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**EMISSION, ABSORPTION AND POLARIZATION OF GYROSYNCHROTRON  
RADIATION OF MILDLY RELATIVISTIC PARTICLES**

by

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EMISSION, ABSORPTION AND POLARIZATION OF GYROSYNCHROTRON  
RADIATION OF MILDLY RELATIVISTIC PARTICLES

ABSTRACT

In this paper, we present approximate analytic expressions for the emissivity and absorption coefficient of synchrotron radiation of mildly relativistic particles with an arbitrary energy spectrum and pitch angle distribution. From these, we derive an expression for the degree of polarization. To accomplish this, we use methods of integration developed in a previous paper.

We then compare the analytic results with numerical results for both thermal and non-thermal (power law) distributions of particles.

## I. INTRODUCTION

The formulas for evaluation of emissivity and absorption coefficient of synchrotron radiation in the ultra-relativistic (synchrotron) and non-relativistic (cyclotron) limits have been known for decades.<sup>1,2</sup> In the intermediate energy range, however, no simple formula exists for an arbitrary distribution of particles. Calculations have been made of the absorption coefficient of a thermal plasma<sup>3,4</sup>, but these results are limited and not very convenient for quantitative calculations. As a result, the usual practice has been to use lengthy numerical calculations.<sup>5,7</sup>

In general, the low harmonics of cyclotron radiation will be self-absorbed, absorbed by the surrounding plasma, or suppressed by the Razin-Tsytovich effect. Consequently, we are interested only in radiation at the higher harmonics, where the optical depth  $\tau_\nu$  is less than or equal to 1.

In a recent paper<sup>6</sup> we presented simple approximate methods for evaluation of the frequency spectrum and dependence on observation angle of the synchrotron radiation at high harmonics from an (essentially) arbitrary distribution of particles in a given magnetic field. In this paper, we shall use the same methods to calculate the emissivity and absorption coefficient of the extraordinary and ordinary modes of synchrotron radiation separately for an arbitrary particle distribution. Also, we will derive the degree of circular polarization from these expressions. (We will refer to the paper in which these methods were developed as Paper I.)

In the next section we describe the general results and in Section III we use them to find emissivity, absorption, and polarization of radiation from particles with Maxwellian and power law energy distributions. Section IV contains a final summary.

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II. GENERAL RESULTS

Consider particles with charge  $e$ , mass  $m_e$ , and distribution  $f(\mu, \gamma)$  where  $f d\mu d\gamma$  is the number density of particles in the energy intervals in the energy interval (in units of  $m_e c^2$ ) from  $\gamma$  to  $\gamma + d\gamma$  and with pitch angle cosine between  $\mu$  and  $\mu + d\mu$ . The emissivity and absorption coefficients for the ordinary (+) and extraordinary (-) modes are<sup>7</sup>,

$$\begin{pmatrix} j_{\pm}(\nu, \theta) \\ \nu^2 \kappa_{\pm}(\nu, \theta) \end{pmatrix} = \frac{2\pi e^2 \nu_b}{c} \frac{\nu}{\nu_b \sin^2 \theta} \int_1^{\infty} d\gamma \int_{-1}^1 d\mu f(\mu, \gamma) \nu \eta_{\pm}(\theta, \gamma, \mu, \nu) \begin{pmatrix} 1 \\ \omega \end{pmatrix}, \quad (1)$$

where

$$\eta_{\pm} = \frac{1}{a_{\pm}^2 + 1} \sum_{m=1}^{\infty} \left\{ a_{\pm} (\cos \theta - \beta \mu) J_m(x_m) - (1 - \beta \mu \cos \theta) \frac{x_m J'_m(x_m)}{m} \right\}^2 \delta(y), \quad (2)$$

$$\omega = \frac{-\beta \gamma^2}{f(\mu, \gamma)} \frac{\partial}{\partial \gamma} \left( \frac{f(\mu, \gamma)}{\beta \gamma^2} \right) + \frac{\beta \cos \theta - \mu}{\gamma \beta^2 (1 - \mu^2)^{1/2}} \frac{1}{f(\mu, \gamma)} \frac{\partial f(\mu, \gamma)}{\partial \mu}, \quad (3)$$

$$y = \frac{m \nu_b}{\gamma} - \nu (1 - \beta \mu \cos \theta), \quad x_m = (\nu \gamma / \nu_b) \beta \sin \theta (1 - \mu^2)^{1/2}, \quad \nu_b = eB / 2m_e c. \quad (4)$$

The quantities  $a_{\pm}$  represent the ratio of the semi-major to the semi-minor axis of the polarization ellipse for the ordinary and extraordinary modes and, in general, are complicated functions of angle  $\theta$ ,  $\nu$ ,  $\nu_b$ , and the plasma frequency  $\nu_p$ . However, when the frequency  $\nu \gg \nu_p$  (consistent with  $\nu \ll 1$ ), these simplify to

$$a_{\pm} = x / [1 \mp (1 + x^2)^{1/2}], \quad x = -2\nu \cos \theta / (\nu_b \sin^2 \theta). \quad (5)$$

Since we are only dealing with the higher harmonics,  $m$  is large and we can replace the sum in equation (2) with an integral. This integral can then be done using the  $\delta$ -function. Also, since  $m$  is large, we can approximate the Bessel's function as<sup>9</sup>.

$$J_m(m_b) = \frac{(1-z^2)^{-1/4} z^m}{(2\pi m)^{1/2}}, \quad z = \frac{ze^{(1-z^2)^{1/2}}}{1+(1-z^2)^{1/2}} \quad (6)$$

which is valid for  $m(1-z^2)^{3/2} \gg 1$ .

