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STATIONARITY OF MAGNETOHYDRODYNAMIC FLUCTUATIONS IN THE SOLAR WIND

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Abstract

Solar wind research and studies of charged particle propagation often assume that the interplanetary magnetic field represents a stationary random process. In this paper we investigate the extent to which ensemble averages of the solar wind magnetic fields follow the asymptotic behavior predicted by the ergodic theorem. Several time periods, including a span of nearly two years, are analyzed. Data intervals which span many solar rotations satisfy the conditions of "weak" stationarity if the effects of solar rotation are included in the asymptotic analysis. Shorter intervals which include a small integral number of interplanetary sectors also satisfy weak stationarity. The results are illustrated using magnetometer data from the ISEE-3, Voyager and IMP spacecraft.

1. Introduction

In many areas of solar wind research it is common to assume, explicitly or implicitly, that the medium being studied is either time stationary or spatially homogeneous (or both). For example, the theories of pitch angle scattering of charged particles [Jokipii, 1966; Hall and Sturrock, 1967; Hasselmann and Wibberenz, 1968; Klimas and Sandri, 1971; Jones, Kaiser and Birmingham, 1973; Fisk et al., 1974] and field line random walk [Jokipii, 1966; Jokipii and Parker, 1968, 1969; Jokipii, 1971] both utilize the concept of an ensemble average of the interplanetary magnetic fields. Such averages are meaningful only if a particular realization of the magnetic field time series does, in fact, represent a realization of a stationary process. Typically one constructs the ensemble average either by utilizing a large number of degrees of freedom in a Blackman-Tukey procedure [Blackman and Tukey, 1958], or by suitably smoothing the output of a fast Fourier transform calculation. Essential to this procedure is the assumption that the underlying probability distribution function which describes the observed magnetic fluctuations is invariant with respect to shifts in the origin of time and/or space.

The purpose of this paper is to investigate quantitatively whether the oft assumed time-stationarity of interplanetary magnetic fluctuations is compatible with the measured properties of the fields. Stationarity of the two-time correlation function is ensured if the first and second moments of the probability distribution are themselves time stationary (i. e. "weak" stationarity). The assumption of weak stationarity is the most frequently encountered approximation in the literature.

The primary emphasis in this paper is to investigate the stationarity of solar wind fluctuations; the relationship of the analysis to the question of homogeneity is deferred to Section 5. In Section 2 we first refine the concept of stationarity and develop the analytic tools necessary to investigate the extent to which the solar wind fields are stationary. In Sections 3 and 4 we apply those techniques to several intervals of solar wind magnetic field data.

2. Stationary Random Processes

Let B(t) be a random variable which depends on time. One can think of it as a single vector component of the interplanetary magnetic field. Although a sample of B(t) over a finite time interval is essentially irreproducible, the set of all possible samples has well defined and physically meaningful average properties. We will designate this average over the ensemble of samples by $\langle f(B) \rangle$, where f is any function of the random variable B(t). The ensemble is completely described in a statistical sense by a heirarchy of N-point probability distribution functions specifying the likelihood that B(t) lies near specified values at N specified times. If both the probability distributions and the ensemble averaged properties of B(t) do not depend on the origin of time, the process B(t) is stationary. In that case the mean of B(t) is

$$a \equiv \langle B(t) \rangle \tag{1}$$

where a is independent of time. The two point correlation function is defined by

$$R(\tau) \equiv \langle \delta B(t_1)\, \delta B(t_2) \rangle \tag{2}$$

where $\tau = t_2 - t_1$ and $\delta B = B - a$.

Stationarity in the strict sense [Cramér, 1940; Pugachiev, 1962; Panchev, 1971; Yaglom, 1962] implies an infinite number of higher order relations, one for each N-point correlation function. Strict stationarity is not of practical value because measurement of N-point correlation functions with $N > 2$ is usually impossible. Consequently, weak stationarity ($N = 2$) is the property of interest. When (1) and (2) are satisfied, a is finite, and $R(\tau)$ is both bounded and vanishes sufficiently rapidly at infinity, it is possible to define the power spectrum of B(t) and to give a meaningful definition to the concept of a correlation time for B(t).

If one examines samples of B(t) over finite time intervals T, some variation in the estimated values of the mean a and $R(\tau)$ will be apparent. However, for sufficiently large intervals T, convergence of the estimated values of a and $R(\tau)$ to the true ensemble average values must occur if the assumption that B(t) is a stationary random process is justified. This intuitive concept can be reformulated in a more rigorous way. Under fairly general assumptions, it is possible to show that the sequence of time averages of a stationary random variable converges to the ensemble average prediction as the averaging interval T is increased without bound. This assertion is known as the ergodic theorem for stationary processes, and has been proven with varying degrees of generality and mathematical rigor by Slutsky [1938], Kolmogoroff [1938], Khinchin [1934], Maruyama [1949], Grenander [1950] and others. General discussions can be found in Panchev [1971], Cramér [1940] and Yaglom [1962]. If we define the time average of B(t) over an interval T as

$$[B]_T \equiv (1/T) \int_0^T dt\, B(t) \tag{3}$$

then, in its simplest version, the ergodic theorem states that $[B]_T$ converges to the ensemble average mean a in the sense that

$$\lim_{T\to\infty} \langle ([B]_T - a)^2 \rangle = 0 \tag{4}$$

provided that the two-point correlation $R(\tau)$ satisfies

$$\lim_{T\to\infty} (1/T)\int_0^T d\tau R(\tau) = 0 \tag{5}$$

More general versions of this theorem state that higher order finite time averages analogous to eq. (3) converge as does $[B]_T$ in eq. (4) provided that certain higher order correlation functions, i. e. "central" moments, satisfy a property analogous to the limit shown in eq. (5). (Central moments are moments of δB rather than B.) For example, if we define the estimate of the variance of B(t) over the interval T to be

$$[\delta B^2]_T \equiv (1/T)\int_0^T dt[B(t) - [B]_T]^2$$

and if the ensemble average variance is defined by $\sigma^2 \equiv \langle\delta B^2\rangle$, then the ergodic theorem states that

$$\lim_{T\to\infty} \langle([\delta B^2]_T - \sigma^2)^2\rangle = 0 \tag{6}$$

provided that, in addition to eq. (5), the following limit is satisfied

$$\lim_{T\to\infty} (1/T)\int_0^T d\tau\langle\{\delta B(t)^2\cdot\delta B(t+\tau)^2 - \sigma^4\}\rangle = 0. \tag{7}$$

The last relation involves a two-point fourth order moment and the assumption that it is independent of t goes beyond the realm of weak stationarity. In the special case of Gaussian distributions, strict stationarity is implied by weak stationarity and all higher order generalizations of eq. (4), including eq. (6), follow from (5) for the two-time correlation function because all the required limits (e.g., eq. 7) depend on moments that are determined completely by $R(\tau)$ [Panchev, 1971; Cramér, 1940].

If $B(t)$ is a bounded function of time and the conditions leading to eq. (4) are satisfied, then $\Delta^2[B]_T \equiv \langle([B]_T - a)^2\rangle$ possesses a Taylor series in powers of $1/T$ about its limiting value of zero. It is a simple matter to extract the asymptotic behavior of the series for several cases of interest. Following Panchev [1971], it follows directly from the definition of $[B]_T$ that

$$\Delta^2[B]_T = (2/T^2)\int_0^T dt \int_0^t d\tau R(\tau) \tag{8}$$

If $R(\tau)$ is a "Lanczos-type" function [Lanczos, 1956; Matthaeus and Goldstein, 1982] so that for some large value of T', $R(t > T') \approx 0$, then the evuluation of (8) is straightforward. This is considerably more restrictive than is required for ergodicity (cf. eq. 5), and corresponds to fluctuations which are completely uncorrelated for $t > T'$. In this case

$$\int_0^t R(\tau)d\tau = \sigma^2 T_c \tag{9}$$

for any $t > T'$, where the correlation time T_c is defined by [Batchelor, 1970]

$$T_c = \int_0^\infty d\tau R(\tau)/R(0) \tag{10}$$

and $R(0) = \sigma^2$. For large $T > T'$, eq. (9) for $\Delta^2[B]_T$ can be rewritten as

$$\Delta^2[B]_T = 2\sigma^2 T_c/T + O((T_c/T)^2) \tag{11}$$

which is the desired form.

