

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

NASA
Technical Memorandum 82887

AVRADCOM
Technical Report 82-C-10

(NASA-TM-82887) KINEMATIC PRECISION OF GEAR
TRAINS (NASA) 40 p HC P33/ME A01 CSCL 13I

82-32733

Unclas
G3/37 28913

Kinematic Precision of Gear Trains

F. L. Litvin and R. N. Goldrich
University of Illinois at Chicago Circle
Chicago, Illinois

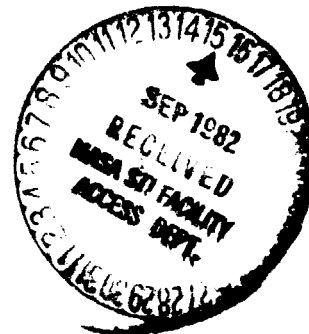
and

John J. Coy
Propulsion Laboratory
AVRADCOM Research and Technology Laboratories
Lewis Research Center
Cleveland, Ohio

and

E. V. Zaretsky
Lewis Research Center
Cleveland, Ohio

Prepared for the
Winter Annual Meeting of the
American Society of Mechanical Engineers
Phoenix, Arizona, November 15-19, 1982



NASA



KINEMATIC PRECISION OF GEAR TRAINS

by F. L. Litvin*, R. N. Goldrich†, J. J. Coy**, and E. V. Zaretsky††

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

ABSTRACT

Kinematic precision is affected by errors which are the result of either intentional adjustments or accidental defects in manufacturing and assembly of gear trains. This paper explains a general method for the determination of kinematic precision of gear trains. The general method is based on the exact kinematic relations for the contact point motions of the gear tooth surfaces under the influence of errors. An approximate method is also explained.

Example applications of the general and approximate methods are demonstrated for gear trains consisting of involute (spur and helical) gears, circular-arc (Wildhaber-Novikov) gears, and spiral-bevel gears. Gear noise measurements from a helicopter transmission are presented and discussed with relation to the kinematic precision theory.

* Professor of Mechanical Engineering, University of Illinois at Chicago Circle, Chicago, Illinois 60680; Member ASME.

† Research Assistant, University of Illinois at Chicago Circle, Chicago, Illinois 60680; Associate Member ASME.

** Propulsion Laboratory, AVRADCOM Research and Technology Laboratories, Lewis Research Center, Cleveland, Ohio 44135; Member ASME.

†† Lewis Research Center, Cleveland, Ohio 44135; Fellow ASME.

INTRODUCTION

Transmission error is a measure of the kinematic precision of gear trains. Transmission error is defined as the departure of the meshed gear pair (or entire gear train) from a constant ratio of angular motions as defined by the ratio of tooth numbers. It is true that in a gear pair each gear has a whole number of teeth and this defines the nominal ratio of angular positions between the two. But the instantaneous ratio during a meshing cycle can vary slightly from the nominal ratio. Transmission error is the measure of instantaneous variation from the ideal nominal value. The precision of gears was investigated by Litvin [1]*, Litvin and Gutman [2], and Michalec [3].

When the mating teeth in a gear train have profiles that transmit motion having no error, they are said to be conjugate pairs. In theory, it is possible to select an arbitrary shape for a driving tooth and then to find a profile for the driven tooth which will give conjugate action. Involute gear teeth happen to have the same form for driving and driven member teeth. A benefit of the involute form is that small errors in center distance between gears will not produce transmission errors. This is not true for other tooth profile forms [4, 5].

In general, there are many causes for transmission error, and they cannot be avoided in practice. Such things as shaft misalignment, profile error, tooth deflections under load, mounting location errors, and gear support deflections may combine to cause transmission error. The effects of transmission error are most often harmful. These are high vibration and noise, pitting and scoring of gear teeth, and reduced reliability of the gear train. Sometimes it is beneficial to introduce small intentional errors into the gear

*Numbers in square brackets denote references.

ORIGINAL PAGE IS
OF POOR QUALITY

tooth profile to compensate for the probable occurrence of accidental and unavoidable errors in the assembled and operating gear train. Tip relief to reduce dynamic loading and combined mismatch in spiral-bevel gears to reduce misalignment sensitivity are two examples of intentional errors which are beneficial.

Baxter has studied the effect of various types of misalignment on tooth contact in bevel and hypoid gears [6]. Townsend, Coy, and Hatvani have examined gear train noise as a test of its precision during an intentional loss of lubricant destruction test [7].

Of course, the effect of all errors (intentional or otherwise) on transmission error must be predicted analytically if the gear design process is to remain rational and not collapse into a confusing heap of empiricism.

Errors of manufacturing and assemblage of gears induce kinematic errors in gear-drives which may be presented by the following function:

$$\Delta\varphi_2(\varphi_1, \Delta Q) \quad (1)$$

Here φ_1 is the angle of rotation of the driving gear, 1,

$$\Delta Q = (\Delta q_1, \Delta q_2, \dots) \quad (2)$$

is the vector of errors, and

$$\Delta\varphi_2 = \varphi_2^0 - \varphi_2 \quad (3)$$

is the kinematic error of the gear drive, represented as the difference between the theoretical and actual angles of rotation of driven gear 2.

In this paper two methods to determine Function (1) are presented: (a) a numerical method for computer solution and (b) an approximate geometric solution which leads to simple, accurate results in an analytical form.

NOMENCLATURE

| | |
|---|---|
| A | shortest distance between gear axes of rotation |
| $\Delta \underline{e}_i$ | eccentricity vector of gear i |
| H_1, H_2 | axial settings of gears (see Fig. 2) |
| M_{ij} | angular velocity ratio, gear i, gear j |
| N_i | number of teeth on gear i |
| $\underline{n}_f^{(i)}$ | vector function representing unit normals of surface of gear i in fixed coordinate system (f) |
| $dn_{\underline{q}}^{(i)}$ | change in unit normal vector due to errors in gear i |
| $dn_{\underline{r}}^{(i)}$ | change in unit normal vector due to point motion <u>relative</u> to gear i |
| $dn_{\underline{tr}}^{(i)}$ | change in unit normal vector due to transfer point motion <u>with</u> gear i |
| $\dot{\underline{n}}_{\underline{abs}}^{(i)}$ | absolute velocity of tip of unit normal vector of surface i |
| $\dot{\underline{n}}_{\underline{r}}^{(i)}$ | similar to $dn_{\underline{r}}^{(i)}$ and $dn_{\underline{tr}}^{(i)}$ but velocities rather than displacements |
| $\dot{\underline{n}}_{\underline{tr}}^{(i)}$ | |
| O_i | center of rotation of gear i |
| $O^{(i)}$ | geometrical center of gear i |
| P | pitch point |
| $\Delta \underline{Q}_i$ | vector of errors |
| Δq_i | components of vector of errors |
| \underline{R} | vector from origin to axis of gear rotation |
| $r_b^{(i)}$ | base circle radius of gear i |
| $\underline{r}_f^{(i)}$ | vector function representing surface of gear i in fixed coordinate system (f) |

**ORIGINAL PAGE IS
OF POOR QUALITY**

| | |
|----------------------|--|
| $\dot{r}_f^{(i)}$ | velocity of contact point |
| S_i | coordinate system i |
| S_f | coordinate system rigidly connected to frame |
| $ds_{abs}^{(i)}$ | absolute displacement of contact point of gear i |
| $ds_q^{(i)}$ | displacement of contact point due to errors in gear i |
| $ds_r^{(i)}$ | displacement of contact point <u>relative</u> to gear i |
| $ds_{tr}^{(i)}$ | displacement of contact point in transfer motion <u>with</u> gear i |
| T | tangent plane |
| u_i | surface coordinate of gear i surface |
| $v_{abs}^{(i)}$ | similar to $ds_{abs}^{(i)}$, $ds_r^{(i)}$, $ds_{tr}^{(i)}$, but velocities rather than displacements |
| $v_r^{(i)}$ | |
| $v_{tr}^{(i)}$ | |
| β_i | angular position of eccentricity vector of gear i |
| β_{i0} | initial angular position of eccentricity vector of gear i |
| $\Delta\delta$ | rotation vector representing position change of gear axis of rotation |
| θ_i | surface coordinate of gear surface i |
| $\Delta\theta^{(i)}$ | change in kinematic error function as measured on shaft of gear i |
| λ | proportionality factor |
| ρ | radius vector |
| Σ_i | surface i |
| φ_1 | angle of rotation of gear 1 |
| φ_2 | actual angle of rotation of gear 2 |
| φ_2^0 | theoretical angle of rotation of gear 2 |

- $\Delta\varphi_2$ kinematic error function
 φ_0 pressure angle
 ω_i angular velocity of gear i

THEORY AND EXACT SOLUTION METHOD FOR KINEMATIC PRECISION

In the process of motion the tooth surfaces of two gears, Σ_1 and Σ_2 (Fig. 1), are in tangency if the following equations are satisfied:

$$\underline{r}_f^{(1)}(u_1, \theta_1, \varphi_1) = \underline{r}_f^{(2)}(u_2, \theta_2, \varphi_2) \quad (4)$$

$$\underline{n}_f^{(1)}(u_1, \theta_1, \varphi_1) = \underline{n}_f^{(2)}(u_2, \theta_2, \varphi_2) \quad (5)$$

Here $\underline{r}_f^{(i)}$ is the position vector of the contact point on gear i; $\underline{n}_f^{(i)}$ is the surface unit normal vector at the contact point M; u_i , and θ_i are the surface coordinates of the gear surfaces; and φ_i is the angle of rotation of gear i. Subscript f denotes a coordinate system which is rigidly connected to the frame.

