systems test as

$$F_s(t) = N \times 0.000904\, t^{B_s} \tag{23}$$

and the cumulative failure rate per component as

$$f_s(t) = 0.000904\, B_s\, t^{(B_s-1)} \quad . \tag{24}$$

If we assume a constant failure rate over the first 12h of a test and that the failure rate at the end of a test program is relatively constant over a 12h period, we can define the average cumulative failure rate at the beginning of the system test as

$$\overline{f}_s(12) = \frac{1}{12} \int_0^{12} 0.000904\, B_s\, t^{(B_s-1)}\, dt \tag{25}$$

or

$$\overline{f}_s(12) = 0.000494 \quad . \tag{26}$$

One would expect a difference in cumulative failure rates to exist when comparing that at the end of the component test program with that at the beginning of the system test program. This should be true at least from the fact that the system test program incorporates the interconnection of the components – so that they must act in concert – as well as the interconnecting hardware plus, generally, some other previously untested equipment. As an approximation, we introduce the term $\overline{f}_i$ (12) to account for this interconnection failure rate during the first 12h of test and define it as

$$\begin{aligned} \overline{f}_i(12) &= \overline{f}_s(12) - \overline{f}_c(187) \\ &= 0.000494 - 0.000487 \\ \overline{f}_i(12) &= 0.000007 \quad . \end{aligned} \tag{27}$$

We take $\bar{f}_i$ (12) as an additive term in the system test program so that no amount of testing at the component level can correct the flaws it represents. Furthermore, hypothesizing that $\bar{f}_i$ (12) results basically from the intergration of the components, we approximate its form as a function of the number of components and designate a term K_6 such that

$$\bar{f}_i(12) = N \times 0.000007/65 = N \times 10^{-7} = K_6 \, . \tag{28}$$

If we attribute the same exponent to f_i as in (24), and assume (pending more complete data analysis) that it equals (B_s-1), then we may write

$$\bar{f}_i(12) = \frac{1}{12} \int_0^{12} A_i \, B_s \, t^{(B_s-1)} \, dt$$

$$\bar{f}_i(12) = \frac{1}{12} A_i(12)^{B_s} = A_i(12)^{(B_s-1)} \tag{29}$$

or

$$A_i = \bar{f}_i(12) \cdot (12)^{(1-B_s)} = K_6(12)^{(1-B_s)} . \tag{30}$$

Then, taking A_s as was done with A_1, we may write

$$f_s(t_s) = A_i \, B_s(t_s)^{(B_s-1)} + A_s \, B_s(t_s)^{(B_s-1)} \, . \tag{31}$$

If

$$f_c(t_{cf}) = \frac{1}{12} \int_0^{12} A_s \, B_s(t_s)^{(B_s-1)} \, dt \quad , \tag{32}$$

$$f_c(t_{cf}) = \frac{1}{12} A_s(12)^{B_s} = A_s(12)^{(B_s-1)} \tag{33}$$

$$A_s = f_c(t_{cf})(12)^{(1-B_s)} \quad , \tag{34}$$

then, from (30), (31) and (34),

$$f_s(t_s) = \left[K_6(12)^{(1-B_s)} + f_c(t_{cf})(12)^{(1-B_s)}\right] B_s(t_s)^{(B_s-1)} \tag{35}$$

$$f_s(t_s) = [K_6 + f_c(t_{cf})] \times (12)^{(1-B_s)} \, B_s(t)^{(B_s-1)} \tag{36}$$

and

$$F_s(t_s) = [K_6 + f_c(t_{cf})] \times (12)^{(1-B_s)} \, t^{B_s} \, . \tag{37}$$

Within the 39 spacecraft that were used to develop (23), were 31 that were included in the PRC data base. From this data base, and again using a 24h time increment for curve fitting, we can describe the orbital performance as

$$F_1(t) = N \times 0.00288\, t^{B_1} \tag{38}$$

and

$$f_1(t) = 0.00288\, B_1\, t^{(B_1-1)} . \tag{39}$$

The average failure rate for the first 12h is

$$\bar{f}_1(12) = \frac{1}{12} \int_0^{12} f_1(t)\, dt \tag{40}$$

$$= 0.000642 . \tag{41}$$

Since $f_s(415.2) = 0.000158$, there is a significant difference between the failure rates at the end of the system-level test program and the beginning of the orbital case. Again, this is not unexpected since the orbital case includes failures due to causes other than these one may associate with the thermal-vacuum test, e.g. failures due to vibroacoustics. Let us designate these "other cause" failure rates as $f_o(t)$. If we assume, pending more complete data analysis, that the failure rates may be transmitted from the system test level to the orbital case, then based on the current data we may write as in (27)

$$\bar{f}_o(12) = \bar{f}_1(12) - f_s(415.2) . \tag{42}$$

Substituting from (41), we have

$$\bar{f}_o(12) = 0.000642 - 0.000158 = 0.000484 \tag{43}$$

If we assume f_o to be a function of spacecraft complexity as measured by the number of components, we can establish a coefficient K_7 such that

$$K_7 = N \times 0.000484/65 = N \times 7.5 \times 10^{-6} . \tag{44}$$

Also, if we assume that the f_o and f_3 terms arise from similar equations having the same exponent, B_3, we may proceed as in (29).

