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Signal Analysis and Error Analysis Studies for a Geopotential Research Mission (GRM)

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ABSTRACT

The Geopotential Research Mission (GRM), formerly known as GRAVSAT/MAGSAT, has been proposed for the mapping of the Earth's gravity field. It is a candidate for a NASA new start in the near future. GRM is composed of a pair of surface-force compensated satellites in identical polar orbits, spaced approximately 3° apart at an altitude of 160km with a system precision of 1 μm/s.

The objectives of this paper are to discuss the signal characteristics and the geopotential parameter recovery capability of the SST Doppler sensor flown on GRM.

Simulation studies of the velocity profiles resulting from the perturbation produced by a 1° X 1°, 1 mgal anomaly as sensed by two GRM spacecraft orbiting altitudes of 160km and 200km respectively are described. It has been found that the amplitude of the gravity signal drops off by a factor of 1.5 when going from an altitude of 160km to 200km. By extrapolation the signal amplitude is further decreased by a factor of 3 when the orbital altitude is increased to 250km. Thus the amplitude of the measurement drops off as the altitude is increased to the point where it is insignificant at the 1 mgal level for altitudes above 200km.

Spectral analysis results show that for a GRM mission altitude of 160km and a system precision of 1 micrometer/sec., gravity field information can be sensed up to 230 cycles per orbital revolution – beyond that frequency the gravity signal is characterized by white noise. It follows that at the GRM mission altitude of 160km and a satellite-to-satellite Doppler system precision of ±1 micrometer per second, 1° X 1° gravity and geoid anomalies can be determined to an accuracy of 3.4 mgals and 8.6cm respectively.

These results are compatible with the scientific requirements for GRM which are: Accuracy – 2.5 mgals; Horizontal Resolution – 100km; Accuracy of Oceanic Geoid Undulation Difference – 10cm (over wavelengths of 100 to 3000 kilometers).

SIGNAL ANALYSIS AND ERROR ANALYSIS STUDIES FOR A GEOPOTENTIAL RESEARCH MISSION (GRM)

I. INTRODUCTION

Studies have been conducted at the Goddard Space Flight Center to investigate the accuracy with which the Earth's gravity field can be determined from a gravity field mapping mission such as the Geopotential Research Mission (GRM), formerly designated as GRAVSAT/MAGSAT. The studies performed at Goddard are threefold: (1) gravity field signal analysis, (2) rapid error analysis which can be used for mean gravity anomaly and mean undulation accuracy estimation for almost continuous data, and (3) full-scale simulation studies. This paper reports on the results of (1) & (2).

II. ANALYSIS

A. Signal Analysis

The perturbation by the gravity field induced on the intersatellite Doppler signal is given by the following relationship:

$$\delta (r \dot{\psi}) = \int_{t_o+\delta t}^{t_1 + \delta t} \frac{1}{r} \frac{\partial T}{\partial \psi} dt - \int_{t_o}^{t_1} \frac{1}{r} \frac{\partial T}{\partial \psi} dt \tag{1.0}$$

where

$T \equiv$ disturbing potential

$r \equiv a + h$

$a \equiv$ Earth's mean equatorial radius

$h \equiv$ satellite orbit altitude

$\psi \equiv$ true anomaly

$\delta t \equiv$ time separation interval of two satellites in identical orbits.

The form of the disturbing potential most useful for formulating (1.0) is given by the formula

$$T(r, \psi) = \frac{a}{4\pi} \iint_{\sigma} \Delta\bar{g} \quad S(r, \psi)\, d\sigma \tag{1.0}$$

where

$$S(r, \psi) = \frac{2a}{\ell} + \frac{a}{r} - \frac{3a\ell}{r^2} - \frac{a^2}{r^2} \cos\psi \left[5 + 3 \ln\left(\frac{r - a\cos\psi + \ell}{2r}\right)\right]$$

$$\ell = [r^2 + a^2 - 2r\,a\cos\psi]^{1/2}$$

$\Delta\bar{g}$ ≡ mean gravity anomaly

$d\sigma$ ≡ element of area.

Using (1.0) and (1.1) for an orbit configuration as depicted in Figure 1, velocity profiles resulting from the perturbation produced by a 1 mgal gravity anomaly as sensed by the 2 satellites are shown in Figure 2. The satellites are spaced 330 km (3°) apart – results are shown for 160 km and 200 km satellite altitudes. The amplitude of the gravity signal drops off by a factor of 1.5 when going from an altitude of 160 km to 200 km. By extrapolation, the amplitude is further decreased by about a factor of 3 when the orbital altitude is increased to 250 km. Thus, the amplitude of the measurement drops off as the altitude is increased to the point where it is insignificant at the 1 mgal level for altitudes above 200 km. By way of contrast, Figure 2A shows the SST signal generated by GRM for one full orbit, under the influence of a full (36, 36) spherical harmonic gravity field (GEM10B). The two satellites are at 160 km altitude, separated by 3 degrees.

The ability to resolve the signal from a (1° x 1°) 1 mgal gravity anomaly for 2 spacecraft orbiting at a 160 km altitude, whose orbits are displaced from the anomaly by 1° and 5°, respectively, is shown in Figure 3. From a 1° orbit displacement, better than 60 percent of the gravity signal is sensed. However, when the orbit is displaced by 5° from the gravity anomaly, only 10 percent of the gravity signal is sensed. This would imply that the gravity anomaly signal from an adjacent block drops off drastically with increased distance. Nevertheless, it is significant that remote gravity anomalies do contribute to the overall gravity signal by as much as the 10 percent indicated.

