# Development of an Autonomous Video <br> Rendezvous and <br> Docking System 



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DEVELOPMENT OF AN
AUTONOMOUS VIDEO
RENDEZVOUS AND
DOCKING SYSTEM

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This report presents the results of a nine-month study by Martin Marietta for the National Aeronautics and Space Administration's George C. Marshall Spacefilght Center. This study, including conceptual design and simulation of three video guidance systems for spacecraft, was performed under contract NAS8-34679, Developaent of an Autonomous Video Rendezvous and Docking System. Significant benefits were obtained trom prev!ous related work under Martin Marietta IR\&D task D-11R.
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I. Summary

Please note the following errors in Development of An Autonomous Video Rendezvous and Docking System:

| Page | Line | Original Text | Correction |
| :---: | :---: | :---: | :---: |
| 111 | 2nd from . bottom | Light Positions Modeled are... | Light Positions are Modeled... |
| I-1 | 6th line of 2nd paragraph | ...20,000 degrees per second... | ...20,000 degrees per hour... |
| IV-11 | 3rd line from bottom | ...1/600th of the... | ...1/400th of the... |
| IV-11 | 2nd line from bottom | ... 380 television lines... | ... 128 television lines... |
| IV-12 | 1st and 2nd lines | ...better resolution, and resolution as... | ...better resolution. <br> [delete remainder of sentence] |
| IV-16 | lst line of ifgure | ...diagran in Figure IV -9. | ...diagram in Figure IV -8. |
| IV-21 | 3rd 1 ine of lst paragraph | ...system. The same resolution was modeled, and the same assumptions were made. | ...system, and the same assumptions were made. but 380-1ine resoiution was modeled. |
| V-5 | 5th line of last paragraph | ...degrees per hour - cove... | ...degrees per hour about... |
| V-10 | 3-Light system line of Table, Resolution column | 380 Television Lines | 128 Television Lines |
| VIII-7 | After definition of $A_{c}$ | ```[Text missing before paragraph that begins, "In these simulations...]``` | $\underline{W}_{\text {, }}$, represented in the program by the 3-element array BODVEL, 18 the chase vehicle angular velocity, expressed in the body coordinate system. |
| VIII-15 | Row defining $g$ and $\dot{g}$, "Meaning" column | Allitude Quaternion | Attitude Quaternion |

A simple docking aid consisting of three flashing lights has proved viable for use in an autonomous video rendezvous and docking system for spacecraft. A telf vision image of this target can be analyzed to determine the relative positions and attitudes of the two spacecraft. The analysis time is only 100 milliseconds because a simple dedicated electronic circuit assists in the analysis.

Control systems using this and two other types of docking aids were evaluated through computer simulation in this study and other approaches were considered. However, this three-light system performed much better than the others. Its accuracy is affected little by tumbling of the target spacecraft, and in the simulations it was able to cope with attitude rates up to 20,000 degrees per hour about the docking axis. Its perfomance with rotation about other axes is determined primarily by the state-estimation and goal-setting portions $c$ : the control system, not by measurement accuracy.

A physical simulation of the three-light control system would be useful to verify the validity of the assumptions and mathematical models used in this study. The simulation should employ scaled target models, a television camera, and hardware video signal processing electronics. The simulation should also model the spacecrafi controi system and the limitations of a practical spacecraft. This level of detail is recommended to force consideration of compatibility problems and may reveal weaknesses in the approach that might not otherwise be detected. This report includes a discussion of a suitable control system, and Appendix A discusses a computer program that can serve as the basis for the physical simulation.
II. Introduction to the Problem

The need for an Automated Rendezvous and Docking System has been shown in a number of mission models, including those in the Mars sample return and large space system studies. Several factors make automation attractive: practical limitations on human interaction, fuel-use and trajectory optimization requirements, safety, communication limitations, and the nted for real-time operation.

The purpose of this study was to identify video techniques that might be suitable for such an automated system, define the equations and algorithms these techniques would use, and evaluate video guidance control systems based on these techniques through computer simulation.

To ensure that practical problems were considered, the simulation was to model not only the sensor but also methods for dealing with a number of practical problems, e.g., maintaining control when the target spacecraft leaves the field of view of the guidanc? sensor. The simulation also was to model the characteristics and limitations of practical spacecraft, because this may reveal subtle incompatibilities. A mission model was then defined to serve as a basis for the simulation.

In this model the chase vehicle is a general-purpose spacecraft for repair, refurbishment, and retrieval of other spacecraft. After it is deployed from the Space Shuttle, it must rendezvous and dock with a passive target spacecraft that is in a circular orbit about the earth at an altitude of 300 km . The chase vehicle is illustrated in Figure II-1 and has the characteristics summarized in Table II-1.

The target spacecraft (Fig. II-2), is similar to the Long Duration Exposure Facility (LDEF). It has an appropriate docking aid and docking fixture, but it is passive during the docking operation. It can neither perform cooperative maneuvers nor maintain a stable attitude. Its only means of cooperation is turning docking-aid lamps on and off in response to radio commands.

Each computer simulation was to begin when the coarse rendezvous system, which nas gotten the chase vehicle to within 1000 feet of the target, hands control to the video system. The coarse rondezvous is assumed accurate enough to guarantee that the target is within the field of view of the video guidance sensor. (Initial target acquisition is considered in the study but is not modeled in the computer simulations.) It is also assumed that, although fuel represents a significant fraction of the vehicle's mass, the mass during the terminal rendeavous can be predicted with reasonable accuracy. Because fuel use can be measured, this assumption is not unreasonable.

The scope of the study did not include system opti..ization, trajectory planning, or detailed hardware design.

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The Body Coordinate System Used in the Simulations Are Shown
Figure II..] Chase Vehicle Modeled jor Computer Simulations

Table II-1 Chase Vehicle Characteristics


* These Characteristies Var; with Fuel Load

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III. Conclusions and Recommendations
A. A THREE-LIGHT DOCKING AID IS RECOMMENDED

Of the three vaeo guidance systems simulated in this study, the best system used the three-light docking aid shown in Figure III-1. Not only was it the simplest of the three systems, it was also the only system that worked reliably in the simulations.


Figure III-1 Fiashing-Light Docking Aid

It was not accuracy that made this system best; it consistently outperformed the others even though the resoluticn and target size that were simulated gave it the poorest accuracy at long range. Its advantage was that it could measure both relative position and relative attitude from a single observation. The other systems had to resolve ambiguities in attitude by combining two successive measurements, an approach the simulation results proved to be dangerous.

The problem with the other two systems could be solved by modifying the docking aids slightly. For example, one of these systems used a ring of lights on the target spacecraft for a docking aid. By observing the target, the guidance system can determine that the camera lies somewhere on a circle about the docking ayis, but it cannot determine the camera's position on the circle. The addition of one light (Fig. III-2) would greatly improve the performance of the guidance system by
uniquely defining the direction to the docking axis. Unfortunatoly, this approach results in a system that is more complicated than the three-light system because the modification requires adding a way of distinguishing between the ring of lights and the added light. Further, target roll about the docking axis still cannot be observed with this system.


Figure III-2 Modification to Ringoof-Lights Docking Aid Target Roll about the Docking Axis Is Still Not Observable, but the Direction to the Dockiige Axis Is Uniquely Defined.

The other system simulated used a docking aid that projects a rainbow of light in each of two direction:. Color sensing and stereo rangefinding are used in this system to uniquely define the chase vehicle's position. Relative attitude, however, is ambiguous because chase vehicle rotation about the line of sight is not observable. The problem could be solved by adding some means for sensing this rotation, but this system is already the most complex of the three.

## B. MANY FACTORS DETERMINE PERFOPMANCE

The video guidance system must do much more than determine the current position and atitude. It must also turn chase vehicle thrusters on and off to steer a safe course from the current position to the target spacecraft. It must ensure that the chase vehicle arrives at the target with the proper attitude and that the two spacecraft contact at low velocity. It must also align the chase vehicle's docking fixture properly with the docking fixture on the target spacecraft. In the process of doing these things, it must it allow the target's image to leave the field of view of its television camera for more than moment.

A control system may perform these furctions poorly despite accurate knowledge of relative position and attitude. In this study we tried to uncover subtle incompatibilities between video guidance techniques and the remainder of the control system by modeling a complete system. If we had not, problems with the "rainbow" and "ring-of-1ight" approaches might have gone undiscovered. The problem in evaluating the systems
this way is that performance depends on the entire system and not fust the vadeo guidance technique. If the system as a whole does not make effective use of the video information, there is a danger that a potentially useful technique will be rejected.

For example, we found that a small change in the goal-setting logic of the contrcl system made a great difference in the ability of the three-light system to cope with tumbling targets. Suitable changes in the other simulations might have made the "rainbow" and "ring-oflights" approaches look more attractive.

While we do not belleve that such changes would have altered the conclusions of the study (the three-light system is still simplest and least sensitive to the rest of the system), we believe that "fine-tuning" of the control system for best performance is worth the effort. We also want to caution the reader that the performance reported here is not necessarily the best the system might be able to provide.

## C. THE HARD PART IS THE LAST EIGHT METERS

A system that measures only the distance and direction to the target is adequate to approach within eight meters of the targe". At this point attitude information becomes vital because offsets among the docking aid, target docking fixture, and target center of mass become major contributors to alignment errors. The offsets among the camera, chase vehicle center of mass, and chase vehicle docking fixture make attitude information doubly important because chase vehicle attitude and position must be controlled. To further complicate the problem, the target may be coning and nutating, making it difficult to anticipate attitude changes.

All three systems performed well at great distances from the target, and all tiree had problems at close range. The ring-of-lights system war totally unable to cope with close range because the relative veloci, y between the chase vehicle and target is small at close range. This system depends on differences between successive observations to measure attitude. With the velocity reduced, it was using observations in which the differences were almost entirely due to random effects.

The "rainbow" system received its best attitude information at close ranse because small movements produced greater changes in orientation of the line of sight. But during the last few critical seconds of the operation, it loses range infomation because the rangefinder cameras cannot see the docking aid.

Although the three-light system maintained good accurary at close range, it still had problems. Small translational movements between light tlashes made it more difficult to keep the entire target witrin the camera's field of view. Target rotation made the problems more severe. Also, because the equat tons this system uses for image interpretation are based on orthographic projection, their accuracy became
poorer as range decreased: this prujection approximates the image's appearance well only when the rampe is much greater than the distance between the lights on the docking aid.

Some of the close-range problems might be handled by using two different docking aids. A large docking aid could be used at long range, and a smaller one with the same geometry could be used for close-in operations. Even if this is done, close range will remain the most difficult problem.
D. TUMBLING TARGETS MAKE CONTROL DIFFICULT

Effective operations with tumbling and coning targets will require sophisticated state estimators and goal-setting logic. The normal tumble rate of the target in the mission model for this study is one revolution per orbit, or 240 degrees per hour. Although the control system used in the simulations was able to cope with roll rates of over 20,000 degrees per hour about the docking axis, the system had trouble in keeping the docking aid within the field of view and in docking with the proper aligrment with rates as small as 1000 degrees per hour in pitch or yaw. At rates of about 4000 degrees per hour in pitch or yaw, docking was extremely difficult. The problem is not measurement accuracy but control. The structure of the control system presented in Chapter VIII is adequate for dealing with rotating targets, but the simplified version used in the simulations is inadequate for high tumble rates.

Some fundamental constraints must be kept in mind in refining the control system. First, the chase vehicle's thrust authority is limited. If the control system tries to minimize stresses on the docking fixtures by staying on the docking axja at all times, the thrusters may not be able to overcome centrifugal force. If the control system tries to minimize problems with centrifugi.l force, it can plan a trajectory that takes the chase vehicle directly to the point where the target's docking fixture is predicted to be on arrival. However, this approach may greatly stress the docking fixtures of both spacecraft. Further, a slight error in estimating tumble parameters may cause the docking fixtures to miss each other. Solving problems like these was beyond the scope of the study reported here. We recomend a separate study to investigate possible solutions.
E. A PHYSICAL SIMULATION SHOULD INCLUDE A CONTROL SYSTEM, CAMERA, AND TARGET

By modeling an entire six-degree-of-freedom control system, this study uncovered problems that might have gone unnoticed if only measurements had been modeled. We highly recommend the same approach in a physical simulation. This will not be difficult to implement because the computer programs in Appendices A through $C$ already include "dummy" subroutines for control of a physical simulator.

The simulator should allow posilioning of a television camera anywhe in a large room under computer control, and have a gimbal set to allow pointing of the camera. It is not necessary to rotate the target model because only relative position and attitude are detectable in the television image. Tumbling of any complexity can be simulated, and it is not necessary to make mechanical changes to switch between simple tumbling about the docking axis and a complex coning and nutating motion. However, the simulated rates must be small enough so only one side of the target need be visible.

Simulator speed is entirely a matter of convenience. Although computer control will be required to allow simulations to be run in a reasonable amount ot time, little is gained by runniny the simulation in exactly the anount of time required for an actual operation in space. Although full-speed operation would show the effects of image smear and after images, implomenting the system with a solid-state camera and flash lamps would inake these factors insignificant.

To simulatu the full dynamic range of 300 meteas to contact, it will probably be necessary to build two or three different target models. One of thesc models would be one or two percent of full scale and would simulate the appoarance of the target at long range. Another would be much larger, porhaps full scale, and would show the detail visible at closc raxice mending on the dimensions of the room and the accuracy of the simuliator, it may be desirable to use a third model for intermediate ranges.

The ra:a!t; ol this study show that a camera with resolution comparable to st indard lelevision will be suitable. A solid-state camera is attrictive tor $\because$ fuce because it does not require high voltage and a fragile hish-v $\quad$ mum camera tube. We therefore recommend the use of a sol-id-silat , diwid in the plysical simulation. This will help to uncover subtle probicom related to camera problems such as blooming, image gramuintic and resolution, lens adjustments, dead picture elements, and $\begin{aligned} & \text { ariat inns among picture elements in dark signal and sensitivity. }\end{aligned}$

The computer used in the simulations should be equipped with interfaces to drive the simulator and to receive information from the television camera. Il flashing lights are used, it will also require an interface to syuchronize light flashes with the computer's calculations. Any video $s$ jpnal processing that would be done in hardware in a real spacecrati : wuld be done in similar hardware iathe simulation to evaluate hardwarc dosigil problems and shortcomings. Aside from these requiremats, the computer does not have to be particularly large or fast. the progrim could, in fact, fit into many personal computers.

# IV. The Three Systems That Were Simulated 

Three separate video guidance systems were simulated in this study. This chapter describes the technical details, computer models and key mathematical equations used in each system to derive relative position and attitude.

One of the most formidable challenges in autonomous video guidance is recognizing and analyzing the target. The image analysis must be done quickly, because a long computation delay degrades the performance of the control system. The analysis must also be accurate, because position data will be used to determine velocity and perhaps other parameters. For example, if the target is tumbling rapidly, the control system may have to estimate parameters for predicting what the target's attitude, attitude rate, and rotation axis will be when the two spacecraft touch.

Whether this is a job for a mainframe computer or an inconspicuous electronics package depends largely on the design of a docking aid on the target spacecraft. In all of the simulated systems, the docking aids were designed to be analyzed with a minimum of onboard intelligence. Each system uses a hardware analyzer that reduces the video data to a small set of parameters. This approach gives these systems high speed without placing a heavy burden on the flight computer.
A. IMAGE OF THREE LIGHTS GIVES ATTITUDE AND POSITION IN FIRST SYSTEM

1. Description of Technique

The first system, developed in 1980 at Martin Marietta under IR\&D task $\mathrm{D}-11 \mathrm{R}$, is the simplest of the three systems. It uses a docking aid that consists of the three lights (Fig. IV-1). By using a simple dedicated electronic circuit and a small routine in the flight computer, this system can analyze the docking aid in approximately $100 \mathrm{millisec}-$ onds. The speed and simplicity are achieved by giving the analyzer a very simple job: findins the coordinates of a spot of light in a television image.

The simplification results from the fact that only one lamp is on at a time. Bright flash lamps are used so that the lamps are by far the brightest objects in the television image. The dedicated electronic analyzer can therefore analyze the image by simply finding the coordinates of the center of bright ness of the image as a whole.

There are several ways to make individual lamps stand out without flashing the lamps in sequence. Colored lights and polarization, for example, can be used to give each lamp a unique "signature."


Figure IV-1 Flashing-light Docking Aid Three Sequentially Flashed Lamps Provide Full Relaitive Attitude and Position Information with a Minimum of Computation. In the Basic Configuration, Lomp Flashes are Directly Controlled by the Chase Vehicle via a Radio Cormanci Link, Bit Variations Not Requiring a Command Link Are Discussed in the Text.

However, sequentially-flashed lamps have three significant advantages:

1) The system can very easily detemine which lamp is which.
2) The flashes can be synchronized to a shutter on the camera so that light fron other sources can gat to the camera only during the flash. This reduces the effective brightness of background "clutter" by a factor of 30 but does not change the effective brightness of the lamp. Further reduction in background lighting can be realized by comparing an image showing the lamp flashing with an impge showing the lamp off.
3) The flash duation is short, about a millisecond. This is short enough to fit into the television camera's retrace interval so that there is no question about whether the flash occurred before or after the corresponding picture elements were scanned. Flashing during the retrace also allows the analyzer to process a different lamp on each video frame, greatly speeding the analysis. If an integrating sensor is used (a charge coupled device or vidicon), the electron image left by the flash will remain in the sensor until the image is scanned.

When the three lamps are flashed in sequence, high attitude rates may cause image distortion. This distortion can be partially corrected by flashing one of the lamps twice, noting the change in the location of the center of brightness, and adjusting the other two light locations accordingly.