In orbital case, we may write

$$\bar{f}_o(12) = \frac{1}{12} \int_0^{12} A_o\, B_1\, t^{(B_1-1)}\, dt = \frac{1}{12} A_o (12)^{B_1}$$

$$\bar{f}_o(12) = A_o (12)^{(1-B_1)} . \tag{45}$$

Substituting the value for K_7 as in (44), we write

$$A_o = \bar{f}_o(12) \times (12)^{(1-B_1)} = K_7(12)^{(1-B_1)} \qquad (46)$$

Then, as in (12),

$$f_1(t) = (A_o + A_1) B_1 t_1^{(B_1-1)} \qquad (47)$$

from which, as was shown in developing (34),

$$A_1 = f_s(t_{sf}) (12)^{(1-B_1)} \qquad (48)$$

and

$$F_1(t_1) = (A_o + A_1) t_1^{B_1} \qquad (49)$$

and

$$f_1(t_1) = (A_o + A_1) B_1 t_1^{(B_1-1)} \qquad (50)$$

Turning for a moment to the question of average utility, $\bar{U}$, we can see that the form of (49) precludes the attainment of $\bar{U}$ below some given level, namely that due to the A_o term which is unaffected by the thermal-vacuum test program. If the A_1 term were zero (i.e., no failures occurred due to causes that could be uncovered by thermal-vacuum testing), then the best average availability that could be achieved would still be no greater than

$$\bar{U} = \frac{1}{t_1} \int_0^{t_1} [1 - 0.273 \exp(-0.0086N)] \left[K_7(12)^{(1-B_1)} t_1^{(B_1)}\right] dt \qquad (51)$$

Using the approaches developed previously, we can describe performance in orbit for three other cases:

1. component test but no system test,
2. system test but no component test, and
3. no test.

Since there is no actual data for these cases, additional assumptions will be needed.

1. Component Test but no System Test

The number of failures during the component test program and the failure rates can be expressed as in (20) and (21) as

$$F_c(t) = N A_c t^{B_c} \tag{52}$$

and

$$f_c(t) = A_c B_c t^{(B_c - 1)} . \tag{53}$$

If there were a system level test, the average failure rate during the first 12h, would be, from (37),

$$\bar{f}_s(12) = \frac{1}{12} \int_0^{12} [K_6 + f_c(t_{cf})] (12)^{(1-B_s)} B_s t^{(B_s-1)} dt$$

$$= [K_6 + f_c(t_{cf})] (12)^{(1-B_s)} (12)^{(B_s-1)} .$$

$$\bar{f}_s(12) = K_6 + f_c(t_{cf}) \tag{54}$$

Some failures would be generated, but we choose to neglect them because they are unassignable.

If now we propogate $\bar{f}_2$ (12) forward to the orbital case as though it were $f_s(t_{sf})$ in (48), (49), and (50), we may define the orbital case characteristics as

$$F_1(t_1) = N (A_0 + A_1) t_1^{B_1}$$

$$F_1(t_1) = N \left[K_7 (12)^{(1-B_1)} + f_s(t_{sf}) (12)^{(1-B_1)} \right] t^{B_1} . \tag{55}$$

Since we define the final system level test failure as the same as the average of the first 12h were there a test, then

$$f_s(t_{sf}) = \bar{f}_s(12) \tag{56}$$

and, from (54), (55) may be rewritten as

$$F_1(t_1) = N \left[K_7 (12)^{(1-B_1)} + (K_6 + f_c(t_{cf})) (12)^{(1-B_1)} \right] t_1^{B_1}$$

$$F_1(t_1) = N [f_c(t_{cf}) + K_6 + K_7] (12)^{(1-B_1)} t_1^{B_1} . \tag{57}$$

Note that $F_1(t_1)$ cannot be less than

$$F_1(t_1) = N [K_6 + K_7] (12)^{(1-B_1)} t_1^{B_1} . \tag{58}$$

Also, given $\bar{U}$ and t_1, t_{cf} may be defined as

$$t_{cf} = (((F_1(t_1)/(N(12)^{(1-B_1)}\, t_1^{\,B_1})) - K_6 - K_7)/(A_c\, B_c))^{(1/(B_c-1))} \tag{59}$$

so that any component test duration equal to or larger than t_{cf} will provide at least the minimum, desired $\bar{U}$. It is possible that the necessary $t_{cf} \leq 0$ in which case no tests will be necessary. It is also possible that no solution exists in which case $\bar{U}$ was chosen below the possible minimum.

