6.4 MAP PROJECTIONS FOR LARGER-SCALE MAPPING*

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ABSTRACT

After decades of using a single map projection, the Polyconic, for its mapping program, the U.S. Geological Survey now uses several long-established projections for its published maps, both large and small scale. For maps at 1:1,000,000-scale and larger, the most common projections are conformal, such as the Transverse Mercator and Lambert Conformal Conic. Projections for these scales should treat the Earth as an ellipsoid. In addition, the USGS has conceived and designed some new projections, including the Space Oblique Mercator, the first map projection designed to permit low-distortion mapping of the Earth from satellite imagery, continuously following the groundtrack. The USGS has programmed nearly all pertinent projection equations for inverse and forward calculations. These are used to plot maps or to transform coordinates from one projection to another. The USGS is also publishing its first comprehensive map projection manual, describing in detail and mathematically all projections used by the agency.

INTRODUCTION

The U.S. Geological Survey was created in 1879, and detailed large-scale mapping of the country soon became one of its primary objectives. It has relied heavily over the years on the former U.S. Coast and Geodetic Survey (USC&GS, now the National Ocean Survey) for guidance on map projections. Until the late 1950's, only the Polyconic projection was used for the primary USGS mapping product, i.e., large-scale quadrangle maps.

The Polyconic projection was apparently invented and certainly promoted by Ferdinand Rudolph Hassler, the first head of what was to become known as the Coast and Geodetic Survey. In the 1950's, the USGS quadrangle projection was changed to the Lambert Conformal Conic and the Transverse Mercator projections, which had been adopted by the Coast and Geodetic Survey in the 1930's for the State Plane Coordinate System. The development of standardized zones based upon the Universal Transverse Mercator and the polar Stereographic led to USGS use of these projections. In addition to these, the regular Mercator, the Oblique Mercator, the Albers Equal-Area Conic, and the Azimuthal Equidistant have been used for other larger-scale mapping by the USGS and other agencies. Although authors and organizations variously define large-

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and intermediate-scale mapping, for the purposes of this paper, the term "larger scale" will apply to scales larger than 1 to 2 million.

When the space age added its impact to mapping, classical projections (Mercator, Lambert Conformal Conic, and Stereographic) were chosen for the mapping of the Earth's Moon, three other planets, and a number of other natural satellites. Some projections, especially the Space Oblique Mercator, originated within USGS to assist mapping from satellite imagery.

TYPES OF PROJECTION

Before describing the projections themselves, I'd like to review briefly the different types. Equal-area or equivalent projections of the globe are used especially by geographers seeking to compare land use, densities, and the like. On an equal-area projection, such as the Albers Equal-Area Conic, a coin laid on one part of the map covers exactly the same area of the actual Earth as the same coin on any other part of the map. Shapes, angles, and scale must be distorted on most parts of such a map, but there are usually certain lines on an equal-area map as well as on other types of projections, along which there is no distortion of any kind. These so-called "standard lines" may be a meridian, one or two parallels, lines which are neither, or not a line but a point.

More commonly used in larger-scale mapping are conformal (orthomorphic) projections such as the Transverse Mercator and the Lambert Conformal Conic. The term means that they are correct in shape, but, unlike the term "equal area," the conformal principle applies only to each infinitesimal element of the map. Angles at each point are correct, and consequently the local scale in every direction around any one point is constant, so the map user can measure distance and direction between near points with a minimum of difficulty. Conformal maps may also be prepared by fitting together small pieces of other conformal maps which have been enlarged or reduced; non-conformal projections require reshaping as well. When the region consists of more than a small element, distortion in shape as well as area becomes appreciable. This is especially serious with the most famous conformal projection -- the Mercator -- because of its widespread use in classrooms, especially in the past. Because there is no angular distortion, all meridians intersect parallels at right angles on a conformal projection, just as they do on the Earth. "Standard lines" may also be applied to a conformal map to eliminate scale and area distortion along these lines and to minimize distortion elsewhere.

Some map projections, such as the Azimuthal Equidistant, are neither equal-area nor conformal, but linear scale is correct along all lines radiating from the center, along meridians, or following other special patterns. In addition, there are compromise projections, almost entirely restricted to small-scale mapping, which are used because they balance distortion in scale, area, and shape.

Projections are often classified by the type of surface onto which the Earth may be mapped. If a cylinder or cone that has been placed around a globe is unrolled, we have the concept of cylindrical or conic projections, such as the Mercator or Lambert Conformal Conic, respectively. If the axis of the cone or cylinder coincides with the polar axis of the globe, the projection has equally spaced straight meridians, parallel on the cylindrical projections and converging on the conics. The parallels intersect the meridians at right angles, being straight on the cylindricals and concentric circular arcs on the conics. The spacing of the parallels is seldom projective. A plane laid tangent to the globe at the pole leads to polar azimuthal projections, such as the polar Stereographic, with the parallels mapped as arcs of concentric circles and meridians as equally spaced radii of the circles. Scale remains constant along each parallel of latitude on a regular cylindrical, conic, or polar azimuthal projection, but it changes from one latitude to another. Directions of all points are correct as seen from the center of an azimuthal projection.

If the cylinder or cone is secant instead of tangent to the globe, the projection conceptually has two lines instead of one which are true to scale. Wrapping the cylinder about a meridian leads to transverse projections. By placing a plane tangent to the Equator instead of a pole, equatorial aspects of azimuthal projections result. Tilting the cylinder, cone, or plane to relate to another point on the Earth leads to an oblique projection, and the meridians and parallels are no longer the straight lines or circular arcs they were in the normal aspect. The lines of constant scale are correspondingly rotated.

