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An Analytical Procedure for Computing Smooth Transitions Between Two Specified Cross Sections With Applications to Blended Wing-Body Configurations

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SUMMARY

An analytical procedure is described for designing smooth transition surfaces for blended wing-body configurations. Starting from two specified cross-section shapes, the procedure generates a gradual transition from one cross-section shape to the other as an analytic blend of the two shapes. The method utilizes a conformal mapping, with subsequent translation and scaling, to transform the specified end shapes to curves that can be combined more smoothly. A sample calculation is applied to a blended wing-body missile-type configuration with a top-mounted inlet.

INTRODUCTION

The construction of aerodynamic configurations requires a detailed numerical description in the form of position coordinates of the surface points. Often these coordinates are read from drawings of the configuration cross sections on graph paper. As a general rule, certain of the cross-section shapes are closely determined by the requirement for packaging internal equipment of a given shape, or by the presence of a jet inlet or exhaust, or by some similar condition. The intermediate cross-section shapes, between those that are thus determined, are usually obtained by making a smooth transition between the predetermined cross sections. When this transition is performed by hand lofting, it becomes tedious and time-consuming.

For some types of configurations or configuration components, however, a computer can be utilized to compute the transition shapes. Reference 1 describes a procedure for computing transition cross-section shapes for forebodies. The method of reference 1, however, treats only cross-section shapes that are single valued when expressed in polar coordinates; thus, it is not applicable to more general design problems.

The present procedure, however, treats the problem of computing cross-section shape transitions for a class of wing-body configurations. It is not intended to perform the same function as a general geometry program, such as that described in reference 2. Such a general geometry program utilizes point-to-point fairing, which is usually by means of low-order polynomials. In essence, it can be used to approximate the geometry of a given configuration, and to display rapidly the geometric effects of altering individual point locations. The present procedure, on the other hand, is intended for application primarily in the design process to generate rapidly a large section of a wing-body configuration in an analytic or semianalytic form. The method uses function averaging rather than point-to-point fairing. Thus, it produces transition surfaces between two given cross-section shapes that represent at each station a true analytic blend of the end shapes without the slope and curvature excursions that are inherent in polynomial fairing. The resulting surfaces, for smooth input cross sections, should have a degree of smoothness (continuous second derivatives) sufficient for accurate calculation of flow properties and radar cross-section parameters.

SYMBOLS

c	transformation parameter (eqs. (1) and (4))
E	exponent function defined by equation (3)
f	transitional transformed shape function at x, defined by equation (2)
f_1, f_2	single-valued functions of ζ determined by transforming the initial and final cross sections, respectively
n	parameter in Kármán-Trefftz calculation
s	complex variable $y + iz$
x, y, z	longitudinal, lateral, and vertical coordinates, respectively
α	exponent function in equation (3)
η, ζ	lateral and vertical coordinates, respectively, in transformed plane
λ	interior angle at corner point
σ	complex variable in transformed plane, $\eta + i\zeta$

Subscripts:

1	initial station
2	final station
i	smallest value
m	maximum value
st	scaled and translated value

ANALYSIS

The cross-section shapes at two x-stations are assumed to be given, and they are assumed to be symmetric about the vertical x,z-plane. Each of the symmetric halves may be represented as a single-valued function y of the vertical coordinate z (as shown in the example of fig. 1(a)), or it may be represented by a more complex shape, such as that of figure 1(b), which might describe a section of a blended wing-body configuration. More extreme cases, such as a section through a fuselage and wing with nacelle, could only be treated with much difficulty; therefore, they are not considered herein.

If both of the shapes can be represented as single-valued functions of z, then a smooth transition between them can be computed by the method of reference 1. However, if at least one of the shapes cannot be represented in such a manner (as that shown in fig. 1(b)), then the method of reference 1 fails and a new approach is required.

This approach involves transforming each of the two end shapes by a conformal mapping into single-valued functions of a similar nature so that they can be easily combined.

First, the general point (y,z) on each of the end shapes is written as a complex variable $s = y + iz$. For this purpose, a proper choice of axes is important. Where a corner exists, as at the wing tip in figure 1(b), the y-axis is taken through that point. For a shape like that of figure 1(a), no corner point exists, but there is a point representing the maximum width, and consequently, the y-axis is taken through that point.

Various choices of the complex mapping function could be made. The Kármán-Trefftz transformation (including the Joukowski transformation as a special case) is normally used because of its versatility and its familiarity as a mapping to open up and round out a complicated shape. (See ref. 3.) The transformation equation is

$$\frac{\sigma + c}{\sigma - c} = \left(\frac{s + nc}{s - nc}\right)^{1/n} \tag{1}$$

where n and c are parameters to be assigned in accordance with the particular shapes to be transformed. This transformation is described in detail in reference 4 (sec. 7.32). For a shape with a corner, like that of figure 1(b), the parameter c is determined by the location of the corner. (Since $\pm nc$ are singular points of the transformation, $s = nc$ or $s = -nc$ is a corner location.) The parameter n is determined by the interior angle λ at the corner in the physical plane in accordance with the relation:

$$n = 2 - \frac{\lambda}{\pi}$$

For a shape like that of figure 1(a), no singularity point exists on the boundary; therefore, the singularity of the transformation is taken inside the boundary. An appropriate location is midway between the point of maximum width and the center of curvature at that point. For smooth shapes the value of c does not have to be determined with great precision. The value of n for this cross section (fig. 1(a)) was assumed to be the same as that for the cross section of figure 1(b).

When the transformation is applied to both of the end shapes, the resulting curves can be represented as single-valued functions of the vertical coordinate ζ. Figure 2 shows, for example, the transformed curves for the cross-section shapes of figure 1. For this example, the similarity of the transformed curves is quite marked despite the distinct dissimilarities of the original shapes. In the event that essential differences in the form of the transformed shapes exist, then a second Kármán-Trefftz transformation could be applied. An example of applying successive mappings is shown in figure 3 where the initial shape is similar to that in figure 1(a) but with a discrete concave corner. An initial transformation rounds out the lower part of the curve, and a second transformation is applied to straighten out the corner.