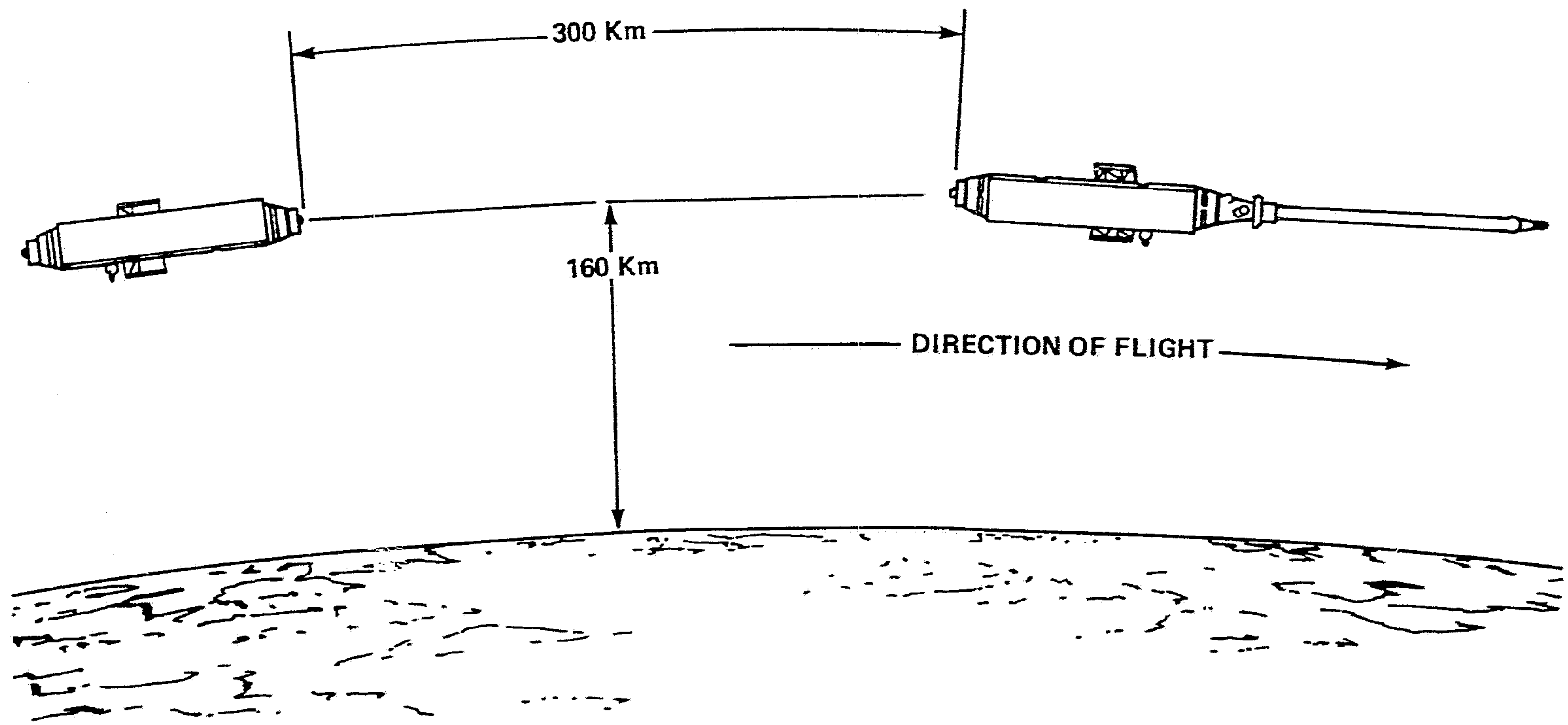

Figure 1. GRM Flight Profile.