THE EARTH AS AN ELLIPSOID

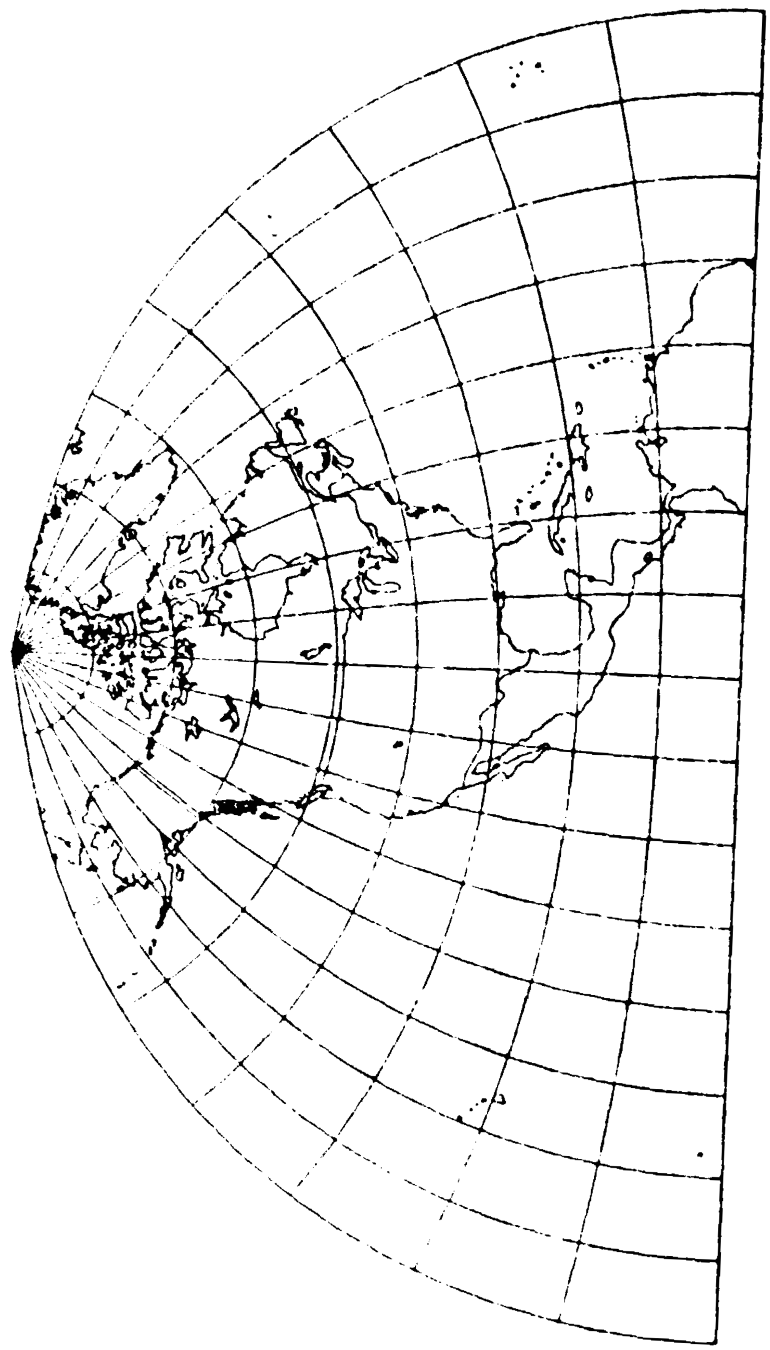
For maps at scales smaller than 1:5,000,000, and which cover regions larger than the United States, the distortions from mapping the spherical Earth on flat paper are much greater than the slight additional corrections needed to compensate for the ellipsoidal shape of the Earth. These corrections may then usually be ignored. The ellipsoid should be, and normally is, used for large-scale mapping of small areas, or for long narrow strips. For such areas, the flattening of the round Earth usually produces less distortion than the use of the sphere instead of the ellipsoid.

A shift from one ellipsoid to another has a negligible effect, even on large-scale maps, upon the projected shapes and positions of meridians and parallels. A greater effect is the translation of latitude and longitude for all points on a map, due to a change in datum, that changes the position of the ellipsoid relative to the Earth. For this reason, the notation in the corner of USGS quadrangles stating "North American Datum 1927" or "1983" is as important as the parameters of the map projection in defining the basis of these maps.

PRINCIPAL PROJECTIONS

Polyconic Projection

About 1820, Hassler began to promote the easily constructed Polyconic



Polyconic Projection

projection as the basis of large-scale mapping. The USGS used this projection for the earliest quadrangles, only changing in the 1950's to other projections, although relabeling the map legend lagged considerably behind the change. The Polyconic is neither equal-area nor conformal. For 7-1/2- and 15-min. quadrangles, however, the distortion is negligible. Along the central meridian, it is free of distortion. Each parallel is also true to scale, but the other meridians are too long, and constantly change scale. The projection is not recommended for maps of considerable east-west extent and, in fact, should not be seriously used for any new maps in view of other projections available.

The parallels of latitude are circular arcs spaced at their true distances along the central meridian, but with radii equal to the length of the element of a cone tangent at the particular parallel. The projection receives its name from the fact that each cone is different. Meridians are marked on each parallel at the true distances, but the meridians are complex curves connecting these points. Lines of constant scale run roughly parallel to the central meridian, but they are curved.

Mercator Projection

The Mercator projection is the most well known of all. It was presented by Mercator for navigational purposes in 1569, because rhumb lines, or lines of constant compass bearings, are plotted as straight lines. This is still the most justifiable use of the projection for a map of regions away from the Equator. It is a normal cylindrical projection, on which the lines of constant scale are straight and run parallel to the Equator. On the Mercator, the scale increases away from the Equator.

The USGS has used the Mercator projection for maps of part of the Pacific Ocean, of Indonesia, and for portions of each of the outer bodies mapped to date, from our Moon to the satellites of Saturn. In some cases, the chosen scale applies to standard parallels placed symmetrically north and south of the Equator. The shape of the map does not change. The scale along the Equator could still be called correct if the stated scale of the map were slightly decreased.

Transverse Mercator Projection

Rotating the cylinder of the Mercator so that it lies tangent (or secant) along a meridian of the globe leads to the very important conformal projection called the Transverse Mercator. The central meridian, the Equator, and each meridian 90° from the central meridian are straight lines. All other meridians and parallels are complex curves. For the sphere and normally for the ellipsoid, the central meridian has a constant scale, but this is usually reduced from the nominal map scale to balance errors in measurement over the rest of the map. The lines of constant scale are straight lines parallel to the central meridian for the sphere, and nearly straight for the ellipsoid. When the scale factor along the central meridian is reduced, the lines of true scale are symmetrical with respect to the central meridian.