For a gearset with kinematic errors, represented by ΔQ_1 and ΔQ_2 , conditions for tangency may be expressed as

$$\underline{r}_f^{(1)}(u_1, \theta_1, \varphi_1, \Delta Q_1) = \underline{r}_f^{(2)}(u_2, \theta_2, \varphi_2, \Delta Q_2) \quad (6)$$

$$\underline{n}_f^{(1)}(u_1, \theta_1, \varphi_1, \Delta Q_1) = \underline{n}_f^{(2)}(u_2, \theta_2, \varphi_2, \Delta Q_2) \quad (7)$$

Equations (6) and (7) yield the functions

$$\varphi_2(\varphi_1, \Delta Q_1, \Delta Q_2) = \varphi_2^0(\varphi_1) + \Delta\varphi_2(\varphi_1, \Delta Q_1, \Delta Q_2) \quad (8)$$

$$u_i(\varphi_1, \Delta Q_1, \Delta Q_2) ; \theta_i(\varphi_1, \Delta Q_1, \Delta Q_2) \quad (9)$$

**ORIGINAL PAGE IS
OF POOR QUALITY**

The functions

$$\underline{r}_f^{(i)}(u_i, \theta_i) ; u_i(\varphi_1, \Delta Q_1, \Delta Q_2), \theta_i(\varphi_1, \Delta Q_1, \Delta Q_2) \quad (i = 1, 2) \quad (10)$$

represent the path of the contact point on gear surface Σ_i corresponding to the meshing of gears with errors of manufacturing and assembly. Functions

$$\underline{r}_f^{(i)}(u_i^0(\varphi_1), \theta_i^0(\varphi_1)) ; u_i^0(\varphi_1), \theta_i^0(\varphi_1) \quad (i = 1, 2) \quad (11)$$

represent the path of the contact point on gear surface Σ_i corresponding to meshing without errors. Comparison of functions (10) and (11) yields the change of the contact point path induced by errors.

Consider the solution of equations (4) and (5) and (6) and (7). Vector equations (4) and (5) yield only five independent scalar equations since

$$|\underline{n}^{(1)}| = |\underline{n}^{(2)}| = 1. \quad \text{These equations may be presented as}$$

$$f_j(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2^0) = 0 \quad (j = 1, 2, \dots, 5) \quad (12)$$

It is assumed that

$$\{f_1, f_2, f_3, f_4, f_5\} \in C^1$$

The symbol C^1 denotes that functions f_j have continuous partial derivatives of the first order (at least) by all arguments.

It is assumed that equation system (12) is satisfied by a set of parameters

$$P^{(1)} = (u_1^{(1)}, \theta_1^{(1)}, \varphi_1^{(1)}, u_2^{(1)}, \theta_2^{(1)}, \varphi_2^{(1)}) \quad (13)$$

and that surfaces Σ_1 and Σ_2 are in tangency at point M_0 . Surfaces Σ_1 and Σ_2 will be in point contact in the neighborhood of M_0 if by the set of parameters $P^{(1)}$ the following Jacobian differs from zero:

$$\frac{D(f_1, f_2, f_3, f_4, f_5)}{D(u_1, \theta_1, u_2, \theta_2, \varphi_2)} = \begin{vmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \varphi_2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_5}{\partial u_1} & \frac{\partial f_5}{\partial \theta_1} & \frac{\partial f_5}{\partial u_2} & \frac{\partial f_5}{\partial \theta_2} & \frac{\partial f_5}{\partial \varphi_2} \end{vmatrix} \neq 0 \quad (14)$$

If inequality (14) is satisfied, equation (12) may be solved in the neighborhood of $p^{(1)}$ with the functions

$$\{u_1(\varphi_1), \theta_1(\varphi_1), u_2(\varphi_1), \theta_2(\varphi_1), \varphi_2^0(\varphi_1)\} \in C^1 \quad (15)$$

The function $\varphi_2^0(\varphi_1)$ represents the ideal law of motion. In most cases (for conjugate tooth action) function $\varphi_2^0(\varphi_1)$ is linear.

Equations (6) and (7) also yield a system of five independent equations in six unknowns $(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2)$

$$g_j(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \quad (j = 1, 2, \dots, 5) \quad (16)$$

It is assumed that this system is satisfied by a set of parameters

$$p^{(2)} = (u_1^{(2)}, \theta_1^{(2)}, \varphi_1^{(1)}, u_2^{(2)}, \theta_2^{(2)}, \varphi_2^{(2)}) \quad (17)$$

with the same value of $\varphi_1^{(1)}$ as in the set $p^{(1)}$. If in the neighborhood of $p^{(2)}$ the Jacobian

$$\frac{D(g_1, g_2, g_3, g_4, g_5)}{D(u_1, \theta_1, u_2, \theta_2, \varphi_2)} \neq 0 \quad (18)$$

then system (16) may be solved with the functions

$$\{u_1(\varphi_1, \Delta Q), \theta_1(\varphi_1, \Delta Q), u_2(\varphi_1, \Delta Q), \theta_2(\varphi_1, \Delta Q), \varphi_2(\varphi_1, \Delta Q)\} \in C^1 \quad (19)$$

**ORIGINAL PAGE IS
OF POOR QUALITY**

Function $\varphi_2(\varphi_1, \Delta Q)$ represents the actual law of motion transformation - the law of transformation of motion which corresponds to errors of manufacturing and assembly. Kinematic errors of the gear drive are represented by the function

$$\Delta\varphi_2 = \varphi_2(\varphi_1, \Delta Q) - \varphi_2^0(\varphi_1) \quad (20)$$

This method of solution can provide, not only the kinematic errors of a gear-set, but also the new path of the contact point (see functions (10)).

In general, the numerical solution of a system of five nonlinear equations is a difficult problem which requires many iterations. To save computer time an effective method of solution was recently proposed by Litvin and Gutman [2]. The principle of this method follows:

The system of equation (16) may be represented as

$$f_1(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, A, H_1, H_2, \Delta Q) = 0 \quad (21)$$

$$f_2(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, A, H_1, H_2, \Delta Q) = 0 \quad (22)$$

$$f_3(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, A, H_1, H_2, \Delta Q) = 0 \quad (23)$$

$$f_4(u_1, \theta_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \quad (24)$$

$$f_5(u_1, \theta_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \quad (25)$$

Equations (21) to (23) are determined from vector equation (6), and equations (24) to (25) from vector equation (7). Here, A represents the shortest distance between the axes of rotation of the two gears and H_1 and H_2 represent the axial settings of the gears (Fig. 2).

ORIGINAL PAGE IS
OF POOR QUALITY

Systems $S_1(x_1, y_1, z_1)$ and $S_2(x_2, y_2, z_2)$ shown in figure 2 are rigidly connected to the driving and driven gears, respectively. Now suppose that some points $M_1(u_1, \theta_1)$ and $M_2(u_2, \theta_2)$ on surfaces Σ_1 and Σ_2 are chosen. With a set of known parameters $(u_1, \theta_1, u_2, \theta_2)$, equations (24) and (25) become a system of two equations in two unknowns which may be expressed as

$$F_1(\varphi_1, \varphi_2) = 0 \quad (26)$$

$$F_2(\varphi_1, \varphi_2) = 0 \quad (27)$$

Upon solving for φ_1 and φ_2 , one checks that the following equations are satisfied:

$$A - K_1(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \quad (28)$$

$$H_1 - K_2(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \quad (29)$$

$$H_2 - K_3(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \quad (30)$$

where A , H_1 , H_2 , and ΔQ are given values.

In general, the solution of the above two systems of equations ((26) and (27) and (28) and (30)) requires an iterative procedure. In practice, one of the four variable parameters $(u_1, \theta_1, u_2, \theta_2)$ is fixed, and the other three are changed such that the two equation systems are satisfied.

The advantage of the above method lies in the ability to divide the system of five equations ((25) to (30)) into two subsystems of two and three equations, and to solve them separately.

This method was applied to investigate the sensitivity of Wildhaber-Novikov gears to errors of center distance mounting [1]. These gears are generated by two rack cutters which have normal sections as shown in Fig. 3.

**ORIGINAL PAGE IS
OF POOR QUALITY**

Surfaces of the rack cutters are in tangency along a straight line $M - M'$ which is parallel to axis Z_a and passes through point M of the normal section. In the normal section, the shape of each rack cutter is a circular arc of radius ρ_i ($i = I, II$). The location of point M is defined by the parameter $\theta = \theta_0$ (Fig. 3). The line $M - M'$ generates a helix on the gear tooth surface which is the path of contact points. Although the procedure described above is primarily for numerical solution, in this case analytical results were obtained.

The investigation showed that the error of center distance ΔA resulted in the change of location of the contact point path. The new location of the path is represented by the equation

$$\sin \theta = \frac{\Delta A - b_{II}}{\rho_I - \rho_{II}}$$

$$R_1 = \sqrt{(\rho_I \sin \theta + r_1)^2 + (\rho_I \cos \theta \sin \lambda)^2} \quad (31)$$

Where R_1 is the radius of gear 1 cylinder on which lies the helix of the new contact point path. Parameters θ , ρ_I , ρ_{II} , and b_{II} are shown in Fig. 3; r_1 is the pitch cylinder radius.

With $\Delta A = 0$,

$$\sin \theta_0 = \frac{b_{II}}{\rho_{II} - \rho_I} \quad (32)$$

where θ_0 is the parameter corresponding to the desired location of the contact point path.

KINEMATIC RELATIONS BETWEEN PARAMETERS OF CONTACT POINT MOTIONS

The following relations are the basis of the second method for the determination of kinematic errors in gear trains. As stated above, the tangency of gear tooth surfaces is represented by equations (4) and (5). Because of the continuity of tangency of these surfaces, it is required that

$$\dot{\underline{r}}_f^{(1)}(u_1, \theta_1, \varphi_1) = \dot{\underline{r}}_f^{(2)}(u_2, \theta_2, \varphi_2) \quad (33)$$

$$\dot{\underline{n}}_f^{(1)}(u_1, \theta_1, \varphi_1) = \dot{\underline{n}}_f^{(2)}(u_2, \theta_2, \varphi_2) \quad (34)$$

Here $\dot{\underline{r}}_f^{(i)}$ ($i = 1, 2$) is the velocity of the surface contact point in absolute motion (with respect to the frame); $\dot{\underline{n}}_f^{(i)}$ is the velocity of the tip of the surface unit normal in absolute motion (also with respect to the frame). Henceforth, $\dot{\underline{r}}_f^{(i)}$ is designated as $\underline{v}_{abs}^{(i)}$ and $\dot{\underline{n}}_f^{(i)}$ as $\underline{n}_{abs}^{(i)}$.

The velocity of absolute motion may be represented as the sum of two components: (a) the velocity of transfer motion (together with the surface) and (b) the velocity of relative motion (with respect to the surface).

