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ABSTRACT

A reliability model is presented for planetary gear trains in which the ring gear is fixed, the sun gear is the input, and the planet arm is the output. The input and output shafts are co-axial and the input and output torques are assumed to be co-axial with these shafts. Thrust and side loading are neglected. This type of gear train is commonly used in main rotor transmissions for helicopters and in other applications which require high reductions in speed. The reliability model is based on the Weibull distribution of the individual reliabilities of the transmission components. The transmission's basic dynamic capacity is defined as the input torque which may be applied for one million input rotations of the sun gear. Load and life are related by a power law. The load-life exponent and basic dynamic capacity are developed as functions of the component capacities.

INTRODUCTION

In recent years, it has been commonly accepted that, in the design of mechanical components and systems, the fixed load-fixed strength approach must be complimented with a more realistic approach [1,2,3]*. It is not prudent to assess a proposed design by calculating a simple factor of safety. The more realistic approach is offered by the methods of probablistic design. In probabilistic design, a proposed design is evaluated in terms of statistically varying load and strength characteristics which more nearly model the true situation. A statistical (probabilistic) approach, which requires knowledge of the nominal loads and strengths as well as the statistical variations in each, allows the designer to assess the reliability or probability of survival of the mechanical system [2,3]. This is not possible with the factor of safety approach.

*Numbers in brackets denote references at the end of the paper.

The utility of a probabilistic approach to design is most apparent in the design of airborne power transmission systems. The requirements of low weight, high power densities, and high speeds must be balanced against requirements of reliability maintainability, and long mean times between overhauls (MBTO's). Currently, there is no suitable probabilistic design methodology for designing lightweight planetary gear trains for helicopter applications.

The probabilistic design approach has been applied to machine systems by Haugen and Smith [2,3] and to the design of epicyclic gear trains by Rao [4]. These design procedures have been based on the use of the Gaussian distribution for both the service load and for the component strengths. Both procedures also assume the existence of an endurance limit which is the limiting stress under which the components and the mechanical system have infinite lives.

It has been shown by Lundberg and Palmgren [5,6] and by Coy, Townsend and Zaretsky [7,8,9] that rolling element bearings and high strength steel gear teeth exhibit a finite life under any level of applied stress. The statistical model for the lives and capacities of these components follows the Weibull distribution [3,5,6,9-11]. The finite life condition of these components is due to the nature of pitting fatigue to which both gears and bearings are subjected. Even in carefully designed gears and bearings where adequate lubrication and no unexpected service conditions exist, pitting fatigue failure will eventually end the useful lives of both bearings and gears [5,6,9]. Therefore, in the present study, pitting fatigue is the mode of failure on which the reliability of each component is based.

In Rao's treatment of the epicyclic gear train [4], service load variation is considered and both tooth bending fatigue and pitting fatigue types of failure are admitted. However, the effects of the planet bearing lives on the system are not treated and the parallel planet load paths are treated as statistically redundant structural load paths which increase the system reliability. In reality, the planet bearings are critical elements in the assembly and broken component debris in a high speed transmission is sufficient to cause total system failure once a single component has failed. Thus, a strict series reliability model is required to adequately model a planetary gear train.

In view of the above mentioned, the object of the research reported herein is to derive a reliability model for planetary gear trains of the type used in helicopter main rotor transmissions. Therefore, the particular kinematic inversion of the gear train treated herein has the ring gear fixed, the sun gear as input and the planet carrier as output. It is assumed that the input and output shafts are co-axial, carrying simple torque loads. The reliability model is based on the reliabilities of the individual gears and bearings and is Weibull in nature. The transmission reliability is presented as a system life for 90 percent probability of survival of the entire assembly based on corresponding lives for the individual components. The transmission's basic dynamic capacity is defined as the input torque which may be applied for one million rotations of the input sun gear with a 90 percent probability of survival. The variation of life with load for a given reliability is modeled with a power law relation. When plotted on log-log coordiates the relation becomes a straight line. The relationship is treated as being uncoupled from the Weibull relationship of reliability to life at a given load [5,8,10]. The load life exponent and basic dynamic capacity are developed as functions of the component capacities.

KINETICS AND KINEMATICS

The gear train under consideration is shown in figure 1 in its most general configuration. There are stepped planet gears: an inner planet gear to mesh with the sun, and an outer planet gear to mesh with the ring. Each inner and outer planet is locked together as a single rigid body with a bearing at its center. The centers of these bearing are connected to a spider which provides the slower output motion. The number of planets may vary but each planet is assumed to be identical with the others. Each planet is assumed to carry an equal share of the total load. Figure 2 shows a single planet in mesh with the sun and the ring and joined to the spider or arm at its center A with a bearing. The radii of the spider and ring are related to the sun and planet radii as

$$R_A = R_S + R_{PS} \tag{1}$$

$$R_R = R_A + R_{PR} = R_S + R_{PS} + R_{PR} \tag{2}$$

The forces acting on the planet gears are shown in figure 3. The force components acting tangent to the pitch circles, F_S and F_R, in terms of the input torque, T_i, are

$$F_S = \frac{T_i}{nR_S} \tag{3}$$

$$F_R = \left(\frac{R_{PS}}{R_{PR}}\right) F_S = \frac{R_{PS}}{R_{PR}}\left[\frac{T_i}{nR_S}\right] \tag{4}$$

The tangential component of the bearing force is

$$F_t = F_S + F_R = \left(\frac{R_{PR} + R_{PS}}{R_{PR}}\right)\left[\frac{T_i}{nR_S}\right] \tag{5}$$

The total bearing load also includes a radial component due to the radial components of the gear tooth loads.

$$F_r = F_R \tan \phi_R - F_s \tan \phi_s \tag{6}$$

The total bearing load is thus

$$F_B = \sqrt{F_t^2 + F_r^2} \qquad (7)$$

It should be noted that this analysis assumes that the planet forces are contained in a radial plane of the transmission so no nutating loads exist on the planet bearings. This can be achieved in the stepped planet transmission by constructing the planets as spools with axial symmetry.

