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# USE OF OPTIMIZATION TO PREDICT THE EFFECT OF SELECTED PARAMETERS ON COMMUTER AIRCRAFT PERFORMANCE 

by
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This Research Was Supported By
NASA Langley Research Center Hampton, Virginia 23665

Grant NAG 1-202


# USE OF OPTIMIZATION TO PREDICT THE EFFECT OF SELECTED PARAMETERS ON COMMUTER AIRCRAFT PERFORMANCE <br> Final Report for NASA Research Grant NAG-1-202 

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#### Abstract

An optimizing computer program, developed as part of this study, determined the turboprop aircraft with lowest direct operating cost for various sets of cruise speed and field length constraints. External variables included wing area, wing aspect ratio and engine sea level static horsepower; tail sizes, climb speed and cruise altitude were varied within the function evaluation program. Direct operating cost was minimized for a 150 n . mi typical mission. Generally, DOC increased with increasing speed and decreasing field length but not by a large amount. Ride roughness, however, increased considerably as speed became higher and field length became shorter.


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## I. Introduction

The resurgence of interest in small, propeller-driven aircraft has sparked renewed analysis of the aerodynamics, structures and propulsion systems of such planes. Along with advanced technology research, which is the bent of much of the recent concern, there remains a need for the answer to a, perhaps, more basic question-that is, for what mission should this airplane be designed? The "mission" includes not just stage length (which is determined by the actual leg distances flown by commuter airlines) but also the speed at which to climb and cruise and the field length from which the aircraft must takeoff and land.

This study, rather than seeking to prescribe a particular design or mission, discovers the relationships between field length and cruise speed and aircraft direct operating cost. To do this, a gradient optimizing computer program was developed to minimize direct operating cost (DOC) as a function of airplane geometry. In this way, one can compare the best airplane operating under one set of constraints with the best operating under another. Best, in this case, means having the minimum DOC.

To compare different airplanes, one can make use of relatively simple techniques for some parameter estimation. For example, a complete stability and control analysis for tall size determination is superfluous for preliminary design when statistical correliations of tail sizes with wing and fuselage characteristics exist for similar airplanes. Thus several such statistical correlations methods appear in the program. However, one must also use more sophisticated procedures when a high degree of accuracy is required or when the particular calculation may have a major influence on the performance index. The program, therefore, has extensive and detailed routines for drag, climb, range and other critical values.

For this study a constant 30 -passenger fuselage and "rubberized" engines based on the General Electric CT-7 were used as a baseline. All aircraft had to have a 600 nautical mile maximum range and were designed to FAR part 25 structural integrity and climb gradient regulations. Direct operating cost was minimized for a typical design mission of 150 nauticai miles. For purposes of $C_{L_{\text {max }}}$ calculation, all aircraft had double-slotted flaps but with no Fowler action.

## II. Procedure

## A. The Optimizer

The optimizer minimizes direct operating cost as a function of wing area, aspect ratio and engine sea-level static horsepower rating through use of a variable metric algorithm which is, in fact, a quast-Newton's method. A true Newton's method utilizes the following strategy for size and direction of step:

$$
\dot{x}_{j+1}=\stackrel{\rightharpoonup}{x}_{j}-H_{j}^{-1} \stackrel{\rightharpoonup}{g}_{j}
$$

where $\vec{x}$ represents the vector of variables, $H_{j}$ is the Hessian (matrix of second derivatives) at step $j$, and $\vec{g}_{j}$ is the gradient vector at step $j$. In the absence of second derivative information, a numerical approximation of the Hessian using known values of the first deriyatives provides an adequate substitute. The variable metric method follows exactly this procedure.

Of course, for such a complicated function as the one in this study (the "function" is a thirty page FORTRAN program), even first derivatives do not exist in closed form. Thus, the program must calculate a gradient estimate using a forward difference approximation. The differencing step size is constrained to be rather large (one percent of the variable value) since noise in the function evaluation leads to incorrect gradients for small steps.

The variable metric method solves unconstrained problems only. Thus, in order to account for the inequality constraints which must hold in order for the aircraft to meet such mission requirements as maximum takeoff distance, minimum engine-out climb gradient, etc., the program uses what is termed the "penalty function" or "soft constraint" approach. In a mathematical sense, this method changes the problem to an unconstrained one by including the constraints in the goal function. The goal function becomes,

$$
\text { GOAL }=D O C+K \mid \text { constraint value - constraint value required }
$$

Where DOC $=$ direct operating cost
$K=$ penalty coefficient
$= \begin{cases}0 & \text { if constraint is met } \\ \text { large } & \text { if constraint is not met. }\end{cases}$
A penalty is added to the goal function for each of the five inequality constraints:

- takeoff distance
- landing distance
- available cruise power
- second segment climb gradient
- enroute slimb gradient


## B. The Function Evaluation

The function evaluation program, which comprises the bulk of the calculations involved in the optimization, acts as a mathematical aircraft model. Thís routiñe determiñes, for à prescribed wing area, sea level static horsepower rating, and wing aspect ratio, the complete geometry, performance, and operating cost of the resulting airplane. For simplicity, it employs preliminary design methodology for estimating such parameters as zero-lift equivalent drag area, tail sizes, $C_{L_{\text {max }}}$, and airplane efficiency factor. The direct operating cost calculation is based on the 1967 ATA DOC method with corrections for inflation and commuter operation. The following outline briefly describes the function evaluation scheme.

