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CORRELATION OF HARD X-RAY AND TYPE III BURSTS  
IN SOLAR FLARES

by

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## ABSTRACT

The observed correlations between x-ray and type III radio emission of solar bursts are described through a bivariate distribution function. Procedures for determining the form of this distribution are described using a sample of data analyzed by Kane (1981). With the help of this distribution a model is constructed to explain the correlation between the x-ray spectral index and the ratio of x-ray to radio intensities. Implications of the model are discussed.

## I. INTRODUCTION

It is well known that electrons accelerated during solar flares produce, among other things, hard x-rays and type III bursts. Hard x-rays are produced primarily by electrons which penetrate to the lower (higher density) regions of the solar atmosphere, while the type III bursts are produced higher up by the electrons which are streaming outward from the sun. Observations show a strong relationship between these two types of radiations, implying a common origin for the electrons. Thus, an understanding of the mechanisms responsible for these radiations can provide insight into the acceleration process and some of the characteristics of the flare plasma, such as the magnetic field structure, density and temperature.

Interpretations of the x-ray observations are simplest. A reasonable estimate of the number of electrons responsible for the observed x-rays can be obtained from the total count of x-rays at lower energies ( $<20$  keV). But a detailed description of the finer aspects of the observation (such as spectral or spatial distribution) is more difficult because of their dependence on the properties of the ambient plasma such as the temperature, density and magnetic field geometry and the variations of these throughout the atmosphere (see e.g. Leach and Petrosian 1981, 1982).

For type III bursts, on the other hand, even simple estimates of the number of electrons involved in the radiation process cannot be made reliably because of the complexities of

the mechanism (see e.g. Goldman and Smith in P.A.Sturrock 1982). Some recent observations (Lin et al 1982) have even questioned the basic premise of the model (Ginzburg and Zheleznyakov 1958) for type III bursts: namely, the excitation of plasma waves (by the streaming electrons) and the conversion of their energy into electromagnetic waves. It is, however, reasonable to assume that the intensity of type III bursts is related to the flux of exciting electrons, even though the relationship may not be a simple linear one as in the case of x-rays. Empirically, such a relationship has been established between electrons and in situ produced type III bursts at one A.U. (Fitzenreiter, Evans, Lin 1976). We assume that this relationship can be extrapolated to the bursts occurring near the sun.

With this assumption it is then clear that any observed relationship between the intensity (or other characteristic) of the type III and x-ray bursts will provide information about the correlation between electrons streaming outward (along open field lines) or beamed (along closed or open field structures) toward the sun.

This relationship, however, has not been fully explored because the observations do not define a clear relationship between the two types of radiation (e.g. there exist many x-ray bursts without detectable type III radiation and vice versa). This can also be understood in regard to the theory because of the complexities of the mechanisms involved. Many parameters enter into the determination of the number of the two types of electrons, such as the spatial, spectral and pitch angle

distribution of the accelerated electrons, the magnetic field structure and the plasma density or temperature.

Because of this, one useful approach is to rely on the statistical behavior of a large pre-selected sample (i.e. a sample having known selection effects) of bursts to bring to light the subtle relationships which are being overwhelmed by the large dispersion in the properties of the bursts. The purpose of this paper is to demonstrate the usefulness of such an analysis by considering the results from a recent statistical study by Kane (1981), who described the correlation between the x-ray and type III emissions of a sample of bursts from OGO-5 and groundbased observation.

In the next section we shall use his results to describe the distribution of the x-ray and type III burst intensities, and in Section III we present an explanation of an interesting correlation associated with the x-ray spectral index using results from our (Leach and Petrosian 1981) program on the transport of the accelerated electrons in the solar atmosphere. A brief summary, including some suggestions for future work, are presented in Section IV.

## II. BURST DISTRIBUTION AND CORRELATIONS

The parameter that most distinctly characterizes a burst is its observed intensity. In the case of solar bursts this is proportional to the intrinsic strength or luminosity of the burst. We shall, therefore, be concerned with the distribution of the x-ray and type III radio intensities. (In astronomical jargon, this is what would be called a luminosity function.) Of course, both x-ray and type III bursts have spectral and temporal variations. Perhaps a good parameter to characterize the strength (or luminosity) of a burst would be its intensity integrated over all photon energies (or frequencies) and over the total duration of the burst. Since these are not readily available from the observations, we shall use instead the intensities at the peak of the burst and at a particular photon energy (or frequency). In the case of x-rays, as the spectrum falls off rapidly with photon energy, the intensity at the lowest energy gives a good measure of the burst strength. It is common practice to use either the intensity or the number of photon counts at ~20 keV. As for type III bursts, it is plausible that the burst strength at the highest frequency (at the earliest appearance of the burst) is a good indicator of the burst strength. This assumption is an uncertain one. However, as we shall be concerned only with a rough measure of the strength of type III bursts (such as whether or not a type III burst was detected for any given x-ray burst), we need not be concerned with this uncertainty.