Consequently,

$$\underline{v}_{abs}^{(1)} = \underline{v}_{tr}^{(1)} + \underline{v}_r^{(1)}, \quad \underline{v}_{abs}^{(2)} = \underline{v}_{tr}^{(2)} + \underline{v}_r^{(2)} \quad (35)$$

$$\underline{n}_{abs}^{(1)} = \underline{n}_{tr}^{(1)} + \underline{n}_r^{(1)}, \quad \underline{n}_{abs}^{(2)} = \underline{n}_{tr}^{(2)} + \underline{n}_r^{(2)} \quad (36)$$

For a surface represented by a vector-function

$$\underline{r}^{(i)}(u_i, \theta_i, \varphi_i) \quad (i = 1, 2) \quad (37)$$

and a surface unit normal

$$\underline{\underline{n}}^{(i)}(u_i, \theta_i, \varphi_i) \quad (i = 1, 2) \quad (38)$$

the following comes from definitions (a) and (b) above:

$$\underline{\underline{v}}_{tr}^{(i)} = \frac{\partial \underline{\underline{r}}^{(i)}}{\partial \varphi_i} \frac{d\varphi_i}{dt}, \quad \underline{\underline{v}}_r^{(i)} = \frac{\partial \underline{\underline{r}}^{(i)}}{\partial u_i} \frac{du_i}{dt} + \frac{\partial \underline{\underline{r}}^{(i)}}{\partial \theta_i} \frac{d\theta_i}{dt} \quad (39)$$

$$\dot{\underline{\underline{n}}}_{tr}^{(i)} = \frac{\partial \underline{\underline{n}}^{(i)}}{\partial \varphi_i} \frac{d\varphi_i}{dt}, \quad \dot{\underline{\underline{n}}}_r^{(i)} = \frac{\partial \underline{\underline{n}}^{(i)}}{\partial u_i} \frac{du_i}{dt} + \frac{\partial \underline{\underline{n}}^{(i)}}{\partial \theta_i} \frac{d\theta_i}{dt} \quad (40)$$

Transfer velocity may also be determined in a kinematical way by supposing that a gear with surface Σ_i rotates about an axis that does not pass through the origin O_i of coordinate system S_f , which is rigidly connected to the frame. Vector $\underline{\underline{\omega}}^{(i)}$ is the vector of angular velocity of the gear's rotation. Then,

$$\underline{\underline{v}}_{tr}^{(i)} = \underline{\underline{\omega}}^{(i)} \times \underline{\underline{r}}^{(i)} + \underline{\underline{R}}^{(i)} \times \underline{\underline{\omega}}^{(i)} \quad (41)$$

where $\underline{\underline{r}}^{(i)}$ is the position vector drawn from origin O_i to the contact point on the tooth surface and $\underline{\underline{R}}^{(i)}$ is a vector drawn from O_i to an arbitrary point on the gear's axis of rotation.

The transfer velocity $\dot{\underline{\underline{n}}}_{tr}$ is represented by the equation

$$\dot{\underline{\underline{n}}}_{tr}^{(i)} = \underline{\underline{\omega}}^{(i)} \times \underline{\underline{n}}^{(i)} \quad (42)$$

Equations (33) to (36) yield the following kinematic relations for two tooth surfaces which are in continuous tangency:

$$\underline{\underline{v}}_{tr}^{(1)} + \underline{\underline{v}}_r^{(1)} = \underline{\underline{v}}_{tr}^{(2)} + \underline{\underline{v}}_r^{(2)} \quad (43)$$

$$\dot{\underline{n}}_{tr}^{(1)} + \dot{\underline{n}}_r^{(1)} = \dot{\underline{n}}_{tr}^{(2)} + \dot{\underline{n}}_r^{(2)} \quad (44)$$

Equations (43) and (44) were first proposed by Litvin [1]. On the basis of these equations, important problems in the theory of gearing were solved, such as avoiding tooth undercutting, deriving the relations between curvatures of two gear tooth surfaces in mesh, and determining the kinematic errors of gear drives which are caused by errors of manufacturing and assembly.

APPROXIMATE METHOD OF CALCULATION OF GEAR DRIVE KINEMATIC ERRORS

As a general rule, kinematic errors of a gear drive determined by the exact method must be obtained in a numerical way using a computer. This is a disadvantage of the exact method. Therefore, an approximate method with the opportunity to obtain accurate results analytically is now presented.

Figure 4 shows two gear surfaces Σ_1 and Σ_2 which are not in tangency due to errors of manufacturing and assembly. Points $M^{(1)}$ and $M^{(2)}$ do not coincide, position vectors $\underline{r}_f^{(1)}$ and $\underline{r}_f^{(2)}$ are not equal, and surface unit normal vectors $\underline{n}_f^{(1)}$ and $\underline{n}_f^{(2)}$ do not coincide. To bring the two surfaces into contact it is sufficient to hold one gear fixed and rotate the other gear by an additional small angle. Since the gear with surface Σ_1 is the driving gear, it is preferable to fix the position of surface Σ_1 and rotate surface Σ_2 to bring it back into contact with Σ_1 . The additional angle of rotation $\Delta\varphi_2$ represents the change of the theoretical angle of rotation φ_2^0 which is exerted by errors of the manufacturing of manufacturing and assembly. The $\Delta\varphi_2$ is as yet an unknown function of the vector of errors $\underline{\Delta Q}$ and varies in the process of motion. Thus

$$\Delta\varphi_2 = f(\varphi_1, \underline{\Delta Q}) \quad (45)$$

**ORIGINAL PAGE IS
OF POOR QUALITY**

The determination of the Function (45) is based on the following kinematic relations, which are analogous to (43) and (44):

$$ds_{\sim tr}^{(1)} + ds_{\sim r}^{(1)} + ds_{\sim q}^{(1)} = ds_{\sim tr}^{(2)} + ds_{\sim r}^{(2)} + ds_{\sim q}^{(2)} \quad (46)$$

$$dn_{\sim tr}^{(1)} + dn_{\sim r}^{(1)} + dn_{\sim q}^{(1)} = dn_{\sim tr}^{(2)} + dn_{\sim r}^{(2)} + dn_{\sim q}^{(2)} \quad (47)$$

where $ds_{\sim q}^{(i)}$ is the displacement of contact point and $dn_{\sim q}^{(i)}$ ($i = 1, 2$) is the change in direction of the surface unit normal due to errors of manufacturing and assembly. To bring the surfaces into contact, it is sufficient to rotate only gear 2, holding gear 1 at rest. Therefore, $ds_{\sim tr}^{(1)}$, and $dn_{\sim tr}^{(1)}$ are zero, and

$$ds_{\sim r}^{(1)} + ds_{\sim q}^{(1)} = ds_{\sim tr}^{(2)} + ds_{\sim r}^{(2)} + ds_{\sim q}^{(2)} \quad (48)$$

$$dn_{\sim r}^{(1)} + dn_{\sim q}^{(1)} = dn_{\sim tr}^{(2)} + dn_{\sim r}^{(2)} + dn_{\sim q}^{(2)} \quad (49)$$

To determine relations between $ds_{\sim tr}^{(2)}$, $ds_{\sim q}^{(1)}$, and $ds_{\sim q}^{(2)}$ take the following scalar products:

$$\underline{n} \cdot (ds_{\sim r}^{(1)} + ds_{\sim q}^{(1)}) = \underline{n} \cdot (ds_{\sim tr}^{(2)} + ds_{\sim r}^{(2)} + ds_{\sim q}^{(2)}) \quad (50)$$

Since vectors $ds_{\sim r}^{(1)}$ and $ds_{\sim r}^{(2)}$ must lie the common tangent plane T, equation (50) is reduced to the following:

$$\underline{n} \cdot ds_{\sim tr}^{(2)} = \underline{n} \cdot (ds_{\sim q}^{(1)} - ds_{\sim q}^{(2)}) \quad (51)$$

The vector $ds_{\sim tr}^{(2)}$ may be represented by the following cross product:

$$d_{tr}^{(2)} = d_{\phi}^{(2)} \times r_2^{(M)} \quad (52)$$

where $d_{\phi}^{(2)}$ is the incremental angle of rotation of gear 2 and $r_2^{(M)}$ is the position vector drawn from an arbitrary point of the axis of rotation to the contact point M.

Equations (51) and (52) yield

$$\left[d_{\phi}^{(2)} r_2^{(M)} \right]_{\underline{n}} = \left[ds_q^{(1)} - ds_q^{(2)} \right]_{\underline{n}} \quad (53)$$

Equation (53) is the basic equation for the determination of kinematic errors of gear drives. Its application will be demonstrated in the following sections.

Analogous scalar products can be composed on the basis of equation (49). It can be proven that these scalar products are zero because the vectors in equation (49) all belong to the tangent plane. Hereinafter, the following notations will apply:

$$\Sigma \Delta q_i^{(1)} \equiv \Delta s_q^{(1)}, \quad \Sigma \Delta q_j^{(2)} \equiv \Delta s_q^{(2)}, \quad (54)$$

where $\Sigma \Delta q_i^{(1)}$ and $\Sigma \Delta q_j^{(2)}$ represent the sum of linear-error vectors due to manufacture and assembly of gears 1 and 2, respectively.

In many cases, however, errors in gear trains do not result from linear displacements, but rather from angular displacements. For instance, kinematic errors may result from the misalignment of gear shafts.

Figure 5 shows the axis of gear 2 rotation a-a in its ideal position. Suppose that, due to an error of assembly, axis a-a is rotated about a nonintersecting axis B-B. Such an error of assembly may be represented by the vector $\Delta \delta$, which is directed along axis B-B, where the direction of $\Delta \delta$ corresponds to the direction of rotation by the right-hand rule.

With the given vector $\underline{\Delta\delta}$ the displacement $\underline{\Delta q}^{(2)}$ of contact point M may be determined as follows:

(a) Vector $\underline{\Delta\delta}$, directed along the axis B-B, is replaced by an equal vector $\underline{\Delta\delta}$, which passes through the origin O_2 and the vector-moment $\underline{R} \times \underline{\Delta\delta}$. Here \underline{R} is a position vector drawn from O_2 to an arbitrary point on the line of action of vector $\underline{\Delta\delta}$ (Fig. 5).

(b) The displacement $\underline{\Delta q}^{(2)}$ corresponding to $\underline{\Delta\delta}$ may be represented by

$$\underline{\Delta q}^{(2)} = \underline{\Delta\delta} \times \underline{r}_2^{(M)} + \underline{R} \times \underline{\Delta\delta} = \underline{\Delta\delta} \times (\underline{r}_2^{(M)} - \underline{R}) \quad (55)$$

A similar equation may be developed to determine the displacement of the contact point M exerted by an angular error corresponding to gear 1.