2. System Test but no Component Test

We take the average failure rate of components, had there been component level tests, as the average failure rate for the beginning of the system level test, both over the first 12h interval.

$$\bar{f}_c(12) = \frac{1}{12} \int_0^{12} A_c\, B_c\, t^{(B_c-1)}\, dt = K_5 \tag{60}$$

Substituting in (36),

$$f_s(t_s) = [K_6(12)^{(1-B_s)} + K_5(12)^{(1-B_s)}]\, B_s\, t_s^{\,(B_c-1)}$$

$$f_s(t_s) = (K_5 + K_6)\,(12)^{(1-B_s)}\, B_s\, t_s^{\,(B_s-1)} \tag{61}$$

and

$$F_s(t_s) = N(K_5 + K_6)\,(12)^{(1-B_s)}\, t^{B_s}. \tag{62}$$

As in (46), $A_1 = f_s(t_{sf})\,(12)^{(1-B_1)}$ and, substituting from (42),

$$A_1 = (K_5 + K_6)\,(12)^{(1-B_s)}\, B_s\, t_s^{\,(B_s-1)}\,(12)^{(1-B_1)}$$

$$A_1 = (K_5 + K_6)\,(12)^{(2-B_s-B_1)}\, B_s\, t_s^{\,(B_s-1)} \tag{63}$$

and from (47),

$$F_1(t_1) = [K_7(12)^{(1-B_1)} + (K_5 + K_6)\,(12)^{(2-B_s-B_1)}\, B_s\, t_s^{\,(B_s-1)}]\, t_1^{\,B_1} \tag{64}$$

For a pre-defined $\bar{U}$ and t_1, we can solve for t_s as

$$t_s = (((F_1(t_1)/t_1^{\,B_1}) - K_7\,(12)^{(1-B_1)})/((K_5 + K_6)\,(12)^{(2-B_s-B_1)}\, B_s)^{(1/(B_s-1))} \tag{65}$$

3. <u>No Test</u>

In this case we assume that the initial component failure rate characteristics propagate to the system test, and the initial system test failure characteristics propagate to the orbital case. As in (60), we define the initial component level failure characteristics as

$$\bar{f}_c(12) = K_5 \quad . \tag{66}$$

The system level failure characteristics are then described as the average of the first 12h of hypothetical system test or, from (61),

$$\bar{f}_s(12) = \frac{1}{12}(K_5 + K_6)(12)^{(1-B_s)}(12)^{B_s}$$

$$\bar{f}_s(12) = K_5 + K_6 \quad . \tag{67}$$

Since the failures that these failure rates infer do not take place during tests, they are not considered.

Since $\bar{f}_s$ (12) is the same as $f_s(t_s)$ in (48), we may write

$$A_1 = (K_5 + K_6)(12)^{(1-B_1)} \tag{68}$$

and, substituting into (49),

$$F_1(t_1) = N[K_7 (12)^{(1-B_1)} + (K_5 + K_6)(12)^{(1-B_1)}]\ t_1{}^{B_1}$$

$$F_1(t_1) = N[K_5 + K_6 + K_7]\ (12)^{(1-B_1)} t_1{}^{B_1} \quad . \tag{69}$$

If one has a mission of duration t_1 and a predefined $\bar{U}$, then, if the number of failures found by (69) is less than those that produce the necessary $\bar{U}$, no test is needed.

Failure Flow Process and Utility for Multiple Flights

In order to demonstrate the approach and not to become too cumbersome, we limit ourselves to no more than three flights (although the process allows for more). Table III indicates the various options that can be exercised

Table III

Available Options

Test Decisions		Flight Decisions		
Component Level, C	System Level, S	First, D_1	Second, D_2	Third, D_3
0_c	0_s	–	0	0
1_c	1_s	1	1	1
			2	2
			3	3
			4	4

where, under C and S:

0_c indicates no component level testing,

0_s indicates no system level testing,

1_c indicates component level testing,

1_s indicates system level testing,

so that (0_c, 0_s) would indicate that neither component nor system level tests were done and (1_c, 0_s) would indicate that component level tests were done but that there was no system level thermal-vacuum test.

Similarly, under decision D_1, D_2, and D_3:

0 indicates no reflight, regardless of the results of the previous flight

1 indicates reflight if the mission is not accomplished (2, 3, and 4 follow from this)

2 indicates reflight without repair, refurbishment, or retest

3 indicates reflight with repair and refurbishment but no retest

4 indicates reflight with repair, refurbishment, and thermal-vacuum retest.

Since there is no program without at least one flight, the decision for D_1 indicated by 0 does not exist.

We can now consider the decision matrix (C, S, D_1, D_2, D_3); each one (with the exception of D_1) has a number of options.

The possibility of multiple flights requires a re-thinking of the concept of utility. Imagine, for instance, that one choses the path (1, 1, 1, 2, 0). This guarantees that the overall average utility will be less than the path (1, 1, 1, 0, 0) which involves one flight rather than two. However, we can be fairly certain that we will gather more data during two flights than during one (given that the first flight of the two is as long as the one flight in the single flight case). Yet, since without repair, refurbishment, and retest we can expect the instantaneous utility to continue decreasing yielding a low, overall average, we do gather additional data.

