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SUMMARY

A linearized, longitudinal model, in state-space format, is formulated for an advanced fighter. The nominal operating point is for a subsonic flight condition. The engine is operating with afterburner on. The model is composed of three subsystem models: the inlet, the engine, and the airframe. A procedure for combining the subsystem models into an integrated model is presented and an integrated controller is developed by using linear quadratic regulator theory. Notable interaction is found in the coupled system.

A procedure, based on eigenvalue sensitivities, is presented which places the feedback gains in a hierarchical arrangement and measures their contribution to the optimal solution. With these numbers, called the gain significance matrix, ineffectual gains can be eliminated thus saving hardware and expense in the realization of the physical controller.

INTRODUCTION

The desire to produce efficient control systems capable of multivariable input/output and optimal mixing of subsystem interaction has led to programs such as INTERACT (integrated research aircraft control technology), AFTI (advanced fighter technology integration), and PROFIT (propulsion-flight control integration technology). Motivation for this work comes from several sources. A primary source is the increased interaction between the airplane and propulsion system in certain flight conditions. This interaction is intensified with advanced systems and more demanding mission requirements. Another incentive is the need to improve efficiencies. As component efficiencies become more difficult to improve, the alternative is to improve overall efficiency through optimal integration and control of components. Finally, the historical separation of major airplane systems in the design process has not fully exploited the capabilities of the overall system; this is important in light of the wide operational envelope of a modern fighter.

The benefits of integrated control design have been demonstrated. For example, Michael and Farrar (ref. 1) developed an integrated inlet-engine controller for the Pratt and Whitney F401 engine and an internal compression supersonic inlet. They were able to improve steady-state thrust by 6 percent and reduce the variation of normal-shock position by a factor of three. This predicts a substantial improvement in performance of the inlet-engine system.

In the same spirit, this report contributes to integrated design methodology by achieving two objectives. First, a state-space model of an airplane comprised of three major subsystem models (inlet, engine, and airframe) is provided as a working tool. A procedure is shown for combining the subsystem models into an integrated model, and then an integrated controller is developed by using linear-quadratic regulator (LQR) theory. The second objective is to provide a method to discern the relative importance of the feedback gains by computing a gain significance matrix, which is a function of the eigenvalue sensitivity to the gains. This matrix indicates the ineffectual gains which can be eliminated; thus, hardware and expense may be saved in the realization of the physical controller. It also indicates sensitive gains so that beneficial interactions can be identified and adverse interactions can be attenuated.

SYMBOLS

A	uncoupled plant matrix
$\hat{A}$	coupled plant matrix
$\tilde{A}$	coupled, closed-loop plant matrix
A_c	inlet capture area, ft^2
A_j	jet area, ft^2
A_w	wing area, ft^2
B	control distribution matrix for uncoupled system
$\hat{B}$	control distribution matrix for coupled system
$\hat{b}_{ik}$	element of $\hat{B}$ matrix
C	output equation state matrix for uncoupled system
$\hat{C}$	output equation state matrix for coupled system
$C_{D,I}$	drag coefficient for inlet
$C_{M,I}$	moment coefficient for inlet
D	output equation control matrix for uncoupled system
$\hat{D}$	output equation control matrix for coupled system
F	feedback gain matrix
$\hat{F},G$	coupling equation matrices
$f_{k\ell}$	element of feedback gain matrix
H	output equation matrix
h	altitude, ft
I	unit (identity) matrix
J	performance index
K_{a2}	distortion factor
k,ℓ,i,j	matrix and vector elements

LQR	linear-quadratic regulator
M	Mach number
m	number of states or eigenvalues and eigenvectors
m_c	compressor surge margin, percent
m_f	fan surge margin, percent
n_c	compressor speed, rpm
n_f	fan speed, rpm
PLA	power lever angle, deg
p_{st}	static pressure, psi
p_t	total pressure, psi
$p_{t,0}$	nominal total pressure, psi
$p_{t,2}$	engine face total pressure, psi
$p_{t,2}/p_t$	pressure recovery, ratio of engine face total pressure to free-stream total pressure
Q	output weighting matrix in performance index
q	pitch rate, rad/sec
R	control weighting matrix in performance index
S_n	gain significance matrix for the nth eigenvalue
1/s	integrator
T	thrust, lb
T_f	fan inlet temperature, °R
T_{fT}	fan turbine inlet temperature, °R
t	time, sec
$t_{1/2}$	time to damp to one-half amplitude, sec
t_2	time to damp to double amplitude, sec
U	control input vector for uncoupled system
$\hat{U}$	control input vector for coupled system
U_n	nth right eigenvector for closed-loop system