The center of bright ness or "centroid" of an image is defined by the coordinates:

$$
\begin{equation*}
X_{c}=k \quad \int_{\text {frame }}^{V(t) X(t) d t /} \int_{\text {frame }}^{V(t) d t} \tag{IV-1}
\end{equation*}
$$

and

$$
\begin{equation*}
Y_{C}=k \quad \int_{\text {frame }}^{V(t) Y(t) d t / \int} \int_{\text {trame }}^{V(t) d t} \tag{IV-2}
\end{equation*}
$$

where:
$k$ is a constant that depends on scan rate and amplitude;
$V$ is the video signal with blanking and synchronizing pulses removed;
$X$ and $Y$ are the horizontal and vertical deflection, varying from a negative value at the left side or bottom of the image to a positive value of equal magnitude at the right side or top of the image;
$t$ is time.

These coordinates can be computed with analog electronic components such as multipliers and integrators, cr they can be approximated with digital components such as counters, adders, and accumulatora. Multiplication of the video signal by the deflection signal is reduced to a gating operation if the video signal is converted to a two-level digital signal with a comparator, but careful selection of the comparator threshold is required to ensure that ihe lights and background gre reliably separated by this technique.

The choice of technology for implementing equations (IV-1) and (IV-2) will depend on a number of factors including environmental factors, reliability requirements, and camera type. In general, it will be conventent to uses an analog epproach with vidicons and other cameras that use an analog deflection signal. A digital approach may be more convenient for systems using self-scanned solid-state arrays. Figure IV-2 illustrates the analog approach.


Frame-Kate
Reset Pulses
Figure IV-2 Analog Centroid Computer
The Centroid Coordinates $X_{c}$ and $Y_{c}$ Are Valid at the End of a Frame. The Boxes Labeled "a+b" Are Analog Dividers.

Besides computing the image centroid, the video preprocessing circuit is required to synchronize the scanning of the camera with the flashing of the lights. This can be done quite easily if the circuit directly commands the lights by a luw-power radio link. This approach has a number of advantages:

1) It is a simple matter to determine which of the three lamps is flashing if each is commanded individually. (The lamps must be uniquely identified for proper image interpretation);
2) Operation with multiple targets in the same area is simplified, as each can be assigned a unique code or radio channel;
3) The lights will flash only when they are needed;
4) Acquisition of the target is simplified;
5) Very little power is required, since the transmitter's operating range is only 300 meters.

An alternate approach to synchronization, phase locking to independently flashing lights, may reduce the cost of the total system when one (or a few) chase vehicles are used for many missions. In such a system, the target cost and complexity are reduced at the expense of a more complicated chase vehicle system.

When target simplicity is most important, the flash-lamp approach will be less attractive. The same pattern of three reference points on the target can be used in a much simpler docking aid, but with retroreflectors and colored filters replacing the flash lamps. A color television
camera is then used to distiuguish among the three points, and a flashing light on the chase vehicle supplies illumination. If lighting is favorable, cost could be further reduced by using painted spots for the reference points. In either case, the reference points are distinguishable by the ratios among the red, blue, and green camera outpute. The color-camera approach allows the video preprocessor to find all three rentroids from a single video frame, sc image distortion from vehicle motion will not need correction.

After finding the coordinates of the three lamps in the image, the system determines the position of the chase vehicle in the instantaneous target reference frame. The calculations the flight computer uses to do this are found in subroutine POSIT of the program listing in Appendix A. The calculations are derived from the positions of the lights in an orthographic projection of the light pattern. This projection is a good approximation to the appearance of the image when the distance from the camera to the lights is many times greater than the distances between lights, and although perspective effects become significant at close range, the equations still properly indicate the direction to the docking axis of the target. The perspective effects do not cause serious problems if the camera and docking aid are set back from the docking fixtures so that the camera is a few meters from the lamps when the two spacecraft are docked.

Orthographic projection was used in deriving the equations to minimize the amount of calculation. A more accurate solution could be developed to consider perspective effects, or a correction algorithm could be used to refine the position estimate provided by the simpler equations presented here. Unless the camera must be mounted at the very front of the chase vehicle, such improvements produce little measurable benefit.

Equations Used
There are two aspects to attitude control with this system: the camera must remain pointed at the docking aid, and the chase vehicle must determine the target spacecraft's attitude and match it to dock proporly.

Camera pointing is the easiest. Yaw and pitch errors are simple trigonometric functions of the coordinates of the center light's image on the television screen:

$$
\begin{equation*}
\theta_{\text {pitch }}=\tan ^{-1}\left(v_{2} / £\right) \tag{IV-3}
\end{equation*}
$$

$\theta_{y a w}=\tan ^{-1}\left(u_{2} / f\right)$
where $\left(u_{1}, v_{1}\right)$ are the coordinates of the ith lamp's image on the camera-focal plane and $f$ is the lens focal length. The roll error can be approximated by the angle between horizontal and the line connecting the images of the two side lamps:

$$
\begin{equation*}
{ }^{\theta} \text { roll }=\operatorname{atan} 2\left(v_{3}-v_{1}, u_{3}-u_{1}\right) \tag{IV-5}
\end{equation*}
$$

where atan2 is the FORTRAN two-argument arctangent function. This formula is acsurate only when the camera is in line with the $x$ axis of the docking aid, but since roll errors are usually unimportant until the instant of docking, and the guidance system forces the chase vehicle toward this axis, the formula works very well in practice. Subroutine RPY of the simulation program uses these formulas to correct attitude after each light-flashing sequence, as lons, as the lights are in the camera's field of view. Alternate formulas that use the "state estimate" from an onboard mathematical dynamics model are used when the light cannot be seen. These formulas are found in subroutine ESTRPY, a complete discussion of which will be found in Chapter VIII.

The more complicated problem of determining target spacecraft attitude is solved by comparing the camera's position in two different coordinate systems. The first system is a reference frame centered at the center docking-aid light and parallel to the target-body fiame (Fig. II-2). This may be called the "docking-aid" frame. The second coordinate system is the so-called "primary" reference frame used for navigation. The latter frame is a nonrotating feame that is centered at the center of mass oi the target spacecraft and 18 parallel to the chase vehicle body axes at the instant the video guidance system takes control. (See Chapter VIII for a more complete discussion of this coordinate system.) In the simulation program, the calculations for deriving target attitude are done in subroutine ATITUD, which calls the lowerlevel subroutines FINDCV, QUATRN, and DIRMAT.

Subroutine FINDCV computes the camera's position in a coordinste system parallel to the primary coordinate system but centered at the center light. The equation it uses is:
$c=A_{c}^{T}\left[\begin{array}{c}-f \\ u_{2} \\ -v_{2}\end{array}\right] \quad \rho / \sqrt{f^{2}+u_{2}^{2}+v_{2}^{2}}$
in which:
C, represented in the program as CVPOS, is the required camera position vector.
$\rho$, represented in the program as RHO, is the measured range from the camera to the docking aid. This value is provided by subroutine POSIT, which was previously discussed.
$u_{2}, v_{2}$, represented in the program as IJC and VC, are the coordinates of the center light's image at the focal plane of the camers as in equations (IV-3) through (IV-5).
$A^{T}$, represented in the programs as ACVT, is the transpose of the chase vehicle's attitude direction cosine matrix, which is obtained from the inertial measurement unit.
f, represented in the program as FOCLEN, is the lens focal length.

To cor : the target's attitude, the guidari: ystem uses $c$ and anoiher "u...., $r$, which defines the camera's $p$ a': 2 in the "docking aid" refert nit frame. In the program, $\frac{r}{}$ is reprac....ed by the array RELPOS and is :ruputed from the image appearance ; broutine poSIT. Subroutine RUATRN computes the target attitide. snion with the following formulas:

$$
\begin{equation*}
\underline{\boldsymbol{w}}=\underline{\underline{s}} \times \underline{\underline{E}} \text { (wross product) } \tag{IV-7}
\end{equation*}
$$

$\dot{\psi}=\tan ^{-1}\left(\frac{\underline{\underline{W}} \underline{\underline{\underline{T}}} \underline{\underline{c}}}{\underline{\underline{c}}}\right)$
$\underline{g}^{\prime}=\left[\left.\begin{array}{c}\underline{w} \sin (\phi / 2) /|\underline{w}| \\ \cos (\phi / 2)\end{array} \right\rvert\,\right.$
(IV-8)
$\underline{S}=A_{c}\left[\begin{array}{l}-2(q i ́ q 2-q \hat{3} q \hat{4}) \\ q i^{2}-q_{2}^{2}+q \hat{j}^{2}-q_{4}^{2} \\ -2\left(q \hat{2} q_{3}^{2}+q i ́ q u\right)\end{array}\right]$
$\theta=\tan ^{-1}\left(S_{3} / S_{2}\right)-\tan ^{-1}\left(\frac{v_{3}-v_{1}}{u_{1}-u_{3}}\right)$
$\underline{q}^{\prime \prime}=\left|\begin{array}{cc}\underline{x} \sin (\theta / 2) /|\underline{r}| \\ \cos (\theta / 2)\end{array}\right|$
$g=q^{\prime} g^{\prime \prime}$ (quaternion product--see Appendix D)
in which:
q, represented in the program as $Q T$, is the required quaternion,
$\underline{r}$ and $\underline{c}$ are the camera position vectors defined previously,
$u_{i}, v_{i}$ are the coordinates of the image of lamp $i$ at the camera focal plane, as before,
$A_{c}$, represented in the program as $A C V$, is the chase vehicle attitude direction cosine matrix from the inertial measurement unit,

The remainder of the variables are intermediate results.
The physical interpretation of this procedure is as follows. The vectors $r$ and $c$ both represent the line of sight, or the camera's position with respect to the docking aid, but they are expressed in two different coordinate systems that have a comon origin. The nnboard computer knows the orlentation of one of those coordinate systems, because that
system is the reference frame for its inertial measurement tult. The other coordinate system, which is parallel to the target'n body axes, 18 unknown.

The computational strategy is to initially assume that the two coordinate systems are identical. If they rere, $\underline{r}$ and $\underset{C}{ }$ should have identical numerical values, and the obser"aiions should agree with preilctions based on projective geometry. The assumption is probably urong, -n they probably won't.

The computer's task is to find a rotation that will give predicted observations that agree with what the camera sees.

Consider what happens if the Jocking aid coordinate system rotates and $r$ retains its orientation with respest to that system (r rotates with the coordinate system). If the docking-aid coordinate system is rotated to align with the primary frame, $\underline{r}$ and $c$ will point in different directions. This is where the computer starts: assuming that the coordinates systems are aligned and noting that $\underline{r}$ and $\underline{c}$ have different numerical values (Fig. IV-3).


Figure IV-3 Interpretation of $\underline{x}$ and $\underline{Q}$

Imediately tice computer senses that sumething is wrong. The vectors $x$ and $c$ don't line up, so the first step is to find a rotation of the docking-aid coordinate system that will line them up.

The smallest rotation that will align $r$ and $c i s$ a rotation about the axis defined by the cross product $\underset{x}{\bar{C}}$ (equation [IV-7]). The magnitude of the cross product is $|\underline{r}| \underline{c}$ sin, , where in the required rota-- Ion angle. Similaily, the dot product roc is|r|c|cosp. The ratio of chese quantities is

```
|\underline{r}|\underline{c}|\operatorname{sin}\phi
```

which aliows the computer to determine $\phi$ (equation [IV-8]).
An axis of rotation and a rotation angle define a quaternion (Appendix D), and this quaternion, $q^{\prime}$, is computed with equation (IV-9).

This quaternion only partially specifies target attitude; any rotation of the docking-ald frame about $\underline{r}$ leaves $\underline{r}$ and $\frac{c}{}$ aligned, so there are an infinite tumber of target at $\bar{t} i t u d e s$ t $\bar{o}$ select from, as Figure IV-4 illustrates. Some other method must now be found to find the proper rotation about the line of sight. The approach here is to use the appearance of lamps 1 and 3 .
(a)

## Figure IV-4 Alternate Attitude Interpretations

An Infinite Number of Target Attitudes Align $r$ and c. Four Possibilities Are Shown in (a) tnrough (d).

A vector from lamp 3 to lamp 1 lies parallel to the docking aid $y$ axis and points in the $-y$ direction. This vector's orientation in the television image can be measured:
$\theta_{1}=\tan ^{-1}\left(\frac{v_{3}-v_{1}}{-\left(u_{3}-u_{1}\right)}\right)$
(The minus sign in the argunent's denominator shows that the posilive horizonlal axis of the image is aligned with the negative $y$ axis of the chase vehicle.)

The vector's orientation can also be predicted. Suppose that $q^{\prime}$ represents the target attitude. If $A_{t}$ is the direc*ion cosine matrix that corresponds to $q^{\prime}$,

$$
\underline{s}=A_{c} A_{t}^{T}\left[\begin{array}{c}
0  \tag{IV-16}\\
-1 \\
0
\end{array}\right]
$$

is a unit vector in the direction of the target's -y axis but expressed in chase vehicle coordinates. The expression
$A_{t}^{T}\left[\begin{array}{l}0 \\ -1 \\ 0\end{array}\right]$
can be evaluated with equation ( $D-5$ ) from Appendix $D$. The result is equation (IV-10).

The projection of $s$ onto the $y-z$ plane of the chase vehicle's coordinate system (which is parallel to the camera focal plane) gives the predicted orientation of the vector in the television image:
$\theta_{2}=\tan ^{-1} \frac{s_{3}}{3_{2}}$
(IV-17)

Rerspective effects may produce errors of approximately one or two degrees at close range, because this procedure uses orthographic projection.

The difference between $\theta_{2}$ and $\theta_{1}$ is the required amount of rotation about the 1 ine of sight, and the axis of rotation is defined by a unit vector in the direction of $r$. The axis and rotation define a second quaternion $q^{\prime \prime}$ (equation [IV-12]).

Finally, the computer calculates the net rotation to get the target attitude. The associated quaternion is found from the quaternion product of $q^{\prime}$ and $q^{\prime \prime}$ in equation (IV-13).
4. Mission Constraints and Compatibility

Except for the camera position requirement-the camera must be set back far enough from the front of the chase velicle to avoid severe image distortion-this approach to video guidance imposes few mission constraints. For example, it can operate in total darkness or full sunlight as long as the sun does not enter the field of view. Unlike the other approaches, this approach allows the docking ald and camera to be offset from the docking fixture, simplifying the design of the system
and reducing the impact on other systems. Further, it computes positinn and attitude fast enough to allow its use with targets that tumble several degrees per second, although other factors may make docking with such a target impractical. (Other parts of she control system may not be able to keep up with such rates, the thrusters may have too little control authority, or the docking mechanism may not be strong enough to withstand the stresses. Moreover, the Kalman filter in the simulated control system was not designed to estimate tumble parameters, and the goal-setting logic used in the system does not optimize trajectoriej.) With improvements in these portions of the control system, the three-light approach might do much better with tumbling targets than the simulation results suggest.

Although the system requires a computer, it is likely that an autonomous spacecraft will require one with any rendezvous system. Very little additional hardware is needed beyond what one might expect to be required for other functions. The additional hardsare consists of a television camera, a handful of electronics weighing perhaps two kilograms including packaging, and a low-power command transmitter with a range of 300 meters. Power requirements will denend on the camera type and other implementation details, but it is reasonable to assume the added hardware can be designed to consume less than ten watts. Because the system is used for only two to four minutes, the energy requirement is negligible when compared to the requirements of other onboard systems.

If the flashing-light approach is used, compatibility with target spacecraft systems must be considered carefully. The spacecraft must provide power to the lights and command receiver, and although the energy requirement is small, it may result in a requirement for solar panels and a power system on a spacecraft that otherwise has no need for them. Other potential compatibility problems with the target spacecraft include interfering with science instruments by blocking their fields of view, overpowering instrunents with bright lights, or seneratins electromagnetic interference. The impacts nn nropulsion, telemetry, coarse rendezvous, launch vehicle, and ground support systems cannot be meaningfully evaluated without specific details of the mission. However, the small size, ruggedness, low power requirement, and speed of the technique will minimize problems.

## r. Measurement Model for Simulation

The simulation program used the perspective projection techniques described in Chapter IX to provide "measurements" from this system. The program simulated motions of the chase vehicle and target between the three light flashes of a single measurement, because this motion produces image distortion that may affect accuracy. The effective resolution of the camera is determined by the random shifting of computed image coordinates in subroutine FLASH. Normally distributed random shifts were added in horizontal and vertical directions. The mean shlft was zero, which models negligible fixed biases, and the standard deviation was $1 / 600$ th of the width of the field of view, which models an effective resolution of 380 television lines. This is comparable with standard commercial television. Better accuracy was obtained with

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better resolution, and resolution as coarse as 128 lines allowed reliable docking with good control.
B. PROJECTED RAINBOWS AND STEREO RANGEFINDING FORM SECOND SYSTEM

1. Description of Technique

In the second system, a beacon on the target spacecraft projects "rainbows" of light. The apparent cclor of the beacon, viewed from the chase vehicle, depends on the angle between the docking axis and the line of sight. If the light from the beacon appears blue, the chase vehicle is to the left of the docking axis; if the light appears yellow, the chase vehicle is on the docking axis; if the light appears red, the chase vehicle is to the right of the axis. The beacon also projects a second rainbow and rapidly alternates between the two. The second rainbow shows whether the chase vehicle is above or below the docking axis. In the simulations, it was assumed that the beacon is directly commanded by the chase vehicle by radio, but other approaches are possible. For example, the chase vehicle could send a radio signal to indicate which rainbow is active, or the chase vehicle's guidance system could synchronize to a one-two-off flashing sequence.

