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THE TURBULENT RECIRCULATING FLOW FIELD IN A CORELESS INDUCTION FURNACE

A comparison of Theoretical Predictions with Measurements

by

N. El-Kaddah and J. Szekely

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ABSTRACT

A mathematical representation has been developed for the electromagnetic force field and the fluid flow field in a coreless induction furnace. The fluid flow field was represented by writing the axisymmetric turbulent Navier-Stokes equation, containing the electromagnetic body force term. The electromagnetic body force field was calculated by using a technique of mutual inductances. The k-ε model was employed for evaluating the turbulent viscosity and the resultant differential equations were solved numerically.

The theoretically predicted velocity fields were in reasonably good agreement with the experimental measurements reported by Hunt and Moore; furthermore, the agreement regarding the turbulent intensities was essentially quantitative. These results indicate the k-ε model does provide a good engineering representation of the turbulent recirculating flows occurring in induction furnaces. At this stage it is not clear whether the discrepancies between measurements and the predictions, which were not very great in any case are attributable either to the model or to the measurement techniques employed.

1. INTRODUCTION

In recent years there has been a growing interest in the development of an improved basic understanding of heat and fluid flow phenomena in coreless induction furnaces. The main motivation for this interest is essentially twofold. One, the actual operation of coreless induction furnaces as means for scrap melting and the refining of molten metals depends critically on the turbulent heat and fluid flow phenomena in this system. The second, more fundamental motivation is that the turbulent recirculating fluid flow phenomena in this system represent a very interesting class of electromagnetically driven flow problems.

Fig. 1 shows a sketch of a typical coreless induction furnace, where it is seen that this consists of a refractory (or non-magnetic) crucible, containing a conducting metallic melt. The outer wall of the crucible is surrounded by water cooled induction coils (connected on a single phase or multi phase arrangement) through which a current is passed. The passage of this current through the coil will in turn induce a current in the melt and the interaction of this induced current with the associated magnetic field will result in an electromagnetic force field, or Lorentz force field, which will cause in a recirculating motion of the melt.

The quantitative representation of this system has two essential components:

(1) The electromagnetic force field has to be calculated

(2) Knowing the distributed body force field the fluid flow field may then be obtained.

The following general observations may be appropriate at this stage:

When the magnetic Reynolds number is small the rate at which the electromagnetic field propagates is much faster than the fluid velocity thus the force field and the fluid flow field calculations may be uncoupled.

The calculation of the electromagnetic force field resulting from a given coil configuration is a classical problem in electrodynamics, which may be readily accomplished for simple geometries, using analytical techniques.[1,2] Indeed, calculations of this type may be readily found in the textbook literature. For more complex geometries, in fact when an accurate comparison with measurements is required, the calculations of the electromagnetic force field requires numerical techniques.

If the flow is laminar in principle, the velocity field could be readily calculated by combining the known body force field with the laminar Navier-Stokes equations.[3] However, this procedure is not readily applicable in the vast majority of practical cases, because the magnitude of the force field and the linear dimension of the system result in a turbulent flow.

Clearly the presence of turbulence precludes the use of simple analytical techniques. The approaches that have been devised to tackle problems of this type may be divided into two major groups:

(A) Starting with the rigorous form of the turbulent Navier-Stokes equations, order of magnitude approximations may be made for the various terms, thus approximate expressions may be deduced for the mean values of the various flow parameters.[4-7]

(B) An alternative, engineering approach, which has been developed by Szekely and Chang, Evans and Tarapore and others[4,8-11] employed various turbulence models such as the k-W or the k-ε model to represent the Reynolds stresses and thus obtained detailed maps of both the velocity fields and of the turublence parameters.

Both these approaches have advantages and drawbacks. The technique of Hunt and Moore[6,7] is certainly elegant and provides a useful insight into relationships between the key system parameters. However, because of the approximations involved the model cannot be used to predict the detailed velocity field, the maps of the turbulence and thus cannot but provide very preliminary order of magniture estimates regarding the key heat, and mass transfer and dispersion phenomena that are of primary interest in the solution of practical engineering problems. The numerical approaches that have been developed for tackling problems of this type are certainly attractive because they are capable of addressing the very questions of practical interest, such as local shear rates, dispersion rates and ultimately the transport controlled reaction kinetics.[8,12-15]

The main drawbacks of these engineering calculations include the substantial computational labor required and the lack of a fundamental basis for the turbulence models employed.

In view of the great potential usefulness of these techniques, from a pragmatic standpoint it would be highly desirable to develop an a posterioiri justification for their use, through a critical comparison of the theoretical predictions with experimental measurements.

Up to the present only a limited range of such measurements is available,which encompases data obtained both from laboratory scale apparatus and industrial scale units. Many of these measurements involved the determination of surface velocities and tracer dispersion rates; while the agreement between predictions and the measured data was quite good, Auguring well for many engineering applications a really rigorous test of these models has not been possible because of the lack of sufficiently accurate measurements.

In a recently published paper, which was kindly provided for us prior to formal publication, Hunt and Moore have reported on an interesting set of measurements in an inductively stirred mercury system [6,7] which should provide a useful basis for a more rigorous test of the model. The purpose of the paper is to report on such an assessment.

2. EXPERIMENTAL

The experimental measurements, which will be used for the purpose of comparison have been reported by Hunt and Moore and therefore only a very brief recapitulation will be given here.

A schematic sketch of the experimental arrangement is shown in Fig. 2. In essence the apparatus consisted of a water cooled stainless steel vessel approximately 0.3m in diameter and 0.4 m high, containing mercury. Agitation was provided by an induction coil containing 11 uniformly spaced turns. The coil current was 1900 A, RMS, having 50 cycles.