## 1. Airplane Geometry and Drag Parameters

In order to compute the airplane geometry (wing span, wing mean aerodynamic chord, vertical and horizontal tail areas) the program assumes as constants:

```
wing average thickness ratio . }1
tail average thickness ratio .1
wing taper ratio .4
horizontal tail aspect ratio 4.0
vertical tail aspect ratio 1.8
wing sweep 0
tail surface sweep angle 20
```

To avoid a complex iteration involving weight and balance, the horizontal and vertical tail lengths are estimated as 32 ft . and 30 ft ., respectively, and a center of gravity range of $25 \%$ of the wing mac is allowed. Using these estimates, the program calculates tail areas as a function of fuselage and wing sizes according to ref. 1.

Once all surface areas are known, the program computes the zero-lift equivalent drag area, $f$. The formula for $f$ of a component has the form

$$
f_{i}=c_{f_{i}} k_{i} S_{w e t}^{i}
$$

where $C_{f}$. friction coefficient; function of Reynolds number
$K=$ form factor; function of fineness ratio or thickness ratio
Swet = component wetted area
1 refers to the ith component such as wing, fuselage, nacelle, etc.

A summation of all component drag areas, plus a $6 \%$ addition for miscellaneous components, gives the total airplane equivalent parasite drag area:

$$
f=\sum_{i} f_{i} / .94
$$

The zero-lift or parasite drag coefficient, $C_{D_{p}}$, is just: $C_{D_{p}}=f / S W$, $\mathrm{S}_{\mathrm{W}}=$ wing reference area.

The program computes airplane efficiency factor, e, from:

$$
e=\frac{1}{\pi R\left(\frac{1}{\pi R u s}+.43 C_{D_{P}}\right)}
$$

where $u=$ induced drag factor due to planform; function of $\mathbb{R}$, taper ratio, sweep.
$s=$ induced drag factor due to fuselage interferences function of wing span/fuselage diameter.

Inclusion of $C_{D p}$ in this formula accounts for the increase in profile drag with angle of attack.
2. Range and Maximum Takeoff Weight

For any combination of wing area, sea level horsepower, and wing aspect ratio (other possible variables assumed constant) there exists a takeoff weight necessary to travel a given distance at a given speed. This routine determines that takeoff weight required for the airplane described by those three variables to cover a maximum range of 600 Nmi . at a prescribed cruising speed. The takeoff weight depends rather heavily on two other variables - cruise altitude and climb speed. Thus, in order to include these as variables, the program performs a two dimensional grid search on altitude and climb speed and saves the combination of the two which uses the least fuel to complete the 600 Nmi . mission.

Determining the maximum takeoff weight is an iterative procedure completed through the use of a one dimensional minimization routine. The minimizer employs a "linear search with parabolic inverse interpolation" with the goal function defined as the square of the difference between the actual range and the desired range.

The range calculation itself has four major parts:
(a) calculation of empty weight
(b) climb
(c) descent
(d) cruise

The weight is calculated using a statistical method based on data from large and small commercial aircraft, (ref. 1).

The time, fuel, and distance to climb are calculated according to:

$$
\text { time to climb }=\int_{h_{\min }}^{h_{\max }} \frac{d h}{R / c}
$$

$$
\begin{aligned}
& \text { fuel to climb }=\int_{h_{\text {min }}}^{h_{\text {max }}} \frac{\text { SHP } * \text { SFC }}{3600 * R / C} d h \\
& \text { distance to } \mathrm{c} 1 \mathrm{l} \boldsymbol{\operatorname { m i g }} \mathrm{E}=\int_{h_{\min }}^{h_{\max }} \frac{V}{R / C} d h
\end{aligned}
$$

where $R / C$ rate of climb in $\mathrm{ft} / \mathrm{sec}$
$h=a l t i t u d e$ in $f t$
SHP = shaft power in horsepower
SFC = specific fuel consumption in 1b/SHP-hr
$V$ = true airpseed in $\mathrm{ft} / \mathrm{s}$

The climb routine numerically evaluates these integrals making the following assumptions:

- climb at constant equivalent airspeed
- climb at maximum continuous power
- SFC constant at maximum continuous power

The numerical integration uses a forward Euler technique and an altitude step size of 200 ft .

