

MEASUREMENT OF THREE-DIMENSIONAL DENSITY
DISTRIBUTIONS BY HOLOGRAPHIC INTERFEROMETRY
AND COMPUTER TOMOGRAPHY

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BY HOLOGRAPHIC INTERFEROMETRY AND COMPUTER TOMOGRAPHY

Traditional Interferometric Techniques for Aerodynamics:

Mach-Zehnder interferometry

Mercury-arc light sources.

These permitted measurement of:

"2-D" boundary layers and shocks

Axisymmetric flows and shocks.

More recent developments include:

Lasers

Holographic Interferometry

Computer tomography.

These permit measurement of:

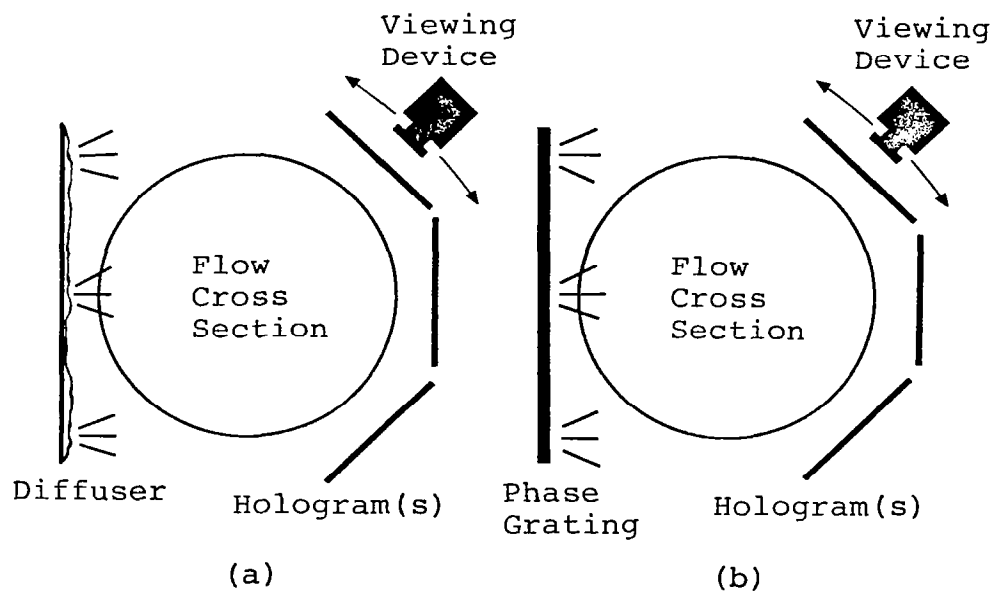
General 2-D and 3-D asymmetrical flows.

Holographic interferometry is described and reviewed by Dr. Trolinger in this workshop.

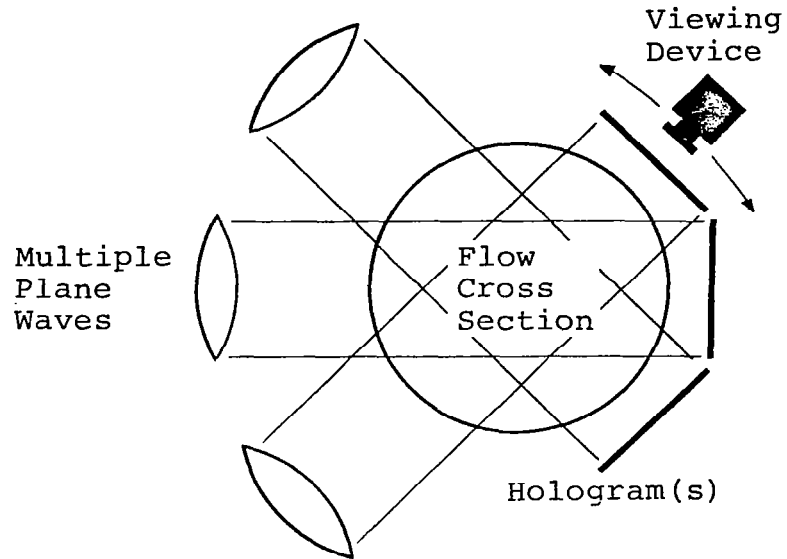
Advantages of the technique include:

- First-order correction for non-flat windows.
- Relative simplicity and ease of alignment.
- Permanent record of wavefront with some post-experiment processing possible.
- Operates readily with pulsed lasers for ~10ns time resolution.
- Multiple viewing directions available when diffused or multi beam illumination is used.