With this approximation the integration over the pitch angle is carried out using the method of steepest descent. This gives excellent results as long as the pitch angle distribution is not extremely anisotropic;  $\partial \ln f(\mu, \gamma) / \partial \mu \gg \psi / v_b$ . (See paper I, eqs. 7 and 19). This also enables us to drop the second term in the expression for  $\omega$  in equation (3) and amounts to a substitution setting  $\mu = \beta \cos \theta$ .

We also use the method of steepest descent for the integral over the energy. Evidently, this is not as well justified as using steepest descent for pitch angle, but it does give an excellent approximation for high harmonics and particle distribution which fall rapidly with increasing energy. (e.g. a power law  $f \propto (\gamma-1)^{-\delta}$  or a thermal distribution  $f \propto e^{-(\gamma-1)/kT}$ ).

Doing these integrations, we find for  $j_{\pm}$  and  $\kappa_{\pm}$ ;

$$\begin{Bmatrix} j_{\pm} \\ v^2 \kappa_{\pm} \end{Bmatrix} = \frac{\pi e^2 v_b}{c} (v/v_b)^{1/2} Y_{\pm}(1-\beta_0 \cos \theta)^{1/4} \gamma_0 z^{2m} \max(t_0) X \begin{Bmatrix} C_j(\gamma_0) \\ C_{\kappa}(\gamma_0) \end{Bmatrix} \quad (7)$$

where

$$Y_{\pm} = \frac{1}{a_{\pm}^2 + 1} \left[ 1 + (a_{\pm}^2 + 1) \frac{\cot^2 \theta}{\gamma_0^2} - \frac{2a_{\pm}}{\gamma_0} (1 - \beta^2 \cos \theta)^{1/2} \right] \quad (8)$$

$$x^{-2} = -\gamma_0^2 \frac{d^2 \ln C}{d\gamma^2} \Big|_{\gamma_0} - \frac{d \ln C}{d \ln \gamma} W(t_0, \theta) \quad (9)$$

$$m = \frac{v}{\gamma_0 v_b} (1 + t_0^2), \quad z_{mx} = \frac{t_0 e^{1/(1+t_0^2)^{1/2}}}{1 + (1+t_0^2)^{1/2}} \quad (10)$$

Note that all these expressions are evaluated at critical energy  $\gamma_0$  (and the corresponding  $\beta_0$  and  $t_0 = \gamma_0 \beta_0 \sin \theta$ ). These are two<sup>1</sup> such critical energies; one for emission and one for absorption. These are obtained from the transcendental equation;

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<sup>1</sup>These critical values are the same for both modes since in our approximation the parts of the expression which are different for each mode are slowly varying functions of  $\mu$  and  $\gamma$  and do not affect the values of the critical parameters  $\mu_0 = \beta_0 \cos \theta$  and  $\gamma_0$ .  
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$$t^2(1+t^2)^{-1/2} + (1 - \beta^2 \cos^2 \theta / t^2) \ln z_{mx} = - (v_b / 2v \sin^2 \theta) (d \ln C / d\gamma) \equiv \epsilon / \sin^2 \theta \quad (11)$$

Here and in equation (7) the function  $C_j$  and  $C_k$  are related to the particle distribution  $f$  as

$$C_j = f(\beta_0 \cos \theta, \gamma) / \gamma, \quad C_k = -\beta \gamma d/d\gamma \left( \frac{f(\beta_0 \cos \theta, \gamma)}{\beta \gamma^2} \right) \quad (12)$$

In the expression (eq. 9) for  $X^{-2}$ , the function  $W$  is a complicated function of  $t_0$  and  $\theta$ . However, as shown in paper I,  $W$  can be approximated by

$$W = 3/2 + 1/(\gamma_0^2 - 1). \quad (13)$$

This is an excellent approximation in the entire region where equations (6) to (11) are valid.

Polarization. For a single particle, the synchrotron radiation can be represented by the Stokes Parameters<sup>7</sup>;

$$\begin{aligned} \eta_I &= \eta_+ + \eta_- , \\ \eta_Q &= [(1 - a_+^2)/(1 + a_+^2)](\eta_+ - \eta_-) + 4\sqrt{\eta_+\eta_-} \cos\delta [a_+/(1+a_+^2)], \\ \eta_U &= 2\sqrt{\eta_+\eta_-} \sin\delta , \\ \eta_V &= [2a_+/(1+a_+^2)](\eta_+ - \eta_-) - 2\sqrt{\eta_+\eta_-} \cos\delta [(1-a_+^2)/(1+a_+^2)] , \end{aligned} \quad (14)$$

where  $\eta_{\pm}$  and  $a_+$  are defined as before, and  $\delta$  is the phase difference between the modes.