A kinematic analysis of this planetary is also required to determine the relative number of load cycles that each component sees as the input sun rotates. This is needed for the fatigue life analysis. The kinematic analysis has been derived in reference [12]. The results are presented in table 1, where the rotation of each component is given in terms of the rotation of the sun gear. All rotations are taken in the coordinate frame of the ring gear which is held fixed. Reading across in the table, for each of the components i, one obtains the terms for the following relative angular motion expression

$$\theta_i = \theta_{i/A} + \theta_A \qquad (8)$$

where A represents the arm or spider.

The itemized angular rotations in this table can be used to relate the number of load cycles of the various components to the number of input sun rotations.

PLANET BEARING RELIABILITY AND CAPACITY

The reliability and capacity of the planetary assembly is a function of the reliabilities and capacities of its components. These quantities have been well defined for the bearings [5,6,13]. The fatigue life model proposed in 1947 by Lundberg and Plamgren [5] is still the commonly accepted theory. The reliability of a single bearing can be expressed in terms of its probability of survival, S, for a life of ℓ rotations by the following relation

$$\log \frac{1}{S} \sim \tau^c \, \ell^{e_B} \, \frac{V}{z^h} \qquad (9)$$

where τ is the critical shearing stress beneath the surface, z is the depth under the surface to the location of the critical stress, and V is stressed volume. The exponents are determined from experimental life testing on groups of bearings run under identical conditions. The Weibull exponent e_B is a measure of the scatter in the distribution of bearing lives.

The above formula for probability of survival reflects the observed effects of stress, stress field, and stress cycles on reliability. Greater stress, τ, decreased reliability. A more shallow stress field (smaller z) decreases reliability. This is true because it is expected that a microcrack beginning at a point of maximum stress under the surface requires some time to propagate to the surface. Therefore, for any given number of stress cycles, there is a higher probability that cracks have propagated to the surface for the more shallow stress field.

The stressed volume V is also an important factor. Pitting initiation occurs near any small stress raising imperfection in the material. The larger the stressed volume, the greater the likelihood of failure.

For a given load and geometry of the bearing the expression can be written in terms of the ℓ_{10} life, which is the life corresponding to a probability of survival of 90 percent.

$$\log \frac{1}{S} = \log \frac{1}{.9} \left(\frac{\ell_B}{\ell_{B10}}\right)^{e_B} \tag{10}$$

The relationship between the bearing life and its load for a 90 percent probability of survival is

$$\ell_{B10} = \left(\frac{C_B}{F_B}\right)^{p_B} \tag{11}$$

where F_B is the load on the bearing, p_b is the load-life exponent and C is the basic dynamic capacity of a single bearing. The basic dynamic capacity is defined as the load which may be endured by 90 percent of the bearings for one million inner race revolutions under certain operating conditions.

To facilitate the combination of lives and capacities of all the transmission components into a single life and capacity for the transmission, the lives and the dynamic capacities of each of the components will be expressed in terms of input sun gear rotations and input sun gear torque.

From table 1, the bearing inner race rotation is given in terms of sun rotations

$$\theta_B = \frac{1}{\frac{R_{PS}}{R_S} + \frac{R_{PR}}{R_R}} \theta_S \tag{12}$$

Using lower case ℓ's to designate component lives in terms of component cycles and upper case L's to designate component lives in terms of input sun gear rotations, equation (12) transforms equation (10) into

$$\log \frac{1}{S_B} = \log \frac{1}{.9} \left\{ \frac{R_S\, R_R\, L_B}{(R_R\, R_{PS} + R_S\, R_{PS})\, \ell_{B10}} \right\}^{e_B} \tag{13}$$

For a 90 percent survival rate for a planet bearing, $S_B = .9$ and $L_B = L_{B10}$ sun gear rotations, substitution into equation (13) yields

$$L_{B10} = \left(\frac{R_R\, R_{PS} + R_S\, R_{PR}}{R_R\, R_S} \right) \ell_{B10} \tag{14}$$

as expected from equation (12).