We note that, in general, results from a statistical analysis are accurate as long as the sample selection is done in a consistent manner. The choice of parameters is important only for any physical interpretation of the results from the statistics.

A. The Distribution Function

Let us consider a sample of bursts chosen and classified according to their x-ray and type III radio intensities denoted by  $X$  and  $R$ , respectively. Let  $f(X,R)$  be the distribution of the bursts (i.e. the bivariate luminosity function) so that  $Af(X,R)dXdR$  is the number of bursts occurring with intensities within the ranges  $X$  to  $X+dX$  and  $R$  to  $R+dR$ . (Here  $A$  is the rate of occurrence of all bursts, since  $f$  is normalized to unity.) The number of bursts observed (per unit of time) is limited by the sensitivity of the instruments. For example, the number of type III bursts with intensity  $R$ , per unit  $R$ , irrespective of their x-ray intensities, is

$$N_0(R) = A \int_0^{\infty} f(X,R) dX ; \quad (1)$$

and the number of such bursts with x-rays detectable by an instrument with a well-defined limiting sensitivity  $X_0$  is

$$N_{X_0}(R) = A \int_{X_0}^{\infty} f(X,R) dX . \quad (2)$$

Similarly, we have

$$N_0(X) = A \int_0^{\infty} f(X,R) dR, \quad N_{R_0}(X) = A \int_{R_0}^{\infty} f(X,R) dR \quad (3)$$



as the number of all the bursts with x-ray intensity  $X$  and the number of those with type III burst intensities  $> R_0$ , respectively.

The purpose of a statistical analysis is to compare such distributions (or the moments of such distributions) with observations in order to derive the intrinsic distribution  $f(X,R)$ . From this and from the nature of the correlation between the x-ray and radio intensities, one can gain insight into the processes by which these radiations are produced and into the mechanisms accelerating the electrons. To demonstrate the usefulness of such a statistical analysis, we use the following observational results obtained by Kane (1981).

#### B. A Sample of Bursts

We now briefly summarize the observational results from Kane (1981) which are relevant to our analysis. Figure numbers preceded by the letter K refer to his paper.

1) Radio distributions. Figure K4 shows that the distribution  $N_0(R)$  and  $N_{X_0}(R)$  are different. From a chi square analysis we find that it is very unlikely (probability  $< 10^{-4}$ ) that such a difference could occur by chance. As a result, the fraction  $f_{X_0}(R) = N_{X_0}(R)/N_0(R)$  (as shown in figure K5) cannot be a constant and the monotonic increase with  $R$  is real.

2) X-ray distributions. The x-ray data, presented in more detail, show similar results: namely, that the distributions  $N_0(X)$  and  $N_{R_0}(X)$  (solid and dashed histograms in figure K7) are different and that their ratio, the fraction  $f_{R_0}(X) = N_{R_0}(X)/N_0(X)$  can be approximated by a monotonically increasing function of  $X$

(figure K8). Again, we find a large chi square (~36 for 10 degrees of freedom), indicating the unlikelihood that these differences occur by chance and that  $f_{R0}(X)$  is a constant.

3) Distribution of the x-ray spectral index. In figure K9 it is shown that the distribution of the x-ray spectral index of all x-ray bursts and those with detectable type III bursts are again different. Using various binnings we find a large chi square (>25) which confirms the reality of the difference. As shown in figure K10, the ratio of the two above-mentioned distributions decreases with increasing spectral index.

We shall use the first two items above (primarily item 2) in the determination of the distribution function  $f(X,R)$  and the observation described in the last item for its implication for flare models.

### C. The Distribution Function $f(X,R)$

With such a limited sample of data, it would be difficult for an inversion of the observations to give a complete description of  $f(X,R)$ . Thus, we must make some simplifying assumptions. First, we consider two of the simplest forms for  $f(X,R)$  with extreme and opposite assumptions.