The descent method assumes:

- descent at constant equivalent airspeed
- descent at constant rate of descent
- idle (minimum) power at $10 \%$ of maximum power

The aircraft rate of descent corresponds to a 300 feet per minute cabin pressure descent where the cabin has a 6000 foot pressure altitude in cruise. The program computes fuel and distance to descend using the following integrals:

$$
\text { fuel to descend }=\int_{h_{\min }}^{h_{\max }} \frac{S H P * S F C}{3600 * R / D} d h
$$

distance to descend $=\int_{h \min }^{h \max } \frac{v}{R / D} d h$
where $h=$ altitude
SFC = specific fuel consumption
R/D = rate of descent, ft/sec
$V$ : descent true airspeed, ft/sec
SHP - shaft power
Since the airplane descends at constant equivalent airspeed and constant rate of descent, the distance integral becoines:

$$
\text { distance to descend }=\frac{V_{E}}{R / \sigma} \int_{h_{\min }}^{h_{\max }}\left(1-6.8634 \times 10^{-6} h\right)^{-2.1324} \mathrm{dh}
$$

where

$$
\begin{aligned}
V_{E} & =\text { equivalent airsyeed for descent } \\
V & =V_{E}\left(1-6.8634 \times 10^{-6} \mathrm{~h}\right)^{-2.1324}
\end{aligned}
$$

Integrating gives:

$$
\begin{aligned}
\text { distance to descend }= & \frac{V_{E}}{R / D}\left(\frac{1}{6.8634 * 1.1324 * 10^{-6}}\right) \\
& \times\left[\left(1-6.8634 \times 10^{-6} \mathrm{~h}\right)^{-1.1324}\right]_{\text {mmin }}^{\text {max }}
\end{aligned}
$$

Letting $h \min =0$.

$$
\begin{aligned}
\text { distance to descend }= & \frac{V_{E}}{R / D}\left(\frac{1}{6.8634 * 1.1324 * 10^{-6}}\right) \\
& \times\left[\left(1-6.8634 * 10^{-6} \mathrm{hmax}\right)^{-1.1324}-1\right]
\end{aligned}
$$

Fuel to descend is numerically calculated using an explicit Euler integration and an altitude step size of 500 ft . The fuel integral begins at the end of the descent and integrates backwards until the aircraft reaches the cruising altitude. The weight at the bottom of descent is the zero fuel weight plus additional fuel weight for an appropriate reserve mission ( $100 \mathrm{n}, \mathrm{mi}$, at best specific range plus 45 minutes at best endurance).

The distance covered in the cruising portion of the mission depends on the weights at the end of climb and at the top of descent. For propellerdriven aircraft,

$$
R=\int_{i 11}^{W 1} 325 \frac{n}{S F C} \frac{d W}{D}
$$

$$
\text { where } \begin{aligned}
n & =\text { propeller efficiency in cruise } \\
D & =\text { drag } \\
\text { Wi } & =\text { weight at beginning of cruise } \\
\text { Wf } & =\text { weight at end of cruise } \\
\text { SFC } & =\text { specific fuel consumption } \\
R & =\text { range, } n \text {. miles }
\end{aligned}
$$

Letting SFC be approximately constant and equal to the average SFC, during cruise, and noting that

$$
D=C_{D_{p}} q S+\frac{w^{2}}{q \pi b^{2} e},
$$

and letting

$$
\begin{aligned}
& A 1=C_{D_{p}} q S \\
& A 2=\frac{1}{q \pi b^{2} e},
\end{aligned}
$$

the integral becomes,

$$
R=325 \frac{n}{S F C} \int_{W f}^{W 1} \frac{d W}{A 1+A 2 W^{2}}
$$

for constant cruise speed and cruise altitude. Integrating gives:

$$
R=325 \frac{\eta}{S F C} \frac{1}{\sqrt{A 1 A 2}}\left[\tan ^{-1} \frac{W}{\sqrt{A 1 / A 2}}\right]_{W f}^{W 1}
$$

or

$$
R=325 \frac{n}{S F C} b \sqrt{\frac{\pi e}{C_{D_{p}} S}}\left[\tan ^{-1} \frac{W}{q b \sqrt{C_{D_{p}} \pi e S}}\right]_{W f}^{W i}
$$

This formula holds only for the case of constant dynamic pressure, $q$. Since the cormuter cruises at constant speed and altitude, it satisfies the condition of invariañt $\bar{q}$.

## 3. Evaluation of Constraint Parameters

Five acceptability criteria constrain the aircraft design.
(a) Maximum Cruise Thrust

To fly at the prescribed cruising speed, the maximum thrust produced by the engines must equal or exceed the cruise drag. Maximum thrust depends on maximum cruise power according to the relation

$$
T H=550 \mathrm{n} \frac{\mathrm{SHP}}{\mathrm{~V}}
$$

where $\eta=$ propeller efficiency
$V=$ true airspeed, $\mathrm{ft} / \mathrm{sec}$
$\mathrm{TH}=$ thrust, lb .
Maximum cruise shaft horsepower is determined as a function of airspeed, cruise altitude, and static sea level power rating. Power caiculations are based on the General Electric CT-7 turboprop engine.
(b) Takeoff Distance