As we shall see below, the convergence of $[B]_T$ to its limiting value can be slow and not dependent solely on T_c. This situation can sometimes be treated by adding a coherent oscillating component with amplitude α and period $T_o = 2\pi/\omega_o$ to a time series whose two-time correlation function is Lanczos type. For example, define $b(t) = B(t) + \alpha\cos(\omega_o t + \phi)$, where ϕ is a random phase which is constant within a given realization. The two-point correlation function corresponding to $b(t)$ is then $R(\tau) + K\cos(\omega_o\tau)$ where $K \doteq \alpha^2/2$ provided that $\alpha^2/2$, the power in the oscillating signal, is much greater at frequency ω_o than the power in the original signal $B(t)$. If $B(t)$ is a stationary process, then so is $b(t)$, and $\langle b(t)\rangle = \langle B(t)\rangle$. Because eq. (5) is satisfied, $[b]_T$ defined in analogy to (3) will converge to $\langle b(t)\rangle$ (cf. eq. 4). In place of (11), the asymptotic behavior of $\Delta^2[b]_T$ is now

$$\Delta^2[b]_T = 2\sigma^2 T_c/T + 4K[\sin^2(\omega_o T/2)]/(\omega_o T)^2 + O((T_c/T)^2) + O((T_o/T)^3) \quad (12)$$

Unlike (11), which depends on time only through T_c/T as $T \to \infty$, eq. (12) for $\Delta^2[b]_T$ now also depends on T_o/T. For $KT_o \gg \pi^2\sigma^2 T_c$, there is a range of T near T_o for which the first two terms in (12) are comparable in magnitude and all higher order terms are negligible. For $T \gg T_o$ the second term becomes negligible and the simple dependence of (11) is recovered.

The relationships (11) and (12) describe in an ensemble average sense how finite time averages of $B(t)$ or $b(t)$ converge to their means. These properties can be extended to higher order if the higher order correlation functions of $B(t)$ are stationary with the appropriate limiting behavior. For example, the "variance of finite time variances", $\Delta^2[\delta B^2]_T \equiv \langle([\delta B^2]_T - \sigma^2)^2\rangle$, converges to zero if the fourth order two-point moment obeys the limit given in eq. (7). The expression for $\Delta^2[\delta B^2]_T$ can also be expanded about 0 for large T in powers of 1/T. If the fourth order moment of the integrand of (7) is stationary and obeys the "Lanczos property", it follows directly that the leading term is proportional to T^*/T where T^* is the correlation time for the fourth-order moment. Adding a coherent oscillation again leads to higher order corrections, but does not prevent statistical convergence for $T > T_o > T^*$. In the general case, the exact form of this expansion is tedious to evaluate and not of central interest in this paper. However, if the probability distribution function of B(t) is stationary and jointly normal, Monin and Yaglom [1975] have shown for the case without a coherent oscillation that

$$T^* = \int_0^\infty d\tau R^2(\tau)/\sigma^4$$

and the expansion for $\Delta^2[\delta B^2]_T$ becomes

$$\Delta^2[\delta B^2]_T = 4\sigma^4 T^*/T + O[(T^*/T)^2] \tag{13}$$

This procedure can be extended to arbitrarily high order. The (ensemble) convergence of the finite time estimates of the Nth order moment of B(t) is guaranteed if all moments up to the $2N^{th}$ are stationary and the central $2N^{th}$ moment obeys a limit equivalent to eqs. (5) and (7). It is always possible to predict the asymptotic dependence of the variance of the finite time N^{th} order moments by postulating the Lanczos property for the $2N^{th}$ central moment.

To examine the stationarity of interplanetary magnetic fluctuations, the convergence described by eqs. (4), (11) and (12) is of primary importance. Recall that the assumption that the one-time and two-time correlations were stationary (eqs. 1 and 2) was used in deriving eqs. (11) and (12). Thus, if this asymptotic behavior can be shown, it is reasonable, if not entirely rigorous, to assert that $\mathbf{B}(t)$ is weakly stationary (and ergodic). In the next section these results will be used to test the consistency of the stationarity hypothesis for solar wind magnetic fluctuations.

3. Application to Magnetic Fluctuations in the Solar Wind: A Two Year Data Interval

The utility of equations (11) and (12) depends on the ability to estimate $\Delta^2[B]_T$ for a given T from a finite data interval. To accomplish this, given a long interval of equally spaced data, we calculate

$$\tilde{\Delta}^2[B_i]_T \equiv \langle([B_i]_T - a)^2\rangle = \frac{1}{M-J+1}\sum_{\ell=1}^{M-J+1} [(1/J)\sum_{n=\ell}^{\ell+J-1}\delta B_i(n\Delta t)]^2 \qquad (14)$$

Equation (14) holds for each vector component of B for a range of values of J; Δt is the time spacing of the magnetic field measurements $B_i(n\Delta t)$, and $T = J\Delta t$. The total length of the data record, $M\Delta t$, must be greater than the maximum interval length $T_{max} \equiv J_{max}\Delta t$ if the estimates are to have reasonable statistical weight. Note that if $J_{max} \to M$, then the right hand side of (14) is forced to converge to 0. This is the reason for requiring $J_{max} << M$. In practice we have found that $J_{max} \leq M/5$ is generally adequate. This gives at least 4M/5 contributions to the averages calculated. In applying this technique to interplanetary magnetic fluctuations, the solar ecliptic coordinate system is used and each of the components is tested separately for stationarity. To avoid confusion, we will use x, y, and z to denote the radial, tangential and normal (to the ecliptic) coordinates, in place of the traditional R, T, and N. The asymptotic behaviors described by eqs. (11), (12) and (14) will be calculated for each vector component separately.

Stationarity of solar wind fluctuations should be an increasingly good approximation the longer the data interval except for complications arising from the influence of the solar rotation. Consequently, we have chosen to analyze a long data interval which includes many solar rotations. The data base of IMP interplanetary magnetometer measurements available from the

National Space Science Data Center contains an interval of 621 days (22 solar rotations) of nearly continuous coverage (87%) of one hour averaged data during the years 1967 to 1968. The 14,900 points were digitally filtered and decimated to yield 4925 three hour averages of each field component. It is important to note that this data contains all tangential discontinuities, shocks, rotational discontinuites, stream interaction regions, etc. that were in the original time series. Our approach is to treat all of these as part of the time series whose stationarity is being investigated. The correlation matrix

$$R_{ij}(\tau) = \langle \delta B_i(t)\, \delta B_j(t+\tau) \rangle \tag{15}$$

was calculated using the Blackman-Tukey algorithm as implemented by Matthaeus and Goldstein [1982], however, we did not use the "frozen-in-flow" hypothesis to relate frequency and wave number space. The diagonal components of $R_{ij}(\tau)$, the x, y, z autocorrelation functions, are shown in Figure 1. The most striking feature of the x and y autocorrelations is that they do not go smoothly to zero, but show an oscillation with a period of about 28 days. This arises from the stream and sector structures seen in the x and y components of the field; structures which are absent in the z component.

Power spectra obtained from the Fourier transforms of the diagonal components of $R_{ij}(\tau)$ are plotted in Figure 2. The oscillations at the solar rotation frequency, $\omega_0 = (2\pi/T_0) = 4.3 \times 10^{-7}$ Hz, are again evident in the x and y components. The correlation times of the components, defined for each by eq. (10), are shown in Table 1 [cf. Matthaeus and Goldstein, 1982]. It is interesting to note that the correlation times for this 621 day dataset are approximately the same as Matthaeus and Goldstein [1982] reported for several five day periods of Voyager 1 and 2 data from 1 to 5 AU.

Because of the absence of low frequency oscillations, $R_{zz}(\tau)$ is well approximated as a Lanczos-type function. Although the coherent power in B_x and B_y at ω_o does not contribute to the correlation lengths of those components, its presence means that neither $R_{xx}(\tau)$ nor $R_{yy}(\tau)$ are Lanczos-type functions. Nonetheless, all three autocorrelation functions appear to satisfy eq. (5), and thus, provided $\underline{B}(t)$ is a stationary vector function, the ergodic theorem (4) should be satisfied.

In Figure 3 we have plotted the results of using eq. (14) to calculate the variance of the means, $\tilde{\Delta}^2[B_i]_T$, of each of the components of $B_i(t)$. These means, calculated over finite times, do converge to their averages, but note that the z component converges more rapidly than the others. Determination of whether the behavior illustrated in Figure 3 indicates stationarity of $\underline{B}(t)$ can best be done by using eq. (12) to model $\Delta^2[B_i]_T$. The variances $\sigma_x{}^2$, $\sigma_y{}^2$, and $\sigma_z{}^2$, their respective correlation times, and the averages of the field components (in units of $\gamma = 10^{-5}$ G) over the entire dataset are given in Table 1. We further divide the variances of B_x and B_y into a contribution $K \simeq \alpha^2/2$ due to power near the spectral enhancements at frequency ω_o, and the remainder, which presumably has little or no contribution from the solar rotation. In this way it is possible to compare directly the values of $\tilde{\Delta}^2[B_i]_T$ caculated from eq. (14) with the stationary random function predictions of eq. (11) for the z component and eq. (12) for the x and y components. There are no free parameters in this comparison (cf. Figure 4). $\tilde{\Delta}^2[B_z]_T$ is in close agreement with the theoretical behavior (eq. 11) for $T > T_C(z)$, where $T_C(i)$ denotes the correlation time of $B_i(t)$. Similarly, $\tilde{\Delta}^2[B_x]_T$ and $\tilde{\Delta}^2[B_y]_T$, the components influenced by solar rotation effects, are well modeled by eq. (12). In Figure 5 we combine the three components into a single weighted curve defined by

$$\Delta^2[\underline{B}]_T/\sigma^2 \equiv \sum_i \Delta^2[B_i]_T/\sum_i \sigma_i^2$$

Both the experimental curve and predicted curves are plotted. These results support strongly the conclusion that this interval is an example of a weakly stationary random vector process.