1) Perfect correlation: This occurs when the ratio of the x-ray to radio intensity is a constant over all bursts. If this constant ratio is, say,  $Y_0$ , then  $f(X,R) = \psi(X) \delta(X - Y_0 R)$ . It is then clear from eqs. (1) through (3) that

$$N_{X_0}(R) = \begin{cases} N_0(R) = A\psi(Y_0 R), & R > X_0/Y_0 \\ 0 & , \quad R < X_0/Y_0 \end{cases} \quad (4a)$$

and

$$N_{R_0}(X) = \begin{cases} N_0(X) = A\psi(X)/Y_0, & X > Y_0R_0 \\ 0 & , \quad X < Y_0R_0 \end{cases} \quad (4b)$$

and that the fractions

$$f_{X_0}(R) = N_{X_0}(R)/N_0(R), \quad f_{R_0}(X) = N_{R_0}(X)/N_0(X) \quad (5)$$

are either zero or unity.

2) No correlation. If there exists no correlation between the x-ray and radio intensities of a burst, then we can write  $f(X,R) = \psi(X)\phi(R)$ . If we further define as cumulative normalized distributions

$$\begin{aligned} \Psi(z) &= \int_z^\infty \psi(z') dz' \\ \Phi(z) &= \int_z^\infty \phi(z') dz' \end{aligned} \quad (6)$$

then, from eqs. (1) through (3), we obtain

$$N_0(R) = A\phi(R), \quad N_{X_0}(R) = A\phi(R)\Psi(X_0), \quad f_{X_0}(R) = \Psi(X_0), \quad (7a)$$

and

$$N_0(X) = A\psi(X), \quad N_{R_0}(X) = A\psi(X)\Phi(R_0), \quad f_{R_0}(X) = \Phi(R_0) \quad (7b)$$

The ratios,  $f_{X_0}(R)$  and  $f_{R_0}(X)$ , of the correlated to the total distribution are then constants independent of R and X. In Figure 2 we show a schematic variation of the distributions in x-rays for the above two opposite assumptions (dotted and dashed

lines) and the general trends of the observation (solid lines) as described in part B above.

It is clear that neither of the above two extreme assumptions is correct. This agrees with our expectations as described in Section I. There must be some correlation (ruling out case 2) because of the common origin of the agents of these radiations (electrons), but this correlation is not perfect (ruling out case 1) because of the variation of other parameters from burst to burst which affects the x-ray or type III intensities ( e.g., the number of escaping electrons, or the excitation of plasma waves, etc.) Our ignorance of the exact nature of such effects forces us to treat the correlations statistically.

3) An example of a distribution that works. The fact that the correlated fractions  $f_{X_0}(R)$  and  $f_{R_0}(X)$  are both increasing functions of  $R$  and  $X$  (figures K5 and K8) implies that statistically the brighter the x-ray burst the stronger its type III radiation, and vice versa. A simple way to describe this fact is by the following distribution function

$$f(X,R) dXdR = \psi(X)g(Y) dXdY , \quad Y = X/R, \quad (8)$$

which assumes that the x-ray intensity is not correlated with the ratio of x-ray to radio brightness. Such a distribution will come about if, for example, the x-ray intensity is directly related to the energy (or number of electrons) released in the acceleration process while the radio intensity depends not only on this but also on the ratio of the escaping to downward flowing

electrons. This is a plausible hypothesis and, as we shall see below, it is also a simple way to interpret the observations<sup>1</sup>.

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<sup>1</sup>Alternatively one can write  $f(X,R)dXdR = \phi(R)h(Y)dRdY$  which can also provide satisfactory explanations for the observations. However, this would imply a more direct relationship between the acceleration mechanism and the small number of escaping electrons which produce the radio burst than the bulk of electrons responsible for the x-rays. In addition, equation (8) is simpler for the interpretation of the data discussed here.  
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Now if, as before, we denote by a capital letter the cumulative distributions

$$\begin{aligned} \Psi(x) &= \int_0^x \psi(x') dx', & \Psi(\infty) &= 1, \\ G(x) &= \int_0^x g(x') dx', & G(\infty) &= 1, \end{aligned} \tag{9}$$

then it can be shown that the distribution in X over all the x-ray bursts is

$$N_0(X) = A \int_{y=0}^{\infty} \psi(x) g(y) dy = A \Psi(X) \tag{10}$$

and the distribution in X of x-ray bursts with detectable type III bursts ( $R > R_0$ ) is

$$N_{R_0}(X) = A \Psi(X) G(X/R_0), \tag{11}$$

so that the fraction of x-ray bursts with detectable radio flux is

$$f_{R_0}(X) = G(X/R_0). \quad (12)$$

Since  $N_0$  and  $N_{R_0}$  (or  $f_{R_0}$ ) are observable distributions, these equations show that one can obtain  $\psi(X)$  and the cumulative distribution  $G(X/R)$  directly from the observations. If the observations are sufficiently detailed, then the differentiation of  $G$  will give the distribution function  $g(Y)$ .