To obtain the load-life relation for the bearing in terms of transmission input parameters, one can substitute the expressions for bearing load in terms of input torque as given by equations (3-7) into equation (11) and substitute all of this into equation (14)

$$L_{B10} = \left[\frac{R_R R_{PS} + R_S R_{PR}}{R_R R_S} \right] \left\{ \frac{n\, R_S\, C_B}{T_i \left[\left(\frac{R_{PR} + R_{PS}}{R_{PR}} \right)^2 + \left(\frac{R_{PS}}{R_{PR}} \tan\phi_R - \tan\phi_S \right)^2 \right]^{1/2}} \right\}^{p_B} \tag{15}$$

The dynamic capacity of a planet bearing is now the input torque on the sun shaft which may be applied with 90 percent of the planetary bearings surviving for one million sun shaft revolutions. From equation (15) the planet bearing system dynamic capacity of $T_i = D_B$ is obtained when $L_{B10} = 1.0$. The result for the dynamic capacity is

$$D_B = \left(\frac{R_{PS} R_R + R_{PR} R_S}{R_R R_S} \right)^{\frac{1}{p_B}} \left\{ \frac{n R_S C_B}{\left[\left(\frac{R_{PR} + R_{PS}}{R_{PR}} \right)^2 + \left(\frac{R_{PS}}{R_{PR}} \tan\phi_R - \tan\phi_S \right)^2 \right]^{1/2}} \right\} \tag{16}$$

The relationship between bearing life in millions of sun rotations and applied sun shaft torque for which 90 percent of the bearings will endure is given by

$$L_{B10} = \left(\frac{D_B}{T_i} \right)^{p_B} \tag{17}$$

The fundamental quantities that describe the reliability and life distribution for single bearings and bearings treated as transmission components have now been determined. Finally, the probability distribution for the reliability of a planet bearing is written as

$$\log \frac{1}{S_B} = \log \frac{1}{.9} \left(\frac{L_B}{L_{B10}}\right)^{e_B} \tag{18}$$

Where L_B is the number of million sun rotations for which the bearing set has the probability of survival, S_B.

SUN GEAR RELIABILITRY AND CAPACITY

Surface fatigue life and dynamic capacity for a spur gear have been the subjects of recent research [7,8,9]. This research has applied the previously mentioned Lundberg-Palmgren reliability model to spur gears.

Tests have shown that the pitting fatigue life of gears follows this reliability relationship, but with a different Weibull exponent, e_G, than that for bearings.

$$\log \frac{1}{S} = \log \frac{1}{.9} \left(\frac{\ell}{\ell_{10}}\right)^{e_G} \tag{19}$$

where S is the probability of survival of a single gear tooth and ℓ is the number of stress cycles imposed on the gear tooth surface.

The load life relationship for a single tooth for a 90 percent probability of survival is

$$\ell_{10} = \left(\frac{C_t}{F}\right)^{p_G} \tag{20}$$

where F is the transmitted tangential tooth load and C_T is the basic dynamic capacity of the tooth which has been developed in reference [9].

$$C_t = B_1 f^a \Sigma\rho^b \ell^c \tag{21}$$

where f is the active tooth face width, $\Sigma\rho$ is the curvature sum at the pitch point, ℓ is the length of heaviest load contact on the tooth and the constant B_1 and exponents a, b, c are based on experimental results from gear life testing. For case hardened AISI 9310 Vacuum Arc Remelt Steel gears, these constants are: $B_1 = 20832$, $a = 0.907$, $b = -1.16$, and $c = 0.093$ in the pound-inch system of units.

At this point, this fundamental gear tooth reliability equation is applied to the sun gear. Since there are n planets, for a number of rotations of the sun gear relative to the planet carrier, $L_{S/A}$, each tooth on the sun sees n $L_{S/A}$ load cycles. From table 1, the number of load cycles, ℓ_S, are expressed in terms of sun rotations, L_S, as:

$$\ell_S = \frac{nL_S}{1 + \frac{R_S R_{PR}}{R_R R_{PS}}} \qquad (22)$$

The probability of survival for the sun gear, S_S, follows from the application of the product law for the number of teeth on the sun gear, N_S.

$$S_S = S^{N_S} \qquad (23)$$

Using equations (19), (22) and (23) the expression for the reliability of the sun gear becomes:

$$\log \frac{1}{S_S} = N_S \log \frac{1}{.9} \left\{ \left(\frac{n R_R R_{PS}}{R_R R_{PS} + R_S R_{PR}} \right) \left(\frac{L_S}{\ell_{S10}} \right) \right\}^{e_G} \qquad (24)$$

where L_S is the number of million sun revolutions corresponding to the probability of survival S_S for the sun gear and ℓ_{S10} is the number of million stress cycles that a single tooth on the sun gear may endure with a 90 percent probability of survival. Equation (24) is directly parallel to Equation (13) for the planet bearings. By similar arguments, the 90 percent life of the sun gear in terms of sun gear rotations is related to the 90 percent life of a single sun gear tooth in terms of tooth load cycles as:

$$L_{S10} = \left(\frac{1}{N_S} \right)^{1/e_G} \left\{ \frac{R_R R_{PS} + R_S R_{PR}}{n R_R R_{PS}} \right\} \ell_{S10} \qquad (25)$$

As with the planet bearings, one can substitute the single tooth load life expression (equation 20) into equation (25) and express the transmitted load in terms of the input transmission torque (equation 3) in order to obtain the expression for the sun gear dynamic capacity, D_S, in terms of an individual sun tooth's dynamic capacity, C_S, when L_{S10} equals 1 million sun rotations:

$$D_S = \left(\frac{1}{N_S} \right)^{\frac{1}{e_G p_G}} \left[\frac{R_R R_{PS} + R_S R_{PR}}{n R_R R_{PS}} \right]^{\frac{1}{p_G}} n R_S C_S \qquad (26)$$

The relationship for sun gear life and applied sun shaft torque for which 90 percent of the sun gears will survive is now given by:

$$L_{S10} = \left(\frac{D_S}{T_i} \right)^{p_G} \qquad (27)$$

Finally, the probability distribution for the reliability of the sun gear can be written as:

$$\log \frac{1}{S_S} = \log \frac{1}{.9} \left(\frac{L_S}{L_{S10}} \right)^{e_g} \tag{28}$$

where L_S is the number of million sun rotations for which the sun gear has the probability of survival S_S.