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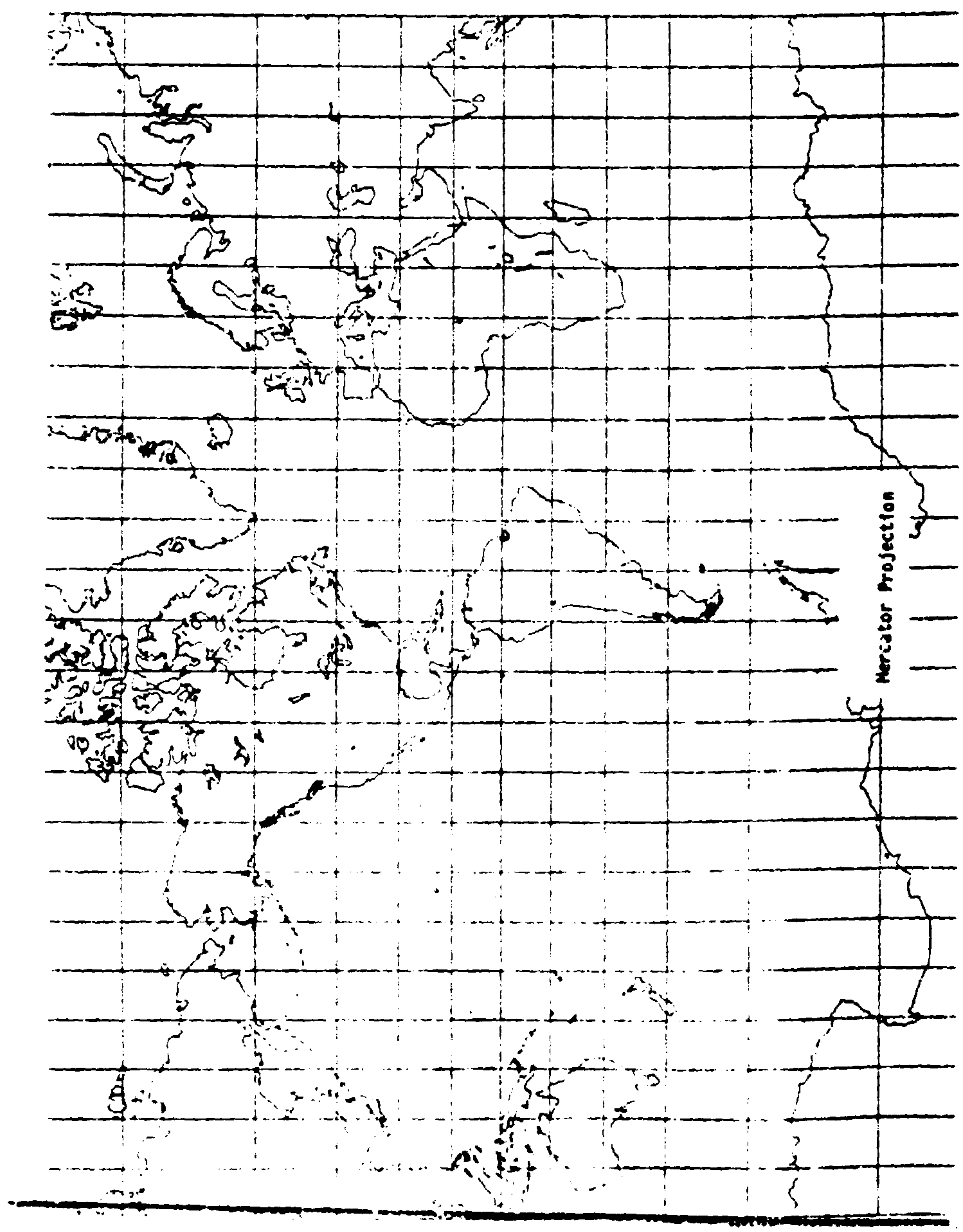

Lambert presented the spherical projection in 1772, but Gauss and later Krüger developed the mathematics for the ellipsoidal form. The projection was almost ignored until the 20th century, when it was adopted for much of the topographic mapping in Europe under the name Gauss-Krüger, and in the United States for the State Plane Coordinate System of States predominantly north and south in extent and for the Universal Transverse Mercator or UTM projection and grid system. The UTM has two special restrictions: (1) The central scale factor is 0.9996, and (2) the Earth is divided into sixty zones, each 6° of longitude wide, with the central meridians placed at every sixth meridian beginning with the 177th West. (There are minor exceptions.)

When USGS stopped using the Polyconic projection for large-scale quadrangle maps, the Transverse Mercator was adopted for maps of areas where it was used for the State Plane Coordinate System. (Recent metric quadrangles are based upon the UTM). In the State Plane Coordinate System, each zone follows county boundaries and is designed to restrict scale variation over the map to one part in ten thousand. The USGS uses the predominant zone for the projection of quadrangles which cross county boundaries. The central scale reductions, which change between zones, vary from 1:160,000 to 1:10,000. Central meridians have been individually selected for each zone.

Equations in series form are used for the Transverse Mercator calculations for the ellipsoid. These are limited to a 6° to 8° band of longitude and cannot be safely extrapolated for use over a whole continent. Simpler closed formulas can then be used if the Earth is assumed to be a sphere. To extend the ellipsoidal form a greater distance from the central meridian, much more complicated formulas are available.

When USGS converted from the Polyconic projection, with a central meridian on each quadrangle, to the State Plane Coordinate System, this included employing the central meridian and other parameters used for the new zone. The discrepancies in measurements on the Polyconic and the Transverse Mercator forms of the same 7-1/2- or 15 min. quadrangle depend especially upon the distance of the quadrangle from the central meridian of the zone. The new quadrangles can be mosaicked for the entire zone.

The 1:250,000-scale 1 x 2 quadrangle series covering 49 of the States was originally to be cast on the UTM. When the Army Map Service prepared these, the UTM central scale factor of 0.9996 was used, but the central meridian of the quadrangle itself was used in place of that of the UTM zone. These central meridians agree with those of the UTM zones for only one-third of the quadrangles. East-west mosaicking within a UTM zone thus cannot be achieved with the existing maps, which are now being updated and distributed by USGS. As these areas are remapped, they are being recast with the UTM central meridians.