With notations (54) the equation (53) for the determination of kinematic errors may be represented as follows:

$$(\underline{\Delta\phi}^{(2)} \times \underline{r}_2^{(M)} + \Sigma \underline{\Delta q}) \cdot \underline{n}^{(M)} = 0 \quad (56)$$

where $\Sigma \underline{\Delta q} = \Sigma \underline{\Delta q}_j^{(2)} - \Sigma \underline{\Delta q}_i^{(1)}$ and $\underline{n}^{(M)}$ is the surface unit normal at the contact point M.

The location of the contact point M and the direction of the unit normal $\underline{n}^{(M)}$ change in the process of motion. A further simplification of equation (56) results by assuming that in all positions the contacting tooth surfaces have a common normal which passes through the contact point M and the pitch point P. This is the fundamental law for uniform motion transmission. For planar gears the pitch point P coincides with the point of tangency of the pitch circles (gear centrodes). The pitch point for bevel gears is located on the line of tangency of pitch cones. In both cases the surface unit normal \underline{n}

is collinear to \overline{PM} and

$$\underline{r}_2^{(M)} = \underline{r}_2^{(P)} + \overline{PM} = \underline{r}_2^{(P)} + \lambda \underline{n}^{(M)} \quad (57)$$

Equations (56) and (57) yield

$$(\underline{\Delta\varphi}^{(2)} \times \underline{r}_2^{(P)} + \Sigma \Delta q) \cdot \underline{n}^{(M)} = 0 \quad (58)$$

because

$$\begin{aligned} (\underline{\Delta\varphi}^{(2)} \times \underline{r}_2^{(M)}) \cdot \underline{n}^{(M)} &= \left[\underline{\Delta\varphi}^{(2)} \times (\underline{r}_2^{(P)} + \lambda \underline{n}^{(M)}) \right] \cdot \underline{n}^{(M)} \\ &= (\underline{\Delta\varphi}^{(2)} \times \underline{r}_2^{(P)}) \cdot \underline{n}^{(M)} + \left[\underline{\Delta\varphi}^{(2)} \lambda \underline{n}^{(M)} \cdot \underline{n}^{(M)} \right] \\ &= (\underline{\Delta\varphi}^{(2)} \times \underline{r}_2^{(P)}) \cdot \underline{n}^{(M)} \end{aligned} \quad (59)$$

Application of equation (58) in place of equation (56) has the advantage that the location of the pitch point may be considered as a constant ($\underline{r}_2^{(P)} = \text{const}$). However, the direction of the surface unit normal is a function of φ_1 . Three types of gears - involute (spur and helical) and Wildhaber-Novikov - are exceptions to this statement. For these gears the unit normal of the gear surfaces at their contact point does not change its direction.

Because of kinematic errors, the angular velocity ratio fluctuates as the gear teeth pass through mesh. Figure 6 shows functions for two types of kinematic errors. The first is a piecewise, nonlinear, periodic function which has a period that depends on the ratio

$$m_{12} = \frac{N_2}{N_1} = \frac{b}{a} \quad (60)$$

where N_i ($i = 1, 2$) is the number of gear teeth and b and a are the

**ORIGINAL PAGE IS
OF POOR QUALITY**

minimum integral numbers with which the ratio m_{12} can be expressed. The angle of rotation of gear 1 corresponding to the period of function $\Delta\varphi^{(2)}(\varphi_1)$ is equal to $2\pi a$.

Such functions of kinematic errors are caused by (a) the eccentricity of gears and (b) the crossing of the theoretical axis with the axis of rotation of gears. (The shortest distance between these axes rotates in the process of motion.)

The second type of function $\Delta b^{(2)}(b_1)$, shown in Fig. 6(b), has a period of $b_1 = \frac{2\pi}{N_1}$. This function is exerted by (a) errors in the generating process of gear teeth and (b) errors of gear axis location which do not change in the process of meshing, etc.

APPLICATION OF THEORY TO ECCENTRICITY OF INVOLUTE SPUR GEARS

Figure 7 shows base circles of radii $r_b^{(1)}$ and $r_b^{(2)}$ for two involute spur gears. The rotation centers of the gears are denoted $O^{(1)}$ and O_2 . If the centers of base circles O_i coincide with centers of rotation $O^{(i)}$ ($i = 1, 2$), and then vectors of gear eccentricity $\underline{\Delta e}_i = \overline{O^{(i)}O_i}$ are zero. The involute curves are in tangency at a point M of the line of action KL .

To model the meshing of gears with eccentricity, gears 1 and 2 are translated from their theoretical positions by $\overline{O^{(i)}O_i} = \underline{\Delta e}_i$ ($i = 1, 2$). Now, the center O_i will be offset from the center of rotation $O^{(i)}$. Because of this displacement of the gears, the tangency of their involute curves is broken: the curves will either interfere with each other (intersect) or lose contact. To bring the involute curves into the contact once again, it is sufficient to rotate gear 2 by a small angle $\Delta\varphi^{(2)}$. According to equation (58) the angle $\Delta\varphi^{(2)}$ may be determined with the equation

$$(\underline{\Delta\varphi}^{(2)} \times \underline{r}_2^{(P)}) \cdot \underline{n}^{(M)} = (\underline{\Delta e}_1 - \underline{\Delta e}_2) \cdot \underline{n}^{(M)} \quad (61)$$

The triple product results in (Fig. 7)

$$(\underline{\Delta\varphi}^{(2)} \times \underline{r}_2^{(P)}) \cdot \underline{n}^{(M)} = \Delta\varphi^{(2)} r_2^{(P)} \cos \psi_0 \quad (62)$$

**ORIGINAL PAGE IS
OF POOR QUALITY**

where b_0 is the pressure angle.

Vectors of eccentricity $\underline{\Delta e}_1$ and $\underline{\Delta e}_2$ form angles β_1 and β_2 with vector

$\overline{O^{(1)}O^{(2)}}$; these angles are measured in the direction of gear rotation (Fig. 7).

The dot products yield

$$\begin{aligned}\underline{\Delta e}_1 \cdot \underline{n}^{(M)} &= \Delta e_1 \sin(\beta_1 + \psi_0) \\ \underline{\Delta e}_2 \cdot \underline{n}^{(M)} &= \Delta e_2 \sin(\psi_0 - \beta_2)\end{aligned}\quad (63)$$

It results from equations (61) to (62) that

$$\Delta b^{(2)} = \frac{\Delta e_1 \sin(\beta_1 + \psi_0) + \Delta e_2 \sin(\beta_2 - \psi_0)}{r_b^{(2)}} \quad (64)$$

where

$$r_b^{(2)} = r_2^{(P)} \cos \psi_0 \quad (65)$$

is the radius of the base circle of gear 2.

The center O_i ($i = 1, 2$) of the base circle rotates in the process of meshing;

$O_i^{(1)}$ and $O_i^{(2)}$ are two instantaneous positions of this center (Fig. 8). Angles

β_1 and β_2 can be represented as follows:

$$\beta_1 = \beta_{10} + \varphi_1, \quad \beta_2 = \beta_{20} + \varphi_2 \quad (66)$$

where β_{10} and β_{20} correspond to the initial positions of centers O_1 and O_2 ,

with $\varphi_1 = \varphi_2 = 0$.

Equations (64) and (66) yield

$$\Delta \varphi^{(2)} = \frac{\Delta e_1 \sin(\varphi_1 + \gamma_1) + \Delta e_2 \sin(\varphi_2 + \gamma_2)}{r_b^{(2)}} \quad (67)$$

where

$$\gamma_1 = (\beta_{10} + \psi_0); \quad \gamma_2 = (\beta_{20} - \psi_0)$$

For convenience, consider the kinematic error function to have zero magnitude at

$\varphi^{(1)} = \varphi^{(2)} = 0$. Then, the kinematic error becomes

ORIGINAL PAGE IS
OF POOR QUALITY

$$\begin{aligned} \Delta\theta^{(2)} &= \Delta\varphi^{(2)}(\varphi_1) - \Delta\varphi^{(2)}(0) \\ &= \frac{\Delta e_1 [\sin(b_1 + \gamma_1) - \sin \gamma_1]}{r_b^{(1)}} m_{21} + \frac{\Delta e_2 [\sin(b_2 + \gamma_2) - \sin \gamma_2]}{r_b^{(1)}} \end{aligned} \quad (68)$$

where

$$m_{21} = \frac{\omega_2}{\omega_1} = \frac{r_b^{(1)}}{r_b^{(2)}} = \frac{N_1}{N_2}$$

Equation (68) represents the kinematic error of a gear train with two gears as the sum of two harmonics. The periods of these harmonics are equal to the periods of complete revolutions of the gears.

Equation (68) may be made symmetric as follows:

$$\begin{aligned} \Delta\theta^{(2)} &= \frac{\Delta e_1 [\sin(\varphi_1 + \gamma_1) - \sin \gamma_1]}{r_b^{(1)}} m_{21} + \frac{\Delta e_2 [\sin(\varphi_2 + \gamma_2) - \sin \gamma_2]}{r_b^{(2)}} m_{22} \\ &= \sum_{i=1}^2 \frac{\Delta e_i [\sin(b_i + \gamma_i) - \sin \gamma_i]}{r_b^{(i)}} m_{2i} \end{aligned} \quad (69)$$

Here, $m_{22} = 1$ and $\varphi_2 = \varphi_1 m_{21}$

Equation (69) can be generalized for a train with n gears as follows:

$$\Delta\theta^{(n)} = \sum_{i=1}^n \frac{\Delta e_i [\sin(\varphi_i + \gamma_i) - \sin \gamma_i]}{r_b^{(i)}} m_{ni} \quad (70)$$

where $\Delta\theta^{(n)}$ is the resulting kinematic error of the gear train represented as the angle of rotation of gear n (the output gear).

A complicated gear train is a combination of pairs of gears. The parameter γ_i may be represented as

$$\gamma_i = \beta_{i0} + \psi_0$$

for the driving gear of the pair, and as

$$\gamma_i = \beta_{i0} - \psi_0$$

for the driven gear of the pair. For instance, for computational purposes, a train of three gears must be replaced by two pairs of gears. The idler (intermediate gear) is considered as the driven gear in the first pair, and as the driving gear in the second pair.