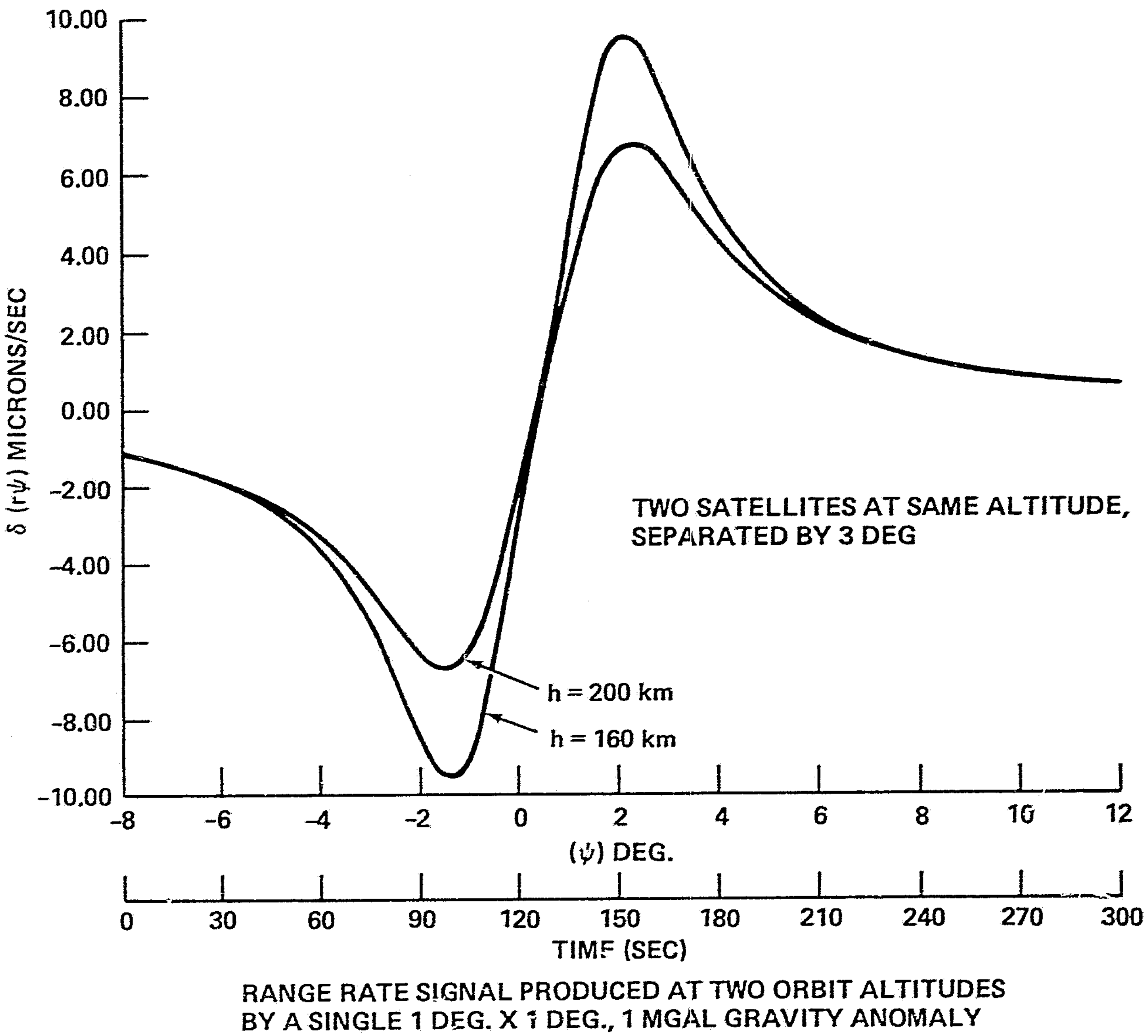

Figure 2. GRM Anomaly Sensing at Two Altitudes.

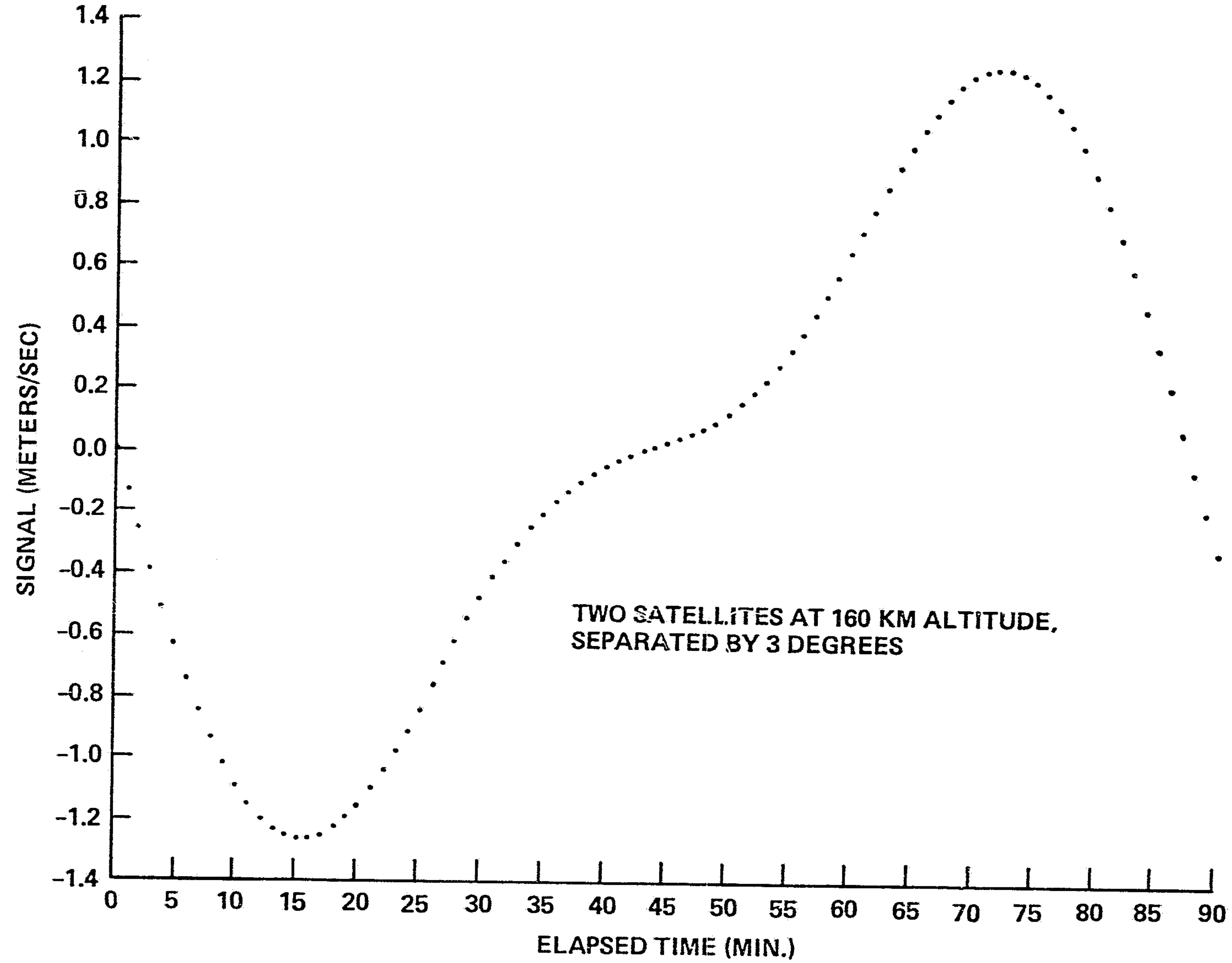

Figure 2A. GRM Full (36, 36) Gravity Field Sensing.