Oblique Mercator Projection

A cylinder may be placed around the sphere so that it is tangent along a great circle which is neither a meridian nor the Equator. In this case, the Oblique Mercator may be conceptually projected for conformal mapping of a region chiefly

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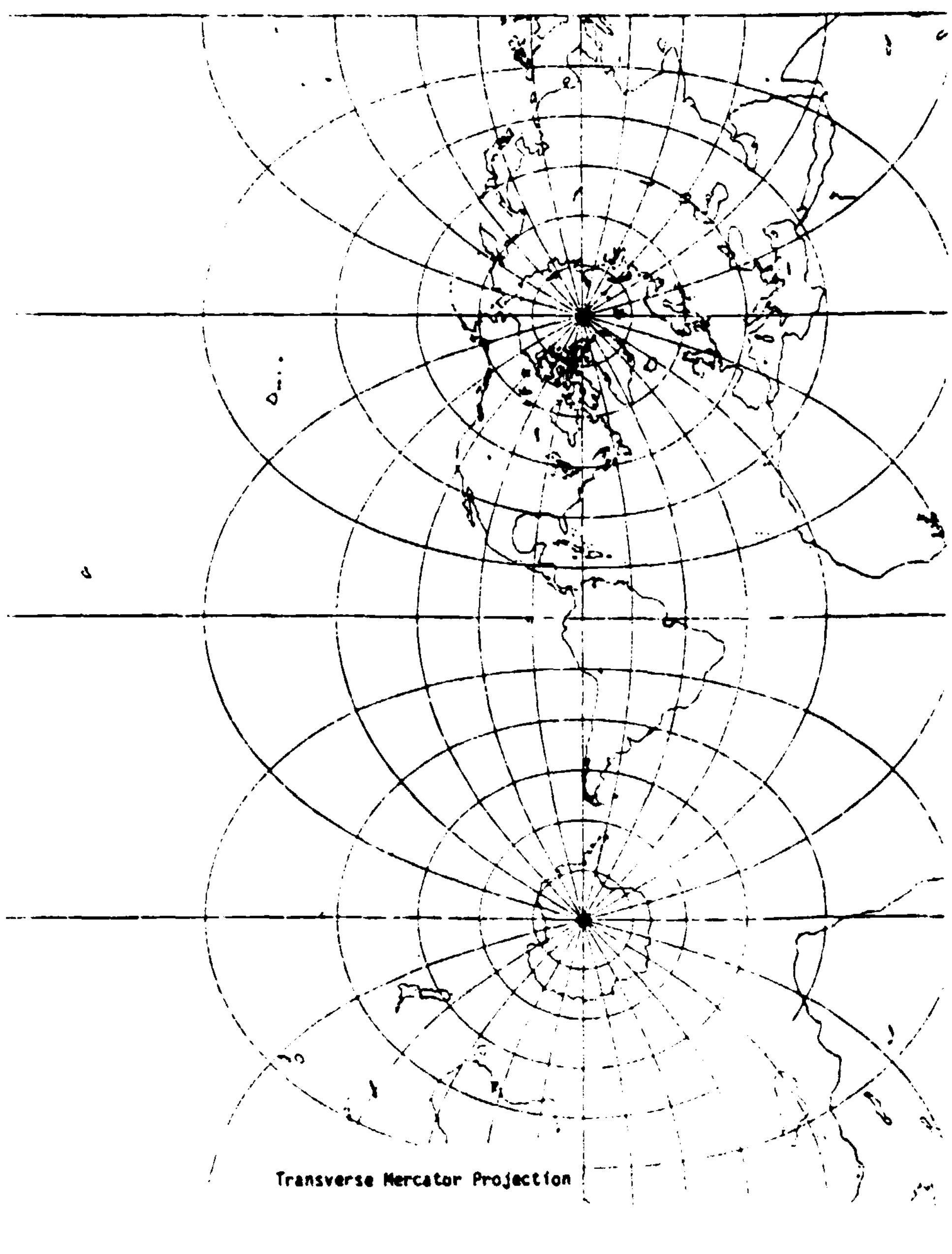
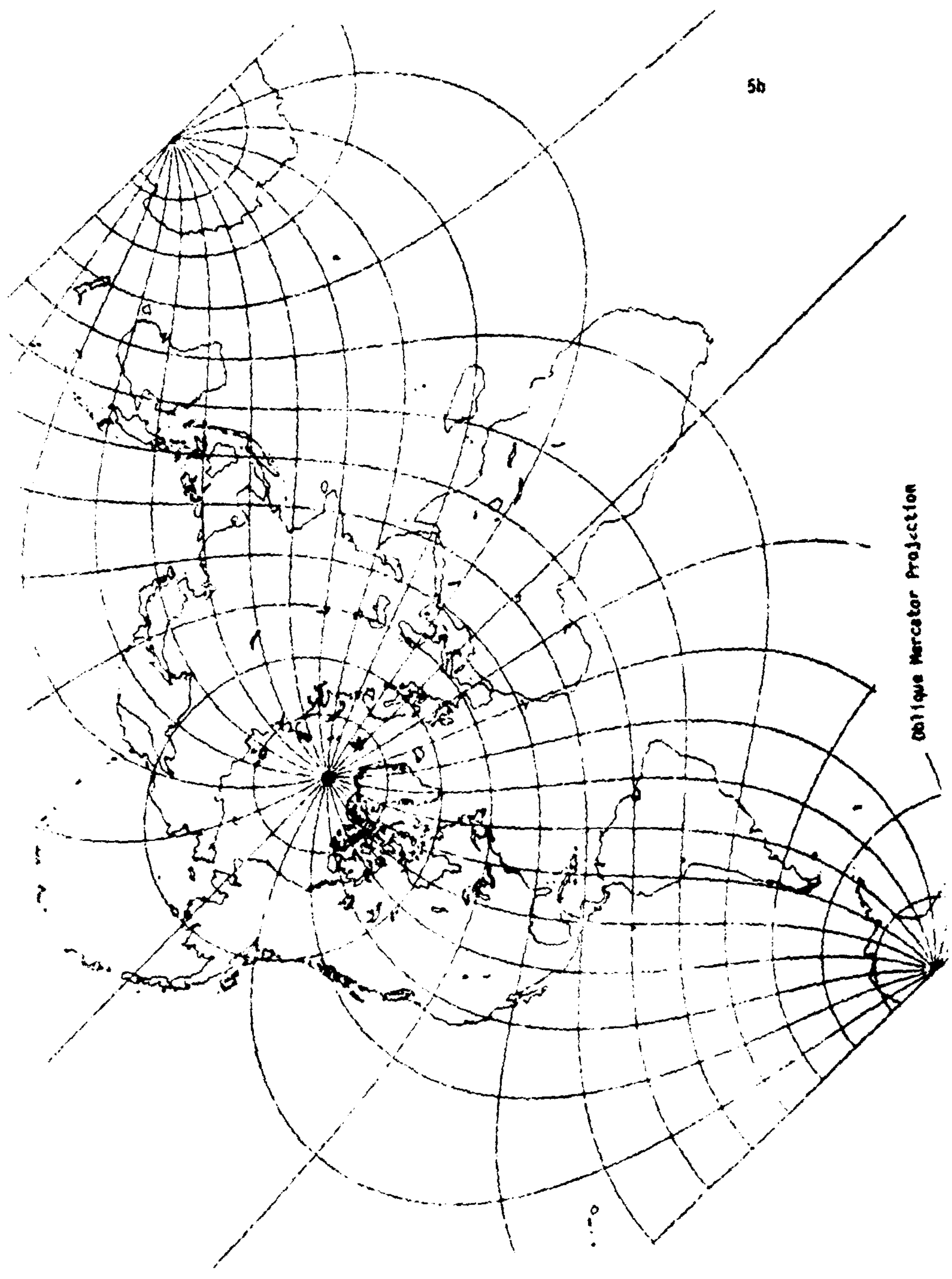


