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Investigation of Geomagnetic Field Forecasting
and Fluid Dynamics of the Core

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TYPE II

1. Problems

None

2. Approach

a) Magnetic Determination of the Depth of the Core-Mantle Boundary Using MAGSAT Data

The pole-strength of earth, $P(r,t)$, defined in equation 1 of Progress Report #6 has been the focus of further study during this reporting period. A slightly different approach has been taken in using this quantity to evaluate, magnetically, the radius of the earth's core-mantle boundary. In Figure A of the last report we plotted the change in pole strength (in megawebers) between 1965 and 1980 at various radii within the earth, the radius at which it is zero being the "magnetic core radius". However, Figure A does not reveal just how small, relatively, this absolute change in P is, so in Figure 1 below we plot, instead the dimensionless ratio of pole strength at 1980 to that at 1965. The magnetic core radius is then where the ratio takes on the value 1. A further refinement is that both data sets have been treated in the same way. The so-called definitive model of Barraclough, Harwood, Leaton, and Malin (BHLM) selected for 1965 was a fit to a massive data set at a truncation level $N=8$, so instead of simply truncating MGST 6/80 from its fitting level of 13 back to $N=8$ (as done in the previous Figure A), we have now utilized calculations kindly made, at our request by Drs. R.A. Langel (GSFC) and Ron Estes (BTS) which re-fit the data used for MGST 6/80 to various

truncation levels, among them $N=8$. This refinement makes no significant changes in the value of the magnetic core radius compared to that found previously.

An important addition to our approach is the calculation of one standard deviation error estimates for $P(r,t)$ for the 1965 BHLM model. The approximate σ 's of the Gauss coefficients (in nT) at $r=a$ are given by Hide and Malin (1981) as

$$\sigma(g_n^m) = \sigma(h_n^m) = 0.56(n+1)^{\frac{1}{2}} \quad . \quad (1)$$

The resulting standard deviation of the pole-strength integral P is then calculated from

$$\sigma(P_1) = \left\{ \sum_{n=1}^N \sum_{m=0}^n (0.56)^2 (n+1) \left[\left(\frac{\partial P_1}{\partial g_n^m} \right)^2 + \left(\frac{\partial P_1}{\partial h_n^m} \right)^2 \right] \right\}^{\frac{1}{2}} \quad . \quad (2)$$

Here P_1 designates P at time $t_1=1965$ and each of the partial derivatives is an integral. To obtain the total one sigma error bar for the ratio P_2/P_1 (where P_2 is P at 1980) we require error bars for the MGST 6/80 model fit to the MAGSAT data. These have just been acquired from Dr. Ron Estes.

Since

$$\begin{aligned} \sigma(P_2/P_1) &= \left\{ \left[\frac{\sigma(P_2)}{P_2} \right]^2 + \left[\frac{\sigma(P_1)}{P_1} \right]^2 \right\}^{\frac{1}{2}} \\ &= \frac{\sigma(P_1)}{P_1} \left\{ 1 + \left[\frac{P_1}{P_2} \frac{\sigma(P_2)}{\sigma(P_1)} \right]^2 \right\}^{\frac{1}{2}} \quad , \quad (3) \end{aligned}$$

we expect that the relatively small fitting errors in MGST 6/80 will be dominated by those in the BHLM model so no appreciable change will be required. In any event, these error estimates have to be regarded as lower bounds on true errors since they do not account for instrument errors and errors associated with extrapolation through the mantle (assumed to be an insulator).

As a partial check on these error bars we also used a random number generator to perturb each g_n^m , h_n^m separately by up to $\pm 0.56(n+1)^{\frac{1}{2}}nT$ and then calculated $P(b, 1965)$ where $b = 3485$ km. Repeating this for 5 different random sets gave differences in P_1 from the value for BHLM that were in every case within the error shown on Figure 1. We, therefore, believe the minimum errors associated with the data fitting process have been reliably determined.

The basic curve of Figure A relied on two main field models only so that, by design, secular variation played only an indirect role (no role whatsoever in MGST 6/80 and only to bring the 20 years of data for BHLM to the common central epoch of 1965). One still wonders whether the closeness of the magnetic core radius, so determined, to the seismic core radius might not be fortuitous. A further, independent test is therefore highly desirable. Accordingly, we have chosen another high quality main field model, GSFC 12/66, and made extensive calculations with it. The results

appear in Figure 1 below and are discussed further in Section 3.

b) Downward Extrapolation through the Electrically Conducting Mantle

In Progress Reports #1, 2, and 4 reference was made to work in progress designed to account for the effects of induced electric currents in the mantle (due to non-zero mantle conduction) for the downward extrapolation of magnetic fields from earth's surface to the core-mantle boundary. The formulation was completed some months ago, but relatively little numerical evaluation and application of the methodology had been made. During the present reporting period (and with financial assistance from an NSF Grant, no NASA funds having been used) the application aspect of this work has been tackled by Dr. Kathryn A. Whaler who visited Boulder for 5 weeks during June and July 1981. The approach used was to choose several selected radially symmetric conductivity models for the mantle and then numerically integrate the magnetic diffusion equations. Some conclusions are described in Section 5 below.

c) Estimate of an upper bound for the time required for earth's liquid core to overturn completely

An approach has been developed for estimating an upper bound on how long it takes earth's core to overturn. While bearing some similarity to recent work of Büsse and Proctor,

which gives methods for bounds on dynamo action, our approach is different in a vital way. These previous results require evaluation of magnetic volume integrals over the core which cannot generally be done since we have no real information on the structure of the magnetic field within the dynamo region. Thus, the results are of theoretical interest, but of limited practical value.

