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THE EFFECT OF BARRIERS ON WAVE PROPAGATION PHENOMENA: WITH APPLICATION FOR AIRCRAFT NOISE SHIELDING

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

ABSTRACT

In recent years the subject of suppressing aerodynamic noise by shielding has actively been pursued both analytically and experimentally. Although there is a reasonably good agreement between the experimental results obtained under laboratory conditions and analytical results, the comparison is far from being satisfactory under flight conditions where several physical factors come into play in this problem. In order to make a reasonable study of the problem, the whole range of the frequency spectrum has to be studied.

In the present study the frequency spectrum has conveniently been divided into two regimes: the low frequency and high frequency regimes. Two separate methods have been developed for application to each regime.

For the long wave length propagation, the acoustic field due to a point source near a solid obstacle may be treated in terms of an inner region where the fluid motion is essentially incompressible, and an outer region which is a

linear acoustic field generated by the hydrodynamic

disturbances in the inner region. This method has been

applied to a case of a finite slotted plate modelled to

represent a wing extended flap for both stationary and

uniformly moving medium.

In the case of short wave length propagation, a very

effective approach utilizing a combination of the method of

ray acoustics, the Kirchhoff's integral formulation and the

stationary phase approximation has been developed. The

examples studied using this method include many limiting

cases. The solutions of these limiting cases agree with the

known solutions. This method, too, has been applied to a

new problem of physical interest. The problem consists of a

semi-infinite plate in a uniform flow velocity with a point

source above the plate and embedded in a different flow

velocity to simulate an engine exhaust jet stream surrounding

the source.

Chapter I

Introduction:

...."Nature operates by the simplest and
most expeditious means"....
-Fermat-

The introduction of more powerful jet engines in civil

air transportation in the 1950s drastically revolutionalized

the aircraft operational speeds and altitudes. The side

effect of this advanced technological advance was the build

up of noise on approach to land and take off paths on

airports.

Although the later generation of engines (turbofans)

are relatively quieter, the increased sizes and power

delivery of such engines still caused unacceptably high

sound pressure levels (generally of the order of 110 dB at

30.5 meters). With a desire to operate aircraft services

close to the city centres and to use shorter runways, the

Short Take Off and Landing (STOL) concept using enhanced

lift produced from the interaction of the engine jet with

the wing and flaps has evolved. This arrangement is used

now to shield engine noise from an observer. The concept

has attracted a wide range of research activities both

analytical and experimental.

The numerous studies on diffraction and noise reduction

by barriers can be divided into three groups roughly:

theoretical studies, experimental studies using scaled

models and full scale experiments under normal enviromental

conditions.

Comparison of experimental (under laboratory

conditions) and theoretical results shows some disagreement

for the lower frequency although there is a good agreement

for the high frequency end of the spectrum.

A full scale experimental study was extensively carried

by Jeffrey and Holbeche (19) using a Delta wing aircraft. In

the long wave length region, particularly, the results

showed a marked departure from those obtained by scaled

model experiments or the theoretical ones. Similar

observation is seen in experiments by Conticelli, De Blasi

and O'Keefe (7). Hoch (16) used the Bertin Aerotrain to study the forward flight effects on aircraft engine noise. The results are in agreement with other studies although, again, there is a disagreement for the low frequency spectrum. Fink (15) reports results that show similar phenomena.

There are many physical factors that exist in the real situation but are generally not included in the analytical studies. In the following discussion, a few of the more important ones are mentioned.

1. Sound Source: In the analytical studies, the source is, in general, simulated mathematically by acoustic poles which are also usually assumed concentrated at a point. In the real situation, the sound is generated aerodynamically over a certain region by the mixing of the high speed turbulent jet with the free stream. Internally, the rotating compressor and the turbine blades are also sources of noise which transmits outside the engine. The

mechanisms of generation and transmission in the source

region are extremely complex and by and large of an unknown

nature. The strength, frequency content and directivity of

the generated sound are not possible to be determined

analytically to any degree of accuracy by the present

knowledge.

2. <u>Transmission Path</u>: Before reaching the atmosphere,

the sound generated at the source region must transmit

through the jet exhaust. Due to the temperature, density

and velocity non-uniformity and the temporal variations of

these quantities caused by turbulence, the transmission

characteristics of noise in this "near field" region is not

easy to determine theoretically.

3. <u>Shrouding Effect</u>: At the interface between the jet

and the free stream, there are strong temperature and

velocity gradients resulting in reflection and refraction

of the sound waves and causing the intensity, directivity

and frequency content to change in the far field from those

predicted by any simple theory assuming a uniform atmospheric

condition.

4. <u>Reflection and other Effects due to the Geometry</u>

<u>of the Wing and the Airframe</u>: For the engine

under the wing configuration, for example, sound may be

strongly reflected by the under surfaces of the wing and the

flaps. This causes a large increase of the sound intensity

received on the ground. For the engine over the wing

configuration, on the other hand, noise is greatly reduced.

During landing, the gap between the main airfoil and the

extended flap may cause "leakage" of sound from the upper

side of the wing to the under side of the wing and thereby

reducing the effectiveness of wing shielding.

5. <u>Frequency on Diffraction</u>: The aircraft noise

spectrum extends from very high frequency sounds to very low

frequency ones. The low frequency sounds are not very

effectively shielded by the wing because of the strong

diffraction of waves of long wave length.

6. <u>Forward Flight Effect</u>: To compare laboratory and

in-flight data, it is important to know how the motion of

the aircraft affects the predicted or measured results.

7. <u>Effect of Shed Vortex:</u> Earlier investigations on wave-barrier interaction did not make any note of the sound that can be generated by shed vortex for finite or semi-infinite surfaces. This issue has occupied several reseachers in the last twenty years: Orzag et al (35); Jones (20); Davis (11); Jeffery et al (18); and Broadbent (2), among others.

These studies have indicated that the trailing edge shed vortex has three roles:

(i)   It acts as a shield to sound from one region of such a vortex to the shadow.

(ii) Such vortex is itself a source of sound.

(iii) The vortex does refract sound if the source of sound is close to the vortex, but it has less refractive effect if the source is far enough, Cooke (8).

8. <u>Effect of Viscous Boundary Layer:</u> Powell (36)

studied the problem of sound caused by boundary layer on the solid surface. This is what many experimental investigators have referred to as scrubbing noise. In this case the dipole source noise as well as the quadrupole sources are present.

Many of the above phenomena are studied in classical theories in the propagation of light waves. The study of light refraction, reflaction and diffraction has a long history. In the field of optics, the basic principles enunciated by Fermat, Huygens and Fresnel have been formulated into a general integral theorem by Kirchhoff. These basic principles, borrowed from the field of optics and later generalized in the general electromagnetic wave propagation after the Maxwell's development of the field equations, have been found applicable in the study of short wave length sound propagation. A combination of these principles has been employed in formulating a method applicable for short wave length acoustic propagation in the present study.

In recent times, with the rapid expansion of the air transportation, the subject of acoustic wave interaction with a barrier has attracted a considerable amount of research. Various theories have been explored in the ensuing search for various solutions to the problem.

The various published theories differ mainly in the way in which the fluctuating flow is assumed to interact with a barrier (with an edge of a plate in particular) to produce sound and fall roughly into the following four categories:

(i)   General theories based on aerodynamic noise based on the Lighthill (1952) acoustic analogy. These have been developed by Curle (10); Ffowcs-Williams and Hawkings (14); Howe (17); Jones (20); Powell (36); Ribner (39) among others.

(ii)   Theories based on the solution of special problems have been tackled by: Broadbent (2,4); Cooke (9); MacDonald (31); Orzag and Crow (35).

(iii)   Theories based on ad hoc models involve postulation

of source distribution whose strengths and

multipole types are generally determined

empirically. This group would include: Clapper et

al (6); Lan et al (22); Larson et al (23) and

Maekawa (32) among others.

(iv)   The experimental work that generally may include

analytical background material may be found in the

work of Conticelli et al (7); Fink (15); Hoch (16);

Jeffrey et al (18); Lush (30); Reshotko et al (38)

and Strout and Atencio (40).

The above list is no way exhaustive but it is

representative of the work that has been performed in

different approaches. In all cases, however, the main

concentration has been the study of short wave phenomena.

The present study, presented in chapter II takes the long

wave length into account to bridge the apparent gap in the

literature.

In the chapters to follow, the wing shielding problem

will be treated in two aspects: One aspect involves sound

of long wave length and the second aspects deals with sound

of short wave length.

The main purpose of the analysis is to develop

mathematical methods and to find physical solutions for the

wave-barrier interaction problems typified by Sommerfeld's

classical treatment of diffraction by a semi-infinite plate.

The particular interest here is in examining the effectiveness

of using the wing as a barrier to shield noise from the

engine installed above the wing under flight conditions.

In chapter II, the shielding of a concentrated

quadrupole sound source by a semi-infinite plate with a

narrow slot is considered. This problem simulates the

shielding by a wing with the flap in the extended position.

The acoustic field as affected by the slot is determined by

the method of matched asymptotic expansions, under the

assumption of long wave length approximation.

In the outer region, far away from the slot, the

presence of the slot can be neglected for a first

approximation. In such a case the solution is easily
obtained from the principle of ray acoustics. Such a
solution, however, would not be satisfactory in the region
close to the slot.

The solution close to the slot is calculated by close
examination of the inner flow. The long wave approximation
together with the Prandtl-Glauert transformation enables one
to derive the inner solution from the Laplace's equation. It
is found that the inner solution contains a second order
term which is a dipole singularity. In order to match the
inner solution to the outer one, it is necessary to calculate
the second order terms for the outer expansion. The dipole
source term in the inner region becomes the generator of the
secondary acoustic disturbances in the far region. It is
observed, in conclusion, that the location of the engine
noise source in relation to the slot is crucial in the
shielding effect of the noise by the wing with a flap.

A method, suitable for deriving solutions for cases of

short wave length, is developed in chapter III. The method

combines the basic techniques of ray acoustics, the

Kirchhoff's integral and the stationary phase approximation.

The basic principle of the method is based on the problem of

a semi-infinite plate with a sinusoidal monopole source

located above it. In the limit of very large wave number

K, the approximate solution can be divided into three

regions with sharp discontinuities. The solution is

reasonably accurate and the order of the neglected term is

$O(\frac{1}{K})$. This solution is not accurate at the boundaries

separating the three regions where a transition layer with

contributions to the solution to the order of $O(\frac{1}{\sqrt{K}})$ are

present.

The method proposed for use is, therefore, to use the

Kirchhoff's integral with a surface of integration lying

outside the transition layers. One can then use the

solution obtained by ray acoustics for ϕ and $\frac{\partial \phi}{\partial n}$ in the

Kirchhoff's integral to determine the function ϕ at any point

P, including that within the transition layer. The integral

generates the second order correction term which is of order $O\left(\frac{1}{\sqrt{k}}\right)$. The method of stationary phase is used to determine the second order approximation directly from the Kirchhoff's integral. In a sense, the method is to treat the Kirchhoff's integral as an integral equation and then solve it by the iteration procedure, using the ray acoustics as a stating first order approximation.

It is found that the stationary point "window" is of the order of $O\left(\frac{1}{\sqrt{k}}\right)$ where k is the wave number of the wave. The acoustic wave received at a point will depend on the proximitly of the stationary point window to the trailing edge. It is then determined that the solution at a point P (observer position) is basically that due to the ray acoustics modified by the shielding factor $F\left(\bar{\beta}\right)$. The shielding factor is a Fresnel integral form and cuts down the amount of the intensity of the wave as the stationary point window moves close to the edge, so that in the limit, the edge blocks the stationary point window leading to the total zone of silence (shadow).

The method is tested by solving two classical problems of wave diffraction. The first one is the interaction of the sound wave from a three dimensional point source with a semi-infinite plate first studied by MacDonald (31) and later extended by Cooke (9) under the condition of large k, which is the same as the condition of large distance used in the present study. The result obtained is in complete agreement with equation 13 on page 9 of Cooke (9). The second problem is the interaction of a plane wave with a semi-infinite plate. The far field solution obtained by the present method is in agreement with equation 2.86 on page 73 in the book by Noble (34) and with the result obtained more recently by Candel (5).

The method developed in chapter III has been applied to solve a problem of practical interest in chapters IV. The problem involves two flows with a semi-infinite plate separating the two uniform flows of different velocities. A source is assumed imbedded in the flow over the upper

surface of the plate. This is intended to simulate the

problem of an engine situated above the wing. The acoustic

field in the region below the wing, caused by the

transmission and diffracted fields is to be determined. The

boundary conditions on the free interface regarding the

continuity of pressure and particle displacements determine

the reflection and transmission coefficients of the wave at

the interface. Using the Kirchhoff's integral for the

semi-circular boundary and the theory developed in chapter

III, the solution is derived without much difficulty. It is

found that, in addition to the shielding factor $F\left(\bar{\beta}\right)$

dependence, such a solution is dependent on the reflection

and refraction of the incident ray at the interface. The

Mach number has the effect of changing the boundaries and

the thickness of the transition zone.

The main conclusions and applications of the developed

principles are outlined in chapter V. From the results

obtained in the problem of a semi-infinite plate, results

for a finite plate as a model for an aircraft wing are

derived and numerical results presented.

The effect of the gaps, angular deflection and other geometrical variations of the wing surface, which represents the real physical situations has been studied and it is observed that the effects of gaps on the far field is equivalent to that produced by localized acoustic poles induced at the position of the slot.

Another major interesting factor regarding the effects of source distribution which has been studied by Thiessen (41); Levine (27); and Embleton (12) is re-examined for the geometry of the semi-infinite plane. The appropriate power radiation in the different regions determined by ray acoustics is presented in integral form.

Finally, the question of how the flow velocity affects the shielding of a semi-infinite plate in a subsonic flow is briefly studied. It is found that the fluid motion causes two important effects:

(i) The boundary of the shadowed region depends only on the location of the source relative to the plate

trailing edge.

(ii) The angular thickness of the transition zone is affected by the fluid motion. The quantitative values of these effects can be numerically evaluated as a function of the Mach number, as shown by a numerical example.

Chapter II

THE SLOTTED WING PROBLEM:

II-1 Background;

In recent years the concept of the Short Take Off and Landing (STOL) has evolved a principle of externally blown flap to generate the additional high lift necessary for such operations. In the approach or take off phase, the jet from the engine interacts with the extended flaps. The slots between the main wing and the flaps become sources of noise as shown in figure (2-A). The engine may be situated above or below the wing. Figure (2-A) shows the case of an engine below the wing but the principle applies for the over-the-wing extenal blowing as well.
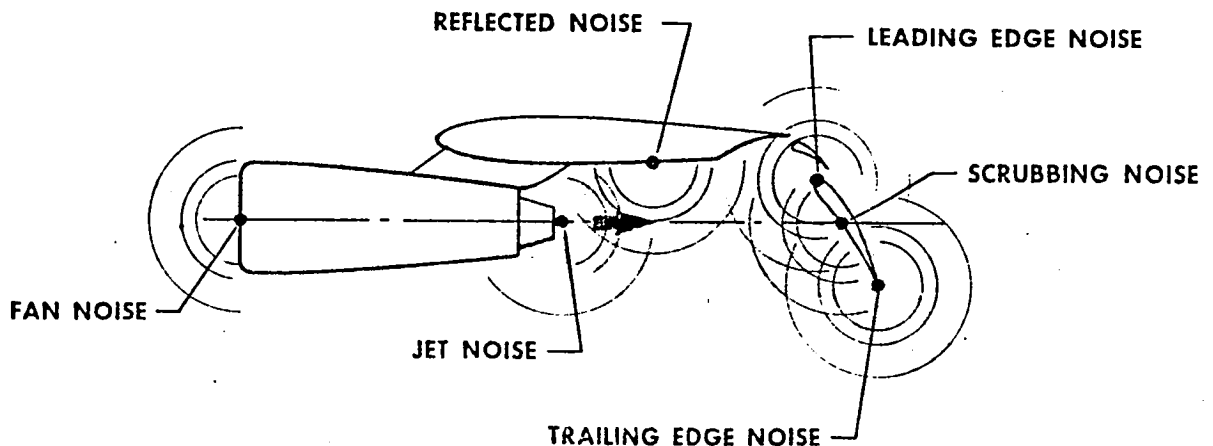
Figure (2-A): Under-the-wing jet blowing.

Although a substantial amount of effort has been made experimentally to determine the noise emitted from the engine jet and the wing/flap interactions (see Lasagna et al (24) and Falarski et al (13)) where it has been observed that the deflection of the flap generates a greater amount of sound intensity, there has been a limited analytical investigation of the problem. Ting (42) and Leppington (25) have investigated curvature effects of barriers on wave diffraction but such work did not involve gaps (slot) effects which is the real situation in the take off and landing phase of aircraft operation. The mathematical model, and method used in the study of this problem is presented in section II-2 below.

II-2 Mathematical Model:

In order to analyse the problem described in section II-1, a model is constructed as in figure (2-B) next page to represent a slotted semi-infinite plate and a quadrupole source S. A uniform flow with Mach number M<1 is assumed. The source S is monochromatic with frequency $f = a_o/\lambda$,

where λ is the wave length and a_o is the speed of the wave.

The dimensions **h, b,** and **L** are assumed much larger than the

wave length of the propagating wave λ . The plate is

assumed to be of zero thickness and zero angle of attack.
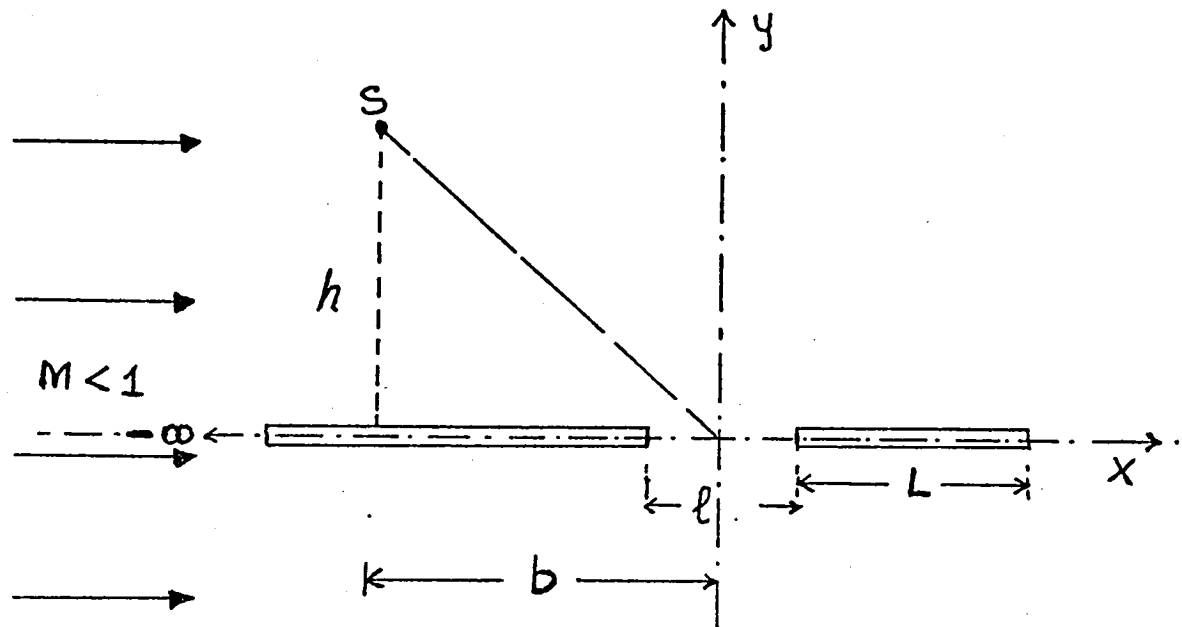
Figure (2-B): Slotted wing and noise source in uniform
subsonic flow.

A method of matched asymptotic expansions using $k\ell=\epsilon$ as

the small parameter will be applied in the solution of this

problem. In the region far from the slot ($r \gg \ell$), the

presence of the slot may be neglected in the first

approximation. The solution for this region is referred to

as the outer solution. For convenience, the outer solution

is constructed by the ray acoustics on the account that by

assumption $\lambda \ll h$, b, and L. As a result, the solution for

the outer region has discontinuities at the boundary of the

regions according to ray acoustics and it is necessary

therefore to relax condition (ii) in section II-3 to allow

such discontinuities. In the small neighbourhood surrounding

the slot, the time variation of the fluid motion is nearly

simultaneous and therefore a quasi-steady approximation may

be applied to the wave equation. The resulting equation is

the steady compressible flow equation which can be reduced

to the form of the Laplace equation by the Prandtl-Glauert

transformation. In solving for the inner solution near the

slot, the exact boundary condition of the slot must be used

however.

In the following analysis the total velocity potential

Φ will be written as the superposition of the uniform flow

potential Ux and a perturbation potential ϕ , so that:

$$\Phi = Ux + \phi \qquad (2\text{-}1)$$

where ϕ is the potential due to the presence of the source

and the slotted plate. The solution of ϕ is sought in the

analysis that follows.

II-3 **Field Equations:**

The governing equation is the convected wave equation:

$$(1-M^2)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}-2ikM\frac{\partial\phi}{\partial x}+k^2\phi=\frac{\partial^2}{\partial x_i\partial x_j}\left[T_{ij}\,\delta(x+b)\delta(y-h)\right] \qquad (2\text{-}2)$$

where ϕ is the velocity potential defined in equation (2-1)

with $\dfrac{\partial^2}{\partial x_i\partial x_j}=\left(\dfrac{\partial^2}{\partial x^2}\ ,\ \dfrac{\partial^2}{\partial x\partial y}\ ,\ \dfrac{\partial^2}{\partial y^2}\right)$ and T_{ij} is the amplitude of

the source strength in tensor notation. The boundary

conditions for this problem are:

(i) $\dfrac{\partial\phi}{\partial y}=0$ on y = 0; $-\infty < X \leqslant -\ell/2$ and

$\qquad\qquad\qquad\qquad\qquad \ell/2 \leqslant X \leqslant (L + \ell/2)$.

(ii) $\phi\ ,\dfrac{\partial\phi}{\partial x}\ ,\ \dfrac{\partial\phi}{\partial y}$ are continuous everywhere except on

the plate surface.

(iii)   The Sommerfeld radiation condition requiring

outgoing wave at infinity must be satisfied.

II-4 **The first order outer solution.**

The first order outer solution will be derived by the

use of ray acoustics approximation. (This is not a

necessary assumption, but it will serve to give an explicit

solution to illustrate the procedure better). The outer

region is, by definition, one which is far from the slot.

In this region, one may assume an outer expansion of the

form:

$$\phi = \widetilde{\phi_0} + \varepsilon \widetilde{\phi_1} + \varepsilon^2 \widetilde{\phi_2} + \cdots \cdot \quad (2-3)$$

The first order solution for $\widetilde{\phi_0}$ corresponds to the case

when $\varepsilon = 0$; that is, $\widetilde{\phi_0}$ can be obtained from equation (2-2)

with the approximation that the slot does not exist.

The solution for $\widetilde{\phi_0}$ for a semi-infinite plate and a

quadrupole source above it is derived by the method of ray

acoustics. The acoustic field due to the source, according

to this method, can be divided into three regions: a region

where both the incident and reflected waves are present;

another region where only the incident wave prevails and

finally one finds a region (below the plate) which is in a

complete shadow, figure (2-C) next page.
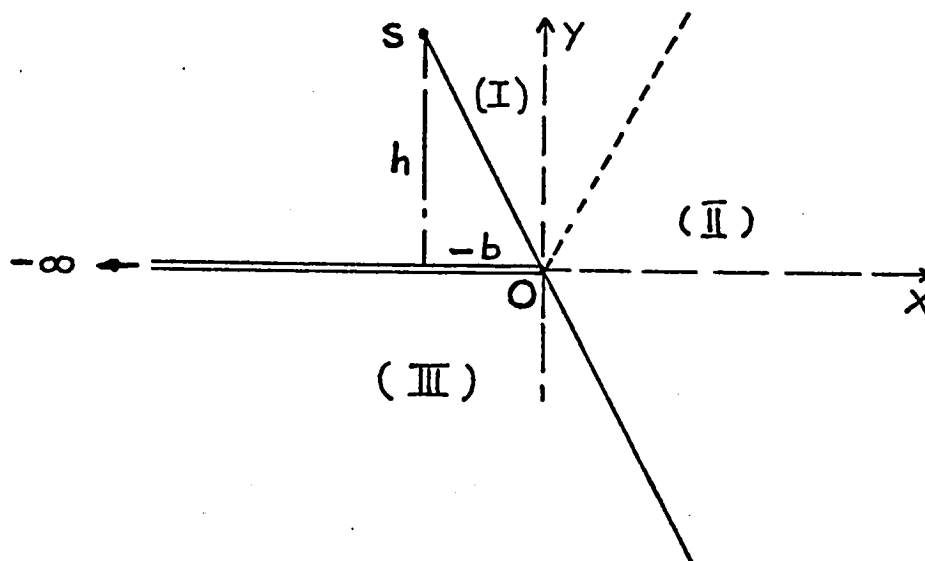
Figure (2-C): Sound emission from a source in the presence of a semi-infinite plate.

The solution for a monopole source at position (−b,h)

in uniform flow is known in the literature (ref. 37), and

is given as:

$$\phi = e^{\frac{ikMX}{1-M^2}} \left[H_0^{(1)} \left(\frac{kr}{1-M^2} \right) \right] \qquad (2\text{-}4)$$

where $r = \left[\left(X+b \right)^2 + \left(1-M^2 \right) \left(y-h \right)^2 \right]^{1/2}$ and $H_0^{(1)} \left(\frac{kr}{1-M^2} \right)$

being the Hankel functions of the first kind and zeroth

order.

The solution for a quadrupole can constructed using

equation (2-4). For the present purpose, only solutions of

$\widetilde{\phi_o}$ in regions I and III are needed. In region I above the

plate, $\widetilde{\phi_o}$ is given by:

$$\widetilde{\phi_o} = \frac{\partial^2}{\partial x_i \partial x_j} \left[T_{ij} \; e^{\frac{ikM(x+b)}{1-M^2}} \left\{ H_o^{(I)}\left(\frac{kr}{1-M^2}\right) \mp H_o^{(I)}\left(\frac{k\bar{r}}{1-M^2}\right) \right\} \right] \qquad (2\text{-}5)$$

where $\bar{r} = \left[(x+b)^2 + (1-M^2)(y+h)^2 \right]^{1/2}$

In equation (2-5) above, the term involving $H_o^{(I)}\left(\frac{kr}{1-M^2}\right)$

gives the potential due to the quadrupole in the absence of

the plate; while the second term involving $H_o^{(I)}\left(\frac{k\bar{r}}{1-M^2}\right)$ is

introduced to satisfy the solid boundary condition on the

plate surface.

The appropriate sign in equation (2-5) is chosen in

such a way that for quadrupoles whose directional

derivatives involving $\frac{\partial^2}{\partial x^2}$ or $\frac{\partial^2}{\partial y^2}$ terms, a plus sign is

selected between the two terms; while a minus sign is used

for quadrupoles whose directional derivatives involve terms

of the form $\frac{\partial^2}{\partial x \partial y}$.