While there are no fit parameters in the comparison shown in Figures 4 and 5, the exact form of the theoretical curves derived from eqs. (11) and (12) depends on the values of $\langle B_i(t)\rangle$, σ_i^2 and $T_c(i)$. These are ensemble averaged quantities and hence are themselves estimates. Small errors in those parameters can only slightly modify the predictions based on (11) and (12). By extrapolating the curves of $\tilde{\Delta}^2[B_i]_T$ shown in Figures (3) and (4), one can infer that errors in the estimated values of $\langle B_i(t)\rangle$ are small.

The ability to estimate σ_i^2 and $T_c(i)$ from finite intervals, depends on how well the second order moments converge. The extent to which $[\delta B_i^2]_T$ converges to its ensemble averaged value σ_i^2 is measured by eq. (6). To investigate this behavior, we have evaluated $\tilde{\Delta}^2[\delta B_i^2]_T = \langle([\delta B_i^2]_T - \sigma_i^2)^2\rangle$ for i = x, y, z, with σ_i^2 evaluated from

$$\sigma_i^2 \approx (1/M) \sum_{\ell=1}^{M} \delta B_i(\ell\Delta t)^2 \tag{16}$$

The values of $\tilde{\Delta}^2[\delta B_i^2]_T$ for the three field components are shown in Figure 6. It is evident that all three components converge, though $\tilde{\Delta}^2[\delta B_y^2]_T$ converges more slowly than do the x and z components. We have not attemped a detailed comparison of the data in Figure 6 either with predictions based on eq. (13) or generalizations which include the influence of coherent power at ω_o.

A linear least squares fit to the quantity $\tilde{\Delta}^2[\delta \underline{B}^2]_T \equiv \sum_i \tilde{\Delta}^2[\delta B_i^2]_T$ vs T gives, over the decade corresponding to the largest T, a dependence of T^{-u}

with $u = 0.949 \pm 0.005$ indicating an overall convergence of second order (vector) moments in moderately good agreement with the expectations of eq. (13). Thus the fourth order moments apparently converge sufficiently rapidly to allow good estimates of second order moments, including correlation functions and power spectra.

The data analyzed in this section contain some 500 correlation times. It appears clear that this length of data permits good estimates of first and second order statistical moments to be made. Our results are consistent with the hypothesis that the magnetic field at 1 AU is a stationary random vector function. However, in many circumstances such a large dataset is either not available or is not relevant to the problem being studied. The following section deals with the issue of stationarity of smaller data intervals.

4. Shorter Data Intervals

Most usage of power spectral techniques in analyses of solar wind data start with data intervals considerably shorter than 22 solar rotations. To the extent that the conclusions of the previous section apply generally in the solar wind, the question which is relevant to the analysis of shorter intervals is whether accurate statistical estimates may be deduced from them. Clearly, this will depend upon the nature of the specific dataset. Poor convergence usually indicates that some type of coherent structure is undersampled in the dataset. The most obvious figure of merit for adequate convergence is the extent to which the scaling of first and second order statistics resembles the asymptotic predictions of the previous section. Intervals containing an integral number of sectors might be expected to admit adequately convergent statistics. However, intervals containing $n + \varepsilon$ sectors with $\varepsilon \approx 1/2$ would probably be a poor choice. In this section we illustrate these intuitive ideas by showing analyses of several smaller data sets. (Variances, means and correlation times for these datasets are given in Table 1.)

In Figure 7 we have plotted the y component of $\underline{B}$ measured by the ISEE 3 magnetometer [Frandsen et al., 1978] during days 7 - 99 of 1979. The interval spans more than three solar rotations, and a regular sector structure is present. When the stationarity analysis is done for this complete interval, as is illustrated in Figure 8, it is clear that the finite time means do not converge well.

The results of analyzing the single sector between days 57 - 66 is also shown in Figure 8. (Each of the plots in Figure 8 is normalized to the variance and correlation length of the specific interval being analyzed.) Not only is the overall convergence better, but nearly all contributions from the solar rotation period are absent. A least squares fit over the last 50 points gives a dependence of $(T_c/T)^{-u}$ with u = 0.965 ± 0.001, which is very close to the slope of 1.0 predicted by (11).

Magnetic field measurements at larger heliocentric distances generally indicate a less pronounced sector structure [Burlaga, et al., 1982]. Thus one might expect that a stationarity analysis of such intervals would show a much reduced influence of solar rotation effects. To demonstrate this, we have analyzed two data intervals at 5 and 10 AU taken by Voyager 1 from days 5 to 55, 1979, and days 235 to 295, 1980. One hour averages were used. In both cases $\tilde{\Delta}^2[B]_T$ vs T (cf. Fig. 8) follows the predictions based on eq. (11) (no solar rotation effects). Least square fits to the slope of the last 30 points of the 5 AU analysis gives $(T_c/T)^{-u}$ with u = 0.992 ± 0.003.

The last question we address is whether second moments, such as variances, spectra and correlation functions, are well estimated for these shorter datasets. In Figure 9 we plot $\tilde{\Delta}^2[\delta \underline{B}^2]_T$ against T for the four datasets discussed above. The curves are separately normalized by the square of the appropriate variance and time is normalized to $T/T_c(i)$. In all cases, the estimates begin to converge rapidly as T begins to exceed 5 to 10 T_c, which suggests that only small errors are made in estimating spectra and correlation functions from this much data. As a further check on the internal consistency of this approach, one should show that Lanczos type correlation functions are associated with good statistical convergence. This was the assumption made in deriving the asymptotic forms (eq. 11 and 12). In Matthaeus and Goldstein [1982] we argued that if the correlation functions were approximately Lanczo-type, then weak stationarity should be a good approximation. In Figure 10 the correlation functions of the four datasets discussed in this section are plotted. The correlation functions of the two Voyager data sets and the single sector of ISEE data are seen to be more nearly Lanczos-type than is the fourth correlation function constructed from the 92 day dataset.

5. Summary and Discussion

In the preceeding sections we have attempted to answer several questions about the time stationarity of interplanetary magnetic fields by examining the consequences of assuming that the field fluctuations are stationary and testing the extent to which these consequences are satisfied. From the discussion in Section 3, it appears clear that the interplanetary magnetic field is statistically time stationary (at least in the weak sense) when the formalism is generalized to include the effects of low frequency coherent oscillations arising from the sector structure and other phenomena driven near the solar rotation period. To the extent to which this two year interval is typical, one can conclude that both the first and second order moments are stationary. This conclusion is entirely insensitive to the question of whether the observed variations are due in part to dynamical evolution in the plasma frame, or are simply a complicated but fixed pattern being convected past the spacecraft. The more difficult question of the nature of the Lagrangian statistical properties, i. e., means and covariances calculated in the frame moving with the mean plasma velocity, remains an important and unresolved issue.

The ergodic theorem for the first moment guarantees that when the correlation function is stationary, finite time estimates of the means converge in a particular way. We have seen that these conditions are well fulfilled. The convergence of the fourth order moment (eq. 7), guarantees in turn, that variances, correlation functions and power spectra can be meaningfully evaluated from appropriately selected finite data intervals. In Section 4 we found that intervals of 5 to 10 correlation times in duration were sufficient so long as the interplanetary sector structure is treated carefully. Optimally, a dataset should contain either a very large number of sectors or a small integral number of sectors.

We have also examined a few intervals which contained isolated interplanetary shocks. The results are what one might expect and we will only summarize the analysis here. Inclusion of a single shock does not greatly affect the convergence of the means, but the convergence of the variances is strongly affected. Thus power spectra and correlation functions for such intervals will not be statistically reliable.

The technique used in Sections 3 and 4 requires evaluation of statistical quantities such as the correlation matrix (15) at varying times but at a single point in space, requiring in effect that we must justify the neglect of the spacecraft motion during the time of observation. The instantaneous rate of change of the signal seen by the spacecraft is predominantly due to the bulk plasma motion and not the speacecraft motion because the solar wind speed is several orders of magnitude greater than the spacecraft speed. It appears, then, that the principal issue is whether during the measured interval the orbital displacement of the speacecraft, ΔR, is small compared to the lengths over which the mean properties of the magnetic field undergo significant variation. We are unaware of any definitive evidence to guide the selection of the latter quantity. However, the heliocentric distance, R, appears to be a plausible candidate for describing the radial variations of the statistics induced by the heliospheric expansion. The quantity $\Delta R/R$ is no greater than 0.01 for the data taken from the IMP and ISEE spacecrafts whose orbital positions span less than 200 earth radii ($\approx 10^{11}$ cm). For the 50 day 1979 Voyager dataset acquired near 5 AU $\Delta R/R \approx 0.06$, while for the 60 day 1980 Voyager dataset $\Delta R/R \approx 0.07$. If much longer data sets than these were used from spacecraft with trajectories similar to the Voyagers, it would not be possible to examine time stationarity without explicitly taking into account spacecraft motion.