In order for the transformed curves to be combined, they must be written as functions having the same direction on the ζ-axis. For this purpose, one of the curves can be translated and scaled as follows. If the domain of the initial function is $\zeta_{1i} < \zeta < \zeta_{1m}$ and the domain of the other end function is $\zeta_{2i} < \zeta < \zeta_{2m}$ then a new independent variable for the second curve is defined by

$$\zeta_{st} = (\zeta - \zeta_{2i})\frac{\zeta_{1m} - \zeta_{1i}}{\zeta_{2m} - \zeta_{2i}} + \zeta_{1i}$$

Denote the resulting functions of ζ as $f_1(\zeta)$ and $f_2(\zeta)$. Various methods can be used to combine these two functions. The simplest of these methods is to form a kind of arithmetic average of the functions. However, experience demonstrates that a smoother, more aerodynamically desirable surface is obtained when geometric averaging is used.

Thus, the combination curve at the x-station is given in terms of the end curves f_1 and f_2 by the formula

$$f(x,\zeta) = \left[f_1(\zeta)\right]^{E(x)}\left[f_2(\zeta)\right]^{1-E(x)} \qquad (x_1 < x < x_2) \qquad (2)$$

where

$$E(x) = \left(\frac{x_2 - x}{x_2 - x_1}\right)^{\alpha} \qquad (3)$$

and α is an arbitrary parameter which may be assigned as a constant or as a smooth function of x. This parameter provides a means of adjusting the rate at which the shape changes. If α is a small fraction, E is approximately one, except very near x_2. Consequently, the factor containing f_1 is dominant except near x_2. Thus, the shape changes slowly near x_1 from the initial shape (f_1). Conversely, when α is relatively large (>2) the transition is rapid at first with little of the change occurring near x_2.

Values of the transformation parameters n and c are required at each x-station. They are obtained from the values at the two end stations by a formula similar to equation (2).

Figure 4(a) shows an example of a transition curve computed from the shapes shown in figure 2, together with the transforms of the two end shapes, for comparison. For each such transition curve, the corresponding cross-section shape is obtained by transforming the coordinates back to the physical plane by the inverse transformation, which is

$$\frac{s + nc}{s - nc} = \left(\frac{\sigma + c}{\sigma - c}\right)^{n} \qquad (4)$$

For the transformed cross sections of figure 4(a), the corresponding physical plane shapes are shown in figure 4(b).

After the required number of transition cross sections have been computed, the results can be displayed as a unit, as illustrated in figure 5. If some tailoring of the calculated transition shape is required, modifications can be accomplished by varying the parameter α in equation (3). The effect of varying α on the overall surface is illustrated, for the sample transition, in figure 6. The effect on a specific cross section, taken at the middle axial station, is shown in figure 7.

Once the transition shapes have all been calculated, they can be independently scaled for the purpose of attaining a specified axial area distribution or for smooth matching with the rest of the configuration.

Whether the rest of the configuration has been designed by a similar method or whether it has been independently specified, the continuity at the joining cross section is assured by specifying that the final cross section of one section match the initial cross section of the next. However, the problem of obtaining a smooth joining involves specific mathematical requirements on the axial scaling and on α, which may be treated as a function of x. This problem is not difficult but is outside the scope of the present paper.

An example of a blended wing-body missile-type configuration with a top-mounted inlet is shown, in plan view, in figure 8. An afterbody determined by the end cross sections of figure 1 was fitted onto a forebody ahead of the inlet. The forebody was specified to have the initial shape of an ellipse, with a terminal shape like that of figure 1(a) but without the superimposed inlet.

It is relatively simple to superimpose a specified camber line on the configuration. Since the camber is given as a function of x, the value of the function at each x-station is simply added to the z-coordinates of the transition cross-section shapes at that station.

CONCLUDING REMARKS

An analytical procedure has been described for generating smooth transition surfaces for blended wing-body configurations. Starting from two specified cross-section shapes, the procedure generates a gradual transition from one cross-section shape to the other as an analytic blend of the two shapes.

The method used a conformal mapping, with subsequent translation and scaling, to transform the end shapes to curves that could be easily combined. The example described was a blended wing-body missile-type configuration with a top-mounted inlet.