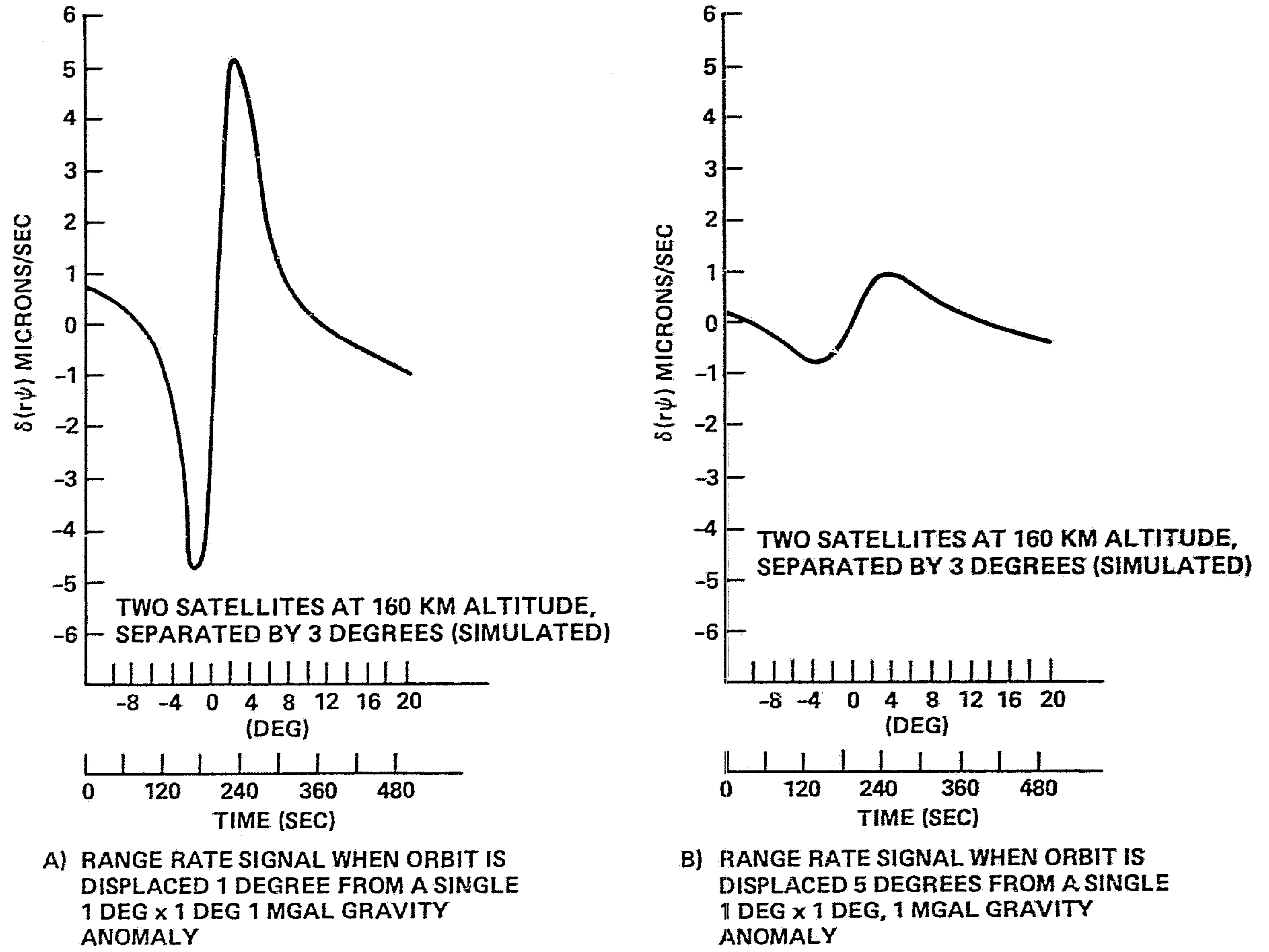

Figure 3. GRM Anomaly Sensing at Two Lateral Separation Distances.

B. Error Analysis

A rapid error analysis procedure developed by C. Jekeli and R. H. Rapp (Ref. 1) has been used for the estimation of mean gravity anomalies and mean undulation accuracies that can be obtained from SST missions such as the future Geopotential Research Mission (GRM). The basic method used is the analysis of the gravity field mapping in the frequency domain with results obtained in terms of mean gravity anomalies and mean geoid undulations. The measurements to be considered are velocity differences, with the errors in the measurements assumed to be uncorrelated.

In order to estimate the expected "signal" for SST Doppler we shall evaluate its power spectrum (Refs. 2, 3) derived from the Earth's anomalous gravity potential. That is,

$$(V+T) = \frac{GM}{r}\left[1 + \sum_{\ell=2}^{N} \sum_{m=0}^{\ell} (a/r)^{\ell}\, \bar{P}_{\ell m}(\sin\varphi)\,(\bar{C}_{\ell m}\cos m\lambda + \bar{S}_{\ell m}\sin m\lambda)\right] \tag{2.0}$$

where

r	is the geocentric distance
φ	is the subsatellite geocentric latitude
λ	is the subsatellite east longitude
a	is the mean radius of the Earth
$\bar{P}_{\ell m}(\sin\varphi)$	are the fully normalized associated Legendre polymonials of degree ℓ and order m (ℓ is also the orbital frequency of a satellite in cycles per revolution) and
$\bar{C}_{\ell m}, \bar{S}_{\ell m}$	are the fully normalized coefficients of a truncated harmonic series expansion representing the geopotential.