The difficulty lies in the definition of overall, average utility, i.e., the ratio of the performance of the actual payload to that of a perfect one. The problem is described graphically in figure 11.

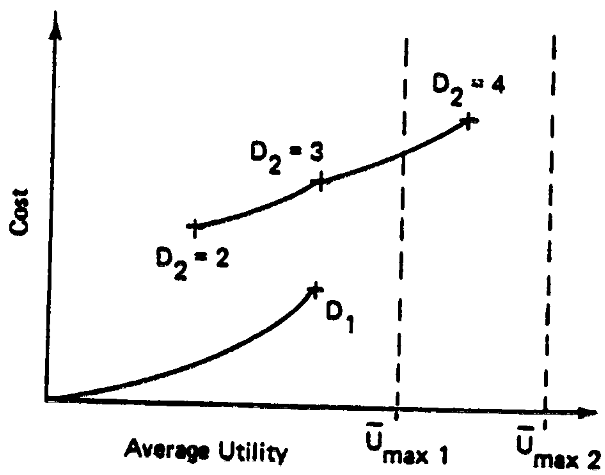

Figure 11

Effect of D_2 Decisions on Cost and Average Utility

If we perform a test program and conduct a flight, we will arrive at some overall average utility for a particular cost at point D_1. If now we simply refly, we have incurred an added cost due to reflight but may expect the performance to continue degrading resulting in a decrease in this utility. This condition corresponds to the point $D_2 = 2$.

If we repair and refurbish, we can expect a further cost increase but will probably increase the average utility by bringing the payload performance nearer to the ideal. This condition is shown as the point $D_2 = 3$ on figure 11. It will be higher and almost certainly to the right of point $D_2 = 2$. It will likely be to the right of the point D_1 but we cannot be sure. Finally, if we include testing, a similar thought process will bring us to the point $D_2 = 4$, again probably to the right of the point D_1.

(Note that $\bar{U}_{max1}$ is the maximum average utility that can be obtained in one flight. It is independent of the thermal-vacuum test program. $\bar{U}_{max2}$ however may be dependent on the performance during the first flight and the success of the repair programs; for instance, the first flight may uncover additional non-thermal-vacuum related failures as the duration is extended, and the repairs before the second flight would correct these flaws that would otherwise have degraded the performance of the second flight.)

If we now construct a graph as in figure 12, we can describe the process in terms of a mission where our goal is to gather a specific amount of data.

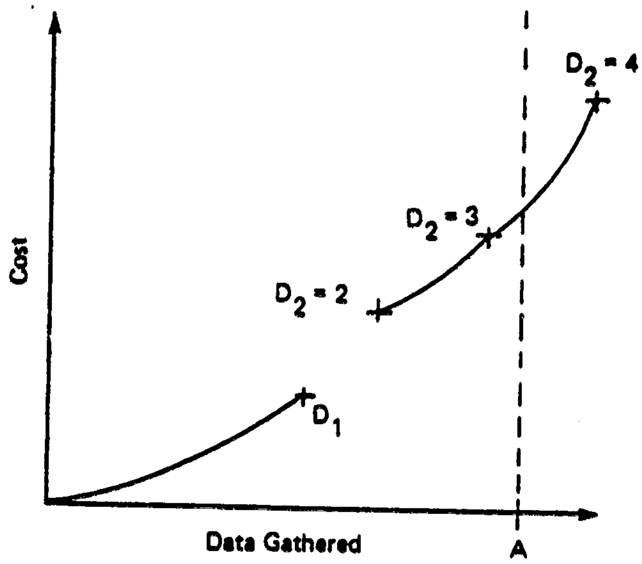

Figure 12

Effect of D_2 Decisions on Cost and Data Gathered

Here, as before, by conducting a particular test program, we arrive at point D_1 having gathered some portion of the desired quantity of data, A. If we now simply refly (without any repairs or retest), we will incur the added cost of reflight but will probably increase the quantity of data that was gathered. Point $D_2 = 2$ would then be above and to the right of point D_1. Decisions 3 and 4 would lead to points $D_2 = 3$ and $D_2 = 4$, increasingly to the right and above D_1. We cannot intuitively determine whether we will cross the line at A, but we would at least move toward it. As can be seen in figure 11, it is possible to progress to the left as a flight program is continued.

Figures 11 and 12 can be thought of as depicting processes important to two different kinds of missions. Figure 11 could be applied to a mission where a level of performance is required as might be the case with a communications relay satellite. Figure 12 could represent a scientific mission such as defining the ultraviolet spectral signatures of a certain group of stars. In either case it is important to know the instantaneous capability of the payload (as might be defined by the number of failures that occur). Therefore, our use of failure information should be done in light of the mission to which it applies. For the purpose of this paper, we will limit ourselves to those cases where average utility is the applicable criterion. Further, it is applied on a per-flight basis.