Our approach reduces the problem to one of magnetic surface integrals over spheres within the core. These, too, cannot be directly calculated, but the integrals in question are the analytic continuation of the same quantities over spheres within the mantle, where we do have magnetic information, so it seems likely that numerical estimates of interest can be inferred.

Without going into much detail, we assert that it is possible to prove, in a straightforward fashion that for some spherical surface of radius r in the core, bounded away from the inner core-outer core boundary and the core-mantle boundary,

$$\int_0^{2\pi} \int_0^{\pi} |u(r, \theta, \phi, t)| r^2 \sin\theta d\theta d\phi > \frac{\eta_0 \int_0^{2\pi} \int_0^{\pi} |\nabla(\mathbf{rBr})|^2 r^2 \sin\theta d\theta d\phi}{|\mathbf{rB} \cdot \nabla(\mathbf{rBr})|_{\text{MAX}}} \quad (4)$$

Here $\eta_0 = 1/\mu_0 \sigma_0$ is the (constant) magnetic diffusivity of the core, and MAX denotes the largest value achieved on the sphere of radius r . Because u is the vertical fluid motion

(whose surface average is zero for incompressible flow, as much fluid moving upwards as downwards), $|u|$ integrated over the spherical surface measures the unsigned flux of fluid volume ($m^3 s^{-1}$) across that surface (just as $\int_0^{2\pi} \int_0^\pi |B_r(r, \theta, \phi, t)| r^2 \sin\theta d\theta d\phi$ measures the unsigned magnetic flux crossing the sphere of radius r). Thus, when the total liquid core volume is divided by the left-hand integral we obtain the overturning time for the core. Now, the two quantities on the right are both continuous functions of radius everywhere and moreover, they can be evaluated in the mantle, and estimated in the inner core. In fact, within the insulating mantle the numerator integral can be evaluated exactly as

$$\eta_0 \int_0^{2\pi} \int_0^\pi |\nabla(rB_r)|^2 r^2 \sin\theta d\theta d\phi = 4\pi a^2 \eta_0 \sum_{n=1}^N \sum_{m=0}^n (n+1)^3 \left(\frac{a}{r}\right)^{2n+2} [(g_n^m)^2 + (h_n^m)^2] \quad (5)$$

where a is the radius of the earth. Thus, the radial dependence of the numerator is readily exposed. It but remains to tabulate $\vec{r} \cdot \nabla(rB_r)$ on the spherical surface in the mantle and evaluate the maximum value at various depths.

Although both the numerator and the denominator in (4) are expected to increase rapidly with depth in the mantle, their ratio should be a rather mild function of radius. It, therefore, becomes possible to extrapolate into the core (since both functions

and their ratio remain continuous across the core-mantle boundary) and estimate a lower bound on the volume flux integral (which upon inversion gives an upper bound on the overturning time).

This work is being jointly funded by this NASA contract and an NSF grant, and we intend to use the MAGSAT data for evaluation.

3. Accomplishments

a) Magnetic determination of the depth to the core-mantle boundary using MAGSAT data

A major accomplishment of this reporting period is the completion of a body of calculations (by Mr. Coerte V. Voochies, a Graduate Research Assistant being supported by this contract) revealing both the radial and time dependence of pole-strength, $P(r,t)$, within the earth. Figure 1 summarizes some of the results. In this figure, the ratio of P evaluated at the MAGSAT epoch to P evaluated at various previous epochs (1930, 1940, 1950, 1960, 1965) is displayed as a function of depth within the mantle (considered to be an insulator). The models used are MGST 6/80 (re-fit to $N=8$) for 1980; BHLM for 1965; and GSFC 12/66 for 1930, 1940, 1950, 1960 (truncated back from 10 to 8, and compared with a re-fit to the data of MGST 6/80 to $N=10$ followed by truncation back to $N=8$). Minimum direct use of secular variation was entailed in the curve labeled 1965.

For the curves using GSEC 12/66, the secular variation and secular acceleration coefficients were used not only as part of the data fitting process, but were also used subsequently to construct Gauss coefficients at the desired epochs.

The final truncation level of 8 chosen was based on the belief that it is the optimum value for evaluating the pole-strength integral at various levels down to the core-mantle boundary. Choosing smaller N excludes too much magnetic flux, whereas choosing larger N results in downward extrapolation (and hence amplification) of too much noise because of the relatively larger uncertainties in the higher order Gauss coefficients.