In the limit of large Faraday rotation (negligible absorption over a path length in which the plane of polarization rotates by  $2\pi$ ), we have the time average values  $\langle \cos\delta \rangle = \langle \sin\delta \rangle = 0$ .

Thus  $\eta_U = 0$  and

$$\eta_Q = c_1(\eta_+ - \eta_-), \quad c_1 \equiv \frac{1 - a_+^2}{1 + a_+^2} = \frac{-1}{(1 + x^2)^{1/2}} \quad (15)$$

$$\eta_V = 2c_2(\eta_+ - \eta_-), \quad c_2 \equiv \frac{a_+^2}{1 + a_+^2} = \frac{-x}{2(1 + x^2)^{1/2}} .$$



From this we obtain the degree of polarization

$$P = \frac{(\eta_Q^2 + \eta_V^2)^{1/2}}{\eta_I} = \frac{|\eta_+ - \eta_-|}{(\eta_+ + \eta_-)} \quad (16)$$

For a distribution of particle we integrate over the distribution as before and find  $P = |j_+ - j_-| / (j_+ + j_-)$  which with the help of equation (7) reduces to

$$P = \frac{[1 - 2x \cos \theta (1 - \beta^2 \theta \cos^2 \theta)^{1/2} / \gamma_0 \sin^2 \theta]}{(1 + x^2)^{1/2} (1 + 2 \cot^2 \theta / \gamma_0^2)} \quad (17)$$

Equations (7) to (13) along with (17) give our results in their general forms and are valid for all particle distributions which are not extremely anisotropic. (The results for the extremely anisotropic situation are more complicated and were described in Paper I. The modification obtaining the emission and absorption coefficients of the two modes separately and the polarization is similar to the modification described above. We will not present these results here because of their complexity and their limited usefulness.)

Given a distribution function subject to these limitations, the first step is evaluation of critical energies  $\gamma_0$  from equation (11). Then equations (6) to (10) and (12) and (13) evaluated at the appropriate  $\gamma_0$ 's give the desired results. This is still a complicated procedure, especially the solution of equation (11) for  $\gamma_0$ .

In the next section we shall show how this is simplified considerably for the two most commonly used particle distributions. Before doing so we

consider the asymptotic limits of these equations.

Asymptotic limits. Let us consider first the case when angle  $\theta$  is not too small (i.e. radiation away from the direction of the field). Then in the two extreme limiting cases, equation (11) simplifies

$$i) \quad \epsilon \ll 1, \quad \gamma_0 \gg 1, \quad \beta_0^3 \gamma_0^3 = 2/3 \epsilon \sin \theta, \quad W = 3/2; \quad (18)$$

$$ii) \quad \epsilon \gg 1, \quad \beta_0 \ll 1, \quad \gamma_0 \approx 1, \quad \beta_0^2 \gamma_0^2 = 4/\epsilon, \quad W = \epsilon/4.$$

The first case is realized at high frequencies and particle distributions which are not extremely non-relativistic which is the case of interest here. The second case is valid for non-relativistic particles and at low frequencies and has limited usefulness.

### III. EMISSIVITY AND ABSORPTION COEFFICIENT OF TWO COMMONLY USED PARTICLE DISTRIBUTIONS

The two particle distributions we use as examples are i) the distribution from a thermal gas, i.e. a Maxwellian distribution in energy and isotropic pitch angle distribution; and ii) the distribution with a power law spectrum at high energies and with a slowly varying pitch angle distribution.

#### A. Thermal Spectrum

This is the type of particle distribution considered by Trubnikov<sup>3</sup>. The distribution of particles at temperature  $kT$  (in units of  $m_e c^2$ ) is

$$f(\mu, \gamma) = C e^{-(\gamma-1)/kT} \gamma (\gamma^2 - 1)^{1/2}, \quad (19)$$

where for  $kT \ll 1$

$$C = (n/2)[2\pi(kT)^3]^{-1/2} (1 - 15 kT/8 + \dots), \quad (20)$$

and  $n$  is the number density of particles. From these and equation (12), it is clear that  $kT\kappa_K = C_j = f/\gamma$  and that  $d\ln C_K/d\gamma = d\ln C_j/d\gamma$ . Thus, it is obvious from equation (11) that the critical  $\gamma_0$  is the same for both  $j_{\pm}$  and  $\kappa_{\pm}$  and, in fact, according to eq. (7),  $j_{\pm} = v^2 kT\kappa_{\pm}$ . We also find

$$\frac{d\ln(f/\gamma)}{d\gamma} = \frac{-1}{kT} \left[ 1 - \frac{kT\gamma}{\gamma^2 - 1} \right], \quad d^2\ln(f/\gamma) = \frac{-(\gamma^2 + 1)}{(\gamma^2 - 1)^2}. \quad (21)$$