Since the average utility for the first flight (designated as $\bar{U}_1$) is established by the decisions at the test levels, it may be defined as a minimum cost approach, i.e., a specified $\bar{U}_1$ for minimum cost. We then proceed with the decisions D_2 and D_3 with the assumption that $\bar{U}_1$ does not fulfill the mission requirements.

From preliminary data analysis, it appears that for cases involving costly payloads, the minimum cost programs involve thermal-vacuum testing at both the component and system levels; we then discuss this case first. Furthermore, it is assumed that flights may be viewed or act upon a payload as an additional test. We therefore can proceed with the concept of a failure flow process, e.g., if an incipient failure doesn't occur during this flight, it may during a test following the flight or during a succeeding flight.

Flowing from flight 1 to flight 2 without any repair, we have

$$f_1(t_{1f}) = \frac{1}{12} \int_0^{12} A_2 B_2 t_2^{(B_2-1)} dt_2 \tag{70}$$

$$f_1(t_{1f}) = \frac{1}{12} A_2 (12)^{B_2} = A_2 (12)^{(B_2-1)} \tag{71}$$

$$A_2 = f_1(t_{1f}) (12)^{(1-B_2)} \tag{72}$$

where $B_2 = B_1$ and t_{1f} is the final flight time of the first flight for the minimal cost-utility strategy using component and systems level testing.

$\bar{U}_{2(2)}$ means the average utility for flight 2, under decision 2, and $\bar{U}_{2(3)}$ means the average utility for flight 2, under decision 3.

Note that these are not utilities that include the previous flight.

As before,

$$f_2(t_2) = [f_1(t_{1f}) + K_8] (12)^{(1-B_2)} B_2 t_2^{(B_2-1)} . \tag{73}$$

Since we currently have no data to speculate on the constant K_8, where K_8 is a constant similar to K_7 but between the first and second flights, we assume that it exists for similar reasons as before and assume $K_8 = K_7$. This assumption means that we are including a flight interconnect type term in our model. It also means that the lift-off has an effect on failures that cannot be accounted for. From (67) we have

$$F_{2(2)}(t_2) = [f_1(t_{1f}) + K_8] (12)^{(1-B_2)} t_2^{B_2} \tag{74}$$

and

$$\bar{U}_{2(2)} = \frac{1}{t_{2f}} \int_{t_{1f}}^{t_{1f}+t_{2f}} (1 - D*)^{(F_{2(2)}(t_2))} dt_2 . \tag{75}$$

This argument generalizes if we consider the various combinations of test options because $f_1(t_{1f})$ can be calculated under these options as explained in the previous sections.

FLIGHT 2, OPTION 3

$B_2 = B_1$ and the assumption is made that after the first flight, we can repair those failures that have occurred in flight, both those which are detectable in the thermal-vacuum test environment and those which are not. Thus we have $K_8/2$ instead of K_8.

The utility is based on

$$f_{2(3)}(t_2) = [f_1(t_{1f}) + \frac{K_8}{2}]\,(12)^{(1-B_2)}\,B_2\,t_2^{(B_2-1)} \tag{76}$$

and

$$F_{2(3)}(t_2) = [f_1(t_{1f}) + \frac{K_8}{2}]\,(12)^{(1-B_2)}\,t_2^{B_2}\ ; \tag{77}$$

hence,

$$\bar{U}_{2(3)} = \frac{1}{t_{2f}} \int_0^{t_{2f}} (1-D*)^{(F_{2(3)}(t_2))}\,dt_2\,. \tag{78}$$

FLIGHT 2, OPTION 4

Here again the utility has started over at 1.0 to begin the flight but with the additional cost of repair, refurbishment, and testing and a change in the failure mode. To account for the change in the failure mode, we use the mechanism as described before; thus we have

$$f_1(t_{1f}) = \frac{1}{12} \int_0^{12} A_{2s}\,B_{2s}\,t_{2s}^{(B_{2s}-1)}\,dt_{2s} \tag{79}$$

$$= \frac{1}{12} A_{2s}\,(12)^{B_{2s}} = A_{2s}\,(12)^{(B_{2s}-1)} \tag{80}$$

or

$$A_{2s} = f_1(t_{1f})\,(12)^{(1-B_{2s})} \tag{81}$$

where $B_{2s} = B_2$.

As before,

$$f_{2s(4)}(t_{2s}) = [f_1(t_{1f})]\,(12)^{(1-B_{2s})}\,B_{2s}\,t_{2s}^{(B_{2s}-1)}\,. \tag{82}$$

and

$$F_{2s(4)}(t_{2s}) = [f_1(t_{1f})]\ (12)^{(1-B_{2s})}\ t_{2s}^{\ (B_{2s}-1)} \ . \qquad (83)$$

We have not assumed a component test stage since it appears most likely that only the few components that have exhibited a failure would be retested, and this is much like the process in a system level test program. After the test stage, these failures flow into the second flight, and we have

$$f_{2s(4)}(t_{2sf}) = \frac{1}{12} \int_0^{12} A_2\ B_2\ t_2^{\ (B_2-1)}\ dt_2 \qquad (84)$$

$$f_{2s(4)}(t_{2sf}) = \frac{1}{12} A_2\ (12)^{B_2} = A_2\ (12)^{(B_2-1)} \ ; \qquad (85)$$

therefore,

$$A_2 = f_{2s(4)}(t_{2sf})\ (12)^{(1-B_2)} \qquad (86)$$

where $B_2 = B_1$.