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April 28, 1982

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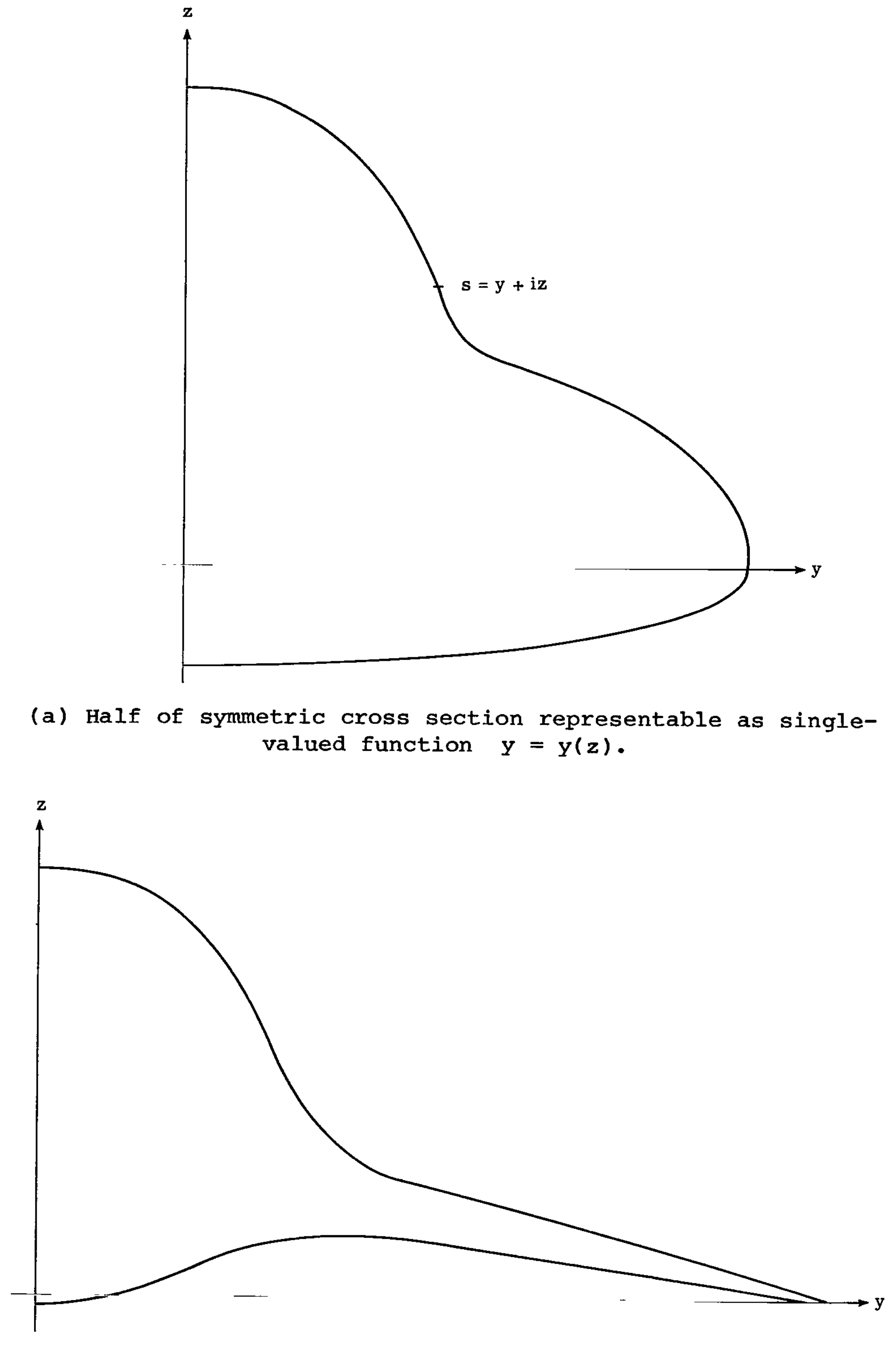
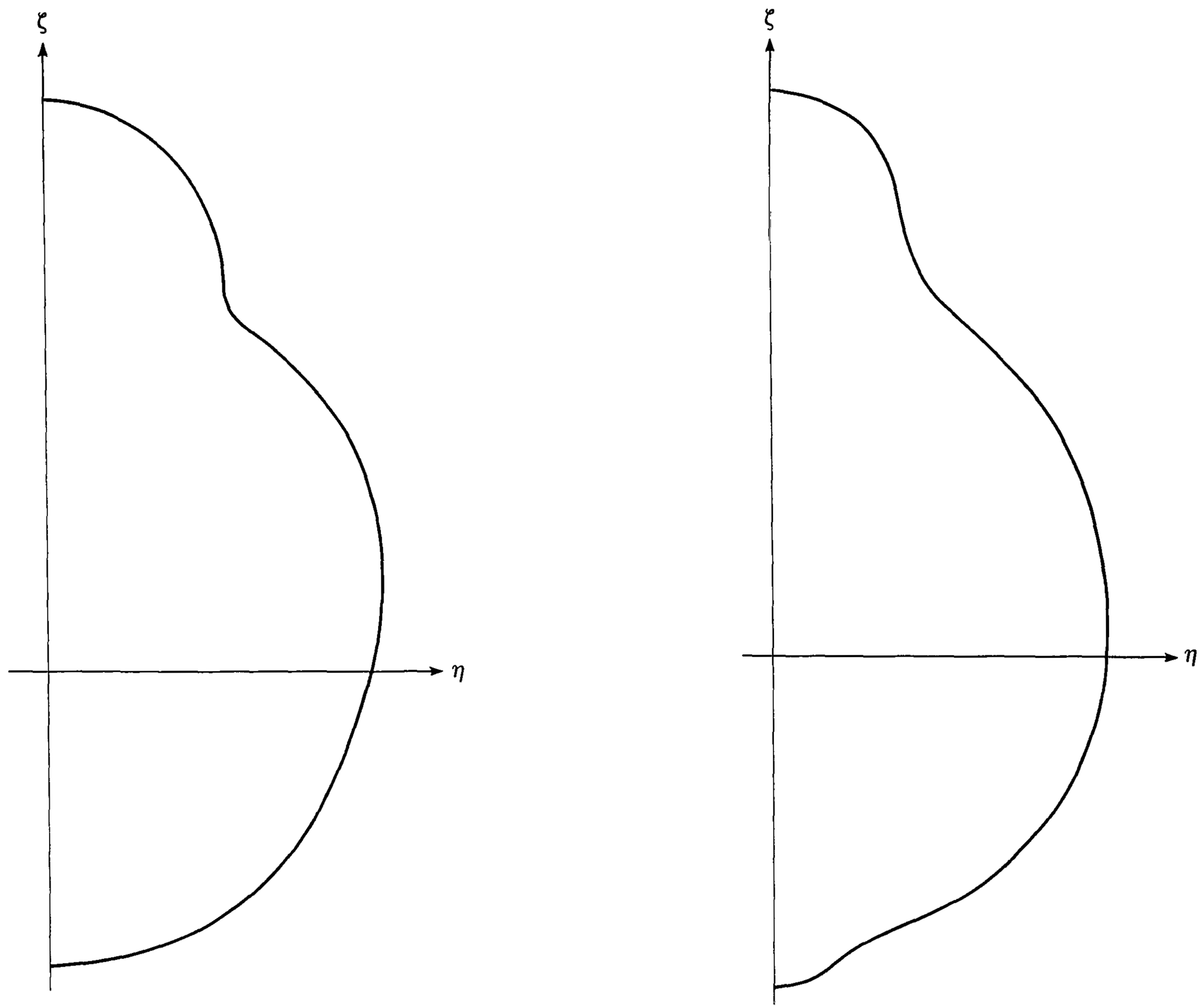


(a) Half of symmetric cross section representable as single-valued function $y = y(z)$.