According to ray acoustics, however, there is no

acoustic perturbation in region III under the plate; so that:

$$\widetilde{\phi}_o = 0 \qquad\qquad (2\text{-}6)$$

From the first order solutions equations (2-5) and (2-6) for

the acoustic perturbation, the velocity field can be

determined. The perturbed velocities for the upper and

lower surfaces of the plate are given by:

$$\left.\begin{array}{l} \text{over the plate}\quad \widetilde{u}_o = \left(\dfrac{\partial \widetilde{\phi}_o}{\partial x}\right)_{y=0} \quad , \quad \widetilde{v}_o = 0 \\[3em] \text{below the plate}\quad \widetilde{u}_o = 0 \quad , \quad \widetilde{v}_o = 0 \end{array}\right\} \qquad (2\text{-}7)$$

The velocity is, therefore, tangential to the plate and

discontinuous across the plate. Such a discontinuity is

acceptable across the plate but is not acceptable across the

slot. Near the slot, a region referred here as the inner

region (see figure 2-D), an inner solution must be

constructed.

The velocity component at the slot is denoted as:

$$\widetilde{U} = \left(\dfrac{\partial \widetilde{\phi}_o}{\partial x}\right)_{\substack{x=0 \\ y=0}} \qquad\qquad (2\text{-}8)$$

where $X = 0$ and $y = 0$ is the location of the centre of the slot. $\tilde{U}$ is an implicit function of the location, strength and direction of the quadrupole source. The velocity field in the <u>inner</u> region should approach $\tilde{U}$ on the upper side of the plate, but will go to zero in the lower part of the plate.

II-5 <u>The first order inner solution.</u>

The inner region is a region of the dimension $O(\ell)$ containing the slot figure (2-D). Since the wave length is much larger than the width of the slot $(\lambda \gg \ell)$, the time variation is nearly simultaneous for all points inside this region; so that the differential equation to the first order approximation is a steady flow equation.
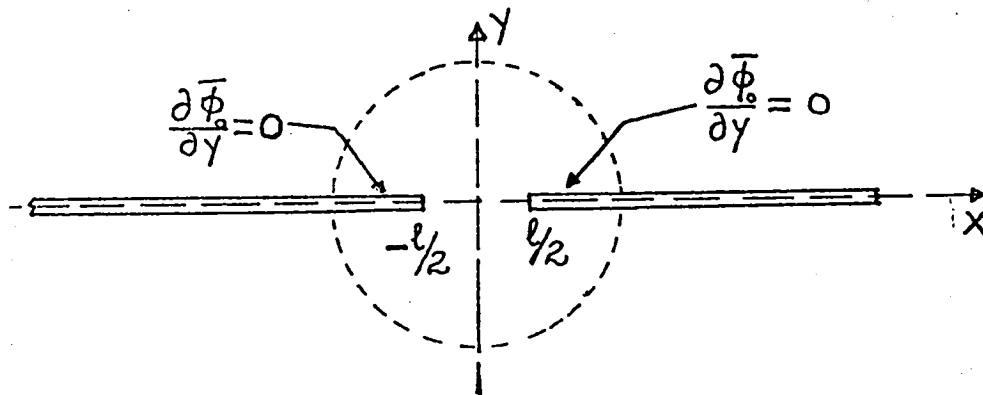
Figure (2-D): Configuration of the slot region.

More precisely, one may write the potential ϕ in terms of the following inner expansion:

$$\phi \simeq \bar{\phi}_0 + \varepsilon \bar{\phi}_1 + \cdots \tag{2-9}$$

Substituting such an expansion into the governing equation (2-2); and after the quasi-steady state approximation is applied, the first order inner solution $\bar{\phi}_0$ satisfies the following Prandtl-Glauert equation:

$$\left(1 - M^2\right) \frac{\partial^2 \bar{\phi}_0}{\partial x^2} + \frac{\partial^2 \bar{\phi}_0}{\partial y^2} = 0 \tag{2-10}$$

For the potential $\bar{\phi}_0$, the exact boundary conditions at the slot will be used. However, since the slot is narrow, the plate may be treated as infinite in both directions. In addition, $\bar{\phi}_0$ must match the outer solution at large distances; so that the boundary conditions may be stated as follows:

(i) $\dfrac{\partial \bar{\phi}_0}{\partial y} = 0$ on $y = 0$, $-\infty < x < \ell/2$ and $\ell/2 < x < \infty$

(ii) $\bar{\phi}_0$, $\dfrac{\partial \bar{\phi}_0}{\partial x}$, $\dfrac{\partial \bar{\phi}_0}{\partial y}$ must be continuous in the flow region.

(iii) $\bar{\phi}_0$ is to match the outer solution.

This last condition requires that the velocity field

obtained from $\overline{\phi_0}$ must match with the outer velocity field,

thus:

$$\frac{\partial \overline{\phi_0}}{\partial x} = \tilde{U} \qquad \text{as} \qquad y \longrightarrow +\infty$$

$$\text{and} \quad \frac{\partial \phi_0}{\partial x} = 0 \qquad \text{as} \qquad y \longrightarrow -\infty$$

The above boundary value problem may be solved by

introducing the complex potential $F_0 (z_1)$ given by:

$$F_0 (z_1) = \phi (z_1) + i \psi (z_1)$$

where $\phi (z_1)$ and $\psi (z_1)$ are the velocity potential and

stream functions respectively. The solution $\overline{F}_0 (z_1)$

adapted from ref. 33 is given by:

$$\overline{F}_0 (z_1) = \tilde{U}/2 \left[z_1 + \sqrt{z_1^2 - \ell^2/4} \right] \qquad (2\text{-}11)$$

where $z_1 = x_1 + i y_1$; $x_1 = x$, $y_1 = y\sqrt{1 - M^2}$, $i = \sqrt{-1}$

The function $\sqrt{z_1^2 - \ell^2/4}$ is defined as

$$\sqrt{z_1^2 - \ell^2/4} = \sqrt{r_+ \, r_-} \; e^{i \frac{\theta_+ + \theta_-}{2}} \equiv a + i b$$

where r_+ , r_- , θ_+ , and θ_- are as shown in the figure (2-E)

below. The functions $\bar{\phi}_o$ and $\bar{\psi}_o$ can now be written as:

$$\bar{\phi}_o(P) = \tilde{U}/2 \left[x_1 + \sqrt{r_+\, r_-}\ \cos \frac{\theta_+ + \theta_-}{2} \right]$$

$$\bar{\psi}_o(P) = \tilde{U}/2 \left[y_1 + \sqrt{r_+\, r_-}\ \sin \frac{\theta_+ + \theta_-}{2} \right]$$

where θ_+ and θ_- are such that the point **P** should not move

across the cut in figure (2-F) next page.

In terms of $\bar{\phi}_o$, the analysis above gives; at large

distance from the slot:

$$\bar{\phi}_o \simeq \tilde{U}x - \frac{\ell^2 \tilde{U}x}{16\left[x^2 + (1-M^2)\, y^2\right]} + \cdots y > 0; \quad \text{(2-12) (a)}$$

$$\bar{\phi}_o \simeq \qquad + \frac{\ell^2 \tilde{U}x}{16\left[x^2 + (1-M^2)\, y^2\right]} + \cdots y < 0; \quad \text{(2-12) (b)}$$
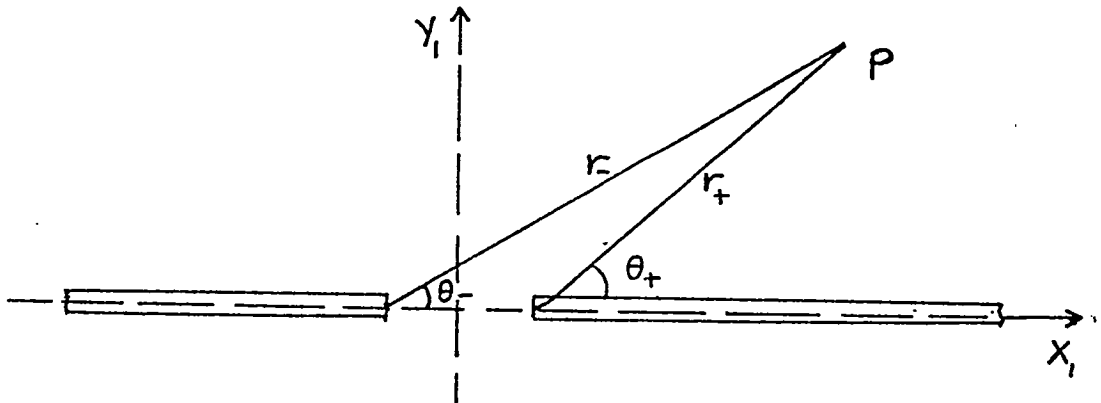
Figure (2-E): Geometry of the slot for large distance
solution calculation.

Y_1

P

r_-

r_+

$a \sim + x_1$

$\theta_+ = \pi$; $\quad a = -\sqrt{x_1^2 - \ell^2/4}$
$\theta_- = \pi$; $\quad b = 0$

θ_-

$\theta_+ = \pi$ | $\theta_- = 0$

θ_+

$\theta_+ = \theta_- = 0$; $a = \sqrt{x_1^2 - \ell^2}$; $b = 0$

$\theta_+ = \pi$; $\quad a = \sqrt{x_1^2 - \ell^2/4}$
$\theta_- = -\pi$; $\quad b = 0$

$-\ell/2$

$b = \sqrt{\frac{\ell^2}{4} - x_1^2}$; $a = 0$

$\ell/2$

$\theta_+ = 2\pi$; $\quad a = -\sqrt{x_1^2 - \ell^2/4}$
$\theta_- = 0$ $\qquad b = 0$

X_1

31

$$\sqrt{z_1^2 - \ell^2/4} = \sqrt{r_+ r_-}\; e^{i\frac{\theta_+ + \theta_-}{2}} = a + ib$$

$a \sim - x_1$
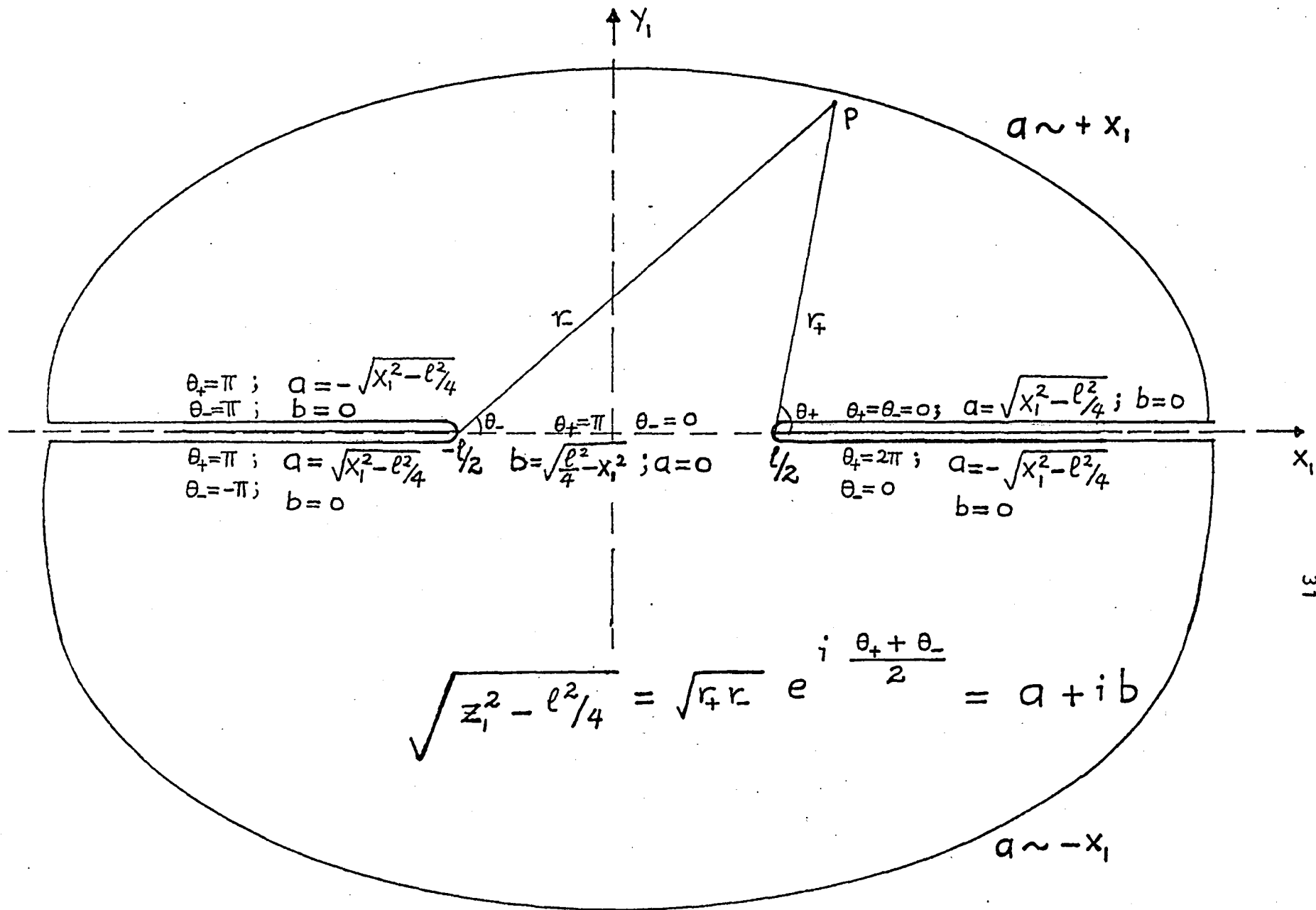
Figure (2-F): The definition of the complex function
$\sqrt{z^2 - \ell^2/4}$

The first term of the solution given by equations (2-12)

matches with the velocity of the first order outer flow

field $\tilde{\phi}_0$; while the second term has a form of a dipole and

represents the effect of the slot.

The inner solution has some rather interesting features.

In equation (2-11), the first term $\tilde{U} z_1 /2$ represents a

uniform flow of velocity $\tilde{U}/2$ along the direction of the

positive X-axis, figure (2-G). The second term $\tilde{U}/2 \sqrt{z_1^2 - \ell^2/4}$

represents a flow past a slot as shown by figure (2-H) below.
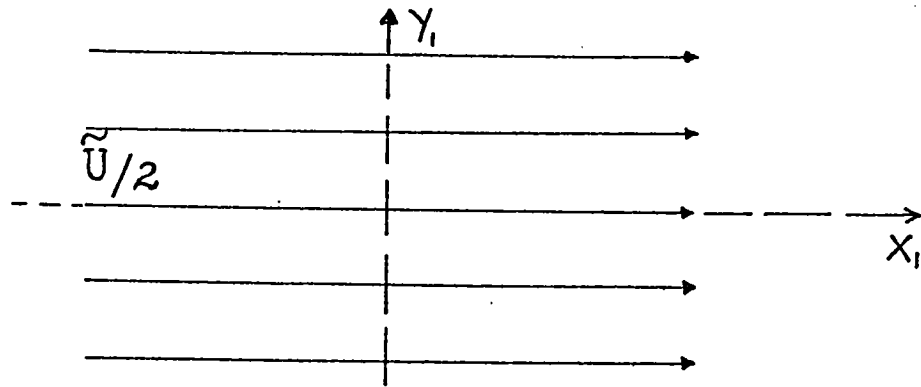
Figure (2-G): Uniform flow.
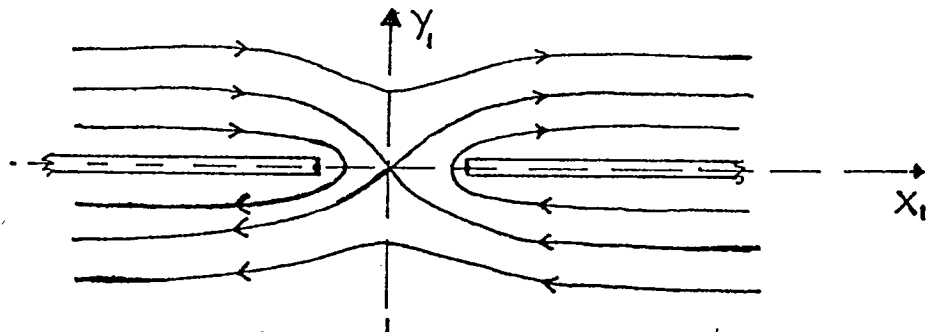
Figure (2-H): Flow past a slot.

The streamlines are adapted from (Ref. 33) with a change

in the branch cut. When the uniform flow represented in

figure (2-G) is superposed to figure (2-H), the flow

velocity in the upper plane approaches the uniform velocity

$\widetilde{U}$ at large distances; while in the lower region, the flows

in the two figures are in opposite direction and tend to

cancel out, so that only the flow velocities of lower order

are present. The resulting flow pattern is shown in figure

(2-I) below. Attention is called to the fact that the flow

depicted here represents the perturbation potential ϕ of

equation (2-1).
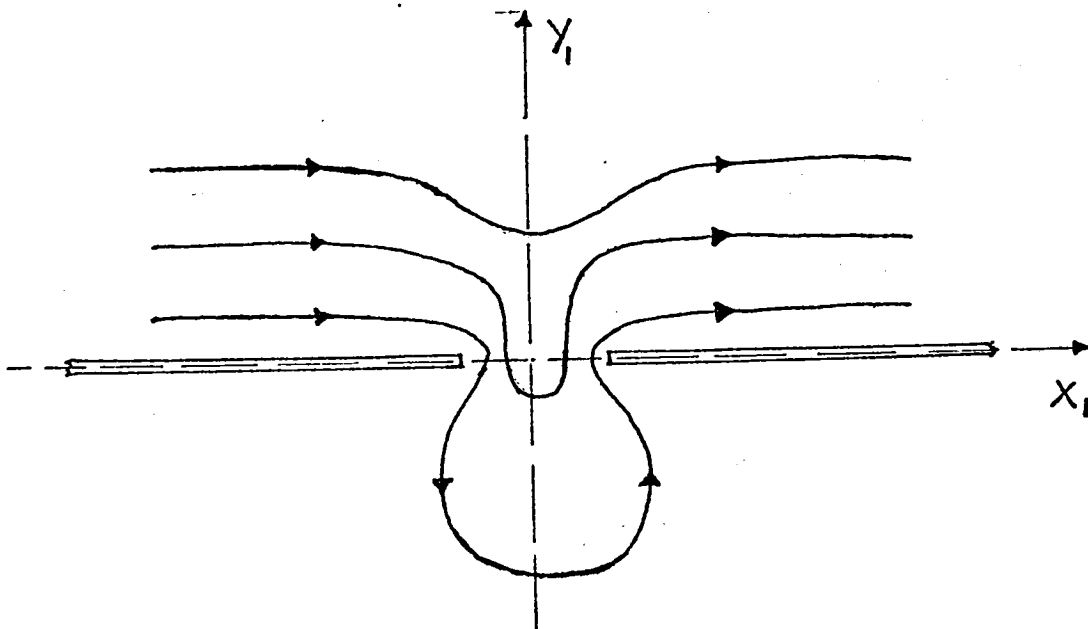
Figure (2-I): Uniform and slot flow superposed.

It is noted that at large distances, the flow field under the plate looks like that of a dipole with a source at $(-\ell/2, 0)$ and a sink at $(\ell/2, 0)$ figure (2-J) below. This flow is represented by the first term in the asymptotic expansion in equation (2-12)(b). In the upper half plane, the streamlines bend downward toward the plate on the left side of the slot and turn away from the plate on the right side of the slot. At large distances, the flow pattern is equivalent to the superposition of a uniform flow of velocity $\tilde{U}$ and a dipole with a sink located on the left edge $(-\ell/2, 0)$ of the slot, figure (2-K).

Figure (2-J): Flow under the plate in the slot region.
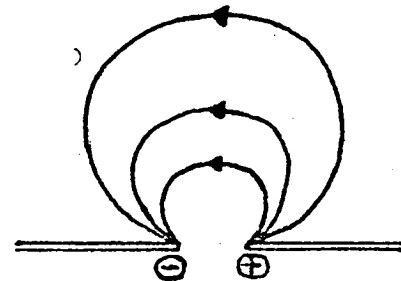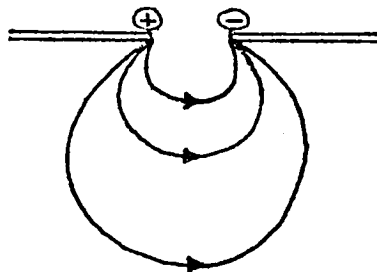
Figure (2-K):

Flow over the plate in the slot region

Thus the far field generated by the slot in a basic

flow of velocity $\tilde{U}$ for $y_1 > 0$; and zero velocity for $y_1 < 0$ is

of a rather peculiar nature, namely, the perturbation at

large distances are the pertubation given by dipoles of

different signs. Note also that at both edges of the gap

($x_1 = \pm \ell/2$, 0) the X — velocity component has a well known

square-root singularities which arise due to the assumption

of the plate being of zero thickness.

On the line $y_1 = 0$, downstream of the plate, the

velocity fields generated by the two dipoles of opposite

signs are discontinuous. The velocity components given by

the two dipoles are both zero at $y_1 = 0$ and hence are

continuous, but the u — components are non-zero and of

opposite signs and hence discontinuous. Since $y_1 = 0$ is the

boundary between the acoustic field generated by the two

dipoles (one on the upper surface and one on the lower

surface of the plate) this discountinuity is acceptable

according to ray acoustics.

There are three important observations to be made about

the inner solution.

(i)   The slot acts like a dipole.

(ii)   The axis of the dipole is along the surface of the

plate (X_1-axis).

(iii) The signs of the dipole are different for the

region $y_1 > 0$ and $y_1 < 0$.

A simple explanation of these features is given as

follows:

(a) Due to the long wave length assumption ($\ell/\lambda \ll 1$),

the effect of the slot is mathematically equivalent to the

scattering of a plane wave by the incompressible flow region

surrounding a small particle (Rayleigh scattering of scalar

waves). The inner solution surrounding the slot, therefore,

acts like a dipole to the outer acoustic field.

(b) By the assumption of a thin plate lying on the X_1-axis,

the dipole must lie on the X_1-axis ($y_1 = 0$) with an axis

directly along the plate. A vertical dipole would generate

a vertical velocity component at $y_1 = 0$ and this cannot

satisfy the boundary conditions imposed on $y_1 = 0$.

(c)   The reason that the dipole has a different sign is

mathematically due to the choice of the branch cut for the

function $\sqrt{z_1^2 - \ell^2/4}$ as shown on page 31.

In the branch cut as in figure (2-L) below, the

infinity point will not be contained in the cut.   In such a

case, the real part of $\sqrt{z_1^2 - \ell^2/4}$ denoted by a will

approach the same value X_1 both for $y_1 \to \infty$ and $y_1 \to -\infty$.
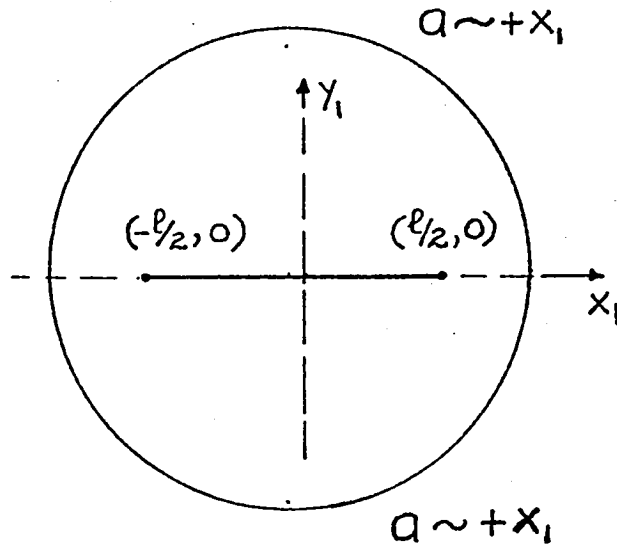
Figure (2-L): Infinity point not in the cut.

In the present problem, however, the matching condition

imposed by the outer solution requires that $a \to X_1$ as $y_1 \to \infty$

and $a \to -X_1$ as $y_1 \to -\infty$.   In order to obtain this result it is

necessary to draw the branch cut from $X_1 = -\ell/2$ to $-\infty$ and from $X_1 = \ell/2$ to $+\infty$. In this way, the infinity points of the two upper and lower planes, in figure (2-F) are on two different Riemann sheets in figure (2-L) so that the behaviour of the function and hence the dipoles have different sign for $y_1 > 0$ and $y_1 < 0$. Physically, the reason that the matching conditions require the real part of $\sqrt{z_1^2 - \ell^2/4}$ to behave like X_1 for $y_1 > 0$ and $-X_1$ for $y_1 < 0$ is that the outer solution for the upper and lower regions are obtained by ray acoustics, and there are strong discontinuities at the interfaces of the different regions.

II-6 The Second Order Outer Solution:

The differential equation for the second order outer solution is:

$$(1-M^2)\frac{\partial^2 \tilde{\phi}_1}{\partial x^2} + \frac{\partial^2 \tilde{\phi}_1}{\partial y^2} - 2ikM\frac{\partial \phi_1}{\partial x} + k^2 \phi_1 = 0 \qquad (2-13)$$

The appropriate boundary conditions are:

(i) $\tilde{\phi}_1$; $\dfrac{\partial \tilde{\phi}_1}{\partial x}$; $\dfrac{\partial \tilde{\phi}_1}{\partial y}$ must be continuous except

at the plate and at the boundaries according to ray

acoustics.

(ii)   Radiation at infinity must be satisfied.

(iii) $\widetilde{\phi}_1$ must match with the inner solution.