As before,

$$f_{2(4)}(t_2) = [f_{2s(4)}(t_{2sf}) + \frac{K_8}{2}]\ (12)^{(1-B_2)}\ B_2\ t_2^{\ (B_2-1)} \qquad (87)$$

and

$$F_{2(4)}(t_2) = [f_{2s(4)}(t_{2sf}) + \frac{K_8}{2}]\ (12)^{(1-B_2)}\ t_2^{\ B_2} \ . \qquad (88)$$

Therefore, we have

$$\bar{U}_{2(4)} = \frac{1}{t_{2f}} \int_0^{t_{2f}} (1-D*)^{(F_{2s(4)}(t_2))}\ dt_2 \ . \qquad (89)$$

This discussion takes care of the calculation for flight 2 after the decision D_2 is made. We now turn to the calculations based on the decision D_3 and options (2, 3,4) before third flight.

FLIGHT 3, OPTION 2

Flowing from flight 2 into flight 3 with $D_3 = 2$, we must base our analysis on each of the three failure rates, $f_{2(2)}(t_2)$, $f_{2(3)}(t_2)$, and $f_{2(4)}(t_2)$, from the previous decision-making process.

Using the final failure rate $f_{2(2)}(t_{2f})$, we have

$$f_{3(2,2)}(t_3) = [f_{2(2)}(t_{2f}) + K_8]\,(12)^{(1-B_3)}\,B_3\,t_3^{(B_3-1)} \tag{90}$$

and

$$F_{3(2,2)}(t_3) = [f_{2(2)}(t_{2f}) + K_8]\,(12)^{(1-B_3)}\,t_3^{B_3} \tag{91}$$

where we assume $B_3 = B_2 = B_1$ until we have data to adjust these estimates. Thus,

$$\bar{U}_{3(2,2)} = \frac{1}{t_{3f}} \int_{t_{1f}+t_{2f}}^{t_{1f}+t_{2f}+t_{3f}} (1-D^*)^{(F_{3(2,2)}(t_3))}\,dt_3 \quad . \tag{92}$$

If option 2 for the third flight follows from option 3 for the second flight, we use $f_{2(3)}(t_{2f})$, and we have

$$f_{3(3,2)}(t_3) = [f_{2(3)}(t_{2f}) + \frac{K_8}{2}]\,(12)^{(1-B_3)}\,B_3\,t_3^{(B_3-1)} \tag{93}$$

and

$$F_{3(3,2)}(t_3) = [f_{2(3)}(t_{2f}) + \frac{K_8}{2}]\,(12)^{(1-B_3)}\,t_3^{B_3} \quad . \tag{94}$$

Then,

$$\bar{U}_{3(3,2)} = \frac{1}{t_{3f}} \int_{t_{2f}}^{t_{2f}+t_{3f}} (1-D^*)^{(F_{3(3,2)}(t_3))}\,dt_3 \quad . \tag{95}$$

If we now consider the case where option 2 for the third flight follows from option 4 for the second flight, we use $f_{2(4)}(t_2)$, and we have

$$f_{3(4,2)}(t_3) = [f_{2(4)}(t_{2f}) + \frac{K_8}{2}]\,(12)^{(1-B_3)}\,B_3\,t_3^{(B_3-1)} \tag{96}$$

and

$$F_{3(4,2)}(t_3) = [f_{2(4)}(t_{2f}) + \frac{K_8}{2}]\,(12)^{(1-B_3)}\,t_3^{B_3} \quad . \tag{97}$$

We have for utility

$$\bar{U}_{3(4,2)} = \frac{1}{t_{3f}} \int_{t_{2f}}^{t_{2f}+t_{3f}} (1-D*)^{(F_{3(4,2)}(t_3))} \, dt_3 \qquad (98)$$

FLIGHT 3, OPTION 3

Following from flight 2 into flight 3 with option 3, we must base our analysis or calculation of utility on each of the three failures rates, $f_{2(2)}(t_2)$, $f_{2(3)}(t_2)$, and $f_{2(4)}(t_2)$, from the previous decision making process. These derivations will be similar to the previous ones, but with an adjustment for repair in the utility calculations.