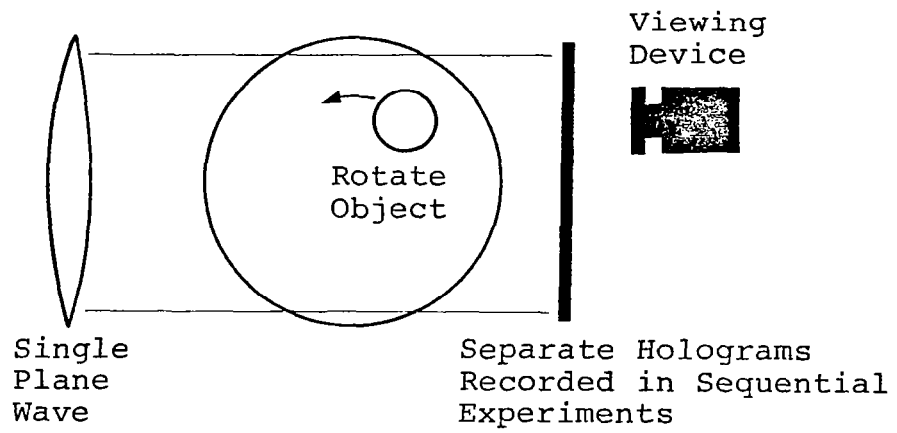
Collection of Multiple Views:



Collection of Multiple Views



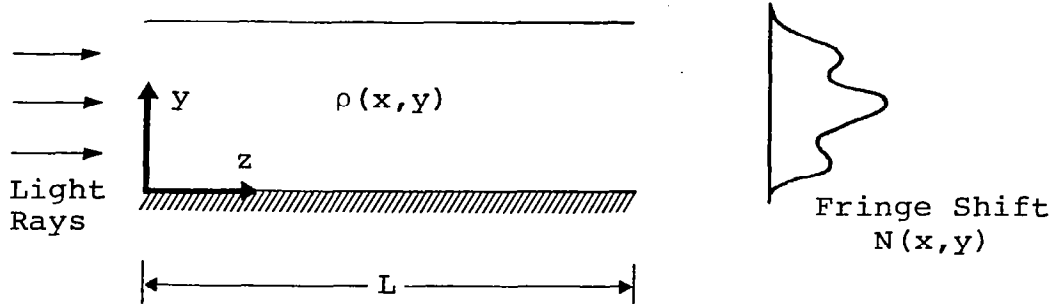
(c)



(d)

Interpretation of multiple-view interferograms requires computer tomography.