Using these equations we can calculate  $\gamma_0$  and  $X$  from equations (11) and (9) respectively. As shown in Paper I, these expressions can be considerably simplified. We find that the following expressions

$$(\gamma_0^2 - 1) = \begin{cases} (2vkT/v_b)(1 + 4.5 vkT\sin^2\theta/v_b)^{-1/3} & kT < 1 \\ (4vkT/3v_b\sin\theta)^{2/3} & kT \approx 1 \end{cases} \quad (22)$$

$$X^2 = (2kT/\gamma_0)(\gamma_0^2 - 1)/(3\gamma_0^2 - 1) \quad kT \lesssim 1$$

have the correct asymptotic limits in agreement with equations (18) and agree with the exact results from equations (9) and (11) to within 30% for most relevant ranges of angles, frequencies and temperatures and better than 10% in the majority of the interesting cases.

The above equations and equations (7), (8), and (10) give a complete description of the emissivity and absorption coefficient from a Maxwellian gas at all temperatures and frequencies. They are valid for  $kT \leq 1$  because at

temperatures  $kT > 1$  the use of the method of steepest descent for integration over the energy becomes invalid. However, the existence of the extremely relativistic thermal gas is in doubt<sup>11</sup>. On Figure 1 we compare the total absorption coefficient  $K = K_+ + K_-$  obtained from these relationships with numerical results from Lamb and Masters<sup>5</sup>. As evident, our analytic results give excellent agreement to the detailed numerical results even at low harmonics.

In the two limiting cases described in the previous section, these equations are considerably simpler. The interesting case,  $\epsilon \ll 1$  corresponds to  $\nu kT/\nu_b \gg 1$ ,  $\gamma_0^3 = 4\nu kT/3\nu_b \sin\theta$ , so that

$$j_{\pm} = 2kT\kappa_{\pm} = (2^{3/2}\pi e^2\nu_b/3c)G(\nu kT/\nu_b) \exp\left\{\frac{-\nu}{\nu_b} \left[\frac{4.5}{\sin\theta} \left(\frac{\nu_b}{\nu kT}\right)^2\right]^{1/3}\right\} \gamma_{\pm} \quad (23)$$

$$\gamma_{\pm} = 1 \mp 2(\sin\theta)^{4/3} (3\nu_b/4\nu kT)^{1/3} .$$

#### B. Power Law Energy Spectrum

Power law spectra are commonly used spectra in astrophysical problems and in other problems when the tail of the Maxwellian distribution begins to deviate from the exponential form. Usually power law spectra are defined with a low energy cut-off. To avoid such discontinuities and the divergence of the number of particles, we assume a spectrum of the form

$$f(\mu, \gamma) = Cg(\mu)[1 + (\gamma - 1)/\epsilon c]^{-\delta} \quad (24)$$

which converge at low energies;

$$C = n(\delta - 1/\epsilon_c), \quad \int_{-1}^{+1} g(\mu)d\mu = 1 . \quad (25)$$

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From this and equation (12) we have  $C_j = f/\gamma$  and

$$C_K = C_j \left[ \frac{\delta}{\gamma-1+\epsilon_c} + \frac{2\gamma^2-1}{\gamma(\gamma^2-1)} \right] \equiv C_j(\gamma) \phi_1(\gamma) . \quad (26)$$

In (24)  $\epsilon_c$  plays the role of the low energy cutoff (in units of  $m_e c^2$ ). For energies much greater than  $\epsilon_c$  the spectrum is a power law with index  $-\delta$  but it tends to a constant value at lower energies. The particles can be classified as ultra-relativistic or non-relativistic if  $\epsilon_c \gg 1$  or  $\epsilon_c \ll 1$ . We are interested primarily in cases with  $\epsilon_c \approx 1$ .

For distributions which are not highly anisotropic (i.e.  $d \ln(\mu)/d\mu \ll v/v_b$ ), we can carry out a calculation similar to that for a thermal gas. From (24) and (26) we find

$$\frac{-d \ln C_j}{d \ln \gamma} = 1 + \frac{\delta \gamma}{\gamma-1+\epsilon_c} , \quad \frac{\gamma^2 d^2 \ln C_j}{d \gamma^2} = 1 + \frac{\delta \gamma^2}{(\gamma-1+\epsilon_c)^2} \quad (27)$$

and

$$\frac{-d \ln C_K}{d \ln \gamma} = \frac{-d \ln C_j}{d \ln \gamma} - \frac{\gamma \phi_2(\gamma)}{\phi_1(\gamma)} , \quad \frac{\gamma^2 d^2 \ln C_K}{d \gamma^2} = \frac{\gamma^2 d^2 \ln C_j}{d \gamma^2} + \frac{\phi_1(\gamma) \phi_3(\gamma) - \phi_2^2(\gamma)}{\phi_1^2(\gamma)} \quad (28)$$

where

$$\phi_2(\gamma) = \frac{d \phi_1(\gamma)}{d \gamma} , \quad \phi_3(\gamma) = \frac{d^2 \phi_1(\gamma)}{d \gamma^2} . \quad (29)$$