The actual measurements taken included the determination of the electromagnetic force field, using search coils and the measurement of both the time smoothed and the fluctuating velocities, using a mechanical probe. In essence this mechanical probe consisted of a perforated spherical shell, made of tantalum and the actual measurement involved the determination of the drag exerted on this sphere. Measurements close to tank wall were made with "wall probe" described elsewhere.[16].

The actual results of the experimental measurements will be discussed in a subsequent section.

3. THE ANALYSIS

The salient features of the analysis will be given in the following:

As mentioned earlier the analytical problem is to calculate the electromagnetic force field and the turbulent fluid flow field in a cylindrical system, agitated by a symmetrically placed induction coil.

3.1 Calculation of the Electromagnetic Force Field

The electromagnetic force field is given by:

$$\underset{\sim}{F} = \underset{\sim}{J} \times \underset{\sim}{B} \tag{1}$$

where

$\underset{\sim}{J}$ is the induced current density and

$\underset{\sim}{B}$ is the magnetic flux density

Thus the problem is the calculated $\underset{\sim}{J}$ and $\underset{\sim}{B}$ for a given geometry and coil current.

For idealized systems, e.g. infinitely long cylinders, and a travelling wave, this task may be accomplished analytically, through the solution of Maxwell's equations.[3] However, for the present system, particularly when a quantitative comparison is desired, an alternative technique, employing the concept of mutual inductances is preferable.

For a cylindrical crucible which is characterized by axial symmetry, the lines of equal current density are circles in planes perpendicular to the axis of the coil, i.e. the θ direction. From Amper's law the magnetic field in an elementary circuit in the melt may be written in terms of the vector potential $\underset{\sim}{A}(\underset{\sim}{B}=\underset{\sim}{\nabla}\times\underset{\sim}{A})$ as:

$$A_\theta = \frac{\mu_o}{4\pi}\left\{\sum_{c=1}^{melt} (J.S)_c \oint \frac{d\ell_s}{r'} + \sum_{k=1}^{coil} I(k) \oint \frac{d\ell_k}{r'}\right\} \quad (2)$$

where A_θ is the vector potential, $d\ell$ is the line element of a circuit of constant current density and S is the cross sectional area of the circuit. I(k) is the coil current.

It should be noted that first term on the rhs of Eq. (2) describes the induced potential while the second term described the applied potential.

From Faraday's law, the induced current in this circuit can be written for a time harmonic field as:

$$\oint J_i \cdot d\ell_i = jw\omega \left\{ \sum_{c=1}^{melt} M_{i,c} (J.S)_i + \sum_{k=1}^{coil} M_{ik} I(k) \right\} \quad (3)$$

where

$$M_{i,c} = \frac{\mu_o}{4\pi} \oint \oint \frac{d\ell_i d\ell_c}{r'} \quad \text{is the mutual inductance} \quad (4)$$

The calculation then proceeds by dividing the melt into elementary circuits (14x14 in the present case), each being represented by Eq. (3) to be solved simultaneously, to obtain the induced current. Once the induced current field is known the vector potential may be obtained from Eq. (2), when then facilitates the computation of the electromagnetic body force field.

3.2 Calculation of the Fluid Flow Field

For cylindrical symmetry, the equation of continuity and motion take the following form:

equation of continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r U_r)+\frac{\partial}{\partial z}(\rho U_z)=0 \quad (5)$$

Momentum balance in the z-direction

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\rho r U_r U_z\right)\right]+\frac{\partial}{\partial z}(\rho U_z^2)=-\frac{\partial p'}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial U_z}{\partial r}\right) +2\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial U_z}{\partial z}\right)+\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\mu_{eff} r\frac{\partial U_r}{\partial r}\right)\right] + \mathrm{Re}(J_\theta \times B_r^*)/2 \quad (6)$$

Momentum balance in the r-direction

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\rho r U_r^2\right)\right]+\frac{\partial}{\partial z}\left(\rho U_r U_z\right)=-\frac{\partial p'}{\partial r}+\frac{2}{r}\frac{\partial}{\partial r}\left(r\mu_{eff}\frac{\partial U_r}{\partial r}\right) +\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial U_r}{\partial z}\right)+\left[\frac{\partial}{\partial z}\left(\mu_{eff}\frac{\partial U_z}{\partial z}\right)\right]-\frac{2U_r\mu_{eff}}{r^2} + \frac{1}{2}\mathrm{Re}(J_\theta \times B_z^*) \quad (7)$$

Here the quanity μ_{eff} is the effective viscosity defined as:

$$\mu_{eff} = \mu + \mu_t \quad (8)$$

where

$$\mu_t = C_\mu \rho k^2/\varepsilon \quad (9)$$

is the turbulent viscosity.

Here k and ε are the turbulent kinetic energy and the turbulent kinetic energy dissipation respectively.

Separate transport equations have to be written down for these, which take the following form:

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$$\frac{\partial}{\partial z}(\rho U_2\phi) + \frac{1}{r}\frac{\partial}{\partial r}(\rho r U_r \phi) - \frac{\partial}{\partial z}\left(\frac{\mu_{eff}}{\sigma\phi}\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial r}\left(r\frac{\mu_{eff}}{\sigma\phi}\frac{\partial\phi}{\partial r}\right) = S\phi \qquad (10)$$

where $S\phi$ is the net rate of generation of turbulent properties per units volume. The source terms S_K and S_ε of transport equation of k and ε are given by

$$S_k = G - D \qquad (11)$$

$$S_\varepsilon = C_1\ \varepsilon/K\ G - C_2\ \rho\ \varepsilon^2/K \qquad (12)$$

where

$$G = \mu_t\left[2\left\{\left(\frac{\partial U_z}{\partial z}\right)^2 + \left(\frac{\partial U_r}{\partial r}\right)^2 + \left(\frac{U_r}{r}\right)^2\right\} + \left(\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z}\right)^2\right] \qquad (13)$$

$$D = \rho\varepsilon \qquad (14)$$

The value used for the constants, viz C_μ(=0.09), C_1(=1.45) and C_2(=1.92) were taken from the work of Spaulding and Launder, (13,17) and were not adjusted in any way in course of the computation.