Finally, we would like to briefly discuss the relationship of these results on time stationarity to the question of homogeneity. Because the mean radial plasma outflow is super-Alfvénic in the solar wind, two-time correlation functions can be interpreted as two-point single time correlation functions, and frequency spectra can be converted into wavenumber spectra. (A more detailed discussion of the "frozen-in-flow" assumption in the analysis of solar wind data can be found in Matthaeus and Goldstein [1982].) Thus, fluctuations that are time stationary are also spatially homogeneous. There are several obvious limitations to this conclusion. The concept of homogeneity cannot be applied to length scales which approach the size of the entire system. As one approaches these scales, the wavenumber-frequency correspondence implied by "frozen-in-flux" breaks down and organized non-statistical phenomena driven by the solar rotation become prominent. These phenomena are not controlled by local dynamics, but rather represent magnetic field configurations reflecting such things as the distribution of coronal holes, active regions, etc. Nonetheless, at higher frequencies, where frozen-in-flow is a valid approximation, our results suggest that spatial homogeneity is also a valid approximation. Thus, at scales smaller than, say, the heliocentric distance of the observer, the interplanetary medium seems well described as statistically stationary and homogeneous. Over larger scales and longer time periods, the observed time behavior is consistent with being stationary with superimposed coherent time variations reflecting the organized macroscopic structure of the heliosphere.

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Figure Captions

Figure 1. Diagonal components of the magnetic field correlation matrix for 621 days of IMP data obtained during 1967 - 1968. Both the radial and normal components (x and y, respectively) show low frequency coherent correlations with a period near that of the solar rotation, while the normal component (z) approaches zero rapidly. The x, y, and z components are plotted with decreasing line thickness.

Figure 2. Power spectra of the x, y, and z components of the magnetic field for the same 621 days of IMP data. Spectra were calculated from the Fourier transform of the correlation functions shown in Fig. 1. Power at the solar rotation period is a distinctive feature of the x and y spectra. For clarity, the x and y components have been multiplied by a factor of 16 and 4, respectively.

Figure 3. The variance of the means ($\tilde{\Delta}^2[B_i]_T$) for the x, y, and z field components of the 621 day interval are calculated for intervals of duration T and plotted as functions of T_c/T, where T_c is the the total correlation time. The variance of the means are normalized by $\sigma_i{}^2$. Convergence of the estimates is evident.

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Figure 4. Comparison of values of $\tilde{\Delta}^2[B_i]_T$ (heavy lines) determined from the 621 days of IMP data with $\Delta^2[b_i]_T$ calculated from the analytic predictions of eqs. 11 and 12. Separate plots for the x, y, and z field components are shown. Equation 11 is used for the z component and eq. 12 is used for the x and y components. The ordinate is normalized by $\sigma_i{}^2$. The abcissa is $T/T_c(i)$ $(i = x, y, z)$.

Figure 5. $\tilde{\Delta}^2[\underline{B}]_T/\sigma^2$ (heavy line) determined from the 621 days of IMP data and $\Delta^2[\underline{b}]_T/\sigma^2$ (thin line) calculated from eqs. 11 and 12 are plotted against T/T_c. There are no free parameters in any of the comparisons shown here or in Fig. 4.

Figure 6. The normalized variance of estimates of the variances of the field components $(\tilde{\Delta}^2[\delta B_i{}^2]_T/\sigma_i{}^4)$ obtained from data intervals of length T and plotted versus T for i = x, y, z. The data is again from the 621 day interval discussed in the text. In the figure, $\tilde{\Delta}^2$ is normalized by the appropriate $\sigma_i{}^4$, and T is normalized by $T_c(i)$.

Figure 7. The tangential (y) component of the magnetic field (in gammas) from the ISEE-3 magnetometer. The interval begins on January 7, 1979 and spans 92 days. A regular sector structure is evident.

<u>Figure 8.</u> Total variance estimates of the means of the field components ($\tilde{\Delta}^2[\underline{B}]_T$) for four datasets: ISEE-3 (days 7 - 99, 1979), ISEE-3 (days 57 - 66, 1979--a single magnetic sector), Voyager 1 (days 5 - 55, 1979) and Voyager 1 (days 235 to 295, 1980). Each is normalized by their respective $\sigma^2 = \sum_i \sigma_i^2$ and plotted versus T/T_c. The 92 day ISEE interval converges less rapidly than the single sector subinterval due to influence of structure at the solar rotation period. Both Voyager intervals converge at a rate very close to that predicted by eq. 11.

<u>Figure 9.</u> $\tilde{\Delta}^2[\delta\underline{B}^2]_T/\sigma^4$ for the two ISEE and two Voyager 1 intervals described in Fig. 8. The rapid decrease for large T/T_c indicates that estimates of variances and spectra are convergent.

<u>Figure 10.</u> The correlation functions (trace of the correlation matrix) for the four intervals described in the text and Fig. 8. As expected, the correlation function of the 92 day ISEE interval is the least Lanczos-type.

