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LOW ENERGY, LEFT-RIGHT SYMMETRY RESTORATION IN SO(N) GUTS

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Abstract

It is shown that a general n-step symmetry breaking pattern of SO(4K+2) down to $SU_C(3)xSU_L(2)xU_Y(1)$, which uses regular subgroups only, does not allow low-energy left-right symmetry restoration. In these theories, the smallest mass scale at which such restoration is possible is $\sim 10^9$ GeV as in the SO(10) case.

We also find that the unification mass in SO(4K+2) GUTS must be at least as large as that in SU(5). These results assume standard values of the Weinberg angle and strong coupling constant.

I. Introduction

The unification group SU(5) of Georgi and Glashow [1] is the smallest simple group which contains the low-energy gauge group $G_{WS} \equiv SU_C(3)xSU_L(2)xU_Y(1)$. Although the SU(5) model has been quite successful in some areas, it leaves some questions unanswered. One of these questions concerns the nature of parity violation. In the SU(5) model, left-right symmetry[2] violation is intrinsic, that is, it is imposed at the outset. This is aesthetically unappealing and leads us to consider theories with spontaneously broken left-right symmetry. The simplest grand unified theory which is left-right symmetric is the SO(10) theory of Fritzsch and Minkowski[3] and Georgi[4]. It contains the subgroup $SO_{LR}(4) \equiv SU_L(2) \times SU_R(2)$ under which the left-handed fermions transform as $(\underline{2},\underline{1})$ and their charge conjugates transform as $(\underline{1},\underline{2})$. Thus, as long as $SO_{LR}(4)$ remains unbroken, left-right symmetry exists (for the phenomenology of $SU_L(2) \times SU_R(2) \times U(1)$ theories, see ref.(5)). At what energy scale is $SO_{LR}(4)$ a good symmetry? Using the method of Georgi, Quinn and Weinberg[6] and known values of the Weinberg angle, θ_W, and of the strong fine structure constant, α_s, (both evaluated at M_W), it has been shown that $SO_{LR}(4)$ symmetry can be restored only at energies larger than 10^9 GeV[7]. The question we ask (and answer) in this paper is the following: can SO(4K+2) (K>2)[8] grand unification groups be found which exhibit low-energy ($O(M_W)$) left-right symmetry restoration? If we assume standard charge, color and weak I-spin assignments for the fermions[9], that only regular subgroups[10] are allowed in the symmetry breaking pattern and that standard values of $\sin^2\theta_W$ and α_s are used, then we find that the answer is no. The lowest mass scale for left-right symmetry restoration is $O(10^9$ GeV) as in the SO(10) case. This result is, in a sense, akin to that of Dawson and Georgi[11] for SU(N) groups. They show that under our assumptions, the unification mass in all such SU(N) models is the same as in the SU(5) case.

This paper is organised as follows: in Sec. II, we collect some general results on n-step symmetry breaking patterns. In Sec. III, we write down the most general symmetry breaking pattern of an SO(4K+2) group to G_{ws} through regular subgroups which could allow low-energy left-right symmetry restoration. Sec. IV uses the known ranges of values for $\sin^2\theta_w(M_w)$ and $\alpha_s(M_w)$ to impose constraints on the left-right symmetry restoration mass scale in the symmetry breaking pattern of Sec. III. Sec. V summarizes our results and lists possible ways to evade the conclusions of our analysis.

II. N-Step Symmetry Breaking in General

Let G be the unification group. As previously stated, we assume standard charge, color and weak I-spin assignments for the fermions. As in ref. (12), we consider an N-step symmetry breaking pattern of G down to G_{ws} of the form:

$$G \xrightarrow{M_1} G_1^C \times G_1^F \times U_1^C(1) \times U_1^F(1) \rightarrow \ldots\ldots \xrightarrow{M_j} G_j^C \times G_j^F \times \prod_{i=1}^{j}$$

$$[U_i^C(1) \times U_i^F(1)] \ldots \xrightarrow{M_N} G_{ws} \tag{2.1}$$

In Eq (2.1), the superscript C (F) indicates that the non-abelian group G_j^C (G_j^F) $(j = 1\ldots N)$ contains $SU_C(3)$ $(SU_L(2))$. We also have

$$G_{j-1}^{(r)} \supseteq G_j^{(r)} \times U_j^{(r)}(1), \quad j=2,---N, \quad r = C \text{ or } F, \tag{2.2a}$$

with

$$G_N^C \equiv SU_C(3),\ G_N^F \equiv SU_L(2),\ U_Y(1) \subseteq \prod_{i=1}^{N} \left[U_i^C(1) \times U_i^F(1)\right]. \qquad (2.2b)$$