This means that as X and y approach the position of the slot

($X = y = 0$); $\widetilde{\phi}_1$ takes the asymptotic values below. One

must note that the discontinuity of the solution on $y = 0$

for $x > \ell/2$ must be admitted just like the discontinuities on

ray acoustics boundaries.

$$\left. \begin{array}{l} \text{over the plate} \quad \widetilde{\phi}_1 \simeq - \dfrac{\ell^2\, Ux}{16(x^2 + (1-M^2)\, y^2)} \\[4ex] \text{below the plate} \quad \widetilde{\phi}_1 \simeq + \dfrac{\ell^2\, Ux}{16(x^2 + (1-M^2)\, y^2)} \end{array} \right\} \qquad (2\text{-}14)$$

The solution for $\widetilde{\phi}_1$ may be obtained by direct observation as

follows:

Considering $\widetilde{\phi}_1$ in region I above the plate, one may write:

$$\widetilde{\phi}_1 = \frac{\partial}{\partial x}\left[A\, e^{\frac{ikMx}{1-M^2}} \; H_o^{(1)}\left(\frac{k}{1-M^2}\sqrt{x^2 + (1-M^2)y^2} \right) \right] \qquad (2\text{-}15)$$

Where A is a constant to be determined. The above expression for $\widetilde{\phi}_1$ represents a two-dimensional acoustic dipole in a uniform flow, and is obviously a solution of the governing equation (2-13). Such a solution also satisfies the radiation condition at infinity since it represents an outgoing wave. To determine the constant A by the marching condition one must expand the expression for small values of X and y to obtain:

$$\varepsilon \, \widetilde{\phi}_1 = \varepsilon \frac{\partial}{\partial x} \left[\frac{2Ai}{\pi} \ln\left(\sqrt{x^2 + (1-M^2)y^2} \right) \right] \qquad (2\text{-}16)$$

$$= \frac{k\ell Ai}{\pi} \frac{\partial}{\partial x} \left[\ln\left(x^2 + (1-M^2)y^2 \right) \right]$$

$$\simeq \frac{2k\ell Ai}{\pi} \frac{x}{\left[x^2 + (1-M^2)y^2 \right]}$$

Comparing expression (2-16) above with the inner solution as presented in equation (2-12) one obtains:

$$\frac{2k\ell Ai}{\pi} = \frac{\widetilde{U}\ell^2}{16}$$

so that

$$A = -\frac{\pi \widetilde{U} \ell i}{32k} \qquad (2\text{-}17)$$

Hence

$$\tilde{\phi}_1 = -\frac{\pi \ell^2 \tilde{U} i}{32} \quad \frac{\partial}{\partial x}\left[e^{\frac{ikMX}{1-M^2}} H_0^{(1)}\left(\frac{k}{1-M^2}\sqrt{x^2+(1-M^2)y^2}\right)\right]$$
(2-18)(a)
$$y > 0$$

Similarly, one finds that in region III below the plate:

$$\tilde{\phi}_1 = +\frac{\pi \ell^2 \tilde{U} i}{32} \quad \frac{\partial}{\partial x}\left[e^{\frac{ikMX}{1-M^2}} H_0^{(1)}\left(\frac{k}{1-M^2}\sqrt{x^2+(1-M^2)y^2}\right)\right]$$
(2-18) (b)
$$y < 0$$

II-7 Discussion and Conclusions:

(1) From the equations (2-18), one observes that the

effect of the slot is equivalent to a dipole situated at the

position of the slot. The strength of the dipole is

directly proportional to $\tilde{U}$ which is the velocity at the

surface location of the slot due to the quadrupole. $\tilde{U}$ is a

scalar function of the strength, directivity and location of

the quadrupole T_{ij} , and may be determined by the use of

equations (2-5) and (2-8), when the value of T_{ij} is

explicitly given. Note that the dipole is of opposite sign

on the two sides of the slot figure (2-I).

(2) The sound field pattern generated by the slot in the

uniform moving medium is sketched in figure (2-M) below and

the relative amplitude pattern for the velocity potential is

in figure (2-N) below.

(3) The strength of the induced dipole is proportional

to (ℓ^2), so that if the slot width ℓ is reduced by half

(say),the strength of the dipole is reduced four times.
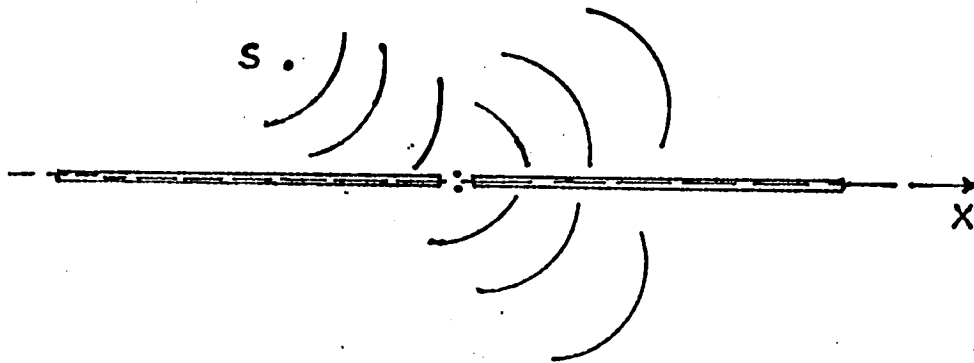
Figure (2-M): Sound field generated by the slot.
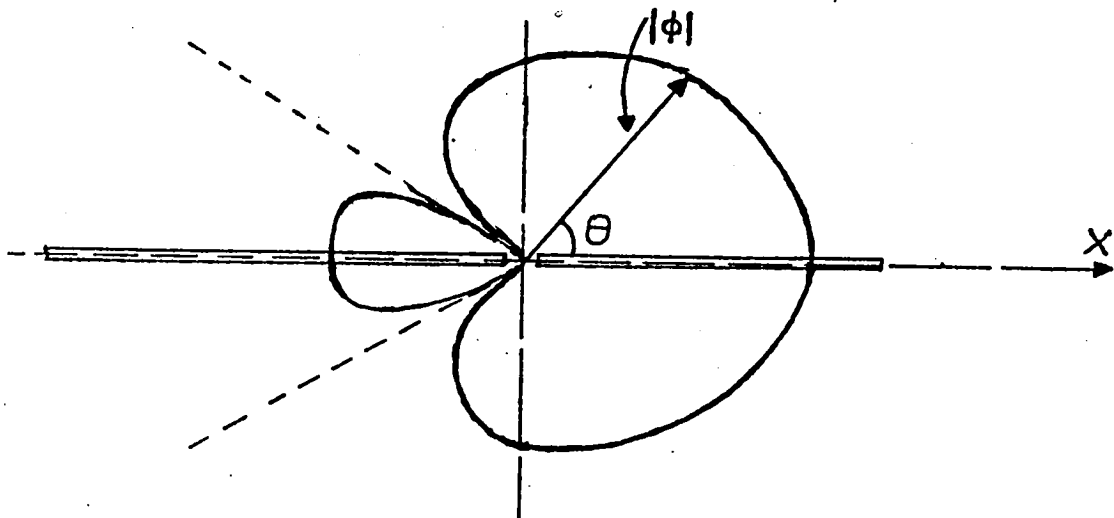
Figure (2-N): Relative amplitude of the velocity
potential $|\phi|$.

(4) The asymptotic behaviour of the Hankel function of argument α is:

$$H_0^{(1)}(\alpha) \sim \frac{e^{i\alpha}}{\sqrt{\alpha}} \qquad \text{as} \quad \alpha \longrightarrow \infty$$

Hence the strength of the induced dipole is proportional to $\frac{1}{\sqrt{r_0}}$ when $r_0 \gg \ell$ and $r_0 = \sqrt{b^2 + (1-M^2)\, h^2}$ as shown in figure (2-O) below. This means that when M is not too small, b has a greater weighting in the determination of the value of r_0, so that an increase in the horizontal distance is more effective in reducing the strength than the increase in vertical distance from the plate to the source.
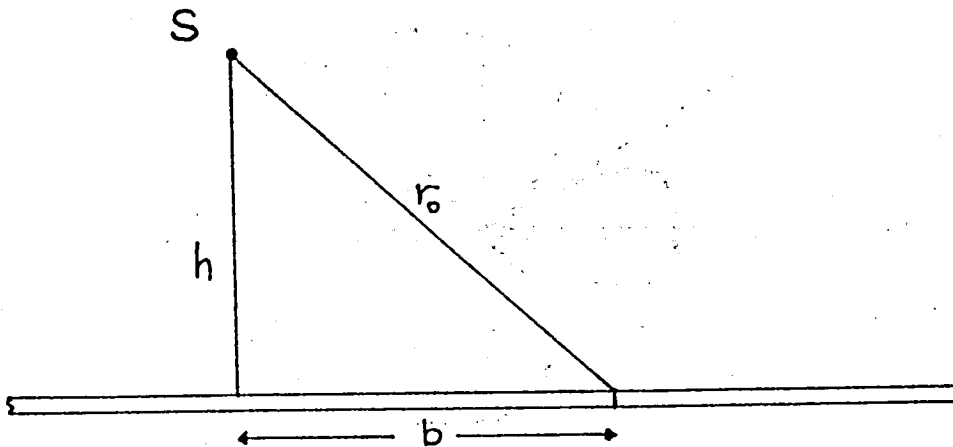
Figure (2-O): Induced dipole is proportional to $\frac{1}{\sqrt{r_0}}$.

(5)   When the original sound source has a directivity

(such as a quadrupole), it is advisible to locate the slot

in the direction of minimum radiation (see figure (2-P) below.
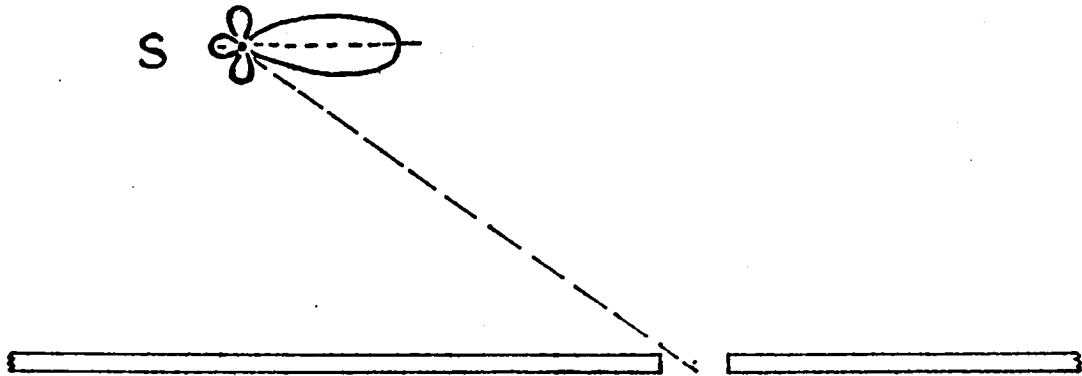
Figure (2-P): Direction of minimum radiation.

Chapter III:

GENERAL METHOD FOR HIGH FREQUENCY WAVES:

III-1 Background:

The development given below indicates a general method
that will be used in solving problems of acoustic wave
propagation in the high frequency end of the wave spectrum.
The method is developed out of a combination of the ray
acoustics, Kirchhoff's integral and the stationary phase
method and applied to the problem of a semi-infinite solid
plate.

The basic principle can be outlined as follows. Consider
a sinusoidal monopole source located above the semi-infinite
plate. In the limit of very large wave number k, the
approximate solution can be constructed by means of ray
acoustics. The solutions are defined in three regions with
sharp discountinuities. The solution is reasonably accurate
and the order neglected is $O(\frac{1}{k})$. This solution is not
accurate at the boundaries separating the three regions
where a transition layer with contributions to the solution

of $O\left(\frac{1}{\sqrt{k}}\right)$ are present. The proposed method uses the Kirchhoff's integral with a surface lying outside the transition layers. The solution obtained by ray acoustics defines the first approximate solution for ϕ and $\frac{\partial \phi}{\partial n}$ that are contained in the Kirchhoff's integral. It is then possible to determine the function ϕ at any point P including that within the transition layer. This integral generates the second order correction which is of order $O\left(\frac{1}{\sqrt{k}}\right)$. The method of stationary phase is used to determine the second order approximation directly by applying the stationary phase method to the integral.

In a sense, the method is to treat the Kirchhoff's integral as an integral equation which is solved by an iterative procedure using the ray acoustics solution as a first approximation.

In order to demonstrate the development and application of the method, a three-dimensional problem is solved as in Section III-2 which follows. From the solution derived for the three-dimensional problem, the solution for a two

dimensional case is deduced and compared to already known

solutions. Such a comparison shows a good agreement for

the present solution with known solutions abtained by more

elaborate mathematical procedures.

III-2 An Illustrative Problem of Diffraction of Sound from

a Three-Dimensional Source.

A three-dimensional source S is situated above a

semi-infinite plate as shown in figure (3-A) below.

Problem Statement: The acoustic field due to a sinusoidal

monopole source located above a semi-infinite plate in a

stationary medium is to be determined.
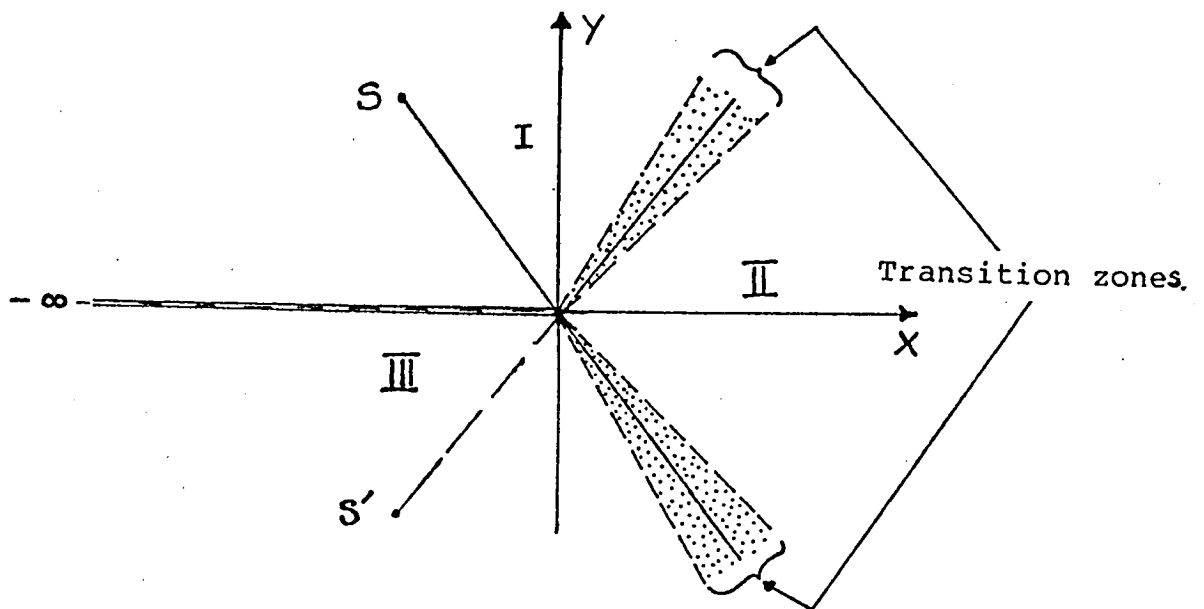
Figure (3-A): Three-dimensional source close to
a semi-infinite plate.

Assumptions: (i) A simple (three-dimensional) source whose

wavelength is short is used.

(ii) A semi-infinite plane extends along the

$-\infty < x \leq 0$ region.

(iii) The surface of the plane is rigid so

that the normal velocity vanishes there.

(iv) A stationary medium surrounding the source

and the semi-infinite plane.

Method of solution: With this arrangement, the first order

solution can be obtained by use of the ray acoustics. Such a

first approximation would provide solutions for regions I,

II and III indicated in figure (3-A) as follows:

$$\text{Region I:} \quad \phi_{I} = -\frac{e^{ikR}}{4\pi R} - \frac{e^{ik\bar{R}}}{4\pi \bar{R}}$$

$$\text{Region II:} \quad \phi_{II} = -\frac{e^{ikR}}{4\pi R} \qquad\qquad (3-1)$$

$$\text{Region III:} \quad \phi_{III} = 0$$

where R and $\bar{R}$ are the distances from the source and from the

image to the observer at P respectively.

This approximation is reasonably accurate as a first

solution approximation of the problem, but will not be

sufficiently accurate in the thin shaded zones marking

transitions from the source reflection reception to the

source affected zone only; and that from the source affected

zone to the shadow zone.

The method now formulated will avoid these difficulties

and hence provide a solution that can be applied at any point

including the shaded (transition) zone. To do this, use is

made of the Kirchhoff's integral as outlined below. Let G

and ϕ be defined inside a closed surface S as in figure

(3-B) below.
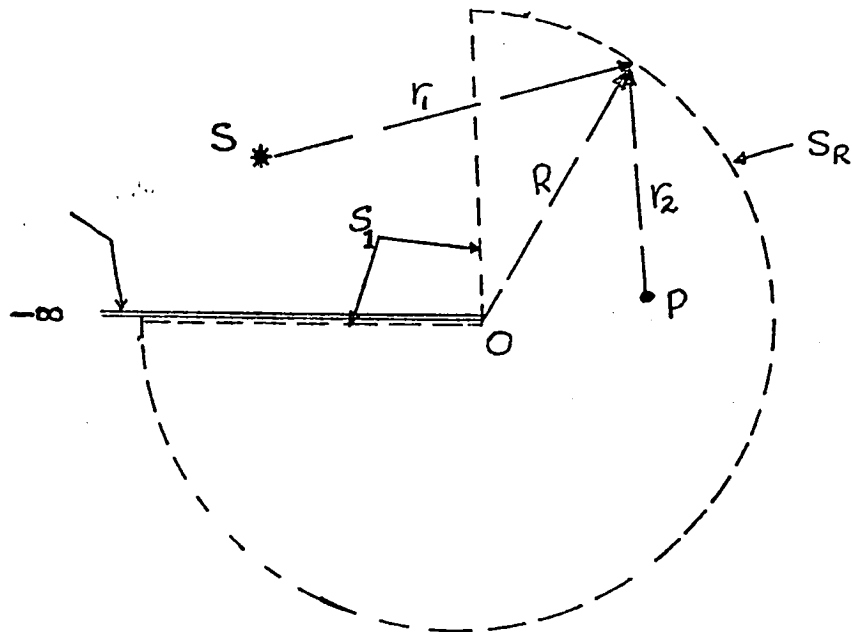
Figure (3-B): Boundaries for the Kirchhoff's integral.

The following equations can be written thus:

$$\nabla^2 G + k^2 G = \delta(\vec{y} - \vec{x}) \qquad (3\text{-}2)$$

$$\nabla^2 \phi + k^2 \phi = 0 \qquad (3\text{-}3)$$

From equations (3-2) and (3-3) one obtains, by the standard

procedure:

$$\phi \nabla^2 G - G \nabla^2 \phi = \phi \delta(\vec{y} - \vec{x}) \qquad (3\text{-}4)$$

Hence

$$\phi(\vec{x}) = \oiint_S \left[\phi \nabla^2 G - G \nabla^2 \phi \right] \cdot \hat{n} \, dS \qquad (3\text{-}5)$$

If the source considered is a point source, and the

acoustic field is three-dimensional, then

$$\phi_s = -\frac{A_0 e^{-ikr_1}}{4\pi r_1} \qquad ; \qquad G = -\frac{e^{-ikr_2}}{4\pi r_2} \qquad (3\text{-}6)$$

where $r_1 = |x - x_s|$ and $r_2 = |x_p - x|$ as shown in the

figure (3-C) next page.
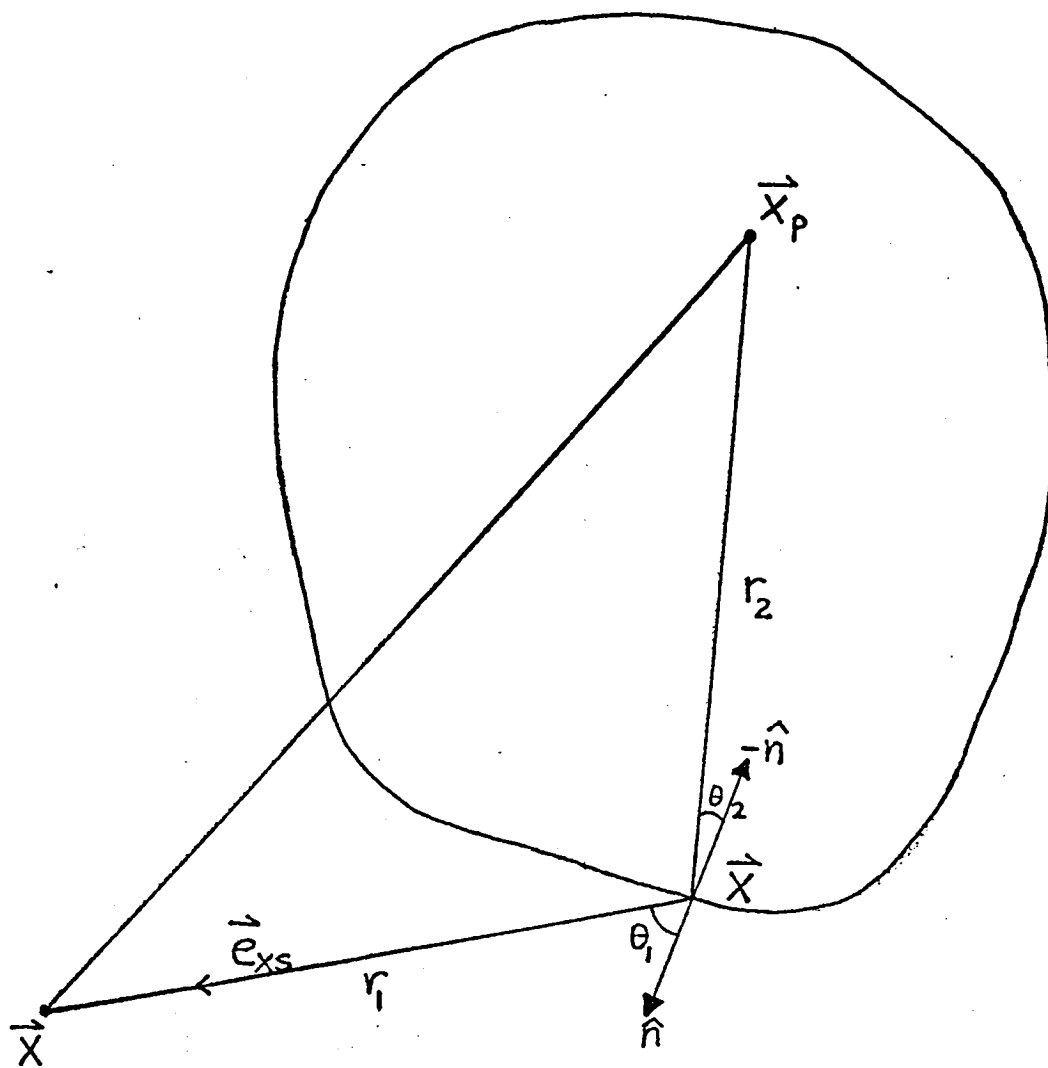
Figure (3-C): Acoustic field from a point source with a boundary present.

$$\nabla \phi = \frac{A_o}{4\pi}\left[\frac{ik}{r_1} + \frac{1}{r_1^2}\right] e^{-ikr_1} \nabla r_1$$

When k is large, and r_1 not too small, the second term is

$O(\frac{1}{k})$ as compared with the first term, and can be neglected.

This gives:

$$\nabla \phi \sim \frac{A_o ik}{4\pi r_1} e^{-ikr_1} \nabla r_1$$

$$\nabla r_1 = \frac{\vec{X}-\vec{X}_s}{|\vec{X}-\vec{X}_s|} = -\frac{\vec{X}_s-\vec{X}}{|\vec{X}-\vec{X}_s|} = -\vec{e}_{xs}$$

where e_{xs} is a unit vector from X to X_s ; so that:

$$\cos \theta_1 = \vec{e}_{xs} \cdot \hat{n} = -\nabla r_1 \cdot \hat{n}$$

and thus:

$$\hat{n} \cdot \nabla \phi \simeq -\frac{A ik}{4\pi r_1} e^{-ikr_1} \cos \theta_1$$

A similar evaluation can be done for the function G; but

in this case the normal vector $\hat{n}$ is negative and this gives:

$$\hat{n} \cdot \nabla G = \frac{ik}{4\pi r_2} e^{-ikr_2} \cos \theta_2$$

The Kirchhoff's integral equation (3-5) now becomes, on substitution of the above:

$$\phi(\vec{x}) = \oiint\limits_{S} -\frac{A_o jk}{16\pi^2} \frac{e^{-ik(r_1+r_2)}}{r_1 r_2} \left(\cos\theta_1 + \cos\theta_2\right) dS \quad (3\text{-}7)$$

With the assumptions of short wave length and hence k is large, use can be made of the stationary phase approximation in evaluating the integral appearing in equation (3-7) as shwon below.

III-3   Solution by Stationary Phase Methos:

The stationary phase method was developed by Lord Kelvin (21) toward the end of the last century and is based on the idea that the value of an integral containing a fast oscillating integral is essentially given by integrating over that part of the path where the phase change of the integrand is the slowest. The contribution from other parts of the integration parts will become very small due to strong cancellations. This method will be applied to the Kirchhoff's integral:

$$\phi(P) = \iint\limits_{S_1\, S_R} \left[\phi\frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n}\right] dS \quad (3\text{-}8)$$

The surface of integration is shown in figure (3-B). It

consists of two surfaces bounding the left upper quadrant

(surface S_1), and a spherical surface (surface S_R) thus

completing the boundary.

In equation (3-8) one may use the ray acoustic solution

for ϕ, and $\dfrac{\partial \phi}{\partial n}$. Those solutions are accurate to the order

of $\left(\dfrac{1}{k}\right)$, except in the transition zone. The boundary shown

in figure (3-B) is chosen to be outside the transition zone.

Since ϕ and $\dfrac{\partial \phi}{\partial n}$ are known the itegral could, in

principle, be integrated directly. Based on the assumptions

stated previously, however, the mathematical procedure can

be reduced immensely by the present method.

The expressions for ϕ_S and G are, respectively:

$$\phi_s = - \frac{e^{ikr_1}}{4\pi r_1} \qquad ; \qquad G = - \frac{e^{ikr_2}}{4\pi r_2} \qquad (3\text{-}9)$$

For large k, the integral shown in equation (3-8) is

approximately given by:

$$\phi(P) \simeq \frac{ik}{16\pi^2} \iint\limits_{S_1 \, S_R} \left[\frac{\partial r_2}{\partial n} - \frac{\partial r_1}{\partial n} \right] \frac{e^{ik(r_1 + r_2)}}{r_1 \, r_2} \, ds \qquad (3\text{-}10)$$

It will first be shown in the following discussion that

for any given wave number **k**, the contribution to the

integral from the spherical surface S_R tends to zero as the

sphere radius **R** tends to infinity. This result is obvious due

to the fact that the distance from a point of integration on

a spherical surface to the source **S** and the observation point

P approach the same value as **R** becomes very large; so that

the difference of the two terms $\left(\dfrac{\partial r_1}{\partial n} - \dfrac{\partial r_2}{\partial n}\right) \longrightarrow 0$ as $R \rightarrow \infty$

Hence

$$\int_{S_R}\left[\frac{\partial r_2}{\partial n} - \frac{\partial r_1}{\partial n}\right] \frac{e^{ik(r_1+r_2)}}{r_1\,r_2}\, dS = o(1)\cdot O\left(\tfrac{1}{R^2}\right)\cdot O(R^2) = o(1): \quad (3\text{-}11)$$
$$\longrightarrow 0 \quad \text{as} \quad R \rightarrow \infty$$

From the ray acoustics solutions, it is observed that the

integral that extends over the under surface of the plate is

zero because this boundary is in the shadow region where the

value of the velocity potential is identically zero.