RING GEAR RELIABILITY AND CAPACITY

The reliability and dynamic capacity of the ring gear is developed in a similar fashion to that of the sun gear. Equations (19), (20) and (21) are equally valid for the ring gear teeth as they are for the sun gear teeth. Different values for tooth life and basic dynamic tooth capacity result from the difference in tooth mesh geometry. With teeth of the same pitch and face width as those on the sun gear, the ring gear should be considerably more reliable and should have a higher basic dynamic capacity due to the conformal contact of the internal gear teeth of the ring with the external teeth of the planet gear. The relationship between the number of load cycles on a ring gear tooth, ℓ_R, and the number of sun rotations, L_R, taken from table 1 is:

$$\ell_R = \frac{n\, L_R}{1 + \frac{R_R\, R_{PS}}{R_S\, R_{PR}}} \tag{29}$$

The probability of survival for the ring gear, S_R, follows from a direct application of the product law for the number of teeth on the ring gear, N_R.

$$S_R = S^{N_R} \tag{30}$$

where S is the probability of survival of a single tooth on the ring gear. Combining equation (19) with equations (29) and (30) yields the expression for the reliability of the ring gear.

$$\log \frac{1}{S_R} = N_R \log \frac{1}{.9} \left\{ \frac{n\, R_S\, R_{PR}\, L_R}{(R_S R_{PR} + R_P R_{PS}) \ell_{R10}} \right\}^{e_G} \tag{31}$$

The 90 percent life of the ring gear in terms of sun gear rotations is related to the 90 percent life of a single ring gear tooth in terms of tooth load cycles by:

$$L_{R10} = \left(\frac{1}{N_R}\right)^{\frac{1}{e_G}} \left\{\frac{R_S R_{PR} + R_P R_{PS}}{n R_S R_{PR}}\right\} \ell_{R10} \tag{32}$$

The basic dynamic capacity, D_R, of the ring gear is the input sun torque for which the ring gear has a 90 percent probability of surviving for one million rotations of the sun gear:

$$D_R = \left(\frac{1}{N_R}\right)^{\frac{1}{e_G p_G}} \left[\frac{R_S R_{PR} + R_P R_{PS}}{n R_S R_{PR}}\right]^{\frac{1}{p_G}} \left\{\frac{n R_S R_{PR}}{R_{PS}} C_R\right\} \tag{33}$$

The relationship for ring gear life in terms of applied sun shaft torque for which 90 percent of the ring gears will survive is now given by

$$L_{R10} = \left(\frac{D_R}{T_i}\right)^{p_G} \tag{34}$$

The basic quantities of reliability and life are now also established for the ring gear. The probability distribution for the reliability of the ring gear can be written as

$$\log \frac{1}{S_R} = \log \frac{1}{.9} \left(\frac{L_R}{L_{R10}}\right)^{e_G} \tag{35}$$

where L_R is the number of million sun rotations for which the ring gear has the probability of survival S_R.

PLANET GEAR RELIABILITY AND CAPACITY

The last set of elements in the planetary transmission which possess finite pitting fatigue lives are the planet gears themselves. These gears mesh with both the sun gear and the ring gear. However, as can be seen in figures 2 and 3, the loads of the two meshes are carried on the opposite sides of the planet teeth. Thus, even if the planets are not stepped and $R_{PS} = R_{PR}$, the pitting damage accumulation from each mesh is independent of the other as long as increased dynamic loading does not occur. It is assumed in this model that this increased dynamic loading occurs after the onset of failure, so the two failure accumulations are counted separately.

The number of load cycles that each planet tooth sees as a function of the number of sun rotations is taken from table 1 as the relative rotation of the planet with respect to the arm:

$$\ell_p = \frac{R_R R_S L_R}{R_R R_{PS} + R_S R_{PR}} \tag{36}$$

This number of load cycles is the same for the teeth meshing with the sun gear and the teeth meshing with the ring gear, though the fatigue damage at the two meshes differ.

The probability of survival for the planet gear, S_P, is the product of the probabilities at each mesh

$$S_P = S_{PS}^{\;N_{PS}} \cdot S_{PR}^{\;N_{PR}} \tag{37}$$

Combining equation (19) for each mesh with equations (36) and (37) yields the expression for the reliability of a planet gear:

$$\log \frac{1}{S_p} = N_{PS} \log \frac{1}{.9} \left\{ \frac{R_R R_S L_P}{(R_S R_{PR} + R_R R_{PS}) \ell_{PS10}} \right\}^{e_G} + N_{PR} \log \frac{1}{.9} \left\{ \frac{R_R R_S L_P}{(R_S R_{PR} + R_R R_{PS}) \ell_{PR10}} \right\}^{e_G} \tag{38}$$