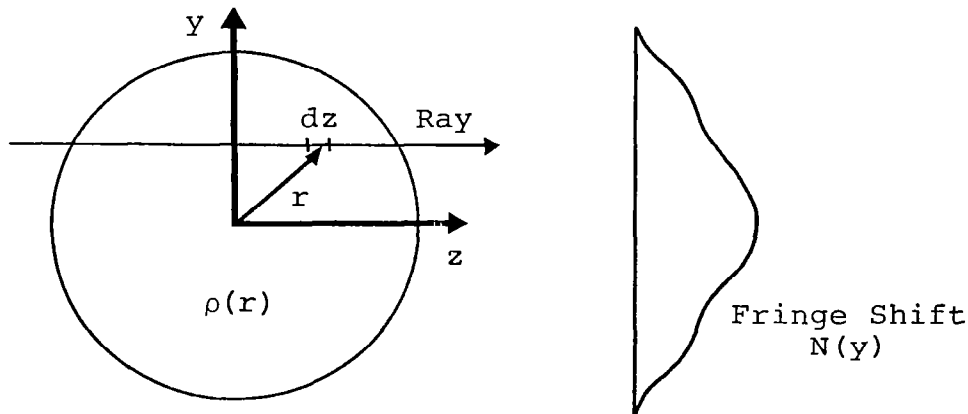
BACKGROUND: "2-D" Fields



$$\lambda N(x, y) = K \int_0^L [\rho(x, y) - \rho_0] dz$$

$$\rho(x, y) - \rho_0 = \frac{\lambda}{KL} N(x, y)$$

BACKGROUND: Axisymmetric Fields

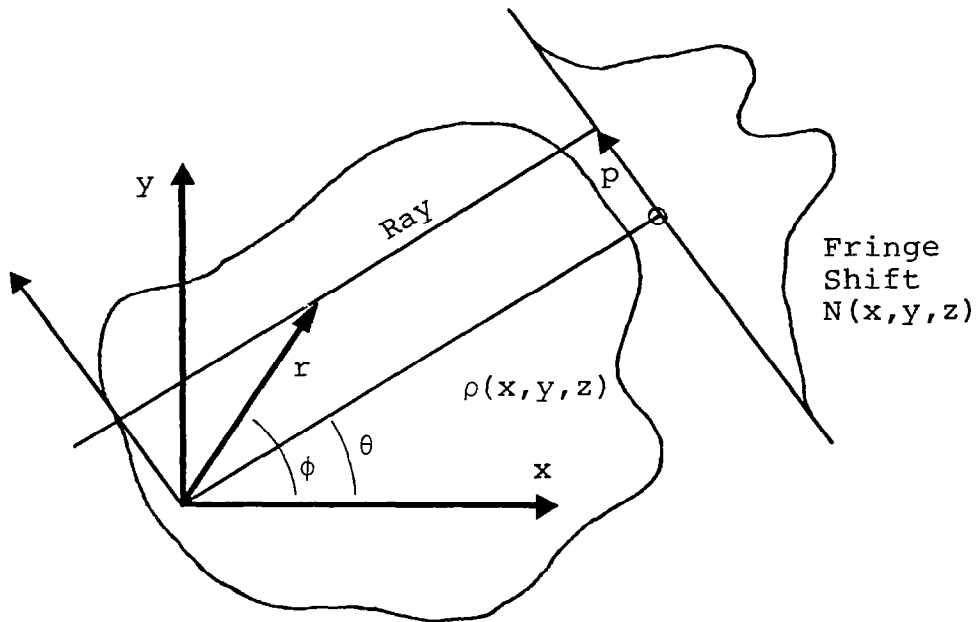


$$\lambda N(y) = 2K \int_y^{\infty} \frac{[\rho(r) - \rho_0] r dr}{(r^2 - y^2)^{1/2}}$$

$$\rho(r) - \rho_0 = -\frac{\lambda}{\pi K} \int_r^{\infty} \frac{(dN/dy) dy}{(y^2 - r^2)^{1/2}}$$

Computer tomography is an important technique for reconstructing 2-D or 3-D fields from measurements of line integrals through the field. It has been under development since the late 1960's. However, its origins are in the mathematical analysis of Radon (1917), work in radio astronomy by Bracewell (1956), and applications to medical imaging by Cormack (1964), and Hounsfield. It is discussed in this workshop in the context of holographic interferometry (Lee, Vest).

BASIC PROBLEM OF TOMOGRAPHY



Line-integral transform:

$$\lambda N(p, \theta) = K \iint [\rho(r, \phi) - \rho_0] \delta[p - r \sin(\phi - \theta)] dx \quad (1)$$

Inversion:

$$\rho(r, \phi) - \rho_0 = \frac{\lambda}{2\pi^2 K} \int_{-\pi/2}^{\pi/2} d\theta \int_{-\infty}^{\infty} \frac{(\delta N / \delta p)}{r \sin(\phi - \theta) - p} \quad (2)$$

The problem of tomography can be stated as a set of linear integral equations. There is an analytical inversion with which continuous and complete data recorded over a 180° range of viewing directions could be used to reconstruct $\rho(r,\phi)-\rho_0$. By doing this for several planes, $z=\text{constant}$, the entire 3-D field could be reconstructed.