Transverse Mercator Projection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Oblique Mercator Projection

extending along this great circle. Nearly all meridians and parallels are complex curves. Here the lines of constant scale run parallel to the oblique great circle path, which may have a reduced scale. Its topographic use by USGS is confined to the State Plane Coordinate System of the southeast extension of Alaska and to the mapping of Landsat data from 1978 to the present, prior to implementation of the Space Oblique Mercator projection on Landsat D. There are several ways of adapting the Oblique Mercator to the ellipsoid, although none is ideal because either the central line does not remain at a precisely constant scale or conformality is not precise. Hotine's adaptation, which is exactly conformal, is used in these USGS applications.

Space Oblique Mercator Projection

Among the more complicated projections is the Space Oblique Mercator, conceived by A.P. Colvocoresses of USGS in 1973 and mathematically implemented in 1978. It is intended specifically for the continuous mapping of imaging from satellites such as Landsat for which a rudimentary form was used 1975-1978. The more accurate form is to be used for Landsat D. The groundtrack for the satellite is held true to scale, and mapping is made basically conformal. Because of the relative motion of Earth and satellite, the groundtrack is curved, and appears almost sinusoidal on the map. Formulas have been published in summarized form, and very recently with detailed derivations as USGS Bulletin 1518. It is designed for use with the ellipsoidal Earth, and with circular or elliptical satellite orbits.