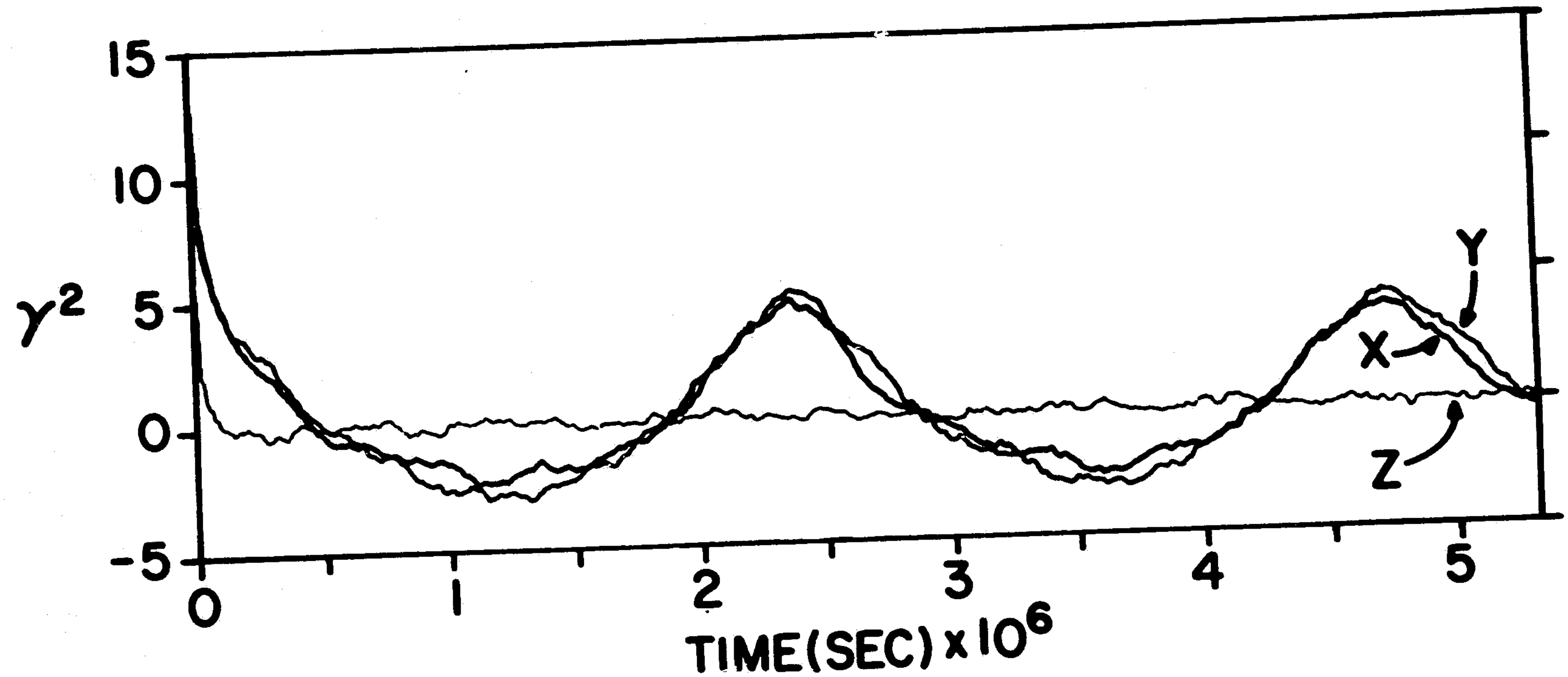


Figure 1

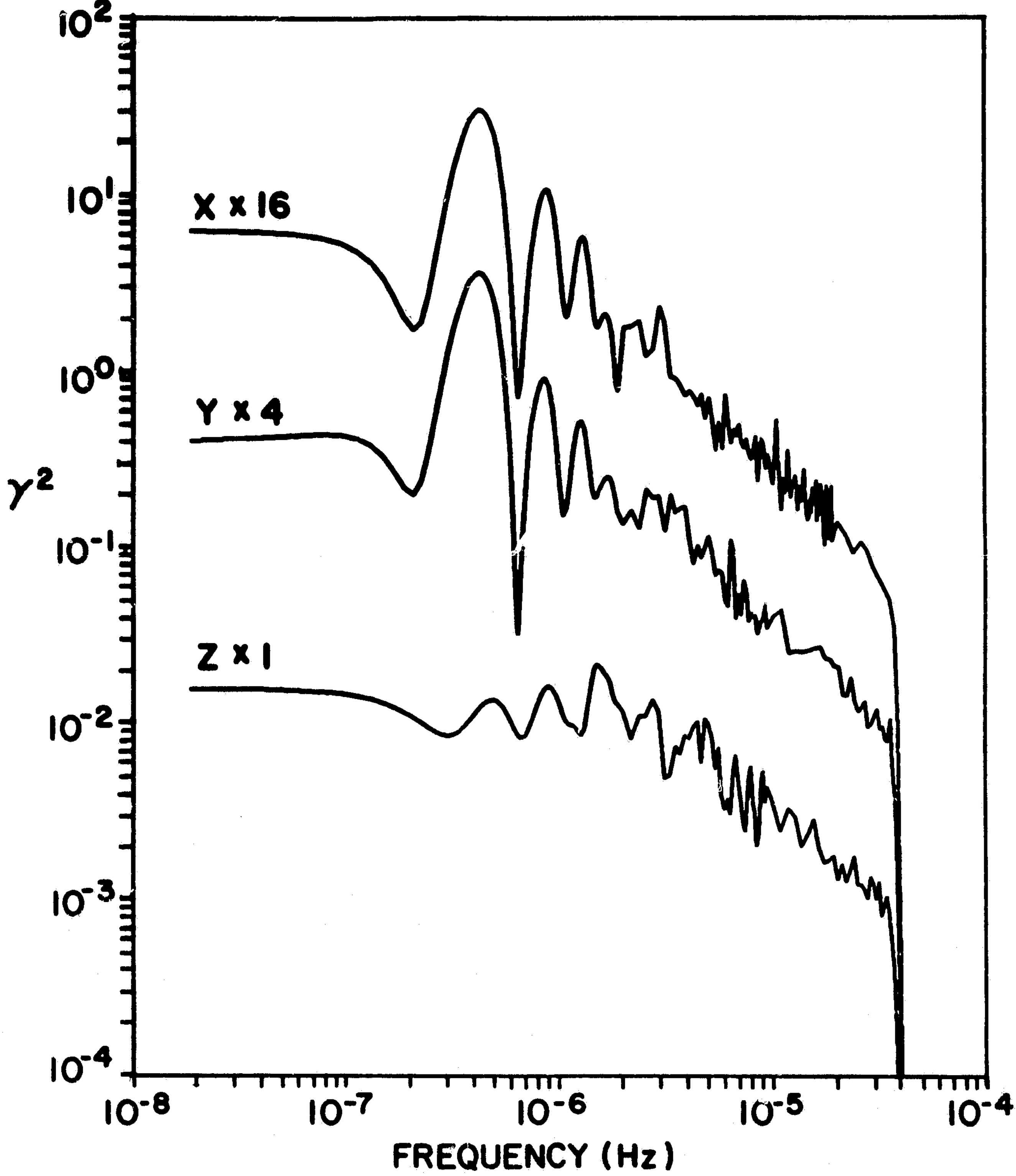

10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
γ^2
X x 16
Y x 4
Z x 1
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
FREQUENCY (Hz)