V velocity, ft/sec

V_n nth left eigenvector for closed-loop system

$w_{a,E}$ engine airflow, lb/sec

$(w_{a,E})_{tr}$ engine airflow trim request, lb/sec

$w_{a,I}$ inlet airflow, lb/sec

w_f fuel flow, lb/hr

$w_{f,A/B}$ afterburner fuel flow, lb/hr

X state vector

$\dot{X}$ state vector differentiated with respect to time

x_R throat ramp position

Y output vector

α angle of attack, rad

β parameter in gain significance analysis

γ flight-path angle, rad

δ_e elevator control, rad

ζ damping coefficient

θ pitch attitude, rad

λ eigenvalue

τ_I inlet duct time constant, sec

τ_R throat ramp time constant, sec

$\tau_{\phi R}$ rotating ramp time constant, sec

ϕ_R rotating ramp position, deg

ω natural frequency, rad/sec

Subscripts:

A airframe

C commanded

E engine

I inlet

n nth

sp short period

Superscript:

T transpose

MODEL DEVELOPMENT

The linear time-invariant airplane model developed in this report is synthesized from three linear subsystem models. The subsystem models are the inlet, the engine, and the airframe aerodynamic models. The integrated system is a model of an advanced fighter with twin turbofan engines. The flight condition about which the models are linearized is subsonic and afterburners are in use. Table 1 summarizes the nominal operating values for most state and output variables.

The airframe model contains only longitudinal aerodynamics. Despite the lack of lateral dynamics, there is still notable interaction among the three subsystems as will be explained in the section "Discussion." The aerodynamic data for the airframe model were generated with an operating-point program based on the data in reference 2. The airframe model, in state-space format, is given in table 2.

The engine and inlet models were taken from reference 3. The engine and inlet models, in state-space format, are given in tables 3 and 4. The linear inlet model is a two-dimensional mixed compression inlet. Since the inlet model was substantially modified from that given in reference 3, a block diagram of the present inlet model is provided in figure 1 to allow a direct comparison with the original model in reference 3. The primary modification to the inlet model is the addition of $C_{M,I}$ and T_f as output variables. The inlet moment coefficient $C_{M,I}$ was estimated with the operating-point program mentioned in the previous paragraph, and the fan inlet temperature T_f was based on the model in reference 4.

The synthesis of the component models is readily accomplished after each is put into state-space format. The analyst needs only to specify the coupling equation which completely defines the subsystem interactions. The coupling equation is

$$U = G\hat{U} + \hat{F}Y \tag{1}$$

where U is the control input vector for all three subsystems, Y is the output vector for all three subsystems, $\hat{F}$ and G are the coupling equation matrices, and $\hat{U}$ is the input vector for the integrated (coupled) system. The U and Y vectors have the form

$$U = [U_A \quad U_I \quad U_E]^T \tag{2a}$$

$$Y = [Y_A \quad Y_I \quad Y_E]^T \tag{2b}$$

The vector $\hat{U}$ contains the system inputs which are independent of any system output. Thus, equation (1) is simply a mathematical statement of the input/output relationships among the subsystems. This relationship is shown diagrammatically in figure 2. The G and $\hat{F}$ matrices are given in table 5.

The integrated system is obtained by substituting the coupling equation (eq. (1)) into the following state-space equations for the three subsystems:

$$\dot{X} = AX + BU \tag{3a}$$

$$Y = CX + DU \tag{3b}$$

In equations (3), U and Y are given by equations (2) and X is in the same format as equations (2):

$$X = \begin{bmatrix} X_A & X_I & X_E \end{bmatrix}^T \tag{4}$$

The matrices A, B, C, and D correspond in the appropriate way; for example, the block diagonal matrix A is

$$A = \mathrm{diag}\begin{bmatrix} A_A & A_I & A_E \end{bmatrix}$$

After substitution of equation (1) into equations (3), the integrated system is given by

$$\dot{X} = \hat{A}X + \hat{B}\hat{U} \tag{6a}$$

$$Y = \hat{C}X + \hat{D}\hat{U} \tag{6b}$$

where X, Y, and $\hat{U}$ are as indicated before and

$$\hat{A} = A + B\hat{F}(I - D\hat{F})^{-1}C \tag{7a}$$

$$\hat{B} = BG + B\hat{F}(I - D\hat{F})^{-1}DG \tag{7b}$$

$$\hat{C} = (I - D\hat{F})^{-1}C \tag{7c}$$

$$\hat{D} = (I - D\hat{F})^{-1}DG \tag{7d}$$

Matrices $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$ are also presented in table 6.