To detect the colors, the system uses a prism spectrometer as illustrated in Figure IV-5. Light from the beacon is refracted in the prism, and the angle of refraction is a function of the wavelength or color of the light. A linear Charge Coupled Device (CCD) array behind the grating detects this deflected light, and the position of the brightest spot along the array indicates the color. The system locates the bright spot with a peak detector that monitors the video signal from the CCD array. The peak detector records the number of clock pulses required to scan out the brightest picture element in the line, and the count is used to compute the wavelength or distance from the docking axis.


Measured
Angle
Off Axis
Light from the Beacon Is Deflected by the Prism by an Angle $\phi$, Which Depends on Wavelength. The Electronico Convert This Deflection to a Measurement of the Chase Vehicle's Angular Separation from the Docking Axis.

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The two television cameras used in the stereo rangefinder are mounted on opposite sides of the chase vehicle (Fig. IV-6) to maximize the interucular distance. Both cameras find the center of brightness, the location of the beacon in the image, by using the centroid detector technique used in the three-light system. The range to the target is computed by comparing the center-of-brightness calculations from the two cameras.


Figure IV-6 Stereo Rangefinder on the Chase Vehicle
The Comeras Are Movonted on Opposite Sides of the Chase Vehicle for Greatest Stereo Effect. The Cameras Are Not Gimballed But Mounted at a Fixed Angle to the Vehicle Frame.

The center-of-brightness information is also used to point the color detector, which has much smaller field of view than the cameras.

Because accuracy at 300 meters calls for precise alignment, the cameras are mounted rigidly to the frame of the chase vehicle; they are not gimballed.

Figure IV-7 is a block diagram of the rangefinder. Two centroid calculators, similar to the one in Figure IV-2, calculate the image-plane coordinates of the ir:age of the beacon, and an onboard computer uses the coordinates to calculate the distance to the target and to keep the cameras pointed at che target. The system provides 30 range measurements per second.


## Figure IV-7 Stereo Rangefinder Block Diagram

The Video Signals from the Cameras Are Processed to Determine the Center of Brightness for the Images of a Beacon on the Target Spacecraft. The Range Is Updated by an Onboard Computer Using Equatio:: (4-18). The Centroid Calculators Are tive Same As in Figure IV-3.
2. Equations Used

The range formula* is:
$\rho=\frac{f d\left(x_{\ell}+x_{r}\right)}{\left(x_{\ell}+x_{r}\right)^{2}-4(\Delta x)^{2}}$
where
$\rho$ is the estimated range,
d is the interocular distance (see Figure IV-8),
$f$ is the camera lens focal length,
$X_{\ell}$ and $X_{r}$ are the distances along the image plane from the center of the field of view to the image center of brightness for the left and right cameras, respectively, and
$\Delta X$ is the tolerance on $X_{\ell}$ and $X_{r}$, which includes:

- The effects of having discrete photosensitive sites in the image plane,
- Uncertainty in camera boresighting;
- Scan position uncertainty;

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- Tolerances in the electronics that calculate the center of brightness of the image,
- İmage blur from motion and lens imperfections.


Figure IV-8 The Stereo Rangefinding System ana Nomenclature
Because the Guidance System Will Keep the Chase Vehicle Cameras
Pointed Toward the Target, the Range Will Equal the Distance along the Perpendicular from the Camera Image Plane to the object. Uncertainty $(\Delta X)$ in the Image Position Causes Uncertainty in Range ( $\Delta R$ ).

In the simulation program (Appendix B) this formula is implemented in subroutine POSIT, where $\rho$ is represented by the variable RHO. If resolution is the only significant limitation on accuracy, the relative measurement error for moderate values of $\rho$ is

$$
\begin{equation*}
\frac{\Delta R}{\rho}=\frac{2 \rho \Delta X}{f d} \tag{IV-19}
\end{equation*}
$$

The candidate system, for which $\Delta X=$ (line length)/750, has a measurement erior of approximately ( $0.2 \rho / \mathrm{d}$ ) percent of the true range.

Figure IV-9 is a graph of percent error versus range for a rangefinder w.th 256 line resolution. Although the accuracy is $\pm 15 \%$ at the start of the rendezvous operation, the system achieves accuracy better than $5 \%$ at ranges where accuracy is critical.

Table IV-1 Derivation of Formulas (IV-18) and (IV-19)

1. Range - Consider the diagram in Figure IV .. The maximum possible range, $R_{1}$, can be determined by noting the similar triangles $A B C$ and $A D E$. Since the ratios among the sides of similar triangles are equal,
$\overline{A E}=\left(R_{1}\right)\left(X_{\ell}-\Delta x\right) / E$
The same reasoning for the right camera gives
$d-\overrightarrow{A E}=\left(R_{1}\right)\left(X_{r}-\Delta x\right) / f$
Combining (IV-20) and (IV-2I) gives
$\frac{\left(R_{1}\right)\left(X_{l}-\Delta x\right)}{f}+\frac{\left(R_{1}\right)\left(X_{r}-\Delta x\right)}{f}=\overline{A E}+d-\overline{A E}=d$
which can be manipulated algebraically to give the maximum possible range:
$\mathrm{R}_{\mathrm{I}}=\mathrm{fd} /\left(\mathrm{X}_{\ell}+\mathrm{X}_{\mathrm{r}}-2 \Delta \mathrm{x}\right)$
The same reasoning gives the minimum possible range:
$\mathrm{R}_{2}=\mathrm{fd} /\left(\mathrm{X}_{\ell}+\mathrm{X}_{\mathrm{r}}+2 \Delta \mathrm{x}\right)$.
The average of $R_{1}$ and $R_{2}$ is the estimated range:
$\rho=\frac{1}{2}\left(R_{1}+R_{2}\right)=\frac{f d\left(X_{\ell}+X_{r}\right)}{\left(X_{\ell}+X_{r}\right)^{2}-4(\Delta x)^{2}}$
2. Range Tolerance - The range tolerance is half the difference between $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ :
$\Delta R=\frac{1}{2}\left(R_{1}-R_{2}\right)=\frac{2 f d \Delta x}{\left(X_{\ell}+X_{r}\right)^{2}-4(\Delta x)^{2}}$
Dividing (IV-26) by (IV-25) gives the relative error
$\frac{\Delta R}{\rho}=\frac{2 \Delta x}{X_{l}+X_{r}}$
(IV-27)

At moderate ranges, $\left(X_{\ell}+X_{r}\right)^{2} \gg 4(\Delta x)^{2}$. In the candidate system, $\left(X_{\ell}+X_{r}\right)^{2}$ is over 40 times as large as $4(\Delta x)^{2}$ for ranges to 300 m .1

So $\Delta x$ can be ignored in (IV-25) with little error:
$\rho=\frac{f d}{X_{\ell}+X_{r}}$

Table IV-1 Concl


Figure IV-9 Percent Error vs Range for Stereo Rangefinding System With a $\overline{0} 0$-degree Field of View and 256 Pixeis Per Line. The Percentage Error Increases Linearly with Range at the Rate of $0.05 \% / \mathrm{m}$.

Because the two cameras used for stereo ranging are not gimballed, they cannot look "cross-eyed" at the beacon. As the beacon moves betweeen the cameras in the last moments before contact, it leaves the fields of view of both cameras. A third camera is therefore used to allow at least pointing, if not ranging, during the last few seconds of the docking operation. This camera is mounted very close to the docking fixture so that the beacon will always be in its field of view.

The third camera has a circuit like the one in Figure IV-2 to compute the beacon's center of brightness. The coordinates from this circuit are used in the flight computer to compute a vector $c$, which defines the camera's position in a coordi,ate system parallel to the primary coordinate system but centered at the beacon. The equation used is identical to equation (IV-6), except that the variable $\rho$ (range) now comes from the stareo rangefinding system. The vector $c$ plays the same role in this juidance system as in the three-light system.

From the data provided by the beacon color analyzer, the system "knows" its angular separation fiom the docking axis. The analyzer provides two angles, one for horizontal error ("yaw") and one for vertical error ("pitch"). Ttese angles are combined with the range information ( $p$ ) tc give $\underline{r}$, the ramera's position in a reference frame that is parallel to the target's body axes but centered at the . acon. In the simulation program in Appendix B, the calculations are found in subroutine POSIT, and $\underline{r}$ is represented by the variable RELPOS.

The flight computer therefore has available to it vectors that are analogous to the $r$ and $c$ vectors of the three-light system.

Unfortunately, the computer has no information to help it resolve the ambiguity in orientation about the line of sight. Unless it can resolve this ambiguity, it cannot determine which way to steer to get to the docking axis.

The solution to this problem is found in combining two separate measurements. Suppose, for example, that one measurement gives $\underline{r}$ and $c$ and that the second measurement gives $r^{\prime}$ and $c^{\prime}$. If the measurements are closely spaced in time, the target ${ }^{\top}$ s attitude will not have time to change appreciably. Therefore only two coordinate systems are involved: the primary system, which is known, and the target system, which is to be computed. If $A_{t}$ is the direction cosine matrix (see Appendix D) giving the target's attitude with respect to the primary frame,
$\underline{r}=A_{t} \underline{c}$, and
(IV-30)
$\underline{r}^{\prime}=A_{t} \underline{c}^{\prime}$.
(IV-31)

This is true because $\frac{r}{}$ and $r^{\prime}$ are expressed in a frame parallel to the target frame and the frame in which $c$ and $c^{\prime}$ are expressed is parallel to the primary frame and has the same origin as the frame for $\underline{r}$ and $\underline{r}^{\prime}$.

The relationship illustrated in equations (IV-30) and (IV-31) holds for any pair of vectors expressed in these two frames. For example, the cross product of $\frac{c}{}$ and $c^{\prime}$ is related to the cross product of $\underline{r}$ and $\underline{r}^{\prime}$ by the same direction cosine matrix. So the relationship holds betweer the two matrices:
$M_{r}=\left[\underline{1}_{r} \vdots \quad \dot{I}_{T} \vdots \underline{k}_{r}\right]$ and
$M_{c}=\left[\begin{array}{lllll}\underline{i} & \vdots & \mathcal{L}_{c} & {\underset{c}{c}}^{c}\end{array}\right]$
(IV-33)
in which
(IV-34)
$\underline{x}_{x}=\frac{\underline{x}}{|\underline{r}|} \quad i_{c}-\frac{c}{|\underline{c}|}$,

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Because of the way $M_{r}$ and $M_{C}$ are constructed (they are $3 \times 3 \mathrm{ma}-$ trices in which each column is a unit vector of a rectangular coordinate system), these matrices have the special property that their inverses are equal to their transposes. The flight computer can therefore find $A_{t}$ from these matrices by the formula:

$$
\begin{equation*}
A_{t}=M_{r} M_{c}^{T} \tag{IV-37}
\end{equation*}
$$

In the simulation, this formula is evaluated in subroutine QUATRN. The validity and usefulness of the formula depends on two requirements:

1) The vector $\underline{r}$ must differ significantly in orientation from $\underline{r}^{\prime}$, and $c$ must differ significantly from $c^{\prime}$. Otherwise, small errors in these vectors may make the results totally random.
2) The target spacecraft must not rotate much between the time $\underline{r}$ and $\underline{c}$ are measured and the time $\underline{r}^{\prime}$ and $c^{\prime}$ are measured.

Unfortunately these two requirements conflict: if the two measurement sets are taken in rapid succession to minimize target rotation between readings, the measurement sets will differ by very little. However, if there is much delay between the two sets, the target may have rotated enough to invalidate the assumptions behind the formula. The delay also requires the chase vehicle to use stale data for an extended per-. iod of time.

In the simulation program we tried only one approach to dealing with this conflict: we attempted to optimize the time interval between measurements. There are alternative strategies.

Oie strategy is to move the chase vehicie in a path perpendicular to the line of sight from time to time to maximize the difference between the measurement sets. This strategy would be combined with an expanded Kalman filter that estimates target tumble parameters. The additional cost in system complexity would be small, but tho approach wastes fuel and places a much heavier burden on the flight computer.

An alternate strategy is to use two color-decoding circuits instead of one. One would be placed midway between the middle camera and the left camera; the other would be placed midway between the middle camera and the right camera. The resulting system would provide two separate color/rangefinder: systems that would allow simultaneous measurement of both sets of data. Becauce of the close spacing of the instruments, the vectors in the two sets wouid be almost parallel until the chase
vehicle gets close to the target. Tins implies that good attitude information is not available at great aistances. If the target's tunble rate is small enough so that elaborate trajectory planning is not required, this should cause few problens.

We did not simulate these approaches, because we felt that chey would result in a system that was so much moze cumplex and expeasive than the three-light system that even if they worked well, they would not be recommended.
3. Mission Constraints and Compatibility

Camera placement is a bigger concern with this approach than with the three-light system. Altiough the beacon, center camera, and color analyzer could be offset from the docking fixtures, there is a risk in doing so. The problem is that the algorithm for computing target attitude is not very rorust. The factors discussed previously (too little chase vehicle motion or excessive target attitude change betweeen measurements) can cause errors in attitude computations. This problem is compounded when the stereo ranging system becomes "blind" during the critical seconds just before docking as the beacon moves between the cameras, out of their fields of view.

The loss of stereo ranging might be cured by making the outboard cameras "crosseyed." Although time did not peimit a simulation of this modification, we believe it will greatly improve performance at close range. However, this change tightens constraints on camera placement and spacecraft design, since it will be harder to avoid obstructing the fields of view of the cameras.

Because the "rainbow" beacon approach requires a significant delay between observations to properly derive target attitude, the approach is inherently slower than the three-light approach. We do not recommend it for use with rapidly tumbling targets. With modifications to the Kalman filter, it may be able to cope with modest tumble rates, but this has not becn demonstrated.

The burden on the onboard computer is greater with this approach than it was with the three-light approach, and it requires more hardware on the target and chase vehicle. If one considers only the hardware that would not be required $W$, nout the video guidance system, the difference In quantity is approximately a factor of five. The power requirement and weight can also be expected to increase by a factor of five. Furthemore, the hamware in the "rainbow" system is more complex and less rugged.

The beacon cannot use flash lamps, because it must produce a continuous spectrum. Although careful design may allow a low-power rainbow proJection system, we duibt that power consumption can be reduced tc withIn a factor of ten of what is possible with a three-light system.

The measurement model for simulating the television cameris in the "rainbow" system was identical to the model used for the threc-light systen. The same resolution was modeled, and the same assumftions were made. The calculations are in subroutine CENTRD.

The model for the rainbow itseli, however, is unique. The program first computes the true angular error from the docking axis about the target $z$ and $y$ axes. This calculation fully takes into consideration the offset between the beacon and the target spacecraft center of mass and the offset between the color analyzer and the center of mass of the chase vehicle.

The validity of the measurement is then tested. If the color decoder cannot "see" the beacon, the program recognizes that the measurement is unuseable, and the guidance system ignores the measurement.

Otherwise, the program adds a random number to the compu:ed angle to simulate imperfections in the system. The accuracy of a "pitch" measurement can be expected to deteriorate when the "yaw" angle becomes large. Similarly, "yaw" measurements will be poor when the "pitch" angle is large. As the colors get crowded close together near the axis about which they are spread out, it becomes harder to separate them. The program models this effect by making the corruption to "yaw" measurements proportional to "pitch" and by making the corruption to "pitch" measurements proportional to "yaw."

The program then rounds of $f$ the simulated measurements to the nearest multiple of 0.01 radian and limits the range of measurements to $\pm 1$ radian. This procedure models the inherent quantization in the angle decoding circuit and the limited rainbow width achievable with a practical rainbow projector.

The computations for "yaw" measurements are in subroutine GETYAN. The computations for "pitch" measurements are found in subroutine GETPCH. The structures of these subroutines are virtually identical. Subrou$t$ ine DOCK selects one or the other of these subroutines for each meassrement to model the fact that both measurement types cannot be taken simultaneously.
C. image of ring of lights leaves roll undefined in third system

1. Description of Technique and Equations Used

Figure $10-10$ shows a third docking aid that can be analyzed with a small dedicated video signal processor. The required circuit (Fig. IV-11) uses the equations in Figure IV-12 to analyze television image of the docking aid in $1 / 30$ second. In the simulation program of Appendix $C$, this hardware analysis is simulated in subroutines LOCATE

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and MCALC. The circuit compites parameters defining an ellipse tha approximates the appearance of the image. These parameters can be interpreted in software to get position and attitude, except that target rotations about the target $x$ axis produce no visible effect. This ambiuguity does not matter if the docking fixture is at the center of the ring of lights and allows mating with an arbitrary rol: misalignment and if the camera is mounted at che center of the chase vehicle's docking fixture.


Figure IV-10 Ring-of-Lights Docking Aid
Ring Encircles Docking Fixture. Spacecraft Body Coordinate System Is Shown. Docking-Aid Coordinate System Is Parallel to Body Frare But Centered at the Niddle of the Ring of Lighte.

Because the video processor analyzes the image as a whole, it is ikportant for the ring of lights to be the only significant source of light in the image. Flashing-light and color-keying approaches can be used with this docking aid in the same way they dere used in the thiee-light sjstem. All of the lights in the ring must flash simultaneously if flash tubes are used.