Figure 2

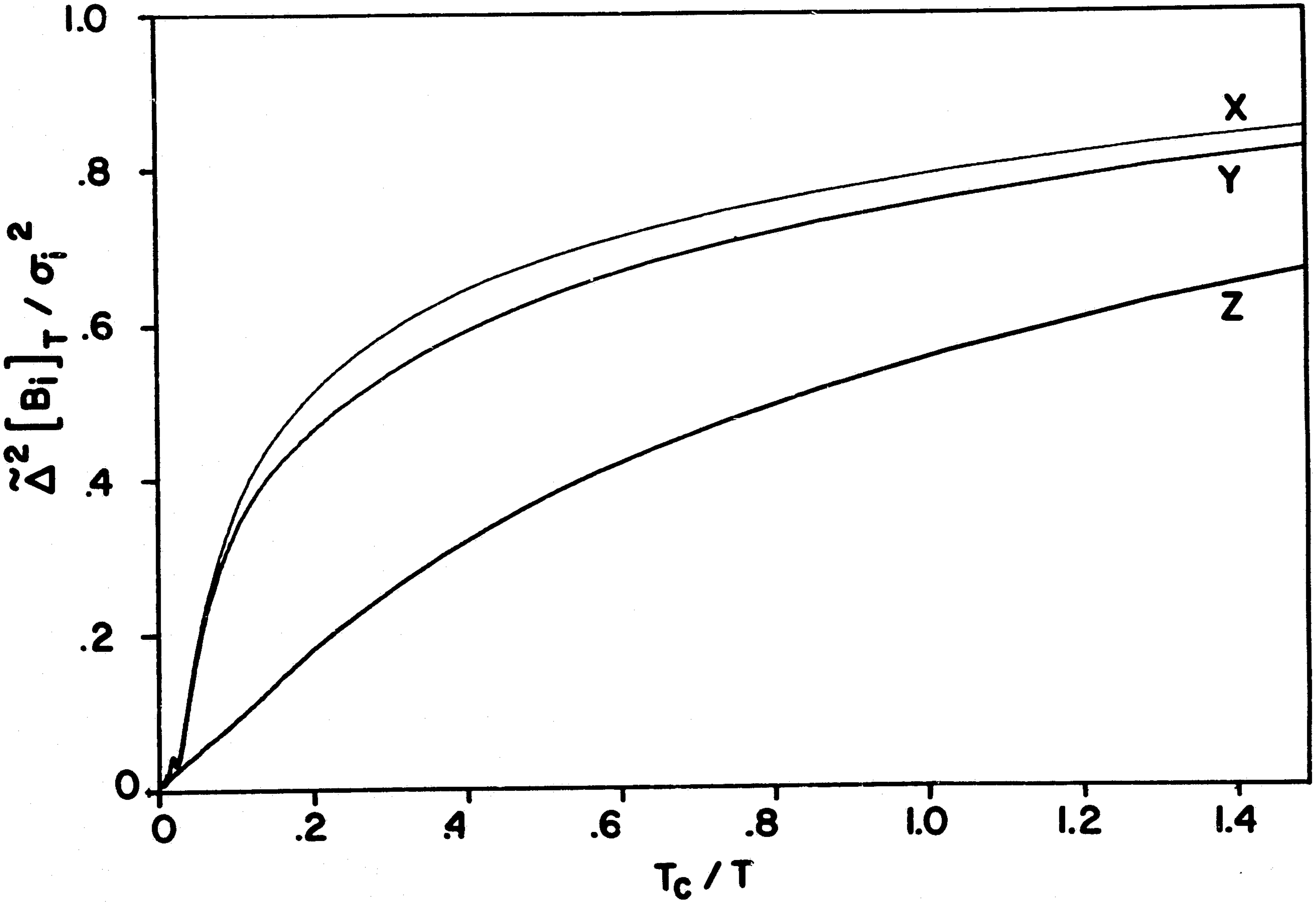


Figure 3

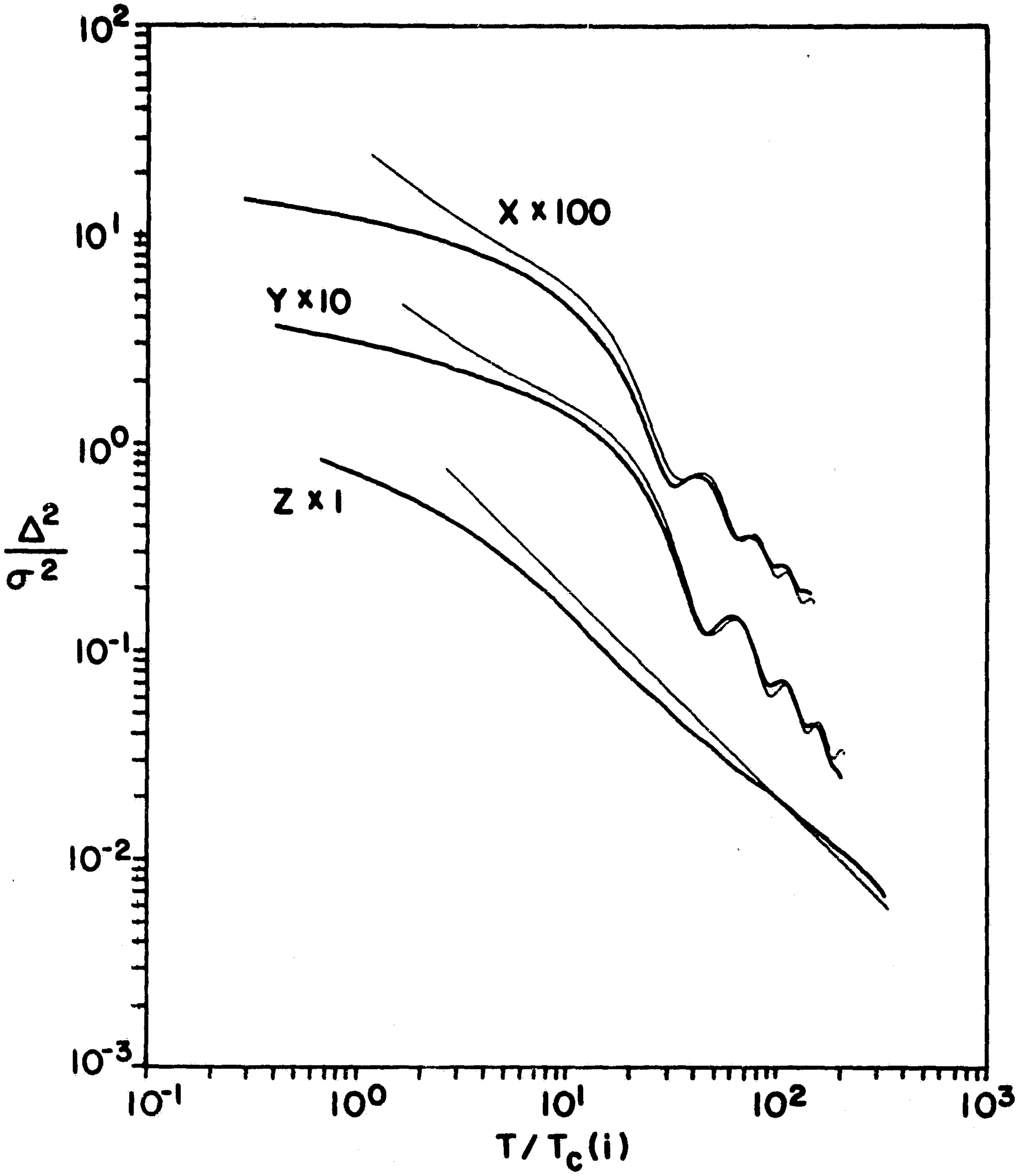

X × 100
Y × 10
Z × 1
$\frac{\Delta^2}{\sigma^2}$
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-1}
10^0
10^1
10^2
10^3
$T / T_c(i)$

Figure 4

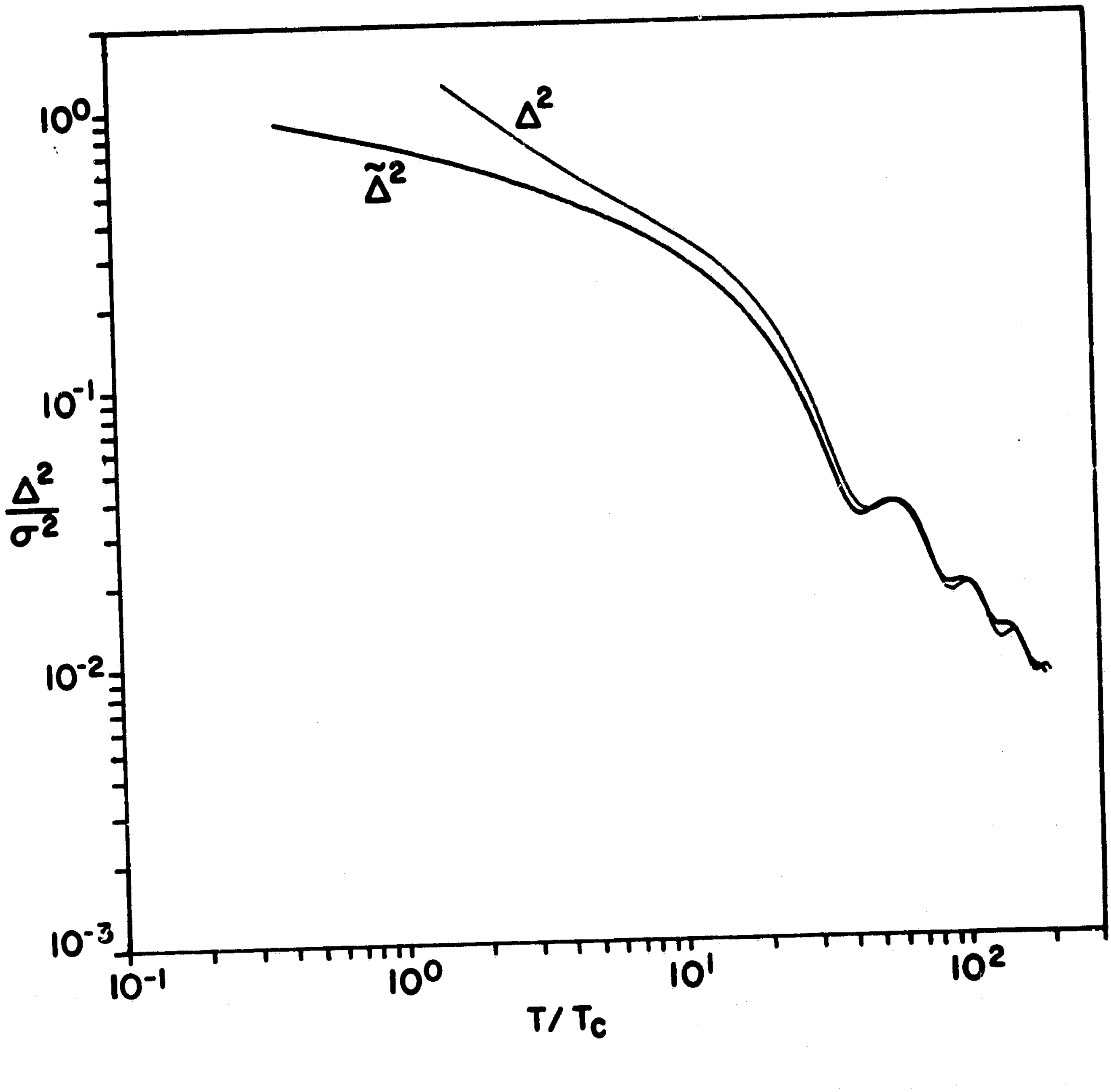
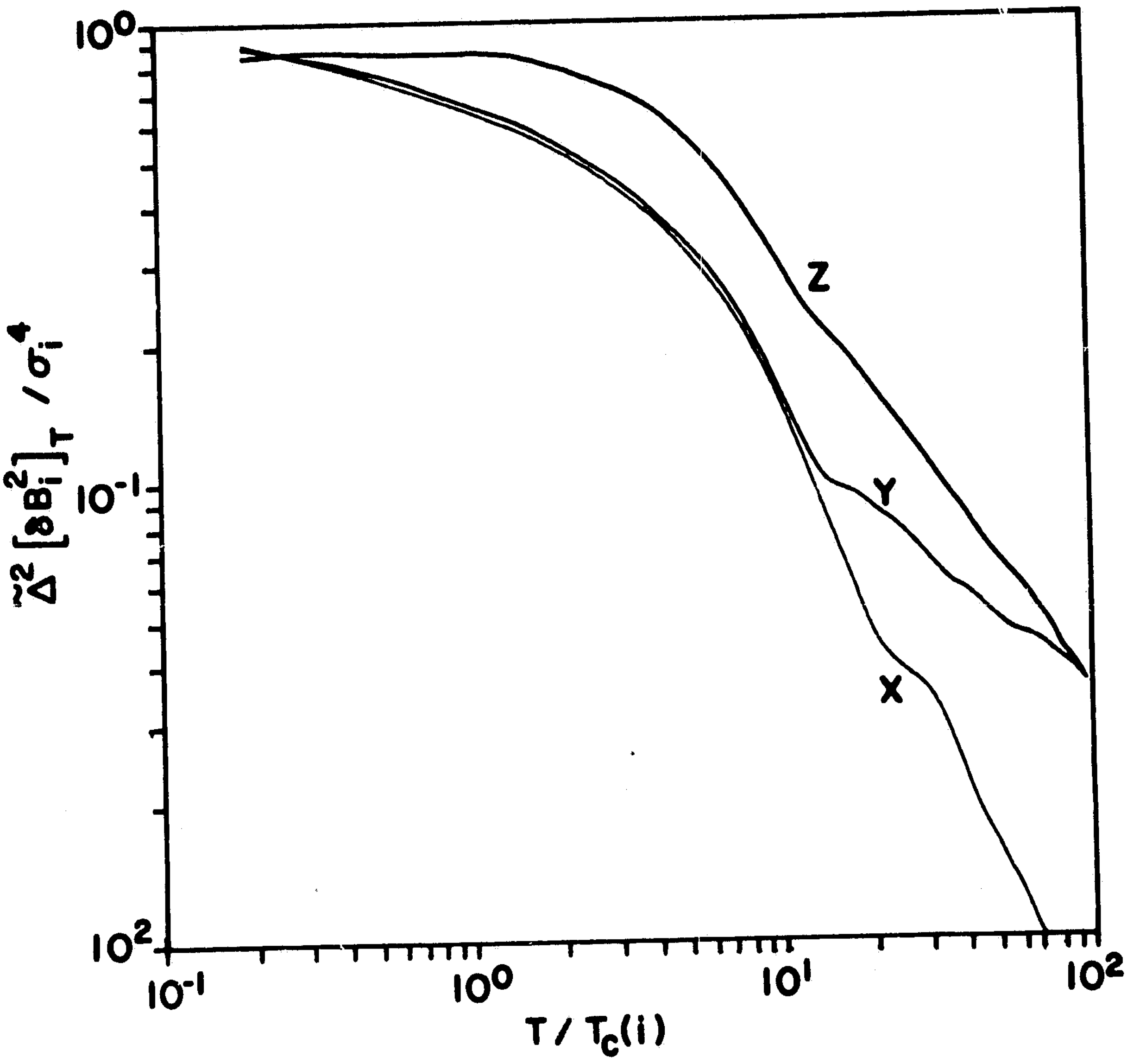