It turns out that for semi-relativistic particle energies,  $\epsilon_c \gtrsim 1$  for  $v/v_b \gg 1$  we can use equation (18), and the values of  $X$  and  $\gamma_0$  obtained from

(11) and (9) can be approximated as follows

$$\beta_0^2 \gamma_0^2 = [4v / (3v_b \sin\theta)] \begin{cases} 1/(1 + \delta) & \text{for } j_{\pm} \\ 1/(2 + \delta) & \text{for } \kappa_{\pm} \end{cases} \quad (30)$$

and

$$X = \begin{cases} (1 + \delta)^{-1/2} & \text{for } j_{\pm} \\ (2 + \delta)^{-1/2} & \text{for } \kappa_{\pm} \end{cases} \quad (31)$$

These expressions are valid to within 30% for  $1 \lambda \epsilon_c \lambda > 0.5$ ,  $\sin\theta > \sin\theta_c \approx \epsilon_c^{1/2}$  and  $v \gtrsim v_b$ . However, they are most accurate at high frequencies  $v \gg v_b$  where  $\gamma_0 \gg 1$  and  $\beta_0 \approx 1$ . In this limit, substitution of (30) and (31) into (7) gives

$$\left. \begin{array}{l} j_{\pm}(v, \theta) \\ v^2 \kappa_{\pm}(v, \theta) \end{array} \right\} = \frac{\sqrt{3\pi} e^2 v_b \sin\theta}{4c} C \epsilon_c g(\beta_0 \cos\theta) Y_{\pm}(\theta) \begin{cases} (3\epsilon_c^2 v_b (\delta+1) \sin\theta / 4v)^{(\delta-1)/2} e^{-(\delta+1)/2} \\ (3\epsilon_c^2 v_b (\delta+2) \sin\theta / 4v)^{\delta/2} e^{-(\delta+2)/2} \left( \frac{(\delta+2)}{\epsilon_c} \right) \end{cases} \quad (32)$$

In this limit  $j_+ = j_-$ ,  $\kappa_+ = \kappa_-$ , and their difference is of higher order in  $\gamma_0$ ;

$$Y_{\pm}(\theta) = 1 \mp \frac{2\sin\theta}{\gamma_0} \quad (33)$$

These expressions have exactly the same dependence on  $v/v_b$  and  $\theta$  as the ultra-relativistic forms<sup>2</sup>.

For extremely non-relativistic particles, that is, for  $\epsilon_c \ll 1$  the above

expressions are valid as long as  $v/v_b \delta \gg 1$ . For the unlikely case of  $v/v_b \delta \ll 1$ , we find that  $\gamma_0 \approx 1$  and  $\beta_0^2 \approx v \epsilon_c / v_b \delta$ . Substitution of this in (7) gives the emissivity identical to that expected from a thermal gas if one identifies  $\epsilon_c / \delta$  with the temperature  $kT$ .

When we substitute eqs. (28) to (31) into (7) and (19), (using  $\theta = 60^\circ$ ,  $\delta = 4$ ), we get the results for  $j_\pm$ ,  $v^2 \kappa_\pm$  and  $P$  seen in Figures 2, 3, and 4. These are shown in comparison with numerical results of Marsh and Dulk<sup>8</sup>. Note that in this comparison we have a distribution with a sharp cut-off point at  $\epsilon = 0.02$ , so the approximations for  $\gamma_0$  at  $v \approx v_b$  are not as good as we would like. Still, at high  $v/v_b$  we get  $\gamma_0$  to within 5% in general.

The values for  $j_\pm$  and  $\kappa_\pm$  are within 60% at high frequencies and the approximation is better at higher  $v/v_b$ . The polarization looks better at lower frequencies (although it still is better than 50% even when  $P$  is small and errors can be magnified). The results for the extraordinary mode are not as good as those for the ordinary mode. (The percentage of error never reaches 30% for either  $j_\pm$  or  $\kappa_\pm$  above  $v/v_b = 10^{0.8}$ ). Also our results are (nearly) systematically higher than the numerical results.

#### IV. SUMMARY

Using a simple method of integration developed previously, we have derived expressions for the emissivity, absorption coefficient and polarization of synchrotron radiation for an arbitrary distribution of particles.

Equations (7) to (13) and (17) give our results in their most general form. And we find that equations (9) and (11) can be simplified considerably as in equations (22), (30), and (31) for the thermal distribution and the power law.

Our results do agree with previous analytic results, and they give good approximations to detailed numerical results. Although our results were derived for high harmonics, they give good agreement down to lower harmonics ( $\nu \approx 6\nu_b$  for  $j_{\pm}$ ,  $\nu \approx 10\nu_b$  for  $\kappa_{\pm}$ ; for total  $j$  and  $\kappa$ , however, the agreement is good down to  $\nu \approx 2\nu_b$ )<sup>6</sup>.