Further details will be described in a paper being prepared for the MAGSAT issue of Geophysical Research Letters being edited by Dr. R.A. Langel. Here we call attention to the fact that each curve compares a previous magnetic model with MAGSAT, and that every curve passes through the value 1 within a few per cent of the accurate seismically determined core radius of 3485 km. The somewhat peculiar shape of the curve for 1930, compared to that for later epochs is believed to be due to its increased dependence on the secular variation and secular acceleration coefficients compared to the later curves. The error bars on the curve for 1965 are those described in the preceding section. Given the size of the (minimum) error, we conclude that this technique finds the core

radius, magnetically over a span of 35 years with acceptable accuracy, the mean of the five determinations being only 11 km (or about 0.3%) larger than the seismic core radius. This significant result builds our confidence in (a) the short term validity of the frozen-flux assumption in the core, (b) the adequacy of treating the mantle as an insulator, (c) the accuracy of the main field models MGST 6/80 for 1980, BHLM for 1965 and GSFC 12/66 for 1960, and (d) the validity of the secular variation and secular acceleration coefficients in GSFC 12/66 for limited interpolation backward in time (while remaining well within the span covered by data).

b) Higher order analytic approximation to the unsigned magnetic flux crossing earth's surface

The principal investigator and Ms. M. Christine Coulter (a Graduate Research Assistant being supported by this contract) have collaborated so as to obtain an improved analytic approximation for $P(a,t)$ compared to that given in the last Progress Report, the result being

$$\begin{aligned}
 P(a,t) = & -4\pi a^2 g_1^0 \left\{ 1 + \frac{1}{2} [(G_1^1)^2 + (H_1^1)^2] \right. \\
 & - \frac{\sqrt{6}}{2} [G_1^1 G_3^1 + H_1^1 H_3^1] + \frac{9}{16} (G_2^0)^2 + \frac{27}{32} [(G_2^2)^2 + (H_2^2)^2] \\
 & + \frac{15\sqrt{15}}{32} [G_2^2 G_4^2 + H_2^2 H_4^2] - \frac{1}{2} G_3^0 + \frac{3}{4} [(G_3^1)^2 + (H_3^1)^2] \\
 & + \frac{5}{4} [(G_3^3)^2 + (H_3^3)^2] + \frac{225}{256} (G_4^0)^2 \\
 & \left. + \frac{175}{128} [(G_4^2)^2 + (H_4^2)^2] + \frac{875}{512} [(G_4^4)^2 + (H_4^4)^2] \right\},
 \end{aligned}$$

where $G_n^m \equiv g_n^m / g_1^0$.

This reveals the fact that, at earth's surface (and undoubtedly elsewhere too), P is a (quadratically) non-linear functional of normalized Gauss coefficients.

In the next step it is hoped to extend this approximation to situations where there is more than one magnetic equator, and then to use the result to help understand the morphology of the geomagnetic field during polarity transitions. Initial conversations with Prof. Michael Fuller (Univ. of California at Santa Barbara) suggest that the above sort of analysis, when properly extended, can contribute needed constraints for modelling reversals.

c) Downward extrapolation through the electrically conducting mantle

The preliminary results revealed by the computations of Dr. Whaler referred to above are as follows: (a) For all but the most extreme conductivity profiles, the correction to main field Gauss coefficients for mantle conduction is insignificant throughout the mantle. (b) The correction to secular variation coefficients is probably marginally significant, but the dependence on conductivity profile is surprisingly weak; thus it is more important to correct SV models than main field models for mantle conduction if they are to be extrapolated downward, but the choice of conductivity profile is of minor concern. (c) Regardless of the conductivity profile chosen, the spectral power in the secular variation is

decreased for all harmonic orders except 1, 2, 4 and the largest corrections occur for a constant conductivity profile. (d) The correction for mantle conductivity does not preferentially amplify the small wave length structures, since the sign of the correction depends upon the ratio of secular acceleration (\ddot{g}_n^m) to secular variation (\dot{g}_n^m), which can be either positive or negative.

4. Significant Results

By comparison, in a consistent fashion, of the magnetic pole-strength of the earth at earlier epochs with the value determined by MAGSAT, it has been demonstrated that the radius of earth's core can be found magnetically with acceptable accuracy, provided the optimum truncation level $N=8$ is chosen for the spherical harmonic models.

5. Publication

Edward R. Benton "Inviscid Frozen-Flux Velocity Components at the Top of Earth's Core From Magnetic Observations at Earth's Surface. Part I. A New Methodology", Geophysical and Astrophysical Fluid Dynamics 18, 157-174, 1981.

6. Recommendations

None

7. Funds Expended through August 31, 1981

\$50,143.

8. Data Utility

The MAGSAT and GSFC magnetic models, as they are currently being produced, continue to be the main data source for this project, and they are eminently suited to our purposes. We expect to utilize the new GSFC 9/80 and 7/81 models in our continuing work.

FIG. 1. RATIO OF POLE STRENGTH OF EARTH AS DETERMINED BY MAGSAT TO THAT AT PREVIOUS EPOCHS AS A FUNCTION OF DEPTH IN THE MANTLE (SEE TEXT PAGE 8 FOR DETAILS)