The parameters $X_{c}$ and $Y_{c}$ from the scene analysis circuit are the coordinates of the center of the ellipse on the camera's focal plane. The morents $I_{x x}, I_{x y}$, and $I_{y y}$ are intermediate resulcs that must be processed further in the flight computer. The computer calculates the sllipse semimajor and semiminor axes (a and $b$, respoctively) and the rotation of the smajor axis from horizontal ( $\theta$ ). The formules are:


Figure IV-11 Possible Hardware to Compute Centroid and Ellipse Parameters

$$
\begin{aligned}
& A V E=\int \underset{\text { Frame }}{V d t}
\end{aligned}
$$

$$
\begin{aligned}
& I_{x y}=\int_{\text {Frame }}^{V o X ~ Y ~} d t / A V E \\
& I_{x x}{ }^{\prime} \\
& I_{x x}=\int_{\text {Frame }} \text { er }^{2} d t / A V E \\
& I_{y y}^{\prime} \\
& I_{x y}{ }^{\prime}
\end{aligned}
$$

Figure IV-12 Equations Solved by Circuit of Figure IV-11
HOLDOUT MEMS


$$
\begin{aligned}
& I_{x x}^{\prime}=I_{x x}-x_{c}^{2} \\
& I_{y y^{\prime}}^{\prime}=I_{y y}-X_{c}^{2} \\
& I_{x y}^{\prime}=I_{x y}-x_{c} \bullet Y_{c}
\end{aligned}
$$

$$
\begin{align*}
& \theta=\frac{1}{2} \tan ^{-1}\left(\frac{2 I_{x y}^{\prime}}{I_{x x}^{\prime}-I_{y y}^{\prime}}\right) \quad \begin{array}{l}
\text { ORIGINAL PAGE IS } \\
\text { OF POOR QUALITY. }
\end{array} \\
& \lambda_{1}=\frac{1}{2}\left(I_{x x}^{\prime}+I_{y y}^{\prime}+\sqrt{\left.\left(I_{x x}^{\prime}-I_{y y}^{\prime}\right)^{2}+4 I_{x y}^{\prime 2}\right)}\right. \\
& \lambda_{2}=\frac{1}{2}\left(I_{x x}^{\prime}+I_{y y}^{\prime}-\sqrt{\left.\left(I_{x x}^{\prime}-I_{y y}^{\prime}\right) 2+4 I_{x y}^{\prime 2}\right)}\right.  \tag{IV-38}\\
& a=k \sqrt{\max \left(\lambda_{1}, \lambda_{2}\right)}  \tag{IV-39}\\
& b=k \sqrt{\max \left(\lambda_{1}, \lambda_{2}\right)} \tag{IV-40}
\end{align*}
$$

where $k$ is a constant that depends slightly on the exact target geometry. In this simulation, which models eight-point light sources, $\mathrm{k}=\sqrt{2}$. These calculations are performed in subroutine EPAR in the simulation program.

Because the roll component of the target's attitude is not observable, the computer assigns it an arbitrary value. Specifically, roll orientation is assumed to place the camera in the $\mathrm{X}-\mathrm{Z}$ plane of the docking aid coordinate system. This coordinate system is parallel to the target spacecraft system but is centered at the middle of the docking aid.

With this arbitrary roll value assigned, a vector $\underline{r}$ can be computed, which is analogous to the vector $\underline{r}$ in the other two video guidance systems discussed previously. The formulas are:

$$
\begin{equation*}
\rho=0.5 \mathrm{~d} f / a \tag{j}
\end{equation*}
$$

$$
\begin{equation*}
\beta=b / a \tag{IV-44}
\end{equation*}
$$

$\underline{\underline{r}}=\left[\begin{array}{l}-\rho \beta \\ 0 \\ \rho \sqrt{1-\beta^{2}}\end{array}\right]$
where:
$P$ is the range to the target,
$\beta$ is the ratio of the lengths of the semiminor axis and semimajor axis of the ellipse,
$\underline{r}$ is the position of the camera in a reference frame that is parallel to the target spacecraft frame but centered at the center of the ring of lights.

In the simulation program, these equations are implemented in subroutite ELIP, and $\underline{r}$ is represented by the array RELPOS.

This system also computes a vector $c$, analogous to tho vector $c$ in the other two systems. This vector specifies the camera's location in a reference frame that is parallel to the "primary" reference frame used for navigation. The origin of this frame is at the center of the ring of lights. Equation (IV-6) is used to compute this vector as in the other two systems, except that $X_{C}$ and $Y_{c}$ from the video-processing hardware fill the role of $u_{2}$ and $v_{2}$ in the equation.

In the other two systems, $\underline{r}$ and $c$ were combined to determine the attitude of the target. The same approach is used in this system, but the details of their use differ.

In the three-light system, there was no ambiguity in the interpretation of an observation. A single measurement was therefore sufficient to define the chase vehicle's position. In the "rainbow" system, there was an ambiguity because target rotations about the line of sight were not observable. Two different measurements were therefore combined to determine the target's attitude uniqiely.

In the ring-of-1ights system, there are two sources of ambiguity. First, target rotation about its $x$ axis is undetectable. This ambiguity is unresolvable but causes no problems if the ring or lights encircles the docking fixture as was discussed previously. The second ambiguity is of greater concern; the target's center of mass may be on either side of the major axis of the ellipse (Fig. IV-13). If the guidance system simply guesses, it will drive the chase vehicle farther from the docking axis. Instead of correcting position errors, it will make them worse.


Figure IV-13 Two Ellipse Interpretations
In ía) the Chase Vehicle Should Steer Down and to the Right to Reach the Docking Axis. In (b) the Chase Vehicle Should Steer Up and to the Left. A Wrong Guess Results in an Unstable Control System.

This problem is also solved by combining two measuremenis, but the method for combining the measurements is completely different.

The procedure starts by picking a plausible interpretation from the equations:

$$
\begin{equation*}
\underline{\mathbf{w}}=\underline{r} \times \underline{c} \quad \text { (cross product) }, \tag{IV-46}
\end{equation*}
$$

$\phi=\tan ^{-1}\left(\frac{\lfloor w \mid}{\underline{r^{\bullet}} \underline{c}}\right)$.
(IV-47)
$\underline{q}^{\prime}=\left[\begin{array}{c}\underline{W} \sin (\phi / 2) /|w| \\ \cos (\phi / 2)\end{array}\right]$
(IV-48)

These equations are identical to equations (IV-7) through (IV-9), which were used in the first two systems, and they serve the same purpose here. The quaternion $q^{\prime}$ defines a target attitude that aligns the $\underline{r}$ and $c$ vectors. This quaternion must now be corrected for rotation about the line of sight. In the first system this was done by comparing the observed target $-y$ axis with a predicted observation that was based on $q^{\prime}$. With the ring-of-lights docking aid, the -y axis cannot be observed. The -x axis, however, is known to lie along the ellipse minor axis in the image, and this fact can be used to form the correction for rotation about the line of sight. The reasoning is identical to that used in deriving equations (IV-10) and (IV-11), but the use of a different axis results in slightly different formulas:

$r=\tan ^{-1}\left(\frac{s_{2}}{s_{3}}\right)-\theta$.
where $s$ is the projection of the target $-x$ axis unit vector onto the chase vehicle's $y-z$ plane. The angle $\gamma$ is the amount of rotation about the line of sight required to make the predicted ellipse appearance match the observed ellipse. The angle $\theta$, the orientation of the ellipse in the image, is computed by the video signal processing hardware shown in Figure IV-11.

The corresponding quaternion is computed with the equations:

$$
g^{\prime \prime}=\left[\begin{array}{c}
\underline{r} \sin (\gamma / 2) /|\underline{r}|  \tag{IV-51}\\
\cos (\gamma / 2)
\end{array}\right]
$$

$$
q=q^{\prime} q \text { " } \quad \begin{aligned}
& \text { ORIGINAL PAGE IS } \\
& \text { OF POOR QUALITY }
\end{aligned}
$$

The alternate interpretatiun, in which the target spacecraft's center of mass lies on the opposiie side of the major axis of the ellipse, is formed by rotating the target 180 degrees about the line of sight. This rotation is done with the formulas:

$$
q^{\prime \prime}=\left[\begin{array}{c}
\underline{r} /|\underline{r}|  \tag{IV-53}\\
0
\end{array}\right]
$$

$q^{*}=q q^{\prime \prime}$.

All these equations are implemented in the simulation program in subroutine QUAIRN, where $q$ and $\underline{q}^{*}$, the possible target attitude quaternions, are represented by the first and second columns of the $4 \times 2$ array QT, respectively.

After the first observation of the target, the guidance system has no basis for preferring one attitude interpretation over the other, but after the second and subsequent observations, it can compare $q$ and $q^{*}$ from the current observation with the values from the previous observation. If the target is not tumbling too rapidly, one pair of attitude measurements will match closely. This pair can be assumed to represent the proper image interpretation.

One problem in implementing this method is that the quaternions all contain arbitrary components representing roll avaut the target $x$ axis. Some means is required for comparing two quaternions while ignoring this roll component in each. The guidance system simulated in this study makes this comparison with the equations:

$$
\begin{align*}
& a=q_{1} Q_{1}+q_{2} Q_{2}+q_{3} Q_{3}+q_{4} Q_{4},  \tag{IV-55}\\
& b=q_{1} Q_{4}+q_{2} Q_{3}-q_{3} Q_{2}-q_{4} Q_{1}  \tag{IV-56}\\
& \delta=\cos ^{-1} \sqrt{a^{2}+b^{2}} \tag{IV-57}
\end{align*}
$$

where $q$ and $Q$ are any two quaternions to be compared, and $\delta$ is smallest possible Euler angle that will account for the difference between $g$ and Q, making the most generous assumptions about the arbitrary roll angle. Here variables $a$ and $b$ are intermediate results and are used only to simplify equation (IV-57). They should not be confused with the ellipse parameters used in other formulas.

Equations (IV-55) through (IV-57) are implemented in the simulation program in subroutine TSTATT. This subroutine is called by subroutine SELECl to test all possible combinations of old and new target attitude interpretations to find the best match. SELEC1 returns pointers that indicate which pair is assumed to represent the true target attitude.

The guidance system uses the selected interpretation of the current observation for control, but it remembers both interpretations tor testing the next observation and for propagating the state estimate. lwo sets of parameters are maintained for the onboard mathematical dynamics model. After each observation it throws away the parameter set that was based on the target attitude incerpretation that the current observation shows to be false. This maintenance of two parameter sets minimizes errors caused by operation with a tumbling target.

## 2. Mission Constraints and Compatibility

Because this system cannot detect roll misalignment of the two spacecraft, it is essential that such a misalignment makes no difference. This requires the camera to be mounted very close to, or even within, the docking fixture. The ring of lights must encircle the docking fixture of the target spacecraft. Also, the docking fixtures must be designed to operate properly with an arbitrary roll misalignment.

The hardware power requirement for this system will probably be only slightly more than that of the three-light system, and the hardware is not much more complex than that system's hardware. The burden on the flight computer is also slightly greater.

In all other: respects, the mission constraints imposed by this system will be essentially the same as those imposed by the three-light system.
3. Measurement Model for Simulation

The simulation program used the perspective projection techniques described in Chapter IX to provide "measurements' from this system. The center of brightness coordinates for each light were corrupted to represent $380-1 i n e$ television resolution. The method used to do this was the same as the method used with the three-light system. The equations in Figure IV-12 were simulated with summations that approximated the required integrals. These calculations are in MCALC of the simulation program.

## V. Simulation Results and Discussion

v. SIMULATION RESULTS AND DISCUSSION
A. THE THREE-LIGHT SYSTEM WORKS BEST

In more than 50 simulations run with the computer programs in Appendices A through $C$, the three-light version consistently outperformed the other rwo systems. Figure $\mathrm{V}-\mathrm{l}$ shows six typical trajectories produced with the three-light system. Each simulation started with the chase vehicle at a random position approximately 300 meters from the target spacecraft and was assigned a random initial velocity. Target attitude varied from one simulation to the next, but the target's attitude rate was zero. In each of these simulations, and several others not shown, docking was successful.

Figure V-2 and V-3 show the results of similar simulations with the "rainbow" and ring-of-lights systems, respectively. Both systems performed reasonably well at distances over 150 meters from the target, but both had trouble at close range. The "rainbow" system successfully docked or came very close tost of the time, but the final alignment was never as good as the three-light system achieved. More serious is the fact that the system occasionally became "confused" and wandered away from the target. This is what the control system is supposed to do if the target leaves the field of view of the television cameras. It is a preprogrammed maneuver to avoid a collision when the system is "blind." However, the system should not have had to take this action with such benign starting conditions.

In some simulations where the simulation time limit was increased, the chase vehicle made another pass at the target when this happened. The onboard mathematical model, described in Chapter VIII, was provided to allow recovery from a temporary loss of imagery. We believe, however, that it is significant that the three-light version did nut have the problem.

Three explanations can account for this behavior of the "rainbow" system:

1) The projected rainbow does not produce valid data when it is viewed from more than one radian from the docking axis. This restriction models what we believe to be a reasonable estimate of the limitations of a practical system;
2) The stereo rangefinder's cameras are mounted at the sides of the chase vehicle, and at close range the image of the beacon moves toward the edge of each camera's field of view. Because of these two limitations, small errors in position and attitud at close range can cause loss of ranging information or incorrect angle measurements;
3) The chase vehicle may be moving too little between successive observations, causing errors in determining the target's attitude.


F'igure V-1
Tupical trajectories with three-light system: a) and b) show entire trajectory; c) through f) ars closeup views of the last 60 m . Ali trajectories had an initial range of approximately 300 m .


Figure V-2
Typical Trajectories with mainbow system: $a ;$ and $b$ ) show entire trajectory; c) through f) are closeup views of the last 60 m . All trajectories had an initial range oj approximately 300 m .


Figure V-3
Typical Trajectories with Ellipse systam: a) and b) show entire trajectory; c) through f) are closely views of the last 60 m . All trajectories had an initial range of approximately 300 m .

The ring-of-lights system never docked successfully. In fact, in mos* of the simulations it appeared to actively avoid the target spacecraft. The problem appears to be in the method it uses to resolve ambiguity in interpreting the image of the ring of lights. This ambiguity (Fig. IV-13), is resolved by combining pairs of successive observations, and the algorithm used is not robust enough to cope with modeled imperfections in the measurements and control system. We have found no simple algorithm change that measurably improves performance.
B. THE THREE-LIGHT SYS IEM WORKS WITH TUMBLING T،akGETS

Because the "rainbow" and ring-of-lights systems did not work well even with an inertialiy stable target, we believed it was pointless to test them wich iumbling targets. We therefore concentrated on testing the three-light system.

Figure V-4 shows trajectories for target roll rates of 540 to 40,000 degrees per hour. The cuntrol system appears to hardle rates up to 20,000 degrees per hour with good accuracy. Atove this rate, performance is degraded by gyroscopic torques caused by attempting to maintain roll alignment with the target at all times. If the chase vehicle did not rotate in synchronism with the target, much better performance could be expected, because the accuracy of the measurementr is not measurably degraded at 20,000 degrees per hour. (Rotation affects the measurement accuracy only by moving the lamps, and the motion during the time an observation takes is only 2 percent of the spacing between lamps.) However, if the chase vehicle does not rotate, the cking fixture must be designed to accept relative rotation between the two spacecraft.

Figures $V-5$ and $V-6$ show the performance of the three-light system with target spacecraft that rotate about their pitch and yaw axes. Agajn the limiting factor was the control system, not measurement accuracy. If a control system is to operate with target attitude rates over 700 degrees per hour above these axes, some form of attitude prediction will probably be needed. This can be implemented without changing the structure of the contrcl system but will require more parameters in the onboard mathematical dynamics model described in Chapter VIII. It may also require an increase in the number of state variables used in the Kalman filter portion of the control system. The result will be an increase of approximately 20 percent in the computational burden on the flight computer. We do not belleve this will be a major problem in system design.


Figure V-4
Trajectories with three-iight system with various target tumble rates about the docking axis: a) with $540^{\circ} / \mathrm{hr}$; b) with $1000^{\circ} / \mathrm{hr} ; c^{\prime}$ with $4000^{\circ} / \mathrm{hr}$; d) with $10,000^{\circ} / \mathrm{hr}$; e) with $12,000^{\circ} / \mathrm{hr}$; and f) witn $20,000^{\circ} / \mathrm{hr}$.
(a)

(c)

(e)

(b)

(d)


Figure V-5
Trajectories with three-light system with various target twmble rates about the pitch axis: a) with $540^{\circ} / \mathrm{hr}$; b) with $1000^{\circ} / \mathrm{hr}$; c) with $2000^{\circ} / \mathrm{hr}$; d) with $4000^{\circ} / \mathrm{hr}$; e) a closeup of d); and f) with $8000^{\circ} / \mathrm{hr}$.

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(a)

(c)

(e)

(b)

(d)

(f)


Figure V-6
Trujectories with three-light system with various twombe rates about the you axis: a) and b) ane a stereo pair with $540^{\circ} / \mathrm{hr}$ tumble rate; a) with $1000^{\circ} \mathrm{hr}$; d) with $2000^{\circ} / \mathrm{hr}$; e) with $4000^{\circ} / \mathrm{hr}$; and f) with $8000^{\circ} / \mathrm{hr}$.

## C. THERE IS ONLY ONE OPTION

The selection of the best system does not require a tradeoff matrix because only the three-light system worked reliability. It is interesting to note, however, that the system would be selected even if the other systems had performed as well. Consider how the systems compare in the 10 characteristics summarized in Table $V-1$. It is better than the other two systems in almost every column of the table.