Allowed takeoff distances range from 3500 feet to 4500 feet. FAR takeoff field lengths depend on the parameter:

$$
\frac{\operatorname{Tow}^{2}}{\sigma_{L_{\text {max }}} S_{w} T H}
$$

where $\sigma=\sqrt{0 / \rho_{0}}$
$S_{w}=$ reference wing area, $f t^{2}$
Takeoff distance is calculated for a hot day (ISA $+30.8^{\circ} \mathrm{F}$ ) sea level.
(c) Landing Distance

The allowed FAR landing distances range from 3500 feet to 4500 feet, and they depend on the square of the airplane's stalling speed. Since commuter airplanes do not usually have the ability to jettison fuel, the studied aircraft must land at cheir takeoff weights.
(d) Second Segment Climb Gradient

To comply with the Federal Air Regulation, part 25, a twinengined airplane must have a second segment climb gradient of 2.4\%. The gradient ts somputed for hot day conditions (ISA $+30.8^{\circ} \mathrm{F}$ ) at takeoff power and with one engine inoperative. The drag in this configuration includes that due to a feathered propeller, due to excess rudder deflection as a consequence of asymmetric thrust, and due to a 25 degree takeoff flap deflection.
(e) Enroute Climb Gradient

According to FAR part 25, a twin-engined a irplane must have a one engine-out enroute climb gradient of $1.1 \%$. Since speed for best climb gradient for aircraft of this type is less than the minimum allowable speed, enroute climb gradient is computed at 1.3 times the stalling speed in the clean configuration. The obstacle clearance height used is 11,000 feet.

## 4. Typical Mission

To optimize the commuter design with respect to operating cost, one must compute DOC (direct operating cost) for a typical shorthaul mission. Using a characteristic stage length of 150 N mi ., the program finds the corresponding block fuel and block time for the airplane designed to meet the restrictions outlined above. Such values as takeoff weight necessary to meet the range are calculated according to the method described in the "Range and Maximum Takeoff Weight" section.

The direct operating cost routine assumes that cormuter pilot pay rates are about one-third that of trunk carrier pllots. It also uses the following cost estimates:

Labor Rate<br>Airframe First Cost<br>Engine First Cost

Fuel Cost
$0 i 1$ Cost
\$12/hr
$\$ 200 / 1 \mathrm{~b}$ of airframe
Taken from Ref. 4; inflated 25\%
\$1.50/gallon
\$10/1b

The cost calculation proceeds as suggested in Ref. 2. Appendix I contains a complete listing of the program.
C. Ride Roughness

Though ride-roughness was not considered in the optimization portion of the study, a relative ride-roughness parameter was computed for each optimum airplane. This parameter, taken from the FAR gust-response/ structures regulations is given by

$$
\Delta n=k \frac{U_{d e} V_{e}}{498(W / S)}
$$

where

$$
\begin{aligned}
U_{d_{e}} & =\text { equivalent gust velocity, } \mathrm{ft} / \mathrm{s} \\
V_{\mathrm{e}} & =\text { equivalent speed, knots } \\
\mathrm{H} / \mathrm{S} & =\text { wing loading, } 1 \mathrm{~b} / \mathrm{ft}^{2} \\
\mathrm{a} & =\mathrm{d} C_{\mathrm{L}} / \mathrm{d} \alpha
\end{aligned}
$$

$$
\begin{aligned}
& K=\frac{.88 \mu}{5.3+\mu} \\
& \mu=\frac{2(W / S)}{p C a g}
\end{aligned}
$$

Reference 7 provides further discussion of this parameter.

## III. Results

The results of the optimization program show that the airplane with the lowest direct operating cost flies at 290 knot TAS with an allowed field length greater than or equal to 4,060 feet, Figure 1. For field lengths less than 3,650 feet, the 250 knot airplane fares best in terms of DOC as the large wings required for short landing distances cause excessive drag at the higher speeds. At greater than 3,650 foot field lengths, 290 knots is the best speed. The best 330 knot airplane, however, with a landing distance of 4,275 feet has only one percent worse direct operating cost than the best airplane overall. Direct operating cost as a function of field length and cruise speed is presented in Figure 1.

The optimization, aside from determining the effect of cruise speed and field length on DOC; produced the following crucial results:

## A. Critical Field Lengths

Although, generally, direct operating cost decreases with increasing field length (for a given speed), for each speed there exists a critical field length beyond which there is no further improvement in DOC; the field length constraint becomes non-active. Two factors contribute to this phenomenon. First, though the wing area can decrease with increased takeoff or landing distance, the aircraft must still maintain a span adequate to meet climb gradient standards. The resulting increase in aspect ratio increases the weight enough to counteract the beneficial effects of the lower wing area. Secondly, a smaller wing area forces the aircraft to an inefficient $C_{L}$ far from that for best $L / D$ (which indicates best specific range for propeller-driven aircraft). A drop in cruise altitude improves the $C_{L}$ but increases the non-lift dependent drag so the altitude modification is not worthwhile.