The 90 percent life of the planet gear in terms of the 90 percent lives of its teeth is thus:

$$L_{P10} = \left[\frac{R_S R_{PR} + R_R R_{PS}}{R_R R_S} \right] \left\{ \frac{\ell_{PS10} \ell_{PR10}}{(N_{PS})^{1/e_G} \ell_{PR10} + (N_{PR})^{1/e_G} \ell_{PS10}} \right\} \tag{39}$$

The basic dynamic capacity, D_P, of a planet gear is the input sun torque for which the planet gear has a 90 percent probability of surviving for one million rotations of the sun gear:

$$D_P = \left[\frac{R_S R_{PR} + R_R R_{PS}}{R_R R_S} \right]^{\frac{1}{p_G}} \left\{ \frac{n R_S R_{PR} C_S C_R}{N_{PS}^{\frac{1}{e_G p_G}} R_{PR} C_R + N_{PR}^{\frac{1}{e_G p_G}} R_{PS} C_S} \right\} \tag{40}$$

The relationship for planet gear life in terms of applied sun shaft torque for which 90 percent of the planet gears will survive is now:

$$L_{P10} = \left(\frac{D_P}{T_i} \right)^{p_G} \tag{41}$$

The fundamental quantities needed to describe the reliability and life distribution for all gears in the transmission have now been determined.

Finally, the probability distribution for the reliability of a planet gear is written as:

$$\log \frac{1}{S_p} = \log \frac{1}{.9} \left(\frac{L_p}{L_{P10}} \right)^{e_G} \tag{42}$$

SYSTEM RELIABILITY AND CAPACITY

The product rule may be used to express the probability of survival of the total system consisting of the planet bearings, the sun gear, the planet gears and the ring gear.

$$S_T = S_B^n S_S S_P^n S_R \tag{43}$$

The probability distribution for the survival of the total transmission can now be obtained by substituting equations (18), (28), (35) and (42) into the natural log of the reciprocal of equation (43).

$$\log \frac{1}{S_T} = \log \frac{1}{.9} \left\{ n\left(\frac{L_T}{L_{P10}}\right)^{e_B} + \left(\frac{L_T}{L_{S10}}\right)^{e_G} + n\left(\frac{L_T}{L_{P10}}\right)^{e_G} + \left(\frac{L_T}{L_{R10}}\right)^{e_G} \right\} \tag{44}$$

Since all the component lives are counted in the same units of sun rotations, this count is now identical for all the components and is thus labeled as L_T in the expression for the probability of survival S_T for the entire transmission.

Unfortunately, equation (44) is not a strict Weibull relationship between system life and system reliability. This equation would represent a true Weibull distribution only if $e_b = e_G$ which is not the case in general. The relationship of equation (44) can be plotted on Weibull coordinates as shown in figure 4.

The examples presented in this plot and the succeeding plots are for the planetary of figure 1 with 20 teeth on the sun, 85 teeth on the ring, 40 teeth on the three planet gears which mesh with the sun and 25 teeth on the three planet gears which mesh with the ring. The gears all have a module of 1.59 mm (Pd = 16) and face widths of 9.19 mm (0.375 in.). The gears are standard 20 degree involute and are all made of AISI 9310 steel. The transmission is loaded with an input sun torque of 48 N-m (425 pound-inches). For the cases treated in this study, the bearing Weibull exponent is 1.2 while the gearing Weibull exponent is 2.5. For this balanced case, each planet bearing has a basic dynamic capacity of 15,250 N (3425 pounds).

This curve can be approximated by a straight line using the least squared error approach over a range such as $0.5 \leq S_T \leq 0.95$. The slope of this straight line approximation is called the system Weibull slope e_T and the system life of the straight line approximation at $S_T = 0.9$ is called the system 90 percent reliability life.

The exact L_{T10} life can be calculated by setting $S_T = 0.9$ in equation (44) and iterating for L_{T10} in the simplified equation:

$$1 = n\left(\frac{L_{T10}}{L_{B10}}\right)^{e_B} + \left(\frac{L_{T10}}{L_{S10}}\right)^{e_G} + n\left(\frac{L_{T10}}{L_{P10}}\right)^{e_G} + \left(\frac{L_{T10}}{L_{R10}}\right)^{e_G} \qquad (45)$$

For the cases studied in this research, the defined Weibull L_{T10} life has not differed from the L_{T10} life calculated from equation (45) by more than one percent. Since this error is considerably less than that between test data for the components and the resulting component Weibull lines, it is felt that the approximation is justified. When one component is weak relative to the rest of the transmission, the reliability model of the entire transmission and the least squares approximation will approach the Weibull model of the weak component. This is shown in figure 5 which is a series of Weibull plots for the planet bearing, sun gear and total transmission for the case of a transmission with $L_{B10} = 1.7$ million cycles, $L_{S10} = 84.3$ million cycles and $L_{T10} = 0.68$ million cycles. This example differs from that of figure 4 in that the planet bearings are weaker with a dynamic capacity of 3460 N (800 pounds). In this case, the bearing life dominates the transmission and the transmission Weibull exponent is 1.2.

If the bearing capacity approaches that of the sun mesh, then the actual transmission reliability curve deviates the most from the least squares Weibull approximation. This is shown in figure 6 which is a series of Weibull plots for the planet bearing, sun gear and total transmission for the case of a transmission with $L_{B10} = 207$ million sun rotations, $L_{S10} = 84.3$ million sun rotations and $L_{T10} = 55.5$ million sun rotations. This is the same case shown in figure 4. For this case, the transmission Weibull exponent is 1.84.