In real experiments several difficulties are encountered:

1. Data are discrete and limited in number.
2. Data are recorded for a finite number of viewing directions.
3. Data are incomplete.
 - a. Range of viewing directions may be less than 180° .
 - b. Opaque objects such as test models may block out part of the data.
4. Data contain experimental error.
5. Refractive ray bending may occur.

Therefore, several computational techniques have been developed for computer tomography.

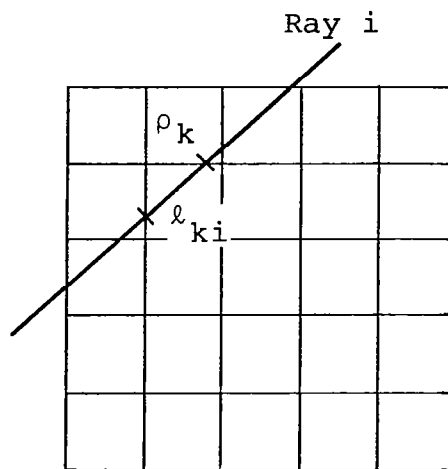
1. IMPLICIT METHODS: The right-hand side of Eq. (1) is approximated and the integral is replaced by a sum. This results in a set of algebraic equations which are solved for unknown coefficients or density values.

a. SERIES EXPANSIONS $\rho - \rho_0 = \sum a_{mn} f_{mn}(r, \phi)$
 where f_{mn} may be Fourier series
 Legendre Polynomial
 Bessel functions
 Sinc functions

This yields a set of equations to solve:

$$\sum A_{ij} f_j = \pi N_i$$

b. DISCRETE ELEMENT REPRESENTATIONS

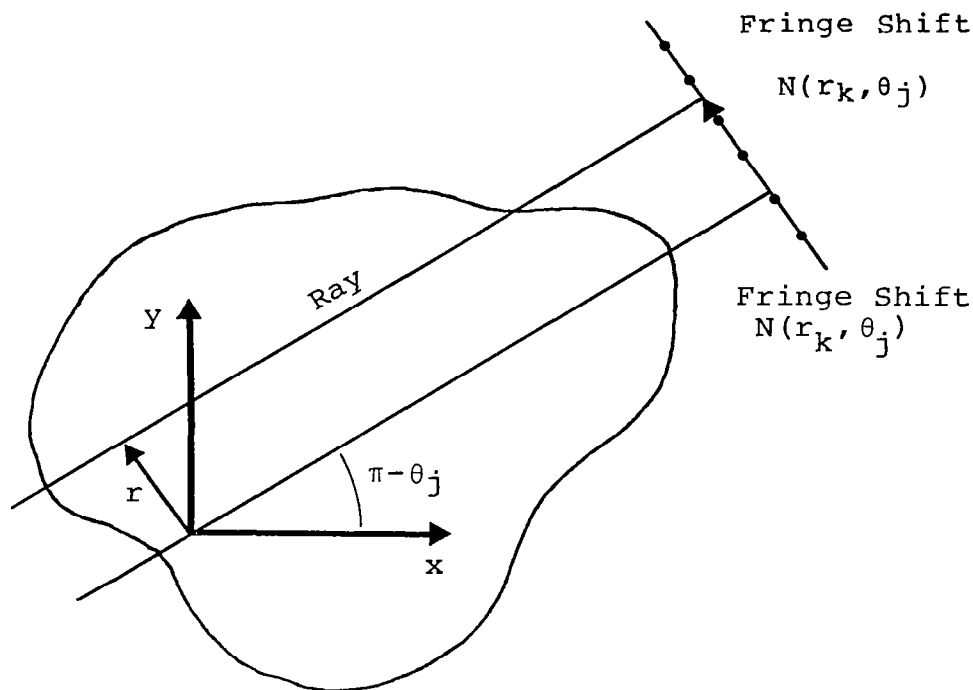


Basic Grid Method
 ART (relaxation)