This demonstrates that the distribution of the x-ray intensities can give the functions  $\psi$  and  $g$  in eq. (8). To further refine these functions or to test their accuracy, one can utilize the observed distribution of the radio intensities. For example, the distribution of radio intensity over all bursts, obtained by substitution of equation (8) into equation (1), is

$$N_0(R) = AR^{-2} \int_{x=0}^{\infty} X \psi(X) g(X/R) dX \quad (13)$$

which should be compared with observations such as those given in figure K2. However, in practice, this requires knowledge of the distribution functions  $\psi$  and  $g$  down to very low values of  $X$ . This is not directly known since the observed distribution of x-ray intensities is limited to bursts with  $X > X_0$ . In order to avoid unnecessary extrapolation, one can compare the observations with  $N_{X_0}(R)$  which is given by eq. (13) with the lower limit of integration changed to  $X_0$  (cf. eq. 2). Alternatively, in order to avoid the propagation and amplification of the errors while taking derivatives in the step leading to  $g(X/R)$ , we can compare the cumulative observed distribution

$$N_{X_0}^C(R) = \int_R^{\infty} N_{X_0}(R) dR \quad (14)$$

which, with the help of eqs. (8) and (9) reduces to

$$N_{X_0}^C(R) = A \int_{X=X_0}^{\infty} \psi(X) G(X/R) dX \quad (15)$$

Thus, using  $\psi$  and  $G$  derived directly from the x-ray distribution, one can deduce the distribution of the radio intensities and compare it with the observed data. We shall not carry out this derivation since the observed distribution of the radio intensities is not sufficiently detailed for this purpose.

#### D. Spectral Index Distributions

As described in Section B.3 above, bursts with flatter x-ray spectra (which also means flatter spectra for the accelerated electrons) are more likely to be accompanied by a type III burst than those with steeper spectra. We show now that when combined with the above distribution, this indicates a correlation between the spectral index and the ratio  $Y = X/R$ . Let us consider the distribution of the ratio  $Y = X/R$ . For all of the x-ray bursts

$$N_0(Y) = Ag(Y) \int_{X_0}^{\infty} \psi(X) dX = Ag(Y) \Psi(X_0), \quad (16)$$

but for those with radio burst intensities larger than  $R_0$

$$N_{R_0}(Y) = \begin{cases} N_0(Y) & \text{if } Y < X_0/R_0 \\ Ag(Y) \Psi(R_0 Y) & \text{if } Y > X_0/R_0 \end{cases} \quad (17)$$

so that the distribution in  $Y$  of the fraction of x-ray bursts which have associated type III burst is

$$f_{Ro}(Y) = N_{Ro}(Y)/N_o(Y) = \begin{cases} \Psi(R_o Y) / \Psi(X_o), & Y > X_o/R_o \\ 1 & , \quad Y < X_o/R_o. \end{cases} \quad (18)$$

This fraction is a decreasing function of  $Y$ . Figure K10 shows that this fraction also decreases monotonically with the spectral index  $\gamma$ . This suggests the existence of a direct relationship between  $Y$  and  $\gamma$ . Since  $X$  and  $R$  are directly related to the number of ingoing and escaping electrons and  $\gamma$  is related to the spectral index of the electrons  $\delta$ , we are led to the hypothesis that the flatter the electron spectrum (the smaller  $\delta$  or  $\gamma$ ) the larger the fraction of escaping or type III producing electrons. In the next section we give support to this hypothesis.

### III. INTERPRETATION OF THE CORRELATION

Many factors enter into the determination of the ratio  $Y=X/R$ . We now describe these factors and their effect on the correlation of  $Y$  with the spectral index  $\gamma$  of the x-rays.