The boundary conditions, which are required to complete the statement of the problem will have to specify zero velocity at the solid surfaces, zero shear at the free surface and the existence of symmetry about the centerline. Regarding the quantities k and ε these were set equal zero at the solid surfaces and their gradient was stipulated to be zero at the free surfaces

It is noted, furthermore that wall functions were used to define the shear stress at the solid surface.[17] These wall functions were based on universial velocity distribution for turbulent boundary layers without pressure gradient and body forces. This is an area where careful experimental measurements could provide a useful refinement.

The governing equations were put in a finite difference form, using a 16x16 grid and the resultant set of simultaneous non-linear algebraic equations was solved using an iterative technique. Typically the computer time requirements were about 120 seconds on MIT's IBM 370 digital computer.

4. COMPARISON OF THE COMPUTED RESULTS WITH MEASUREMENTS

Since the principal concern here is a critical comparison of the experimentally measured and the theoretically predicted velocity fields and turbulence characteristics the treatment presented will be confined to this aspect of the problem. In any case the calculation of an electromagnetic force field for a given coil configuration may be readily undertaken as an essentially routine matter.

Figs. 3 and 4 show the experimentally measured and the theoretically predicted maps of the velocity vector. Inspection of these two figures shows quantitative agreement regarding the nature of the flow and the position of the recirculating loops. While this form of representation is not ideal for making a fully quantitative comparison between the numerical values of the velocity vector, these magnitudes seem comparable.

Fig. 5 shows the experimentally measured profile of the axial velocity, determined at a vertical position, corresponding to the eye of the upper vortex. Also shown, with the broken line is the computed velocity profile. It is seen that these two profiles are quite similar, but that the two curves do not coincide, Whether this discrepancy is attributable either to possible shortcomings of the model or to experimental errors will be discussed subsequently

Fig. 6 shows a plot of the local value of the turbulence intensity, as a function of the radial position. Here again the measurements are given with the circles, while the theoretical predictions are designated by the full line. It is of interest to note that here the agreement is very good, rather better than that found for the velocity profiles.

5. DISCUSSION

In the paper a comparison is made between experimentally measured fluid velocity profiles and profiles of the turbulence intensity as reported by Hunt and Moore for an inductively stirred mercury pool, and theoretical predictions based on the concept of mutual inductances and the two equation k-ε model.

This comparison was thought to be instructive, because these measurements, kindly supplied by Messrs H nt and Moore represent the most detailed data on inductively stirred systems available up to the present.

The principal findings of the work may be summarized as follows:

(1) The technique of mutual inductances was able to predict the electromagnetic force field quite readily, thus this facet of the approach was unlikely to introduce a serious error.

(2) The theoretical predictions regarding the general nature of the flow were found to be in very good agreement with the measurements.

(3) The profiles of the time smoothed velocity near the wall were reasonably well predicted by the model although there were certain discrepancies, patricularly at a large distance from the wall.

(4) The agreement between measurements and predictions, regarding the profiles of the turbulence intensities was very good.

Before considering these points in detail some general observations may be appropriate.

As far as the mathematical modelling of turbulent recirculating flows is concerned it is fair to say that important reservations must be expressed regarding the fundamental basis of the k-ϵ model and that of the wall functions.

It should be stressed, however, that at present there appears to be no generally applicable alternative for doing engineering type calculations for turbulent recirculating flows where details are required of the turbulence energy distribution and of the local heat or mass transfer rates.

Thus in a pragmatic sense the application of this model has to be justified through a direct comparison with measurements. On the basis of information available in the literature the use of this model has been quite successful in a number of instances.

When applying turbulent recirculating flow models to representing electromagnetically driven flows an additional complication may arise because of the possible damping or accentuation of anistropy, as caused by the interaction between the electromagnetic force field and the turbulence field.

On the basis of prior experience one would expect turbulence models of the type employed here to predict the overall flow field rather well, because neither the particular features of the turbulence model employed nor the damping effect of the electromagnetic field on the turbulence are likely to be very important in the bulk. More specifically in the bulk of the fluid the convective transport of momentum is likely to dominate. It follows that a much more critical test of the model would be provided by the assessment of the predictions in the near wall region.

In considering the experimental technique employed, it has to be recognized that the characterization of electromagnetically driven turbulent recirculating flows is notiorously difficult, because of the problems inherent in the use of traditional measuring equipment. The technique employed by Moore and Hunt is thought to be ingeneous, but perhaps not subjected to a lengthy enough testing period to eliminate all possible experimental errors. The relatively large size of the probe or even the alternative probing devices employed near the wall compared to a standard hot film device) makes the reliability of the measurements in the very near wall regions somewhat problematic, where accurate data would be most desirable.

It follows that it is not really quite clear whether the discrepancies between measurements and predictions are unequivocally attributable to the shortcomings of the model or to the measurement techniques.

Let us now comment briefly on the specific findings of this study, concerned with the details of the velocity fields and of the spatial distribution of the turbulence intensity.