$$\sum l_{ki} [\rho_k - \rho_0] = \frac{\lambda N_i}{K}$$

2. EXPLICIT METHODS: The right-hand side of Eq. (2) is approximated and the integral is replaced by a sum. This results in a direct representation of $\rho(r, \phi)$ which is obtained by operations entirely in the object domain. The most common method is:

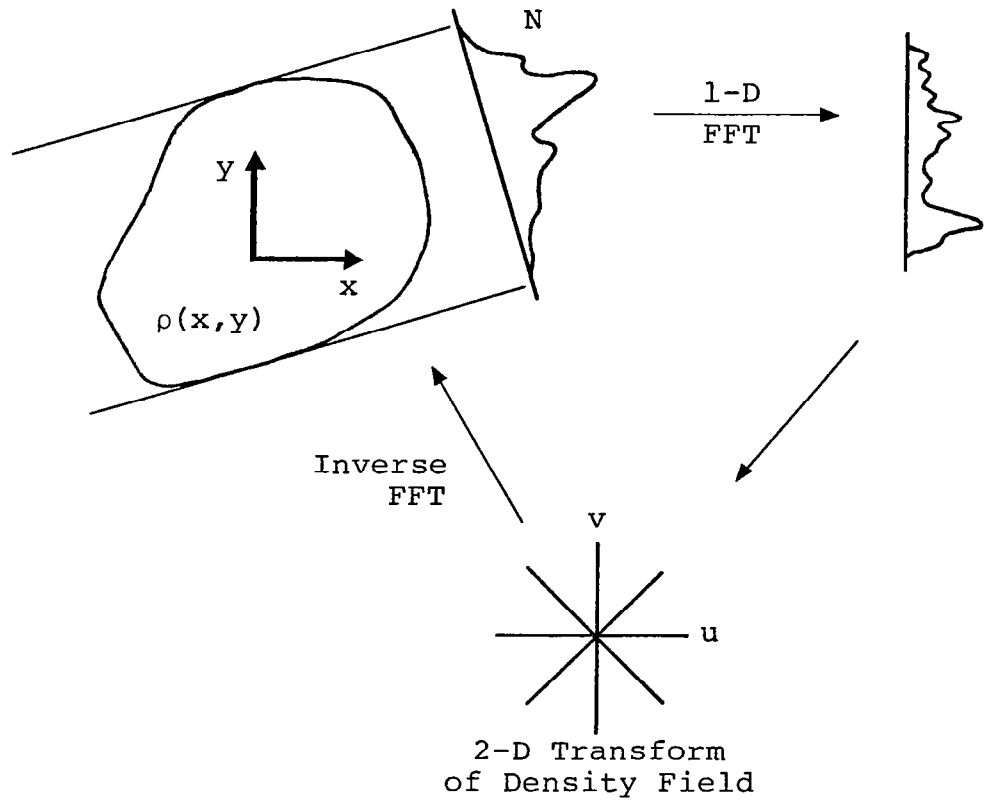
CONVOLUTION METHOD (FILTERED BACKPROJECTION)



$$\rho(x, y) - \rho_0 = \sum \sum N(r_k, \theta_j) w(x \cos \theta_j + y \sin \theta_j - p_k)$$

where w is a weight, or filter, applied to the fringe shift of each ray passing through the point in question.

3. **FOURIER TRANSFORM METHODS:** These are based on the Central Section Theorem of Bracewell which states that the 1-D Fourier transform of a projection, i.e. $N(p;\theta)$ for fixed θ , equals the 2-D Fourier transform along a corresponding radial line in the 2-D Fourier transform of the density field.