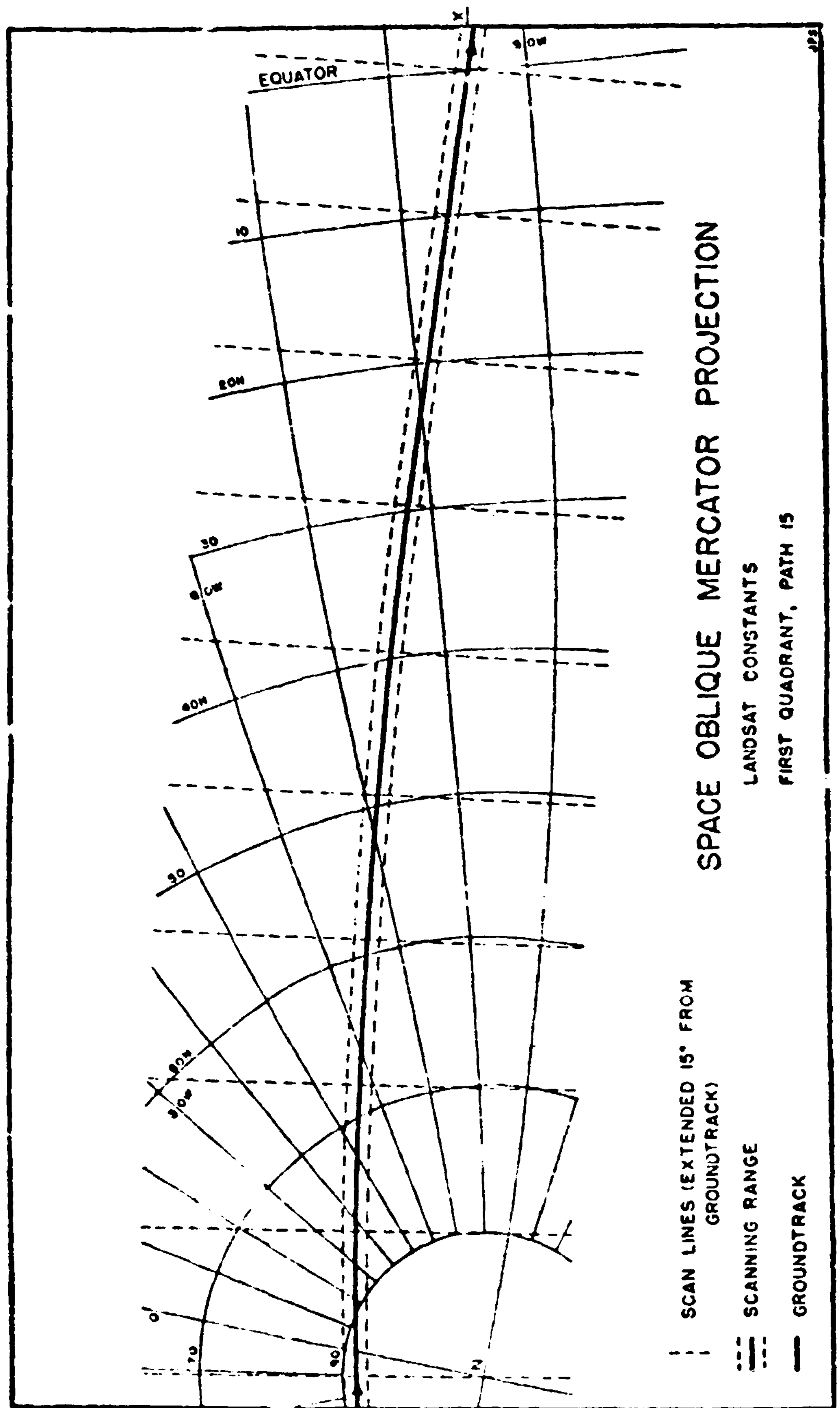
Cassini Projection

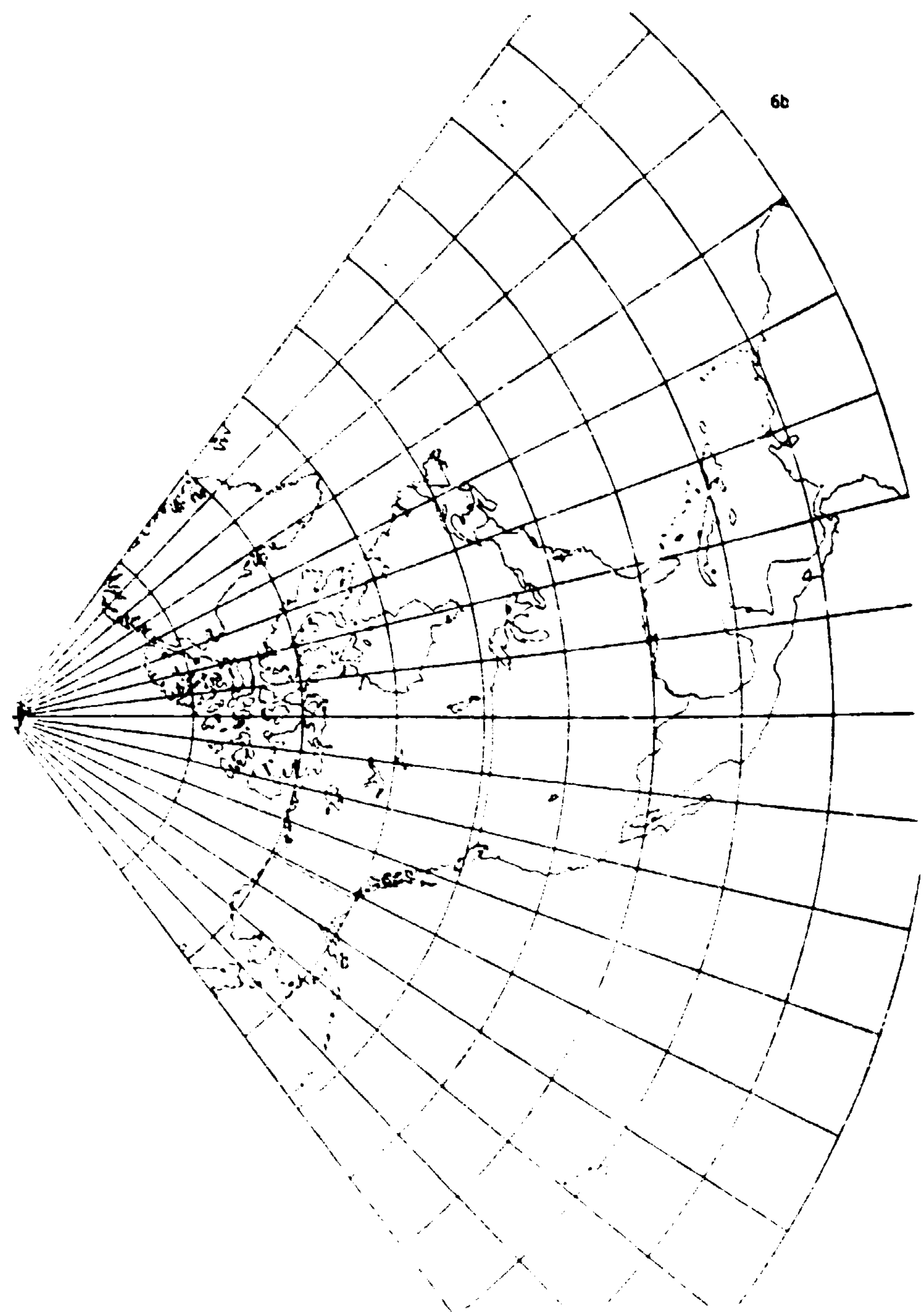
A non-conformal transverse cylindrical projection called the Cassini was used for British topographic mapping until about 1920, when it began to be replaced with the Transverse Mercator. On the Cassini, scale is true along the central meridian and along lines on the map perpendicular to the central meridian. In a direction parallel to this meridian, scale is constant, but it is increasingly too large as the distance from the meridian increases. Thus shape, angles, and area are distorted. It may also be cast obliquely. The projection was used for Landsat from 1972 to 1975, but is hardly used at present.

Lambert Conformal Conic Projection

Nearly all the States predominantly east and west in extent use the Lambert Conformal Conic as the projection for the State Plane Coordinate System. It was therefore adopted by USGS for post-1950 quadrangle mapping of these areas. This projection, presented by Lambert in 1772, maps parallels as concentric circular arcs and the meridians as equally spaced radii of those circles. One pole is at the center of the circles, while the other pole is at infinity. The parallels are more closely spaced between the normally two standard parallels, which have no distortion.

For the USGS map of the conterminous United States, the standard parallels are 33° and 45° N., and USGS has made base maps of each of the 48 States using these standard parallels at a scale of 1:500,000. The base maps therefore


SPACE OBLIQUE MERCATOR PROJECTION
LANDSAT CONSTANTS
FIRST QUADRANT, PATH 15
SCAN LINES (EXTENDED 15° FROM GROUNDTRACK)
SCANNING RANGE
GROUNDTRACK
EQUATOR



Lambert Conformal Conic Projection

match along the boundaries. Each zone of the State Plane Coordinate System has its own standard parallels. These are the parameters used for USGS quadrangles which have been produced on this projection; thus, quadrangles edge-match within the zone.