In Eq (2.2b), Y denotes the hypercharge operator of the Weinberg-Salam theory.

Thus, in Eq (2.1), the unification mass (at which color and flavor are first separated) is M_1 and the weak I-spin mass scale is $M_W \equiv M_{N+1}$.

Next, we use the renormalization group equations(13) for the various gauge couplings to obtain equations for $\alpha_s(M_W)$, $\alpha_I(M_W)$ $\left(\alpha_s \equiv \frac{g_s^2}{4\pi},\ \alpha_I \equiv \frac{g_I^2}{4\pi},\right.$ where g_s and g_I are the gauge couplings of the groups $SU_C(3)$ and $SU_L(2)$ respectively) in terms of the intermediate mass scales in Eq (2.1). Following ref. (12), we define

$$A^2 \equiv \frac{Tr(Y^2)}{Tr(I_3^2)}, \qquad (2.3a)$$

$$\Gamma \equiv \frac{6\pi\alpha_e^{-1}}{11}\left[1 - (1 + A^2)\sin^2\theta_W\right], \qquad (2.3b)$$

$$\Lambda \equiv \frac{6\pi\alpha_e^{-1}}{11}\left[\sin^2\theta_W - \frac{\alpha_e}{\alpha_s}\right], \qquad (2.3c)$$

$$x_i \equiv \ln\frac{M_i}{M_{i+1}} \quad i=1,\text{---}N, \qquad (2.3d)$$

where α_e is the electromagnetic fine structure constant, I_3 is the diagonal generator of $SU_L(2)$,

$$\sin^2\theta_W \equiv \frac{\alpha_e}{\alpha_I}, \qquad (2.4)$$

and all coupling constants are evaluated at $Q^2 = (2M_W)^2$. For standard charge assignments, A^2 is given by its value in the SU(5) model, i.e.

$$A^2 = 5/3. \qquad (2.5)$$

Using the results of ref. (12), we may write:

$$\Gamma = \sum_{j=1}^{N} a_j x_j \tag{2.6a}$$

$$\Lambda = \sum_{j=1}^{N} b_j x_j, \tag{2.6b}$$

where

$$a_j \equiv C_j^F (A^2 - [N_j^F]^2) - C_j^C [N_j^C]^2 \tag{2.7a}$$

$$b_j \equiv C_j^C - C_j^F. \tag{2.7b}$$

Here, $C_j^{(r)}$, $[N_j^{(r)}]^2$ (r = C or F) are the eigenvalue of the second Casimir operator acting on the adjoint representation of $G_j^{(r)}$ and the embedding coefficient of the hypercharge Y into $G_j^{(r)}$, respectively. $[N_j^{(r)}]^2$ is a measure of the fraction of generators of $G_j^{(r)}$ which go into the makeup of Y. If we write

$$Y = Y_j^{(r)} + Y', \tag{2.8}$$

with $Y_j^{(r)}$ (Y') contained (not contained) in $G_j^{(r)}$, then

$$[N_j^{(r)}]^2 \equiv \frac{\mathrm{Tr}\,[(Y_j^{(r)})^2]}{\mathrm{Tr}\,[I_3^2]}. \tag{2.9}$$