Designate the kinematic error exerted by the eccentricity Δe_i of gear number i as

$$\Delta \theta_i = \frac{\Delta e_i [\sin(\varphi_i + \gamma_i) - \sin \gamma_i]}{r_b^{(i)}} \quad (71)$$

where $\Delta \theta_i$ is the error of the rotation angle φ_i . The maximum possible value of this error is

$$\Delta \theta_{i,\max} - \Delta \theta_{i,\min} = \frac{2 \Delta e_i}{r_b^{(i)}} \quad (72)$$

The kinematic error of the train may be represented as

$$\Delta \theta^{(n)} = \sum_{i=1}^n \Delta \theta_i m_{ni} \quad (73)$$

Usually gear trains are applied for the reduction of angular velocities and thus m_{ni} is less than 1. It results from equation (73) that the last gears of a train (numbers n , $n - 1$, $n - 2$) induce the largest part of the resulting kinematic error $\Delta \theta^{(n)}$. Therefore, the precision of these gears must be higher than the others.

**ORIGINAL PAGE IS
OF POOR QUALITY**

The largest value of the kinematic error function $\Delta\theta^{(n)}$ and its distribution above and below the abscissa depend on the combination of parameters γ_i ($i = 1, 2, \dots, n$). Figure 9 shows the distribution of a function $\Delta\theta_i$ (φ_i) exerted by eccentricity of gear i of the train.

The resulting errors of a gear train may be compensated for in part, by definite rules of assembly of gears with eccentricity. For instance, for gears with tooth numbers $N_1 = N_2$ and equal eccentricities $\Delta e_1 = \Delta e_2$ the resulting kinematic error will be approximately zero if eccentricity vectors $\underline{\Delta e}_1$ and $\underline{\Delta e}_2$ (Fig. 7) are directed opposite each other.

APPLICATION OF THEORY TO ECCENTRICITY OF SPIRAL BEVEL GEARS

For spatial gears the word "eccentricity" is used to describe that the geometric axis of a gear is parallel to, but does not coincide with, its axis of rotation (Fig. 10). As the eccentric gear rotates its geometric axis generates a cylindrical surface of radius Δe . The eccentricity vector $\underline{\Delta e}$ is a vector which rotates about the gear axis. The initial position of vector $\underline{\Delta e}$ (its position at the beginning of motion) is given by angle α (Fig. 10).

Figure 11 shows coordinate systems $S_1(X_1, Y_1, Z_1)$ and $S_f(X_f, Y_f, Z_f)$, which are rigidly connected to gear 1 and the frame, respectively. System S_h is an auxiliary coordinate system, which is also rigidly connected to the frame. Driving gear 1 rotates about axis Z_h . The position of $\underline{\Delta e}_1$ in coordinate system S_1 is given by the angle α_1 , which is made by $\underline{\Delta e}_1$ and axis X_1 . The current position of $\underline{\Delta e}_1$ in coordinate system S_f (or S_h) is defined by the angle $(\varphi_1 + \alpha_1)$ and the matrix equality

$$\begin{aligned}
 [\Delta e_f^{(1)}] &= [L_{fh}] [\Delta e_h^{(1)}] \\
 &= \begin{bmatrix} \cos \gamma_1 & 0 & \sin \gamma_1 \\ 0 & 1 & 0 \\ -\sin \gamma_1 & 0 & \cos \gamma_1 \end{bmatrix} \begin{bmatrix} \Delta e_1 \cos(\varphi_1 + \alpha_1) \\ -\Delta e_1 \sin(\varphi_1 + \alpha_1) \\ 0 \end{bmatrix} \quad (74)
 \end{aligned}$$

**ORIGINAL PAGE IS
OF POOR QUALITY**

Here $[\Delta e_h^{(1)}]$ is the matrix of vector Δe_h in terms of its projections on the axes of coordinate system S_h . The 3 by 3 matrix $[L_{fn}]$ transforms elements of the column matrix $\Delta e_h^{(1)}$ to coordinate system S_f from coordinate system S_h .

Matrix equality (74) yields

$$[\Delta e_f^{(1)}] = \begin{bmatrix} \Delta e_1 \cos(\varphi_1 + \alpha_1) \cos \gamma_1 \\ -\Delta e_1 \sin(\varphi_1 + \alpha_1) \\ -\Delta e_1 \cos(\varphi_1 + \alpha_1) \sin \gamma_1 \end{bmatrix} \quad (75)$$

The vector of eccentricity of the driven gear can be defined in a similar way. Figure 12 shows coordinate systems S_2 and S_f rigidly connected to gear 2 and the frame, respectively. The auxiliary coordinate system S_p is also rigidly connected to the frame.

Vector $\Delta e_f^{(2)}$ is represented by matrix equalities

$$\begin{aligned} [\Delta e_f^{(2)}] &= [L_{fp}] [\Delta e_p^{(2)}] \\ &= \begin{bmatrix} \cos \gamma_2 & 0 & -\sin \gamma_2 \\ 0 & 1 & 0 \\ \sin \gamma_2 & 0 & \cos \gamma_2 \end{bmatrix} \begin{bmatrix} \Delta e_2 \cos(\varphi_2 + \alpha_2) \\ \Delta e_2 \sin(\varphi_2 + \alpha_2) \\ 0 \end{bmatrix} \end{aligned} \quad (76)$$

which after matrix multiplication gives

$$[\Delta e_f^{(2)}] = \begin{bmatrix} \Delta e_2 \cos(\varphi_2 + \alpha_2) \cos \gamma_2 \\ \Delta e_2 \sin(\varphi_2 + \alpha_2) \\ \Delta e_2 \cos(\varphi_2 + \alpha_2) \sin \gamma_2 \end{bmatrix} \quad (77)$$

Kinematic errors induced by gear eccentricities may now be found by applying Equation (58) as follows:

**ORIGINAL PAGE IS
OF POOR QUALITY**

$$\begin{bmatrix} \Delta \varphi_f^{(2)} \\ p_f^{(2)} \\ n_f^{(2)} \end{bmatrix} = \begin{bmatrix} \Delta e_f^{(1)} \\ - \Delta e_f^{(2)} \end{bmatrix} \cdot n_f^{(2)} \quad (78)$$

Here $\Delta e_f^{(1)}$ and $\Delta e_f^{(2)}$ are given by matrices (75) and (77); vector $\Delta \varphi_f^{(2)}$ (Fig. 12) is represented by the matrix

$$\begin{aligned} [\Delta \varphi_f^{(2)}] &= [L_{fp}] [\Delta \varphi_p^{(2)}] \\ &= \begin{bmatrix} \cos \gamma_2 & 0 & -\sin \gamma_2 \\ 0 & 1 & 0 \\ \sin \gamma_2 & 0 & \cos \gamma_2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \Delta \varphi_2 \end{bmatrix} \\ &= \begin{bmatrix} -\Delta \varphi_2 & \sin \gamma_2 \\ 0 \\ \Delta \varphi_2 & \cos \gamma_2 \end{bmatrix} \end{aligned} \quad (79)$$

Vector $p_f^{(2)}$ represents the position vector of the contact point which belongs to the line of action and n_f represents the common unit normal of the gear surfaces at their point of tangency.

Equations (78) and (79) yield

$$\Delta \varphi_2 = \frac{n_x \Sigma \Delta e_x + n_y \Sigma \Delta e_y + n_z \Sigma \Delta e_z}{-y \cos \gamma_2 + (x \cos \gamma_2 + z \sin \gamma_2) n_y - y \sin \gamma_2 n_z} \quad (80)$$

where $\Sigma \Delta e_x = \Delta e_x^{(1)} - \Delta e_x^{(2)}$, $\Sigma \Delta e_y = \Delta e_y^{(1)} - \Delta e_y^{(2)}$, $\Sigma \Delta e_z = \Delta e_z^{(1)} - \Delta e_z^{(2)}$ and the subscript f was dropped.

Projections n_y and n_z of the surface unit normal n , and coordinates x , y , and z of the contact point change in the process of motion. But since the changes in these variables are relatively small, they may be neglected (Fig. 13).

$$n_f = \sin c_c i_f + \cos c_c (\cos \beta j_f + \sin \beta k_f) \quad (81)$$

$$x_f = 0, y_f = 0, z_f = L$$

Equations (80) and (81) yield

$$\Delta\varphi_2(\varphi_1) = \frac{A(\varphi_1)}{L \sin \gamma_2 \cos \psi_c \cos \beta} \quad (82)$$

where

$$A(\varphi_1) = a_1 \sin(\varphi_1 + \alpha_1) + b_1 \cos(\varphi_1 + \alpha_1) +$$

$$a_2 \sin(\varphi_2 + \alpha_2) + b_2 \cos(\varphi_2 + \alpha)$$

$$a_1 = -\Delta e_1 \cos \psi_c \cos \beta$$

$$a_2 = -\Delta e_2 \cos \psi_c \cos \beta$$

$$b_1 = \Delta e_1 (\cos \gamma_1 \sin \psi_c - \sin \gamma_1 \cos \psi_c \sin \beta)$$

$$b_2 = -\Delta e_2 (\cos \gamma_2 \sin \psi_c + \sin \gamma_2 \cos \psi_c \sin \beta)$$

$$\varphi_2 = \varphi_1 \frac{N_1}{N_2}$$

It is concluded from the form of equation (82) that kinematic errors induced by the eccentricity of spiral-bevel gears may be represented as the sum of four harmonics: the period of two harmonics coincides with the period of revolution of gear 1; the period of the other two coincides with the period of revolution of gear 2.

The function $\Delta\varphi_2(\varphi_1)$ represented by equation (82) is a smoothed, continuous function which serves as a first approximation. In reality the true function $\varphi_2(\varphi_1)$ breaks as different sets of teeth come into mesh. This break can be discovered if $\Delta\varphi_2(\varphi_1)$ is determined by equation (80).

GEAR TRAIN VIBRATION AND NOISE MEASUREMENT

To illustrate the principles discussed on the subject of gear train precision, Figs. 14 and 15 are used. These figures show some frequency spectrum measurements made on a helicopter transmission running in a test stand [7]. The transmission had a spiral-bevel input stage with 19 teeth on the pinion

and 71 teeth on the gear. The pinion was turning at 6200 rpm and the output shaft at 355.5 rpm. The output stage was a spur planetary arrangement with a 27-tooth sun, 3 planet gears, each with 35 teeth and a 99-tooth ring gear which was splined to the transmission housing. An accelerometer was mounted on the case immediately outside the spline.