1) Effect of the emission mechanism. Although there are many factors which affect the intensities  $X$  and  $R$ , it is difficult to see how these factors might be strongly correlated with the spectral index of the x-rays. We assume, therefore, that the observed correlation between  $Y$  and  $\gamma$  is primarily due to the correlation between the spectral index of the accelerated



electrons  $\delta$  (which is related in a simple way to  $\gamma$ ) and the ratio  $Y_e$  of the number of x-ray producing electrons,  $n_X$ , to the number of type III producing electrons,  $n_R$ ;  $Y_e = n_X/n_R$ . We, therefore, concentrate on factors effecting this ratio.

2) Magnetic field structure. One of the most important factors is the geometry of the magnetic field at the acceleration site as depicted in Figure 1. The electrons accelerated within a closed magnetic field region will not be seen streaming outward to produce type III radiation since diffusion across the field lines is slow (the gyroradius of a typical electron,  $\sim 1$  cm, is much smaller than the relevant scales of  $>10^9$  cm). Thus, if  $Y_c$  is the fraction of the electrons accelerated on closed field lines, then  $Y_e \geq Y_c$ . The fact that there exists type III radiation and that there exist electrons at one A.U. associated with flares indicates that some accelerated electrons must find themselves on open field lines and that the fraction  $Y_c < 1$ . However, note that it is difficult to conceive of a direct relationship between the geometry of the field (and the fraction  $Y_c$ ) and the spectral index  $\delta$ .

3) Pitch angle distribution. The electrons on the closed field lines produce only x-rays, in accordance with the thick target model, and thus their pitch angle distribution is unimportant. Of the electrons on the open field lines those directed outward will produce type III radiation and insignificant amounts of x-rays. Those directed inward toward the photosphere either will lose all their energy and in the process produce some x-rays or will be reflected and stream out to

produce type III radiation. If  $Y_D$  is the fraction of the electrons on the open field lines which are directed downward and if  $Y_R$  is the fraction of these electrons which are reflected, then  $n_X \approx Y_C + Y_D(1 - Y_R)$  and  $n_R \approx (1 - Y_C) [(1 - Y_D) + Y_D Y_R]$ .

The fraction  $Y_D$  depends on the acceleration process, and it is difficult to see why this fraction (or for that matter,  $Y_C$ ) would depend on the spectral index  $\delta$ . The only remaining factor is  $Y_R$  and, as we show below, this fraction does depend on, among other factors, the index  $\delta$ . One of these factors is the pitch angle distribution of the electrons. If electrons are strongly beamed, it would seem less likely that they could be turned around to reach the regions of low coronal density and there produce the type III radiation.

4) The ambient density variation and the variation of the magnetic field also play an important role. If the magnetic field converges rapidly with penetration toward the photosphere, the downward directed electrons could be reflected before they lose much of their energy. In addition, if the density at the acceleration region is high, the electrons will rapidly isotropize and lose their energy and few will be left streaming outward. As shown in Leach and Petrosian (1981) the parameter which enters here is a combined measure of those two effects and is the derivative of the logarithm of the magnetic field with respect to the column depth traversed,  $d \ln B / d \tau$ .  $\tau$  is the dimensionless column depth, measured from the point of injection (cf. figure 1).

5) Spectral index. The following simple argument illustrates the correlation between the spectral index of the electrons and the fraction  $Y_R$ , and that the sense of this

correlation is the same as that implied by the observations. Consider down-flowing electrons injected with a given spectral index  $\delta$ . These electrons, as they spiral along the field lines, interact with the ambient plasma and lose energy through coulomb collisions. Some of these electrons, either because they have undergone multiple collisions or because of the convergence of the magnetic field lines, will be reflected. The higher the energy of the electron the slower it loses energy by collisions and, therefore, the more likely it is to be adiabatically reflected by the magnetic field. Thus, on the average, the higher energy electrons are the more likely to be reflected and to retain a larger fraction of their energy in the process. Therefore, on the average, the reflected (i.e. escaping) electrons are those which started with higher energies. (We assume once electrons are injected into the flare plasma they are not subjected to further acceleration.) The flatter the electron spectrum (the smaller the value of  $\delta$ ) the greater is the fraction of the higher energy electrons and, consequently, the greater the value of  $Y_R$ , i.e. the greater the number of escaping electrons.

In an earlier paper (Leach and Petrosian 1981) we described a method for evaluating the characteristics of the accelerated electrons injected either into an open or into a closed magnetic field region. Using this method we have calculated the fraction of electrons with energies above 20 keV which escape (i.e., which are reflected and return to the injection point) for a given spectral and pitch angle distribution of electrons accelerated

into an open magnetic tube. As described in item 4 above, the only other parameter necessary for a complete description of the problem is  $d\ln B/d\tau$ , the rate of convergence of the magnetic field  $B$  with column depth.