The formalism of Appendix B of ref. (12) gives a straight-forward way of calculating $[N_j^{(r)}]^2$ for any group (for the SU(N) case, these may be found in ref. (11) and ref. (14). We list the values of $C_j^{(r)}$ and $[N_j^{(r)}]^2$ below:

$$C_j^{(r)} = \begin{cases} N & G_j^{(r)} \equiv SU(N) \\ N-2 & G_j^{(r)} \equiv SO(N) \\ 0 & G_j^{(r)} \equiv U(1), \end{cases} \tag{2.10a}$$

$$[N_j^C]^2 = \begin{cases} 2(\frac{1}{3} - \frac{1}{n}) & G_j^C \equiv SU_C(n) \\ \frac{2}{3} & G_j^C \equiv SO_C(n) \end{cases}, \quad [N_j^F]^2 = \begin{cases} 2(\frac{1}{2} - \frac{1}{m}) & G_j^F \equiv SU_F(m) \\ 1 & G_j^F \equiv SO_F(m) \end{cases} \tag{2.10b}$$

Using Eqs (2.10a,b), we evaluate a_j, b_j of Eqs (2.7a,b) for the intermediate subgroups which will be relevant to later discussions. Let K_j denote the intermediate symmetry group which is unbroken at the jth-step of symmetry breaking. Then we have:

$$a_j = -\frac{2}{3}\Delta_j, \quad b_j = \Delta_j \text{ if } K_j \equiv SO_C(n_j) \times SO_F(m_j) \tag{2.11a}$$

$$a_j = -\frac{2}{3}\Delta_j + \frac{2}{3}, \quad b_j = \Delta_j + 2 \text{ if } K_j \equiv SU_C(n_j) \times SO_F(m_j) \times U_j^C(1) \tag{2.11b}$$

$$a_j = -\frac{2}{3}\Delta_j + \frac{10}{3}, \quad b_j = \Delta_j - 2 \text{ if } K_j \equiv SO_C(n_j) \times SU_F(m_j) \times U_j^F(1) \tag{2.11c}$$

$$a_j = -\frac{2}{3}\Delta_j + 4, \quad b_j = \Delta_j \text{ if } K_j \equiv SU_C(n_j) \times SU_F(m_j) \times U_j^C(1) \times U_j^F(1), \tag{2.11d}$$

where

$$\Delta_j \equiv n_j - m_j. \tag{2.12}$$

III. N-step Symmetry Breaking for SO(4k+2)

We now let $G \equiv SO(4K+2)$ and consider an N-step symmetry breaking pattern of G down to G_{WS}, subject to the contraint that only regular subgroups of G be allowed to appear. From Dynkin[15], we see that the subgroups $G_j^{(r)}$ can only be of the form $SO(2\ell)$, $SU(\ell)$ ($\ell \leq 2K+1$). This constraint also implies that once an $SO(2\ell)$ group has broken down to an $SU(m)$ subgroup, this $SU(m)$ can only break down into subgroups of the form $SU(n_1) \times SU(n_2) \times U(1)$ ($n_1+n_2 \leq m$).

We consider the following symmetry breaking pattern:

$$\xrightarrow{M_1} SO_C(n_1) \times SO_F(m_1) \rightarrow \ldots \xrightarrow{M_{\alpha-1}} SO_C(n_{\alpha-1}) \times SO_F(m_{\alpha-1}) \xrightarrow{M_\alpha} SU_C(n_\alpha) \times SO_F(m_\alpha) \times U^C_\alpha(1)$$

$$\rightarrow \ldots \xrightarrow{M_{\beta-1}} SU_C(n_{\beta-1}) \times SO_F(m_{\beta-1}) \times \prod_{i=\alpha}^{\beta-1} U^C_i(1) \xrightarrow{M_\beta} SU_C(m_\beta) \times SU_F(n_\beta) \times U^F_\beta(1) \times \prod_{i=\alpha}^{\beta} U^C_i(1)$$
$$\rightarrow \ldots \xrightarrow{M_N} G_{WS} \qquad (3.1)$$