The Lambert is also used for the map sheets of the International Map of the World series at a scale of 1:1,000,000. The projection for these sheets was changed from a Modified Polyconic in 1962.

Albers Equal-Area Conic Projection

The Albers Equal-Area Conic is probably seen more than the Lambert Conformal Conic as the projection for maps of the United States and its larger regions. In 1805, Albers showed that by proper spacing of the parallels on a conic projection, there is no area distortion, and along one or two standard parallels there is no scale or angular distortion. The projection was nearly dormant until Oscar S. Adams of USC&GS began encouraging its use for equal-area maps of the United States in the early part of the 20th century.

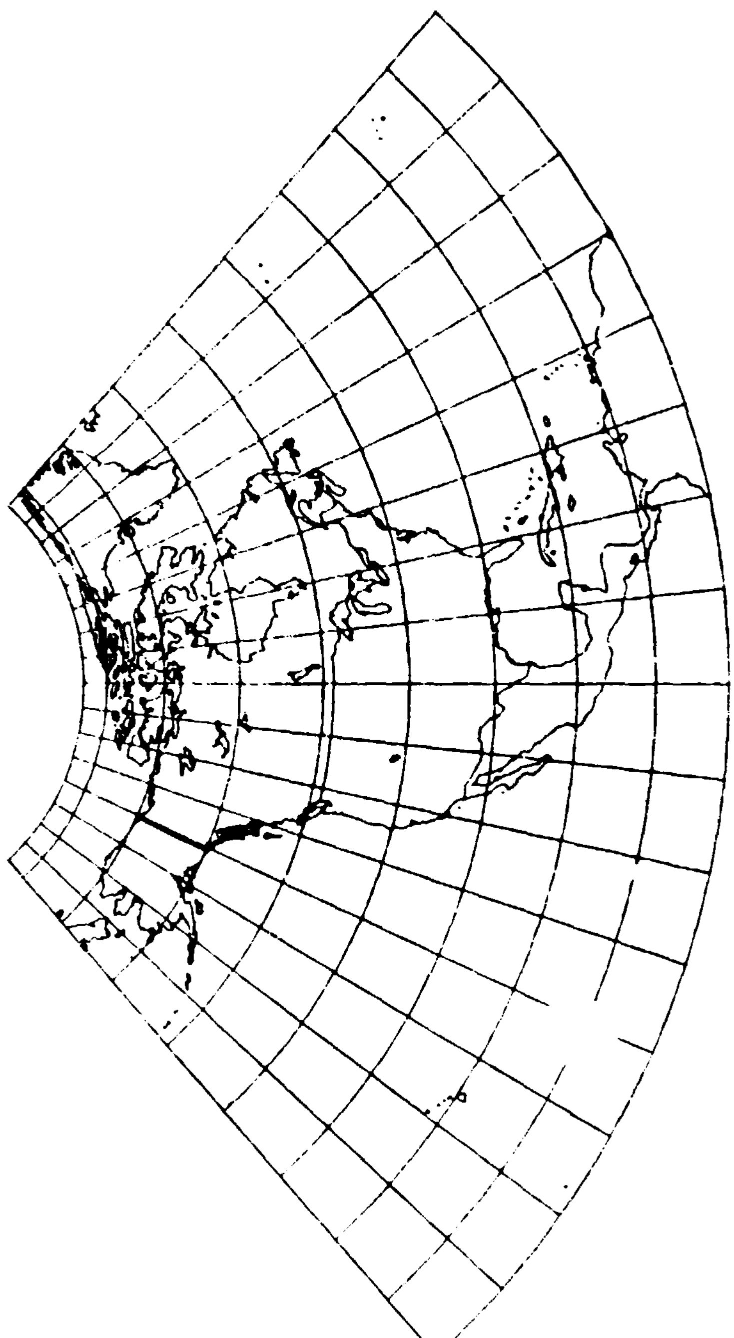
Adams's tables of coordinates for the 48 States are based upon standard parallels of 29-1/2° and 45-1/2° N. It should be noted that the United States on the Albers projection cannot be distinguished from a Lambert Conformal Conic version if the projection is not identified, except by careful measurements. Like the Lambert, the Albers, which is the equal-area counterpart, has concentric arcs of circles for parallels, and equally spaced radii as meridians. The parallels are not equally spaced, but they are farthest apart in the latitudes between the standard parallels and closer together to the north and south. The pole is not at the center of the circles, but is normally an arc itself.

Scale along any given parallel is constant, as on the Lambert, with scale too small between the standard parallels, and too large beyond them. The scale along the meridians is just the opposite, to maintain equal area. While Adams recommended that standard parallels be placed one-sixth of the displayed length of the central meridian from the northern and southern limits of the map, this is empirical. The standard parallels may be selected in other ways.

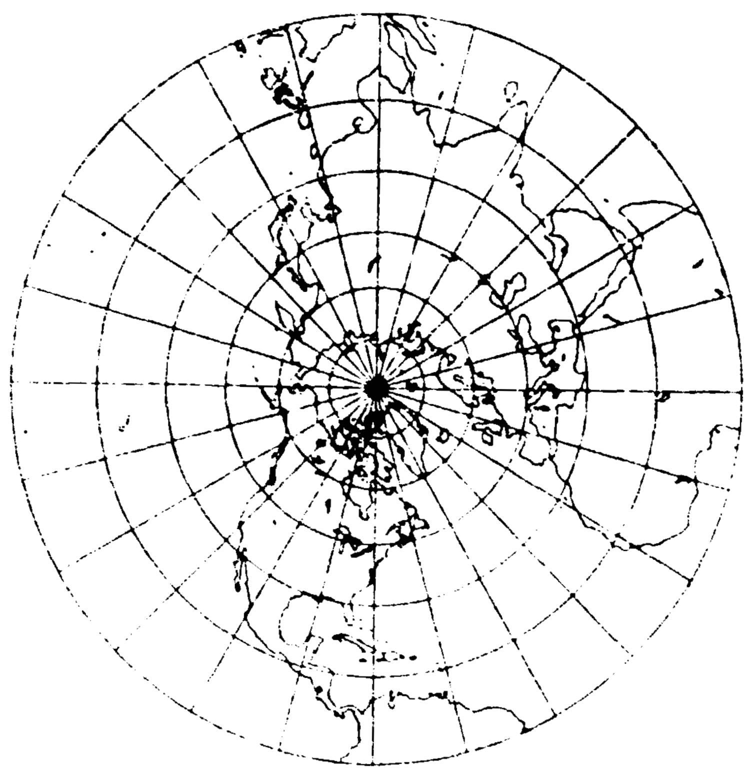
Since meridians intersect parallels on the Albers at right angles, it may at first be thought that there is no angular distortion. It exists, however, for any angle other than that between the meridian and parallel, except at the standard parallels.