Δ^2
$\tilde{\Delta}^2$
10^0
10^{-1}
10^{-2}
10^{-3}
$\frac{\Delta^2}{\sigma^2}$
10^{-1}
10^0
10^1
10^2
T/T_c

Figure 5



10^0
10^{-1}
10^{2}
10^{-1}
10^0
10^1
10^2
$\tilde{\Delta}^2 [\delta B_i^2]_T / \sigma_i^4$
$T / T_c(i)$
Z
Y
X

Figure 6

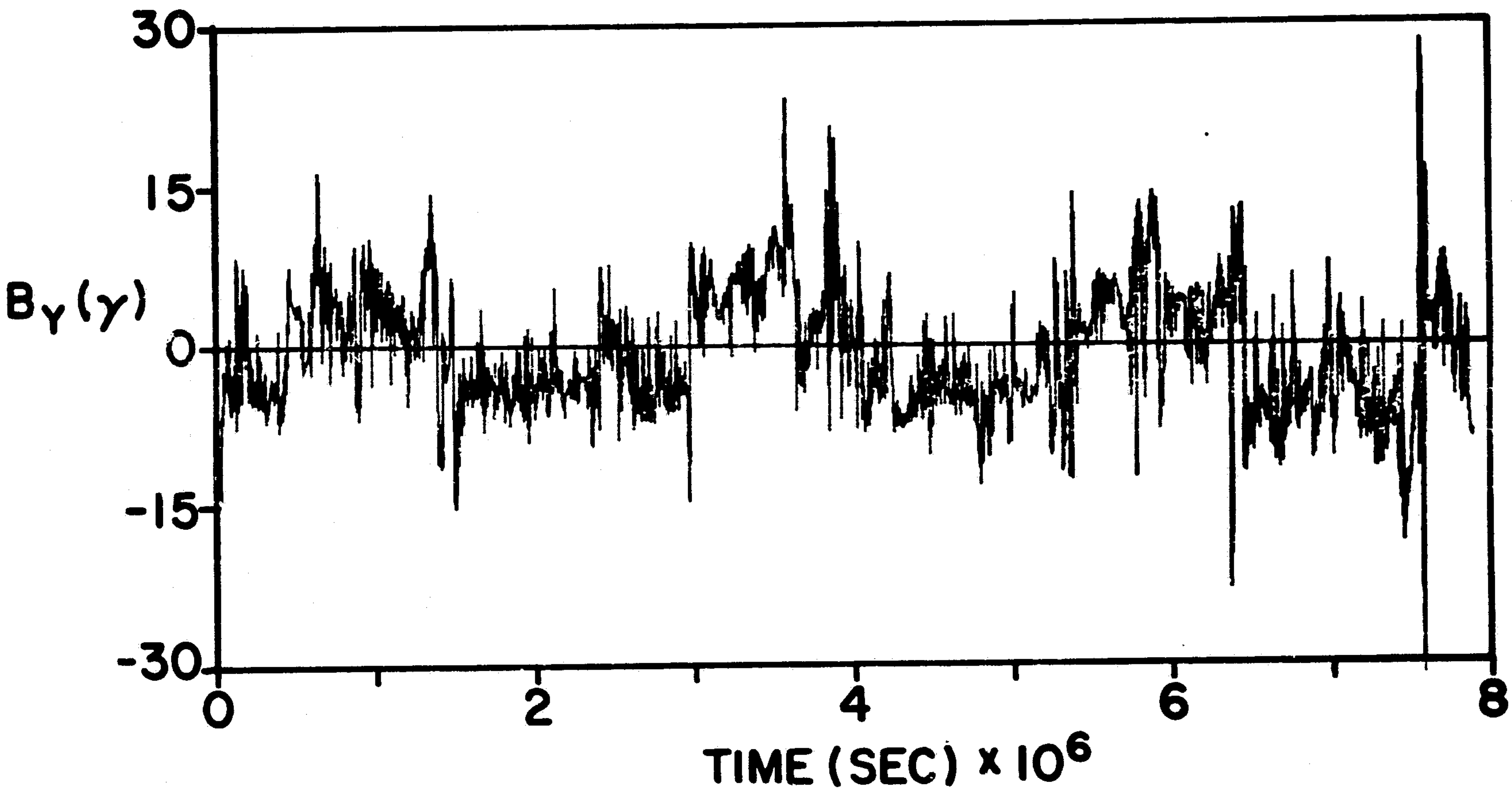


Figure 7

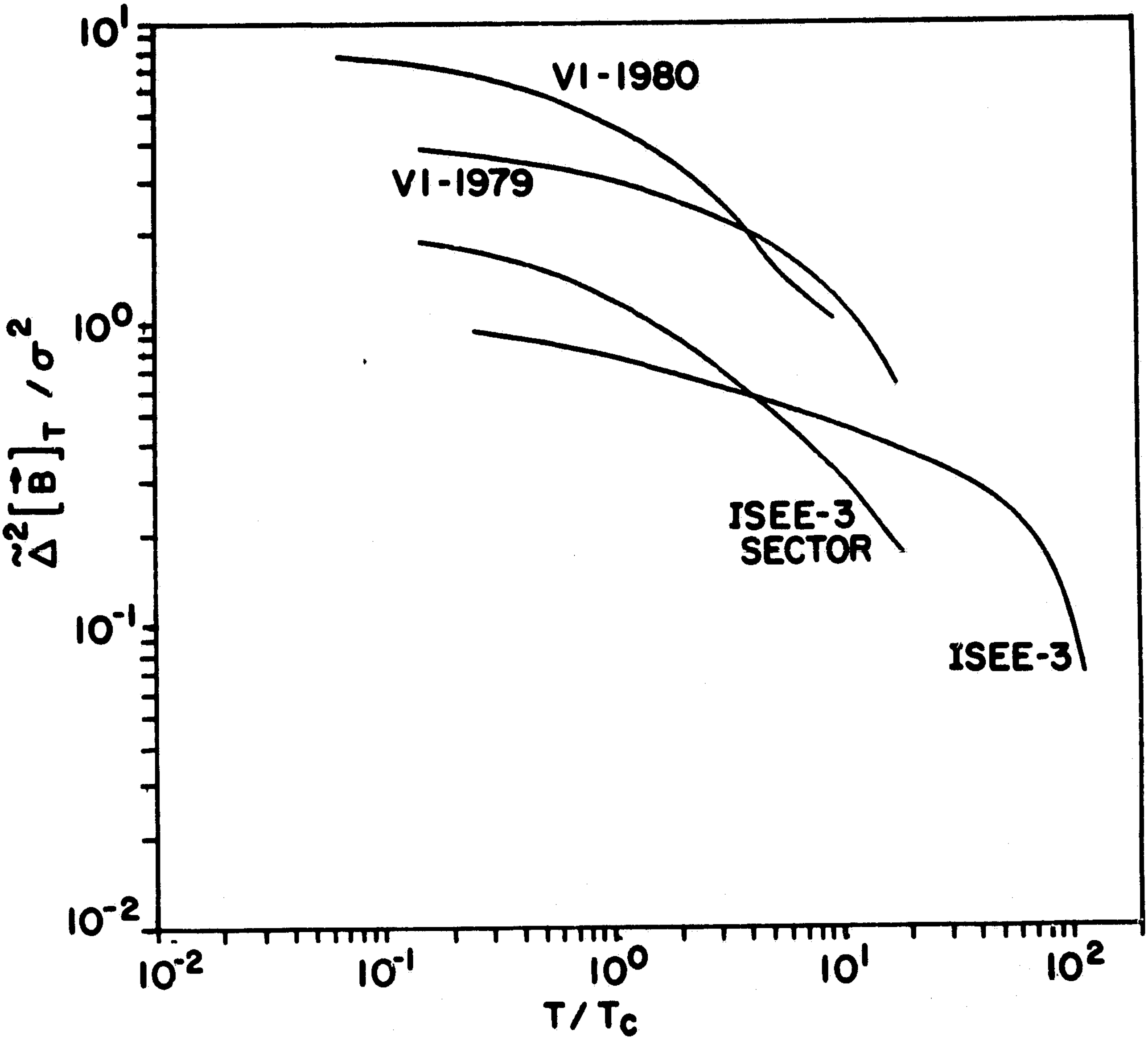

VI-1980
VI-1979
ISEE-3
SECTOR
ISEE-3
10^1
10^0
10^{-1}
10^{-2}
10^{-2}
10^{-1}
10^0
10^1
10^2
$\tilde{\Delta}^2[\vec{B}]_T\ /\sigma^2$
T/T_c