For this pattern, Eqs (2.6a,b) become:

$$\Gamma = \frac{-2}{3} \sum_{i=1}^{N-1} \Delta_i x_i + \frac{2}{3} \sum_{i=\alpha}^{\beta-1} x_i + 4 \sum_{i=\beta}^{N-1} x_i + \frac{10}{3} x_N \qquad (3.2a)$$

$$\Lambda = \sum_{i=1}^{N-1} \Delta_i x_i + 2 \sum_{i=\alpha}^{\beta-1} x_i + \qquad + x_N, \qquad (3.2b)$$

where Δ_i is defined as in Eq (2.12)[16]. The relevant quantity in our analysis will be Ω, defined by:

$$\Omega \equiv \frac{1}{4} \left[\Gamma + \frac{2}{3} \Lambda\right] = \frac{6\pi\alpha_e^{-1}}{11} \frac{1}{4} \left[1 - 2 \sin^2\theta_w - \frac{2}{3} \frac{\alpha_e}{\alpha_s}\right], \qquad (3.3)$$

where all couplings are evaluated at $Q^2 \approx (2M_w)^2$. Dawson and Georgi[11] have shown that if M_G denotes the unification mass in the $G \equiv SU(N)$ case, then

$$\Omega = \ln \frac{M_G}{M_w}. \qquad (3.4)$$

From Eqs (3.2a, b) we find[17]

$$\Omega = \sum_{i=\beta}^{N} x_i + 1/2 \sum_{i=\alpha}^{\beta-1} x_i. \qquad (3.5)$$

If we set

$$x_i = 0 \quad i = 1, \ldots, \beta-1, \qquad (3.6)$$

then only groups of the form

$$SU_C(n_j) \times SU_F(m_j) \times \Pi\left[U_j^C(1) \times U_j^F(1)\right] \qquad (3.7)$$

can appear in Eq (3.1). The unification mass M_β is given by

$$\ln \frac{M_\beta}{M_W} = \sum_{i=\beta}^{N} x_i = \Omega, \qquad (3.8)$$

which is the SU(N) result stated above. That this should be the case can be seen by realizing that all subgroups of the form in Eq(3.7) are contained within the SU(2K+1) subgroup of SO(4K+2). Thus, the fact that they are also embedded in SO(4K+2) becomes irrelevant.

IV. Constraints on Mass Scales

We now proceed to find constraints on some of the intermediate mass scales appearing in Eq(3.1). We are especially interested in constraints on M_β. the scale at which the flavor group changes from an orthogonal group to a unitary one. This change signals the breakdown of left-right symmetry amongst the fermions since $SO_F(m)$ treats both particles and their charge conjugates in an identical fashion. Thus, it is at M_β that the flavor interactions become left-handed.

We shall use values of $\sin^2\theta_W(M_W)$ and $\alpha_s^{-1}(M_W)$ in the ranges[18]

$$\sin^2\theta_W(M_W) = 0.19 - 0.24 \qquad (4.1a)$$

$$\alpha_s^{-1}(M_W) = 7.5 - 9.3 \qquad (4.1b)$$

We shall also take $\alpha_e^{-1}(M_W)$ to be[18]

$$\alpha_e^{-1}(M_W) = 128.5. \tag{4.2}$$

The quantity that we are interested in is

$$\ln \frac{M_\beta}{M_W} = \sum_{i=\beta}^{N} x_i \equiv \Phi \tag{4.3}$$

Since all the x_i (i=1, ---, N) are non-negative, we may use Eq(3.5) to find the crude bound:

$$\Phi \leq \Omega, \tag{4.4}$$

with equality if and only if all x_i ($i = \alpha$, --, β-1) vanish. In this case only groups of the form $SO_C(n_i) \times SO_F(m_j)$ appear in Eq (3.1). Since we have the bound

$$\Omega \geq 28 \tag{4.5}$$

for $\sin^2\theta_W$ and α_s^{-1} as in Eqs (4.1a,b), this implies that when x_i (i=α,..β-1) vanish, left-right symmetry can only be restored for $M_\beta \gtrsim 10^{14}$ GeV. This would also imply that the unification mass, M_1, of Eq(3.1) could be larger than 10^{14-15} GeV. This result agrees with those found in ref. (19) where the two-step case

$$SO(N) \xrightarrow{M_1} SO_C(n_1) \times SO_F(m_1) \xrightarrow{M_2} G_{WS} \tag{4.6}$$

is treated.

