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TEMPERATURE GRADIENT AND ELECTRIC FIELD DRIVEN ELECTROSTATIC INSTABILITIES

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ABSTRACT

We investigate the stability of electrostatic waves to thermodynamic and electric potential gradients. The major virtue of this analysis, other than its overall generality, is that thermodynamic gradients drive instabilities even when the internal electric field vanishes. This result does not emerge from previous analyses because skewing of the distribution function was not included in the dielectric.

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The role of low-frequency electrostatic instabilities in inhibiting transport within laser produced plasmas has been the focus of substantial theoretical interest in the last few years. For example, ion acoustic and electrostatic ion cyclotion instabilities are known to be driven by non-thermal features of the electron distribution function associated with electric currents and/or heat fluxes,[1-5] and can severely inhibit transport within a plasma. The importance of transport inhibition in astrophysical plasmas is also being recognized - a number of articles addressing current and heat flux limitations by electrostatic instabilities during solar flares having also appeared in the astrophysical literature.[6-10]

Since transport inhibition in both laboratory and natural plasma systems appears to be of universal importance, a general linear analysis merits close attention. Despite numerous articles on this subject only instabilities directly dirven by induced electric fields have been considered. For example, Kindel and Kennel[2] have investigated the ion acoustic and electrostatic ion cyclotron instabilities driven by resistive electric fields, $E_{resistive} = \eta j_{\parallel}$, in plasmas for which the net current is non-zero. Another example is the so-called "heat flux" instabilities which are driven solely by a combination of thermoelectric fields, $E_{thermo} = 0.71 \frac{\nabla_{\parallel} T}{e}$, and electron pressure electric fields, $E_{\nabla_{\parallel} P} = \frac{\nabla_{\parallel} P_e}{me}$, in plasmas for which the net current <u>is</u> zero.[11-13]
Under the zero current condition it is noted in these articles that the internal electric field, $E_{\parallel}$, adjusts such that $E_{\parallel} = - E_{thermo} - E_{\nabla_{\parallel} P_e}$, resulting in a non-thermal feature in the electron distribution function. This feature has been referred to as a "return current."[5] There is, of course, an additional non-thermal skewing of the electron distribution function directly related to the <u>temperature and pressure gradients</u>. However, unlike the $E_{\parallel}$ - field induced "return current", the skewing of the distribution function by temperature and pressure gradients was not included in the plasma dielectric.

Therefore, any resulting plasma instabilities are driven solely by internal electric fields--regardless of the source of these fields (eg. temperature and pressure gradients, induction fields, etc.). Strictly speaking then, "heat flux" instabilities, as addressed in the literature, are not categorically different from "current driven" instabilities since the driving agent in both cases is an internal electric field.

In this note we include, in the plasma dielectric, the skewing of the electron distribution function by temperature and pressure gradients and investigate the resulting modifications to the stability threshold. It should be stressed that retention of the nonthermal skewing allows us to study a class of instabilities that are categorically different from those driven by internal electric fields, viz., temperature and pressure gradient instabilities, which can occur even if the internal electric field is zero. Our analysis will also relax the zero current constraint which <u>a-priori</u> defines the magnitude of the internal electric field, $E_{\parallel}$. In general, the internal electric field does not adjust such that $j_{\parallel} = 0$. This is especially true during non-steady electrodynamic conditions when currents can be created by induction. A determination of the self-consistent internal electric field is highly model dependent, depending on a variety of sources of emf (eg. temperature and pressure gradients, suprathermal particle beams, induction fields, etc.) and thus cannot be reasonably addressed in this note. Therefore, in our analysis, the internal electric field will be explicitly treated as an undetermined parameter, along with the temperature and pressure gradients, in order to isolate the different sources of instability. The primary goal of this analysis is to determine, as a function of the electron-ion temperature ratio, the threshold electric field, temperature gradient, and pressure gradient above which ion-acoustic and/or electrostatic ion cyclotron waves are destabilized. The major virtue of this analysis, other than its overall generality, is the realization that under certain conditions, electron temperature and/or pressure gradients can drive plasma instabilities even when the internal electric field vanishes. This important result does not emerge from previous analyses because the evaluated plasma dielectric did not include the gradient-induced skewing of the electron distribution function.