The table entries shown in the cost column are only for comparison and aie based on the known costs of hardware of similar complexity. Although the figures do not include such indirect costs as software development and impact on other systems, these costs can be expected to vary roughly in proportion to the hardware costs. None of the systems requires the expensive packaging associated with high voltage, none requires a zoom lens, and all can operate in the red/near-infrared portion of the spectrum for which inexpensive sensors are available. All of the systems will require some form of iris or threshold control, and this control will have comparable complexity in all three systems. Because the color sensor and rainbow beacon of the "rainbow" system are the only elements in any of the systems based on totally untried technology, the "rainbow" sysiem represents by far the greatest risk.

All three systems have an algorithm hazard in common; they must be able to determine when the target is not within the television cameras' field of view. When the distance to the target is great, the video signal amplitude will fall with the square of the distance to the target, and it may be difficult to determine whether the target is further away than expected or is out of the field of view. We recommend a physical simulation to demonstrate that a hardware system can solve this problem.

In addition to this common hazard, the "rainbow" and ring-of-1ights systems have the problems discussed previously that caused them to perform poorly in the simulations.

The speed of the three-light system is set primarily by the video frame rate, and the figure shown in Table $V-1$ is based on standard video rates. A special-purpose camera might be developed for higher speed. The other two systems, in contrast, are limited in speed by more fundamental constraints. In both of these systems, two successive observations are combined to define target attitude. Enough time must be allowed between observations to ensure that the observations are significantly different from one another. Therefore these two systems can never achieve the speed of the three-light system.

Accuracy and television resolution are closely related. The resolution specified for the three-light system was based on simulations that showed the system works with resolution three times poorer. A factor of three safety margin was added to increase confidence while adding little to the cost; a $380-1$ ine resolution is approximately that of standard television cameras.

Tuble V-1 Tradeoff Matrix for Three Rembezvous and Dockiny Systems

|  | Hardware Cost <br> (1) | Algorithm Hazalds (: ) | decuracy | Speed | Sire <br> and <br> Weight <br> (equif <br> (4) | Power Required | Min/Max Range | Kesolution Required | Rugged- <br> ness <br> and <br> Hard- <br> ware <br> Relia- <br> bility | System Compat1bility |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3-Light <br> System | \$300,000 | Moderate | 3. at 20 <br> Meters <br> 47\% at 300 Meters | Up to 10 Measurements per Second | $0.02 \mathrm{~m}^{3}$ 5 kg | 10-15 Watts | $0-300 \mathrm{~m}$ | ```380 Television lines``` | Good | Good |
| Rainbow System | \$500, 000 | Severs | 4* at 20 Meters 7\% at 300 Neters | Up to 1 <br> Measure- <br> ment <br> per <br> Second <br> (3) | $0.04 \mathrm{~m}^{3}$ $1: \mathrm{kg}$ | 15 Watts | 0-300m | ```380 Television lines``` | Fair | Fair |
| Ellipse <br> Svitem | \$300,000 | Severe | 41\% at 20 Meters <br> 23* at 300 <br> Neters | Up to 1 <br> Neasutement per second (3) | $\begin{aligned} & 0.03 \mathrm{~m} \\ & 8 \mathrm{~kg} \end{aligned}$ | 20 Wates | 0-300m | ```380 Television Lines``` | Good | Severe Probleas |

(1) For Comparisun Unly - Based on Cost per kg of Similar Hardware
(2) See Text
(3) Conatraint Impozed by Having to take Two Measurements; if Neasurements Are Too Close, Noise Will Be Dominant
(4) Chase Vehicle Sustem

The resolution for the other two systems was specified at a level high enough to convince us that accuracy was not the dominant cause of fallure. Simulations with much better resolution demonstrated no better performance.

All three systems are rugged, but the "rainbow" system uses a rainbow projector and a color detector that may require fragile parts and precise alignment. The types and the quantity of components make this system rate the lowest in terms of hardware ruggedness and reliability. The differences among the systems are so great in other respects, however, that chis factor is insignificant.

Ail three systems place similar constraints on the system:

1) Nothing must be allowed to block the field of view of the cameras or color detector;
2) The propulsion system must be able to support the added mass of the guidance system;
3) The power system must provide power for the additional hardware:
4) The coarse rendezvous system must provide a sufficiently accurate estimate of the relative positions and velocities of the two spacecraft to give the video system time to search for the target, lock on, and begin to track.

Additional constraints are discussed in Chapter IV. However, of the three systems the three-light system imposes the fewest constraints; it has only one camera, and the camera does not have to be placed at the docking fixture, it requires the smallest amount of power and weighs the least. Even if the other systems worked well, they would place more constraints on the two spacecraft.
D. THE IARDWARE TECHNOLOGY IS AVAILABLE

The three-light system does not require any new developments in hardware to make it practical. It can use television cameras of standard resolution and speed operating in a convenient spect:al band. The burden on the flight computer is modest and will not require a special computer design.

We do not recommend, however, that fully autonomous operation be tried with the first such system built. We recommend that the system be used first as an aid in remote piloting of the chase vehicle. As the system proves its ability, it can be given greater autonomy from mission to mission. This approach of gradually increasi.ng autonomy minimizes risks and allows the collection of valuable data that cat be used to improve system perfonnance.

$$
v-11
$$

VI. Two Other Candidate Systems

## A. CORRELATION SYSTEM WAS TOO EXPENSIVE

In most correlation guidance schemes, a reference scene is compared to the live scene from a television camera. The center of the reference scene is placed at different locations on the live scene, and the correlator computes the quality of the match with each alignment. The system uses the coordinates that $g$ ive the best match between scenes for horizontal and vertical error signals.

The rendezvous system cannot afford the time to do that because the misalignment can be in six degrees of freedom, not just two. The rendezvous system would have to match scenes with several different horizontal displacements for several individual vertical displacem ats, for each zoom lens setting for each of several rotations of the camera about the optical axis, for each of several reference scenes showing the target from different angles. The number of comparisons required for each measurement is astounding, perh:ps ten billion at the very least, and each comparison may require 2000 or more operations.

The number of comparisons must be reduced, and t.is is possible. The guidance system can use the centroid-tracking and ellipse-fitting techniques described in Chapter IV to greatly reduce the number of degrees of freedom. The system we considered used these techniques to control the camera's zoom lens and gimbal set. This was done to keep the target image centered in the camera's field of view, to maintain a constant image size, and to keep the major axis of the best-fit ellipse horizontal. The eccentricity of the best-fit ellipse was to be used to select a subset of reference scenes for comparison. The correlator would then need only to select the reference scene that best matched the live scene.

Without question this system was the most complex of the five systems that were evaluated, though it was less complex than pattern interpretation systems and more tolerant of sensor imperfections. It had the advantage of requiring no special docking aids on the target spacecraft. With modifications to its database of comparison images, it could, in principle, be used with any target. However, the scheme could not compete with the other concepts in accuracy or cost.
B. CURVE-FITTING SCHEME DUPLICATED MSFC EFFORT
R. Dabney of NASA's George C. Marshall Spaceflight Center describes a video guidance scheme in MSFC memorandum EDI5-81-71. His system used the same ring-of-lights docking aid as one of the systems described in Chapter IV, but its method for fitting an ellipse to the ring's image
was considerably different, and a different approach was used to cierive guidance information from ellipse parameters.

This system was not evaluated under this study, because more could be gained from the study of systems that had not already been analyzed and simulated, as this one had.

## VII. Other Techniques

 That Were InvestigatedTable VII-l summarizes the approaches that wero considerej for use in the video guidance system. Manv of these are discussed in detail in Chapters IV and VI. This chaptior discusses the rejected apnroaches.

Table VII-1 Techniques Investi,jated

- Interpreting an Image of Three Lights
- Fitting an Eliipse to the Image of a Ring of Lights
- Correlation
- Recognition of Corners and Edges
- Projected Pattern from Target
- Recognition of a Bar Pattern on the Target Spacecraft
- Detection of Target Color, which Varies with Ooservex's Position
- Stereo Rangefinc: :ug
- Stadimetry
- Rangefinding by Optical Focusing
A. CORNER/EDGE RECOGNITION IS TOO EXPENSIVE

Many of the studies on robotic vision have concentrated on the detection of edges and corners. While this approach holds promise in many robotics applications, it is not recommended for an autonomous video rendezvous system. There are two redsons:

1) Fast algorithms are easily fooled. The guidace system must contend with an image of extremely high contrast and with shadows that are almost totally black. Although contrast aids iu the detection of corners and edges when the target's geometry is simple, the variable collection of instruments on the target gorcecraft will result in a complex patern of shadows that will min. $1^{\text {+ }}$ the interpretation very difficult.
2) Better algorithms cannot compete in speed or cost. Speed is vital when the system must operate with tumbling targets, and software analysis of a complex image cannot come close to the speed of the other algorithms considered. Further, the better algorithms store the entire image, which may require as much as a quarter of a million bytes of memory. A detailed cost analysis is not required to conclude that such a system will be far more expensive than one that uses one of the selected algorithms.

The Universal Product Code has been highly successful in automating checkout linez in supermarketr. In principle, a pimilar bar-code pattern could be used for locating key reference points on a spacecraft. Although this approach would greatly simplify the target, it greatly complicates the overall system. First, it requires good lighting to ensure that the bar patterns are visible. Because the target may be tumbing, sunlight will be unreilatle. While the problem is not insurmountable (a backlit pattern might be used, for example), the approach remains more susceptible to lighting problems than the selected approaches.

The hardest problem to solve, however, is resolving the details of the pattern. The pattern must be small if it is to be effective for identifying reference points, and the small size makes it difficult to use, especially when the system must operate at all ranges from zero ts wer 300 meters. Other techniques such as those discussed in Chapter IV, circumvent these problems and provise the same capabilities at a lower cost.
C. RANGEFINDING BY OPTICAL FOCUSING IS HARD TO USE

Optical focusing, at first glance, appears to be a simple method for determining range, but a number of practical problems make it very difficult to implement:

1) The system does not know what is in focus. A practical guidance system needs to detemine the range to a known referefce point on the target spacecraft to adjust approach speed for a oft contact with the target. A simple focusing system attempts to adjust for maximum sharpness of the image as a whole, and there is no guarantee that a known reference point will be in sharp focus. While this $1 ;$ it a problem at great distances from the target, it may cause stvere problems in deriving velocity at close range, and proper alignment of the docking fixture may require additional handware that partially duplicates the function of the rangefinder.
2) A telephoto lens is required at long range. Lenses with wide fields of view and modest $F$ numbers have great depth of focus; one lens setting may produce sharp focusing for any range from a few meters to infinity. A large-aperture telephoto lens is therefure required t; make effective use of optical focusing. Such lenses, however, are expensive and their narrow ilelds of view introduce pointing problems. A gimballed pointing mount may be required, along with a complex state estimation algorithm to keep the target centered in the field of view of such a lens.
3) Automatic focusing is a noisy operation. Sharp focusing is detected by measuring the high-frequency content in the video signal,
because sharpest focusing is associated with the highest high-spa-tial-frequency content in an image. Finding the point of sharpest focus is analogous to finding a point of zero slope on a curve of sharpness versus lens setting. Both operaticns involve the calculus operation of differentiation. This operation is always "noisy" in that it emphasizes signal corruptions, and the double differentiation is especially noisy. The result is that an accurate system will be slow, expensive, or both.

Although it may be argued that accurate information is not required at great distances from the target, the simulations have shown that systems which are not accurate do not work well. The problem 13 that the guidance system cannot determine the approach rate. Therefore, it allows the chase vehicle to drift away from the target or drives the vehicle toward the target at such a high speed that it cannot stop.

## VIII. A Generic Video Guidance System

The simulation programs in Appendices A thru $C$ are adaptations of a single original program called DSIM, which was developed by Martin Marietta to test our autonomous viden rendezvous guidance system. The program and the guidance system were developed under IR\&D project D-llR. Because a major portion of this guidance system is cnamon to all three simulations, a detailed explanation of the system is presented.

Figure VIII-1 is a block diagram of the control system, and each block corresponds, in general, with a subroutine or set of subroutinez in the simulation programs. The exceptions to this rule are:

1) Telemetry uplink and downlink have not been implemented in these simulations;
2) Several subroutines simulate television images of the docking aid and process them to obtain position and target aititude. These subroutines, which differ among the simulation programs, were written to perform specific functions rather than to correspond to specific hardware modules in the block diagram;
3) Subrou'ine DOCK in each simulation is the "wiring" among the blocks; it sequences operations and passes data anong blocks.

Table VIII-1
Relationship between Block-Diagram Slocks and Subroutines in the Simulation Programs

| Block Name | Subroutines* |
| :---: | :---: |
| Kalman Filter | IHCORY, (COMPG, ESTCOV, KALGAN, UPDSTA, UPDCOV) |
| Mathematical Dynamics Model | PROPES, RPY, ESTRPY |
| Inertial Measurement Unit | IMU, (TIRMAT, ANGVEC) |
| Goal-Seteing Logic | SETGOL |
| Control Law | THRUST, (CNTLAW, ACCEL, FIRTHR) |
| Thrusters | SELECT, (TABLE1, IABLE2, TAELE3) |
| Chase Vehicle Dynamics | 9ROPTR, 'COKPK1, COMPK2, COMPK3, CUTPK4, POINT, STPRIM, ANGVEC, IINACL, MPRIEE, FORCE, DIRMAT, TORQUE, LPRIME, MAKROT, QPRIME) |
| Target Spacecraft Dynamics | IRGATT |

Table VIII-1 shows the correspondence between program subroutines and blocks in the block diagram. Subroutine nares are shown in parentheses if they are not the primary routines invclved in implementing a block. These routines are called on by the primary routines to implement lowlevel details of a function.


Figure VIII-1 Genern Video Rendezvous Guidance Systems
Only Major Signal Paths Are Shown. The Simulations Reported Here Do Not Use TM Upiink and Downlink, and Cnly Minimal Forms of the Kalman Filter', Mathematical Model, and Goal-Setting Logic Were Implemented.
A. THE KALMAN FJLTER IMPROVES ACCURACY AND DERIVES RATES

The Kalman filter combines observations to fine-tune the parameters in a mathematical dynamics model, providing accurate estimates of position and velocity. In these simulations, the filter adjusts six parameters, the $x, y$, and 2 components of position a:d velocity of the chase vehicle relative to the target. It can alsu be expanded to estimate such other parameters as target tumbling characteristics.

Although a mathematical dynamics model is an essential feature of a Kalman filter, the model is considered to be a separate entity in this control system for two reasons:

1) The model would be used in the ccntrol system even if the filter were n'st. The primary reason the system maintains the model is to allow it to operate properly even when feedback is interrupted by obstruction of the camera's view of the target. The model is not simply a piece of the Kalman filter;
2) The model contains parameters that are not adjust 2 d by the filter. The parameters include the latest chase vehicle attitude quaternion and angular velocity vector from the inertisl measurrment unit, the latest target attitude measurement, information about the geometry of both spacecraft, an estimate of the mass of the chase vehicle, optical system parameters, and other data.

The subroutine that implements the Kalman filter is called INCORP. Fach time a new measurement is taken, this subroutine updates the state estimate ( $x$ ) and the state covariance matrix ( $P$ ) by calculating

$$
\begin{align*}
& K+P G^{T}\left(R+G P G^{T}\right)^{-1}  \tag{VIII-1}\\
& \underline{x}-\underline{x}+K(\underline{z}-g)  \tag{VIII-2}\\
& P+(I-K G) P \tag{VIII-3}
\end{align*}
$$

where
$K$, represented in the programs as KGAIN , is a $6 \times 3$ matrix referred to as the Kalman gain matrix,
$P$, repres nted in the program as $P$, is a $6 x G$ matrix, the state covar ance matrix,

G, represented in the programs as $G$, is a $3 \times 6$ matrix, the partial derivative of the predicted measurement with respect to the state. Since the measurements in all three simulations are the first three elements of the state vector, $G$ is a constant with the value

$$
\left[\begin{array}{llllll}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{array}\right]
$$

It is, however, "computed" in subroutine COMFG to maximize the usefulness of the program for other simulations in which $G$ is not constant,
$R$, represented in the programs as $R$, is a $3 \times 3$ matrix, the measurement covariance matrix. This matrix is calculated from an empirical formula in subroutine ESTCOV. The formula was derived by observing the measurement errors at various ranges in several rendezvous simulations and fitting a curve to a graph of the mean square error versus range. In the design of a flight system a similar approach could be used, but test data and analytically derived tolerances would be used instead of simulation results,
x, represented in the programs as ESTATE, is the estimated chase vehicie state. This is a sixelement vector, the first three elements of which represent position along the $x, y$, and $z$ axes of the socalled "primary" reference frame. The remaining elements are the velocity components along these axes. (The primary frame is a non-
rotating right-handed rectangular coordinate system centered at the target spacecraft's cencer of mass but aligned with the body axes of the chase vohicle at the instant the video guidance system takes control),
z, represented in the programs as CVPOS, is a three-element measurement vector representing the measured position of the chase vehicle's center of mass in the primary reference frame,
g, which is not explicitly represented in the programs, is the predicted observation. In these implementations of the control system, g is simply the first three elements of $x$,

I, the $3 \times 3$ identity matrix, is not explicitly represented in the programs.

The Kalman filter form used here does not have the best roundoff and stability characteristics for flight software, but is ideal for simulations because it is sasy to modify.
B. THE MATHEMATICAL DYNAMICS MODEI. ALLOWS DEAD RECKONING

The mathematical model embodies the guidance system's knowledge of the target spacecraft and chase vehicle. Some of this information--positions of the cameras, docking aids and fixtures with respect to the spacecraft centers of mass, the optical focal lengths, the thruster locations, orientations and thrust levels, and similar information--is known in advance. In the simulations, this information is passed to all subroutines that need it through common blocks. All these common blocks appear in subroutines INIPAR, where the variables are initialized and the variables' definitions, which vary from one simulation to the next, are given in comments with the type declarations in that subroutine.