We can find a better bound on Φ as follows: from Eq(3.5), we have

$$\Phi + \tfrac{1}{2}\sum_{i=\alpha}^{\beta-1} x_i = \Omega. \tag{4.7}$$

Let us compute $\ln \frac{M_\alpha}{M_\beta}$:

$$\ln \frac{M_\alpha}{M_\beta} = \sum_{i=\alpha}^{\beta} x_i = \sum_{i=\alpha}^{\beta-1} x_i + x_\beta = 2\,(\Omega - \Phi) + x_\beta. \tag{4.8}$$

Using Eq(4.3), we find

$$\ln \frac{M_\alpha}{M_W} = \ln \frac{M_\alpha}{M_\beta} + \ln \frac{M_\beta}{M_W} = 2\,\Omega - \Phi + x_\beta \geq 2\,\Omega - \Phi, \tag{4.9}$$

since $x_\beta \geq 0$. If we now make the reasonable assumption that the unification mass, M_1, must be less than the Planck mass $M_P \sim 10^{19}$ GeV $\sim 10^{17}\, M_W$, we arrive at the constraint:

$$\ln \frac{M_P}{M_W} \simeq 39 \geq \sum_{i=1}^{N} x_i \geq \sum_{i=\alpha}^{N} x_i = \ln \frac{M_\alpha}{M_W} \geq 2\,\Omega - \Phi, \tag{4.10a}$$

or

$$\Phi \geq 2\,\Omega - 39 \gtrsim 17, \tag{4.10b}$$

or

$$M_\beta \gtrsim 10^9 \text{ GeV}, \tag{4.10c}$$

where Eq (4.5) was used in Eq (4.10b). Thus we see that the pattern of Eq (3.1) does not allow low-energy left-right symmetry restoration. Since the pattern of Eq (3.1) is the most general one (subject to our earlier constraints) which could give rise to low-energy left-right symmetry restoration, we must conclude that this phenomenon is not compatible with our assumptions.

We may extract one more piece of information from this analysis; using Eqs (4.4,4.9), we find that

$$\ln \frac{M_\alpha}{M_W} \geq \Omega \tag{4.11}$$

This implies that the unification mass for the pattern of Eq (3.1) can in general be no smaller than 10^{14-15} GeV.

V. Conclusions

Given our assumptions on the assignment of fermion quantum numbers, the form of the symmetry breaking pattern of SO(4K+2) down to G_{WS} and the values of $\sin^2\theta_W$ and α_s^{-1}, the mass scale at which left-right symmetry restoration occurs must be $\gtrsim 10^9$ GeV. In this respect, the general SO(4K+2) case and the SO(10) case are identical. If we want left-right symmetry to be restored at energies of the order of M_W, we must relax some of the assumptions made here. The possibilities are as follows:

1. We may allow non-standard assignment of fermion quantum numbers. In ref. (12,20), an SO(14) based GUT, with non-standard charge assignments is examined. In this theory, renormalization group arguments allow the appearance of $SO_{LR}(4)$ at mass scales M_β such that $3M_W \lesssim M_\beta \lesssim 10^2 M_W$.

2. We can argue that $\sin^2\theta_W(M_W)$, $\alpha_s^{-1}(M_W)$ do not have to lie in the ranges given in Eqs(4.1a,b). Rizzo and Senjanović[21] have argued that $\sin^2\theta_W$ may be as large as 0.27-0.31, when right handed current effects are taken into account. This would then allow M_β to be $O(M_W)$.

3. Non-regular subgroups of SO(4K+2) could be allowed in the symmetry breaking pattern[22]. This possibility will be treated in a later work.

4 [illegible] include Higgs boson effects in the renormalization group equations (see ref (23)).

We also found that unification mass scale in the SO(4K+2) theories has to be at least as large as that in SU(5). If proton decay is not seen in the near future, it may be because Nature prefers an SO(4K+2) unification group.