The steady state electron distribution function, $f_e(v, \epsilon_E, \epsilon_T, \epsilon_P)$, is modeled by a Maxwellian plus small non-thermal components associated with the presence of an internal electric field, $\epsilon_E = E_\parallel/E_D$, a temperature gradient, $\epsilon_T = \frac{\nabla_\parallel T_e}{e E_D}$, and a pressure gradient, $\epsilon_P = \frac{\nabla_\parallel P}{neE_D}$.

$$f_e(\vec{v}, \epsilon_E, \epsilon_T, \epsilon_p) = (\pi v_{te}^2)^{-\frac{3}{2}} \exp\left\{-\left(\frac{v}{v_e}\right)^2\left(1 + \sum_\alpha \epsilon_\alpha D_\alpha(v) \cos\theta\right)\right\} \qquad (1)$$

corresponding to the first two terms of a Legendre expansion in pitch angle, θ. In equation (1), $v_e = (2T_e/m_e)^{\frac{1}{2}}$, $v^2 = v_\parallel^2 + v_\perp^2$, $v_\parallel = \frac{\vec{v}\cdot\vec{B}}{|\vec{B}|}$, $\alpha = E, T_e, P_e$, E_D is the Dreicer field $E_D = 4\pi n_e e^3 \ln\Lambda/T_e$ and where:

$$D_\alpha(v) = \sum_m a_{m\alpha}\left(\frac{v}{v_e}\right)^{2m+1} \qquad (2)$$

is the analytic form in the weak anisotropy limit which follows from an expansion of the distribution function in Sonine polynomials (cf., Braginskii[14]). The coefficients $a_{m\alpha}$ are obtained by a least squares fit to the results of Cohen et al[15] and Spitzer and Harm[16] and are tabulated in Table I. Figure 1 illustrates the structure of f_e for the two cases which isolate the non-thermal features associated with electric fields (ie, $\epsilon_E = 0.15$, $\epsilon_T = \epsilon_P = 0$) and temperature gradients (ie, $\epsilon_T = 0.15$, $\epsilon_E = \epsilon_P = 0$). Note that for each case there is a region in velocity space for which $\partial f_e/\partial v > 0$, a necessary condition for instability. This occurs for the higher velocity electrons. Furthermore, note that the current, $j_{\parallel}$, and heat flux, $Q_{\parallel}$, are not necessarily zero. This follows from the first and third velocity moments of equation (1) which result in Onsager's relations, viz.,

$$j_{\parallel} = n_e v_e\ (5.28\ \epsilon_E + 5.28\ \epsilon_p + 3.70\ \epsilon_T) \tag{3}$$

$$Q_{\parallel} = -\frac{3}{2} n_e m_e v_e^3 (5.60\ \epsilon_E + 5.60\ \epsilon_p + 6.77\ \epsilon_T). \tag{4}$$

Although the model electron distribution function used in this note does not include: (1) Feedback from potentially excited plasma turbulence and (2) a runaway region in velocity space (i.e., for $\frac{v}{v_e} > \epsilon^{-\frac{1}{2}}$), it is a reasonable choice for marginally stable systems provided $\epsilon_\alpha \ll 1$.

The dispersion function for electrostatic waves in a $\beta \ll 1$ magnetoplasma is:

$$D(\omega, \vec{k}) = 1 + \sum_{\sigma=e,i} \chi_\sigma \tag{5}$$

where the electron and ion susceptibilities are given by:

$$\chi_e(\omega,\vec{k}) = \frac{1}{k^2\lambda_{De}^2}\left\{1 + \xi_e Z(\xi_e) - \sum_\alpha \epsilon_\alpha \left[\sum_{m=0}^{\infty}\left(A_{m\alpha}\, \xi_e^{2m}\, Z(\xi_e) + B_{m\alpha}\, \xi_e^{2m+1}\right)\right]\right\} \tag{6}$$