We have, similar to the previous equations,

$$f_{3(2,3)}(t_3) = [f_{2(2)}(t_{2f}) + \frac{K_8}{2}] \, (12)^{(1-B_3)} \, B_3 \, t_3^{(B_3-1)} \qquad (99)$$

and

$$F_{3(2,3)}(t_3) = [f_{2(2)}(t_{2f}) + \frac{K_8}{2}] \, (12)^{(1-B_3)} \, t_3^{B_3} \qquad (100)$$

where K_8 has been adjusted by a multiple of 1/2 due to one repair in the failure flow process. K_8 is adjusted by a multiple of $(1/2)^n$ (where n is defined as the total number of repairs in the process). We use this method to account for learning. We have for utility,

$$\bar{U}_{3(2,3)} = \frac{1}{t_{3f}} \int_{o}^{t_{3f}} (1-D*)^{(F_{3(2,3)}(t_3))} \, dt_3 \; . \qquad (101)$$

If option 3 for the third flight follows from option 3 for the second flight, we use $f_{2(3)}(t_{2f})$ and have

$$f_{3(3,3)}(t_3) = [f_{2(3)}(t_{2f}) + \frac{K_8}{2^2}] \, (12)^{(1-B_3)} \, B_3 \, t_3^{(B_3-1)} \qquad (102)$$

and

$$F_{3(3,3)}(t_3) = [f_{2(3)}(t_{2f}) + \frac{K_8}{4}] \, (12)^{(1-B_3)} \, t_3^{B_3} \qquad (103)$$

with

$$\bar{U}_{3(3,3)} = \frac{1}{t_{3f}} \int_{o}^{t_{3f}} (1-D*)^{(F_{3(3,3)}(t_3))} \, dt_3 \; . \qquad (104)$$

If option 3 for the third flight follows the failure flow process from option 4 for the second flight, then we use $f_{2(4)}(t_{2f})$; then,

$$f_{3(4,3)}(t_3) = [f_{2(4)}(t_{2f}) + \frac{K_8}{4}]\,(12)^{(1-B_3)}\,B_3\,t_3^{(B_3-1)} \tag{105}$$

and

$$F_{3(4,3)}(t_3) = [f_{2(2)}(t_{2f}) + \frac{K_8}{4}]\,(12)^{(1-B_3)}\,t_3^{B_3} \tag{106}$$

with

$$\bar{U}_{3(4,3)} = \frac{1}{t_{3f}} \int_0^{t_{3f}} (1-D^*)^{(F_{3(4,3)}(t_3))}\,dt_3 \tag{107}$$

FLIGHT 3, OPTION 4

Under this decision, we must take each of the final failure rates, $f_{2(2)}(t_{2f})$, $f_{2(3)}(t_{2f})$, and $f_{2(4)}(t_{2f})$, and process them through a re-test stage and then through the third flight stage.

Starting with $f_{2(2)}(t_{2f})$, we have

$$f_{4(2)}(t_{2f}) = \frac{1}{12} \int_0^{12} A_{3s}\,B_{3s}\,t_{3s}^{(B_{3s}-1)}\,dt_{3s} \tag{108}$$

where $B_{3s} = B_2$.

If we make derivations similar to equations (79) thru (83), we have

$$f_{3s(2,4)}(t_{3s}) = [f_{2(2)}(t_{2f})]\,(12)^{(1-B_{3s})}\,B_{3s}\,t_{3s}^{(B_{3s}-1)} \tag{109}$$

and

$$F_{3s(2,4)}(t_{3s}) = [f_{2(2)}(t_{2f})]\,(12)^{(1-B_{3s})}\,t_{3s}^{B_{3s}}\,. \tag{110}$$

Extending $f_{3s(2,4)}(t_{3s})$ into the third flight, and using a derivation similar to equations (78) thru (81), we have

$$f_{3(2,4)}(t_3) = [f_{3s(2,4)}(t_{3sf}) + \frac{K_8}{2}]\,(12)^{(1-B_3)}\,B_3\,t_3^{(B_3-1)} \tag{111}$$

and

$$F_{3(2,4)}(t_3) = [f_{3s(2,4)}(t_{3sf}) + \frac{K_8}{2}]\,(12)^{(1-B_3)}\, t_3^{\,B_3} \qquad (112)$$

with

$$\bar{U}_{3(2,4)} = \frac{1}{t_{3f}} \int_o^{t_{3f}} (1-D^*)^{(F_{3(2,4)}(t_3))}\, dt_3 \;. \qquad (113)$$

We extend $f_{2(3)}(t_{2f})$ and $f_{2(4)}(t_{2f})$ into the system level test and obtain $f_{3s(3,4)}(t_{3s})$ and $f_{3s(4,4)}(t_{3s})$ as in equation (113). From these equations we obtain $f_{3(3,4)}(t_3)$ and $f_{3(4,4)}(t_3)$ as in equation (111) and $F_{3(3,4)}(t_3)$ and $F_{3(4,4)}(t_3)$ as in equation (112).

These failure functions yield

$$\bar{U}_{3(3,4)} = \frac{1}{t_{3f}} \int_o^{t_{3f}} (1-D^*)^{(F_{3(3,4)}(t_3))}\, dt_3 \qquad (114)$$

and

$$\bar{U}_{3(4,4)} = \frac{1}{t_{3f}} \int_o^{t_{3f}} (1-D^*)^{(F_{3(4,4)}(t_3))}\, dt_3 \;. \qquad (115)$$

As a final comment on utility, we mention that their confidence intervals are calculated as in the previous section where time allowances are made for the various repair, refurbish, no repair, etc. options.