Equation (3-8) then reduces to the form:

$$\phi(P) \simeq \frac{ik}{16\pi^2}\int_{S_1}\int_{S_R}\left[\frac{\partial r_2}{\partial \xi} - \frac{\partial r_1}{\partial \xi}\right] \frac{e^{ik(r_1+r_2)}}{r_1\,r_2}\, dS \qquad (3\text{-}12)$$

where S_R denotes the half plane $\eta - \zeta$ as shown in figure (3-D).
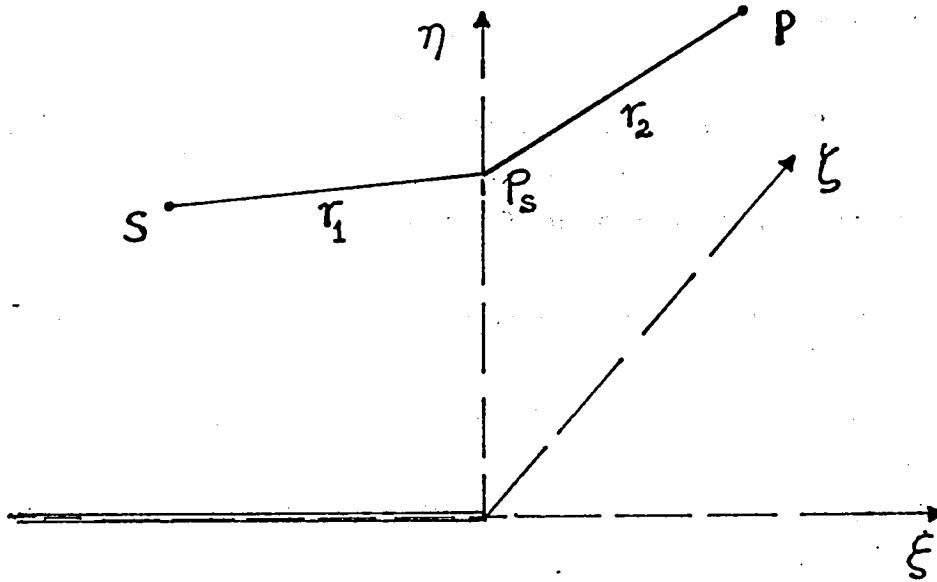
Figure (3-D): Half plane (η, ζ).

Let (X_s , Y_s , Z_s) denote the position of the source

and (X_p , Y_p , Z_p) the position of the observer and

(ξ , η , ζ) the point of integration. The expression for

r_1 and r_2 appearing in equation (3-10) are defined by:

$$r_1 = \sqrt{(\xi - X_s)^2 + (\eta - Y_s)^2 + (\zeta - Z_s)^2}$$

$$r_2 = \sqrt{(X_p - \xi)^2 + (Y_p - \eta)^2 + (Z_p - \zeta)^2}$$

(3-13) (a,b)

The method of stationary phase can now be applied if it is assumed $k \gg 1$. For convenience, equation (3-10) is expressed in the form:

$$\phi(P) = ik \int_0^\infty d\eta \left[\int_{-\infty}^\infty F(\eta,\zeta) \, e^{ikf(\eta,\zeta)} \, d\zeta \right] \qquad (3\text{-}14)$$

where

$$F(\eta,\zeta) = \frac{1}{16\pi^2} \left[\frac{\partial r_2}{\partial \xi} - \frac{\partial r_1}{\partial \xi} \right] \frac{1}{r_1 r_2} \qquad (3\text{-}15) \ (a)$$

and

$$f(\eta,\zeta) = r_1 + r_2 \qquad (3\text{-}15) \ (b)$$

The function $f = r_1 + r_2$ has a simple physical meaning. It represents the total distance an acoustic ray travels from the source point S to the observer point P through the point P_O on the integration path. As the integration point P_O moves on the $(\eta - \zeta)$ plane, the phase angle $k(r_1 + r_2)$ of the exponential function in equation (3-10) varies; resulting in strong cancellations of the wave for large values of k.

The attention will now be directed to the main

contribution of the integral in the neighbourhood of the

stationary phase point (η_o, ζ_o) defined by:

$$\frac{\partial}{\partial \eta}(r_1 + r_2) = 0 \quad ; \quad \frac{\partial}{\partial \zeta}(r_1 + r_2) = 0 \qquad (3\text{-}16)$$

Obviously, (η_o, ζ_o) is the point that lies on the straight

line joining the points **S** and **P**, as shown in figure (3-E)

below.
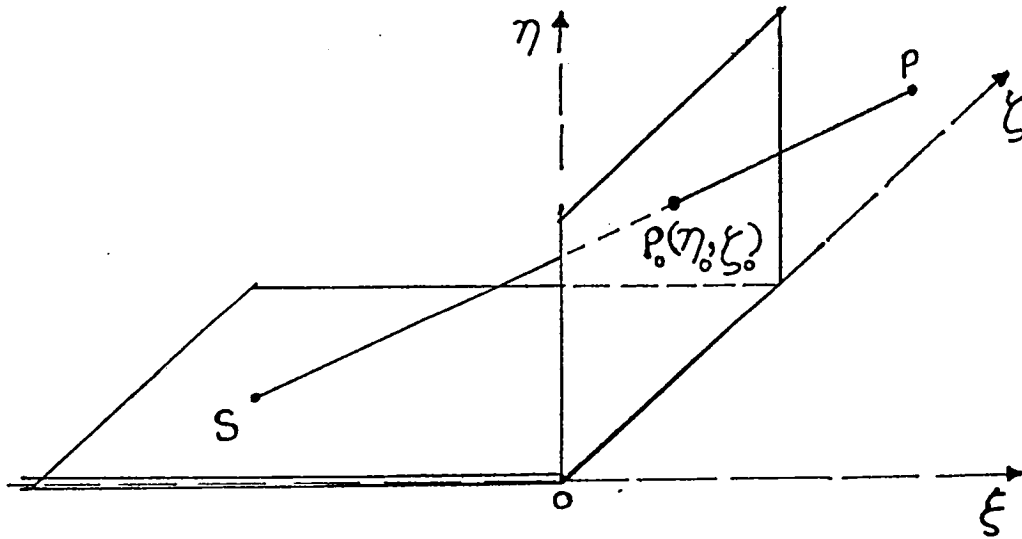
Figure (3-E): Line joining the points S and P
passes through the stationary point.

To apply the stationary point method, the function

$f(\eta, \zeta)$ is expanded into a double Taylor series around

the point (η_o, ζ_o) thus:

$$f(\eta, \zeta) = f(\eta_o, \zeta_o) + \frac{1}{2}\left[A(\eta - \eta_o)^2 + 2B(\eta - \eta_o)(\zeta - \zeta_o) + (\zeta - \zeta_o)^2\right] + \cdots \quad (3\text{-}17)$$

where $\dfrac{\partial f}{\partial \eta} = \dfrac{\partial f}{\partial \zeta}$ at $(\eta_o \ \zeta_o)$

From equations (3-13) it is found that:

$$\frac{\partial}{\partial \eta}(r_1 + r_2) = \frac{\eta - y_s}{r_1} - \frac{y_P - \eta}{r_2} \left.\vphantom{\frac{\frac{\eta}{r_1}}{\frac{\zeta}{r_1}}}\right\} \quad \text{(3-18) (a, b)}$$

$$\frac{\partial}{\partial \zeta}(r_1 + r_2) = \frac{\zeta - z_s}{r_1} - \frac{z_P - \zeta}{r_2}$$

and

$$\frac{\partial^2}{\partial \eta^2}(r_1 + r_2) = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) - \left[\frac{(\eta - y_s)^2}{r_1^3} + \frac{(y_P - \eta)^2}{r_2^3}\right]$$

$$\frac{\partial^2}{\partial \eta \partial \zeta}(r_1 + r_2) = -\left[\frac{(\zeta - z_s)(\eta - y_s)}{r_1^3} + \frac{(z_P - \zeta)(y_P - \eta)}{r_2^3}\right]$$

$$\frac{\partial^2}{\partial \zeta^2}(r_1 + r_2) = \left(\frac{1}{r_1} + \frac{1}{r_2}\right) - \left[\frac{(\zeta - z_s)^2}{r_1^3} + \frac{(z_P - \zeta)^2}{r_2^3}\right]$$

$$\text{(3-19) (a,b,c)}$$

At the stationary phase point $\eta = \eta_o$; $\zeta = \zeta_o$, so that

$$\frac{\partial}{\partial \eta}(r_1 + r_2) = \frac{\eta - y_s}{r_1} - \frac{y_P - \eta}{r_2} = 0$$

and

$$\frac{\partial}{\partial \zeta}(r_1 + r_2) = \frac{\zeta - z_s}{r_1} - \frac{z_P - \zeta}{r_2} = 0$$

$$\text{(3-20) (a,b)}$$

It follows from equations (3-13 a,b) that at $(\eta_o \ , \ \zeta_o)$

$$\frac{\xi - X_s}{r_1} = \sqrt{1 - \frac{(\eta - Y_s)^2}{r_2^2} - \frac{(\zeta - z_s)^2}{r_1^2}} = \sqrt{1 - \frac{(Y_p - \eta)^2}{r_2^2} - \frac{(z_p - \zeta)^2}{r_1^2}}$$

$$= \frac{X_p - \xi}{r_2} \tag{3-21}$$

thus,

$$\frac{\xi_o - X_s}{X_p - \xi_o} = \frac{\eta_o - Y_s}{Y_p - \eta_o} = \frac{\zeta_o - z_s}{X_p - \zeta_o} \tag{3-22}$$

That is, the three points (X_s , Y_s , z_s), (ξ_o, η_o, ζ_o)

and (X_p, Y_p, z_p) all lie on the same straight line. Using

equations (3-13 a, b), the expressions for $\frac{\partial^2}{\partial \eta^2}(r_1 + r_2)$,

$\frac{\partial^2}{\partial \eta \partial \zeta}(r_1 + r_2)$, and $\frac{\partial^2}{\partial \zeta}(r_1 + r_2)$ may be written as:

$$A \equiv \left[\frac{\partial^2}{\partial \zeta^2}(r_1 + r_2)\right]_o = \frac{r}{(r_1 r_2)_o}\left[1 - \frac{(z_p - z_s)^2}{r^2}\right]$$

$$B \equiv \left[\frac{\partial^2}{\partial \eta \partial \zeta}(r_1 + r_2)\right] = -\frac{r}{(r_1 r_2)_o}\frac{(z_p - z_s)(Y_p - Y_s)}{r^2} \tag{3-23} (a,b,c)$$

$$C \equiv \left[\frac{\partial^2}{\partial \eta^2}(r_1 + r_2)\right] = \frac{r}{(r_1 r_2)_o}\left[1 - \frac{(Y_p - Y_s)^2}{r^2}\right]$$

where r = $\sqrt{(X_p - X_s)^2 + (Y_p - Y_s)^2 + (z_p - z_s)^2}$ and the subscript O

denotes evaluation at $\eta = \eta_o$ and $\zeta = \zeta_o$.

To carry out the integration separately with respect to η and ζ one can write equation (3-17) in the form:

$$k(r_1+r_2) = kr + \frac{k}{2}\left[\sqrt{A}\,(\zeta-\zeta_0) + \frac{B}{\sqrt{A}}(\eta-\eta_0)\right]^2 + \frac{k}{2}\left(C-\frac{B^2}{A}\right)(\eta-\eta_0)^2 \quad \textbf{(3-24)}(a)$$

$$= kr + \alpha_0^2 + \beta^2 \cdot \qquad\qquad (3\text{-}24)\ (b)$$

where the new variables α_0 and β are defined by:

$$\left.\begin{array}{l} \alpha_0^2 = \dfrac{k}{2}\left[\sqrt{A}\,(\zeta-\zeta_0) + \dfrac{B}{\sqrt{A}}(\eta-\eta_0)\right]^2 \\[4mm] \beta^2 = \dfrac{k}{2}\left[C - \dfrac{B^2}{A}\right](\eta-\eta_0)^2 \end{array}\right\} \qquad (3\text{-}35)(a,b)$$

then

$$d\alpha_0 = \sqrt{\frac{kA}{2}}\ d\zeta \quad ; \quad d\beta = \sqrt{\frac{k}{2}}\ \sqrt{C - \frac{B^2}{A}}\ d\eta$$

By substuting the expressions for A,B,C, one finds that:

$$d\beta = \sqrt{\frac{k}{2}}\ \frac{1}{\sqrt{A}}\ \frac{(X_P - X_S)}{(r_1 r_2)_0}\ d\eta$$

and

$$d\alpha_0 d\beta = \frac{k}{2}\ \frac{(X_P - X_S)}{(r_1 r_2)_0}\ d\eta\, d\zeta$$

The integral of ϕ then becomes:

$$
\left.
\begin{aligned}
\phi &= \frac{ik}{16\pi^2} \iint \frac{1}{r_1 r_2}\left[\frac{\xi - x_s}{r_1} + \frac{x_P - \xi}{r_2} \right] e^{ik(r_1 + r_2)} \, d\eta \, d\zeta \\[2mm]
&= \frac{ik}{16\pi^2} \iint \frac{2}{(r_1 r_2)_0} \, \frac{x_P - x_s}{r} \, e^{ik(r_1 + r_2)} \, d\eta \, d\zeta \\[2mm]
&= \frac{-i\,e^{ikr}}{4\pi^2 r} \int_{\bar{\beta}}^{\infty} \left[\int_{-\infty}^{\infty} e^{i\alpha^2} \, d\alpha_0 \right] e^{i\beta^2} \, d\beta
\end{aligned}
\right\}
\qquad (3\text{-}26)
$$

where the end point value $\bar{\beta}$ will be defined later. Note that

in obtaining equation (3-26) one has to use the relation that

at the point $(\eta_0, \, \zeta_0)$,

$$
\frac{\xi - x_s}{r_1} = \frac{x_P - \xi}{r_2} = \frac{x_P - x_s}{r} \qquad (3\text{-}27)
$$

The definate integral in equation (3-26) can be readily

integrated, yielding:

$$
\int_{-\infty}^{\infty} e^{i\alpha_1^2} \, d\alpha_0 = \frac{\pi}{2}(1+i) \qquad (3\text{-}28)
$$

For the integral involving β one may define a function $F(\bar{\beta})$ by:

$$F(\bar{\beta}) = \frac{1-i}{2} \sqrt{\frac{2}{\pi}} \int_{\bar{\beta}}^{\infty} e^{i\beta^2} d\beta \qquad (3-29)$$

Upon substitution of equations (3-28) and (3-29) then equation (3-26) becomes:

$$\phi = -\frac{e^{ikr}}{4\pi r^2} F(\bar{\beta}) \qquad (3-30)$$

If the common forms of Fresnel integrals are used:

$$\left. \begin{array}{l} C(z) = \displaystyle\int_0^z \cos\left(\frac{\pi t^2}{2}\right) dt \\[4mm] S(z) = \displaystyle\int_0^z \sin\left(\frac{\pi t^2}{2}\right) dt \end{array} \right\} \qquad (3-31)$$

$F(\bar{\beta})$ may be written as:

$$F(\bar{\beta}) = \frac{1}{2} - \frac{1-i}{2}\left[C\left(\sqrt{\frac{2}{\pi}}\,\bar{\beta}\right) + S\left(\sqrt{\frac{2}{\pi}}\,\bar{\beta}\right) \right] \qquad (3-32)$$

The asymptotic behaviour of $C(z)$ and $S(z)$ are given by the following equations.

$$C(z) \simeq \frac{1}{2} + \frac{\sin\left(\frac{\pi}{2}z^2\right)}{\pi z} + O\left(\frac{1}{z^3}\right) \qquad (3\text{-}33)\ (a)$$

$$(z \to \infty,\ z > 0)$$

$$S(z) \simeq \frac{1}{2} - \frac{\cos\left(\frac{\pi}{2}z^2\right)}{\pi z} + O\left(\frac{1}{z^3}\right) \qquad (3\text{-}33)\ (b)$$

It can then be easily seen that:

$$\left. \begin{array}{l} F(\bar{\beta}) \simeq 1 + \dfrac{1+i}{2}\ \dfrac{e^{i\bar{\beta}^2}}{\sqrt{2\pi}\ \bar{\beta}} + O\left(\dfrac{1}{\bar{\beta}^3}\right); (\bar{\beta} > 0) \\[4mm] F(\bar{\beta}) \simeq 1 - \dfrac{1+i}{2}\ \dfrac{e^{i\bar{\beta}^2}}{\sqrt{2\pi}\ \bar{\beta}} + O\left(\dfrac{1}{\bar{\beta}^3}\right); (\bar{\beta} < 0) \end{array} \right\} \quad (3\text{-}34)\,(a,b)$$

In deriving the second expression in equation (3-34) (a) and

(b) above, use has been made of the relations $C(-Z) = -C(Z)$

and $S(-Z) = -S(-Z)$.

The Lower Limit $\bar{\beta}$:

One recals that the equation (3-26) is integrated using

the new variables defined by:

$$\left. \begin{array}{l} \alpha_o = \sqrt{\dfrac{k}{2}}\left[\sqrt{A}\,(\xi - \zeta_o) + \dfrac{B}{\sqrt{A}}\,(\eta - \eta_o)\right] \\[4mm] \beta = \sqrt{\dfrac{k}{2}}\,\sqrt{\left(C - \dfrac{B^2}{A}\right)}\,(\eta - \eta_o) \end{array} \right\} \qquad (3\text{-}35)\ (a,b)$$

The line $\alpha_0 = 0$ is the locus of point on which $\frac{\partial f}{\partial \zeta} = 0$.

This means that along such a line, the function f has

stationary values with respect to the variable ζ . The

range of the limit of integration for $\bar{\beta}$ is, therefore, from

P^* to $\bar{\beta} = \infty$ as shown in figure (3-F) below.
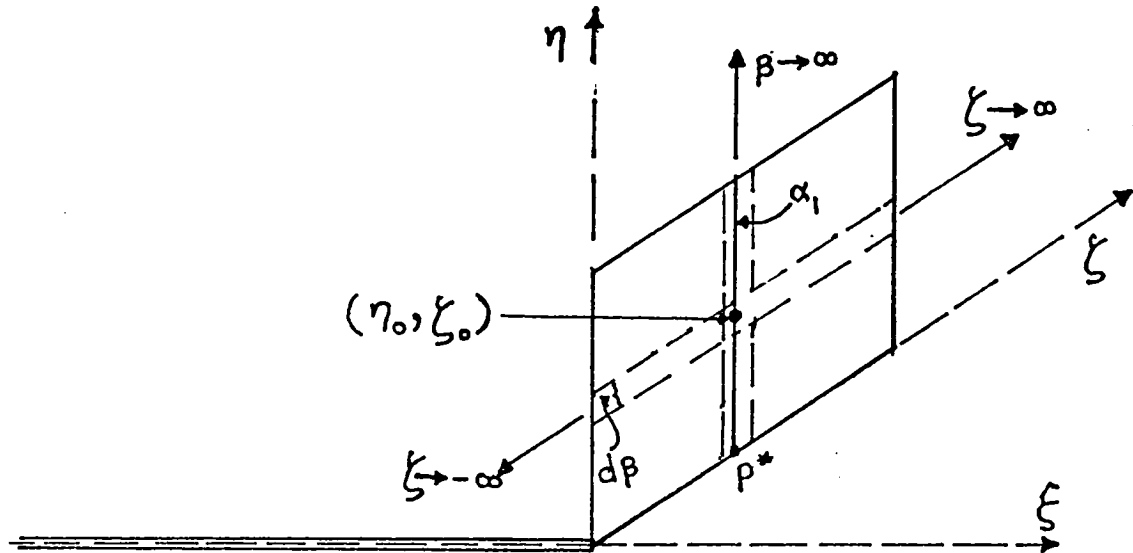
Figure (3-F): The range of the limit of integration
for $\bar{\beta}$.

The point P^* lies on the trailing edge of the plate where

$\frac{\partial f}{\partial \zeta} = 0$. From equation (3-24) (b) it can be seen that

$$\bar{\beta}^2 = k r_0 - k r \qquad (3-36)$$

where $r_0 = r_1 + r_2$ at the point P^*.

To derive the expression r_0, one may write r_1 and r_2 in the form:

$$r_1 = \sqrt{\rho_1^2 + (\zeta - z_s)^2} \qquad (3\text{-}37)\ (a)$$

$$r_2 = \sqrt{\rho_2^2 + (z_p - \zeta)^2} \qquad (3\text{-}37)\ (b)$$

where ρ_1 and ρ_2 are the projections of r_1 and r_2 on the

(ξ, η) plane figure (3-G) below.
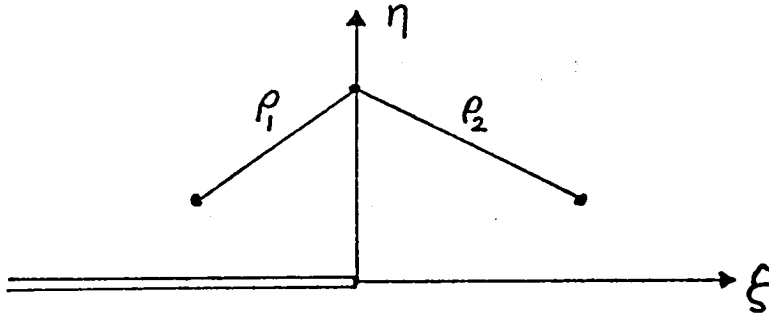
Figure (3-G): Acoustic lengths on the projection
of the plane (η, ξ).

At P^*, ζ satisfied the equation: $\dfrac{\partial f}{\partial \zeta} = \dfrac{\partial}{\partial \zeta}(r_1 + r_2)$ (3-38)

Using equations (3-37)(a),(b), this becomes:

$$\frac{\zeta^* - z_s}{\sqrt{\rho_1^2 + (\zeta^* - z_s)^2}} - \frac{z_p - \zeta^*}{\sqrt{\rho_2^2 + (z_p - \zeta^*)^2}} = 0 \qquad (3\text{-}39)$$

which can be written in the form:

$$\frac{\zeta^* - z_s}{\sqrt{\rho_1^2 + (\zeta^* - z_s)^2}} = \frac{z_p - \zeta^*}{\sqrt{\rho_2^2 + (z_p - \zeta^*)^2}} = \gamma \text{ (say)} \qquad (3-40)$$

Hence

$$\left.\begin{aligned}
\zeta^* - z_s &= \frac{\gamma}{\sqrt{1 - \gamma^2}}\, \rho_1 \\[2ex]
z_p - \zeta^* &= \frac{\gamma}{\sqrt{1 - \gamma^2}}\, \rho_2 \\[2ex]
z_p - z_s &= \frac{\gamma}{\sqrt{1 - \gamma^2}}\, (\rho_1 + \rho_2) \\[2ex]
\gamma &= \frac{z_p - z_s}{\sqrt{(\rho_1 + \rho_2)^2 + (z_p - z_s)^2}}
\end{aligned}\right\} \qquad (3-41) \; (a,b,c,d)$$

At the point $\zeta = \zeta^*$ one has, therefore, the following

relation:

$$\frac{\zeta - z_s}{\sqrt{\rho_1^2 + (\zeta - z_s)^2}} = \frac{z_p - \zeta}{\sqrt{\rho_2^2 + (z_p - \zeta)^2}} = \frac{z_p - z_s}{\sqrt{(\rho_1 + \rho_2)^2 + (z_p - z_s)^2}} \qquad (3-42)$$

Equation (3-42) is presented graphically by figure (3-H).
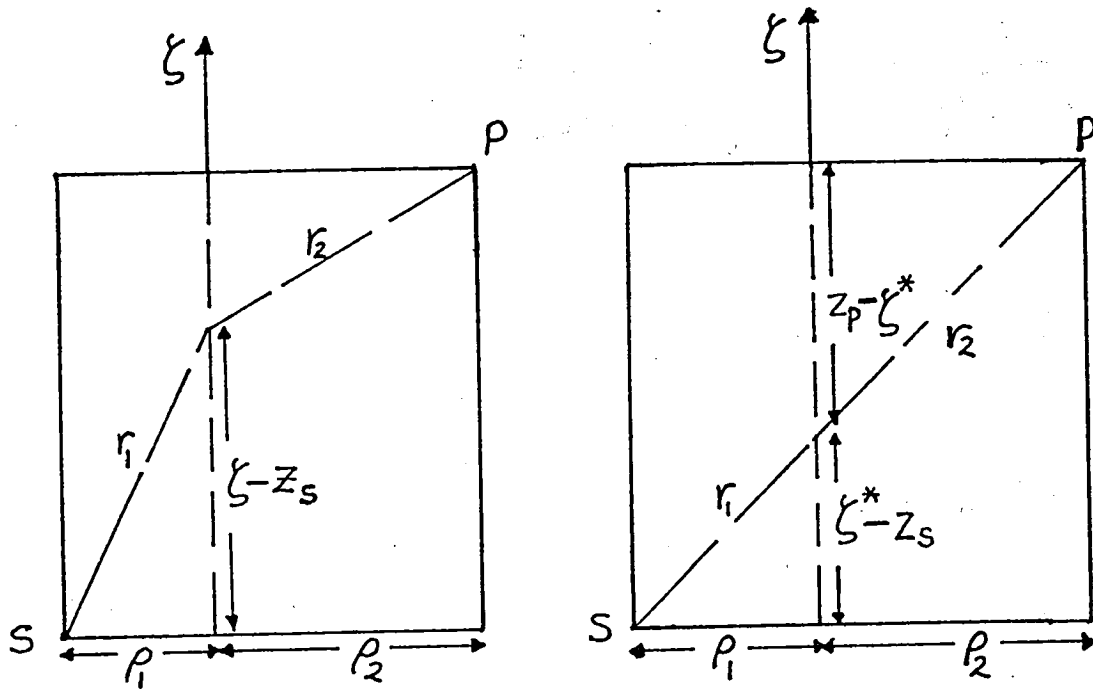
Figure (3-H): Graphical sketch of acoustic length in relation to the stationary point.

The distances r_1 and r_2 at P^* are seen to be:

$$r_0 = \sqrt{\left(\rho_s + \rho_p\right)^2 + \left(z_p - z_s\right)^2} \qquad (3\text{-}43)$$

so that $\overline{\beta}^2$ can then be expressed as:

$$\overline{\beta}^2 = k\left[\sqrt{\left(\rho_s + \rho_p\right)^2 + \left(z_p - z_s\right)^2} - \sqrt{\left(x_p - x_s\right)^2 + \left(y_p - y_s\right)^2 + \left(z_p - z_s\right)^2}\right] \qquad (3\text{-}44)$$

III-4 Conclusions and Discussions:

Based on the geometrical configuration shown in figure

(3-I) the main results of the above analysis can be

summarized.
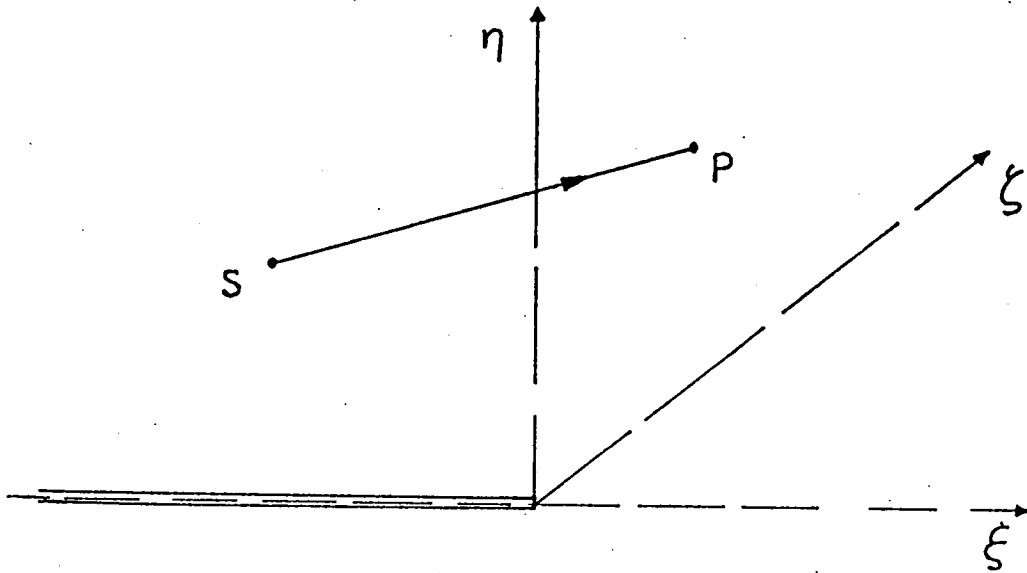
Figure (3-I): Wave reception at P due to three-dimensional
source in the presence of a semi-infinite
plate.