and

$$\chi_i(\omega,\vec{k}) = 2\left(\frac{\omega_{pi}}{kv_i}\right)^2 \sum_{m=-\infty}^{\infty} e^{-\mu_i} I_m(\mu_i)\ [1 + \xi_i\, Z(\xi_i - m\xi_{ci})] \tag{7}$$

with $\lambda_{De}^2 = \frac{T_e}{4\pi m e^2}$, $\omega_{pi}^2 = \frac{4\pi n e^2}{m_e}$, $\xi_\alpha = \frac{\omega}{k_{\parallel} v_\alpha}$, $\xi_{ci} = \frac{\Omega_i}{k_{\parallel} v_i}$, $\mu_i = \frac{1}{2}(k_\perp v_i/\Omega_i)^2$, $\Omega_i = \frac{eB}{m_i c}$, and where I_n is the modified Bessel function, while $Z(\xi)$ is the plasma dispersion function. In deriving these susceptibilities equation (1) has been used for the electron distribution function and $f_i = (\pi v_i^2)^{-3/2} \exp(-v^2/v_i^2)$ for the ions. The electron susceptibility coefficients, Am_α and Bm_α, in equation (6) depend upon the electron distribution through the $a_{m\alpha}$ in equation (2)(c.f. Morrison and Ionson[17]) and are tabulated in Table I.

Following the usual procedure of letting $\omega \rightarrow \omega + i\Gamma$ with $\frac{\Gamma}{\omega} \rightarrow 0$, the dispersion function given by equations (5)-(7) results in the following zero growth rate condition:

$$\epsilon = \sum_\alpha \epsilon_\alpha A_{o\alpha} = \xi_e \left[1 + \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \left(\frac{T_e}{T_i}\right)^{3/2} \sum_m \Gamma_m(\mu_i) \exp\left[(\xi_i - m\xi_{ci})^2 + \xi_e^2\right]\right] \quad (8)$$

This condition, which follows from $\mathrm{Im}(D(\omega,\vec{k})) = 0$, corresponds to the zero growth rate relation between the magnitude of the electron distribution function's nonthermal features, characterized by $\epsilon = \sum_\alpha \epsilon_\alpha A_{o\alpha}$, and marginally unstable waves of frequency ω and wavenumber $\vec{k}$. In deriving equation (8) we have used $\xi_e \ll 1$ thereby allowing us to define a generalized instability parameter, ϵ --a convenient measure of the electron's tendency to drive electric field and pressure and temperature gradient instabilities. The absolute marginal stability condition is found by minimizing equation (8) for ϵ with respect to μ_i and ξ_e for fixed m_i/m_e and T_e/T_i. The resulting absolute marginal stability condition is given by:

$$\epsilon^* = \begin{cases} \left[1 + \frac{2D}{\Gamma_1}\left(\frac{m_e}{m_i}\frac{T_i}{T_e}\right)^{\frac{1}{2}}\left(5\,[\text{illegible}]^{?/3}\left(\frac{T_e}{T_i}\right)^{3/2}\right)\right]\left[1 + \frac{1}{\Gamma_1}\left(1 + \frac{T_e}{T_i} - \Gamma_o - \frac{4}{3}\Gamma_1 + \frac{2}{3}\Gamma_2\right)\right] D & \text{for } .01 < \frac{T_e}{T_i} \le 8 \quad (9) \\[2ex] \dfrac{\left\{\frac{T_i}{T_e}\frac{m_e}{m_i}\,\ell n\left[2\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}\left(\frac{T_e}{T_i}\right)\right]^{3/2}\right\}^{\frac{1}{2}}}{1 - \frac{23}{2}\left(\frac{T_i}{T_e}\right)^{4/3}} & \text{for } \frac{T_e}{T_i} > 8 \quad (10) \end{cases}$$

where $D = \left\{ \frac{T_i}{T_e} \frac{m_e}{m_i} \ln \left[2\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \left(\frac{T_e}{T_i}\right)^{3/2} \Gamma_1 \right] \right\}^{\frac{1}{2}}$, $\Gamma_n = e^{-\mu_i^*} I_n(\mu_i^*)$.