(b) Cross section for which y is not representable as single-valued function of z.

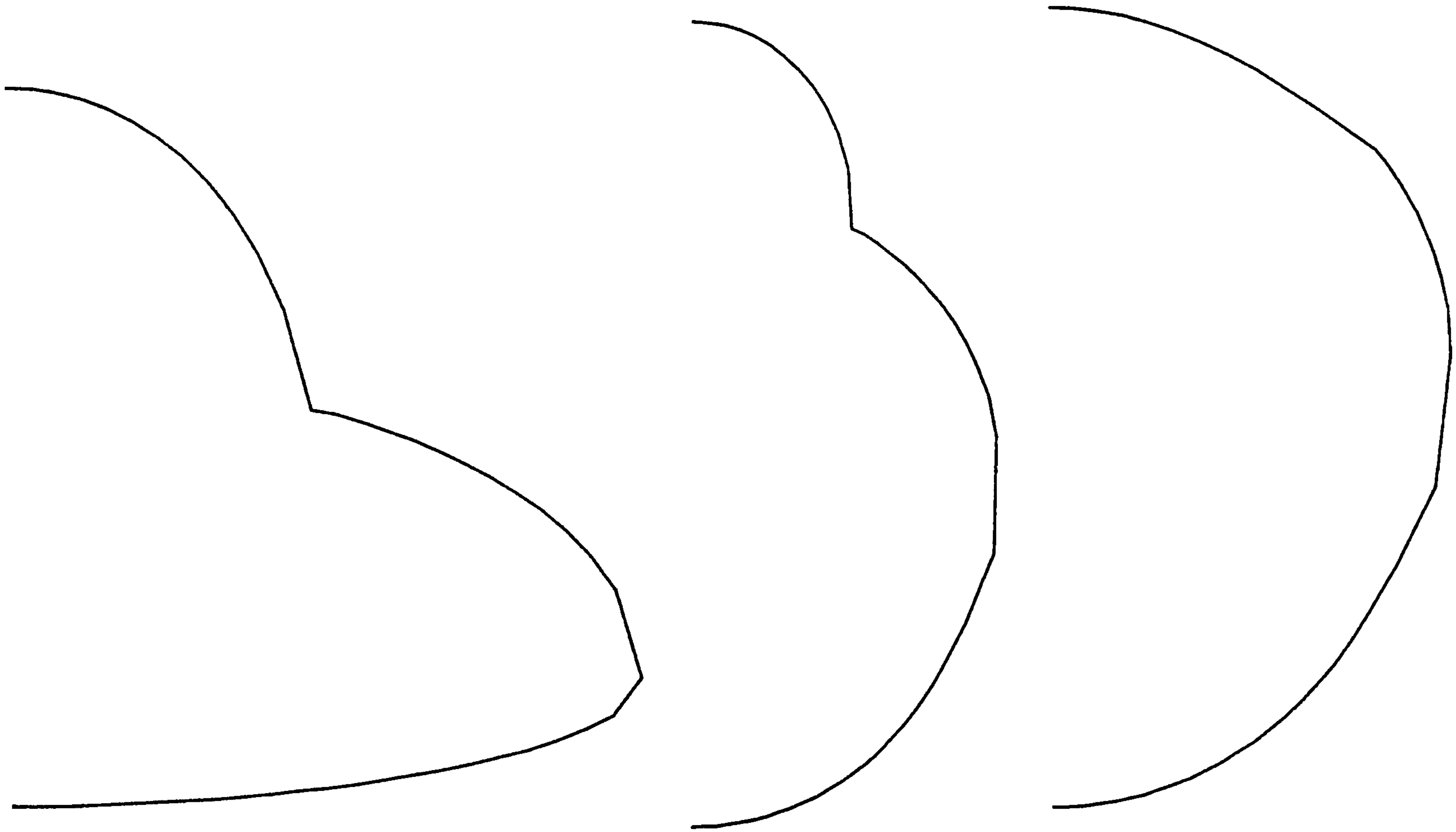
Figure 1.- Configuration cross-section examples.



(a) Transform of figure 1(a).

(b) Transform of figure 1(b).

Figure 2.- Transforms of cross-section curves of figure 1.