For this straight line transmission Weibull curve, the reliability of the transmission is approximated by:

$$\log \frac{1}{S_T} = \log \frac{1}{.9} \left(\frac{L_T}{L_{T10}}\right)^{e_T} \qquad (46)$$

The basic dynamic capacity for the transmission, D_T, is the sun input torque required to produce a system 90 percent reliability life, L_{T10} of one million sun rotations. By letting $S_T = 0.9$ in equation (44) and substituting equations (17), (27), (34) and (41), one has for $L_{T10} = 1$;

$$1 = n\left[\frac{D_T}{D_B}\right]^{p_B e_B} + \left[\frac{D_T}{D_S}\right]^{p_G e_G} + n\left[\frac{D_T}{D_P}\right]^{p_G e_G} + \left[\frac{D_T}{D_R}\right]^{p_G e_G} \qquad (47)$$

The basic dynamic capacity of the transmission can be found by iterating this expression since the component exponents and capacities are known. It can also be found from equation (45) by determining a sequence of L_{T10}'s corresponding to a sequence of input sun torques, T_i's, and plotting the natural log of T_i versus the natural log of L_{T10}. The value of T_i corresponding to L_{T10} = 1 million sun rotations is the transmission basic dynamic capacity. A plot of log T_i versus log L_{T10} is shown in figure 7 from the transmission example of figure 6. The slope of this curve is the negative of the load life exponent P_T for the transmission. The case shown is that of nearly equal lives and capacities in which the deviation from a straight line relation is maximized. For the cases studied in this research $P_B = 3.3$ and $P_G = 4.3$. For this example the basic dynamic capacities are: D_B = 242 N-m (2140 pound inches), D_S = 135 N-m (1190 pound inches), and D_T = 134 N-m (1180 pound inches). As for the transmission Weibull model, an approximate load-life curve is obtained by a least squares fit over a range of input torques (i.e., $0.1\ D_T \leq T_i \leq D_T$). With this approximation the load-life relation for the system is given by:

$$L_{T10} = \left(\frac{D_T}{T_i}\right)^{P_T} \tag{48}$$

For the example plotted in figure 7, the transmission load-life exponent, P_T, is 3.8. As for the Weibull model, a weak component will dominate the transmission dynamic capacity and the system capacity and load-life factor will approach that of the weakest component.

SUMMARY

A reliability model for the planetary gear train has been derived for use in the probabalistic design of this type of transmission. This gear train has the ring gear fixed, the sun gear as input and the planet carrier as output. The input and output shafts are assumed to be co-axial with the applied torques and each other; no side loading is considered.

The reliability model is based on reliability models of the bearing and gear mesh components which are two dimensional Weibull distributions of reliability as a function of life. The transmission's 90 percent reliability life and basic dynamic capacity are presented in terms of input sun rotations and torque. This life and capacity are given as exact functions of the component lives and capacities. However, due to the different distributions for the bearing and gearing components, the Weibull model for the planetary transmission is an approximate model. In this model, the transmission's 90 percent reliability life, Weibull exponent, basic dynamic capacity and load-life exponent are presented.

The following results were obtained:

1. A system reliability model for planetary sput gear trains including the planet bearings and the possibility of stepped planets was formulated;

2. The fact that Weibull reliability distributions with different Weibull exponents do not follow the law of mathematical closure was disclosed; and,

3. Straight line Weibull and load-life exponents were formulated for a system Weibull model containing different types of components.

NOMENCLATURE

C	basic dynamic component capacity (N)
D	basic dynamic system capacity (N-m)
e	Weibull exponent
f	gear face width (m)
F	force (N)
ℓ	life in millions of component load cycles
L	life in millions of sun gear rotations
n	number of planets
N	number of gear teeth
p	load-life exponent
R	gear radius (m)
S	probability of survival (reliability)
T_i	input torque (N-m)
V	stressed volume (m^3)
z	depth to maximum shear stress (m)
$\Sigma\rho$	curvature sum at pitch point (m^{-1})
θ	angular rotation
τ	maximum shear stress (P_a)
ϕ	pitch line pressure angle

Subscripts

A	planet carrier or arm
B	planet bearing
r	radial direction
P	planet gear
PR	planet gear meshing with ring gear
PS	planet gear meshing with sun gear
R	ring gear
S	sun gear
T	transmission
t	tangential direction
10	corresponding 90 percent probability of survival