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References and Footnotes

1. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

2. If all particles are left-handed, this is equivalent to charge conjugation symmetry.

3. H. Fritzsch and P. Minkowski, Ann. of Phys. (N.Y.), 193 (1975).

4. H. Georgi, Particles and Fields (1974) (APS/DPF Williamsburg) ed. C. E. Carlson (AIP, New York, 1975).

5. R. N. Mohapatra and G. Senjanović Phys. Rev. Lett 44, 912 (1980); Phys. Rev. D23, 165 (1981); T. Rizzo, Iowa Preprint ISJ 672 (1981).

6. H. Georgi, H. Quinn, S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

7. H. Georgi, D. V. Nanopoulos, Phys. Lett. 82B, 392 (1979); Nucl. Phys. B155, 52 (1979); B159, 16 (1979); S. Rajpoot, Phys. Rev. D22, 2245 (1980); Q. Shafi, M. Sondermann, Ch. Wetterich, Phys. Lett. 92B, 304 (1980).

8. SO(4K+2) (K>2) is used since only these have fermion representations compatible with the requirements of anomaly freedom, complexity with respect to $SU_c(3) \times SU_L(2) \times U_Y(1)$, having only $\underline{1}$, $\underline{3}$, $\underline{\overline{3}}$'s of color, irreducibility and the ability to contain more than one fermion generation.

9. This means that only $\underline{1}$, $\underline{3}$, $\underline{\bar{3}}$ of $SU_C(3)$, $\underline{1}$, $\underline{2}$ of $SU_L(2)$ and quarks with charges $Q = -1/3$, $+2/3$, leptons with charges $Q = -1,0$ are allowed.

10. Regular subgroups have as generators subsets of the generators of the unification group. Non-regular subgroups have non-trivial linear combinations of these generators as their generators.

11. S. Dawson and H. Georgi, Phys. Rev. Lett. 43, 821 (1979).

12. G. Feldman and R. Holman, John Hopkins Preprint JHU-HET 8201; this is an extension of the SU(7) model of J. E. Kim, Phys. Rev. Lett. 45, 1916 (1980), Phys. Rev. D23, 2706 (1981).

13. H. D. Politzer, Phys. Rep. 14C, 131 (1974); we neglect two-loop and higher effects as well as Higgs boson effects.

14. F. Bordi, R. Holman and C. W. Kim, John Hopkins Preprint JHU-HET 8110 (Phys Rev D, in press).

15. E. B. Dynkin, Amer. Math. Soc. Transl. Ser. 2, 6, 111 (1957).

16. This result is true for any group G that breaks down as in Eq (3.1).

17. This is also the result for Ω in a symmetry breaking pattern where $SO_F(m_j) \to SU_F(m_{j+1}) \times U^F{}_{j+1}(1)$ before $SO_C(n_k) \to SU_C(n_{k+1}) \times U^C{}_{k+1}(1)$.

18. J. E. Kim, P. Langacker, M. Levine and H. H. Williams, Rev. Mod. Phys. 53, 211 (1981); P. Langacker, Phys. Rep. 72, 185 (1981); W. Marciano and A. Sirlin, Phys. Rev. Lett. 40, 163, (1981).

19. F. Bordi, R. Holman and C. W. Kim, Phys. Lett. 106B, 58 (1981).

20. R. Holman, Johns Hopkins Preprint, JHU-HET 8205 (1982); other SO(14) based GUT models are presented in J. E. Kim, Seoul preprint on Supersymmetric Grand Unification in SO(14); Y. P. Kuang and S. -H. H. Tye, Cornell Univ. Preprint CLNS 82/529; L. F. Li and D. G. Unger Carnegie-Mellon preprint CMU-HEG 82-2; F. W. Lczek and A. Zee, Princeton Univ. Preprint "Spinors and Families" (1979).

21. T. Rizzo and Senjanović, Phys. Rev. Lett. 46, 1315 (1981); Phys. Rev. D24, 704 (1981).

22. Our results are also true if the non-regular SO(2ℓ+1) subgroups of SO(4K+2) are used.

23. R. N. Mohapatra and M. Popović, CUNY Preprint CCNY-HEP-81/11; J. C. Pati and A. Mohanty (Private Communication).