In Table 1 we show our results from these calculations. The numbers represent the ratio  $Y_R$  or the fraction of downflowing electrons which have been reflected and have returned to  $\tau = 0$ . We give the rate of convergence of the magnetic field in terms of  $d\ln B/d\tau$  as explained above. The values given correspond to a factor of zero to a factor of 5 increase in the strength of the magnetic field between  $\tau = 0$  and the transition zone. The injected energy ( $E$ ) spectrum and pitch angle ( $\alpha$ ) distributions are given by  $F_0(E,\alpha) = KE^{-\delta} \exp - \alpha^2 / \alpha_0^2$ .

The important effect which we would like to point out is the rapid increase in the fraction  $Y_R$  with decreasing spectral index  $\delta$ , especially for highly beamed distributions ( $\alpha^2 \ll 1$ ) and for uniform magnetic fields ( $d\ln B/d\tau \ll 1$ ). If our assumptions that all the other factors which also affect the ratio  $Y_e$  and  $Y_d$  as described above are not correlated with  $\delta$ , then this gives a good explanation for the dependence of the distribution of  $Y = X/R$  on the spectral index.

In the next section we discuss some of the implications of these results.

#### IV. DISCUSSION AND SUMMARY

Using the analysis by Kane (1981) of x-ray and type III radio bursts as a guide, we have discussed the x-ray and type III burst intensity distributions. We have shown that the observed

correlations in these distributions rule out the two most simple distributions, namely, the distributions with perfect and with zero correlation between the x-ray and type III burst intensities.

We have also shown that the qualitative features of the observed distributions and correlations can be described by the simp\_e distribution function given by equation (8). The observed distribution of the x-ray intensities (X) gives the function  $\psi(X)$ . The ratio of the distribution (in X) of the x-ray bursts with detectable type III bursts to the distribution of all bursts, ( $f_{RO}(X)$ , cf. eq. 12) gives the integral of the function  $g(X/R)$ . We have also demonstrated how such distributions can be refined or tested using the distribution of a sample of type III burst intensities. More detailed data is necessary for the last steps. We intend to use a sample of x-ray bursts observed by the SMM HXRBS experiment for a more detailed description.

Finally, we have shown that the above distribution and the observed systematic variation of  $f_{RO}(X)$  with the x-ray spectral index  $\gamma$  imply a direct relationship between the ratio Y of x-ray to type III intensities and  $\gamma$ . The ratio Y (= X/R) is related to the ratio  $Y_e$  of the number of electrons which produce x-rays to the number of outstreaming ones which produce the type III radiation. Therefore, the correlation between Y and  $\gamma$  implies a correlation between the electron ratio  $Y_e$  and the electron spectral index  $\delta$ . We have discussed the various flare parameters which affect the ratio  $Y_e$ .

We find that the only situation where  $Y_e$  would vary with  $\delta$  is when electrons are injected into an open magnetic tube (with

an initial specified pitch angle distribution) in the downward direction. Table 1 shows that the variation of  $Y_e$  with  $\delta$  is strongest for highly beamed electrons in a nearly uniform magnetic flux tube. Increasing the rate of convergence of the magnetic field reduces this variation. This variation is reduced further as the pitch angle distribution of the injected electrons becomes broader. For an isotropic pitch angle distribution the variation will almost vanish. The ratio  $Y_e$  is also, to some extent, dependent upon the fraction  $Y_c$  of electrons accelerated along closed field lines. However, since there is no obvious relationship between  $Y_c$  and  $\delta$ , this and other similar factors will not alter the above conclusions.

At the present time there is not sufficient data to draw any quantitative conclusions with regard to this model. With more accurate x-ray and radio intensities measured for a larger sample of bursts, one could test the hypothesis set forth here by studying the correlation between X/R and  $\gamma$  directly. Our analysis appears to indicate that the electrons accelerated along open field lines must be strongly beamed in the downward direction and that the magnetic field cannot vary rapidly. The second of these conclusions agrees with high spatial resolution observations of microwave radiation during the impulsive phase of flares. However, these observations favor a more isotropic pitch angle distribution for the electrons injected in a closed magnetic loop (cf. Petrosian 1982). Obviously this dichotomy needs further study.