The power spectrum of the velocity can be determined by utilizing the Law of Conservation of energy.

$$\tfrac{1}{2}v^2 - U = E \tag{3.0}$$

where U is the total gravitational potential, v is the velocity, and E is the total (constant) energy. Decomposing U into the reference potential V and the disturbing potential T of the Earth, we have

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$$\tfrac{1}{2}v^2 - V - T = E \tag{3.1}$$

The variation of (3.2) then reads

$$v\,\delta v = \delta T \tag{4.1}$$

where $v^2 = GM/r$.

From Equation (2.0), together with (4.1) we obtain the variation of the satellite velocity magnitude as:

$$\delta v = v \sum_{\ell=2}^{N} \sum_{m=0}^{\ell} (a/r)^{\ell}\, \overline{P}_{\ell m}(\sin\varphi)\ (\overline{C}_{\ell m}\cos m\lambda + \overline{S}_{\ell m}\sin m\lambda) \tag{4.2}$$

The velocity power spectrum of δv is now obtained by squaring Equation (4.2) and integrating over the unit sphere:

$$V_{\ell}^{2}(\delta v) = v^2 (a/r)^{2\ell} \sum_{m=0}^{\ell} \overline{P}_{\ell m}^{\,2}(\sin\varphi)\ (\overline{C}_{\ell m}^{\,2} + \overline{S}_{\ell m}^{\,2}) \tag{4.3}$$

Using Kaula's rule (Ref. 4), the summation containing the $\overline{C}$, $\overline{S}$ terms can be estimated as follows:

$$\sum_{m=0}^{\ell} (\overline{C}_{\ell m}^{2} + \overline{S}_{\ell m}^{2}) = (10^{-5}/\ell^2)^2 \tag{4.4}$$

Then

$$V_{\ell}^{2}(\delta v) = v^2 (a/r)^{2\ell} (2\ell+1)(10^{-5}/\ell^2)^2 \qquad \ell > 2 \tag{4.5}$$

In order to derive the velocity variations for the low-low SST configuration, Equation (4.1) is to be modified as follows:

$$\delta T = (T_2 - T_1) = v\,\delta \dot{v}_H \tag{4.6}$$

where T_1 and T_2 are the disturbance potentials at satellite positions 1 and 2 in the orbit with an angular separation, ψ_{12}.

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The corresponding variance in the horizontal component of velocity is now to be derived as follows:

$$\sigma_\ell^2\,(\delta T) = \sigma_\ell^2(T_1) + \sigma_\ell^2(T_2) - 2\sigma_\ell(T_1T_2) \qquad (4.7)$$

By assuming that $\sigma_\ell^2(T_1) = \sigma_\ell^2(T_2) = \sigma_\ell^2(T)$ and $\sigma_\ell(T_1T_2) = P_\ell(\cos\psi_{12})\,\sigma_\ell^2(T)$ (Ref 2) one obtains

$$\sigma_\ell^2(\delta T) = 2\sigma_\ell^2(T)\,[1 - P_\ell(\cos\psi_{12})] \qquad (4.8)$$

and from Equation (2.0) together with (4.6) and (4.8),

$$\sigma_\ell^2(\delta\,\sigma_H) = 2v^2\,(a/r)^{2\ell}\,[1 - P_\ell(\cos\psi_{12})]\,(2\ell + 1)\,(10^{-5}/\ell^2)^2 \qquad (4.9)$$

Equation 4.9 represents the horizontal velocity "signal", variance.

The intersatellite velocity signal spectra is given in Figure 4. It can be seen that for the planned GRM altitude of 160 km and the SST system precision of $\pm 1\ \mu$m/s, gravity signals at the orbital frequency of 180 cycles/revolution ($\ell = 180$) can be sensed by GRM. Also shown in this figure is the level of signal attenuation with increasing orbital altitude.

The frequency spectrum only provides information on the specific frequency (spectral line) at which the SST velocity signal will sense the gravity field. It does not sense the maximum frequency at which a SST system with a specified signal precision and a specified orbit altitude can sense gravity field information. Determining the total gravity signal sensitivity for a SST system having a given level of precision necessitates integrating the velocity cpectrum. That is,

$$\sigma_{\ell,\infty}(\delta v_H) = \left[\int_{\ell_1}^{\infty} \sigma_\ell^2\,(\delta v_H)\,d\ell\right]^{1/2} \qquad (5.0)$$

The lower integration limit can be interpreted as that orbital frequency at which the gravity signal-to-noise ratio is unity for a given SST system precision. In Figure 5, it can be seen that for the system precision of 1 μm/sec and orbital altitude of 160 km, the gravity signal cut-off frequency