A second class of information in the mathematical model is measured data that does not pass through the Kalman filter. Although the nature and use of this information vary among the three simulations, all three programs maintain target attitude, chase vehicle attitude, and the coordinates of the lamp images' centers of brightness.

The third class of information, which is identical in all three simulations, is the information processed by the Kalman filter. In the three simulations this class included only the position and velocity of the chase vehicle. These collectively referred to as the state estimate (ESTATE) and the state estimate covariance matrix (P). Additional parameters could be added to improve performance with tumbling targets.

In addition to a data base, the mathematical model has procedures for updating and using the data. The largest of these is subroutine PROPES, which uses numerical integration tn propagate the state estimate and covariance matrix between observations. The formulas it uses are

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$\underline{x}=A_{c}^{T} K_{2} \underline{f} / m$
$\Delta x=\left(\underline{\dot{x}}+\frac{1}{2} \underline{x} \Delta t\right) \Delta t$
(VIII-5)
$\Delta \dot{x}=\underline{x} \Delta t$
$\Delta \mathrm{P}=\left(\mathrm{FP}+\mathrm{PF}+N V N^{\mathrm{T}}\right) \Delta t$
(VIII-7)
where
$\underline{x}, \dot{x}$ and $\ddot{x}$ represent the position, velocity, and acceleration vectors, respectively. The position vector is the first three elements of the array ESTATE, and the velocity vector is the second three elements. The program variable name for $x$ is ACCEL,

A is the transpose of the chase vehicle direction cosine matrix, as measured by the inertial measurement unit, which is modeled in subroutine IMU. The variable name in the program is ACVT,

K2 is a constant matrix relating the torce magnitudes of all the thrusters to the force vectors they produce. The variable name in the programs is AK2,
f is a 14-element array in which each element corresponds to the magnitude of the force currently produced by one thruster. The programs refer to this array as $F$,
$m$ is an estimate of the chase vehicle mass. The programs refer to this variable as AVGMAS,
$t$ is the time interval over which the state estimate is to be propagated. The variable is referred to in the programs as STEP,
$P$ is the state estimate covariance matrix. The correspor g variable name in the programs is $p$,

F is the partial derivative of $d(E S T A T E) / d t$ with respect to ESTATE, which, in this context, is the constant matrix.
$\left[\begin{array}{llllll}0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0\end{array}\right]$
This matrix does not appear explicitly in the programs because its effect is simply to select elements of $P$ to be added together. The selection and addition were done more efficieatly by avoiding matrix arithmetic,

NVN ${ }^{T}$ is a matrix that gives the covariance of stray (unmodeled) accelerations of the chase vehicle. These accelerations arise from uncertainty in thruster force magnitude from attitude changes between the time $A^{T}$ is measurel and the time it is used, from gravity gradient acceleration, from thruster misalignment, and from roundoff errors in computation. Because this matrix is sparse (only three elements out of 36 are nonzero), it is handled without using matrix arithmetic and is not explicitly represented in the programs.

The mathematical model is also responsible for attitude control. Subroutine RPY is used if the docking aid is within the field of view. It returns the changes in roll, pitch, and yaw required to align the chase vehicle's $x$ axis with the line of sight between the camera and the docking aid. Subroutine ESTRPY is used when the docking aid cannot be seen. It estimates the changes in roll, pitch, and yaw required to align the chase vehicle's $x$ axis with the line connecting the chase vehicle's center of mass and the target spacecraft's center of mass. ESTRPY operates on the state estimate (ESTATE) whereas RPY uses the coordinates of the center of brightness of the docking aid's image. ESTRPY is the same in all three simulations, but RPY is slightly different in the three versions because of the differences in the docking aids among the three systems.
C. THE INERTIAL MEASUREMENT UNIT PROVIDES ATTITUDE AND ATTITUDE RATE INFORMATION

The guidance system uses the so-called "primary" reference frame to maintain estimates of relative position and velocity, and of the attitudes of both spacecraft. The primary frame is a nonrotating coordinate system that is initially aligned with the chase vehicle's body axes at the instant the video guidance system takes control. The center of the coordinate system is at the target spacecraft's conter of mass. Since both spacecraft rotate with respect to this frame, the guidance system must measure the chase vehicle's attitude with each video measurement to properly interpret the imagery. Subroutine IMU supplies simulated attitude measurements from the inertial messurement unit for this purpose. The attitude is returned in the form of a direction cosine matrix defining the attitude with respect to tio primary irame.

The transpose of tie direction cosine matrix and an angular velocity vector, used for control system damping, are also returned from the IMU.

The attitude is determined by examining the "true" stace of the chase vehicle, which is maintained in the 14 -element array STATE. Elements 10 through 13 of this array are the current attitude quaternion and define the attitude with respect to the "truth" coordinate system. Subroutine IMU subtracts the initial attitude from the current attitude to compute attitude with respect to the "primary" reference frame used for control. It then converts from quaternion notation to direction cosine notation.

The angular velocity vector is found from elements 7 through 9 of STATE, which are the angular momentum vector, and from element 14 of STATE, which is the chase vehicle mass. The equations are
$I=I_{0}+\left(m-m_{0}\right) d I / d m$
${\underset{b}{w}}_{\omega_{b}}=I^{-1} A_{c}$
(VIII-9)
where
I, represented in the programs as the $3 \times 3$ matrix INERTA, is the total chase vehicle inertia,
$I_{0}$, represented in the programs as the $3 \times 3$ matrix INERCV, is the inertia of the chase vehicle without fuel,
$m, m_{0}$, are the mass of the chase vehicle with the current fuel load and the mass with no fuel, respectively. In the programs m is represented as STATE (10), and $m_{o}$ is MEMPTY,
$\mathrm{d} / \mathrm{dm}$, represented in the programs by the $3 x 3$ matrix FULDIS, is the amount of additional inertia added for each unit of fuel,
represented in the progiams by the three-element array ATRATE (in subroutine IMU) or BODVEL (in subroutine ANOVEC), is the angular velocity vector expressed in the chase vehicle's body coordinate system,

L, represented in the programs by elements 7 through 9 of STATE, is the angular momentum vector of the chase vehicle, expressed in the "truth" coordinate system,
A. represented in the program by the $3 \times 3$ matrix TACV, is the direction cosine matrix that defines the current chase vehicle attitude vith respect to the "truth" coordinate system.

In these simulations the $I M U$ returns exact values for attitude and angular momentum. Although it would be easy to add corruptions to these quantities, this has not been done because the inertial measurement unit of a real spacecraft can be very accurate for the few minutes a rendezvous operation requires, and IMU errors will not be a major contributor te the total error.
D. GOAL-SETTING LOGIC PROVIDES INTELLIGENCE

The goal-setting $\log i c$ is a software module in the flight computer and is responsible for planning, safety, and hazard recognition. For example, it examines the state estimate and covariance matrix provided by the mathematical dynamics model, decides how close it should come to the target, and where its aim point should be with respect to the current position. The simulations use a minimal version of this software;

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a practical flight system might also decide what trajectory will minimize fuel use, determine if the target is tumbling so fast that docking is impossible, or decide to ask for human assistance via telemetry.

The subroutine that embodies the goal-setting logic is SETGOL. The three simulation programs use slightly different versions of this routine because the locations of lights and cameras differ among the simulations. The essential logic, however, is identical.

First, the subroutine picks a desired location for the docking fixture of the chase vehicle. This point (Fig. VIII-2) will be just beyond the end of the target spacecraft's docking fixture. It can be expressed in target coordinates as $\underline{r}=h_{d t}-d$ ), where $h_{d t}$ is the location of ine end of the target docking fixture with respect to the target's center of mass and $d=(d, 0,0)^{T}$ is a safety margin.


Figure VIII-2 Physical Significance of Equation (8-10)
Multiplicatione by $A_{c}$ and $A_{t}^{T}$ Are Done to Convert All Vectors to the Chase Vehicle's Current Body Coordinate System. Attitude Control, a Separate Operation, Keeps $h_{d c}$ Pointed Toward the Target So That, by the Time the Chase Vehicle Reaches Its Goal the Attitude Is Correct.

The value of $d$ is computed from the state estimate covariance matrix ( $P$ ) and has a value of approximately three times the standard deviation of the position estimate. If the chase vehicle is estimated to be more than 15 meters from the target's $x$ axis, $d$ is increased to encourage an approach along the $x$ axis.

When range and velocity are sufficiently small, dis set to zero. If this were not done, the chase vehicle might hover indefinitely a few centimeters from contact.

After $r$ has been determined, the subroutine finds a goal for the chase vehicle's center of mass, expressed in chase vehicle coordinates, with the formula

$$
\begin{equation*}
\underline{v}=A_{c}\left(A_{t}^{T}\left(\underline{r}-\underline{h}_{d c}\right)-\underline{x}\right) \tag{-10}
\end{equation*}
$$

where
v, represented in the programs as $V 3$, is the goal for the chase vehicle's center of mass,

Ac, represented in the programs as $A C V$, is the chase vehicle's attitude direction cosine matrix, as measured by the inertial measurement unit,
$\Lambda_{t}$, represented in the programs as TRNAT, is the transpose of the target's attitude direction cosine matrix, which is derived from the appearance of the target in the television images,
$x$, represented in the programs as the first three elements of ESTATE, is the chase vehicle's estimated position in the "primary" reference frame,
$h_{d c}$, represented in the programs as $H D C$, is the location of the chase vehicle's docking fixture with respect to the chase vehicle's center of mass.

Finally, the subroutine defies a "box" or region of space around the goal. The control law will use this box in selecting thrusters. If the chase vehicle is already within the box, the control law will attempt to pievent it from leaving. If it is not within the box, the control law will attempt to force it to enter the box before a deadine time has been reached. If the chase venicle is within the box and is not in danger of immediately drifting out of the box, the control law will not activate any thrusters.

The box concept has two advantages. First, it allows the control law to work on a single axis at a time and io ignore any interaction. Second, it allows control tolerances to be adjusted as a function of the distance to the target. This prevents waste of fuel at greater distances from the target where acrurate positioning is not important. The subroutines implement only a very simple version of the box-sizing strategy.
E. THE CONTROL LAW DETERMINES IDEAL THRUST VECTORS

The control law, subroutine CNTLAW, converts the velocity estimate (elements 4 through 6 of ESTATE) to the chase vehlcie body coordinates and then calls on the function ACCEL six times, once for each translational and rotational axis. ACCEL returns a value of zero or $\pm 1.0$
to indicate the state of an idealized thruster (off, on with positive thrust, or on with negative thrust). The value is selected to ensure that limit cycling along the axis the thruster controls is confined to the "box" defined by the goal-setting logic. The algorithm behind sCCEL is difficult to state in plain English, but the "pseudocode" flowchart in Figure VIII-3 shows the essential logic. For a given set of input parameters, ACCEL defines a simple phase-plane control law. The input parameters have the effect of adjusting the phase-plane decision boundaries to accommodate different thrust authorities, decision intervals, limit cycle envelopes and control bandwidths.


Figure VIII-3 Essential Logic of Function ACCEL
The Logic of the Subroutine is Complicated Slightly by the Fact That
The Thrust Directions Required to Implement. "Brake,"
"Accelerate," and "Thruet Toward Goal" Depend on the Current
Position and Volocity.

After $\underline{r}$ has been determined, the subroutine finds a goal for the chase vehicle's center of mass, expressed in chase vehicle coordinates, with the formula
$\underline{v}=A_{c}\left(A_{t}^{T}\left(\underline{r}-\underline{h}_{d c}\right)-\underline{x}\right)$
(VII[-10)
where
v, represented in the programs as $V 3$, is the goal for the chase vehicle's center of mass,
$A_{C}$, represented in the programs as $A C V$, is the chase vehicle's attitude direction cosine matrix, as measured by the inertial measurement unit,
$A_{t}$, represented in the programs as TRNAT, is the transpose of the target's attitude direction cosine matrix, which is derived from the appearance of the target in the television images,
$x$, represented in the programs as the first three elements of ESTATE, is the chase vehicle's estimated position in the "primary" reference frame,
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to indicate the state of an idealized thruster (off, on with positive thrist, or on with negative thrust). The value is selected to ensure that limit cycling wlong the axis the thruster controls is confined to the "box" defined by the goal-setting logic. The algorithm behind ACCEL is difficult to state in plain English. but the "pseudocode" flowchart in Figure VIII-3 shows the essential logic. For a given set of input parameters, ACCEL defines a simple phase-plane control law. The input parameters have the effect of adjusting the phase-plane derision boundaries to accommodate different thrust authorities, decision intervals, limit cycle envelopes and control bandwidths.


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The Logic of the Subroutine is Complioated Slightly by the Fact That
ihe Thrust Directions Required to Imqlement "Braike,"
"Accelerate," and "Thrust Toward Goal" Depend on the Current Position and Velocity.

A practical spacecraft has a limited number of thrusters and cannot command an arbitrary acceleration ir an azbitrary direction. The simulation models this restriction by giving the chase vehicle only 14 thrusters. (Physic-lly there are 15 , but two of them are controlled as a unit., Combinations of the 14 thrusters can be selected to provide accelerations in any combination of the three translational axes and torques about any combination of these axes. The magnitudes of the accelerations and torques may, however, significantly differ from the ideal values. The logic in the control law accounts for this fact by knowing both the maximum and the minimim possible value for each acceleration and torque.

There are three possible commands ( $-1,0$, and +1 ) for ach axis, and there are six axes for which commands cin be given ( $x, y, z$, yaw, roll, and pitch). This means there are only $3^{6}$ or 729 different conmand combinations that can be given. Subroutine SELECT converts the set of commands to an index and then looks up an appropriate set of thrusters in a table. In flight hardware this could be done by using read-only memory to convert acceleration commands to thruster commands.
(i. CHASE VEHICLE DYNAMICS RESPOND TO THRUSTER COMMANDS

The simulation programs use a set of five coupled differential equations to compute the chase vehicle's response to the thrusters. These equations propagate the vehicle's so-called "true state", which includes position, velocity, angular momentum, attitude, and mass. Because some of these quantities are vectors, the number of elements in the state is 14. These quantities are represented in the programs by the array STATE, whose elements are defined in Table VIII-2.

The array STATE differs significantly from the array ESTATE, the state estimate used for navigation. ESTATE contains only position and velocity estimates and therefore has only six elements. These six elements are not simply estimates of the first six elements of STATE because two different coordinate systems are used.

The true state is measured with espect to a nonretating coordinate system centered at the target spacecraft's center of mass. The orientation of this coordinate system is fixed at the instant the simulation begins, with the +2 axis pointing away from the center of the earth, the $+y$ axis aligned with the orbit angular moment im vector, and the x axis defined by the cross product of unit vectors along the ty and $+z$ axes. This orientation was chos?n to simplify the calcillation of gravity gradient acceleictions.

Table VIII-2
Definitions of Elements of True-State Array STATE

| Element Number | Definition |
| :---: | :---: |
| $\begin{aligned} & 1 \\ & 2 \\ & 3 \end{aligned}$ | $\begin{aligned} & X \text { Position (m) } \\ & Y \text { Position (m) } \\ & \text { Z Position (m) } \end{aligned}$ |
| $\begin{aligned} & 4 \\ & 5 \\ & 6 \end{aligned}$ | X Velocity (m/s) <br> $Y$ Velocity (m/s) <br> Z Velocity (m/s) |
| $\begin{aligned} & 7 \\ & 8 \\ & 9 \end{aligned}$ | Angular Momentum about $X$ Axis ( $k g \mathrm{~m}^{2} / \mathrm{s}$ ) <br> Angular Momentum about $Y$ Axis ( $k g \mathrm{~m}^{2} / \mathrm{s}$ ) <br> Angular Momentum about 2 Axis ( $\mathrm{kg} \mathrm{m}^{2} / \mathrm{s}$ ) |
| $\begin{aligned} & 10 \\ & 11 \\ & 12 \\ & 13 \end{aligned}$ | $q_{1}$ of Attitude Quaternion <br> $Q_{2}$ of Attitude Quaternion <br> $q_{3}$ of Attitude Quaternion <br> $9_{4}$ of Attitude Quaternion |
| 14 | mass (kg) |

The state estimate (ESTATE) uses a different coordinate system to demonstrate tiat the guidance system has no need for the target-orbit information that defines the truth coordinate system. In a flight situation the guidance system may, in fact, have no access to this info mation.

The differential equations, sumarized in Table VIII-3, are integrated in subroutine PROPT' with fourth-order Runge-Kutta numerical integration. This integrat ion algorithealculates the state at a future time from the current state and the first derivative of the state at various points within the integration interval. Since a minimum of four evaluations of the first derivative are used for any interval, a separate subruatine, STPRIM, is used to calculate the derivatives. This subroutine, in turn, calls subroutines that evaluate the equations in Table VIII-3. Table VIII-4 shows the meanings of the symbols in the formulas and the names of the variables used in the simulation program to represent them.

The orbit is modeled as a 300-kiloneter circular earth orbit. Because of the choice of coordinate system, the orbit has only a minor secondorder effect; it detemines the magnitude and direction of the gravity gradient acceleration, which is a relative acceleration between the two spacecraft caused by the fact that gravity acts most strongly on the spacecraft closest to the earth. The maximum magnitude of this differential acceleration is approximately 0.0001 g .