These results are limited to pitch angle distributions which are not extremely anisotropic and energy spectra that decrease rapidly with increasing energy. They also are only applicable for emissivities and absorption coefficients away from magnetic field times. Examples of other cases can be found in Paper I.

A more detailed numerical comparison will be found in a subsequent paper to be published in The Astrophysical Journal.

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## FIGURE CAPTIONS

Figure 1. The total synchrotron absorption for a thermal source at  $\theta = \pi/2$  for  $kT = 0.04$  (20 keV electrons). Points are from analytic expressions; the solid lines are numerical results of Lamb and Masters<sup>5</sup>.

Figure 2. Synchrotron emissivity of each mode divided by magnetic field and total particle number.  $\text{Log}(j_{\perp}/BN)$  vs  $\text{Log}(\nu/\nu_b)$ , at  $\theta = 60^\circ$ ,  $\delta = 4$ . The ordinary mode has been shifted down by a factor of 10 for clarity. The solid lines are numerical results of Dulk and Marsh. The O's are our analytic results. (The same will be true in Figures 3 and 4.)

Figure 3. Self absorption of each mode,  $\text{Log}(Bk_{\perp}/N)$  vs  $\text{Log}(\nu/\nu_b)$ . The low points at  $\nu/\nu_b = 10$  are due to the fact that we used a different approximation for  $\gamma_0$  on the first two points; one which is better at low  $\nu/\nu_b$ .

Figure 4. Degree of Circular Polarization vs  $\text{Log}(\nu/\nu_b)$ .

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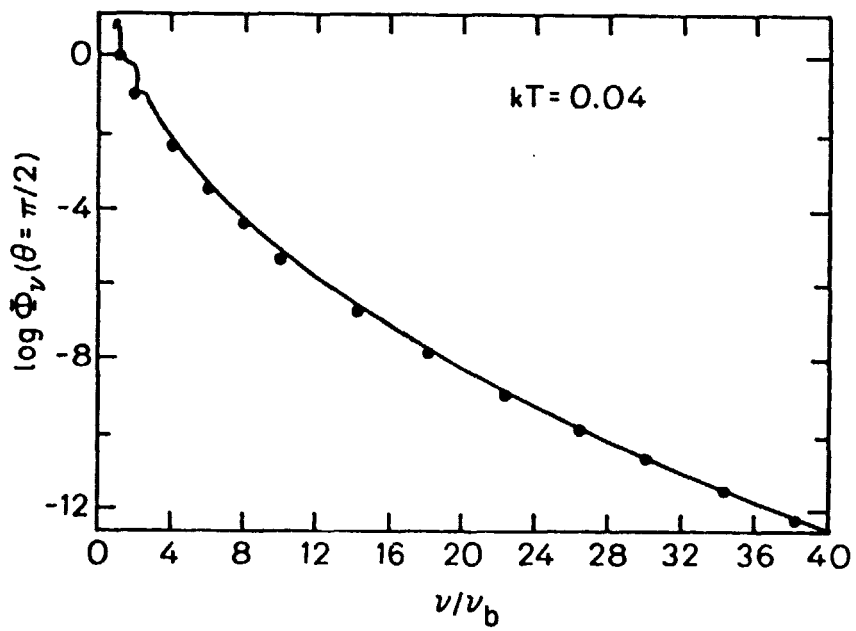