Figure 14 shows a broadband frequency spectrum measurement of the vibration signal. The spur mesh frequency was 583 Hz and the spiral bevel mesh frequency was 1963 Hz. The spiral-bevel vibration signature was much stronger than the spur signature. This indicates that the meshing accuracy was better for the spur mesh than for the spiral-bevel mesh. There are also other peaks in the spectrum at multiples of the fundamental frequencies of 1963 Hz and 583 Hz. These other peaks are the higher harmonics due to the noise and vibration pulsations as the teeth mesh being different from the pure sinusoidal shape as shown in Fig. 9.

Figure 15, an expanded region of the autospectrum plot given in Fig. 14, shows many peaks that are symmetrically located about the spur gear mesh fundamental frequency peak at 583 Hz. These peaks locate the sideband frequencies which are due to sources of modulation in the time dependent vibration waveform. Each source of modulation may produce one pair of sidebands if it is a harmonic modulator. If nonharmonic, the side bands will repeat many times, as in the case in Fig. 15.

SUMMARY OF RESULTS

Kinematic precision is affected by errors that are the result of either intentional adjustments or accidental defects in the manufacturing and assembly of gear trains. A general method for the determination of kinematic precision of gear trains has been explained. The general method is based on the exact kinematic relations for the contact point motions of the gear tooth sur-

ORIGINAL PAGE IS
OF POOR QUALITY

faces under the influence of errors. An approximate method was also explained.

Example applications of the general and approximate methods were demonstrated for gear trains consisting of involute (spur and helical) gears, circular arc (Wildhaber-Novikov) gears, and spiral-bevel gears. Gear noise measurements from a helicopter transmission were presented and discussed with relation to the kinematic precision theory. The following results were obtained:

1. The exact numerical iterative procedure for finding kinematic errors, Δb_2 , is as follows: From equation (20) find

$$\Delta \varphi_2 = \varphi_2(\varphi_1, \Delta Q) - \varphi_2^0(\varphi_1)$$

where the angles φ_2 and φ_2^0 have been determined from an iterative solution of the nonlinear algebraic system of five equations, which are separable into two systems of two and three equations each as follows:

$$F_i(\varphi_1, \varphi_2) = 0, \quad i = 1, 2$$

$$\begin{cases} A - K_1(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \\ H_1 - K_2(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \\ H_2 - K_3(u_1, \theta_1, \varphi_1, u_2, \theta_2, \varphi_2, \Delta Q) = 0 \end{cases}$$

where A , H_1 , H_2 , and ΔQ are given values and φ_2^0 is determined by solving with $\Delta Q = 0$, whereas φ_2 is determined by the full solution of the five equations.

2. The approximate equation for kinematic error $\Delta \varphi^{(2)}$ is

$$(\Delta b_2^{(2)} \times r_2^{(P)} \times S \Delta q) \cdot n^{(M)} = 0$$

where $r_2^{(P)}$ is the radius vector to the pitch point, $n^{(M)}$ is the surface normal vector at the contact point M , and $S \Delta q$ is the sum of known error vectors.

3. Application of the formulas showed that Wildhaber-Novikov gears are sensitive to any errors which cause changes in the center distance. A formula for the location of the gear-tooth contact point was given.

4. It was found that for a pair of spur gears the kinematic error function due to eccentricities is a sum of two simple harmonics. For a multistage speed reducer, it was concluded that accuracy in the final stages has the most impact on kinematic error. For gears with approximately equal known eccentricities, the kinematic error may be compensated for by directing the eccentricities opposite one another.

5. For a pair of spiral-bevel gears, the kinematic error function due to eccentricities is a sum of four harmonics.

REFERENCES

1. Litvin, F., Theory of Gearing, 2nd ed., Nauka, 1968 (in Russian).
2. Litvin, F. and Gutman, Ye., "Methods of Synthesis and Analysis for Hypoid Gear Drives of 'Formate' and 'Helixform'," P. 1, P. 2, P. 3, Journal of Mechanical Design, Vol. 103, No. 1, Jan. 1981, pp. 83-113.
3. Michalec, G. W., Precision Gearing: Theory and Practice, John Wiley & Sons, Inc., New York, 1966.
4. Dudley, D. W., ed., Gear Handbook, McGraw-Hill, New York, 1962.
5. Buckingham, E., Analytical Mechanics of Gears, Dover Publications, New York, 1949.
6. Baxter, M., "Effect of Misalignment on Tooth Action of Bevel and Hypoid Gears," ASME Paper 61-MD-20, 1961.
7. Townsend, D. P., Coy, J. J., and Hatvani, B. R., "OH-58 Helicopter Transmission Failure Analysis," NASA TM X-71867, Jan. 1976.

ORIGINAL PAGE IS
OF POOR QUALITY

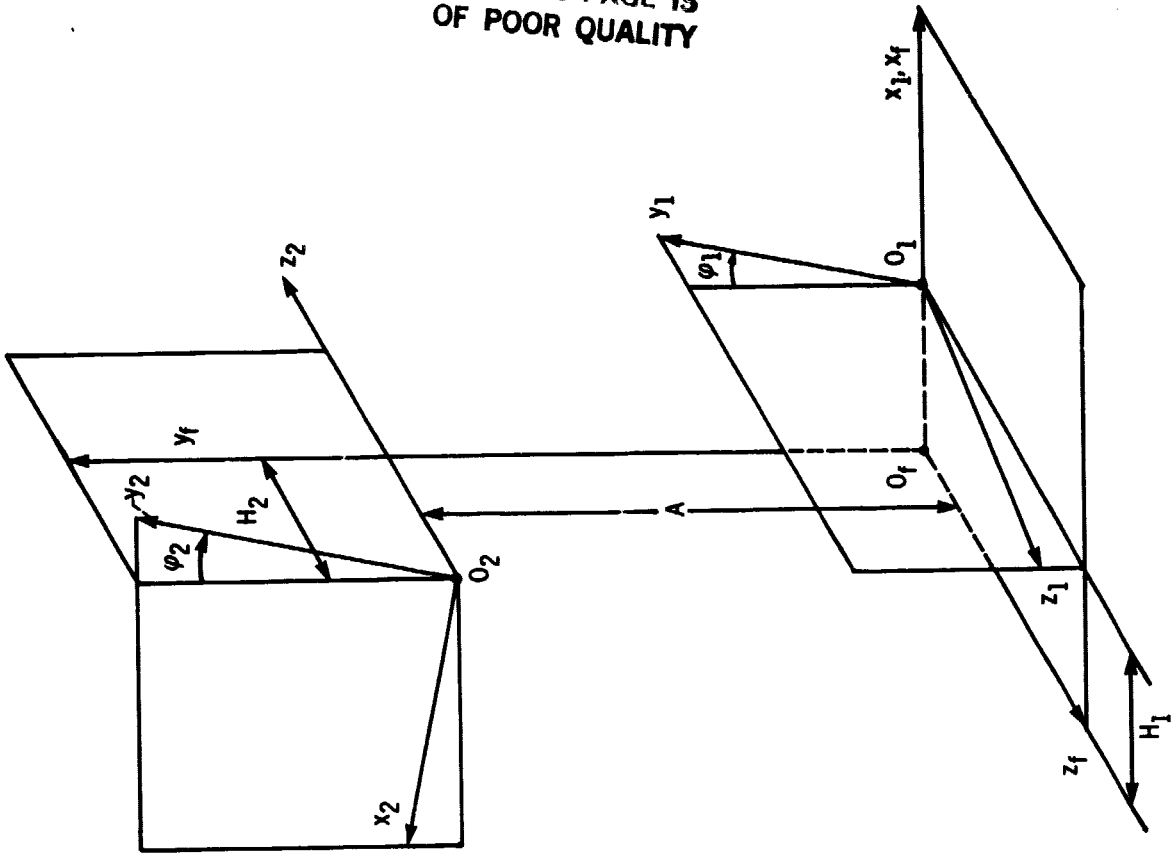


Figure 2. - Relationship between coordinate systems.

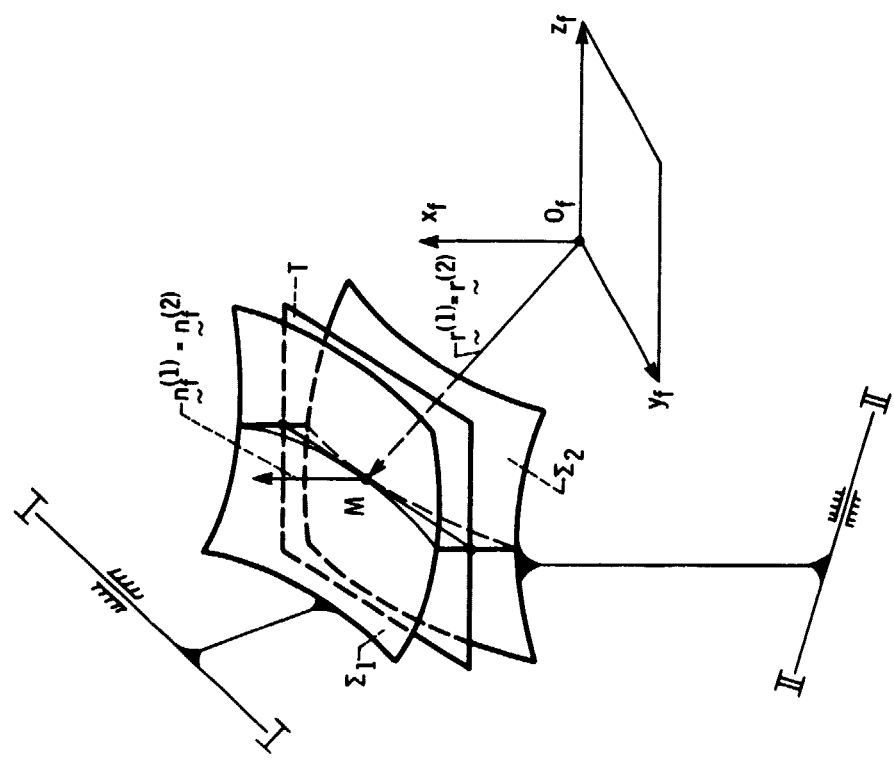


Figure 1. - Gear teeth in contact.