## B. Active Constraints and Optimal Variable Values

A rough rule of thumb governing the selection of aircraft geometry states that the landing field length requirement determines the wing area and the other operative constraint, whichever one it is, fixes the proper combination of aspect ratio (span) and engine power. In fact, though wing area is not quite independent of cruise speed for a given field length, wing loading (takeoff weight divided by wing area) does not vary with speed. Thus the landing distance has only secondary effect on aspect ratio and horsepower required.

Table 1 presents a list of the active constraints-that is, those limiting the design-for each cruise speed and field length tested. The table includes the critical field length for each speed. At the lower soeeds, the required enroute climb gradient sizes the aspect ratio and engine power Since, previously, commuter aircraft have not been designed to meet FAR part 25 regulations, they have not encountered as much difficulty with the one-engine-out enroute climb restriction. Though enroute climb rarely presents a problem for turbofan aircraft, the turboprop airplane, because its speed for best climb is lower than the minimum allowable speed (30\% above the stall speed), is often restricted by this regulation if it is designed according to part 25 rules.

At the highest cruise speed, in most cases, minimum cruise power to fly at 330 knot determines both engine power and aspect ratio. Obviously, increasing the horsepower increases the maximum cruise speed, but, though not as important a factor in the power-restricted cases, increasing the aspect ratio also increases the maximum cruise speed due to the reduced induced drag. So, whether the second active constraint is minimum enroute climb gradient or power to cruise at a given cruise velocity, several combinations of aspect ratio and engine power exist to satisfy that constraint. The optimizer chooses the best, or lowest cost, combination of the two.

At a cruise speed of 330 knot and landing distance 4,275 feet or more, enroute climb gradient rather than available cruise power becomes the second operational constraint. This occurs because the wing area has decreased enough that the cruise drag (and, therefore, cruise power required) has also decreased to the extent that power to climb is greater than the power to maintain a 330 knot cruise speed.

Figure 2 shows the variations of optimal wing area, aspect ratio, and horsepower with field length and cruise velocity. As expected, wing area decreases as the field length gets longer. The aspect ratio, however, increases in an attempt to keep the same span in order to maintain the same climb gradient or induced drag. The 250 knot airplanes have higher aspect ratios than the 290 knot planes because they must meet identical climb gradients but with lower power levels. The slower airplanes have lower power ratings but higher spans than the 290 knot aircraft. The 330 knot airplanes have aspect ratios lying between those of the other two speed aircraft since the cruise speed constraint affects choice of aspect ratio differently from the enroute climb constraint.

Figure 2 c provides an interesting insight into the effects of differing active constraints on optimum engine power. As wing areas decrease with increasing field length, the aspect ratios increase but, in general, not enough to maintain constant span. If enroute climb is critical, then, the engine power must increase for the airplane to meet the climb gradient for reduced span. At 250 knot and 290 knot this indeed happens. However, if meeting the required cruise velocity is critical, the smaller wing area reduces the parasite drag much more than the smaller span increases induced drag. Therefore, the aircraft requires less power to overcome the cruise drag, and the curve indicating a 330 knot aircraft follows this trend.

## C. Sensitivity Studies

1. Grid Search About an Optimal Point. Although the optimizing program chooses a lowest-cost airplane for a given set of constraint parameters, it gives little information about the effects of small changes in variable values about that optimum. Figures $3 \mathrm{a}-\mathrm{c}$ show cost for values of wing area, aspect ratio, and engine power above and below those calculated as the optimum for cruise speed equal to 330 knot and a field length of 4,000 feet. Constraint barriers are included in these figures to indicate areas of impossible choices. At the smallest wing area ( $345 \mathrm{ft}^{2}$ ) no airplane can meet the 4,000 foot field length constraint whereas, at a wing area of $385 \mathrm{ft}^{2}$, all airplanes easily fall below the field length requirement.

As these figures lllustrate, the optimizer chooses the lowest cost configuration which can meet all requirements. At the optimum point, the design is bounded by both cruise power and field length, and, as a consequence, it cannot move in a direction of lower cost. (See Figure 3b.)

The "kinks" in the highest power curves of Figures 3 b and 3 c occur because the program allows only discrete values of cruise altitude which leads to slight discontinuties in the goal function.
2. Non-Optimal Operation. The previous discussion deals with aircraft operation under the conditions for which that aircraft is designed. Possibly, however, a commuter operator would like to have the ability to fly his airplanes at a fast speed even if he normally flies much more slowly.

Figure 4 shows the cost penalty incurred for two cases of non-optimal operation. The costs for the optimum airplanes designed for cruise at 330 knots and field lengths of 3,500 and 4,000 feet, but actually flown at several lower cruise speeds over the 150 nautical mile typical stage length, are shown. Although the cost does decrease as the airplane slows down, it does not reach the economy level achieved for the optimized airplane at each speed. The difference in DDC between the optimized aircraft and the high-speed airplane flown at a lower speed reaches as high as $1.4 \%$ for airplanes meeting a 4,000 foot landing distance and as high as $5 \%$ for airplanes with 3,500 foot field lengths. The non-optimized airplanes cost more to operate at a given speed since their larger engines and higher wing areas contribute to higher weight and drag and, thus, to more fuel burned per mission.