While there was good agreement between the predicted and the experimentally measured velocity profiles a certain discrepancy was evident, both in the immediate vicinity of the wall and in the bulk of the fluid.

It should be remarked that the experimentally obtained velocity profile do not satisfy the equation of continuity i.e.

$$\int_0^R U_z 2\pi r dr = 0 \qquad (15)$$

thus some questions may be raised regarding the accuracy of the absolute values of the measured velocities. At this stage one may comment that the calibration curves for the price were not exactly linear, which in conjunction with the straight line relationship actually employed provided a significant error band especially at low velocities. It should be stressed here that this should not be taken as a criticism of the experimental technique because the problems of velocity measurements in liquid metal systems are generally appreciated by workers in this field.

It is of interest to note that perhaps unexpectedly very good agreement between the theoretically predicted and the experimentally measured turbulence intensities. The k-ε model explicitly assumes local anisotrophy and serious questions may be raised, whether this condition has been met in an inductively stirred system. One may speculate that some self-cancelling errors may have come into play, alternatively perhaps the experimental technique was not sensitive enough to pick up some of the higher harmonics. It should be remarked, furthermore, that the errors introduced due to the non-linearity of the calibration curve will affect the absolute values of the velocity far more than the velocity ratios. Since the turbulence intensity curves, shown in Fig. 6 represent ratios, these would not be less affected by calibration errors.

In conclusion one must state that notwithstanding its short-comings as far as lacking a fundamental basis, the k-ε model appears to provide a reasonable prediction for both the gross features and the detailed velocity fields and turbulence energy distribution in an inductively stirred system.

The principal discrepancy that one should expect would lie in the near wall regions. Should one require more precise information regarding the behavior of these regions such as the knowledge of local heat or mass transfer rates then the best alternative would be to measure these quantities directly. Work of this type is in progress in the author's laboratory at present.

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References

(1) W.K.H. Panofsky and M. Phillips, Classical Electricity and Magnetism, Addison-Wesley, Reading, Mass. (1955).

(2) J.D. Jackson, Classical Electrodynamics, John Wiley and Sons (1962).

(3) W.F. Hughes and F.J. Young, The Electromagnetodynamics of Fluids, John Wiley and Sons (1966).

(4) J. Szekely and C.W. Chang, Ironmaking and Steelmaking, (1977), 4, 190.

(5) D. Khaletzky Thesis, Univ. de Grenoble, France (1976).

(6) D.J. Moore and J.C.R. Hunt, to be published in Proc. 3rd Bar Sheva Symposium on MHD flows and turbulence.

(7) D.J. Moore and J.C.R. Hunt, Proc. of 1981 Seminar on Metal Flow in Induction Melting Furnaces, (1981)

(8) J. Szekely and K. Nakanishi, Met. Trans. B., (1975), 6B, 245.

(9) J. Szekely and C.W. Chang, Ironmaking and Steelmaking, (1977), 4, 196.

(10) E.D. Tarapore and J.W. Evans, Met. Trans. B., (1976), 7B, 343.

(11) E.D. Tarapore and J.W. Evans, Met. Trans. B., (1977), 8B.

(12) A.D. Gosman, W.M. Pun, A.K. Runckal, D.B. Spalding and M. Wolfshtein, Heat and Mass Transfer in Recirculating Flow, Academic Press, (1969).

(13) B.E. Launder and D.B. Spalding, Mathematical Models of Turbulence, Academic Press, (1972).

(14) B.E. Launder, Heat and Mass Transport in Turbulence, P. Bradshaw, ed. p 232 Springer, Berlin (1976).

(15) K. Nakanishi, J. Szekely, T. Fujii, Y. Mihara and [illegible], Met. Trans. B., (1975), 6B, 111.

(16) D.J. Moore Ph.D. Thesis, Cambridge, Univ. (1982).

(17) B.E. Launder and D.B. Spalding, Computer Meth. Appl. Mech. Engr. (1974), 3, 269.

Symbols

$\bar{A}$ vector potential (Wb/m)

B magnetic flux density (Wb/m^2)

F_s electromagnetic stirring force (N/m^3)

I coil current (A)

J induced eddy current (A/m^2)

K turbulent kinetic energy (m^2s^2)

p pressure (N/m^2)

r radial coordinate

S cross section area of elementary circuit (m^2)

z axial coordinate

μ_o magnetic permeability (H/m)

μ_{eff} effective viscosity (kg/m.s)

ρ density of the melt (kg/m^3)

ϵ turbulent energy dissipation (m^2s^3)

σ electric conductivity $(Ohm.m)^{-1}$

Figure Captions

1. Sketch of an induction furnace
2. Salient features of the apparatus
3. Experimentally measured velocity field 1900 Amp. coil current
4. Computed velocity field for 1900 Amp coil current
5. A comparison of the experimentally measured (full line) and theoretically predicted broken line) radial variation of the axial velocity component at a height of 22.5 cm.
6. A comparison of the experimentally measured (circles) and the theoretically predicted (full line) radial profile of the turbulence intensity.