The formulae that have been developed in the analysis take

into account only the radiation from the source **S**. To

obtain the complete solution, the contribution from the

reflected waves from the upper surface, which can be calculated

by the same procedure, must be included.

The formulae for calculating the acoustic field by a

source at **S** are as follows:

Let $\phi_0(P)$ denoted the potential at the point P due to the

source S in the absence of the plate.

$$\phi_0(P) = - \frac{e^{ikr}}{4\pi r} \qquad (3-45)$$

The velocity potential $\phi(P)$ in the presence of the plate will

be:

$$\phi(P) = \phi_0(P) \ F(\bar{\beta}) \qquad (3-46)$$

where $F(\bar{\beta})$ is the diffraction factor. The function $F(\bar{\beta})$

is defined by the integral:

$$F(\bar{\beta}) = \frac{1-i}{2} \sqrt{\frac{2}{\pi}} \int_{\bar{\beta}}^{\infty} e^{i\beta^2} d\beta \qquad (3-47) \ (a)$$

$$= \frac{1}{2} - \frac{1-i}{2} \left[C\left(\sqrt{\frac{2}{\pi}} \bar{\beta}\right) + i S\left(\sqrt{\frac{2}{\pi}} \bar{\beta}\right) \right] \qquad (3-47) \ (b)$$

$$= \left\{ \begin{array}{ll} 0 & \bar{\beta} \longrightarrow \infty \\ \frac{1}{2} & \bar{\beta} \longrightarrow 0 \\ 1 & \bar{\beta} \longrightarrow -\infty \end{array} \right\} \qquad (3-47) \ (c)$$

The following three cases for the values of $\bar{\beta}$ are considered.

Case 1 $\bar{\beta}=0$:

This corresponds to the case when the line drawn from

the observer point P to the source passes through the edge

of the plate. In such a case,

$$F(\overline{\beta}) = 1/2 \text{ and therefore}$$

$$\phi = 1/2 \, \phi_o(P) \tag{3-48}$$

This means that half of the radiation emitted by the source

is received by the observer.

Case 2 $\overline{\beta} < 0$:

In this case, the observer is on the upper side of the

line $\overline{\beta} = 0$. The edge of the plate lies below the

stationary phase point. The diffration factor $F(\overline{\beta})$ is

greater than half, and increases to unity, as the observation

point moves further upwards. In the limit, $\phi(P)$ takes the

value $\phi_o(P)$ so that the sound emitted is not obstructed.

Case 3 $\beta > 0$:

This occurs when the observer is at the lower side of

the $\overline{\beta} = 0$ line. The value of $\overline{\beta}$ is positive and continues

to increase as the observation point P moves downward. The

value $F(\overline{\beta})$ decreases as $\overline{\beta}$ increases, and in the limit ϕ

tends to zero, so that the observer is in the shadow region.

For any given value of $\bar{\beta}$, the variation of the

diffraction factor $F(\bar{\beta})$ is shown in figure (3-J) below.

As the observer moves from P_+ in the transmission region to

P_- in the shadow region, the sound varies from complete

reception to zero. The transition of $F(\bar{\beta})$ from zero to

unity occurs in the range of approximately (ref. 1)

$$-20 \leq \bar{\beta} \leq 20$$
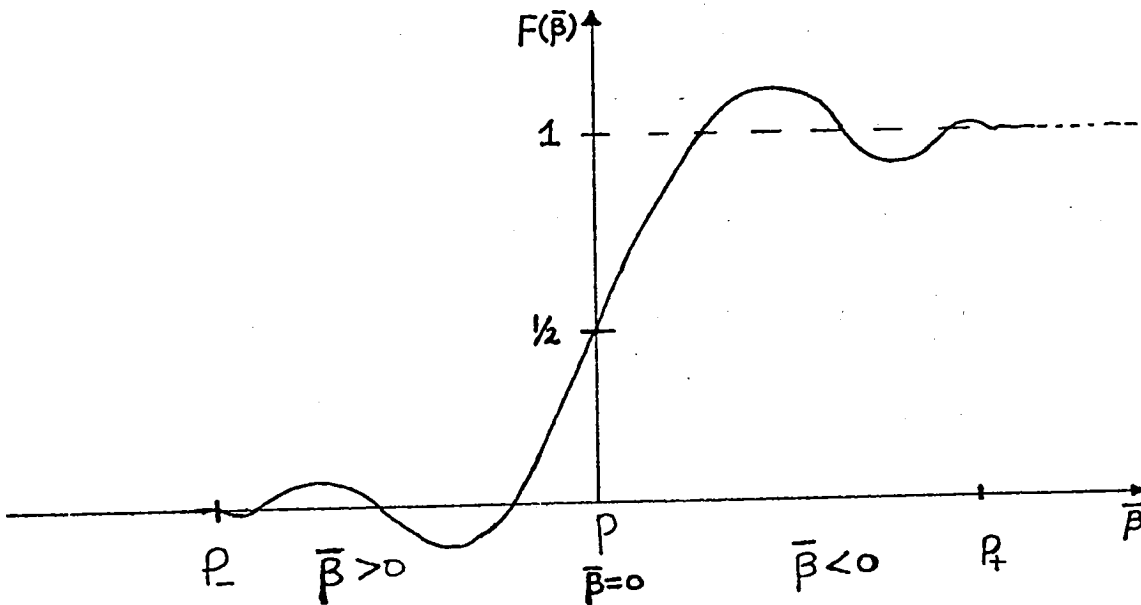
Figure (3-J): Variation of the diffraction factor $F(\bar{\beta})$.

One may note that for large values of k, the value of

$\phi(P)$ depends on the integration over a small region around

a stationary phase point. If the edge of the plate lies

inside this region, it will make no effect on the transmission

of sound from the source to the observer.

III-5 Applications and Comparisons with known Solutions:

Cooke (9) has derived an approximate expression for the

far field due to a three-dimensional source over a

semi-infinite plate. He gives the solution for ϕ for the

direct radiation from the source as:

$$\phi(P) = \frac{e^{ikr}}{r}\left[\frac{1}{2} - \frac{1-i}{2}\left[C(\beta) + i\,S(\beta)\right]\right] \tag{3-49}$$

The solution derived in the present study is given by the

multiplication of the results obtained in equations (3-46)

and (3-47)(b), so that

$$\phi(P) = -\frac{e^{ikr}}{4\pi r}\left[\frac{1}{2} - \frac{1-i}{2}\left[C\left(\sqrt{\tfrac{2}{\pi}}\,\bar{\beta}\right) + i\,S\left(\sqrt{\tfrac{2}{\pi}}\,\bar{\beta}\right)\right]\right] \tag{3-50}$$

Apart from the difference in the factor $\left(-\dfrac{1}{4\pi}\right)$, the

solution obtained in the present study agrees with that

obtained by Cooke.

The two-dimensional problem shown in figure (3-K) below

may be considered as a special case of the above

three-dimensional problem, and it's solution may be obtained

by letting $Z = Z_0 = 0$ and $\rho_s \to \infty$ in equations (3-44) and

(3-46). In taking the limit, the equations

$$\phi(P) = \phi(P) \, F(\bar{\beta})$$

$$F(\bar{\beta}) = \frac{1-i}{2} \sqrt{\frac{2}{\pi}} \int_{\bar{\beta}}^{\infty} e^{i\beta^2} d\beta$$

remain unchanged. The two important quantities that need

to be calculated when $\rho_s \to \infty$ are r, the distance from the point

P to the source, and r_0, the acoustic length given by equation

(3-43).
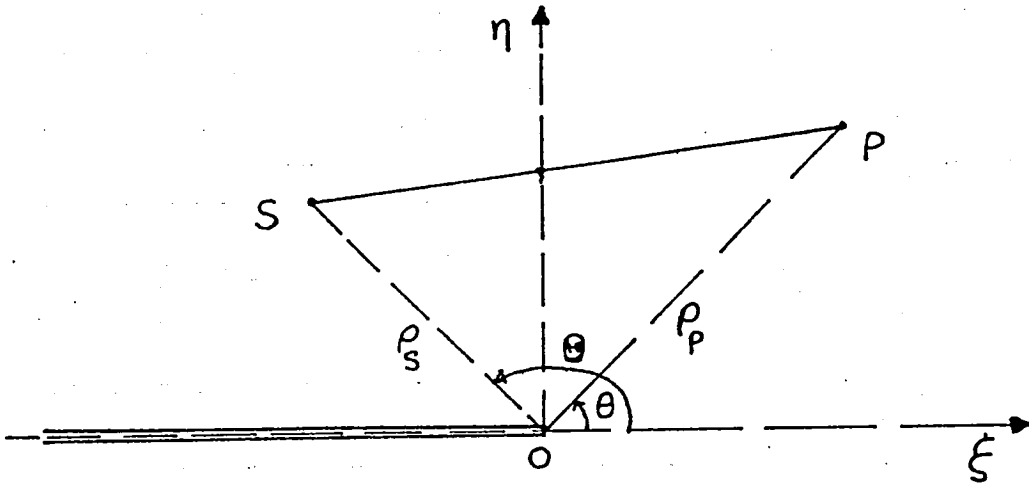
Figure (3-K): Two-dimensional problem sketch.

In terms of the notations used in figure (3-K), one finds:

$$r = \sqrt{\rho_s^2 + \rho_p^2 - 2\rho_s\rho \, \cos(\Theta - \theta)}$$

$$\simeq \rho_s - \rho_p \cos(\Theta - \theta) \qquad as \qquad \rho_s \to \infty$$

and $\quad r_0 = \rho_s + \rho_p$

Using these values, the expressions of $\phi(P)$ and $\bar{\beta}$ are

simplified to the forms:

$$\phi_0(P) = -\frac{e^{ik\rho_s}}{4\pi\rho_s} \; e^{-ik\rho\cos(\Theta - \theta)} \left. \right\}$$
$$\simeq e^{-ik\rho\cos(\Theta - \theta)} \qquad \qquad (3\text{-}44)$$

(after removing the non-essential constant multiplier by

re-normalization) and:

$$\bar{\beta} = \sqrt{k(r_0 - r)} = \sqrt{k\rho(1 + \cos(\Theta - \theta))}$$

$$= \sqrt{2k\rho} \, \cos\left(\frac{\Theta - \theta}{2}\right)$$

Note that the sign of $\bar{\beta}$ is either negative or positive

depending on whether the plate tip ($\beta = \bar{\beta}$) is below or

above the stationary phase point $\beta = 0$, figure (3-L) shown

on the next page. For the incident wave, $\bar{\beta}$ is below the

point $\beta = 0$, so that it's value is negative and is given by:

$$\bar{\beta} = -\sqrt{2k\rho} \, \cos\left(\frac{\Theta - \theta}{2}\right)$$
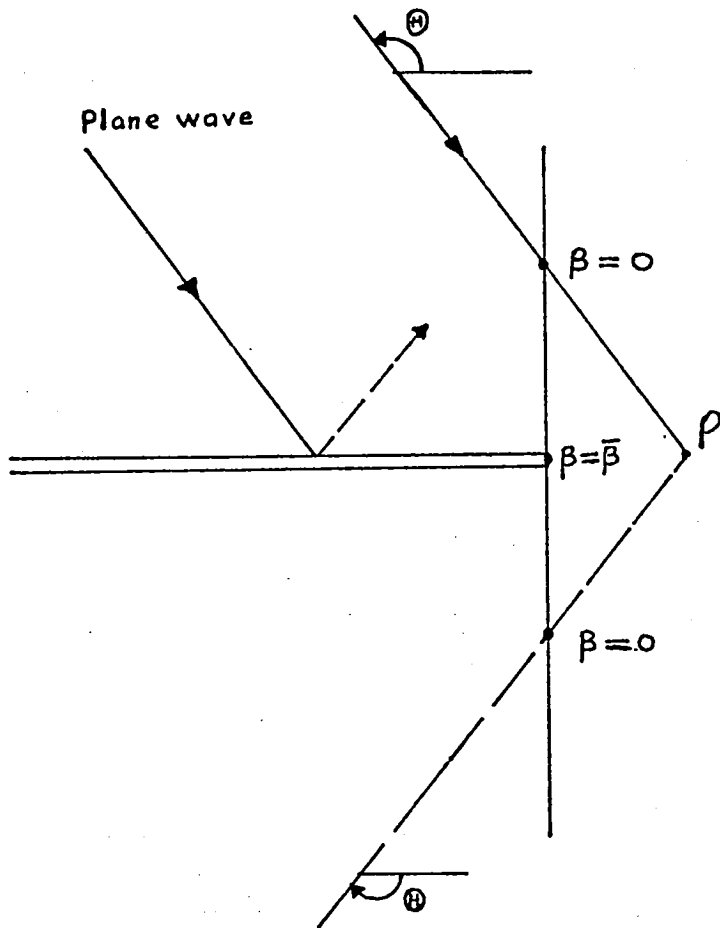
Figure (3-L): Plane wave problem showing ranges of integration ($\bar{\beta}$).

For the reflected waves, Θ must be changed to $-\Theta$ and the value of $\bar{\beta}$ is positive as given by:

$$\bar{\beta} = \sqrt{2k\rho}\ \cos\left(\frac{\Theta-\theta}{2}\right)$$

The total velocity potential due to both waves is then:

$$\phi_T(P) = e^{-ik\rho\cos(\theta-\Theta)}\ F\left\{-\sqrt{2k\rho}\ \cos\left(\frac{\theta-\Theta}{2}\right)\right\} +$$

$$e^{-ik\rho\cos\left(\frac{\theta+\Theta}{2}\right)}\ F\left\{\sqrt{2k\rho}\ \cos\left(\frac{\theta+\Theta}{2}\right)\right\} \tag{3-45}$$

This result is identical to that given by equation (2.86) on page (73) of Noble[*]. When θ is sufficiently far from $\Theta-\pi$ and $\pi-\Theta$, the asymptotic expression for ϕ_T for large values of $k\rho$ may be obtained using the asymptotic expression of F given by equations (3-34) (a,b). With the aid of the trigonometric identities: $2\cos^2\frac{(\theta\pm\Theta)}{2} = 1 + \cos(\theta\pm\Theta)$

$$2\cos\frac{(\theta+\Theta)}{2}\cos\frac{(\theta-\Theta)}{2} = \cos\theta + \cos\Theta$$

---

* Noble B., "Methods based on the Wiener-Hopf technique" (1958); where ρ is denoted by r and $F(\nu)$ is replaced by

$$\frac{e^{-i\pi/4}}{\sqrt{\pi}}\ F(\nu) \quad \text{with} \quad F(\nu) = \int_\nu^\infty e^{i\beta^2}\,d\beta$$

it is found that

$$\phi_T \simeq e^{-ik\rho \cos(\theta - \Theta)} + e^{-ik\rho \cos(\theta + \Theta)} + \phi_d$$

where

$$\phi_d \simeq (2\pi k\rho)^{-1/2} \frac{2 \sin \frac{\theta}{2} \sin \frac{\Theta}{2}}{\cos \theta + \cos \Theta} e^{-ik\rho + i\pi/4}$$

ϕ_d is the asymptotic expansion of the diffracted wave and is

the same expression expressed by Candel (5) except for the

sign in the complex exponent.

Chapter IV

TWO-STREAM-FLOW ACOUSTIC PROBLEM

IV-1 Nature of the Problem:

In conventional and short-take off and landing aircraft, the high velocity exhaust gas from the engine is usually the dominating noise source. The so-called "mixing noise", which is produced by the turbulent eddies in a region near the nozzle exit, together with the turbine and other engine noise is transmitted through a jet stream into the ambient atmosphere, which, in most cases, is in relative motion with respect to the aircraft. Figure (4-A) below shows the situation when the engine is installed over the wing.

EXTERNALLY BLOWN FLAP
OVER THE WING
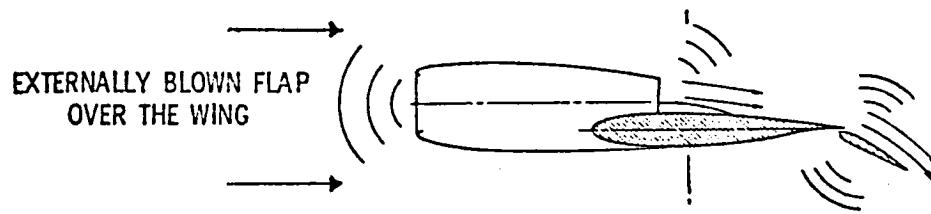
Figure (4-A): Over the wing external blowing.

Since the shielding of the wing is more effective on the

high frequency end of the noise spectrum, the effect of

shielding is studied based on the model problem shown in

figure (4-B) below.
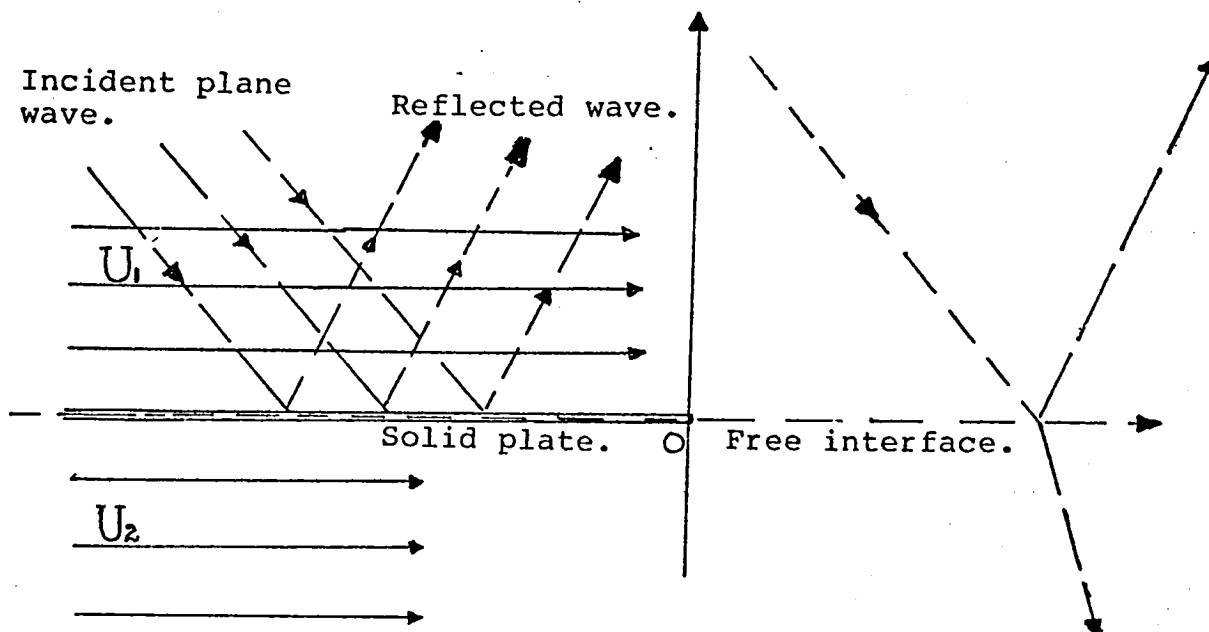
Figure (4-B): Two stream flow with plane wave
and solid plate.

The problem consists of a wing surface which separates

two of different velocities. In the region above the wing,

the flow represents the exhaust of jet. The source of the

noise is embedded in this flow at some distance from the

surface. The flow below the surface represents the forward

motion of the aircraft, and the interest here is in studying

the acoustic field below the wing.

The following assumptions will be used for the study of

this problem.

(i)   The problem is treated as two dimensional.

(ii)   The wing surface is approximated by a semi-infinite

        thin plate lying along the negative X-axis.

(iii)   The dorminant frequency of the source is high

        enough so that the short wave length approximation

        may be used.

(iv)   To simplify the mathematical treatment, the

        incoming sound wave is treated as a plane wave as

        an approximation to the case when the source is

        very far from the edge of the plate.

(v)   The jet stream is attached to the wing surface and

        extends over the entire half plane.

The method to be used in studying this problem is

basically that discussed in chapter III. The first

approximation of the problem is obtained by applying the

method of ray acoustics. The boundanry surface consists of

not only solid surface but also the interface between the two

streams where transmission and reflection take place.

In solving this classical problem (transmission of sound

through a shear layer), no consideration will be attempted

in the stability of the problem. The study here will be

confined to the study of the sound field below the plate, so

that the boundary chosen for the Kirchhoff's integral consists

of the half circle bounded by the flat plate and the free

layer.

The solution for this problem is obtained for the case

of a large wave number. Since the problem involves no

characteristic length, this solution will be valid for any

value of k at a large distance also. The effects of the free

stream, the deflection and reflection at the interface are

contained in the results obtained. However, no numerical

calculations have been attempted.

IV-2   Problem Formulation:

The problem outlined in section IV-1 above is

represented in figure (4-B). The field equations are

appropriately given as:

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2} - \frac{1}{a_0}\left(\frac{\partial}{\partial t} + U_1\frac{\partial}{\partial x}\right)^2 \phi_1 = 0 \qquad (4\text{-}1)$$

$$\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial y^2} + \frac{\partial^2 \phi_2}{\partial z^2} - \frac{1}{a_0}\left(\frac{\partial}{\partial t} + U_2\frac{\partial}{\partial x}\right)^2 \phi_2 = 0 \qquad (4\text{-}2)$$

where ϕ_1 and ϕ_2 are velocity potentials above and below the

$y = 0$ plane respectively. The boundary conditions are:

(i) $\quad \dfrac{\partial \phi_1}{\partial y} = \dfrac{\partial \phi_2}{\partial y} = 0 \qquad$ on $y = 0$; $-\infty < x < 0$

(ii) $\quad P_1 = P_2$ and $\quad \eta_1 = \eta_2$ on $y = 0$; $0 < x < \infty$ where P_1

and P_2 are the pressures and η_1 and η_2 the particle

displacements above and below the $y = 0$, $x > 0$ plane

calculated from ϕ_1 and ϕ_2

(iii) The Sommerfeld radiation condition is to be satisfied.

The above problem will be solved by applying the

Kirchhoff's integral over the lower half plane. To do this

one has to find the boundary value of ϕ_2 at the interface by

the analysis given in section IV-3.

IV-3 (1)   The Dispersion Relation of a Plane wave in a

Uniform Stream:

The convected wave equation in a stream of velocity U

is $\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} - \dfrac{1}{a_0}\left(\dfrac{\partial}{\partial t} + U\dfrac{\partial}{\partial x}\right)\phi = 0$ (4-3)

If $\phi = e^{i(k_1 x + k_2 y) - i\omega t}$ then equation (4-3) becomes,

with the definition of $k = \dfrac{\omega}{a_0}$; $M = \dfrac{U}{a_0}$

$$\left(k_1 + \dfrac{Mk}{1-M^2}\right)^2 + \dfrac{k_2^2}{1-M^2} = \dfrac{k^2}{(1-M^2)^2}$$ (4-4)

The locus of relation (4-4) is an ellipse as shown in figure

(4-C).
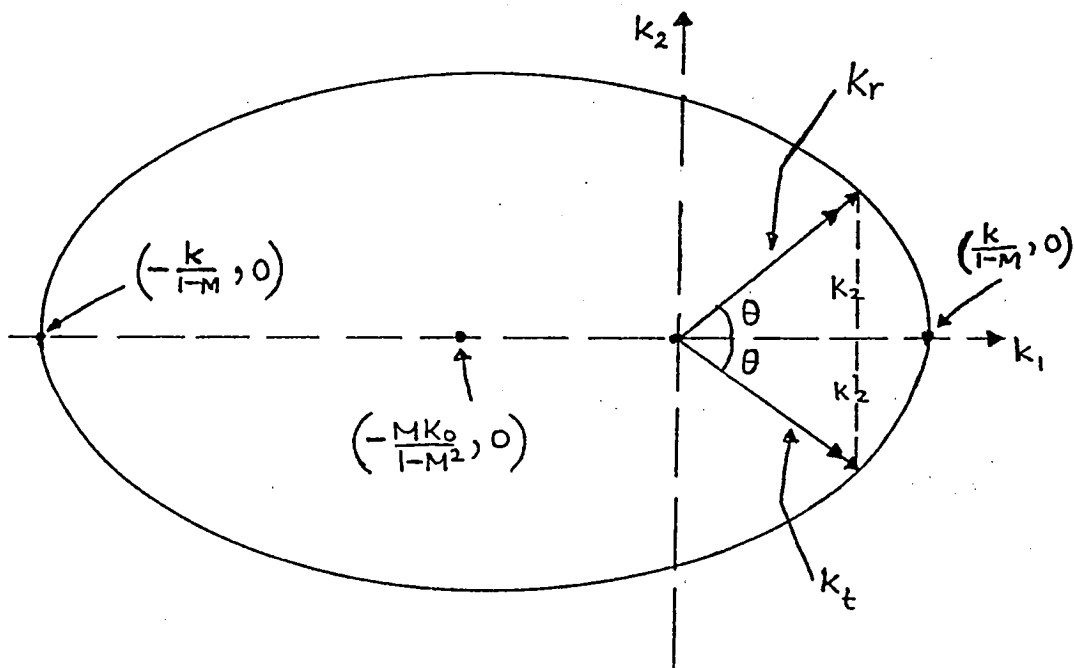
Figure (4-C): An ellipse showing the locus of the
incident wave number k_1.

It is to be noted that for any given value of K_1 , two

solutionns of K_2 are possible. The one denoted by $\vec{K}_r$ has a

possitive K_2 and corresponds to a plane wave propagating

upward away from the $y = 0$ plane; while the one denoted by $\vec{K}_t$

has a negative value of K_2 and corresponds to a plane wave

propagating downward into the half-plane.

(2) Reflection and deflection at the interface of two uniform

flows with different velocities U_1 and U_2:

If two flows are involved as in section IV-2, then there

will be equations for the two conic sections (ellipses)

determined in the manner outlined in the last section. The

two ellipses which intersect one another are shown in figure

(4-D) below.
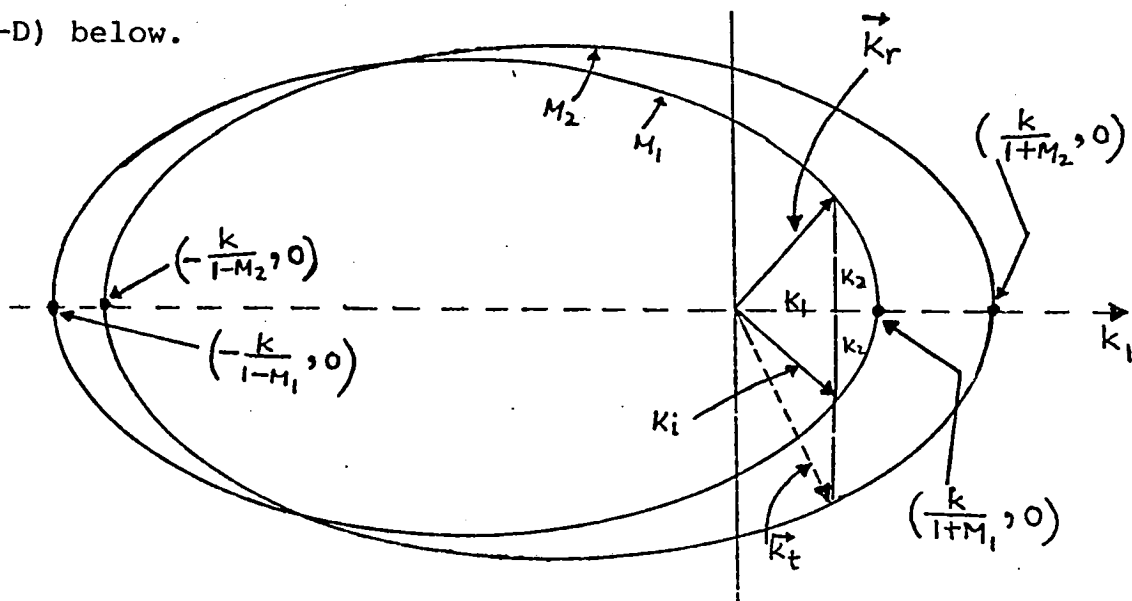
Figure (4-D): Superposed ellipses showing the two
stream flow acoustic structure.