## D. Ride Roughness

Figure 5 shows ride roughness parameter, $\Delta n$, as a function of field length for the airplanes generated by the optimizing portion of this study. The plot does not form a family of smooth curves with speed as a parameter due to the discrete altitudes allowed in the optimizing routine. In particular, the lowest field length, 330 knot airplane flies at 25,000 feet because of the extremely sub-optimal $C_{L}$ produced at lower altitude. As the field length increases and the wing becomes smaller, the airolanes come down to

20,000 feet since smaller engines provide the required nower at the lower altitude.

Also shown in figure 5 are the rouqhness parameters of several comparable-mission aircraft. Notice that, according to the method employed here, three of the optimum conmuters would, in fact, have more favorable gust response than the $D C-9-30$. If indeed true, this suggests that commuters need only increase wing loading by using a good flap system, or slow down to 250 kts , in order to solve the ride roughness problem. Because of qualitative assessments of commuter ride roughness, however, some skepticism remains as to the validity of using this particular parameter to compare these somewhat different aircraft.

The theory for response to a sharp-edged gust (see Jones, ref. 8) can be applied to the optimum aircraft and to the DC-9-30. Figure 6 shows the theoretical curves for maximum $\Delta C_{L}$ vs. mass ratio and those points corresponding to the optimal commuters from this study. The ordinate for this plot, maximum $\Delta C_{L}$, is obtained from:

$$
\begin{aligned}
\Delta n & =\frac{\Delta L}{L}=\frac{\Delta C_{L}}{C_{L}} \\
\Delta C_{L} & =\Delta n C_{L}
\end{aligned}
$$

Since, however, $\Delta n$ is computed for a $30 \mathrm{ft} / \mathrm{sec}$ equivalent airspeed gust and the theory shows response to a unit gust, the result must be normalized by:

$$
\operatorname{maximum} \Delta C_{L}=\Delta n C_{L} \frac{V}{U}
$$

Where $U=$ true gust velocity for which an is computed
$V=$ true airplane speed

The theoretical plot indicates, first of all, that the gust response method of reference 7 coincides quite well with the theory for a sharpedged gust. Secondly, it shows that the $0 C-9$, with comparable mass ratio
and aspect ratio to the optimal commuters, should indeed have comparable gust response. Even with this evidence, however, a need remains for verification of $\Delta n$ as a useful gust response parameter, especially since the theoretical curve in figure 6 has been computed only up to mass ratios of 280 - far less than those of present-day airplanes.

Figure 7 shows a diagram of the optimal airplane for 330 knot cruise speed and a 4,000 feet field length.

## IV. Conclusions

- Increasing cruise speed (beyond 290 knot) and decreasing allowable field length tend to increase direct operating cost, but only a six percent difference in DOC exists between the best and worst airplanes studied. This occurs because each airplane is optimized with respect to direct operating cost for its particular mission.
- One=engine=out enroute climb gradient requirements restrict the conmuter aircraft with turboprops more than they do a turbofan aircraft because the cormuter's speed for best climb gradient is less than its enroute minimum allowable speed.
- The major drawback to increasing speed and decreasing runway length is the increased ride roughness due to both higher velocity and lower wing loading. The worst ride roughness calculated for an optimal airplane represents a 45 percent increase in the relative parameter $\Delta n$ over the lowest value.
- Some work remains to verify the FAR value $\Delta \mathrm{n}$, as a useful parameter for comparing airplane ride roughness.


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## Table 1. Active Constraints

| Cruise Speed (kts) | Field Length Constraint (ft) | Active Constraint ${ }^{1}$ |
| :---: | :---: | :---: |
| 250 | 3,500 | Enroute climb gradient |
|  | $3,725^{2}$ | 11 |
| 290 | 3,500 | " |
|  | 3,750 | 11 |
|  | 4,000 | 11 |
|  | $4,060^{2}$ | 11 |
| 330 | 3,500 | Maximum cruise power |
|  | 3,750 | . |
|  | 4,000 | 11 |
|  | 4,275 ${ }^{2}$ | enroute climb gradient |

1. This column contains the second active constraint. The first active constraint is landing distance at the field length listed in column 2.
2. Critical field length above which field length does not determine wing arna, power or aspect ratio.


Figure 1. Direct Operating Cost vs. Field Lenath

Figure 2. Optimal Variable Values vs. Field Length

## 



Figure 3. Optimal Point Sensitivities
Cruise Speed $=330 \mathrm{kt}$, traximarl Field Length $=400$
(c)


Finure 4. C.nst for Mon-notimal Oneration [330 kt desinns flown at lower soeeds]


Fig̣ure 5. Ride Rounhness vs. Field Lenath

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## Appendix I

## Program Listing

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SUBPROGRAM TO COHPUTE THE GRADIENT OF THE FUACTION AT A FOIITT.