SELECTION OF TOMOGRAPHY ALGORITHM

There is no simple way to determine what computer tomography algorithm is "best". The behavior of algorithms is highly dependent on the structure of the density field, the amount and format of available data, the amount of "noise" in the data, and the nature of the desired information about the field.

A few characteristics of the two methods with which we have had the most success are:

1. CONVOLUTION METHODS

- a. Moderately simple to program.
- b. Modest storage requirements.
- c. Low execution time.
- d. Requires 180° range of viewing angles.
- e. Requires equally-spaced data in each view.

2. SERIES EXPANSION METHODS

- a. Can be moderately complex to program.
- b. Storage requirements can be moderately high.
- c. Low execution time if efficiently programmed.
- d. Can be used with limited range of viewing angles.
- e. Does not require equally-spaced data.

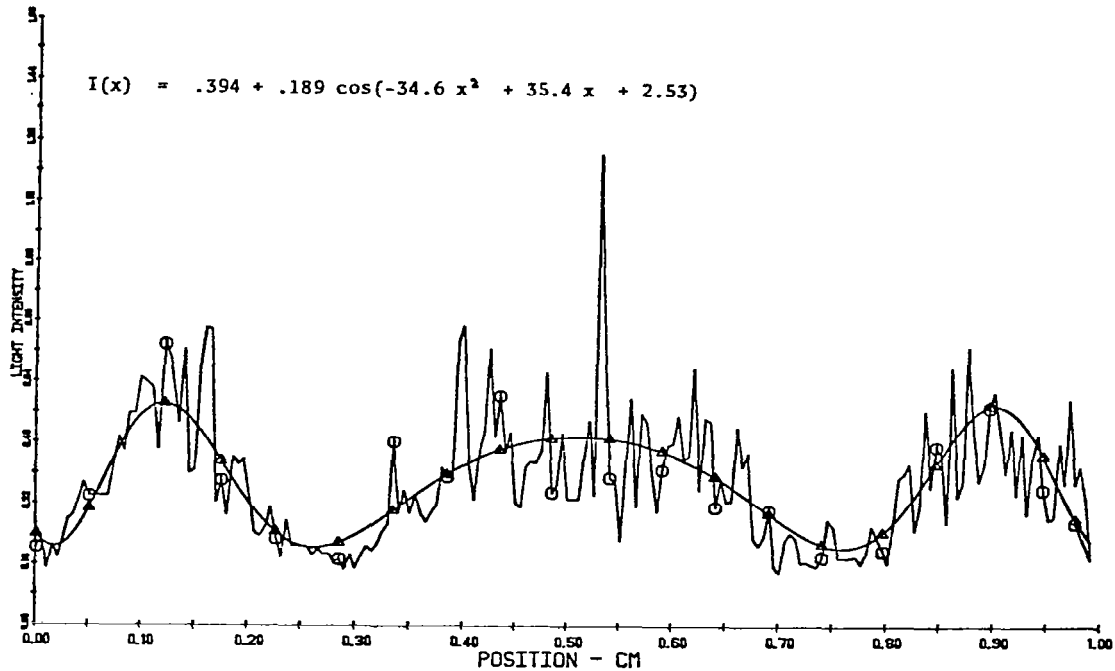
CURRENT AND FUTURE RESEARCH TOPICS

1. Development of automated fringe readout system which permits accurate interpolation.
 - a. Digital TV/Computer systems
 - b. Heterodyne systems
2. Development of optimum reconstruction procedures when an opaque test model is present in the field.
3. Interferometry and tomography with strongly refracting fields and shocks.
4. Gaining experience with well-instrumented experiments.