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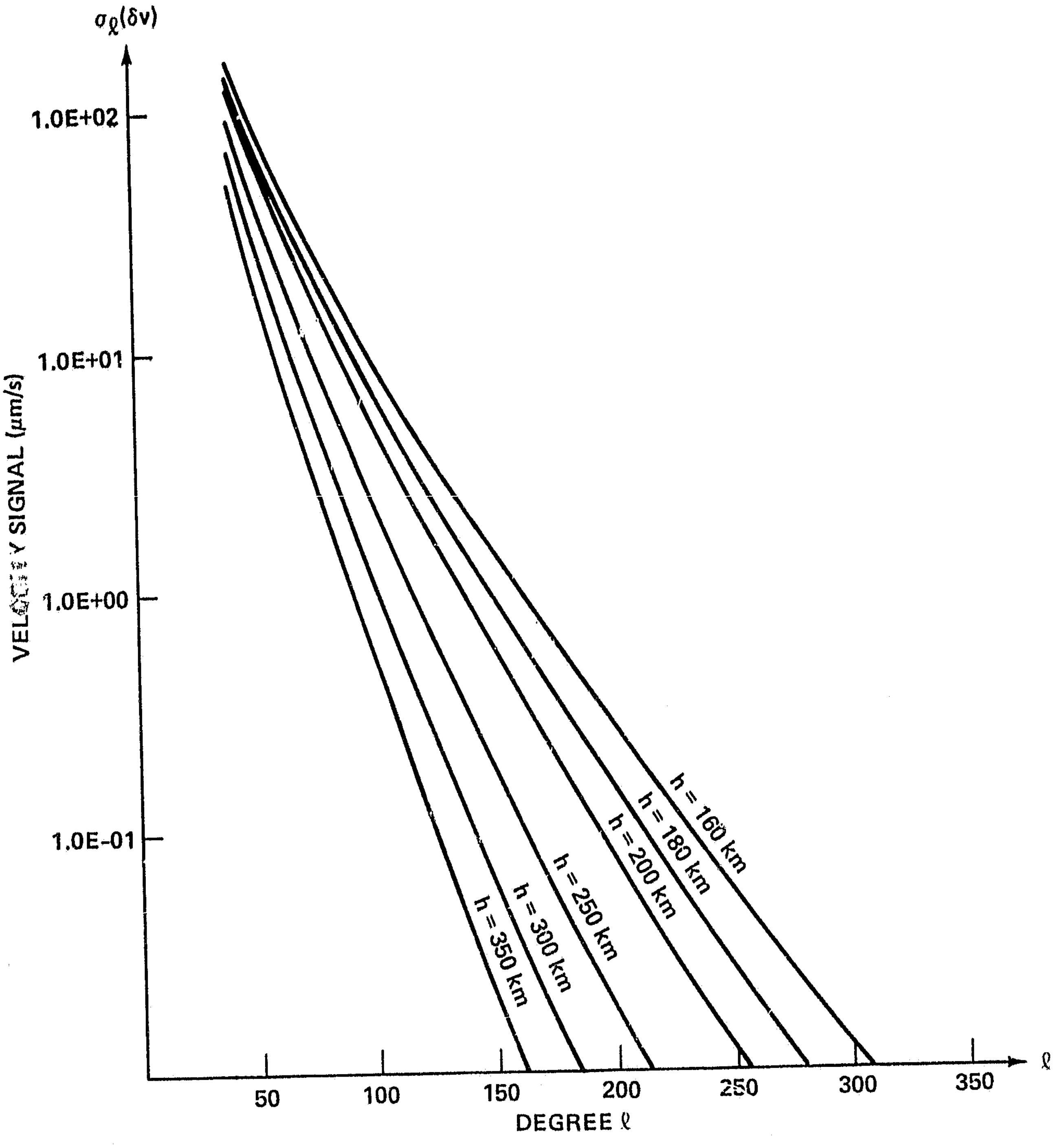

Figure 4. GRM Velocity Spectrum for Variable Orbit Altitudes.