ϵ^* is the minimum value of ϵ above which the plasma is unstable to the electrostatic waves and $\mu_i^* \simeq 1.2$ has been used[17]. Note that equation (9) for ϵ^* corresponds to an electrostatic ion cyclotron instability (which has the lower threshold for the temperature ratios shown) whereas equation (10) corresponds to an ion acoustic instability. These results are illustrated in Figure 2.

The major emphasis in this Note has been to determine the form of a general instability parameter, $\epsilon^* = 2.6\ (\epsilon_E + \epsilon_p) + 0.20\ \epsilon_T$ as a function of $\frac{T_e}{T_i}$ where $\epsilon_E = \frac{E_\parallel}{E_D}$, $\epsilon_T = \frac{\vec{\nabla}_\parallel T}{e E_D}$, and $\epsilon_p = \frac{\nabla_\parallel P_e}{n_e E_D}$. These results reduce to those of Kindel and Kennel(2) for $\epsilon_T = \epsilon_p = 0$. However, since Forslund[11] and Singer[12] did not include gradient induced "skewing" of the electron distribution function in the dielectric, their results are somewhat different than ours. Specifically, for the zero net current condition, i.e., $\epsilon_E + \epsilon_p = -.71\ \epsilon_T$, they find that $\epsilon^*_{F.S.} = -1.85\ \epsilon_T$ while we find that a more accurate result is $\epsilon^* = -1.65\ \epsilon_T$.

More importantly, however, our results indicate the possibility of instability even when the internal electric field is zero (i.e. instability can occur when $\epsilon_E = \epsilon_p = 0$ at $\epsilon^* = 0.20\ \epsilon_T$). This interesting result only emerges when one includes the gradient induced skewing of the electrons in the dielectric and thus did not appear in previous analyses of this problem.

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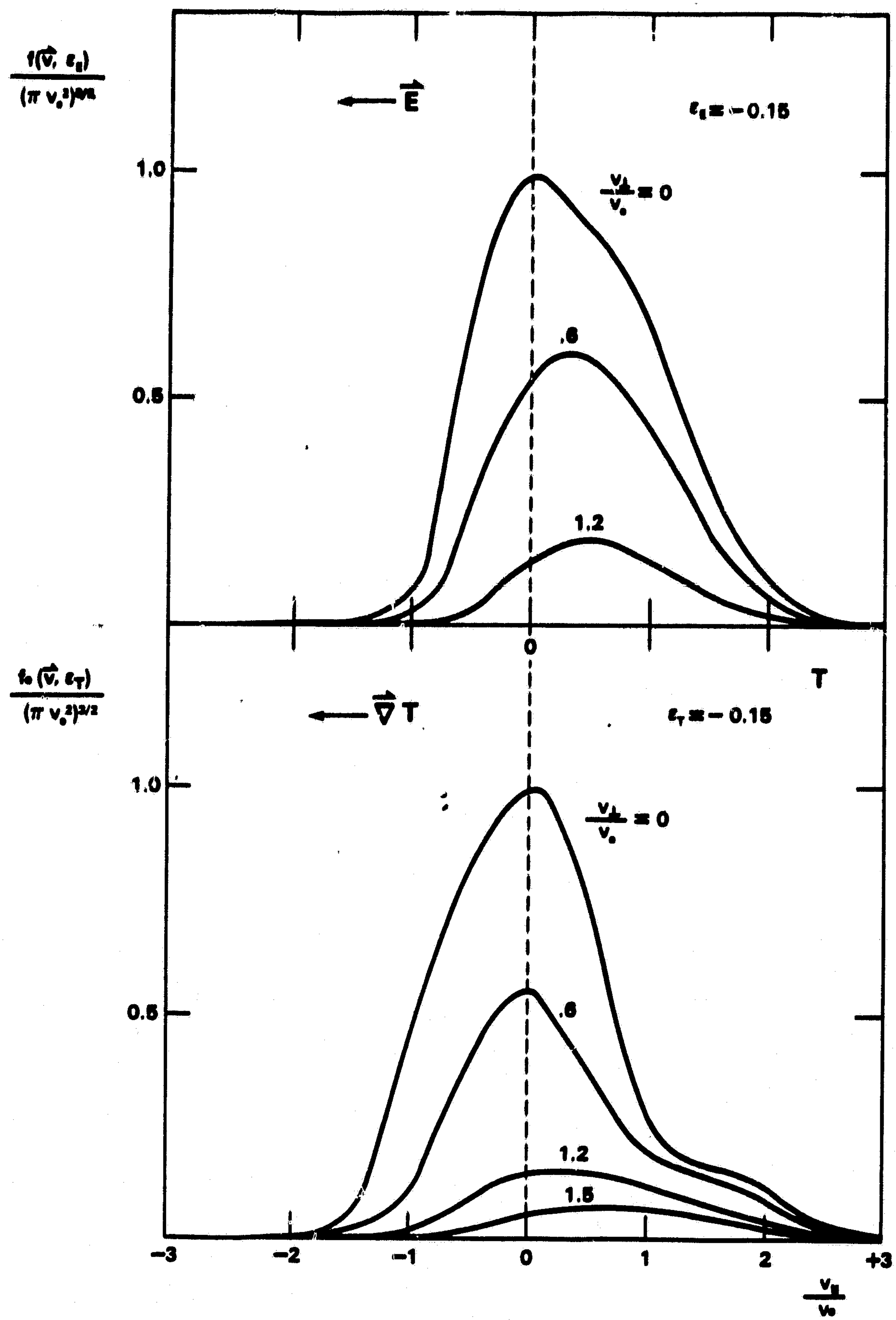
TABLE I. Electron distribution function and electron susceptibility coefficients (c.f., equations (1), (2), and (8)).