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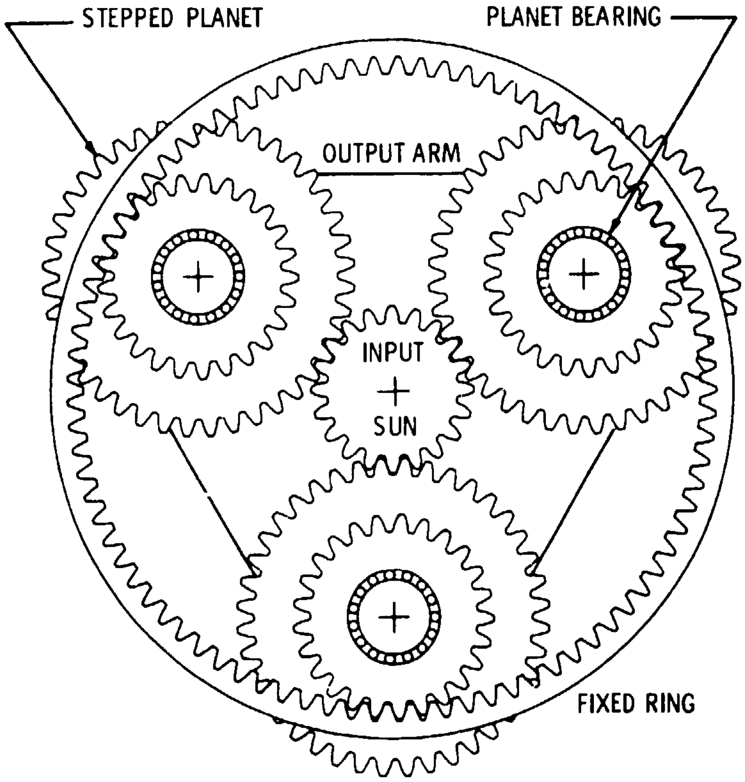

Figure 1. - Planetary gear train.

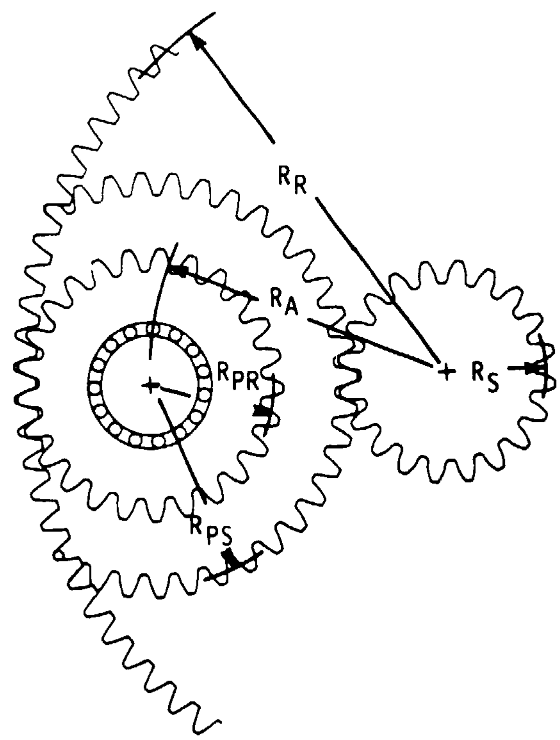

Figure 2. - Planetary geometry.

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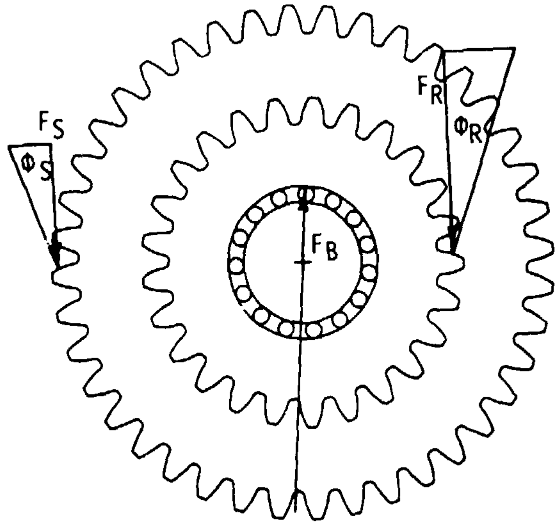

Figure 3. - Planet gear forces.

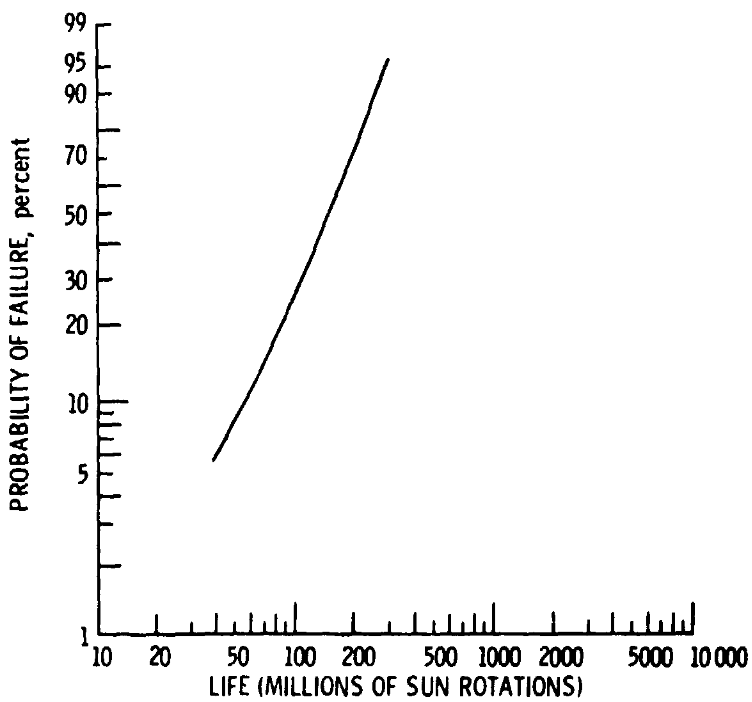

Figure 4. - Weibull graph of S_T versus L_T for a full planetary system.