RETURN
END



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ISN 0027
KSN 0028

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COMPILER OPTIOHS - NARE $=$ MAIN, OPT $=02$, LIMECNT $=58, S I Z E=0000 \mathrm{~K}$, , HONREF
 Do $2001=250.300 .25$
CLISTD $=$ FLOAT(I)
BEGIN SEARCH LOOP ON CRUISE ALTITUDE. us DO $100 \mathrm{~J}=20000,30000,5000$
HHAX $=$ FLOAT(J)
HHIAX $=$ FLOAT(J)
GRITEI6,300 HITRX, CLHISPD
FORIUTC $/ f,:$ HCRUS,VCLM $=0,2514.41$
ICOUNT $=$ ICOUNT +1
ICOHNT $=$ ICOUNT +1
IMTMUH $=1$



CALCULATE DESIGN EGUIVALENT STRUCTURAL SPEEO AT 10000 FT.
VECH $=1.07=$ VCRUSmSORT $(.0017553 / .0023769)$

Q = .5*RHO VCRUS**2
C GUESS TOH ALD FIHD ERTPTY WEIGHT.

> X2 $=25000$. STEP $=\times 2 / 50$. HAXRH $=600$. COHN $=1$. $H=$ HHAX VCL $=$ CLHSPD.
15N 0002

$0=N$
088
08
気雷

$n .0$
0.8
20
$n$

ISN 0024



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##  

| 1SN 0075 |  | 6050100 |
| :---: | :---: | :---: |
| ISN 0076 | 99 | IF (X2 .EE. TOWH) 60 T0 100 |
| ISN 0078 |  | CALL HAXIHR(LC, VCRUS,HDDIFF) |
| ISN 0879 |  | IF IDIFF. LT. O. .AND. HHAX .ME. 20000.) 60 TO 100 |
| ISN 0081 |  | HCH = INC |
| ISH1 0032 |  | AFHTI = NTAF |
| ISN 0083 |  | OHEFLI $=$ 2FN |
| ISH 0084 |  | DIST1 $=$ FI |
| ISN 0035 |  | TORA $=\times 2$ |
| ISN 0036 |  | hCRUS $=$ hriax |
| ISN 0037 |  | CORTIINE |
| ISN 0088 |  | IF (TOWH .GE. TOH) 6050200 |
| ISH 0090 |  | TOA $=$ TOH |
| ISN 0091 |  | vClits $=$ curspo |
| ISN 0092 |  | KCT $=$ HCH |
| ISN 0093 |  | AFHT $=$ AFHTI |
| ISN 009.4 |  | OHEPL $=$ OHEPLI |
| ISN 0095 |  | OIST $=$ DIST |
| 2Std 0096 | 200 | COATIITUE |
| 1EN 0097 |  | EICRP |
| ISH 0098 | c | CALL IhFo I |
| ISH1 0099 |  | Stop |
|  | c |  |
| ISN 0100 |  |  |






Mf"


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ぶ
 C．SOURCE，EECDIC，NOLIST，NOOECK，LOAD，PAP，HOEDIT，ID，$N$
 COHONOCHARAC／CDP COINOHVGEOH／／XHAC，SHE，SHG，BH，SVE，SVG，BV
COHMOIVAERONEQ，CLALPH

## DESIGN STRUCTURAL SPEED DETERHIMED AT 10000 FT．

$$
\text { R110 }=.0017553
$$


$H 7=.04 \#$ TON
$Z 2=70 H \geqslant 11$.
$22=10 W *(1 .-F 9)$


$\mathrm{XKK}_{2}=2.08 \mathrm{VV} /(5.3+\mathrm{V} 2) \quad 18 \mathrm{~V}$
F1 $=1 .+(X K 1$ MCLALPHINUGVEQ $) /(841.12$（21／S）$)$
$f 3=1 .+1 \times 2 \times C L A L P I M U 9 \times V E Q) /(84) .12$（10w／5）
$\begin{aligned} & \text { F }=2.1 \\ & \text { IF（F5 ．6T．} 3.8 \text { ）} F 5=3.8\end{aligned}$
IF（F5，LT． 2.5 ）$F 5=2.5$
IF（F3，LT．F5）F3 $=85$
HING HEIGHT；AVERAGE THICKNESS $=.15$ $4^{\circ}$
リ4
 $H_{1}=(1.642 * X I I+4.22) * S$
HRITE $6, *) X I I, M 1$
HORIZONTAL TAIL HEIGHT；AVERAGE THICKNESS＝． 12
 $W_{2}=(1.5 * \times 12+5.25)$ SHIE
ISN 0002