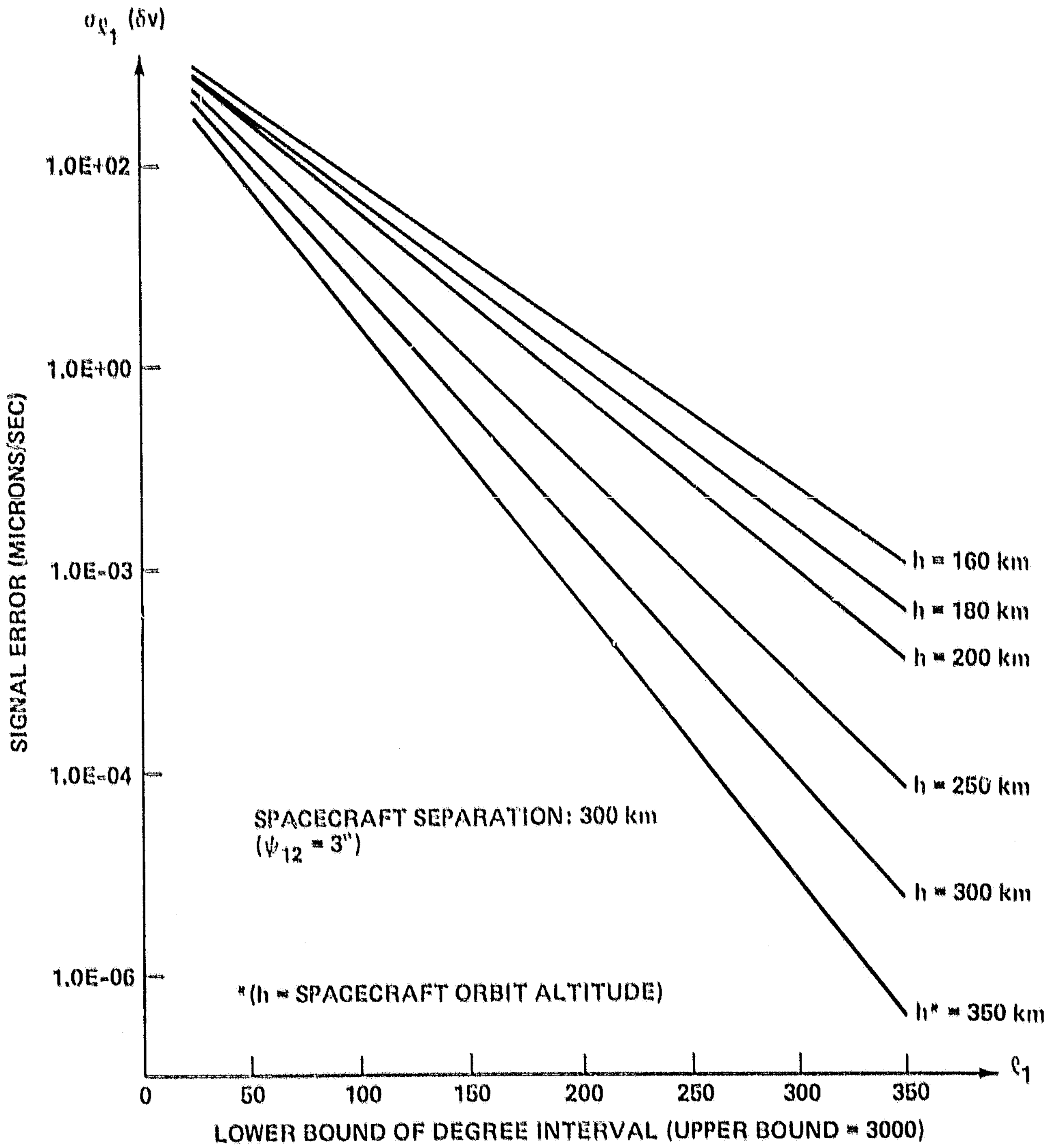

Figure 5. GRM Velocity Signal Error.

occurs at the orbital frequency of about 230 cycles per revolution ($\ell = 230$). All gravity information above that frequency (i.e. limited by the SST sensor precision) would be characterized by white noise. Thus, a higher "cut-off" frequency at a given orbit altitude necessitates a SST sensor of a higher precision.

To assess the accuracy with which geophysical parameters (such as gravity or geoid anomalies) can be determined when using the previously selected GRM sensor precision of ± 1 μm/s, the models developed by Jekeli and Rapp (Ref. 1) are used.

The total error budget associated with these parameters is the combination of the errors of commission and that a omission (generally the larger terms). The models for these error sources read as follows:

(a) The error of commission in terms of mean gravity anomalies is given by:

$$\sigma_C(\delta\bar{g}) = (\gamma/a)^{1/2} \; (\Delta A/4\pi)^{1/2} \; \sigma(\delta v_H) \left[\sum_{\ell=2}^{\ell_1} \frac{\beta_\ell^2 (\ell-1)^2 (2\ell+1)}{2[1-P_\ell(\cos\psi_{12})]} \left(\frac{r}{a}\right)^{2\ell+1} \right]^{1/2} \quad (6.0)$$

(b) The error of omission in terms of mean gravity anomalies is given by:

$$\sigma_O(\delta\bar{g}) = \gamma \times 10^{-5} \left[\sum_{\ell_1+1}^{\infty} \beta_\ell^2 \; \frac{(\ell-1)(2\ell+1)}{\ell^4} \right]^{1/2} \quad (6.1)$$

where

$$\beta_\ell \equiv \cot \frac{\psi_o}{2} \left[\frac{P_{\ell 1}(\cos\psi_o)}{\ell(\ell+1)} \right] \quad (6.2)$$

$$\psi_o = [S^\circ \times S^\circ/\pi]^{1/2} \quad (6.3)$$

$$\gamma = (GM/a^2) \quad (6.4)$$

ℓ_1 = Value obtained from Figure 5 for which the signal-to-noise ratio is unity for a specified SST system precision $\sigma(\delta v_H)$.

ΔA = Area of block in units of radians squared.

ψ_o = The radius of a circular cap that has about the same area ΔA as a block whose sides are S° (in degrees).

ψ_{12} = Angular separation of the two GRM spacecraft (in degrees).

a = Mean equatorial radius of the Earth.

S° = Side of anomaly block (in degrees).

and

(c) <u>The total error for mean gravity</u> anomalies is as follows:

$$\sigma_T(\delta\overline{g}) = [\sigma_C^{\,2}(\delta\overline{g}) + \sigma_O^{\,2}(\delta\overline{g})]^{1/2} \tag{6.5}$$

In terms of geoid anomaly errors, the error models are as follows.

(a) <u>The error of commission</u> in terms of mean geoid anomalies is given by:

$$\sigma_C(\overline{N}) = (a/\gamma)^{1/2}\,(\Delta A/4\pi)^{1/2}\,\sigma(\delta v_H)\left[\sum_{\ell=2}^{\ell_1} \frac{\beta_\ell^2\,(2\ell+1)}{2[1-P_\ell(\cos\psi_{12})]}\left(\frac{r}{a}\right)^{2\ell+1}\right]^{1/2} \tag{7.0}$$

(b) <u>The error of omission</u> in terms of mean geoid anomalies is given by:

$$\sigma_O(\overline{N}) = a \times 10^{-5}\,[\sum_{\ell_1+1}^{\infty} \beta_\ell^2\,(2\ell+1/\ell^4)]^{1/2} \tag{7.1}$$

(c) <u>The total error</u> for mean geoid anomalies is:

$$\sigma_T(\overline{N}) = [\sigma_C^{\,2}(\overline{N}) + \sigma_O^{\,2}(\overline{N})]^{1/2} \tag{7.2}$$

Figures 6 and 7 show the total expected errors in terms of the gravity and geoid anomaly parameterization as obtained from Equations (6.5) and (7.2). The errors shown are for the GRM mission SST sensor precision $\sigma(\delta v_H) = \pm 1$ μm/s, satellite separation distance $\psi_{12} = 3°$ as a function of orbital altitude and block size.

It is shown that by using a pair of spacecraft in a 160 km polar orbit separated by 3°, one can map the geopotential in terms of 1° x 1° blocks to about 3½ milligals or 9 cm respectively.