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Table VIII-3 Major Equations for Tme-State Computation

| Equation |  | Subroutine Where Computed |
| :---: | :---: | :---: |
| $\underline{\mathrm{E}}=\mathrm{A}^{\mathrm{T}} \mathrm{K}_{2} \underline{\mathrm{~F}}$ | (VIII-11) | FCRCE |
| $\dot{m}=-k_{1}^{T} \underline{F}$ | (VIII-12) | MPRIME |
| $\underline{\underline{v}}=\underline{a}_{88}+\underline{\underline{i}} / \mathrm{m}$ | (VIII-13) | LINACL |
|  | (VIII-14) | ANGVEC |
| $s_{6}=\left[\begin{array}{cccc}0 & b_{3} & -\omega_{b_{2}} & w_{b_{1}} \\ -w_{b_{3}} & 0 & \omega_{b_{1}} & \omega_{b_{2}} \\ \omega_{b_{2}} & -\omega_{b_{1}} & 0 & \omega_{b_{3}} \\ -\omega_{2} & -\omega_{b_{1}} & -w_{b_{3}} & a\end{array}\right]$ | (VIII-15) | MAKROT |
| $\underline{N}=A^{T} \mathrm{r}_{-} \mathrm{F}$. | (VIII-16) | TURQUE |
| $\underline{L}=\underline{N}-\left(A^{\top} \underline{w}_{\boldsymbol{w}}\right) \times L$ | (VIII-17) | LPRIME |
| $\dot{q}=0.5 \Omega \mathrm{q}$ | (VIII-18) | QPRIME |
| $\underline{\dot{x}}=\underline{v}$ | (V111-39) | STPRIM |

Table VIII-4 Definition of Variables in Table VIII-3

| Algebraic <br> Symbol | Dimension | Program Variables | Units | Coordinate System | Meaning |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A, A^{\text {T }}$ | $3 \times 3$ | A, TRNA | -- | Truth | Direction Cosine Matrix Corresponding to q |
| f | $3 \times 1$ | NTFORC | N | Truth | Net Force Vector from Thrusters |
| F | $14 \div 1$ | E | N | -- | Element i is the <br> Magnitude of Thrust <br> Produced by Thruster 1 |
| I, $\mathrm{I}^{-1}$ | $3 \times 3$ | INERTA, INVIN | $\begin{aligned} & {\mathrm{kg} \cdot \mathrm{~m}^{2}}_{\mathrm{kg}^{-1} \cdot \mathrm{~m}^{-2}} \end{aligned}$ | Body | Moment of Inertia and Its Inverse |
| dI/dm | $3 \times 3$ | FULDIS | $\mathrm{m}^{2}$ | Body | Sensitivity of I to Additional Fuel Mass |
| $\mathrm{k}_{1}^{\mathrm{T}}$ | $1 \times 14$ | AK1 | $k_{3} \cdot N^{-1} s^{-1}$ | -- | Element i is Fuel Mass Consumed, per NewtonSecond of Thrust, for Thruster i |
| $\mathrm{K}_{2}$ | $3 \times 14$ | AK2 | -- | Body | Column i is a Unit Vector in the Direction of the Thrust Produced by Thruster 1 |
| $\mathrm{K}_{3}$ | $3 \times 14$ | AK3 | m | Body | Column i is a Vector <br> Corresponding to <br> Thruster i. Its <br> Direction is the <br> Direction of the Torque <br> Produced by the Thruster. <br> Its Magnitude is the <br> Amount of Torque per <br> Newton of Force. |
| L, L | $3 \times 1$ | Siate (7) <br> (et seq.), <br> DSTATE(7) <br> (et seq.) <br> DLDT | $\begin{aligned} & \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s}, \\ & \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2} \end{aligned}$ | Truth | Angular Momentum about Center of Mass and its Rate of Change |

## Table VIII-4 (concl)

| Algebraic Symbol | Dimension | Program Variables | Units | Coordinate System | Meaning |
| :---: | :---: | :---: | :---: | :---: | :---: |
| m , in | Scalar | $\begin{aligned} & \operatorname{STATE}(14), \\ & \text { DSTATE (14) } \\ & \text { or DMDT } \end{aligned}$ | $\begin{aligned} & \mathrm{kg}, \\ & \mathrm{~kg} / \mathrm{s} \end{aligned}$ | -- | Total Chase Vehicle Mass and its Rate of Change |
| $\underline{N}$ | $3 \times 1$ | N | $\mathrm{N} \cdot \mathrm{m}$ | Truth | Torque about Center of Mass |
| $\underline{\mathrm{q}}, \underline{\mathrm{q}}$ | $4 \times 1$ | STATE (10) <br> (et seq), DSTATE(10) (et seq) or QPRM | $\begin{aligned} & --, \\ & s^{-1} \end{aligned}$ | Truth | Allitude Quaternion and its Rate of Chang: |
| $\underline{v}$, $\underline{\underline{v}}$ | $3 \times 1$ | STATE(4) <br> (et seq), <br> DSTATE(4) <br> (et seq) <br> or ACCEL | $\begin{aligned} & \mathrm{m} / \mathrm{s}, \\ & \mathrm{~m} / \mathrm{s}^{2} \end{aligned}$ | Truth | Velocity and its Rate of Change |
| $\underline{\underline{x}} \underline{\underline{\underline{x}}}$ | $3 \times 1$ | STATE (1) <br> (et seq), <br> DSTATE(1) <br> (et seq) | $\begin{aligned} & \mathrm{m} \\ & \mathrm{~m} / \mathrm{s} \end{aligned}$ | Truth | Position and its Rate of Change |
| $\stackrel{\omega}{b}^{\text {b }}$ | $3 \times 1$ | BUDVEL | $\mathrm{s}^{-1}$ | Body | Angular Velocity |
| 3 | . 4 | OMEGA | $s^{-1}$ | Body | Matrix Formed from Elements of $\mu b$ to Implement Equation (D-6) |

fi. TARcet attitide is modeled as deterministic

Subrout ine TRGATT computes the target spacecraft's attitude as a function of time. In the simulations several versions of this subroutine were used to simulate different initial attitudes and tumbling at different rates about different axes. In each case a simple tumbling was assumed so that system performa ce could be evaluated as a function of simple parameters.

The dynamics model used in these programs is readily adapted to different targets and chase vehicles. For example, different masses, thruster orientations, number of thrusters, thruster forces, docking fixture locations, camera locations, iuel loads, and engine types can be modeled without changing the program logic. Different control laws can be used by changing the logic of subroutine CNTLAW, and different methods for deriving rates can be invest, gated by replacing the Kalman filter subroutines. These investigations were beyond the scope of the study reported here.

## IX. Computer Models of Measurements and Noise

A. LIGHT POSITIONS ARE MODELED WITH PERSPECTIVE PROJECTION

In each program, lights are assumed to be point light sources, and the tuages from the cameras are simulated by perspective projection. The calculations are identical for each light in every program, except that the vectors defining camera and lamp positions vary in value. The basi.c strategy is to compute the position of the lamp in the chase vehicle's camera coordinate system. This system is parallel to the chase vehicle's body coordinate system but offset by $h_{c}$, which is the separation between the camera and the chase vehicle's center of mass. The formula for determining a lamp's position in the camera coordinate system is
$\underline{r}=A_{c}\left(A_{t}^{T} \underline{h}_{t}-\ldots-\underline{h}_{c}\right.$

The physical interpretation of this formula is shown in Figure $1 X-1$.


Figure IX-1 Physical Interpretation of Equation (IX-1)

The position of the lamp's image on the television screen can be determined from
for the horizontal component and from

$$
\begin{equation*}
v=r_{3} f / r_{1} \tag{1X-3}
\end{equation*}
$$

for the vertical component, where $r_{1}, r_{2}$, and $r_{3}$ are the $x, y$, and $z$ componeats of $r$. The direction cosine matrix $A_{c}$ is derived from the true state, which is represented in the programs as the array STATE. Elements 10 through 13 of this array are the attitude quaternion. It is converted to a direction cosine matrix with equation ( $D-5$ ) from Appendix D. The vector $x$ is the current position of the chase vehicle's center of mass with respect to the "truth" coordinate system defined in Chapter VIII. This vector is the first three elements of the array STATE. The vectors $\underline{h}_{c}$ and $\underline{h}_{t}$, which vary from one simulation to the next, are the positions of the camera and lamp, respectiveiy: $h_{c}$ is the camera position with respect to the clase vehicle's center of mass, and $h_{t}$ is the docking aid's position with respect to the target spacecraft's center of mass. The direction cosine matrix $A_{t}$ is the target's attitude, which is computed as a function of time in subroutine TRGATT, as described in Chapter VIII.

This measurement model's accuracy depends on the validity of several assumptions:

1) The focal plane of the camera is flat and parallel to the $y-z$ plane of the chase vehicle's coordinate system;
2) The lens and the scanning of the camera do not significantly distort the image;
3) The lamp's image is small enough to be considered a pint;
4) The lamp is bright enough to be considered the only signi:icant object in the image.

Equations (IX-1) through (IX-3) are used several times in each simulation. In the program that simulates the system with three flashing lights, for example, the equations are used three times per obscrvation with different values of $h_{t}$. Similarly, in the program that. simulates the system with stereo ranging, the equations are used three times with different values of $\underline{h}_{c}$.
B. RANDOM NOISE CORRUPTS IMAGE COORDINATES

To simulate the imperfections of a practical guidance system, the simulation programs add random numbers to the coordinates computed by equations (IX-2) and (IX-3). The random numbers have a Caussian distribution with a mean of zero and a standard deviation selected to represent the root-mean-square uncertainty in the ideal locations of the lamp images.

This uncertainty arises from several factors:

1) Imperfect electronics are used to compute the center of brightness of the image. The operational amplifiers, analog multipliers, and other components corrupt the video and deflection signals with biases and random errors;
2) Scanning is not perfect. In some cameras a sawtooth analog deflection voltage or current is used for scanning. Misalignment of deflection coils, imperfections in deflection drive amplifiers and similar factors may cause errors in the center-of-brightness calculations. Some cameras use discrete picture elements, which may cause the center of brightness to shift due to quantization errors. Finally, solid-state cameras use no analog deflection voltage, and the use of a synthetic deflection voltage in the center of brightness computation may introduce errors caused by errors in deflection amplitude and synchronization with the actual scanning of the camera;
3) Lenses introduce errors. The distortions, aberrations, and focusing limitations of practical lenses can add additional corruptions to the center of brightness calculations;
4) Camera positioning is not perfect. The mathematical model of equa~ tion (IX-l) does not allow fo: misalignment of the camera axes from the spacecraft axes, errors in the camera's mounting location, or the transient changes in position and alignment caused by thermal expansion and vibration;
5) Camera sensitivity varies from picture element to picture element. In some solid-state cameras the variation is as much as 15 percent. Further, some solid-state cameras have blemishes--picture elements that always give full output or no output no matter how much light shines on them. Charge-coupled device (CCD) cameras tend to "bleed" from one scanning line to the next when a bright light is in the field of view, while other caneras are troubled with blooming and after-images. Many cameras have s.gnificant "dark" output that varies from picture element to picture element and may vary greatly with temperatur?.

Meaningful numbers cannot be assigned to all tisse error sources, especially because there are no detailed designs ror the spacecraft, guidance system, and mission. The noise model should therefore be considered a specification of the maximum allowable signal corruption rather than a prediction of system performarse with some specified set of hardvire.

Appendix A
Computer Program to Simulate Three-Light System

The program listinf, in this appendix is provided to document the simulation methods used in analyzing the three-light videc guidance system. It vas written to run on a Prime 550 computer under the PRIMOS operating system, but it has few hardware-dependent subroutines. If it is to be run on another computer, the following information will prove useful:

1) Several library routines are used, and these are not shown in the listing. The routines include ACOS and ASIN, which compute the inverse trigonmetric functions $\cos ^{-1} x$ and $\sin ^{-1} x$, and a random number generator, RANFN. The latter function computes normally distributed random values with a specified mean and standard deviation. In addition, the matrix arithmetic routines MADD (addition), MSUB (subtraction), MMLT (multiplication), MINV (inversion), MSCL (multiplication by a scalar), MIDN (setting an array to the identity matrix), and MTRN (forming the transpose of a matrix) are used from the Prime library MATHLB;
2) File handing may present conversion problems even if the program is to be run on another Prime 550 computer because logical unit numbers, file names, and amount cf disk storage vary from installation to installation. Standard Prime subroutines are used to open and close files. These subroutines (TSRC\$\$, EXST\$A, CLOS\$A, DELE $\$$ A) are from the Prime library APPLIB;
3) Run time is approximately real time if the computer is dedicated to a single user. That is, a simulation of three minutes of flight will take approximately three minutes;
4) The perspective drawings shown in this report are not created directly by this program. They are drawn by a second program that uses the data file created by this program. This allows the creation of stereo plots and views from different perspectives;
5) Several WRI'CE statements in subroutine DOCK are rendered inactive by a character $C$ in the first column of the text. Removing this character will provide a printout at the user's terminal for monitoring the progress of a similation, but this will make the simulation run more slowly;
6) The goal in writing the program was to make the program easy to modify. As a result, many of the operations are donc inefficiently, and a speed improvement by a factor of two or three might be achieved.

The first part of the listing is the text of a terminal session, which includes compilation, loading, and execution of the program.






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SUBROUTINE DOCK MP, ESTATE, STATE, TM, DOCKED. F'

INPUT EUTATATE, DOKRAD, STATE, T, F, INERCV OUTPUTATE, T, F, DOCKED, ESTATE, P

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## APPENDIX A -- THPEE-LIGHT VERSION OF SIMULATION PROGRAM <br> Page 1!

 REMEASURED DIRECTION COSINE MATRIX IF TARGET SPACECRAFt:REAL TYPOS ( 3 ) CHASE VEHICLE POSITION IN PRIMARY REFERENCE FRAME REAL DEDEY WHICH STRATEGY LOGIC WANTS CHASE VEHICLE IN GOAL 'BOX' REAL RADIUSAD FROM TARGET :T WHICH CHASE VEHICLE IS CONSIDERED DOCKED REAL CHANGE IN CENTER-LAMP IMAGE COORDINATES BETWEEN FLASHES ESTIMATE OF STATE VECTOR
TEAL GORES, EXR, GYL,GYE, VEHICLE THRUSTERS AFTER SELECTION
REAL HDC (3), HDT (3) POCKING FIXTURES POSITIONS IN RESPECTIVE SPACECRAET CODRDINATE
INTEGER OUT
FORTRAN UNIT NUMBER FOR OUTPUT FILE

REAL RELPOS CAMERA COORDINATES IN CURRENT DOCKING AID FRAME CENTERED
REAL REARED RANGE, CAMERA TO TARGET DOCKING AID CENTER LIGHT
REAL RPYERR ( 3 POL
REAL ASSURED PITCH, AND YAW ERRORS IN RADIANS REAL STATE (14) CHASE VEHICLE STATE VECTOR
REAL ELAPSED TROUT 3 , 3 )

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APPENDIX A - THREE-LIGHT VERSION OF SIMULATION PROGRAM

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APPENDIX A -- THREE-LIGHT VERSION OF SIMLATION PROGRAH

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appendix a -- three-light version of simlation program page 29

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REAL R $(3.3$ )
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ESTIRNCE

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aPPENDIX A - THREE-LIGHT VERSION OF SIMUATION PROGRAM 32

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APPENDIX A -- THREE-LIGHT VERSION IF SIMULATION PROGRAM
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A SUBROUTINE THRUST AF, ESTATE, GXL, GXH, GYL, GYH, GZL, GZH. TLIM, ACV, (* ABRA- SELECT THRUSTERE *)
CALLLNSTLAW, DEFINF, FIRTHR, SELECT
INPUT
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REAL E(14)
REAL EUSTATE COMMAND ENTRIES FROM THRUSTER SELECT LOOIC
REAL STATE ESTIMATE
REAL FORCES FROM THRUSTERS AFTER SELECTION

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REAL TMIMIMIT

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APPEREIX A - THREELICHT VERSION OF SIMLATION PROGRAM

appendix a -- Three-light version of simulation program page 45
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appendix a -- three-Light version of simulation program

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| C |  |
| $\begin{aligned} & \mathrm{c} \\ & \mathrm{c} \end{aligned}$ | CALLSTMMAT, MADD, MSCL, ANGVEC |
| c | CALLED BY |
| c | dock |
| c | Input |
| ${ }_{c}^{\text {c }}$ | GO, STATE, FULDIS, INERCV, MEMPIY |
| c | OUTACV. ACUT, ATRATE |
|  | REAL ACV(3, 3). ACVT (3, 3) |
| ${ }_{C}^{C}$ | DIRECTION COSINE MATRIX AND ITS TRANSPOSE FOR 'MEASURED' ATTITUDE |
|  | REAF CHASE VEHICLE WITH RESPECT TO PRIMARY REFERENCE FRAME |
| c | MEASURED ATTITUDE RATES AbOUT GHASE VEhicle axes |
| c | REAL FENSDR ${ }^{\text {d }}$ DESCAIBING FUEL DISTR:BUTION |
|  | REAL INERCU(3.3) |
| c | REAENSOR DESCRIEING INERTIA OF EMPTY Chase vehicle |
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|  | REAL INEFTA S3. 3 ) |
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| c | MEMASS OF EMPTY CHAJE VEHICLE |
| c | INITIAL' CHASE VEHICLE ATTITUDE GHIATERNION IN 'TRUTH' REFERENCE FRAME |
| c | AND CURRENT PiRIMARY REFERENCE FRAME, RESPECTIVELY |
| c | AL ETATE (14) |
|  | REAL TACV (3.3), TACVT(3, 3) |
| $\stackrel{C}{c}$ | CHASE VEMICLE DIRECTION COSINE MATRIX WITH RESPECT TO TRUTH COORDIMATE SYSTEM AND INVERSE OF THIS MATRIX |
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COMTON/REF/OO/FULDIS, INERCV. MEMPTY


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APPENDIX A -- THREE-LIGHT VERSION OF SIMLATION PROGRAM
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> Appendix B Computer Program to Simulate System That Uses
> "Rainbow" Beacon and Rangefinder

APPENDIX B--COMPITER PROGRAM TO SIMILLATE SYSTEM THAT USES "RAINBOW" BEACON AND RANGEFINDER

The program listing in this appendix is provided to document the simulation methods used in analyzing the "rainbow" beacon video guidance system. The discussion presented in Appendix $A$ is equally applicable to this prosran. The text of the terminal session, which is presented In Appendix $A$, is not repeated here because the user commands and dia= log are essentially identical as those in Appendix A.