Figure 8

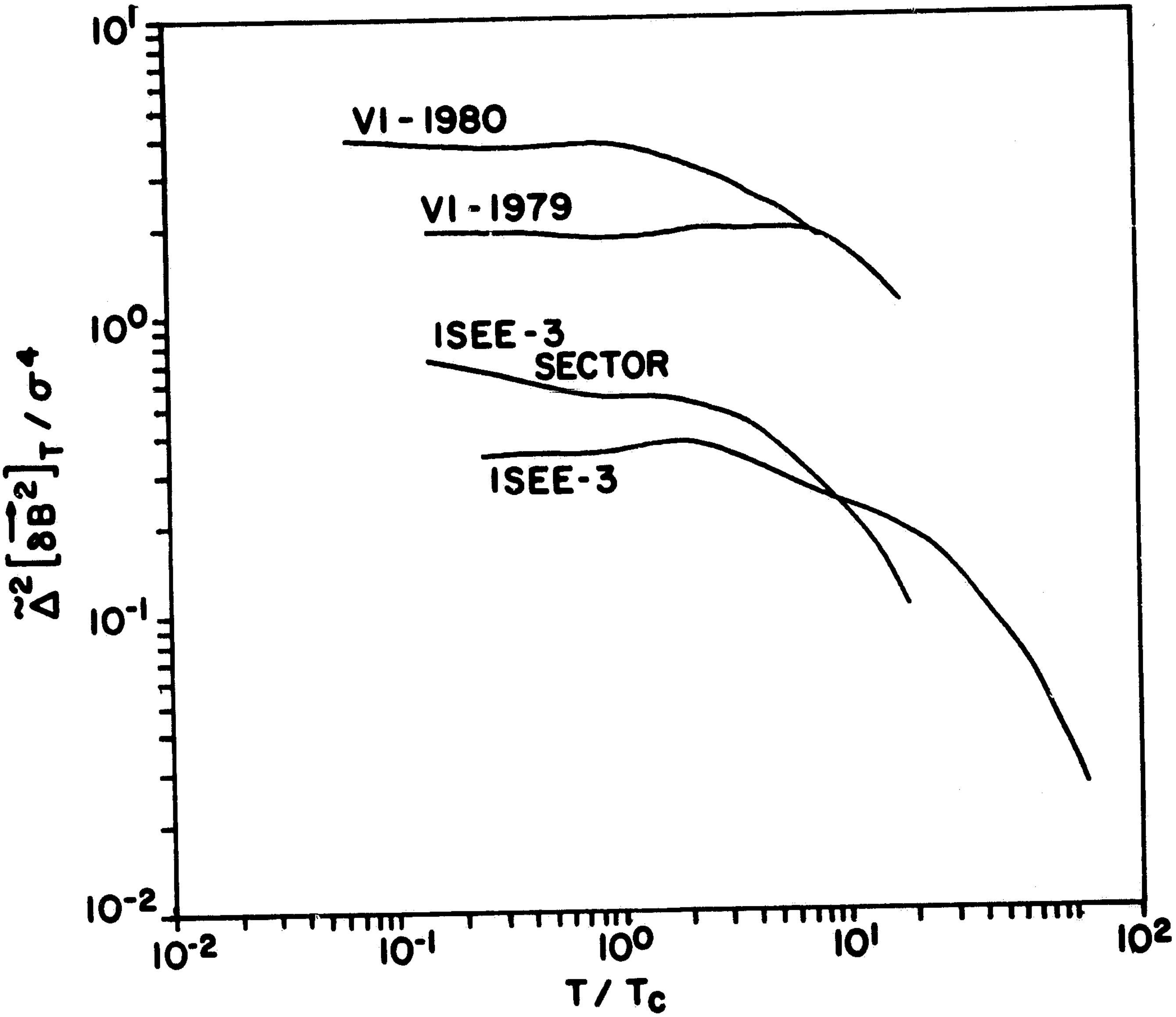

VI - 1980
VI - 1979
ISEE - 3
SECTOR
ISEE - 3
10^1
10^0
10^-1
10^-2
10^-2
10^-1
10^0
10^1
10^2
T / Tc

Figure 9

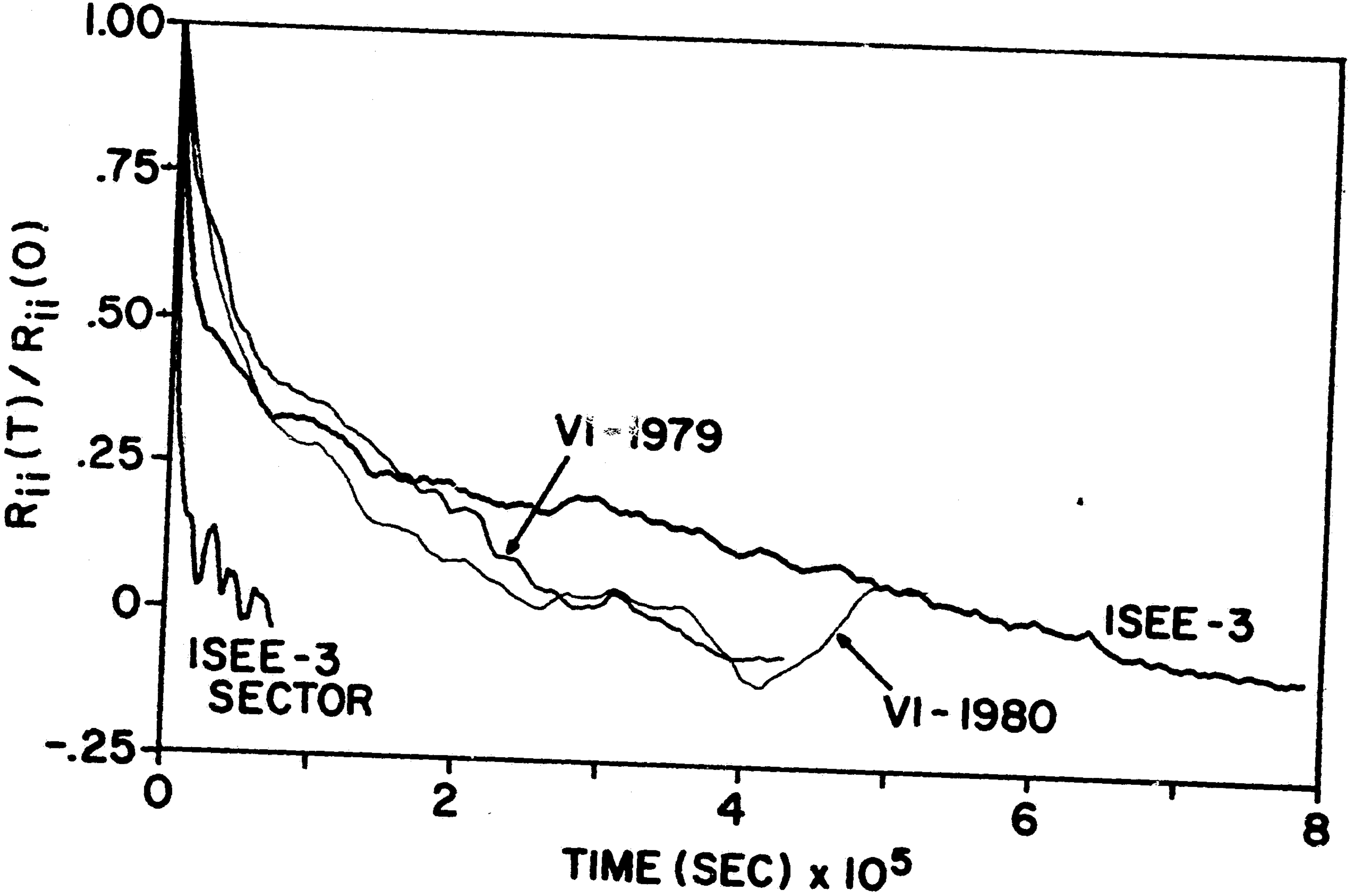

1.00
.75
.50
.25
0
-.25
$R_{ii}(T)/R_{ii}(O)$
VI - 1979
ISEE-3
SECTOR
ISEE - 3
VI - 1980
0
2
4
6
8
TIME (SEC) x 10^5

Figure 10

Table 1

Variances and Correlation Times for the Data Intervals Analyzed

	Means (γ)			Variances (γ^2)			Correlation Times (10^4 sec)			
	B_x	B_y	B_z	σ_x^2	σ_y^2	σ_z^2	T_c	$T_c(x)$	$T_c(y)$	$T_c(z)$
IMP (621 days) 1967 - 1968	.49	-.66	-.046	11.22	13.07	6.31	5.6	7.4	5.3	3.2
ISEE-3 1979 days 7 to 99	.48	-.72	-.28	22.55	31.4	17.6	5.48	6.5	7.3	0.93
ISEE-3 1979 days 57 to 66	3.4	-4.5	-1.09	7.47	8.28	12.36	0.8	1.2	1.04	0.53
Voyager-1 1979 days 5 to 55	-.03	.03	.09	0.08	0.50	0.175	5.0	20.0	2.9	3.5
Voyager-1 1980 days 235 to 295	-.03	.10	-.17	0.014	0.141	0.063	11.2	6.9	15.7	2.9