The equations are given below:

$$\left(k_1 + \frac{M_1 k}{1-M_1^2}\right)^2 + \frac{k_2^2}{1-M_1^2} = \frac{k^2}{(1-M_1^2)^2} \quad ; \quad Y > 0 \tag{4-5}$$

$$\left(k_1 + \frac{M_1 k}{1-M_2^2}\right)^2 + \frac{k_2^2}{1-M_2^2} = \frac{k^2}{(1-M_2^2)^2} \quad ; \quad Y < 0 \tag{4-6}$$

At the interface $Y = 0$ separating the two streams U_1 and U_2 an incident wave will produce a transmitted wave as well as a reflected one. As will be shown in the next section, the K_1 component of these three waves must be equal. With a given incident wave K_i propagating towards the interface, the reflected wave K_r in the upper half plane $Y > 0$ is determined from the ellipse $M = M_1$ which is the Mach number of the upper stream, and the transmitted wave $\vec{K_t}$ in the lower half-plane $Y < 0$ is determined from the other ellipse denoted by $M = M_2$ which is the Mach number of the lower stream. Thus, in the case $M_1 > M_2$ as shown in figure (4-D), sound waves are deflected toward the shadow region under the plate due to the presence of the velocity discontinuity.

(3)   <u>The Transmission and Reflection Coefficients:</u>

In this case the solution of equations (4-1) and (4-2)

<u>in the absence of the plate</u> (that is, if the interface $y = 0$

extends from $X = -\infty$ to $X = +\infty$) are considered. The results

are used to determine the boundary condition at $Y = 0$, $X > 0$

in the Kirchhoff's integral to solve the two-stream problem.

In the region $y > 0$, the solution of equation (4-1)

consists of an incident and a reflected wave while in the

region $y < 0$ the solution of equation (4-2) consists of

transmitted wave only. If one selects:

$$\phi_1 = \phi_i + \phi_r \qquad (4-7)$$

$$\phi_2 = \phi_t$$

where

$$\phi_i = A_i \, e^{i(k_1 x - k_2 y - \omega t)}$$

$$\phi_r = A_r \, e^{i(k_1 x + k_2 y - \omega t)}$$

$$\phi_t = A_t \, e^{i(k_1 x + k_2' y - \omega t)} \qquad (4-8)$$

where A_1, A_r , A_t are the wave amplitudes for the incidence,

reflection and transmission,

The boundary conditions on the interface ($y = 0$)

require that:

(i)   There must be a pressure balance across the interface;

hence $P_1 = P_2$

where $P_1 = -\rho_o \left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} \right) \phi_1$

and $P_2 = -\rho_o \left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x} \right) \phi_2$

or

$$\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} \right) \phi_1 = \left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x} \right) \phi_2 \quad \text{on} \quad Y = 0 \quad (4-9)$$

(ii)   The particle displacement of the two streams should

be the same at the interface.   This means that

$$\eta_1 = \eta_2 = \eta$$

where the expression of the interface is taken to be:

$$g(x, y, t) = y - \eta(x, t) \qquad (4-10)$$

and the equation of $g(x, y, t)$ is:

$$\frac{\partial g}{\partial t} + \vec{U} \cdot \nabla g = 0 \qquad (4-11)$$

When this equation is linearized and applied separately

to the upper and the lower streams, the following kinematic

conditions result. $V_1 = \frac{\partial \eta_1}{\partial t} + U_1 \frac{\partial \eta_1}{\partial x} \; ; \; V_1 = \frac{\partial \phi_1}{\partial y} \quad \text{on} \quad y > 0$

$V_2 = \frac{\partial \eta_2}{\partial t} + U_2 \frac{\partial \eta_2}{\partial x} \; ; \; V_2 = \frac{\partial \phi_2}{\partial y} \quad \text{on} \quad y < 0$

On matching these conditions, one obtains:

$$\left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x}\right)\frac{\partial \phi}{\partial y} = \left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x}\right)\frac{\partial \phi_2}{\partial y} \quad at \quad y=o \qquad (4\text{-}12)$$

On substituting expressions (4-7) into equations (4-9) and (4-12) and setting $y = 0$ in the result, each term in the resulting equations will contain the factor $e^{ik_1 x - j\omega t}$

A necessary condition to obtain a consistent solution is, therefore, to choose the same k_1 for all three wave solutions ϕ_1, ϕ_r, and ϕ_t. This is what has been used in writing equation (4-10) and in determining $\vec{k}_r$ and $\vec{k}_t$ using the ellipses in section IV-3 (2).

Since the wave number $\vec{k}_i$ of the incident wave is given as a boundary condition, the value $\vec{k}_1$, $\vec{k}_2$ and $\vec{k}_2'$ for all three expressions in equation (4-8) are determined. The two equalities resulting from the two conditions at the interface, namely, equations (4-9) and (4-12), serve to determine the reflection coefficient $R_c = \dfrac{A_r}{A_i}$ and the transmission coefficient $T_c = \dfrac{A_t}{A_i}$. Let $D_1 = \omega + U_1 K_1$ and $D_2 = \omega + U_2 K_1$.

Equations (4-9) and (4-12) yield:

$$D_1 (A_i + A_r) = D_2 A_t$$

$$k_2 D_2 (A_i - A_r) = k_2' D_1 A_t$$

(4-13)

then

$$R_c = \frac{K_2 D_2^2 - K_2^1 D_1^2}{K_2 D_2^2 + K_2^1 D_1^2}$$

(4-14)

$$T_c = \frac{2 D_1 D_2 K_2}{K_2 D_2^2 + K_2^1 D_1^2}$$

IV-4 Solution of the Two-Stream Problem:

The interest here is in determining the wave transmission

and refraction in the region under the wing (see figure 4-E);

for this reason the Kirchhoff's integral is applied using a

contour enclosing the lower half plane, and composed of the

semi-infinite plate, the interface and a semi-circle of

infinite radius. In addition, use is made of the short wave

length approximation (that is, $k = \frac{\omega}{a_0} \gg 1$) to find the

asymptotic solution of ϕ_2 valid to the order $O(\frac{1}{\sqrt{k}})$.
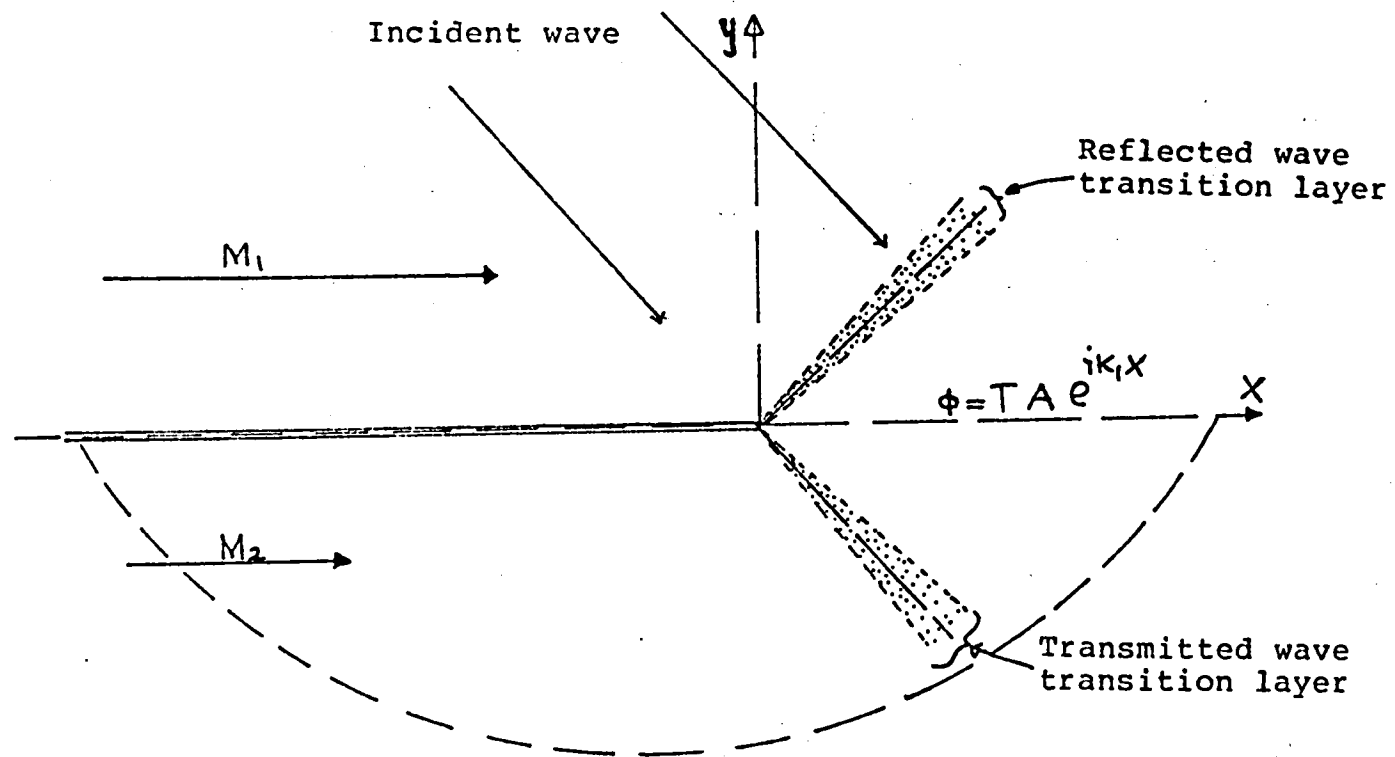
Figure (4-E): Transition zones in ray acoustics.

Two important approximations are then made possible.

These are:

(1) The method of stationary phase can be used to evaluate

the Kirchhoff's integral. As shown in chapter III, the point

of stationary phase is located on the part of the control

surface which lies between the acoustic source and the point

of observation. Thus, in the present problem the point of

stationary phase lies on the surface composed of the solid

plate and the interface. Any contribution from integral over

the semi-circular arc is of higher order and need not be

considered.

(2) As shown in figure (4-E) the region of the transition

layers approach two narrow sectors as k becomes sufficiently

large. Outside these two sectors, the solution given by the

ray acoustics is accurate to the extent that the neglected

error terms are of the order $O\left(\frac{1}{k}\right)$. The solution sought for

the transmission transition layer is of lower order (that

is $O\left(\frac{1}{\sqrt{k}}\right)$), and hence can be determined by the Kirchhoff's

integral using the boundary value on the $y = 0$ axis given by

the ray acoustics.

With these in mind, one may reformulate the problem in

a way that one can solve it by the general method outlined

in chapter III. Since the interest in this problem is in the

transmitted wave across the interface, the field equations

will be developed for the lower flow regime.

IV-5 <u>Field equations and solution procedure.</u>

$$\left(1-M^2\right)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - 2kM_2 i\,\frac{\partial\phi}{\partial x} + k^2\phi = 0 \qquad (4\text{-}15)$$

where $k = \dfrac{\omega}{a_o}$

The appropriate boundary conditions are:

(i)   At the interface, $y = 0$; $x > 0$, the solution of ϕ_2

is the same as the one given by the ray acoustics, namely, the

form expressed in equation (4-7) as discussed in section IV-3.

$$\phi_2 = A_t\,e^{i\left(k_1 x + k_2' y - \omega t\right)}$$

with A_t and k_2' likewise determined by the method outlined in

that section. Thus:

$$\phi_2 = A_t\,e^{ik_1 x} \quad \text{and} \quad \frac{\partial\phi_2}{\partial y} = iA_t k_2'\,e^{ik_1 x} \quad \text{at} \quad \begin{array}{l} y=0 \\ x>0 \end{array} \qquad (4\text{-}16)$$

(ii) At the backside of the plate, (that is, in $Y < 0$,

$X < 0$), the sound is completely shielded according to the ray

acoustics. So,

$$\phi_2 = \frac{\partial \phi_2}{\partial y} = 0 \quad \text{at} \quad Y = 0, X < 0 \qquad (4\text{-}17)$$

(iii) The radiation condition is satisfied at $Y = -\infty$.

Let

$$\phi = e^{\frac{ikMX}{1-M^2}} \psi \qquad (4\text{-}18)$$

This gives, on substitution into equation (4-3):

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{1-M^2} \frac{\partial^2 \psi}{\partial y^2} + \frac{k^2}{(1-M_2^2)^2} \psi = 0 \qquad (4\text{-}19)$$

with the boundary conditions of the form:

(i) $\psi = A_t e^{i\Omega x}$; $\dfrac{\partial \psi}{\partial y} = i A_t k_2' e^{i\Omega x}$; $\begin{array}{l} y=0 \\ x>0 \end{array}$

where $\Omega = k_1 - \dfrac{M_2 k}{1-M_2^2}$

(ii) $\psi = \dfrac{\partial \psi}{\partial y} = 0$ on $Y = 0, X < 0$ \qquad (4\text{-}21)

(iii) The radiation condition at $Y = -\infty$

On introducing new variables:

$$\bar{X} = X; \quad \bar{Y} = Y\sqrt{1-M_2^2} ; \quad \bar{K} = \frac{K}{\sqrt{1-M_2^2}} ; \quad \bar{K}_2' = \frac{K_2'}{\sqrt{1-M_2^2}}$$

The equation (4-19) reduces to the standard form:

$$\frac{\partial^2 \psi}{\partial \bar{X}^2} + \frac{\partial^2 \psi}{\partial \bar{Y}^2} + k^2 \psi = 0 \qquad (4\text{-}22)$$

with the boundary conditions:

(i) $\quad \psi = A_t e^{i\Omega\bar{x}} \quad ; \quad \dfrac{\partial \psi}{\partial \bar{y}} = i A_t \bar{k}_2 e^{i\Omega\bar{x}}$ on $\quad \bar{y} = 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \bar{x} > 0$

(ii) $\quad \psi = \dfrac{\partial \psi}{\partial y} = 0 \quad$ on $\quad \bar{y} = 0 \quad , \quad \bar{x} < 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (4-23)
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (a,b,c)

(iii) $\quad$ The radiation condition is satisfied at $\bar{y} = -\infty$

Equations (4-22) and (4-23) constitute a standard wave

scattering problem which has been discussed thoroughly in

chapter III.

The solution of these equations is:

$$\psi(\bar{x}, \bar{y}, \bar{k}) = A_t e^{i\left(\Omega\bar{x} + k_2'\bar{y}\right)} F(\bar{\beta}) \qquad (4\text{-}24)$$

where $\qquad \bar{k}_2' = -\sqrt{\sigma^2 - \Omega^2}$

and $\qquad F(\bar{\beta}) = \dfrac{1+i}{\pi} \displaystyle\int_{\bar{\beta}}^{\infty} e^{-u^2} du$

with $\qquad \bar{\beta}^2 = \bar{k}^2 \left[\bar{r}_m - \bar{r}_o\right] \qquad\qquad$ as shown in figure

(4-F) next page. The final solution for 0 is obtained by a

direct substitution:

$$\phi = T_c A_i \, e^{i\left[k_1 \dot{x} - \sqrt{k^2 + 2kM_2 k_1 - (1 - M_2^2)k_1^2} \, y\right]} F(\bar{\beta})$$

$$(4\text{-}25)$$

where $\bar{r}_o \, , \, \bar{\theta} \, , \, \bar{\Theta} \, , \, \sigma \quad$ are defined next page.
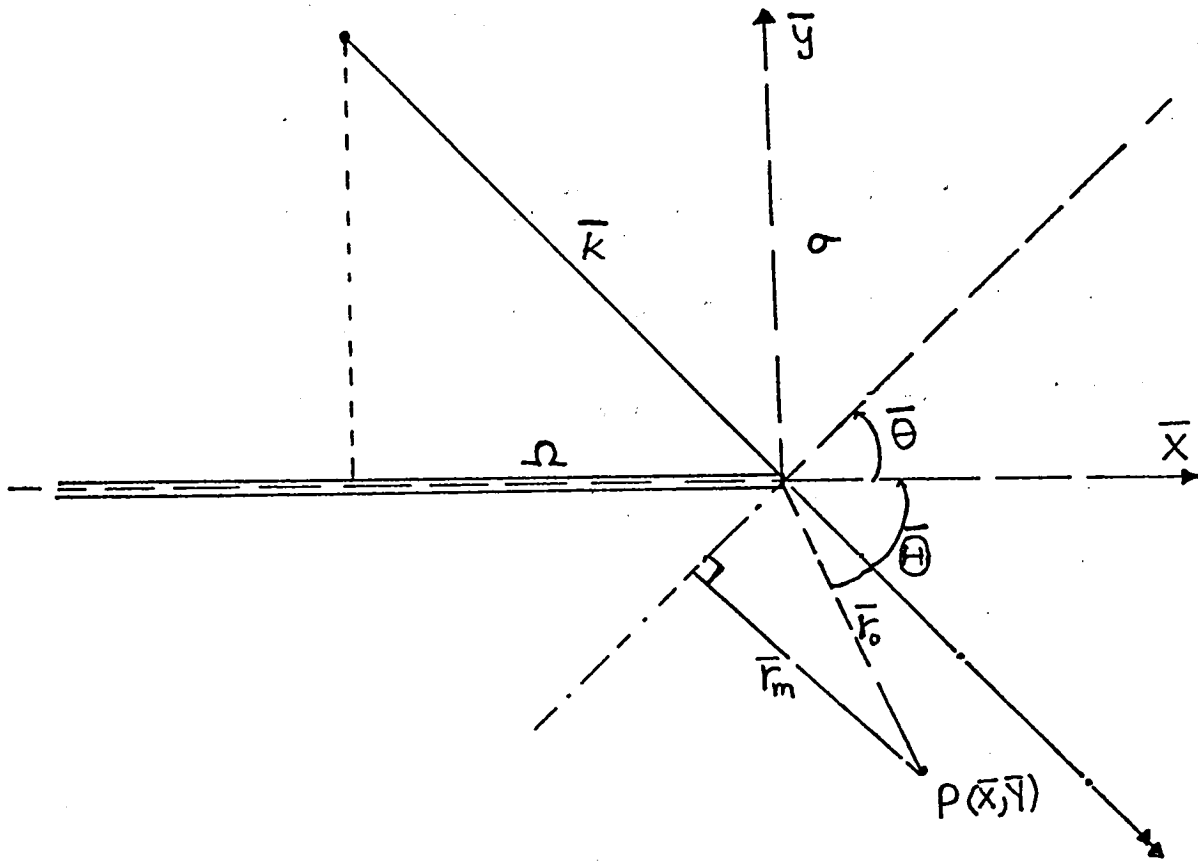
Figure (4-F): The geometry showing the transformation
as in the solution in equation (4-25).

$$\bar{\beta} = \sqrt{\frac{2k\bar{r_0}}{1-M_2^2}} \; \sin\left(\frac{\bar{\theta} - \bar{\Theta}}{2}\right)$$

$$\bar{r_0} = \sqrt{x^2 + (1-M_2^2)y^2} \qquad ; \quad \bar{k} = \frac{k}{1-M_2^2}$$

$$\sin \bar{\theta} = -\frac{y}{\bar{r_0}} = -\frac{\sqrt{(1-M_2^2)}\; y}{\sqrt{x^2 + (1-M_2^2)y^2}}$$

$$\sin \bar{\Theta} = \frac{\sigma}{k}$$

$$\sigma = \sqrt{\frac{k^2 + 2k_1 k M_2 - (1-M_2^2)k_1^2}{(1-M_2^2)}}$$

Chapter V

CONCLUSIONS:

The methodology that has been designed and used in the previous chapter proves extreemely powerful in handling the problem of current interest in noise shielding by aircraft wing. Additional features and utility of the mathematical method formulated can be illustrated in the following aspects of noise shielding.

V-1 Shielding by a Finite Plate:

The solution method for a semi-infinite plate can be used to obtain the solution for a finite wing. The following discussion is based on the two-dimensional case. As shown in figure (5-A) next page, the potential $\phi(P)$ at a point P below the semi-infinite plate is due to the direct radiation and given by:

$$\phi(P) = + \phi_o F(\bar{\beta}) \qquad (5-1)$$

where $F(\bar{\beta})$ is the diffraction factor, and

$$\bar{\beta}^2 = k\left[\vec{SO} + \vec{OP} - \vec{SP}\right] \qquad (5-2)(a)$$

$$\vec{\beta}^2 = k\left[\overrightarrow{SO} + \overrightarrow{OP} - \overrightarrow{SP}\right]$$
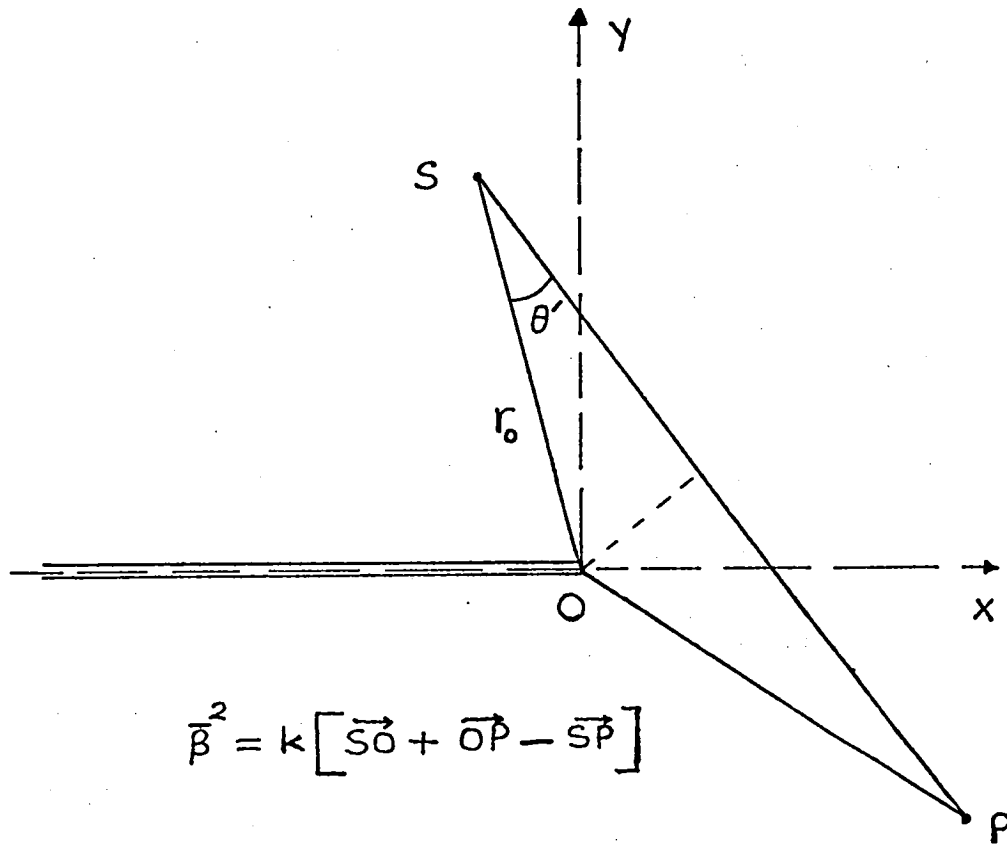
Figure (5-A): Geometry defining the diffraction factor for the two-dimensional case.

When P is very far away, $\overline{\beta}$ is given approximately by:

$$\overline{\beta}^2 = k\left[r_0 - r_0 \cos\theta'\right] = 2kr_0 \sin^2\frac{\theta'}{2} \qquad \text{(5-2)(b)}$$

or

$$\overline{\beta} = -\sqrt{2kr_0}\ \sin\frac{\theta'}{2} \qquad \text{(5-3)}$$

In this formula, θ' is the angle between the line $\overrightarrow{SP}$ and the line $\overrightarrow{OS}$ figure (5-B).

$\theta' \simeq \theta$ if P is in the far field.

Figure (5-B): Far field approximation of wave reception at P.

When P is very far away, this angle becomes approximately equal to θ , which is the angle between the line $\overrightarrow{OP}$ and a line passing through the two points S and O. According to

ray acoustics, the line OP defines the boundary line between the propagation and the shadow regions.

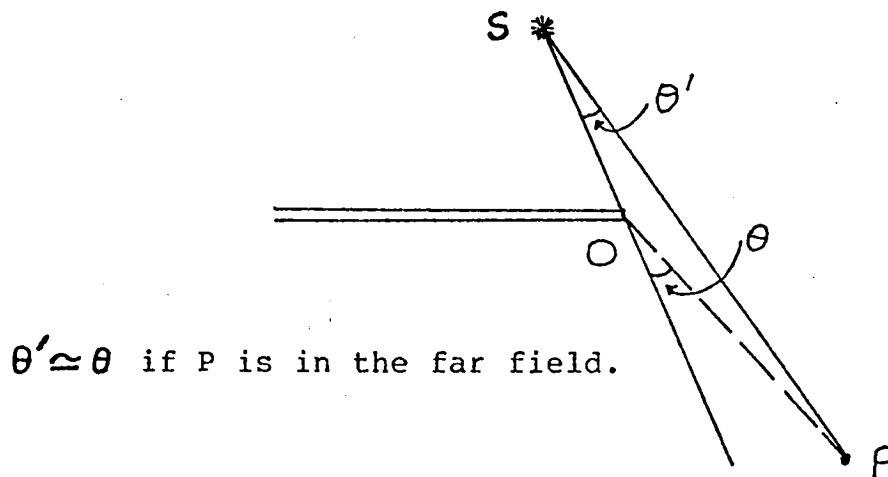The potential at P, to the first order of accuracy, may be written as:

$$\phi = +\phi_0 F(\bar{\beta})$$

with

$$\bar{\beta} = -\sqrt{2kr_0} \; \text{Sin} \; \frac{\theta}{2}$$

As shown in figure (5-C) next page, the transition from full transmission to complete blockage takes place inside a sector centred at the trailing edge of the plate. The angular width of the sector decreases as the wave number K increases. Figure (5-D) next page shows the acoustic field due to the reflected wave. Again for large K, the transition takes place within a narrow sector.