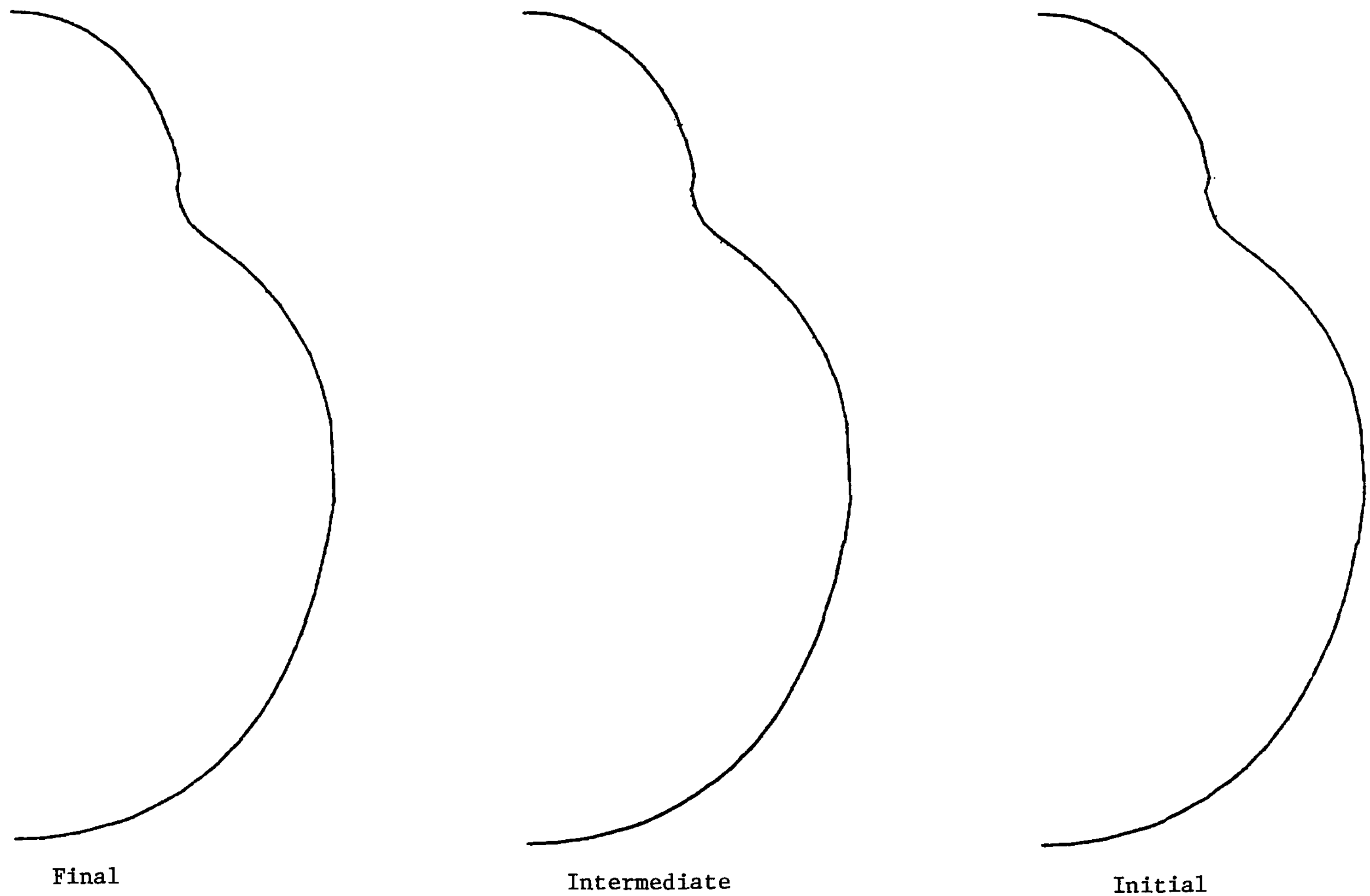


(a) Cross section in physical plane.

(b) Result of initial transformation.

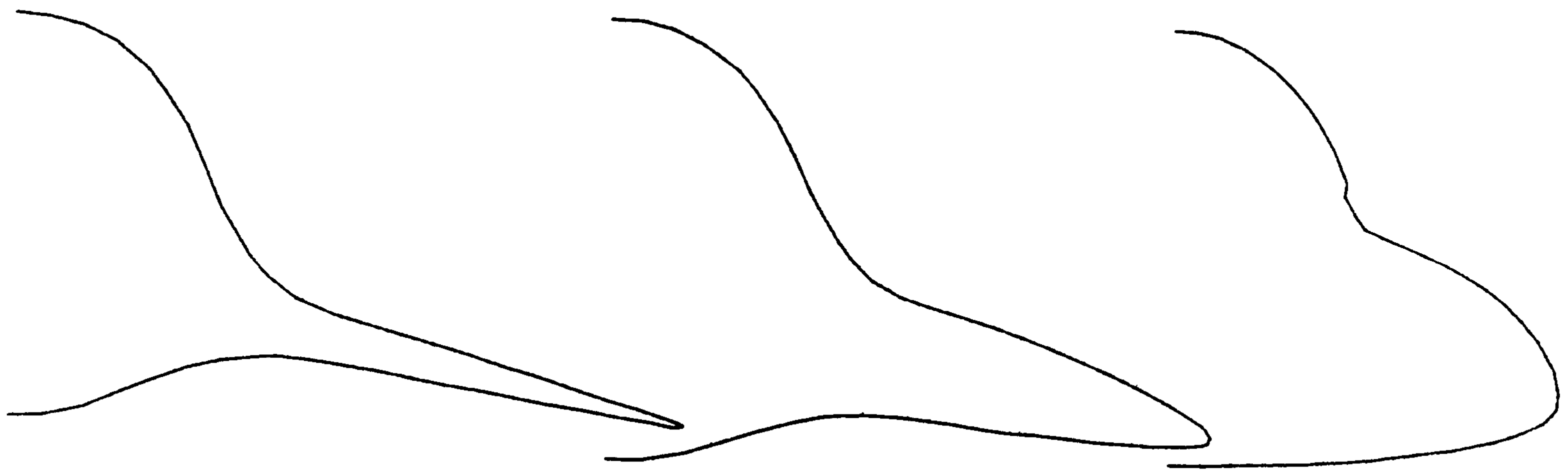
(c) Result of second transformation.

Figure 3.- Effect of applying successive Kármán-Trefftz mappings.



(a) Transformed cross sections.

Figure 4.- Transformed cross sections and corresponding shapes in physical plane.



(b) Physical plane shapes corresponding to transforms of figure 4(a).

Figure 4.- Concluded.