CONCLUSIONS

As is evident from Figures 6 and 7, the planned GRM mission requirements can be met using the SST low-low approach and a 1 μm/s Doppler system, that is, global gravity and geoid anomalies

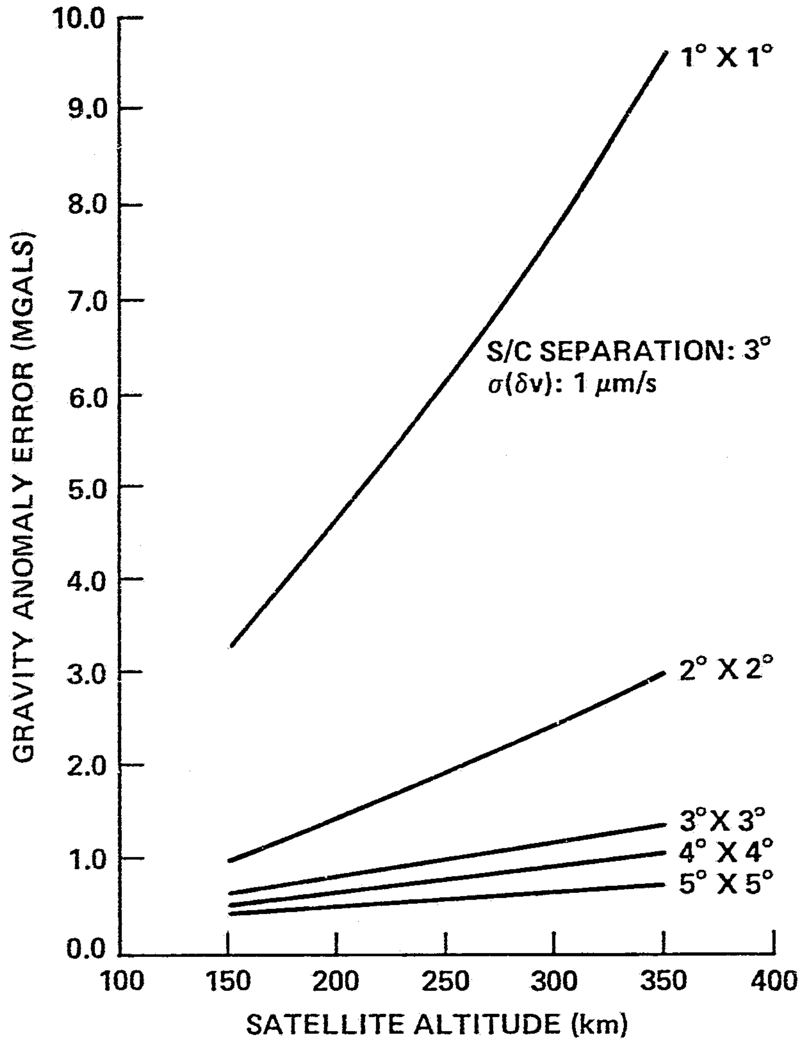

Figure 6. Gravity Anomaly Error (mgals).

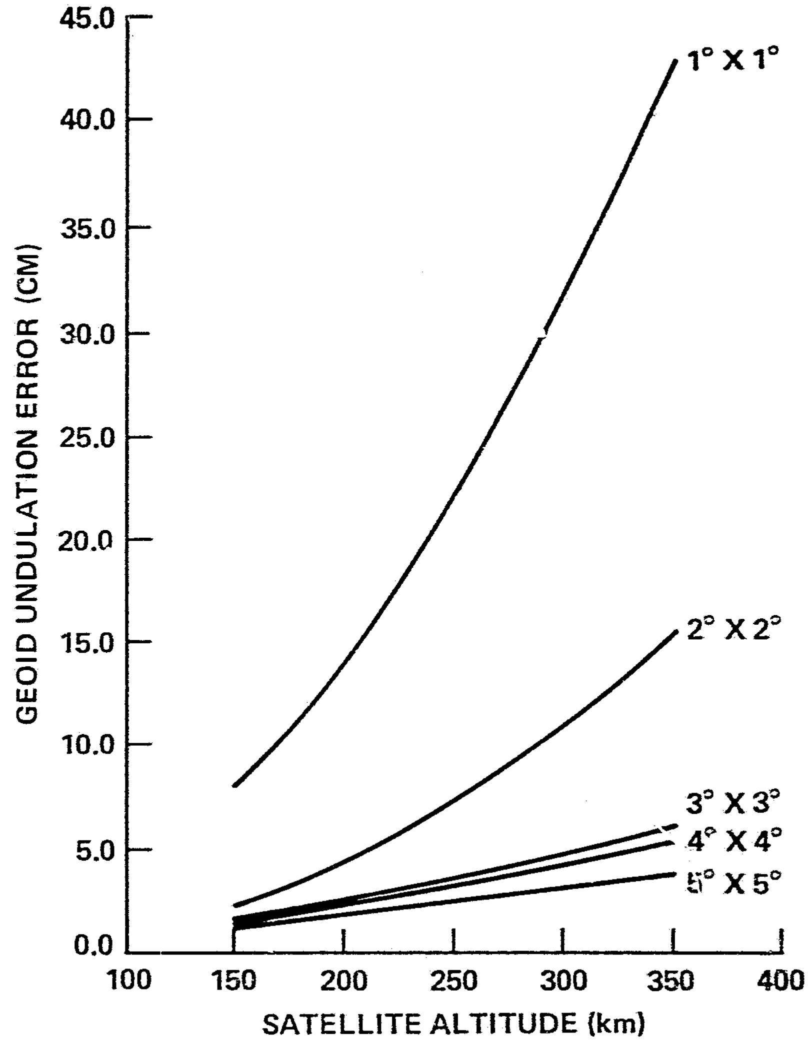

Figure 7. Geoid Undulation Error (cm).

can be recovered with a resolution of 100 km and a precision of 3 mgals and 9 cm respectively.

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