Figure 1

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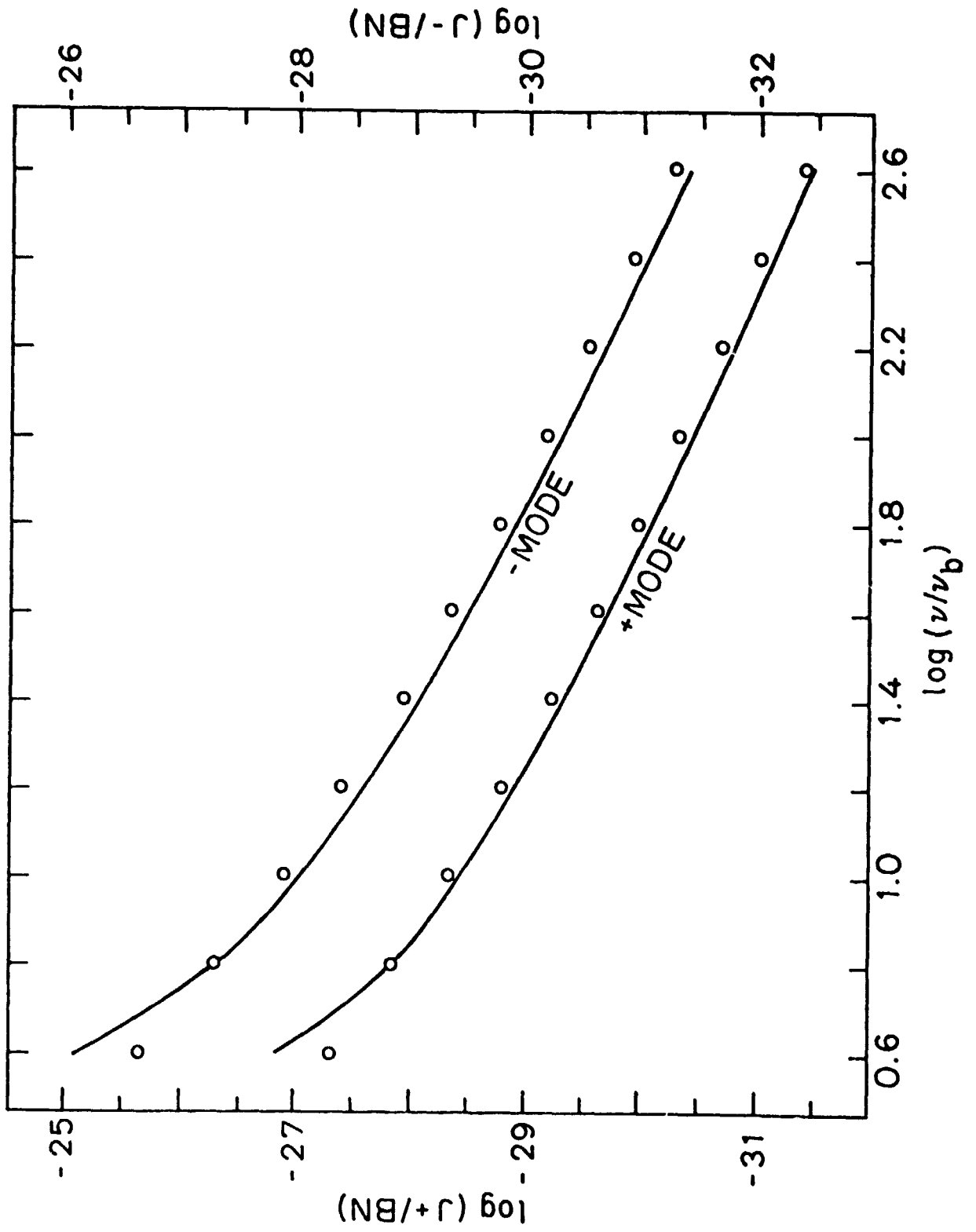