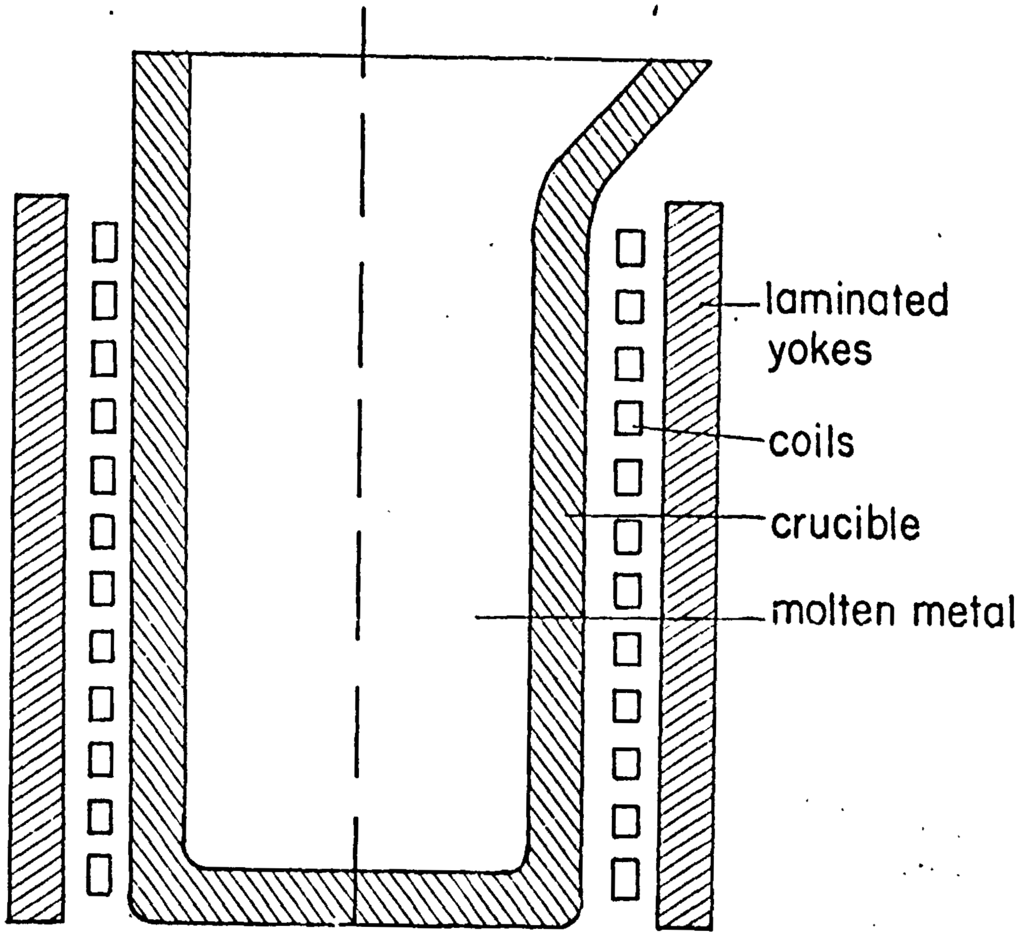

Fig. 1

fig (2

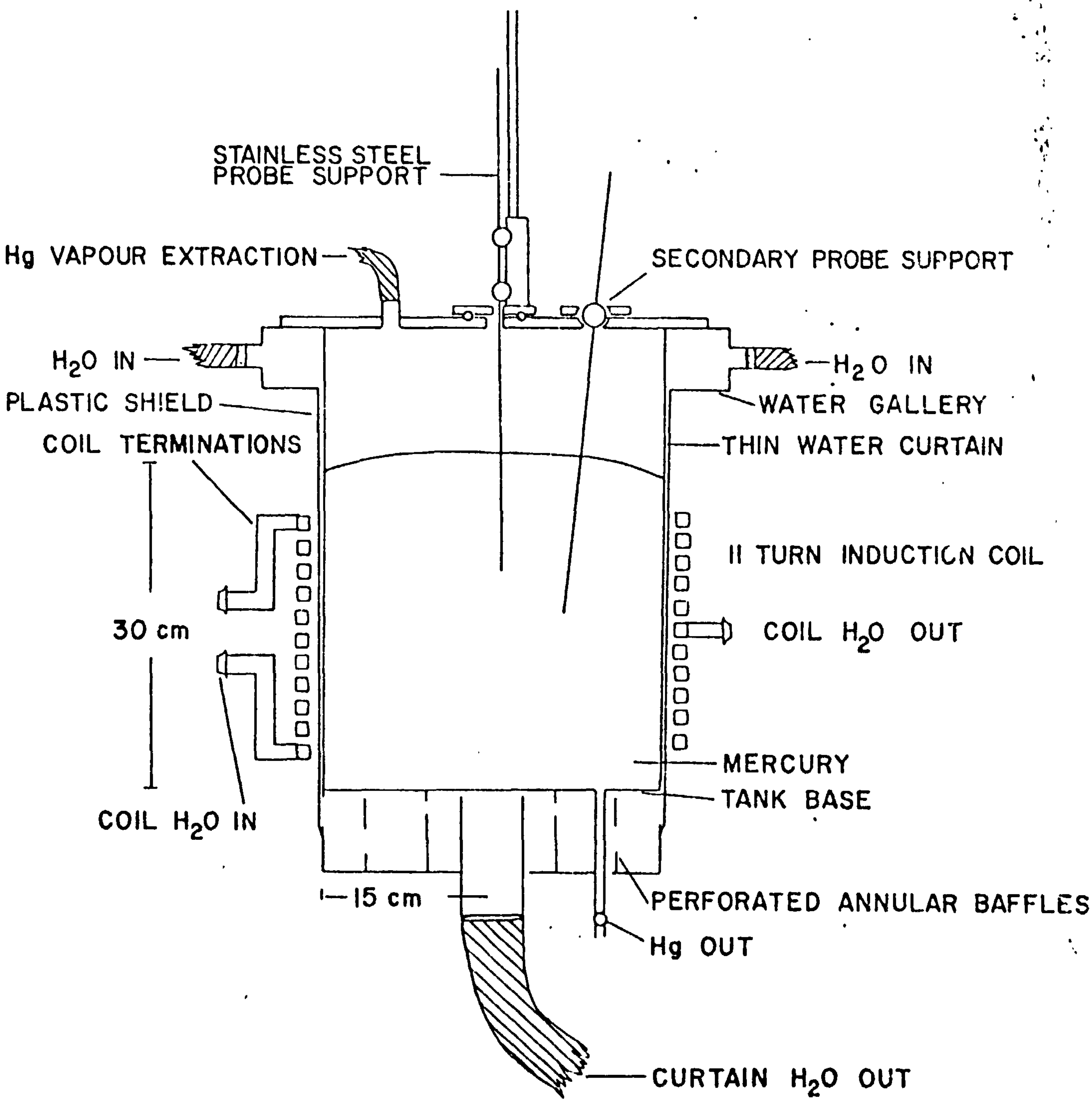
STAINLESS STEEL
PROBE SUPPORT
Hg VAPOUR EXTRACTION
SECONDARY PROBE SUPPORT
H2O IN
H2 O IN
PLASTIC SHIELD
WATER GALLERY
COIL TERMINATIONS
THIN WATER CURTAIN
II TURN INDUCTION COIL
30 cm
COIL H2O OUT
MERCURY
TANK BASE
COIL H2O IN
15 cm
PERFORATED ANNULAR BAFFLES
Hg OUT
CURTAIN H2O OUT