Stereographic Projection

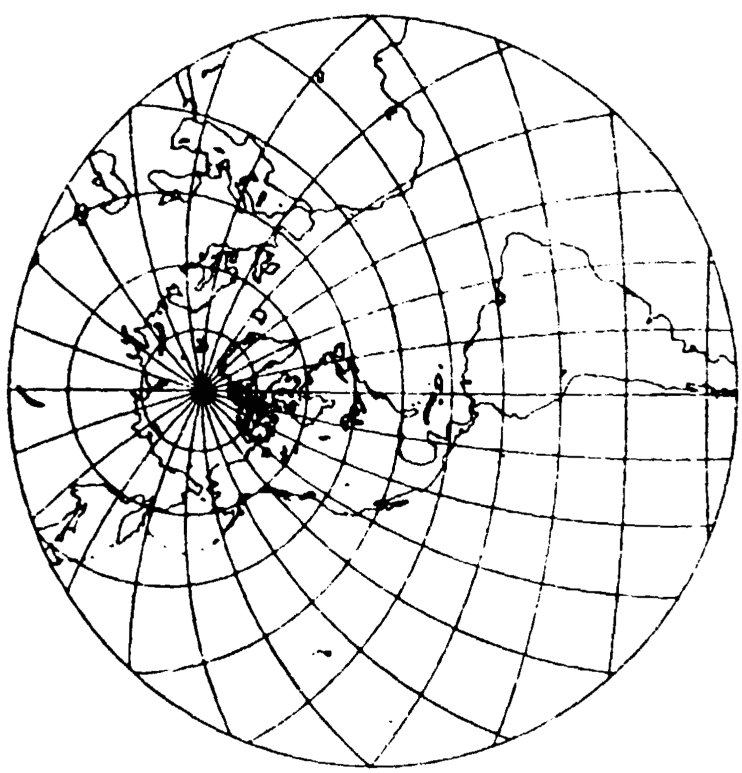
For larger-scale maps of polar regions, the Stereographic projection is commonly used. This is a perspective projection of the sphere onto a tangent or secant plane. The point of perspective lies on the opposite side of the globe. For the sphere, the Stereographic is both perspective and conformal. For the ellipsoid, it may be one or the other, but not quite both; in practice, it is used conformally in every case. All great and small



Albers Equal-Area Conic Projection



Polar Stereographic Projection



Oblique Stereographic Projection

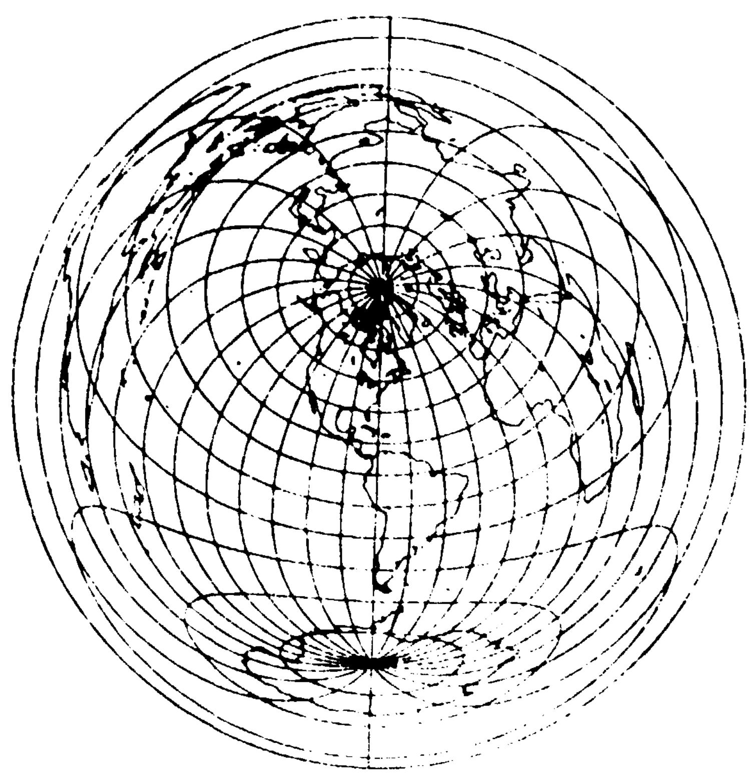
circles on the sphere are shown as circles or straight lines on the map. This includes all meridians and parallels.

The Stereographic is also azimuthal, and lines of constant scale are circles centered on the projection center. In the polar aspect, a parallel other than the pole is therefore often made true to scale to balance scale variation. The USGS has used the projection for maps of the entire Antarctic continent as well as for 1:250,000-scale quadrangles of portions. It is also used for polar portions of the International Map of the World, and for portions of extraterrestrial bodies centered at the poles or at basins elsewhere on the bodies.

Azimuthal Equidistant Projection

Familiar to many air-age atlas users is an azimuthal projection which shows both distances and directions correctly from the chosen center of the map, whether the North Pole or a major city. Scale in other directions varies, and is too great except at the center; therefore shape and area are distorted. In addition to several spherical applications, the Azimuthal Equidistant projection has been used in the ellipsoidal form for the Plane Coordinate System of Micronesia, for which each major island provides a center for one of the zones.

Because of the increasing digitization of data from and onto many different maps, most of these as well as several other map projections have been programmed by USGS in both forward and inverse form, to transform geodetic coordinates into rectangular coordinates, and vice versa. A bulletin describing in detail all projections which have been used by USGS, including numerical examples of formulas, is soon to be published. We hope that such analyses of both the past usage and capabilities of map projections will serve the public at large as well as our own mapping development.



Oblique Azimuthal Equidistant Projection