As stated earlier, the purpose of this paper has been to

demonstrate the usefulness of the statistical analysis. We hope that with a more detailed and larger sample, we can answer some of the questions raised by this preliminary analysis.

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## FIGURE CAPTIONS

Figure 1. The geometry of open and closed field lines. Electrons in a closed loop emit only x-rays. Those injected at  $\tau = 0$  on an open field line emit x-rays at lower, high density regions, but some are reflected back to  $\tau = 0$  and stream out to produce type III radiation.  $\tau$  is a dimensionless column depth.

Figure 2. Schematic distribution of x-ray or radio burst intensities  $N_o(X)$  or  $N_o(R)$ , and illustrations of three distributions of correlated bursts,  $N_{RO}(X)$  or  $N_{XO}(R)$ , and the fractions  $f_{RO}(X)$  or  $f_{XO}(R)$ . The solid line gives the trend of observations as described by Kane (1981). The dotted lines and dashed lines refer to the perfectly correlated and the completely uncorrelated distributions as discussed in C.1 and C.2 of Section II.

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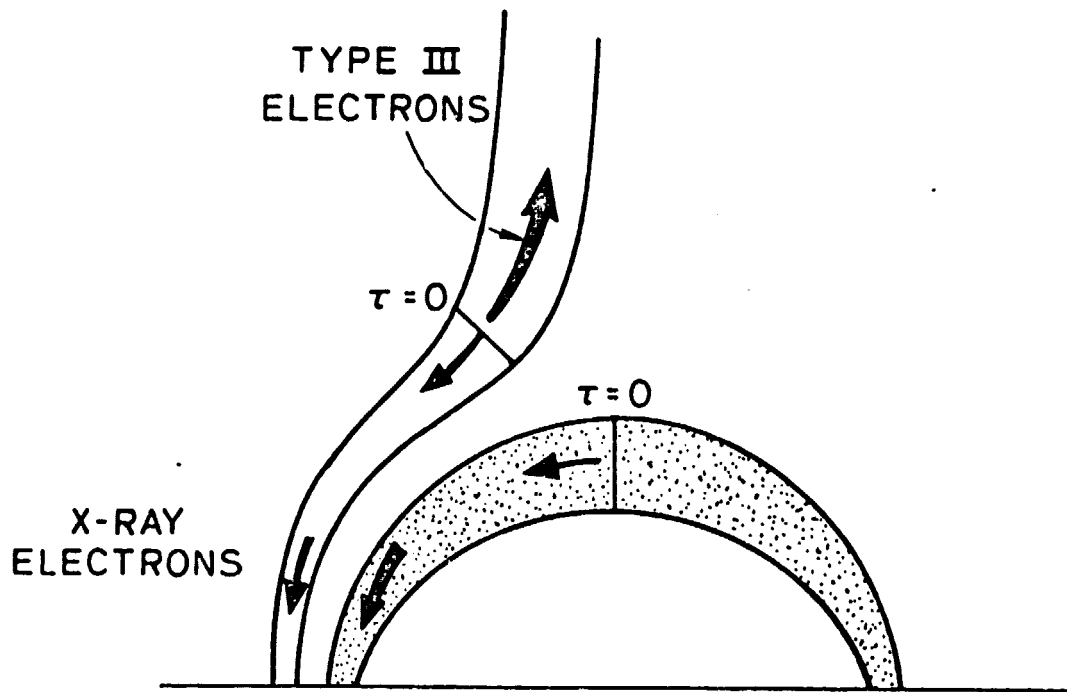