Fig. 2

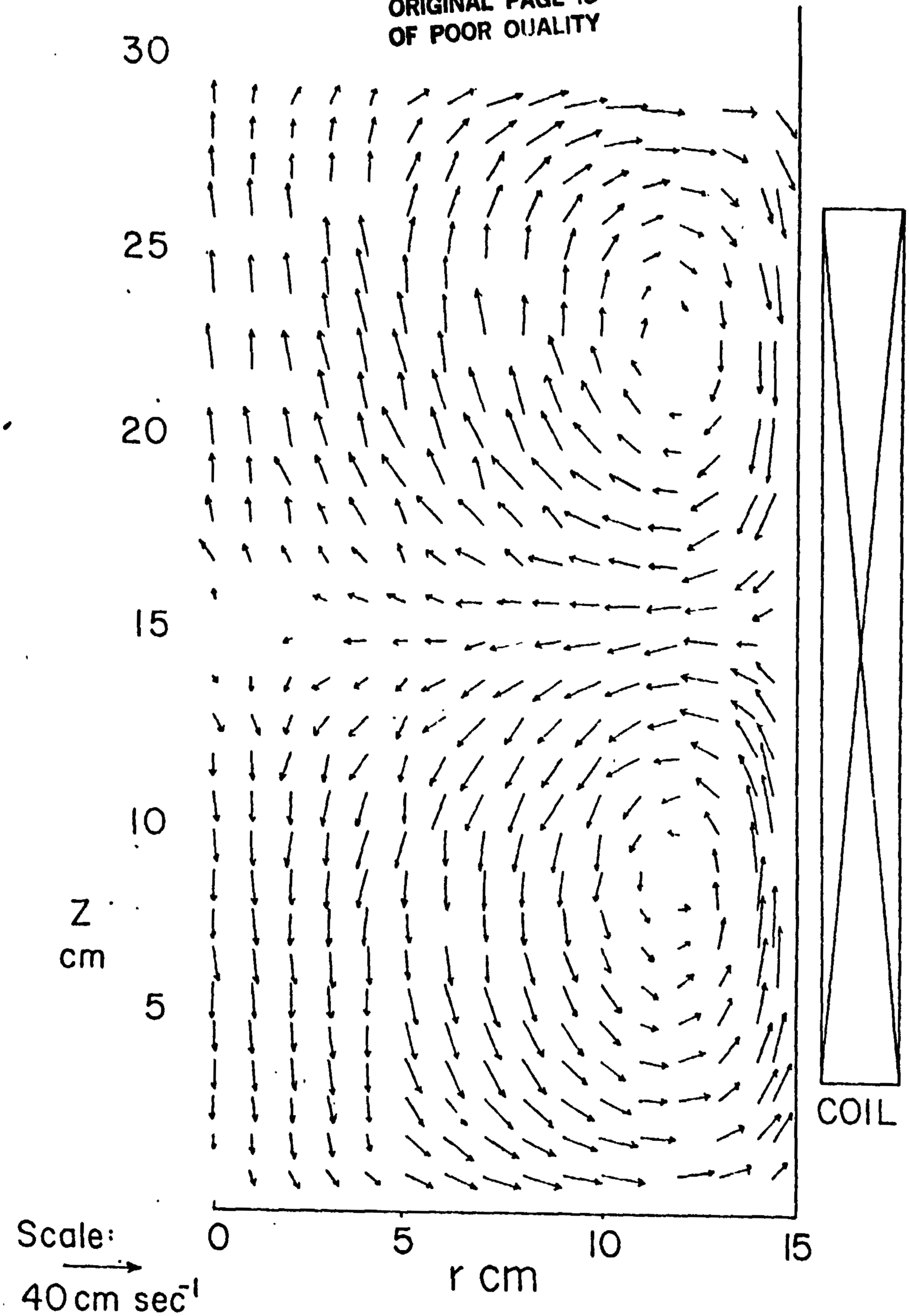

Fig. 3

fig (3)