ORIGINAL PAGE IS
OF POOR QUALITY

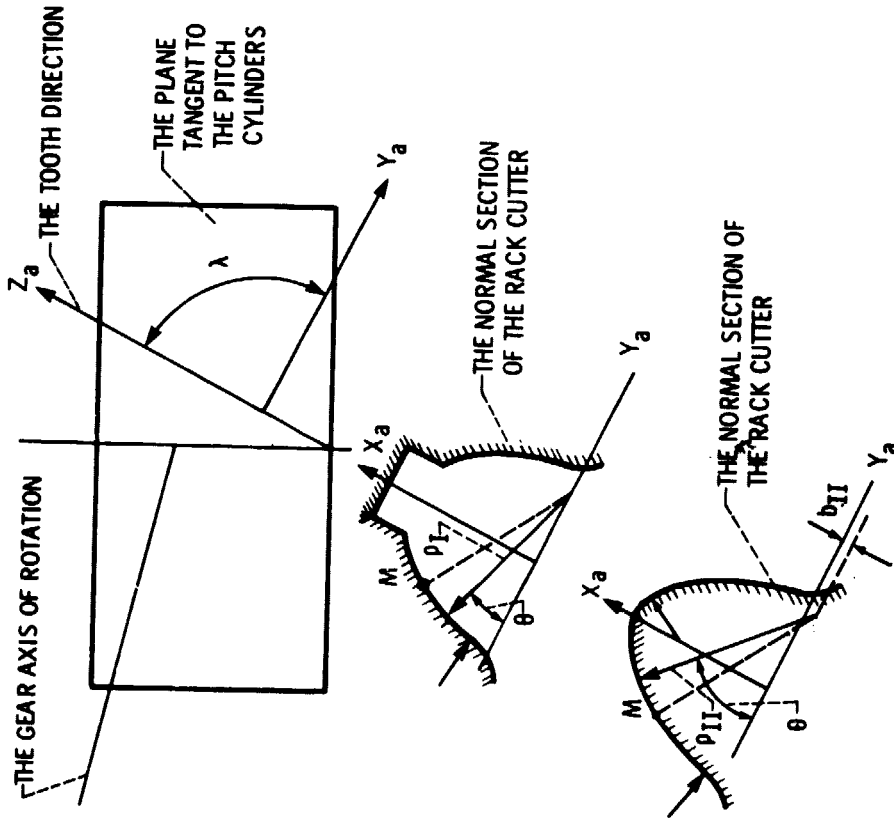


Figure 3 - Generation of circular arc gears.

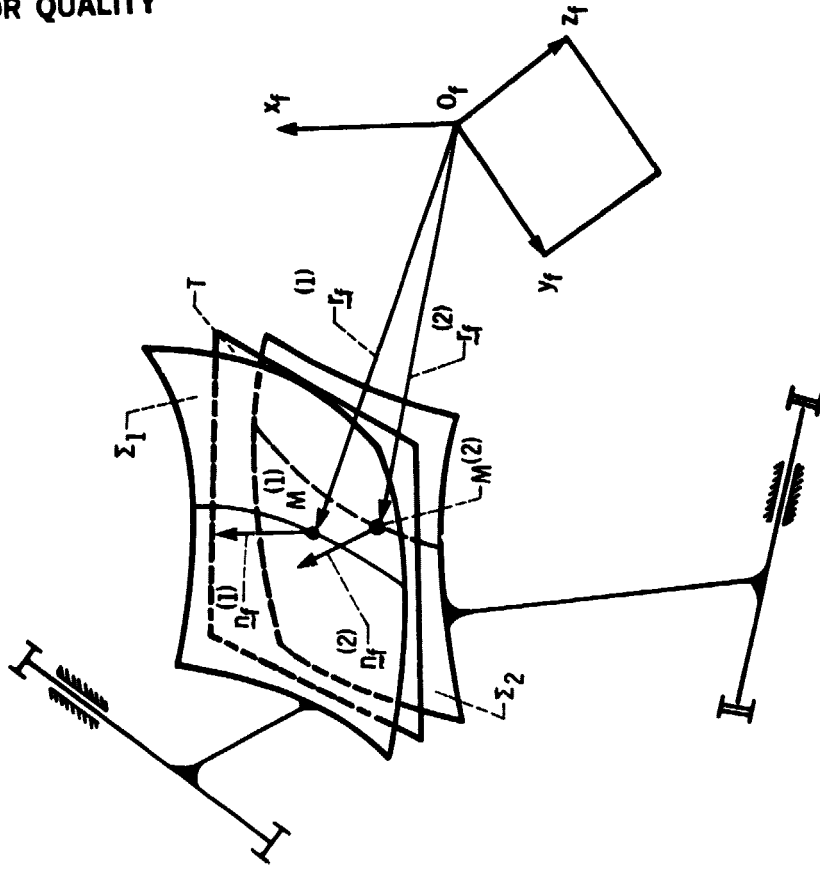
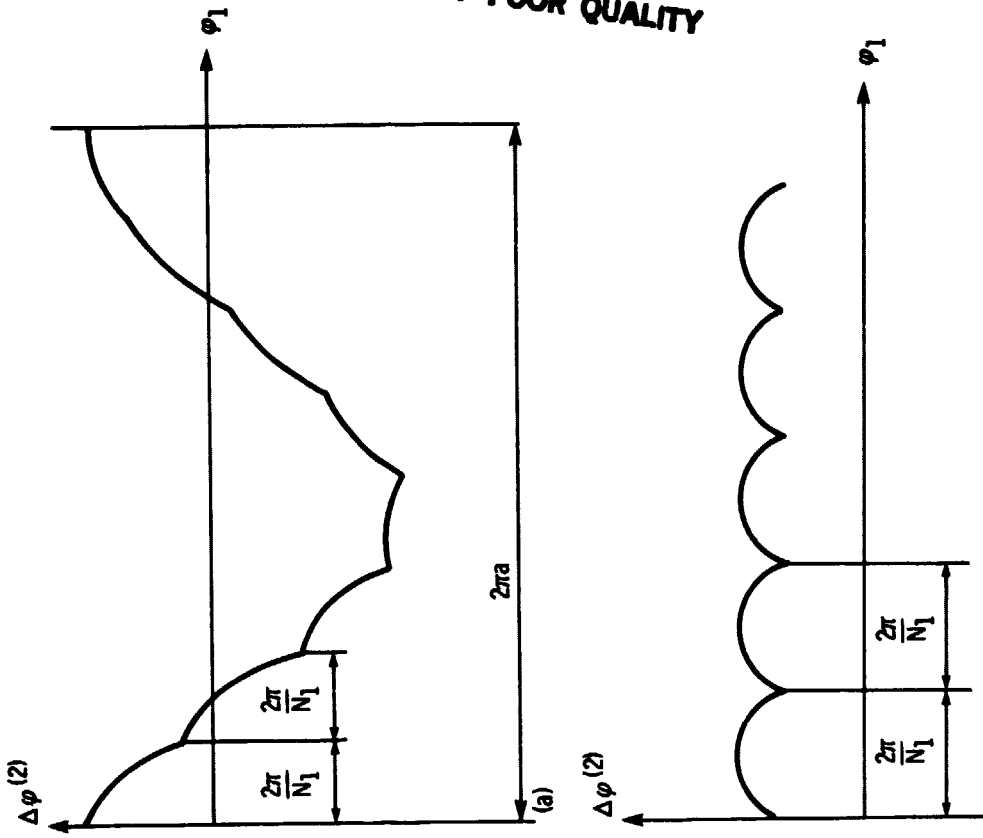


Figure 4 - Gear teeth not in contact.

ORIGINAL PAGE IS
OF POOR QUALITY



(a) Periodic with gear rotation.
(b) Periodic with tooth mesh cycle.
Figure 6. - Kinematic errors.

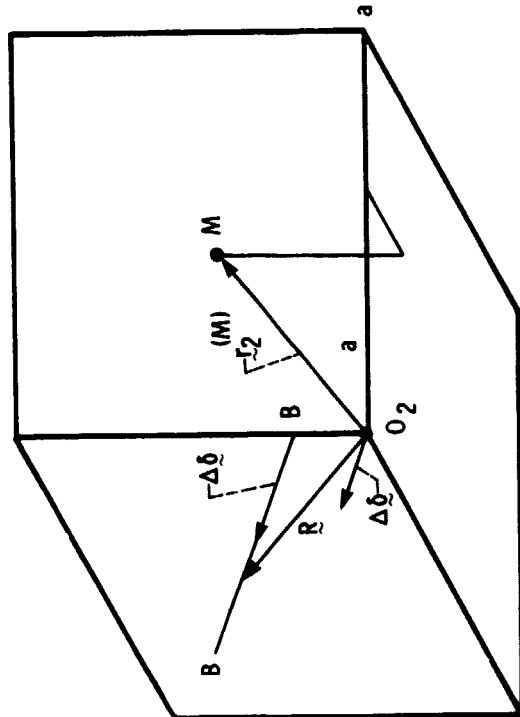


Figure 5. - Errors in gear rotation axis.

ORIGINAL PAGE IS
OF POOR QUALITY

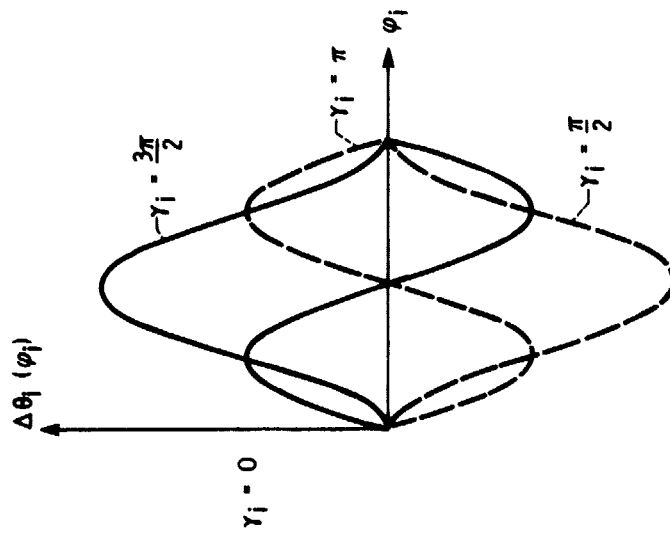


Figure 9. - Kinematic error function.