For example, if there is no repair and two flights are made, where t_1 is the time in the first flight and t_2 is the time for the second flight, then we have the total time interval from (o, t_1+t_2).

To calculate the variability for average utility after two flights, we calculate failures using the assumption of process dependence at the end of the time (t_1+t_2).

If repair is made after the first flight, then we calculate the failure made at t_2.

Another scheme to place bounds on average utility can be devised by using the Product Limit procedure and regression to calculate upper and lower bounds on the failure mode functions $F(t)$. To illustrate this relationship, we construct the curve $F(t)$ and its upper and lower confidence bounds versus average utility achieved (see figure 13). Note that as the cumulative number of failures increases, the average utility, $\bar{U}$, decreases.

Suppose one obtains $\bar{U} = 0.8$ in flight 1. To find $Pr(\bar{U} > \bar{U}_a) = 0.95$, $\bar{U}_a < 0.8$, we calculate 90% Product Limit confidence bounds, $F_U(t)$, $F_L(t)$ by regression using a real data base. We then project up to $F_U(t)$ from 0.8, over to $F(t)$, and then down to $\bar{U}$ at $\bar{U}_a$ where $Pr(\bar{U} \geq \bar{U}_a) = 0.95$.

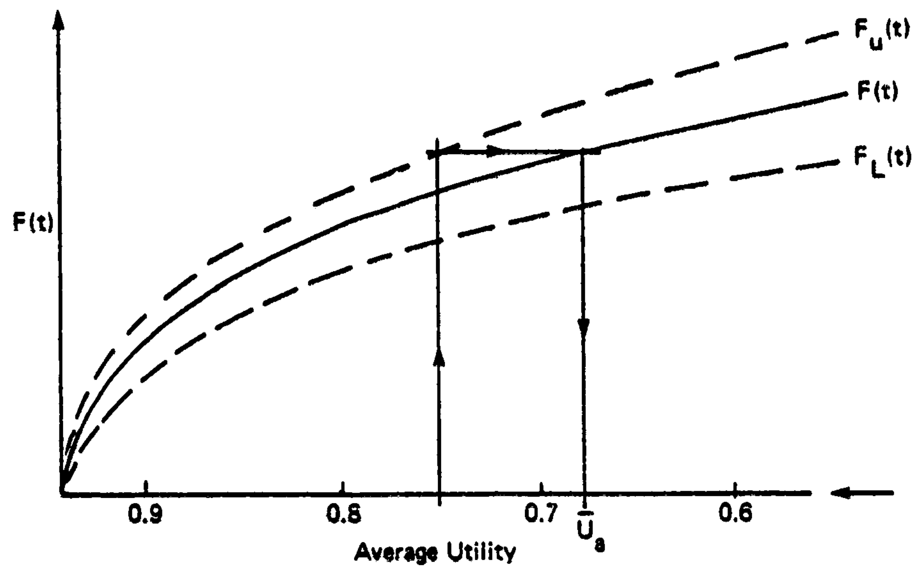

Figure 13

Bounds for $\overline{\text{Average Utility}}$

This argument can be reversed to yield a test program which gives the desired utility based on a minimal cost thermal-vacuum program. What one does is to reverse the path, find the utility one needs to obtain, and then refers to the minimal cost curve for thermal-vacuum test program to determine the particular test program.

RECOMMENDATIONS FOR FUTURE STUDIES

1. Study the question of complexity to see how it affects the calculation of utility.

2. Use the science of Information Theory to better quantify ideas about obtaining and using information from Space Shuttle experiments. Build guides for management to use in planning missions in terms of the actual information needed to complete an experiment.

3. Apply utility to individual components in an experiment having several components to determine the model of overall utility for the total number of components.

4. Study the process dependent assumption; obtain data to either verify or change this assumption.

5. Gain data from Space Shuttle missions to model and establish the relationships of failure between individual flights and to measure the interconnect effects in flight as well as for test.

6. Trace failure flow data through component and system levels thermal-vacuum tests and on into flight.

7. The timing of testing; is testing prior to flight 1 equivalent to testing between flights? Does the timing matter?

8. Is there a relationship between complexity and refurbishment costs? An examination of the LANDSAT data might reveal one.

9. Is there an ultimate $\bar{U}_{max}$ less than 1.0? Our model and the intuition it is based upon suggests that there is. If so, what is that level and why?

10. The cost figures appear to be changing rapidly. An updated version of The STS Reimbursement Guide is necessary to calculate accurate costs. Too, since updating is likely, they should be easy to change in the computer program.

11. A more careful look at scientific decision making is needed. We have shifted the model's output toward information that we believe is more attuned to managers' needs. The question now is whether they will.

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