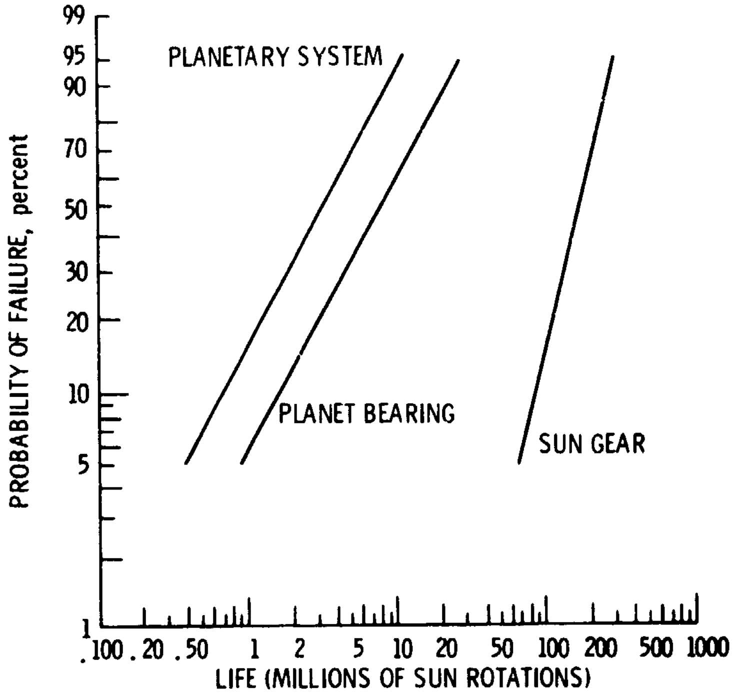

Figure 5. - Weibull graphs for sun gear, planet bearing and transmission for a planetary with a weak planet bearing.

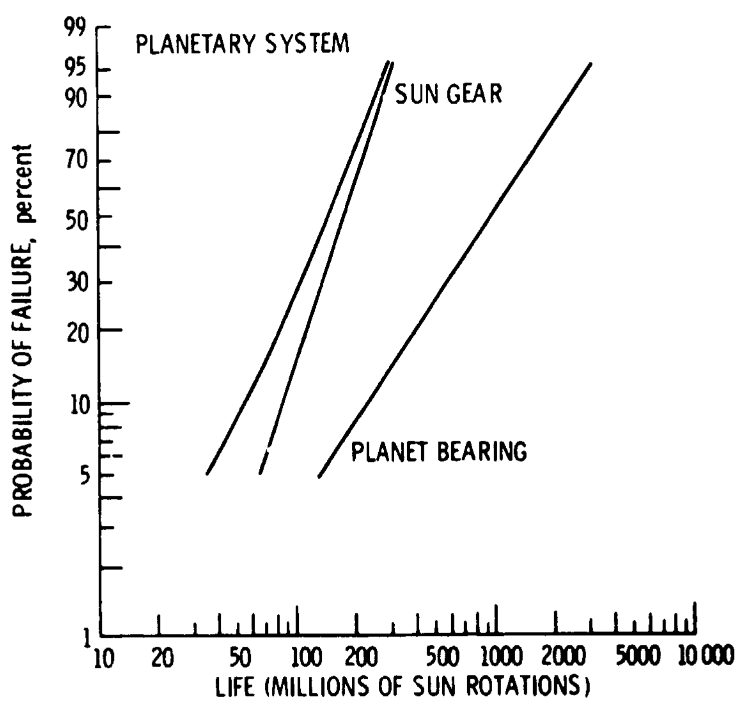

Figure 6. - Weibull graphs for sun gear, planet bearing, and transmission for a planetary with balanced components.

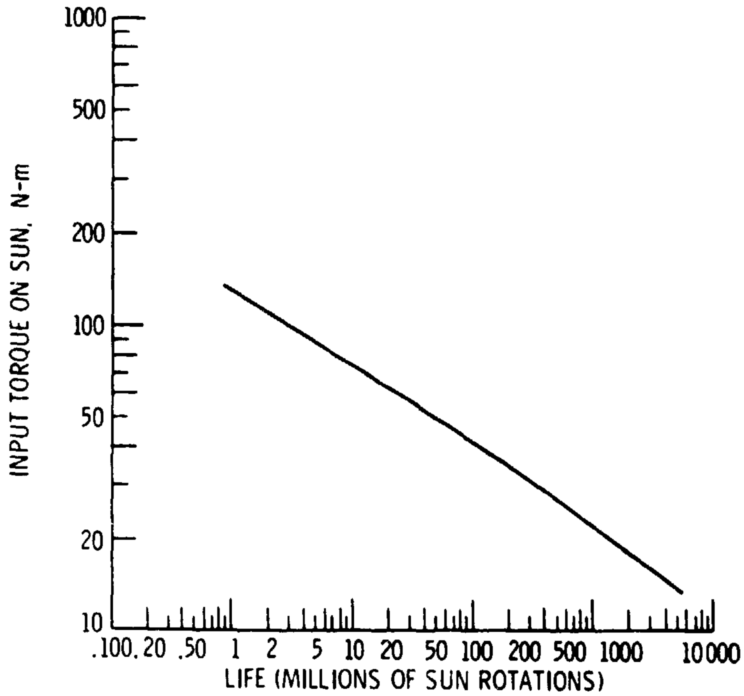

Figure 7. - Load-life curve for a full planetary system.