m	$a_{mE}=a_{mP}$	a_{mt}	$A_{mE}=A_{mP}$	A_{mT}	$B_{mE}=B_{mP}$	B_{mT}
0	0	0	2.6	0.20	-0.48	1.0
1	-7.84	4.75	-2.6	-0.20	-7.6	4.5
2	2.37	-3.62	8.4	-6.0	2.3	-3.5
3	-0.516	0.626	-2.5	3.8	-0.52	0.62
4	5.72×10^{-2}	-7.67×10^{-2}	0.52	-0.66	5.6×10^{-2}	-8.8×10^{-2}
5	-3.17×10^{-3}	3.42×10^{-3}	-6.0×10^{-2}	7.8×10^{-2}	-3.2×10^{-3}	3.4×10^{-3}
6	7.04×10^{-5}	-3.14×10^{-5}	3.2×10^{-3}	-3.4×10^{-3}	7.2×10^{-5}	-3.1×10^{-5}
7	-	-	-7.2×10^{-5}	3.1×10^{-5}	-	-

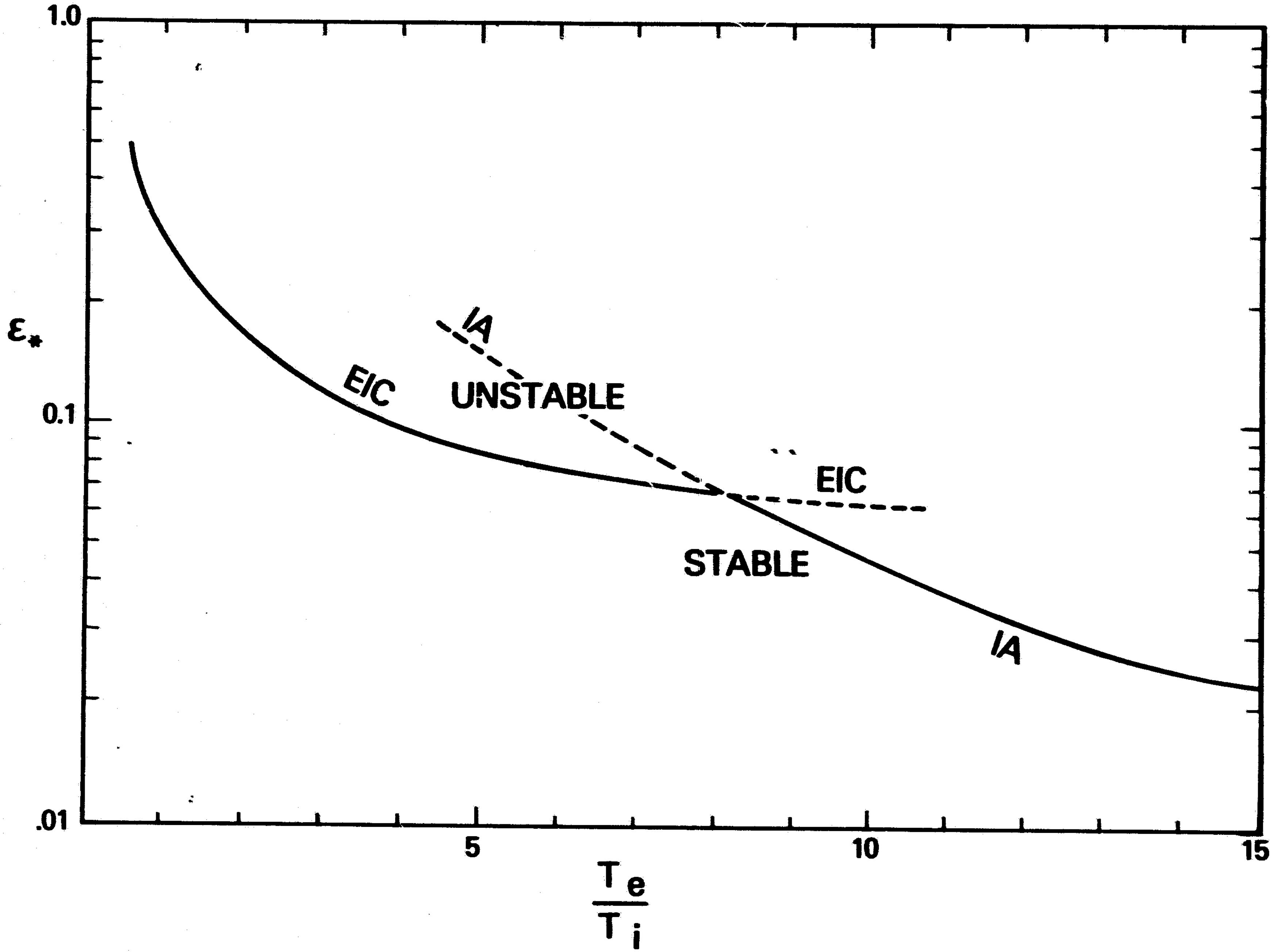
FIGURE CAPTIONS

Figure 1. Electron distribution function profiles as a function of the velocity component parallel to the magnetic field for several values of the perpendicular component. The upper curve assumes only an electric field is present, the lower that only a temperature gradient exists.

Figure 2. Instability threshold, ϵ_w, as a function of electron to ion temperature ratio.



$\frac{f(\vec{v}, \varepsilon_E)}{(\pi v_o^2)^{3/2}}$
$\leftarrow \vec{E}$
$\varepsilon_E = -0.15$
1.0
0.5
$\frac{v_\perp}{v_o} = 0$
.6
1.2
$\frac{f_o(\vec{v}, \varepsilon_T)}{(\pi v_o^2)^{3/2}}$
$\leftarrow \vec{\nabla} T$
$\varepsilon_T = -0.15$
T
1.0
0.5
$\frac{v_\perp}{v_o} = 0$
.6
1.2
1.5
−3
−2
−1
0
1
2
+3
$\frac{v_\parallel}{v_o}$


1.0
0.1
.01
ε*
5
10
15
Te/Ti
EIC
IA
UNSTABLE
EIC
STABLE
IA