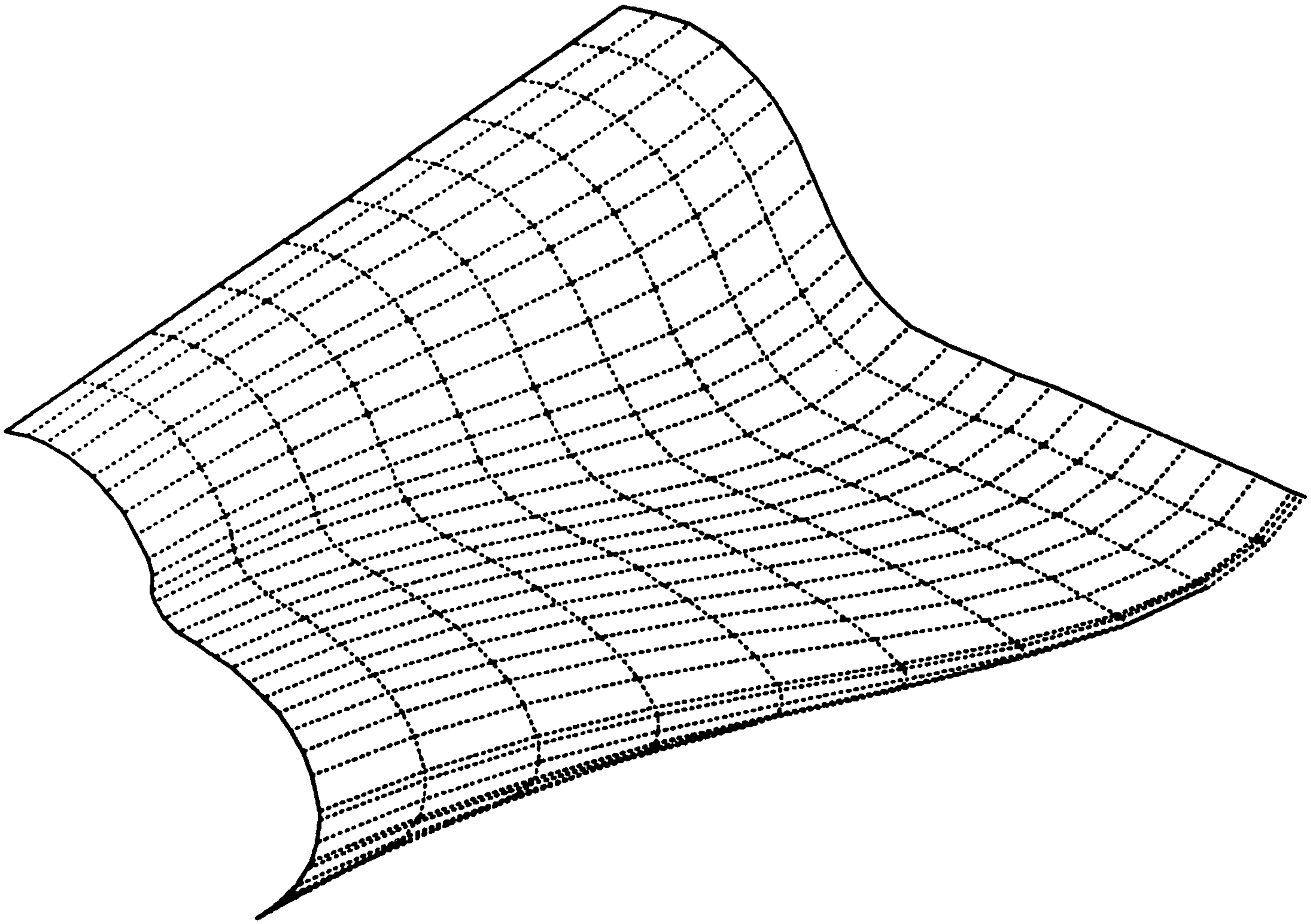
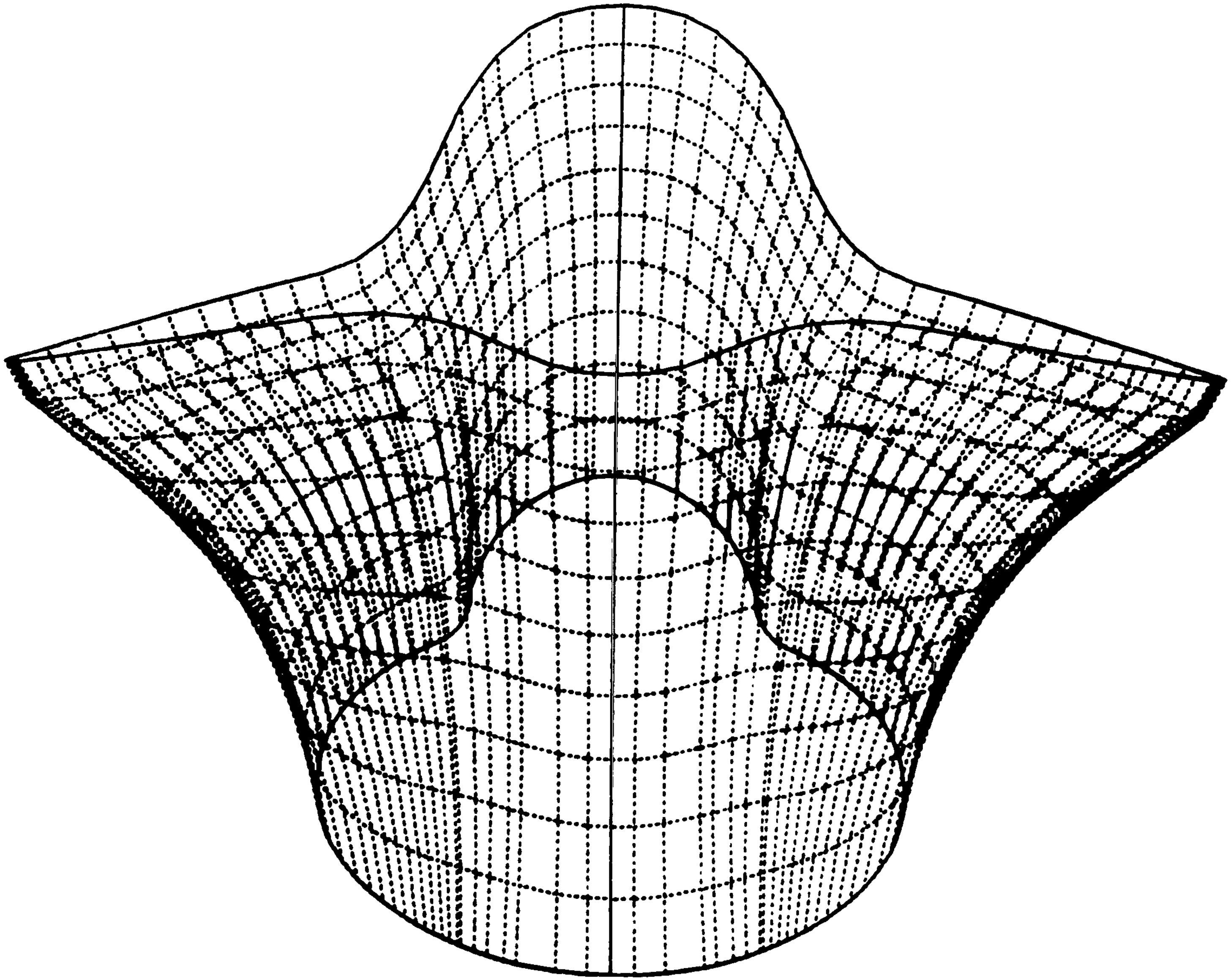