CURRENT RESEARCH AT THE UNIVERSITY OF MICHIGAN

1. Fringe readout system using a CCD array camera and LSI 11/23 computer. Noisy data are fitted to fringe functions with several degrees of freedom by nonlinear regression analysis. (In progress)

JBS990.DS11 NON-LINEAR PHASE VARIATION, HIGH NOISE

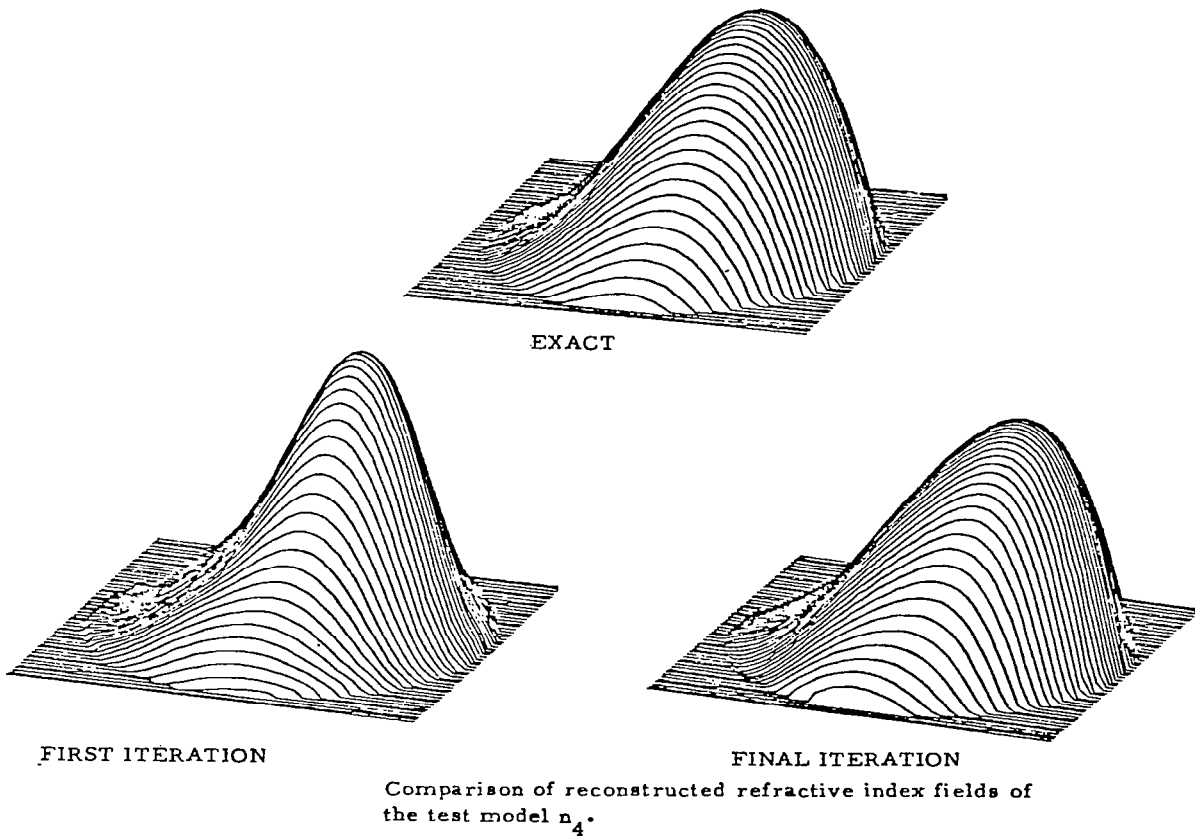


The Sampled Data with the curve that was fit to it for Data Set JBS990.DS11.

2. Reconstruction with an opaque test object in the field (in progress).
 - a. Development of a convolution algorithm which iterates between the line-integral-transform plane and the density field plane to correct for the missing data.
 - b. Development of series-expansion algorithms which meet all criteria for reconstruction of the density field outside a convex region surrounding the opaque object.

3. Interferometry of strongly-refracting density fields and shocks. (Initial study completed.)

Developed an algorithm which corrects reconstructions for refraction by iteratively using geometric optical ray tracing through estimates obtained by series-expansion tomographic reconstruction.



4. Well-instrumented experiment. (Proposed)

We hope to study the structure of a turbulent jet in a turbulent cross flow in an experiment scaled to model the mixing of dilutant jets in gas turbine engines.