The implication of the above observation is that for the case of large K, the solution of $\phi(P)$ for a finite plate can be obtained by combining two solutions: one solution corresponds to waves diffracted around the trailing edge and the other corresponds to waves diffracted around the leading edge.
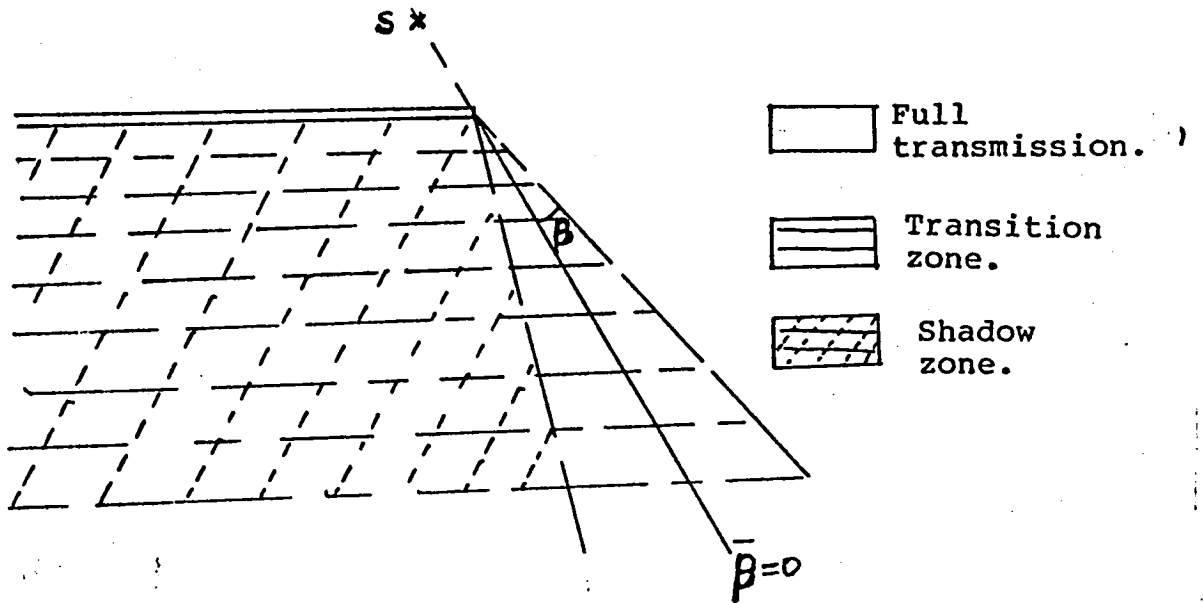
Figure (5-C): Incident wave field variation.
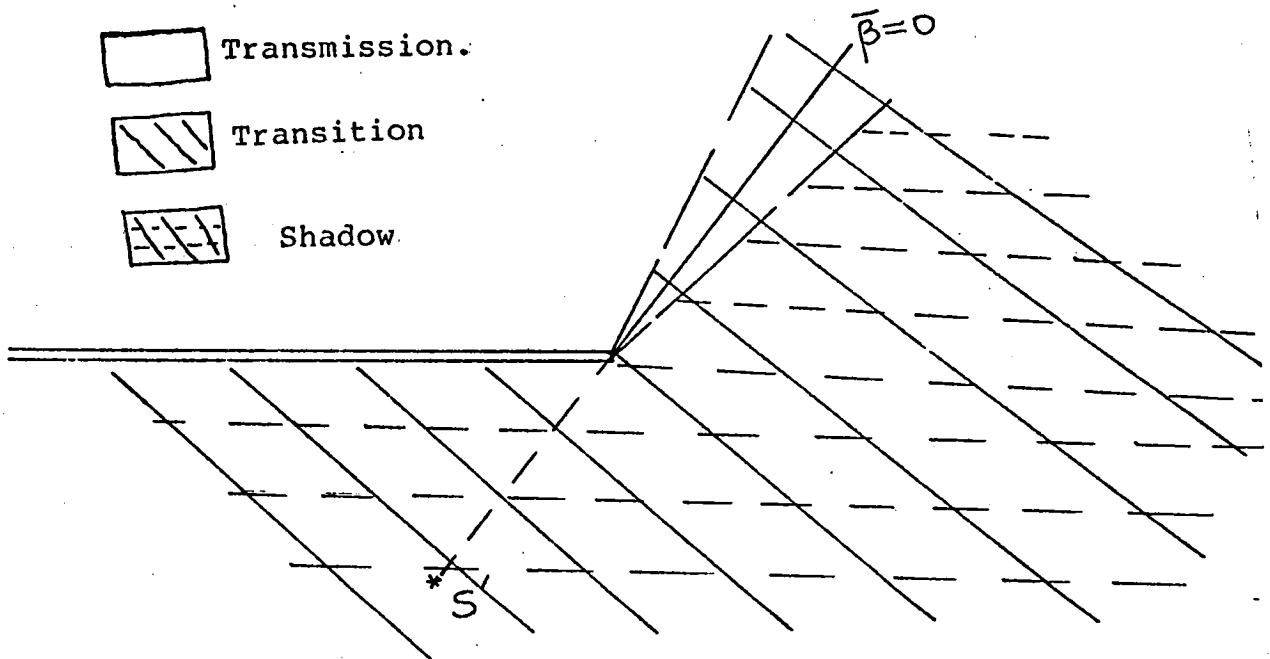
Figure (5-D): Reflected wave field variation.

It can be shown that, for a long plate, the resulting solution

will not significantly violate the boundary conditions imposed

on the problem (figure 5-E) below. Using this approach the

diffracted field around a plate of finite length has been

calculated numerically. Figures (5-F, 5-G and 5-H) next three

pages show the results of such calculations.
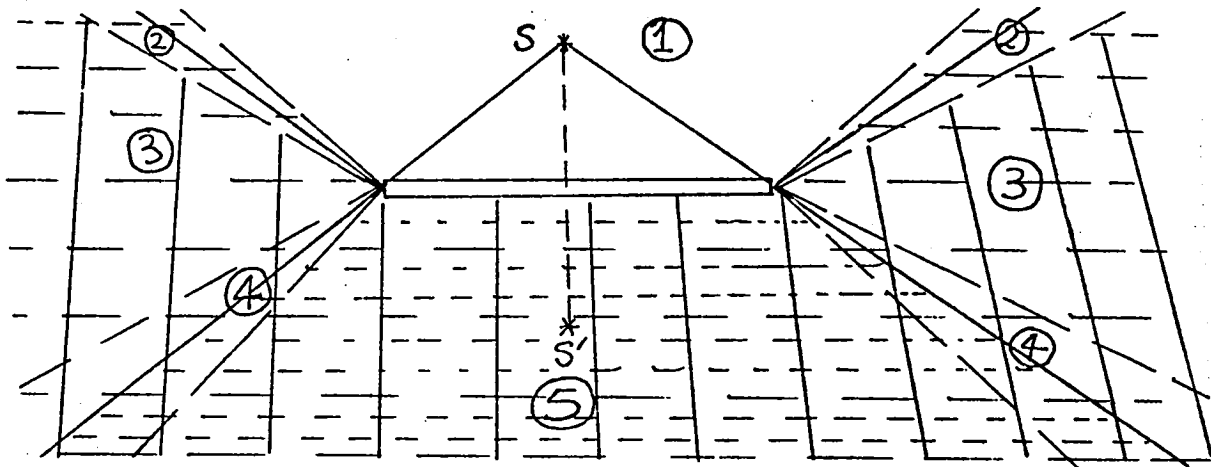
Figure (5-E): Configuration of a finite plate and a
simple source above it.

1. Incident and Reflected wave.
2. Transition between 1 and 3.
3. Incident wave only.
4. Transition between 3 and 5
5. Complete shadow.
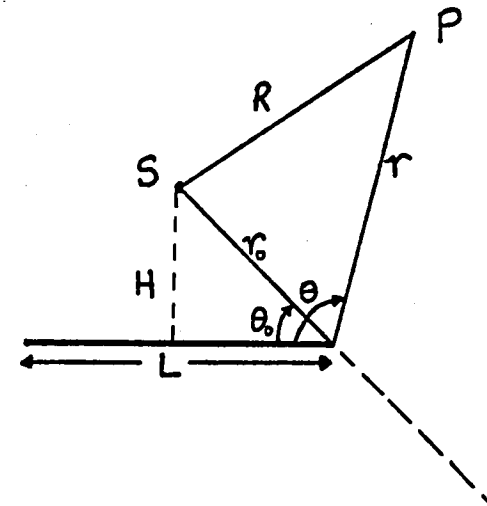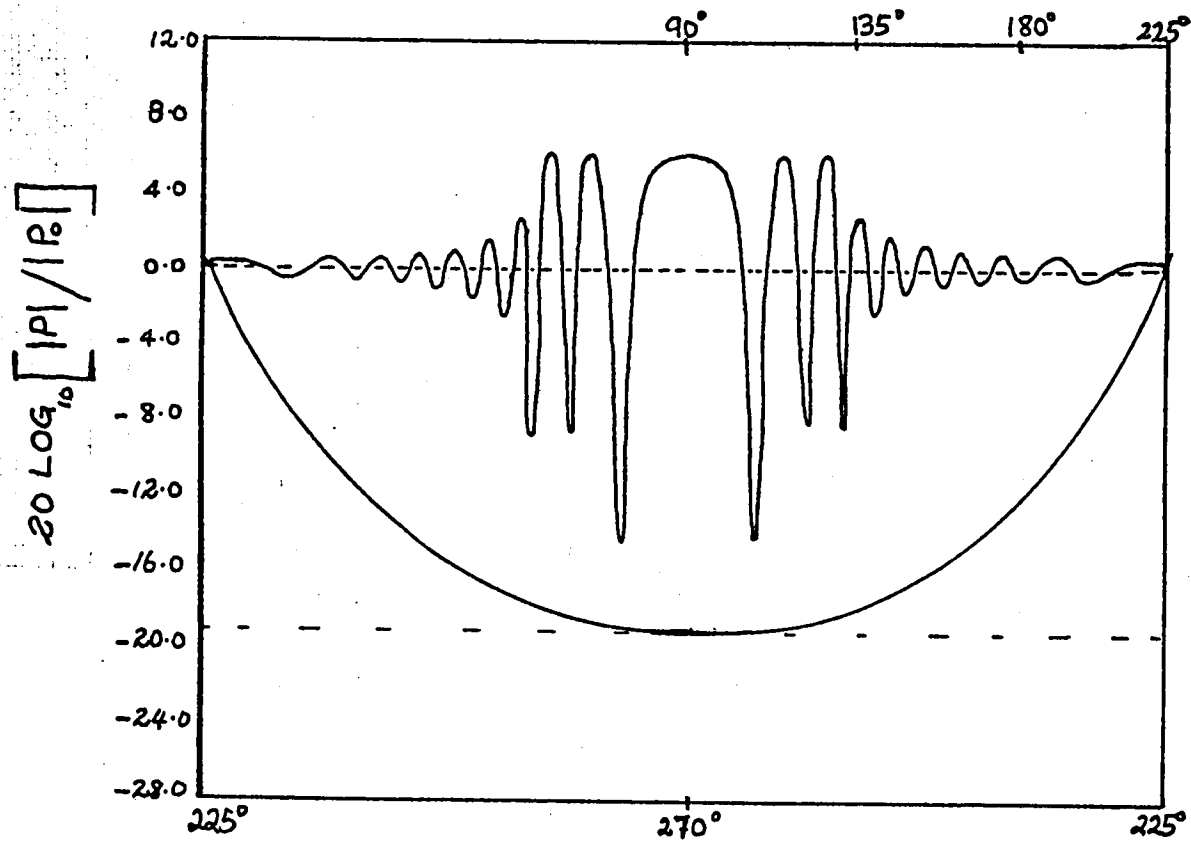
Figure (5-F): Finite plate solution for $\lambda = 1$ with the source centrally located.

$H = 5.0$

$\lambda = 1.0$

$L = 10.0$

$r = 1000.0$

$\theta_0 = 45°$
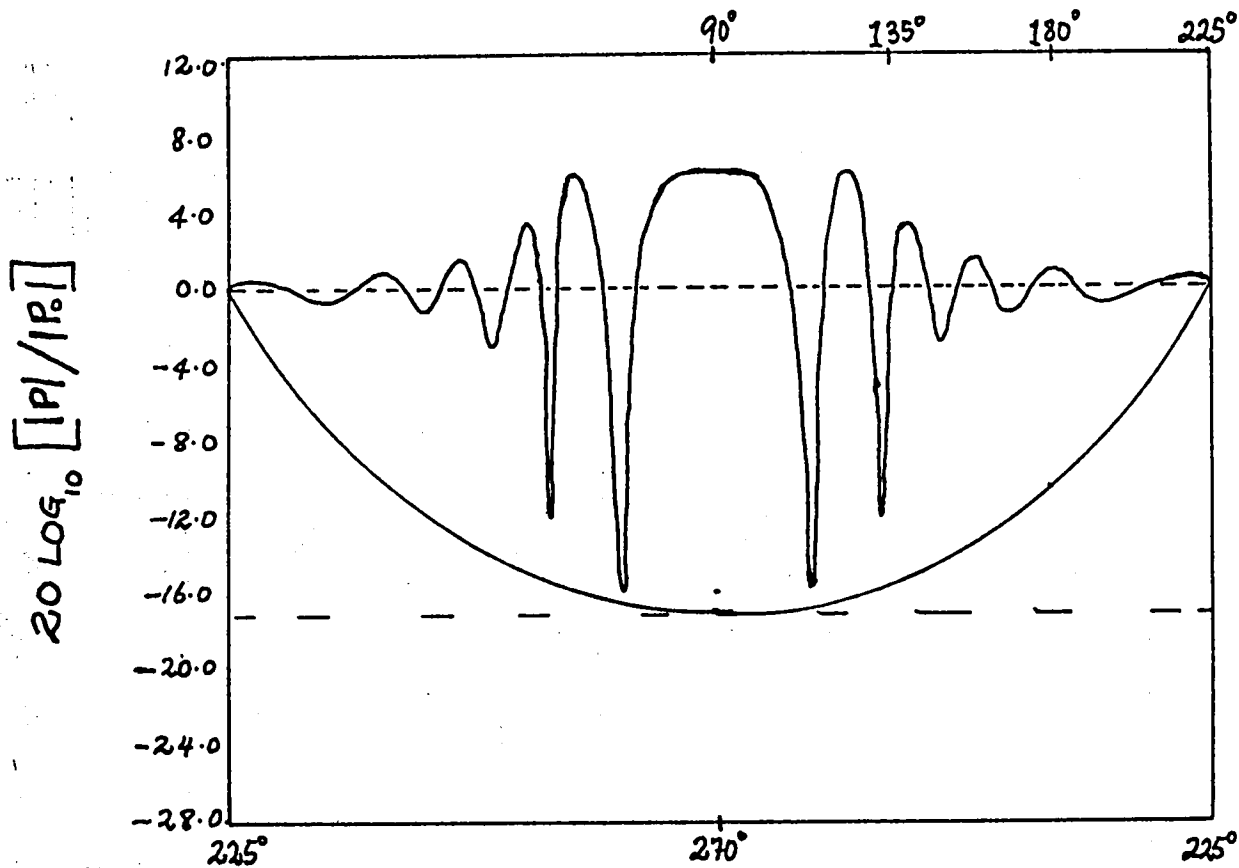
Figure (5-G): Finite plate solution for $\lambda = 2$ with the source centrally located.
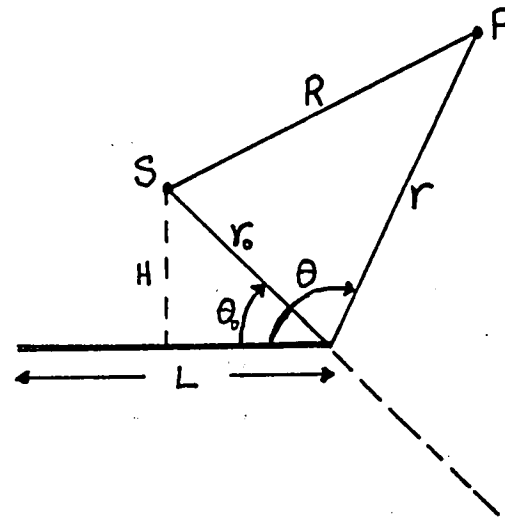
$H = 5.0$

$\lambda = 2.0$

$L = 10.0$

$r = 1000.0$

$\theta_0 = 45°$
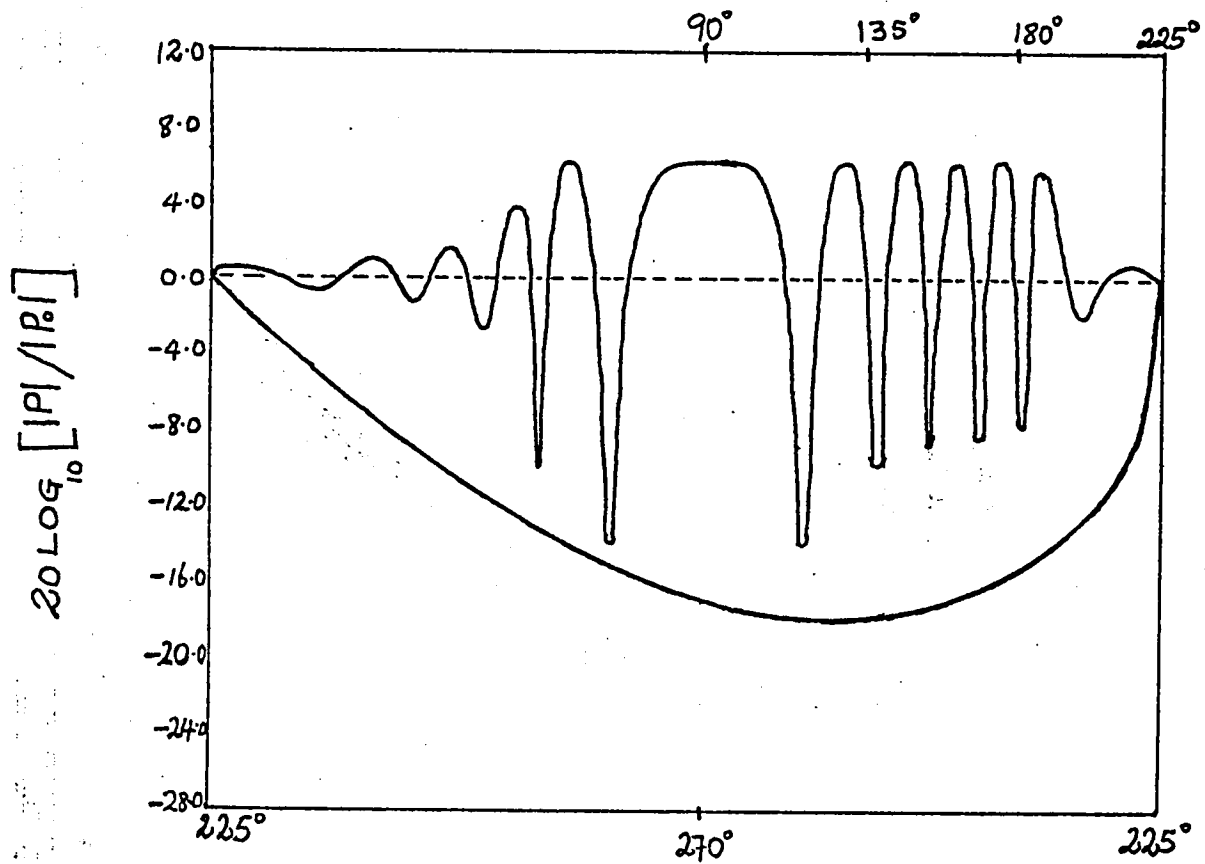
Figure (5-H): Finite plate solution for $\lambda = 2$ with the source not centrally located.
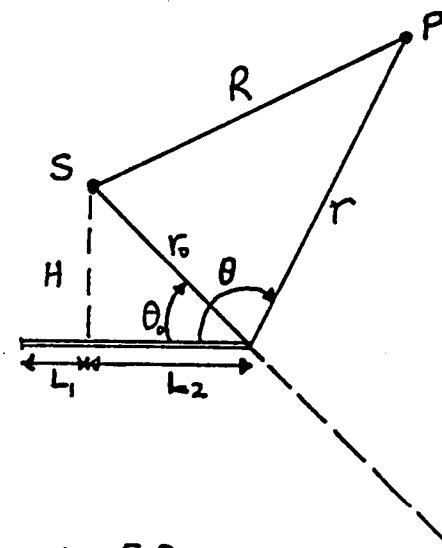
$H = 5.0$

$L_1 = 5.0$

$L_2 = 20.0$

$\lambda = 2.0$

$r = 1000.0$

V-2 Effect due to Gaps, Angular Deflection and other

Geometrical Variation of the Wing Surface:

The audible range of the aircraft sound lies essentially

between 20 to 20,000 Hz, corresponding to wave length of

about 16 metres to 2 centimetres. If the dimension of any

geometrical variation for the assumed plate and a gap on the

plate surface is less than the dominant wave length of the

sound, the fluid pressure and velocity variation over that

part of the wing surface is nearly simultaneous. Therefore,

the solution to the region of the acoustic field may be

approximated by the solution of the steady compressible flow

equation. A local solution obtained by this equation is

needed in order to smooth out any descontinuities in velocity

pressure or mass flow due to ignoring such geometrical

variation in the first approximation. In the gap problem

discussed in chapter II, for example, the tangent velocity

over the upper and the lower surface given by the wave

equation and neglecting the presence of the gap will be

discontinuous and a local solution near the gap is necessary

to remove this discontinuity. In the far field, this local

steady solution around the gap acts like localized acoustic

poles. The strength and phase of these induced poles depend

on the nature of the original sound source.

V-3 Effect of Source Distribution on Power Radiation:

The problems of acoustic interactions with barriers poses

the more fundamental question of the nature the acoustic

power is distributed in the region of interest. The clear

understanding of this will facilitate the approach to the noise

shielding.

It is known that the interaction of acoustic waves by

any barrier has the effect of inducing distributed sources

on the surface of the barrier and hence the problem then

becomes one of understanding the nature of interactions from

such sources and the effect these have on the power generated

from the original source. The study on such problems is

nearly a century old beginning with the study of such problems

in the field of radio waves. Recently, however, the effect of a boundary on the radiated power has been studied by Thiessen (41), Embleton (12) and Levine (27). In such studies, it has been indicated that the distribution of sources affects the impedance of the power transmission. In this study, the radiative power of a three-dimensional monopole source placed in a stationary medium in the presence of a semi-infinite plate will be briefly discussed. The mathematical expressions given below are in terms of the spherical coordinates whose centre is the point on trailing edge that lies in the same plane as the source point S (figure 5-I). The line through the point O into the paper is chosen as the polar axis whose azimuth and polar angles are θ and ν respectively.
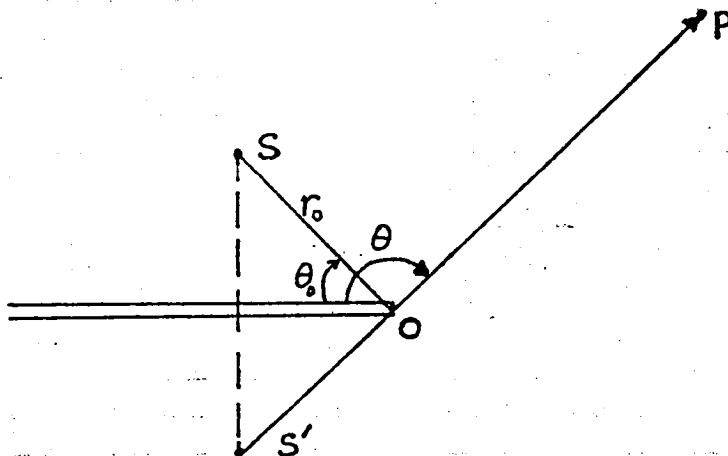
Figure (5-I): Spherical geometry for source distribution solution.

The ranges of such angles are:

$$0 \leqslant \theta \leqslant 2\pi \quad ; \quad 0 \leqslant \nu \leqslant \pi/2$$

The potential $\phi(p,t)$ at a point $P(r, \psi, \theta)$ is obtained

as in chapter III as:

$$\phi = -\frac{e^{i(kr_1 - \omega t)}}{4\pi r_1} F(\bar{\beta}_1) - \frac{e^{i(kr_2 - \omega t)}}{4\pi r_2} F(\bar{\beta}_2) \quad (5\text{-}4)$$

where r_1 and r_2 are the distances from the point P to the

points S and S'. $F(\bar{\beta}_1)$ and $F(\beta_2)$ are the diffraction

factors with $\bar{\beta}_1$ and $\bar{\beta}_2$ given by the expressions (assuming

r very large):

$$\left.\begin{array}{l} \bar{\beta}_1 = -\sqrt{2kr_0 \sin\nu}\ \cos\left(\dfrac{\theta - \theta_0}{2}\right) \\[4mm] \bar{\beta}_2 = +\sqrt{2kr_0 \sin\nu}\ \cos\left(\dfrac{\theta + \theta_0}{2}\right) \end{array}\right\} \quad (5\text{-}5)$$

The function F may be written in the form $F = A_1 e^{i\alpha_1}$.

Let $A_1(\bar{\beta}_1)$ and $\alpha_1(\beta_1)$ be denoted by A_1 and α_1 ; and $A_2(\bar{\beta}_2)$

$\alpha_2(\beta_2)$ by A_2 and α_2. Then the following expressions for

large r may be easily obtained:

$$p' = -\frac{i\rho_0 \omega A_1}{4\pi r} e^{i(kr_1 - \omega t + \alpha_1)} - \frac{i\rho_0 \omega A_2}{4\pi r} e^{i(kr_2 - \omega t + \alpha_2)} \quad (5\text{-}6)$$

$$I \simeq \frac{1}{\rho_0 a_0} <p'^2> = \frac{\rho_0 \omega^2}{32 \pi^2 a_0 r} \left[A_1^2 + A_2^2 + A_1 A_2 \cos \left[k(r_1 - r_2) + (\alpha_1 - \alpha_2) \right] \right]$$

$$(5\text{-}7)$$

where, for large r: $\quad r_1 - r_2 \simeq -2r_0 \sin \theta_0 \, \sin \nu \, \sin \theta$

The total radiated power **W** is then given by the expression:

$$W = \frac{\rho_0 \omega^2}{32 \pi^2 a_0} \int\limits_{\nu=0}^{\pi} \int\limits_{\theta=0}^{2\pi} \left[A_1^2 + A_2^2 + A_1 A_2 \cos \left[k(r_1 - r_2) + (\alpha_1 - \alpha_2) \right] \right] \sin \nu \, d\nu \, d\theta$$

$$(5\text{-}8)$$

which may be evaluated numerically. For the purpose of the

present discussion, it suffices to comment on the following

simple example. Consider the source to be at a distance **h**

above the edge of the plate (figure 5-J).
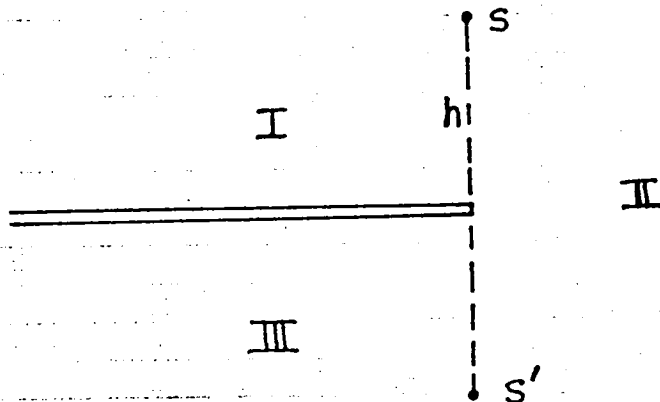
Figure (5-J): Set up for power approximation for distributed sources.