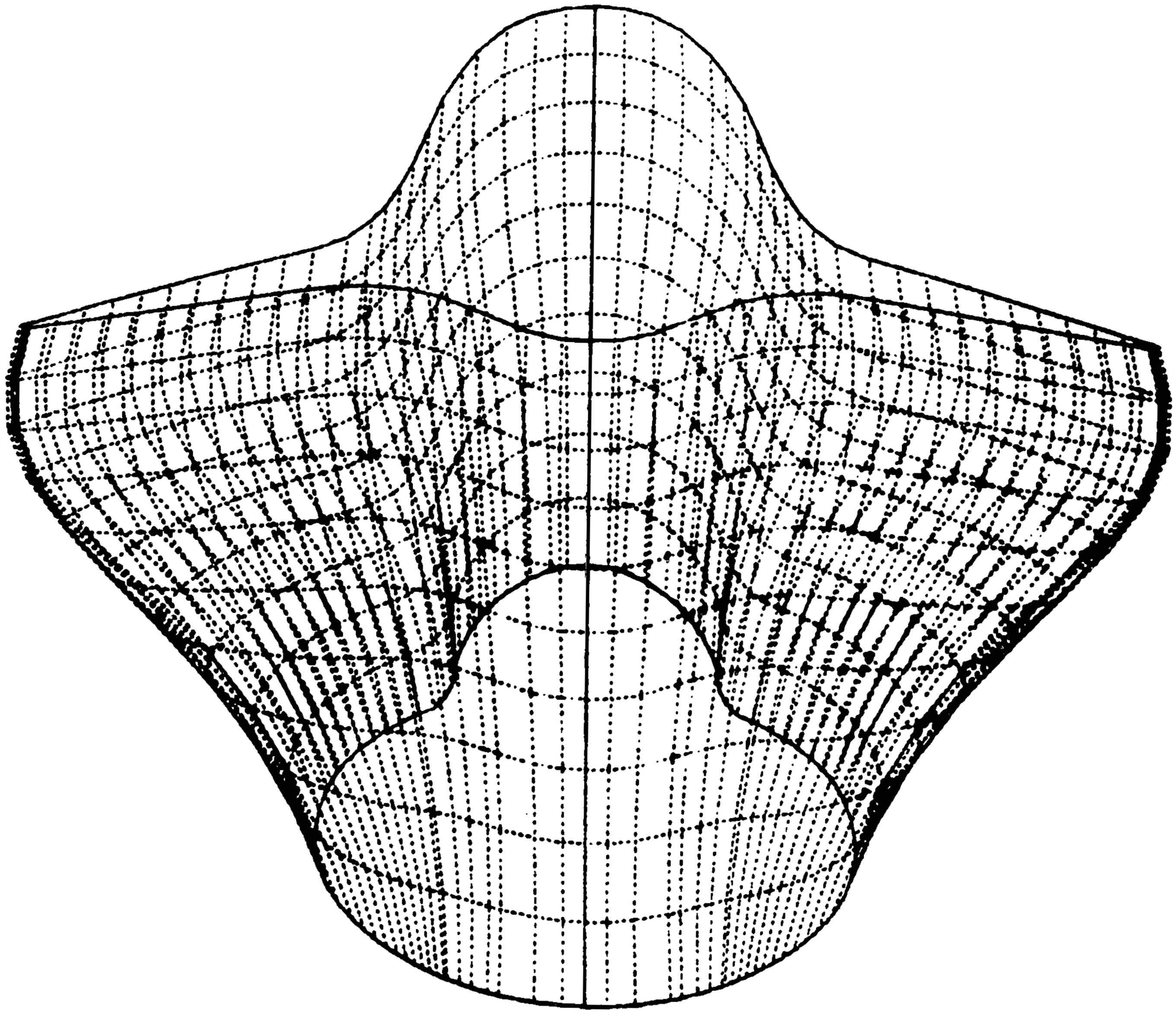