## APPENDIX B -- RAINBOW VERSION OF SIMULATION PRGGRAM

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NKUE STATE
    STATE( 1) X POSITION
    STATE ( 2) Y POSITION
    STATE( 3) Z POSITION
    STATE ( 4) X VELOCITY
    STATE ( S) Y VELOCITY
    STATE( 6) Z VELOCITY
    STATE( 7) ANGULAR MOMENTUM ALSDUT }
    STATE( B) ANGULAR MOMENTUM ABOUT Y
    STATE( 9) ANGULAR MOMENTUM ABOUT Z
    STATE(10) Q1\
    STATE(11)
    STATE(12) Q3 > ATTITUDE W.R.T. 'TRUTH' AXES
    STATE(13) Q4 (14) MAS, INCL FUEL
    (X,Y,Z) = 'TRUTH' AXES
STATE ESTIMATE
ESTATE(1) X' POSITION
ESTATE(2. Y' POSITION
ESTATE(3) Z' PQSITION
ESTATE (4) X, VELIOCITY
ESTATE (5) Y , VELOCITY
ESTATE(S) Z'VELOCITY
    (X',Y', Z') = 'PRIMARY REF. FRAME' AXES
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APPENDI: E -- RAINBOL VERSION OF SIMLAGTION PROGRAM






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APPENDIX B - RAINBON VERSION OF SIMLATION PROCRAM


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appendix a -- rainbow version of simulation program

## PAGE 11



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REALEASUED ATTITUDE RATES ABOUT CHASE VEHICLE AXES


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APPENDIX B -- RAINBOW VERSION OF SIMLATION PRTGORAM

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APPENDIX B - RAINBOW VERSION OF SIMLATION PROGRAM

PAGE 31 APPENDIX B - RAINBOW VERSION OF SIMLATION PROGRAM



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APPENDIX B -- RAINBOW VERSION OF SIMRAIION FRDGFAM
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SUBROUTINE PROPESTP, ESTATE, F, STEP, ACVT'
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APPENDIX B -- RAINBCN VERSIDN CF SIMK ATIDN PROGRAM PAGE 43




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appendix b -- rainbow vers ' V OF Simlation program
APPENDIX B -- RAINBOH VERSION OF SIRULATION PROGRAM

## PACE 53



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APPENDIX B -- RAINBDW VERSION OF SIMULATION PROGRAM PAGE 57
 APPENDIX B -- RAINBON VERSION OF SIMLLATION PROGRAM

PAGE 56
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appendix $\quad$ - -- rainbow version of simyation program


APPENDIX B -- RAINBON VERSICN OF SIMLATION PROGRAM



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appendix b -- rainbon version of simlation program

| PAGE 62 |  |
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| SUBROUT INE LPRIME(N, BODVEL, ANCHNT, DLDT, TRNA) <br> (© 9025 - COMPUTE TIME DERIVATIVE OF ANGULAR MOMENTUM ") |  |
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APPENDIX B -- RAINBOW VERSION OF GIMULATION PRIGRAM PAGF 67



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i. FUNCTION DPRD(X,Y,IDIM)
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        GUATFEN
        INPUTS:
        X, Y,IDIM
        DUTPITTS:
        DrRD
        REAL NFRD
        (* DQT PRODUCT OF VECTORS }X\mathrm{ AND }Y*\mathrm{ )
    REAL X(3),Y(3)
        (* VECTORS FOR WHICH DOT PRODUCT IS TO BE DETERMINED *)
        INTEGER IDIM,I 
        (* DIMENSION OF VECTORS X AND Y, AND DO LOOP INDEX, RESPECTIVELY *)
    REAL SUM
        (* INTERMEDIATE RESULT WITH NO SIMPLE INTERPRETATION *)
        (* COMP--
        SUM=0.O
    DO 10 I=1, IDIM
        SUM=SUM+X(I)*Y(I)
    10 CONTINUE
    DPRD=SUM
    RETURN
C
    END
```

Appendix CComputer Program toSimulate System That UsesRing of Lights

A- PENDIX C--COMPUTER PROGRAM TO SIMULATE SYSTEM THAT USES RING OF LIGHTS

The program listing in this appendix is provided to document the simulation methods used in analyzing the "ring-of-lights" video guidance system. The discussion presented in Appendix A is equally applicable to tans program. The cext of the teminal session, shich is presented in Appendix A, is not repeated here because the user commands and dia$10 g$ are essentially identical to chose in Appendix A.

# APPENDIX C -- RING-OF-LIGHTS VERSION OF SIMULATION PROGRAM 

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ESTATE(3) & \(z\) POSITION \\
ESTATE(4) & \(X\) VELOCITY \\
ESTATE (5) & \(Y\) VELOCITY \\
ESTATE (6) & \(z\) VELOCITY
\end{tabular}
    \(\left(X^{\prime}, Y^{\prime}, Z^{\prime}\right)={ }^{\prime} P R I M A F Y\) REF. FRAME' AXFS
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APPENDIX C - hIME-DF-LIGTS VERSICN OF SIMLATION PROGRAM



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APPENDIX C -- RING-OF-LIGHTS VERSION OF SIMLLATION PROGRAM
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APPENDIX C - RING-OF-LIGHTS VERSION OF SIMLATION PROGAAM


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APPENDIX C -- RING-OF-LIGHTS VERSION DF SIMULATION PROGRAM
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Page 45



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APPENDIX $C$ - RIMG-OF-LiGHTS VERSION OF SIMLATIION PROGRAM






APPENDIX $C$-- RInî-OF-lights version of simulation program

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## Appendix D

Attitude Parameterization

The three computer simulations performed in this study required frequent conversions among coordinate systems. For example, the camera locations are fixed in the chase vehicle's body coordinate system, but this system is constantly rotating with respect to the inertial frame used for navigation. Similarly, the docking aid's position is fixed in the target spacecraft's coordinate system, which also rotates with respect to the inertial frame. After each observation, the onboard computer must compute the position of the chase vehicle's center of mass with respect to the target's center of mass. Because the camera and the docking aid are not at the target's center of mass, the computer must convert all the position data to a consistent coordinate system.

One of the most convenient ways to convert a vector from one coordinate system to another is to multiply the vector by the direction cosine matrix that expresses the relationship between the two coordinate systems.

Because the target's direction cosine matrix is not known in advance, it must be measured. The calculations required to do this are simplified somewhat if quaternions are used as an intermediate step. Quaternions were also found useful for a number of other operations in the simulation programs.

Direction sosine matrices and quaternions are alternative ways to express exactly the same information, and the computations to convert from one to the other are not difficult. Subroutine DIRMAT in the computer programs in Appendices $A$ through $C$ converts from quaternion notation to direction cosine matrix notation. Subroutine TOQUAT in Appendix $C$ converts from direction cosine matrix notation to quaternion notation.

Suppose that $A$ is a $3 \times 3$ direction cosine matrix that expresses the attitude of a rotated coordinate system with respect to a fixed system, and that $\underline{v}_{r}$ and $v_{f}$ are $3 \times 1$ matri.es that express the vector $v i n$ the rotated and $\bar{f} i x e d$ frames, respectively. Then $v_{r}$ and $v_{f}$ are related by the equations
$\underline{v}_{r}=A \underline{v}_{f}$,
$\underline{v}_{\mathrm{f}}=\mathrm{A}^{\mathrm{T}} \underline{v}_{\mathrm{r}}$.

The second enuation makes use of the fact that direction cosine matrices are members of a special class of matrices whose inverses are equal to their tracsposes. The superscript $T$ can therefore be thought of as denoting eit'er the inverse or the transpose in this context.

The rows and colums of a direction cosine matrix have a simple physical interpretation: the columns are the unit vectors that define the fixed coordinate system, expressed in the rotated coordinate system. Similarly, the rows are the unit vectors defining the rotated coordinate system, expressed in the fixed coordinate system.

For example, if the fixed coordinate system is defined by the vectors $\underline{i}, \underline{j}$, and $\underline{k}$, and the rotated system's unit vectors are $\underline{u}, \underline{v}$, and $\underline{w}$, and If $\bar{A}=\left[a_{r} \bar{s}\right]$, $i$ can be expressed in the fixed system as ${ }^{-1}(1,0,0)^{\top}$ or in the rotated system as $\left(a_{11}, a_{21}, a_{31}\right)^{T}$. Likewise, u can be expressed in the rotated frame as (1,0,0) ${ }^{\mathrm{T}}$ or in the fixed frame as $\left(a_{11}, a_{12}, a_{13}\right)^{T}$.

There is a simple formula for computing the direction cosine matrix that represents the results of two successive rotations. If $A_{21}$ represents the attitude of coordinate system 2 with respect to system 1 , and A32 represents the attitude of coordinate ystem 3 with respect to system 2, then $A_{31}$, which gives the attitude of system 3 with respect to system 1 , can be computed from the formula
$A_{31}=A_{32} A_{21}$.
Quaternions are based on an entirely different view of attitude. There is a theorem that states that no natter how a rotated frame is oriented with respect to a fixed refer:ace frame, the orientation can be accounted for by assuming a single rotation about a single axis. The angle of rotation is called the Euler angle ( $\Phi$ ), and the axis is called the Euler axis (e). The quaternion representing the attitude of the rotated frame is a $4 \times 1$ matrix*

$$
q=\left[\begin{array}{l}
e_{1} \sin (\Phi / 2)  \tag{D-4}\\
e_{2} \sin (\Phi / 2) \\
e_{3} \sin (\Phi / 2) \\
\cos (\Phi / 2)
\end{array}\right] \equiv\left[\begin{array}{l}
q_{1} \\
q_{2} \\
q_{3} \\
q_{4}
\end{array}\right]
$$

where $e_{1}, e_{2}$, and $e_{3}$ are the components of the Euler axis in the fixed coordinate system.

The direction cosine matrix can be expressed in terms of the quaternion elements as

$$
A=\left[\begin{array}{lll}
q^{2}-q_{2}^{2}-q_{3}^{2}+q_{4}^{2} & 2\left(q_{1} q_{2}+q_{3} q_{4}\right) & 2\left(q_{1} q_{3}-q_{2} q_{4}\right) \\
2\left(q_{1} q_{2}-q_{3} q_{4}\right) & -q_{1}^{2}+q_{2}^{2}-q_{3}^{2}+q_{4}^{2} & 2\left(q_{2} q_{3}+q_{1} q_{4}\right) \\
2\left(q_{1} q_{3}+q_{2} q_{4}\right) & 2\left(q_{2} q_{3}-q_{1} q_{4}\right) & -q_{1}^{2}-q_{2}^{2}+q_{3}^{2}+q_{4}^{2}
\end{array}\right](D-5)
$$

and subroutine DIRMAT is a stralghtforward implementation of this formula. The reverse operation can be performed by ayy of four equivalent sets of simple formulas, but an appropriate set must be used to avoid subtraction of nearly equal quantities or division by zero. Subroutine TOQUAT contains all four sets of formulas and the logic required to select an approprlate set.

[^5]Quaternions are useful when orientation is known (or sought) in terms of the Euler angle and axis. They also prove especially useful in numerical integration of angular velocity because there is a simple formula for the integration and because they are easy to renormalize.

Renormalization is required because roundoff and other errors in numerical integration cause the norm of the quaternion to drift. [The norm is the square root of the sum of the squares of the four elements, and must be exactly 1.0 for the definition in equation ( $D-4$ ) to be valid.] Renormalization is a simple matter of dividing each element of the quaternion by the norm.

Numerical integration with direction cosine matrices requires only about 30 percent more additions and multiplications, but normalizaticn is much more difficult. If a direction cosine matrix is not normalized, after several integration steps it begins to represent a mapping into a distortedcoordinate system and not a simple rotation. This happens because roundoff errors leave rows and columns representing vectors that are not of unit length or vectors that do not define the axes of a rectangular coordinate system.

The formula used in the simulations for integrating angular velocity is

$$
\begin{equation*}
\mathrm{dg} / \mathrm{dt}=\frac{1}{2} \Omega q \tag{D-6}
\end{equation*}
$$

where:
$\Omega \equiv\left[\begin{array}{cccc}0 & \omega_{w} & -\omega_{v} & \omega_{u} \\ -\omega_{w} & 0 & \omega_{u} & \omega_{v} \\ \omega_{v} & -\omega_{u} & 0 & \omega_{w} \\ -\omega_{u} & -\omega_{v} & -\omega_{w} & 0\end{array}\right]$,
a matrix made up of angular velocity components about the spacecraft body axes. The integration starts with an initial value for $q$ and uses equation ( $D-4$ ) to calculate $\underline{q}$ at future times.

A quaternion version of equation ( $D-3$ ) can be used to combine the effects of multiple rotations. If $q 21$ represents the attitude of coordinate system 2 with respect to system 1 , and 932 represents the attitude of coordinate system 3 with respect to system 2 , then $q 31$, which gives the attitude of system 3 with respect to system 1 , can be computed from the formula
$q_{31}=q_{21}$
where $Q$ is a $4 \times 4$ matrix formed from the elements of $q 32$

$$
Q \equiv\left[\begin{array}{llll}
q_{4} & q_{3} & -q_{2} & q_{1}  \tag{D-9}\\
-q_{3} & q_{4} & q_{1} & q_{2} \\
q_{2} & -q_{1} & q_{4} & q_{3} \\
-q_{1} & -q_{2} & -q_{3} & q_{4}
\end{array}\right],
$$

In which
$q_{32} \equiv\left[\begin{array}{l}q_{1} \\ q_{2} \\ q_{3} \\ q_{4}\end{array}\right]$.
An alternate formula that gives identical results is
$931=Q^{\prime} 932$
in which $Q^{\prime}$ is a $4 \times 4$ matrix formed from the elements of 921 :

$q_{21}=\left[\begin{array}{l}q_{1} \\ q_{2}^{\prime} \\ q_{3}^{\prime} \\ q_{4^{\prime}}^{\prime}\end{array}\right]$
Equations ( $D-8$ ) and ( $D-11$ ) can be considered definitions of quaternion "multiplication." The multiplication defined is (q21 q32), which does not equal ( $832 q_{21}$ ). Quaternion multiplication has an identity quaternion $(0001)^{T}$ that is analogous to the identity matrix in matrix arithmetic. The product of this quaternion with any quaternion $q$ is $q$, whether $q$ is on the left or on the right in the multiplication. Also, any quaternion has an inverse $q^{-1}$. The product $q q^{-1}$ $=q^{-1} q$ equals the identity quaternion. The inverse is formed by simply negating the first three elements of the quaternion.

> Appendix E
> Adapting the Kalman Filter for Other Simulations

The Kalman filter used in the three simulations presented in this report can be adapted for other rendezvous and docking simulations if the appropriate changes are made.

The Kalman filter accepts a 3 -element measurement vector representing the measured position of the chase vehicle's center of mass in the primary reference frame. (The primary reference frame is a nonrotating right-handed rectangular coordinate system that is centered at the target spacecraft's center of mass but aligned with the budy axes of the chase vehicle the instant the video guidance system takes contri.) The measurement data in the simulation programs are obtained from video information. The Kalman filter can be used for simulations that use any ot eer sensor and target that produces $X, Y$, and $Z$ position data, provided that the necessary changes are made. Subroutine ESTCOV, which estimates the measurement covariance, must be altered so the measurement co ariance matrix is reasonable for the accuracy of the sensor and target used. There are two convenient methods of derivirig the measurement covariance matrix. It can be calculated by fitting a curve to data from a Monte Carlo simulation or it can be derived from an analytical determination of the sensor's accuracy. The simulation programs in this report used curve-fitting and a Monte Carlo simulation to determine the equation for VARNCE, the estimated variance per axis. The estimated measurement covariance matrix $R$ is then assigned to be a diagonal matrix whuse elements along the diagonal are equal to VARNCE. It is important to realize that system performance is the issue-as long as the Kalman filter performs well, an accurate measurement covariance estimate is not necessary.

If a different chase vehicle is desired for other simulations, a thorough reevaluation of the kalman filter equations is recommended. The dynamics equations and the assumptions made tc implement the kalman filter could be totallv wrong for another chase vehicle, and overlooking these differetices could cause unsatisfactory system performance.

The numerical scheme that propagates the state estimate and the state covariance matrix between measurements is valid only for small time intervals between measurements. If intervals between measurcments of about 1 second or greater are desired, a more accurate integration technique must be used. To use an alternative integration scheme, subroutine PROPES must be changed to implement .he new integration technique.

Other design changes not mentioned here require a thorough reevaluation of the Kalman filter equations.


[^0]:    *Equations IV-18 and IV-19 are derived in Table IV-1.

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[^2]:    APPENDIX B - RAINBLI VERSIDN OF SIMLLATION PROGRAM
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[^3]:    END

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[^5]:    *In mathematics texts where quaternions are viewed as hypercomplex numbers, the elements are usually shown in a differeui viviou. The viúa given here is used consistently throughout this report.