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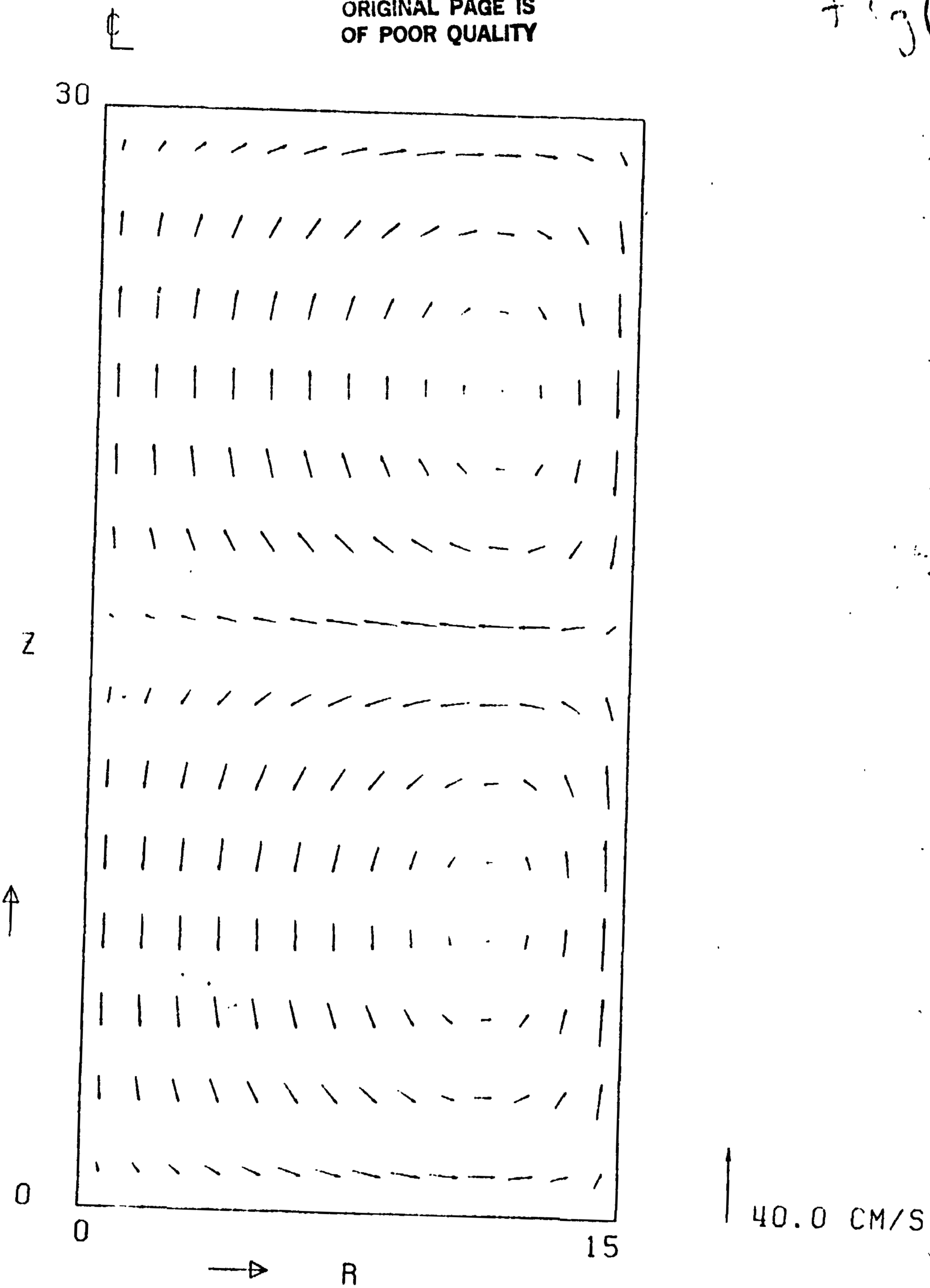
fig (4,