Figure 5.- Perspective view of surface generated as a transition between the cross-section shapes of figure 1.



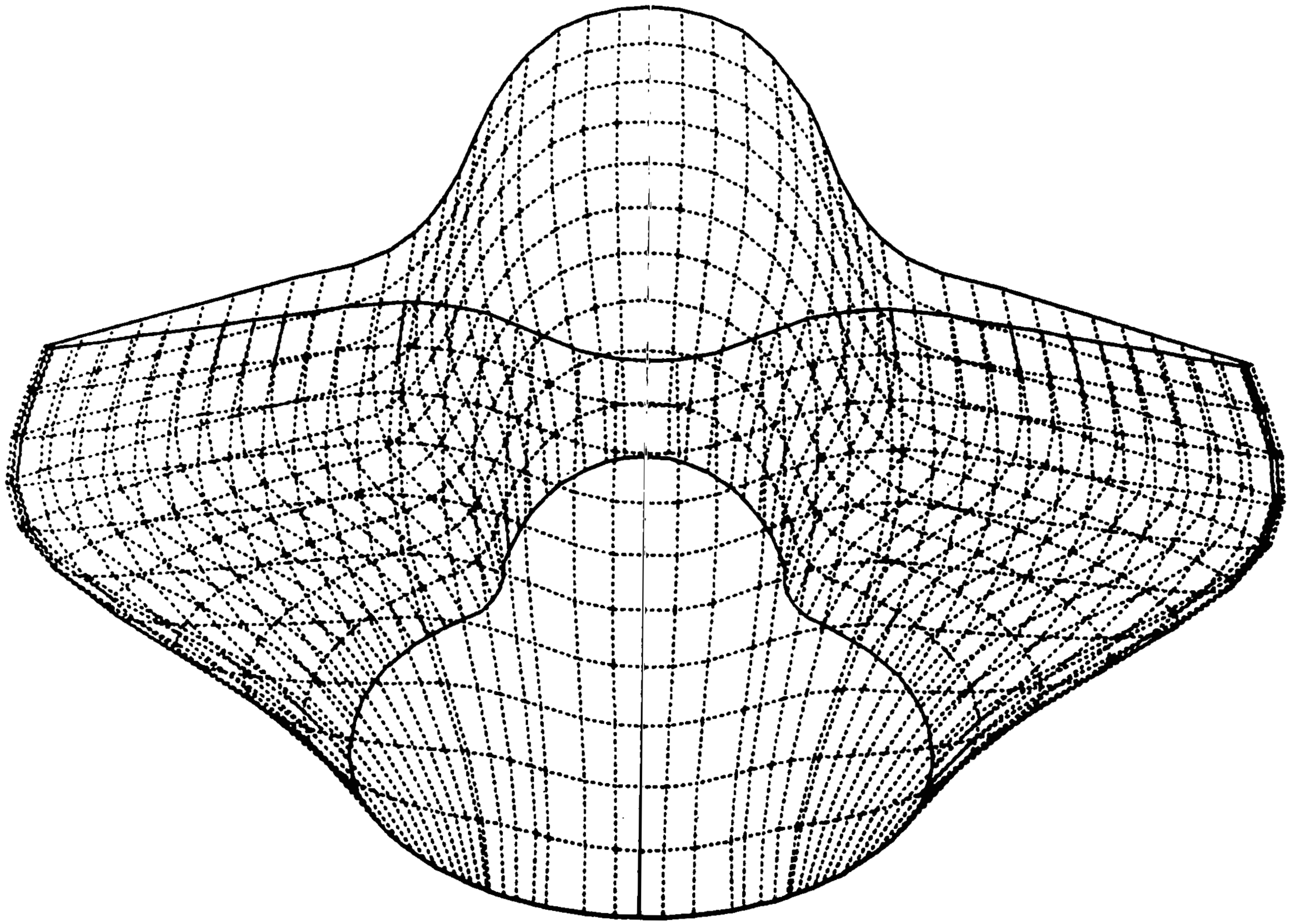
(a) $\alpha = 1.5$.

Figure 6.- Influence on configuration shape of varying parameter α.



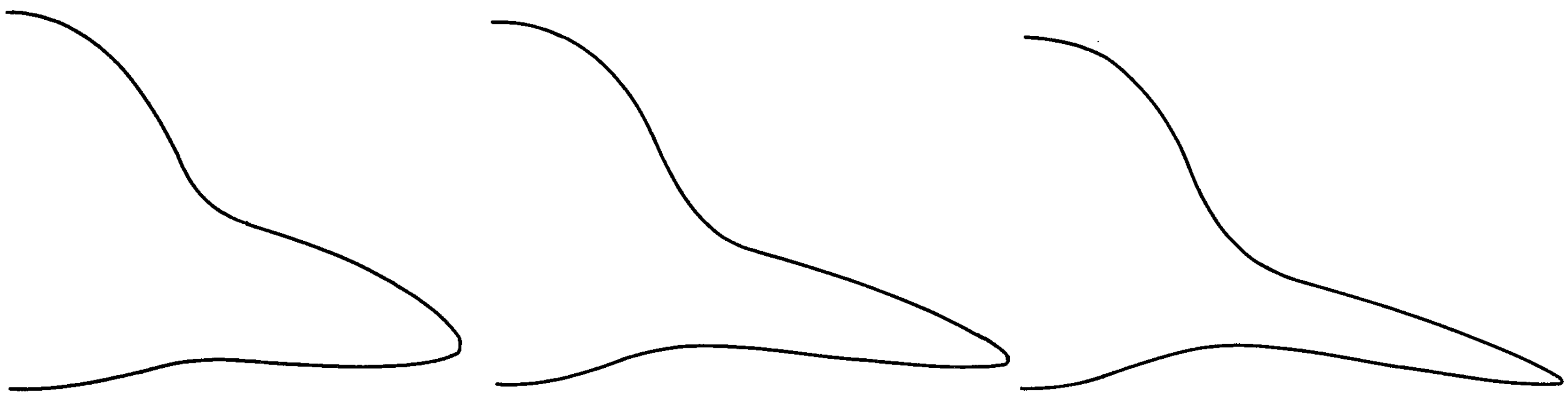
(b) $\alpha = 2.5$.

Figure 6.- Continued.



(c) $\alpha = 3.5$.

Figure 6.- Concluded.



(a) $\alpha = 1.0$.

(b) $\alpha = 2.0$.

(c) $\alpha = 3.0$.

Figure 7.- Effect of varying α on midstation cross-section shape.

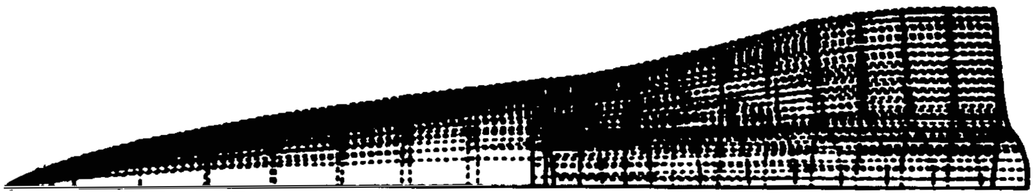


Figure 8.- Transition section smoothly fitted on to forebody.

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