Figure 2

ORIGINAL PAGE IS  
OF POOR QUALITY

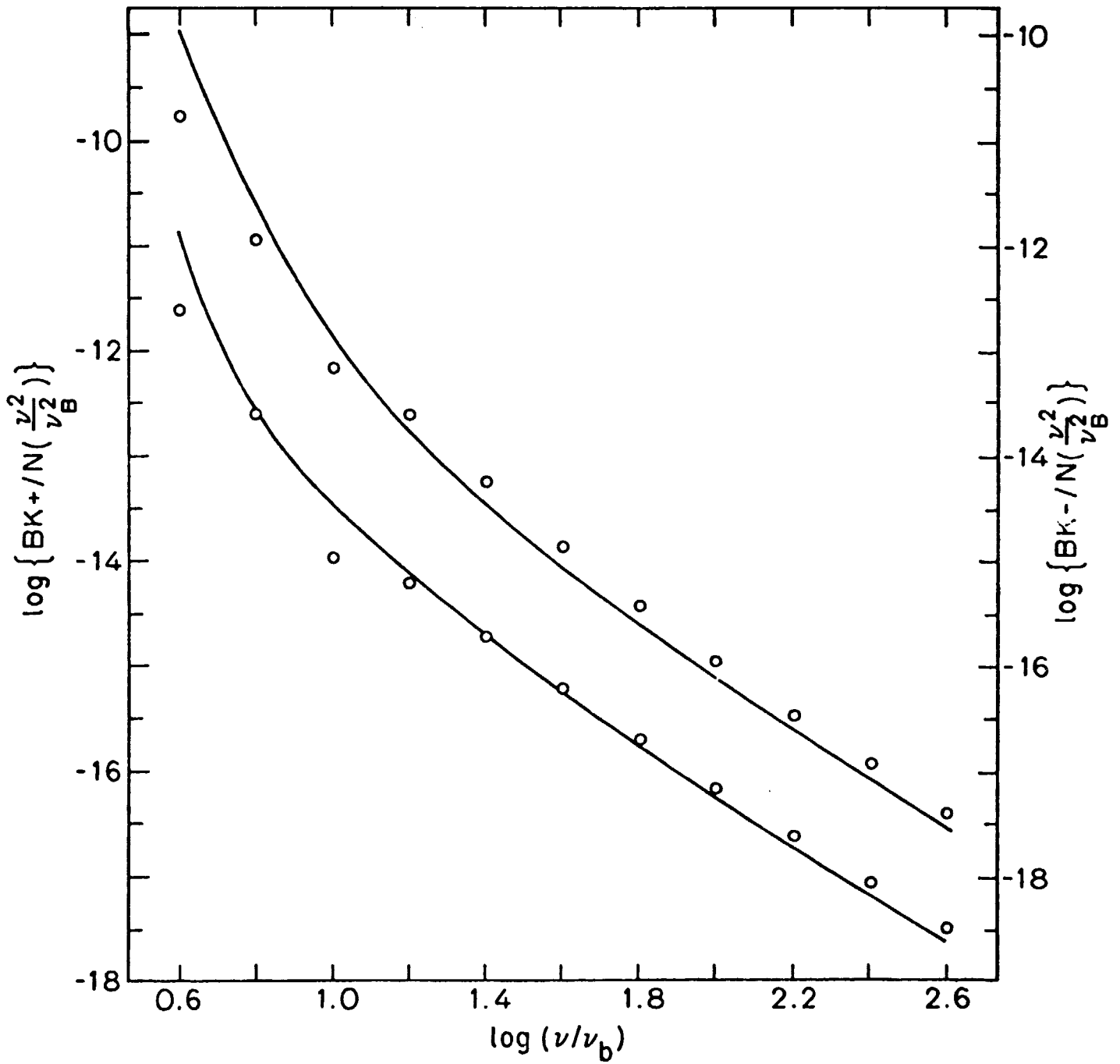


Figure 3

ORIGINAL PAGE IS  
OF POOR QUALITY

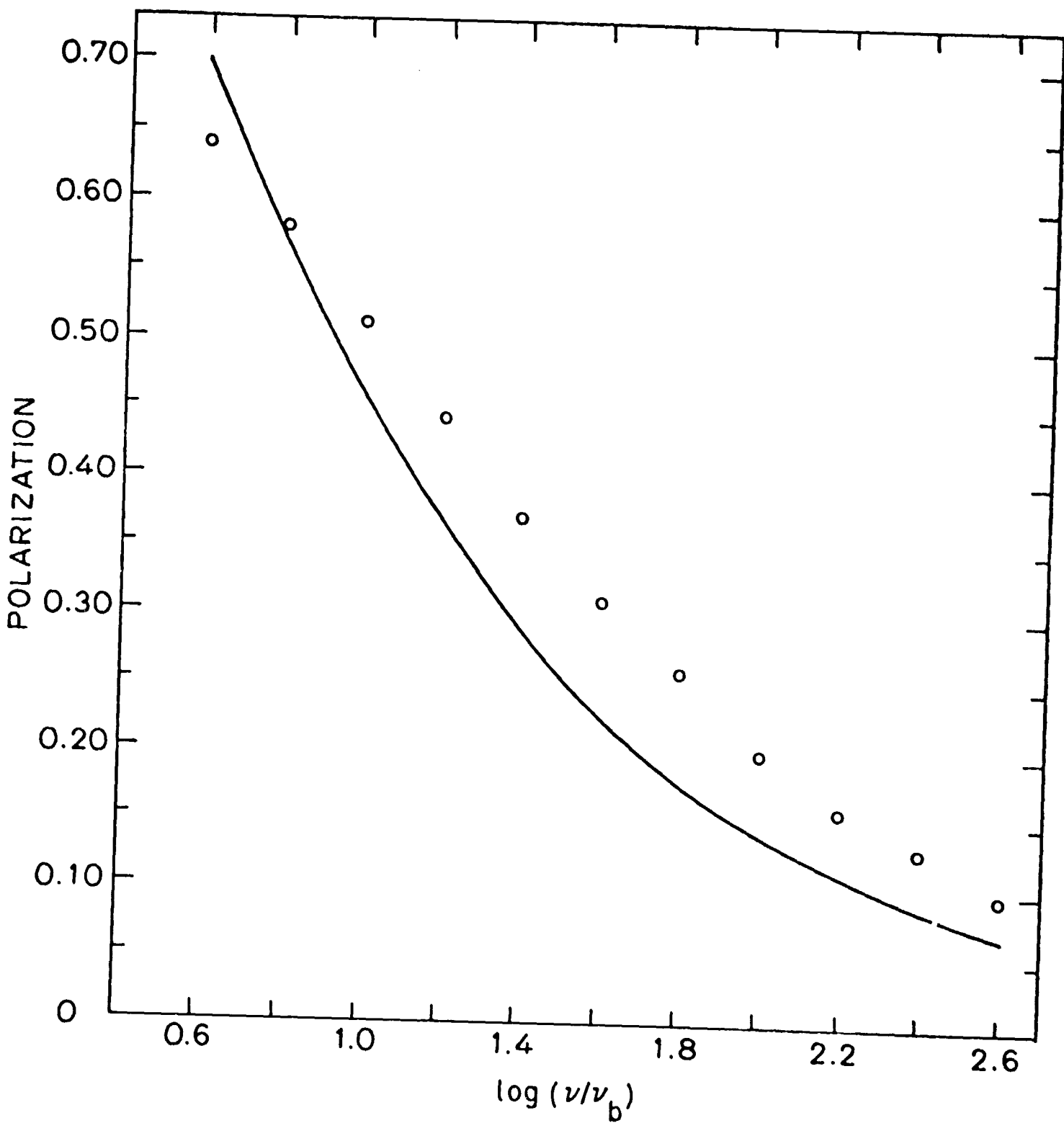


Figure 4