ISN 0003
ISN 0004
ISH 0005
ISH 0006
ISN 0007
ISN 0008
ISN 0009
ISN 0010

－45－

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|  | C | VERTICAL TAIL AND RUDOER heisht; Vertical ave t/c = .12 |
| :---: | :---: | :---: |
| 15 N 0032 |  |  |
| 1SN 0033 |  |  |
| ISN 0034 |  | H4 $=1.6 \times \mathrm{H} 3 / 3$. |
|  | $C$ $C$ | SURFACE CONIROLS HEIGHT |
| ISN 0035 |  | W5 = 1.7*(SHC+SVG) |
| ISN 0036 | C | IF (FI. .LT. 2.5) Fi $=$ F3 |
|  | C C C | FUSELAGE HEIGHT |
| ISN 0038 |  | SLSHPH $=$ SLSHP/2. |
| ISH 0039 |  |  |
| 15N 0040 |  | N9 = 6528. + HENG |
| 1511001 |  | HO $=1948 .+$ HELG |
| IS110042 | C |  |
| ISN 0043 |  | T2 $=11-T 7$ |
| ISN $004{ }^{\circ}$ |  | IF (TE, LT, 0.) $72=0$. |
| 1511 0046 |  |  |
| ISN 0047 |  | W6 $=(.102$ WXI6 +1.051 ) 11472.47 |
|  | C $C$ $C$ $C$ | determine zfu. coipare hith estimated zfh. if not same, ITERATE. |
| ISN 0048 |  |  |
| ISN 0049 |  | IF (ABSC(Z2 - Zi)/Zi) .GT. .0001) GO 7010 |
| ISN 0051 |  | $\mathbf{2 F W}=22$ |
| ISN 0052 |  | WTAF $=22-7270$. - NENG |
| ISN 0053 | C | RETURN |
| ISN 0054 |  | EHD |




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## 1. P PRGE IS Or FOOR QUALITY



LEVEL 21.8( NN1 74)



)

|  | $X_{2}=X_{2}+.5 \text { STEPE(Fi-F3)/(F3-2.XF24F1) }$ <br> CALL DISTN(X2,F2, HRHAX,VCLIH3,150, D,STAGEH,HC,FI,TC,FC, WCRUS) <br> IF (F2 LE. CONV) 60 TO 99 <br> STEP = STEP/4. <br> 607010 |
| :---: | :---: |
| c <br> C MHERE F2 IS betheen or greater than fi ard f3, hake xz the value c FOR A HINEMUH |  |
| C IF (F3 IT Fit co 70 |  |
|  | $x_{2}=x_{1}$ |
|  | $F 2=F 1$ |
|  | $\mathrm{XI}_{1}=\mathrm{XI}_{1}-$ STEP |
|  | CALL DISTNIXI,FI,HHAX, VCLITR, WO, $Q$, STAGER,HC,FI,TC, FC, VCRUS GO TO 3 |
| $c$ |  |
| 2 | $\mathrm{XI}_{1}=\mathrm{X}_{2}$ |
|  | $F 1=52$ |
|  | $x_{2}=x_{3}$ |
|  | $F 2=F 3$ |
|  | X3 $=\mathbf{X 3}+5$ SEP |
|  | CKLL DISTHIX $3, F 3$, HHAX,VCLIFB, $20, Q, S T A E E R, W C, F I, T C, F G, V C R U S\}$ 60 T0 3 |
| c |  |
| 82 | \&RITE(6,302) |
| 302 |  |
| $c$ |  |
| $c$ |  |
| 99 | COCTITALE |
|  | BLOCKF $=\mathrm{FD}+\mathrm{FC}+(\mathrm{HC}-\mathrm{HO})+.0024 T 04$ |
|  | BLOCKT $=$ T0/60. *T/3600. + FI*6072.1(VCRUSW5600.)*.25 |
| c |  |
|  | RETURH |
|  | ERO |




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## AIRFRAHE HATERIAL



MAIMTEHANCE BURDEN
BUREEN $=1.8 *$（AFLAB + EHIGLAB）


> CEHIS PER SEAT STATUTE HILE

CEHIS $=($ CRHCST + FULCST + XINCST + TOTMAI + EEPR $) \times 100.130$ ．

CENTS PER SEAT NAUTICAL HILE


RETURK
ERE

ISN 0022
$15 ⿰ ㇒ ⿻ 二 乚 ⿴ 囗 十 一 23$
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ISH 0025
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0
8
0
5
ISH 0027
ISH 0028
ISN 0029



$$
1,1,1,1,1,1,
$$

COHPILER OPTIOAS－NAHE $=~ M A I H, ~ O R Y=02, ~ L I H E C H T=58, S I 2 E=000 O K, ~$
SOURCE，EBCDIC，HOLIST，HOOECK，LOAD，HAP，MOEDIT，ID，HOXREF


FUKCTIOI EVALUATIOA FOR FINDING THE TOA FOR THE REGUIRED RAMCE．


IKANT •N゙ビ．OS 60 TO 10
DAV $=$ CDPagns $+($ LIC
SIPAV $=$ DAVIVCRUS $/(550$. m．05）

SIPAV $=$ DAVAVCRUS／（550．W．05）
SIFTUX $=$ PONER（SLSHP，VCRUS，HHUXI
$320^{\circ} 2+10=315$
XIUdifS／AYdIIS $=2$



if airplane cariot clits，let the rabge de equal to gniuc． $F I=$ GC
 RETURI
ERD

$c^{10}$ $c^{20 .}$

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