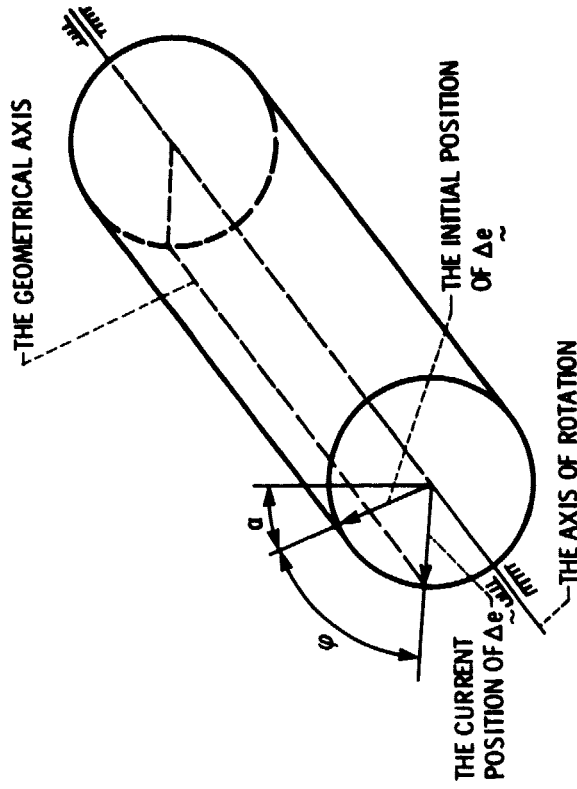


Figure 10. - Eccentricity between geometric and rotational axes.

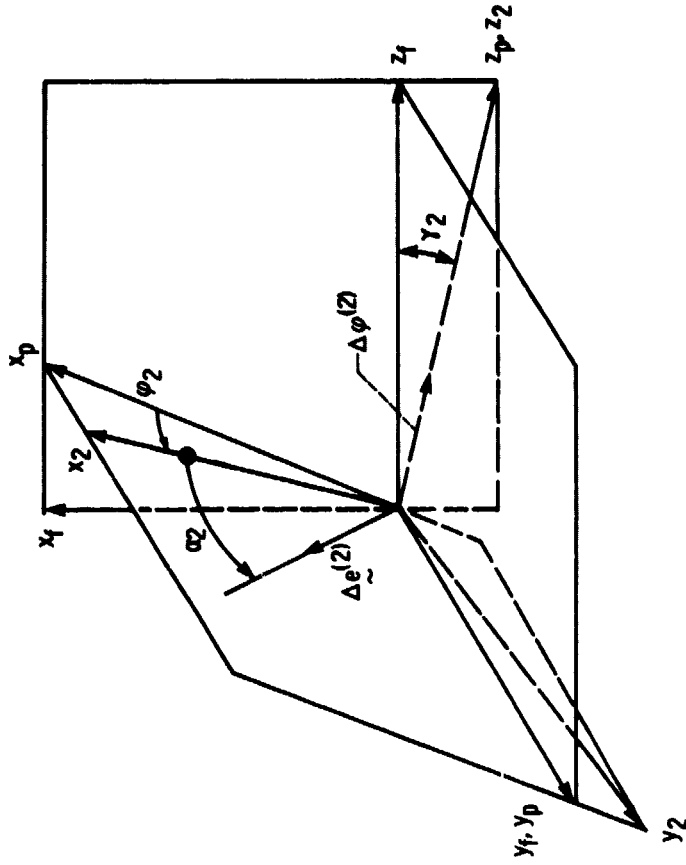


Figure 12. - Coordinate frames used to describe eccentricity for gear 2.

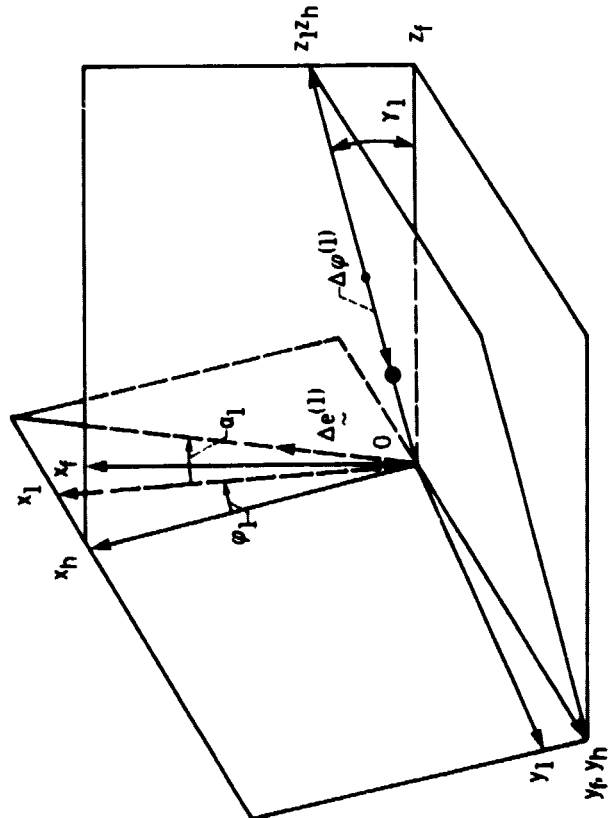


Figure 11. - Coordinate frames used to describe eccentricity for gear 1.

ORIGINAL PAGE IS
OF POOR QUALITY

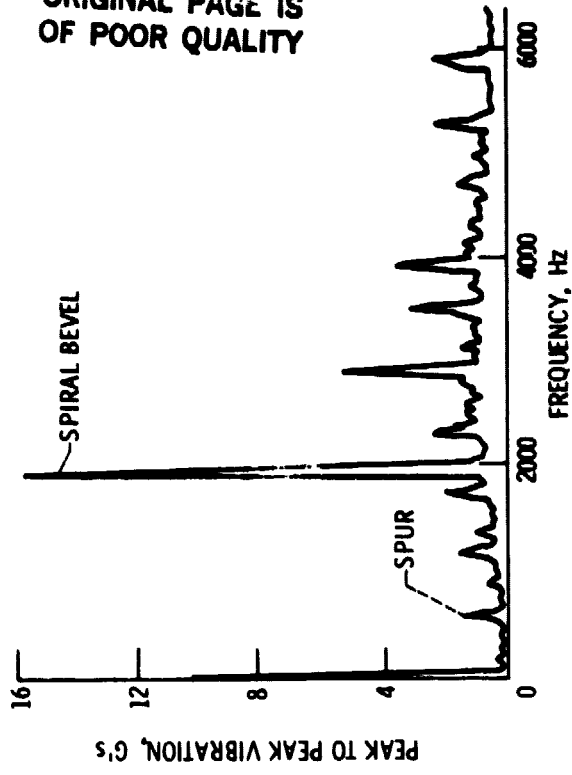


Figure 14. - Baseband frequency spectrum showing spiral bevel amplitude compared with spur.

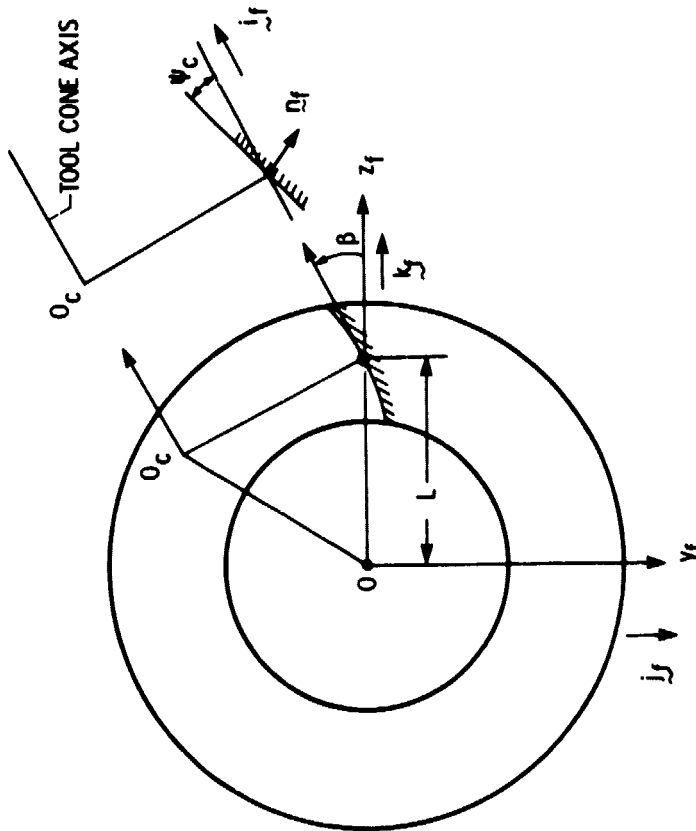


Figure 13. - Spiral bevel gear showing surface unit normal vector \vec{n}_f .

ORIGINAL PAGE IS
OF POOR QUALITY

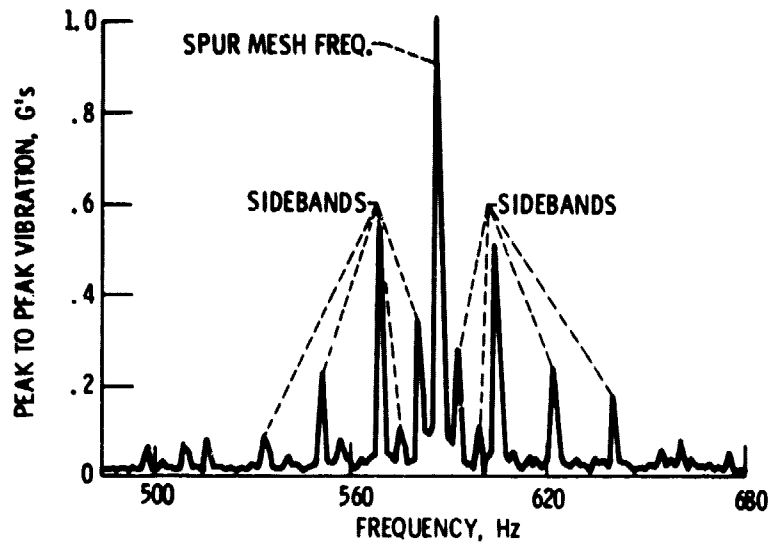


Figure 15. - Narrow band frequency spectrum showing sidebands around the spur mesh frequency.