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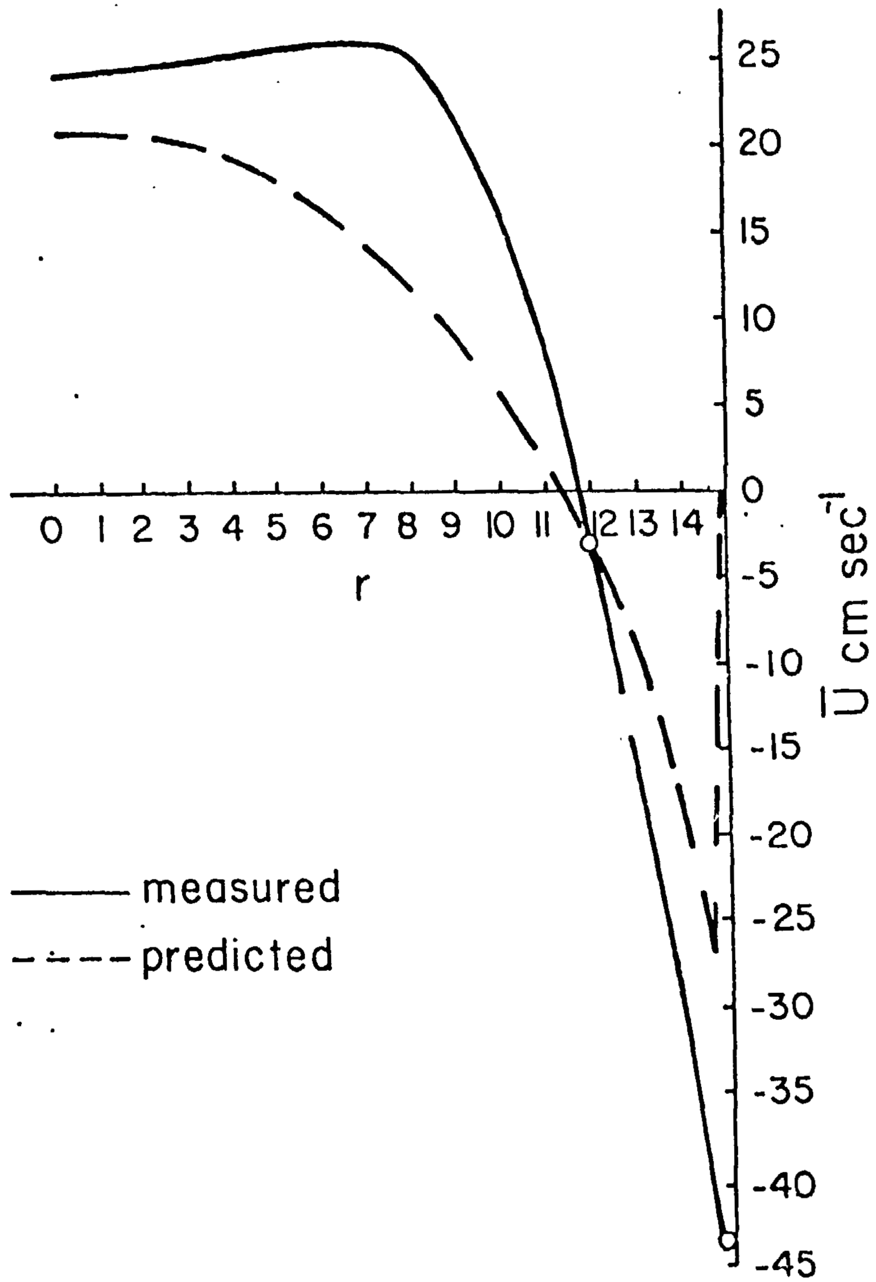
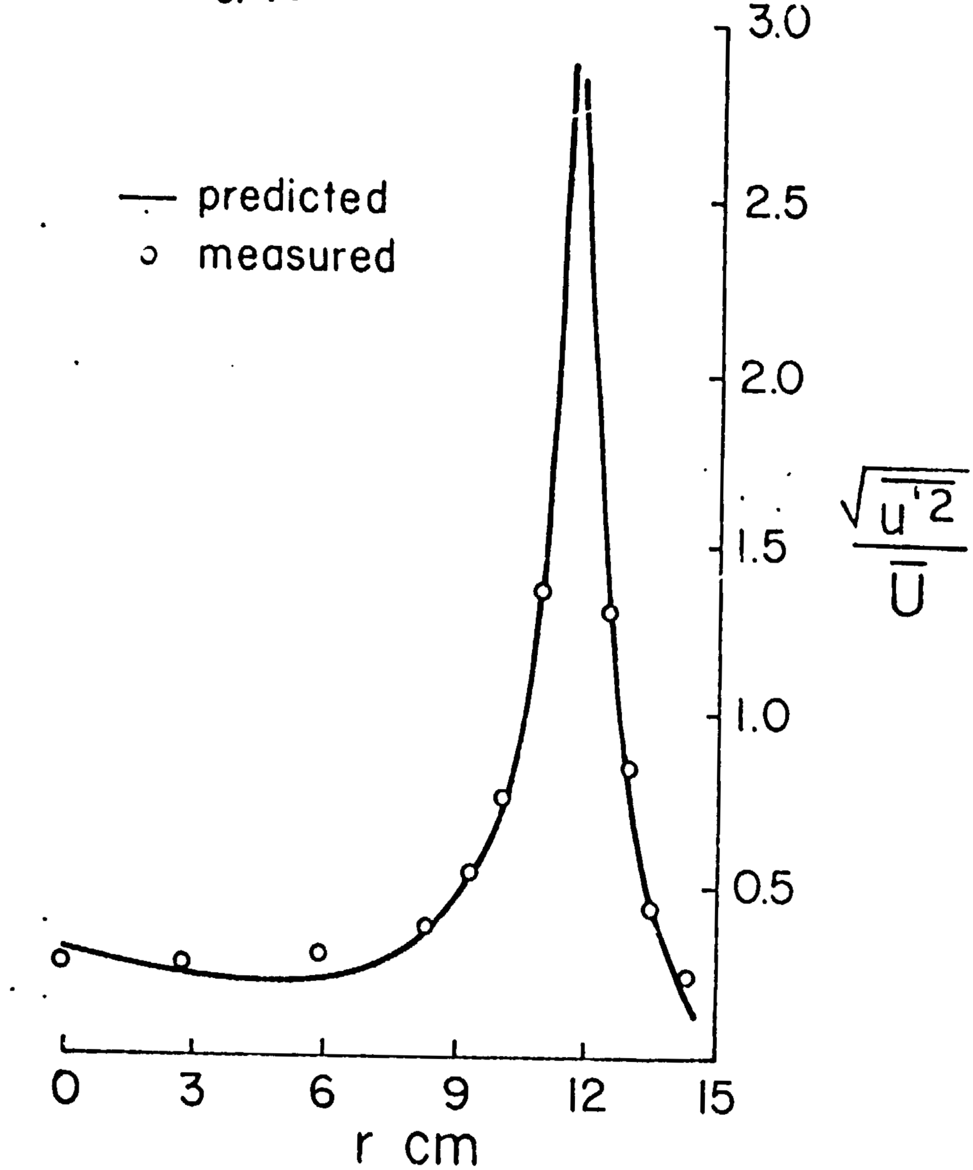


Fig. 5

fig (5)



— predicted
○ measured
3.0
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2.0
1.5
1.0
0.5
$\frac{\sqrt{\overline{u'^2}}}{\overline{U}}$
0
3
6
9
12
15
r cm

Fig. 6

fig(6)