Assuming the wave number is very large, then the ray acoustic

solution gives a good approximate solution.

The acoustic field then comprises of three regions with

sharp boundaries. In region I, which is the region of direct

radiation and reflection, sound is received from both the

original source S and its image s' so that:

$$\phi = -\frac{e^{i(kr_1 - \omega t)}}{4\pi r_1} - \frac{e^{i(kr_2 - \omega t)}}{4\pi r_2} \qquad (5-9)$$

The total radiated power from two identical sources with a

separation distance $2h$ in a free space without boundaries can

be easily be found.

$$W_{ss'} = \frac{\rho_o \omega^2}{4\pi a_o} \left[1 + \frac{1}{2} \frac{Sin\,\mu}{\mu} \right]$$

where $\mu = 2kh$

The power radiated in region I is 1/4 of this value and

is hence equal to:

$$W_I = \frac{\rho_o \omega^2}{16\pi a_o} \left[1 + \frac{1}{2} \frac{Sin\,\mu}{\mu} \right] \qquad (5-10)(a)$$

In region II, which is the region of direct radiation with

no reflection, the radiated power is equal to 1/2 of the

power radiated by a source in the free space and is given by:

$$W_{II} = \frac{\rho_0 \omega^2}{16\pi a_0} \tag{5-10)(b)}$$

In region III, where there is no radiation

$$W_{III} = 0 \tag{5-10)(c)}$$

The total power radiated by the simple source in the

presence of the plate is then:

$$W_t = W_I + W_{II} + W_{III}$$

$$= \frac{\rho_0 \omega^2}{8\pi a_0}\left[1 + \frac{1}{2}\frac{Sin\,\mu}{\mu}\right] \tag{5-11}$$

so that $\quad \dfrac{W_T}{W_0} = 1 + \dfrac{1}{2}\dfrac{Sin\,\mu}{\mu}$

where $\quad W_0 = \dfrac{\rho_0 \omega^2}{8\pi a_0}\quad$ is the power radiated by a simple source

in the free space. In figure (5-K), the variation of $\dfrac{W_T}{W_0}$

is plotted as a function of μ .

Figure (5-K): Power variation as a function of the wave number k and height h.
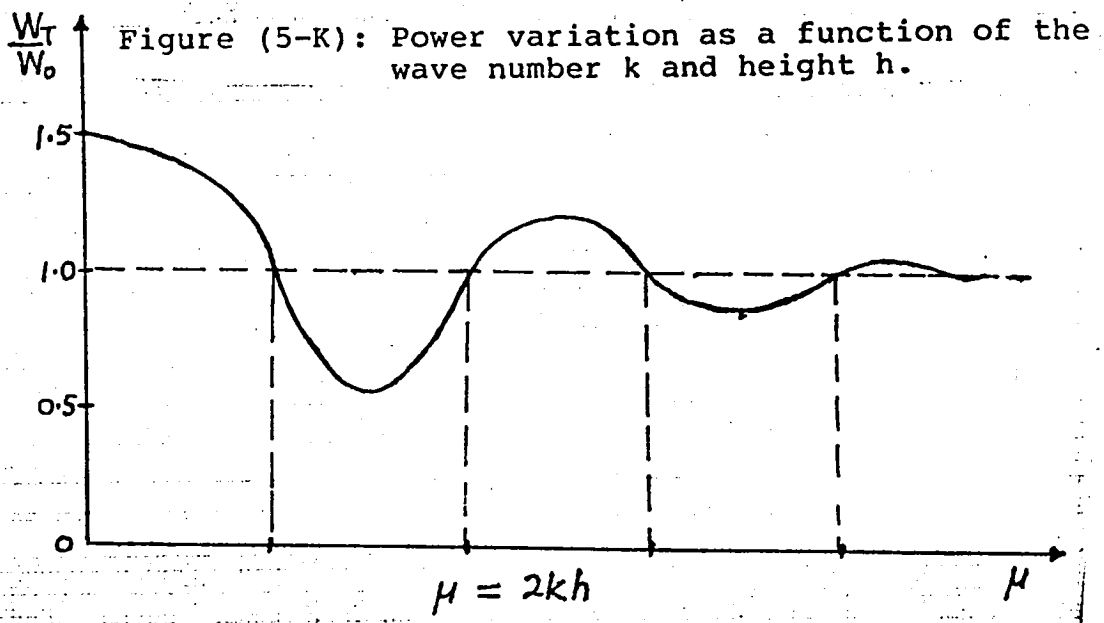
It is seen that, as the source approaches the plate (that is

$h \to 0$ or $\mu \to 0$), the radiated power approaches a value

which is 50% greater than the value for the source without

the plate, so that

$$\frac{W_T}{W_0} \to 1 \cdot 5 \quad \text{as} \quad \mu \to 0$$

On the other hand, when h is much larger than the

acoustic wave length, then

$$\frac{W_T}{W_0} \to 1 \quad \text{as} \quad \mu \to \infty$$

Between these two limits, the value of $\frac{W_T}{W_0}$ oscillates

through a series of maxima and minima about the value $\frac{W_T}{W_0} = 1$

The crossover points with $\frac{W_T}{W_0} = 1$ are $\mu = n\pi$ where $n = 1, 2, \cdots$

The corresponding values of h is ($\lambda = \frac{2\pi}{k} =$ wave length)

with $\quad h_n = n\lambda/4 \quad , \quad (n = 1, 2, \cdots)$

As h is increased, $\frac{W_T}{W_0}$ changes from greater than unit

1 to less than 1 as h passes the series of values h_n,

$$n = 1, 2, \cdots$$

The acoustic impedance as seen by the source is, therefore,

greater or smaller than that in the absence of the plate

depending on where the position of the source is. This effect

on the radiating power is due to the reflection effect of the

plate.

V-4   The Effect of the Uniform Flow:

Consider a monopole in a uniform flow in the presence

of a semi-infinite plate as shown in figure (5-L).
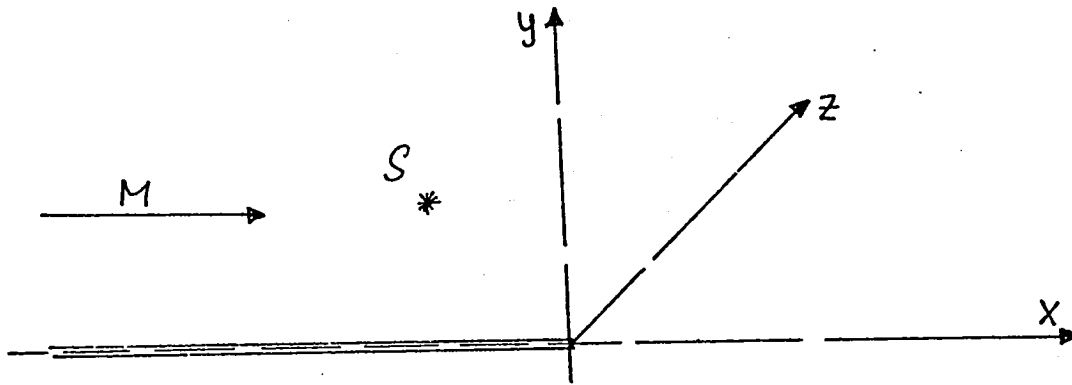
Figure (5-L): A monopole in a uniform flow in the
presence of a semi-infinite plate.

The governing diffrential equation and the boundary

conditions for the potential ϕ are given by

$$(1-M^2)\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} - 2ikM\frac{\partial\phi}{\partial x} + k^2\phi = \delta(x-x_0)\delta(y-y_0)\delta(z-z_0) \quad (5-12)$$

where $\quad k = \dfrac{\omega}{a_0}$

The boundary conditions are:

(i) $\dfrac{\partial \phi}{\partial y} = 0$ at $y = 0$; $x < 0$

(ii) ϕ and its derivatives are continuous elsewhere.

(iii) The Sommerfeld radiation condition at infinity is

satisfied.

This problem can be reduced to the standard form by

letting

$$\phi = e^{-\dfrac{jkMx}{(1-M^2)}} \psi(\bar{x}, \bar{y}, \bar{z}, \bar{k})$$ (5-13)

where: $\bar{x} = x$; $\bar{y} = y\sqrt{1-M^2}$; $\bar{z} = z\sqrt{1-M^2}$; $\bar{k} = \dfrac{k}{1-M^2}$

The equation and the boundary conditions for ψ are then:

$$\frac{\partial^2 \psi}{\partial \bar{x}^2} + \frac{\partial^2 \psi}{\partial \bar{y}^2} + \frac{\partial^2 \psi}{\partial \bar{z}^2} + \bar{k}^2 \psi = \delta(\bar{x}-\bar{x}_0)\delta(\bar{y}-\bar{y}_0)\delta(\bar{z}-\bar{z}_0)$$ (5-14)

(i) $\dfrac{\partial \psi}{\partial \bar{y}} = 0$ at $y = 0$; $\bar{x} < 0$

(ii) ψ and it derivatives are condinuous elsewhere.

(iii) Sommerfeld radiation condition at infinity is

satisfied.

Therefore, the diffraction problem for ψ in the new variables

is identical to that for a monopole source in a stationary

medium.

If the potential due to the incident wave only is considered the solution of ψ is

$$\psi = \psi_o \, F(\bar{\beta}) \qquad (5\text{-}15)$$

where $\psi = -\dfrac{e^{i\bar{k}\bar{r}}}{4\pi\bar{r}}$ and $F(\bar{\beta})$ is the diffraction factor.

Assuming, for simplicity, that both the source and the observation point lie on the X-Y plane, then $\bar{\beta}$ is given by the relation (figure 5-A):

$$\bar{\beta}^2 = \bar{k}\left(\overrightarrow{SO} + \overrightarrow{OP} - \overrightarrow{SP}\right)$$

In term of the original variables, the expression is

$$\bar{\beta}^2 = \frac{\bar{k}}{1-M^2}\left[\sqrt{x_o^2+(1-M^2)y_o^2}+\sqrt{x^2+(1-M^2)y^2}-\sqrt{(x-x_o)^2+(1-M^2)(y-y_o)^2}\right] \quad (5\text{-}16)$$

It will first be shown that along the straight line SOP′ shown in figure (5-M) the value of $\bar{\beta}$ is equal to zero for all values of K and M.
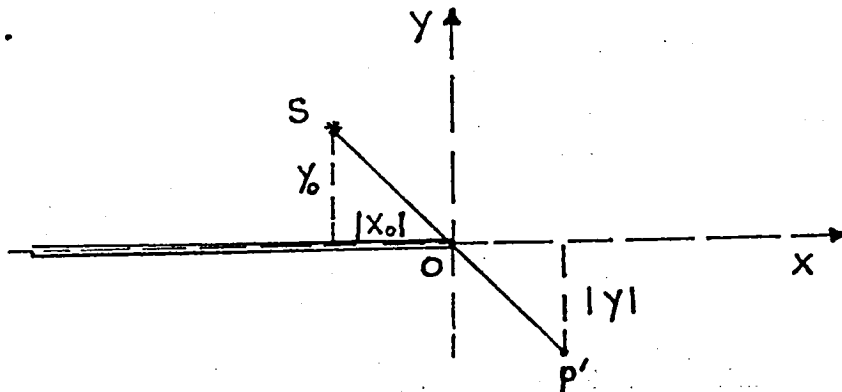
Figure (5-M): Geometry to show the boundary according to ray acoustics.

Obviously, if SOP' is a straight line, its end point (X_o, Y_o)

and (X, Y) satisfy the relations:

$$X = -\Gamma X_o \quad ; \quad Y = -\Gamma Y_o$$

where Γ is a positive constant. By a direct substitution,

it is found that:

$$\sqrt{X_o^2 + (1-M^2)Y_o^2} + \sqrt{X^2 + (1-M^2)Y^2} - \sqrt{(X-X_o)^2 + (1-M^2)(Y-Y_o)^2}$$

$$\text{(5-17)}$$

$$= \sqrt{X_o^2 + (1-M^2)Y_o^2} + \Gamma\sqrt{X_o^2 + (1-M^2)Y_o^2} - \sqrt{(1-\Gamma^2)\left[X_o^2 + (1-M^2)Y_o^2\right]} = 0$$

This proves that $\bar{\beta}$ has a zero value everywhere on the line

SOP' for all values of the wave number k and flow Mach number

M. When, in particular, the wave number becomes very large

($k \longrightarrow \infty$), then the transition layer collapses onto this

line SOP'. Therefore, the line SOP' is the boundary line

between the regions of transmission and the shadow according

to ray acoustics for all Mach numbers as long as the wave

number k is large. Latter in this section, the effect of the

Mach number on the transition layer thickness will be shown.

From this, it is concluded that the line drawn from the source to the trailing edge of the plate completely defines the position of the ray acoustics boundaries irrespective of whether the medium is stationary or moving figure (5-N). In the special case of a plane wave incident on the plate, the above conclusion agrees with the result of Candel (5) as shown below.

The incident field due to a monopole at (r_o, θ_o) and $Z = 0$ is given by the expression:

$$\phi_i = - \frac{e^{-ikM\left[(x + r_o \cos\theta_o) - \sqrt{(x + r_o \cos\theta_o)^2 + (1-M^2)(y - r_o \sin\theta_o)^2}\right]}}{\sqrt{(x + r_o \cos\theta_o)^2 + (1-M^2)(y - r_o \sin\theta_o)^2}} \qquad (5-18)$$
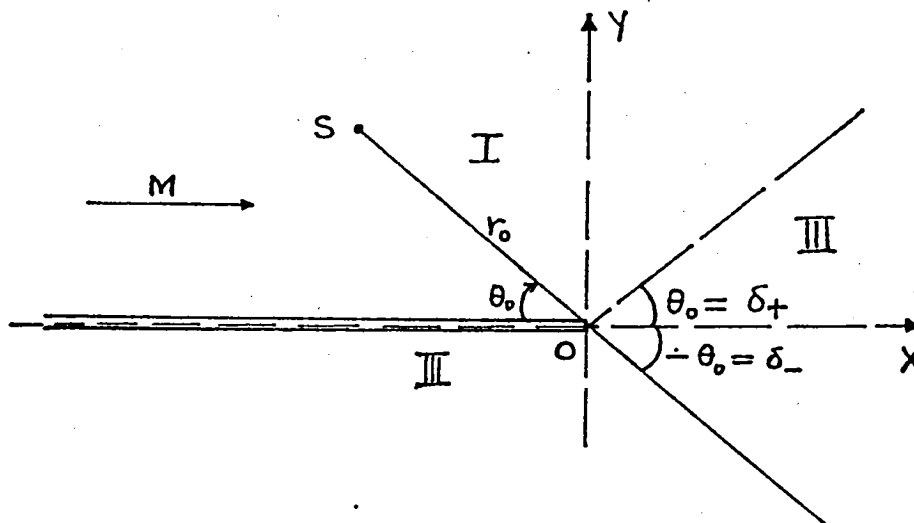
Figure (5-N): A monopole source field in two-dimensional geometry in the presence of a uniform flow.

When the source is moved to infinity along the line OS

figure (5-N), the incident wave on the plate due to the

concentrated source becomes identical to that of a plane wave

with its potential given by the asymptotic expression of

equation (5-18) with $r_o \rightarrow \infty$ as:

$$\phi_i \simeq e^{-ik\left[\frac{x}{1-M^2}\left(M - \frac{\cos\theta_o}{\sqrt{1-M^2\sin^2\theta_o}}\right) + \frac{y\sin\theta_o}{\sqrt{1-M^2\sin^2\theta_o}}\right]} \qquad (5\text{-}19)$$

The potential due to a plane wave of incidence angle Θ

figure (5-O) is given by the expression in the equation 19 of

Candel (5) as:

$$\phi_i = e^{-ik\left[\frac{x\cos\Theta}{1-M\cos\Theta} + \frac{y\sin\Theta}{1-M\cos\Theta}\right]} \qquad (5\text{-}20)$$
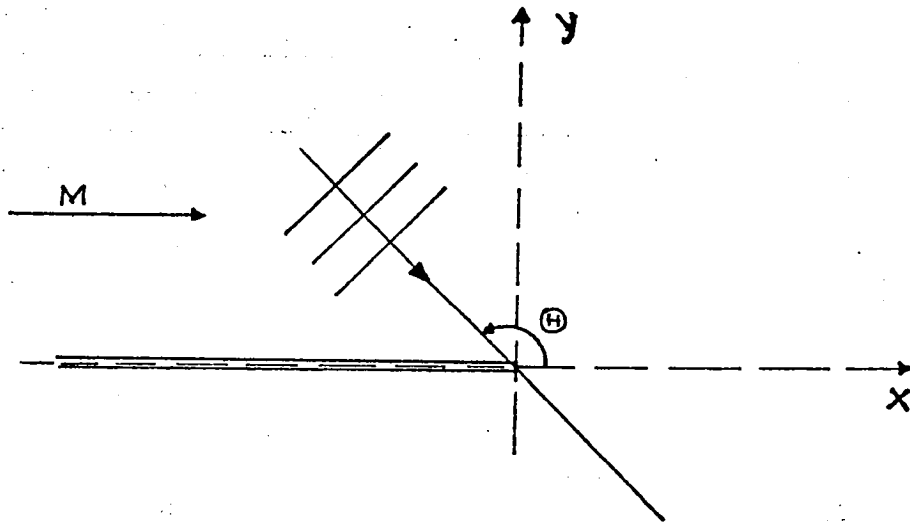
Figure (5-O): A plane wave field in two-dimensional
geometry in the presence of a uniform flow.

Comparing equation (5-19) and (5-20) it is seen that

the incidence angle $\oplus$ is equal to $\pi - \theta_o$ only when $M = 0$. In

general, when $M \neq 0$, the incidence angle $\oplus$ of the plane wave

given by equation (5-20) is a function of θ_o and the

relationship is obtained by comparing the exponents contained

in equations (5-19) and (5-20) thus:

$$\frac{1}{1-M}\left[M - \frac{\cos\theta_o}{\sqrt{1-M^2\sin^2\theta_o}}\right] = \frac{\cos\oplus}{1-M\cos\oplus} \qquad (5\text{-}21)(a)$$

and

$$\frac{\sin\theta_o}{\sqrt{1-M^2\sin^2\theta_o}} = \frac{\sin\oplus}{1-M\cos\oplus} \qquad (5\text{-}21)(b)$$

The following relation is then obtained from equation (5-21)

(b) as:

$$\tan\theta_o = \frac{\sin\oplus}{M-\cos\oplus} \qquad (5\text{-}22)$$

This relation is also found to satisfy equation (5-21)(a).

According to the conclusion mentioned above, the angle

θ_o is also the angle of the ray acoustic boundaries which

were determined and denoted as δ_+ and δ_- by Candel (5),

figure (5-N). Thus:

$$\tan\delta_\pm = \tan(\pm\theta_o) = \pm\frac{\sin\oplus}{M-\cos\oplus} \qquad (5\text{-}23)$$

This relationship between $\delta_{\pm}$ and Θ derived by the present general method is identical to equation 35 obtained by Candel (5) for the case of the plane wave.

The effect of the Mach number on the thickness of the transition layer is now examined. An angle α is introduced, for this purpose, using the line $\bar{\beta} = 0$ as the baseline (figure 5-P). The angle α is defined as positive on the side adjacent to the radiation zone, and is negative on the shadow region.

In terms of α, the asymptotic expression of $\bar{\beta}^2$ for large r is obtained by a lengthy but straightforward calculation.
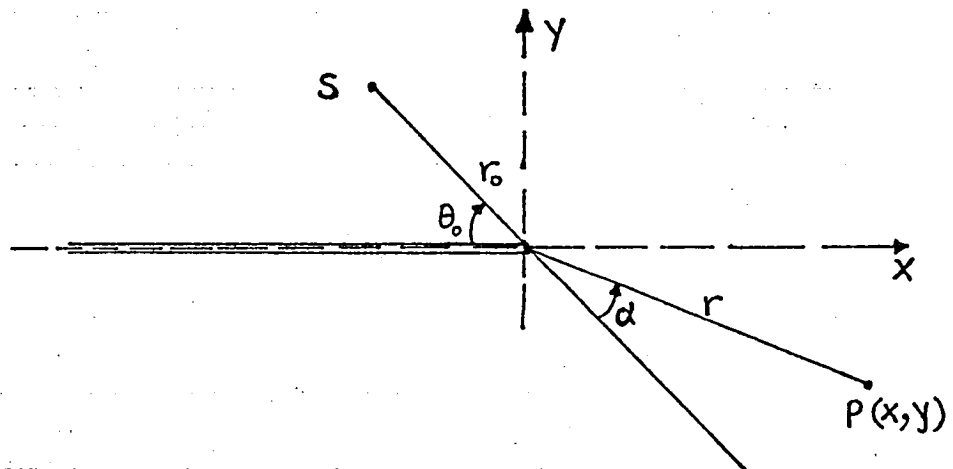
Figure (5-P): Transition zone layout.

The final result is given as:

$$\bar{\beta}^2 = \frac{k r_0}{1-M^2}\left[\sqrt{1-M^2 \sin^2\theta_0} + \frac{M^2 \sin(\theta_0-\alpha)\sin\theta_0-\cos\alpha}{\sqrt{1-M^2\sin^2(\theta_0-\alpha)}}\right] \qquad (5-24)$$

$$+ \; O\left(r_0/r\right)$$

This result is derived by assuming S and P are in the same plane. The sign of $\bar{\beta}$ is chosen so that $\bar{\beta}<0$ when $\alpha>0$ and $\bar{\beta}>0$ when $\alpha<0$. It follows from this choice that:

(i) $\qquad \bar{\beta}\longrightarrow -\infty$ and $F(\bar{\beta})\longrightarrow 1$ as $\alpha\longrightarrow\infty$

(ii) $\qquad \bar{\beta}=0,\; F(\bar{\beta})=1/2$ at $\alpha=0$ $\qquad\qquad$ (5-25)

(iii) $\qquad \bar{\beta}\longrightarrow +\infty,\; F(\bar{\beta})\longrightarrow 0$ as $\alpha\longrightarrow -\infty$

From the mathematical table of $F(\bar{\beta})$ (Ref. 1), it is found that $F(\bar{\beta})$ is practically equal to 1 when $\bar{\beta}\simeq -20$. The corresponding value of α, denoted by α_+ defines the position of the "bright" edge of the transition layer. Similarly, when $\bar{\beta}\simeq +20$, $F(\bar{\beta})$ is practically equal to zero, and the corresponding α, denoted by α_-, defines the position of the "dark" edge of the transition layer. As one can see from the expression for $\bar{\beta}^2$, α_+ and α_-

are functions of kr_o, θ_o and M. In table I and figure (5-Q) next page, the values of α_+ and α_- are listed. These values are calculated by assuming $kr_o = 2500$, $\theta_o = \pi/4$ for various values of the Mach numbers.

From this example, it is seen that the thickness of the transition layer at the trailing edge of the plate becomes narrower when the Mach number increases, and the decrease in thickness is more pronounced on the side adjacent to the shadow region (under the plate). The effect of the Mach number is, therefore, to increase the size of the shadow region and hence make the shielding more effective. Table II shows such calculations.

Table I

Transition layer thickness as a function of the Mach number.

Mach number (M)	O	0.2	0.4	0.6	0.8	0.95
α_+ (degrees)	32.9	32.7	32.2	31.2	29.5	27.7
α_- (degrees)	-32.9	-32.0	-29.5	-25.5	-20.3	-15.7
Layer thickness (degrees)	65.8	64.7	61.7	56.7	49.8	43.4
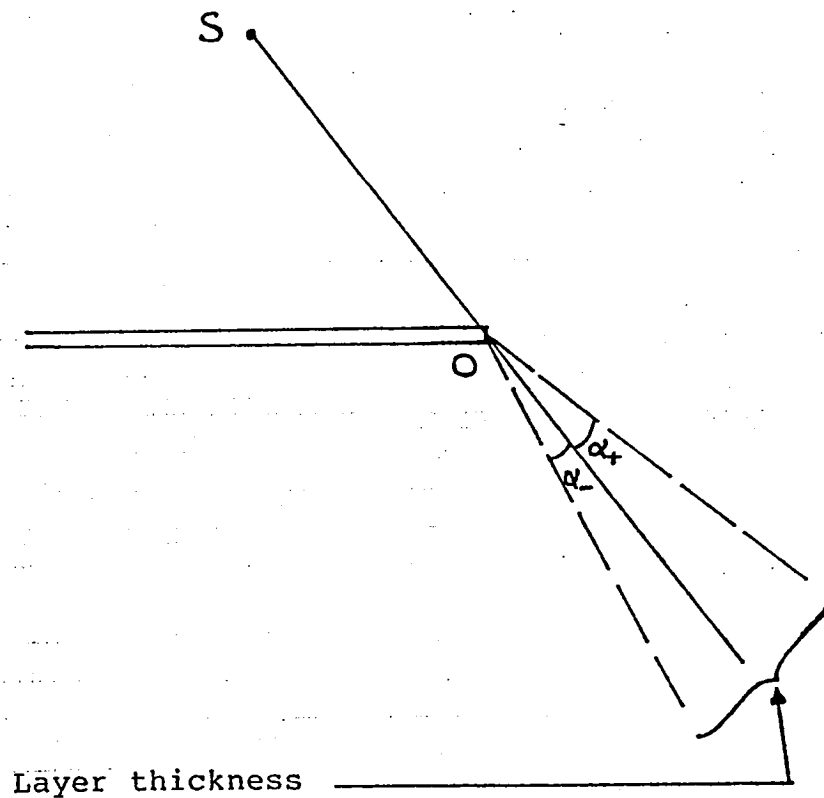
Figure (5-Q): Transition layer thickness.

Table II

Values of $\bar{\beta}$ when $kr_o = 2500$ and $\theta_o = \pi/4$

α	M = 0	M = .2	M = .4	M = .6	M = .8	M = .95
-35	21.263	21.820				
-30	18.301	18.759	20.318	23.763		
-25	15.304	15.667	16.891	19.546	25.706	
-20	12.278	12.551	13.463	15.402	19.670	
-15	9.229	9.419	10.049	11.358	14.096	
-10	6.162	6.279	6.660	7.437	8.982	
-5	3.084	3.137	3.308	3.649	4.297	
0	0	-1.456^{-03}	-1.019^{-03}	-1.651^{-03}	-2.543^{-03}	-3.863^{-03}
5	-3.084	-3.125	-3.258	-3.512	-3.959	-4.522
10	-6.162	-6.235	-6.464	-6.894	-7.628	-8.514
15	-9.229	-9.322	-9.614	-10.154	-11.053	-12.105
20	-12.278	-12.382	-12.708	-13.304	-14.277	-15.388
25	-15.304	-15.411	-15.747	-16.355	-17.335	-18.439
30	-18.301	-18.405	-18.732	-19.321	-20.264	-21.307
35	-21.263	-21.361	-21.666	-22.216		

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