Figure 1

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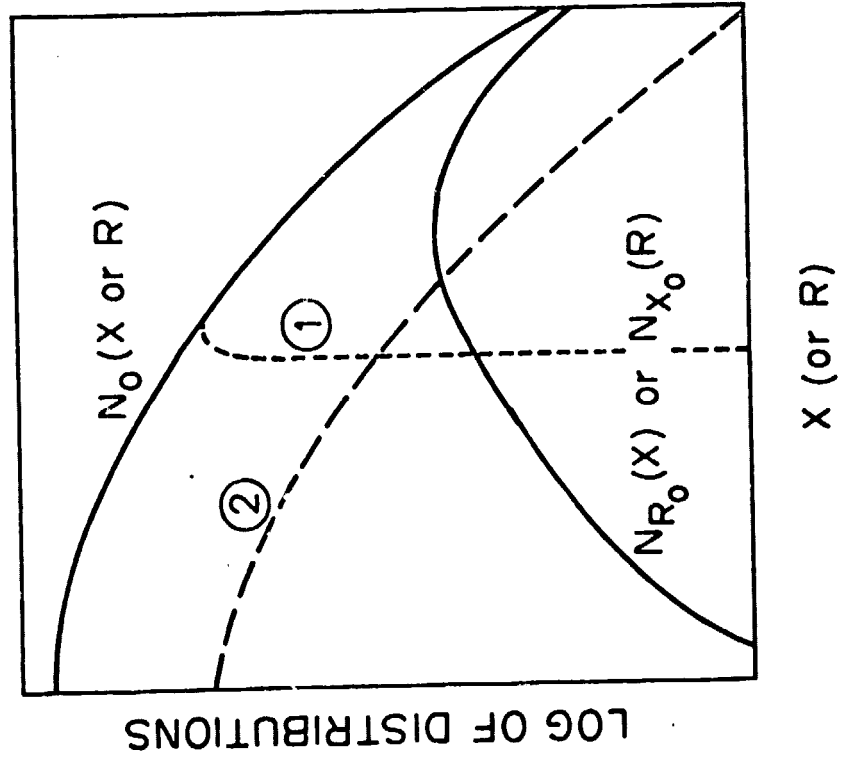
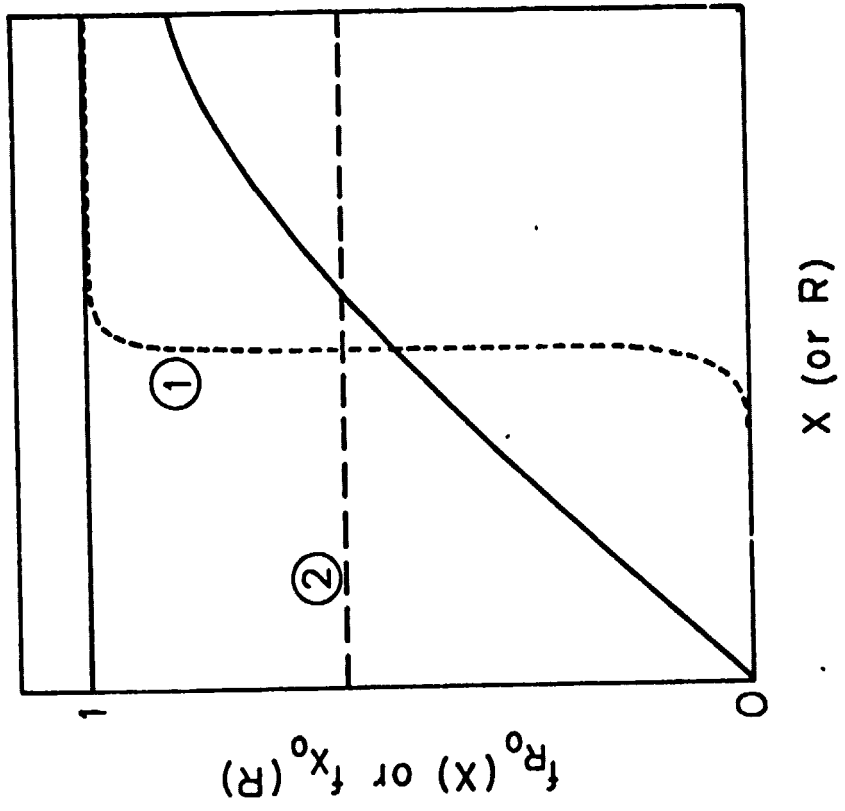


Figure 2

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TABLE I

Variation of Log  $Y_R$  with Various Parameters

$\delta$ \ $\frac{d \ln B}{d \tau}$ *	$\alpha_0^2 = 0.04$				0.25		$\infty^\dagger$	
	0	5	8	11.5	0	11.5	0	11.5
-2	-2.77			-.70				
-3	-3.02			-.90	-2.3	-.75	-1.20	-.5
-4	-3.26			-1.1				
-5	-3.55	-2.0	-1.62	-1.27	-2.73	-1.07	-1.40	-.7
-6	-3.8			-1.54				

\*  $d \ln B / d \tau$  in units of  $10^3$ . The four values correspond to an increase in the magnetic field between  $\tau = 0$  and the transition zone (which is situated at a column depth of  $N_{TZ} = 7 \times 10^{19} \text{ cm}^{-2}$ ) of factors of 1, 2, 3 and 5.

$^\dagger$  Uniform pitch angle distribution in the downward direction.