The solution to equations (6) is accomplished by using standard LQR theory. In this report, ORACLS software was used from reference 5. The ORACLS library provides routines to solve the regulator problem of the form

$$\dot{X} = AX + BU \qquad (8a)$$

$$Y = HX \qquad (8b)$$

$$J = \int (X^T QX + U^T RU)\ dt \qquad (8c)$$

Since the desired problem is not in this form, another control variable transformation is required to eliminate the cross-product terms (eq. (10b)) in the performance index. This procedure is covered in reference 5. The desired form is

$$\dot{X} = \hat{A}X + \hat{B}\hat{U} \qquad (9a)$$

$$Y = \hat{C}X + \hat{D}\hat{U} \qquad (9b)$$

$$J = \int (Y^T QY + \hat{U}^T R\hat{U})\ dt \qquad (9c)$$

Equation (9c) can be expanded to show the cross-product terms by substituting equation (9b) into equation (9c) to get

$$J = \int \left[\left(\hat{C}X + \hat{D}\hat{U} \right)^T Q \left(\hat{C}X + \hat{D}\hat{U} \right) + \hat{U}^T R\hat{U} \right] dt \qquad (10a)$$

and expanding gives

$$J = \int \left[X^T \hat{C}^T Q\hat{C}X + 2X^T \hat{C}^T Q\hat{D}\hat{U} + \hat{U}^T \left(R + \hat{D}^T Q\hat{D} \right) \hat{U} \right] dt \qquad (10b)$$

The final statement of the LQR problem is the specification of the weighting matrices. These matrices are basically design parameters chosen by the control designer to achieve the desired response characteristics. The criteria for choosing these matrices are primarily based upon experience. Since methodology is more important than strict model fidelity for this report, two simple criteria were used to select weights. The first, suggested in reference 6, takes the ith weight to be the inverse of the squared maximum allowable deviation of the ith parameter. The second required that the short-period frequency and damping meet the Military Specification requirements (ref. 7) for the category and class of airplane. The first criterion was used to obtain initial estimates of the weighting matrices. The second criterion was satisfied through a trial-and-error process of adjusting the initial weights. The final weighting matrices Q and R are given by the following equations:

$$Q = \text{diag}[1.0E\text{-}05 \quad 0. \quad 0.1 \quad 0.25 \quad 0. \quad 0.4 \quad 9. \quad 0.01 \quad 1. \quad 1. \quad 0. \quad 0.03 \quad 0.04 \quad 0.04 \quad 0.002]$$

$$R = \text{diag}[4. \quad 0.04 \quad 0.02 \quad 1.]$$

FEEDBACK GAIN ANALYSIS

The solution to the LQR problem for the integrated system is a feedback gain matrix. This matrix defines the optimal control law with full state feedback. It is desirable to eliminate ineffectual gains since they represent added weight and hardware in the physical system. Of course, eliminating any element of the gain matrix means the solution is no longer the optimal solution; however, the solution may still be close enough to optimal from an engineering point of view. With this in mind, a method is shown that will determine which gains are essential, which gains can be eliminated, and in what order they should be eliminated. In addition, a relative measure of the penalty for removing the gain is provided.

The procedure is based upon the closed-loop eigenvalue sensitivity to gain matrix elements. Sensitivity problems have been considered by many authors. (For example, see refs. 8 through 12.) The sensitivity of the nth eigenvalue λ_n to the gain matrix element $f_{k\ell}$ is given by

$$\frac{\partial \lambda_n}{\partial f_{k\ell}} = \left(\sum_{i=1}^{m} V_{n,i} \hat{b}_{ik} \right) U_{n,\ell} \tag{11}$$

where V_n and U_n are the nth left and right eigenvectors of the closed-loop-system matrix, respectively, and $\hat{b}_{ik}$ is the (i,k) element of the control input distribution matrix. This result is developed in the appendix.

In order to compare the sensitivities in a hierarchical arrangement, a gain significance matrix S_n is defined for each eigenvalue. For the nth eigenvalue, the (k,ℓ) element of S_n is given by

$$S_{n,k\ell} = \left| \frac{\partial \lambda_n}{\partial f_{k\ell}} \frac{f_{k\ell}}{\lambda_n} \right| \tag{12}$$

The elements of this matrix represent the nondimensional modulus of the sensitivity of the nth eigenvalue to each gain matrix element. This can be interpreted as a relative indication of the penalty for changing any particular gain and a hierarchy for eliminating gains. In general, if $S_{n,k\ell}$ for all eigenvalues is much less than 1, the removal of the corresponding gain $f_{k\ell}$ will not appreciably affect the optimal solution. If $S_{n,k\ell}$ for any eigenvalues is greater than 1, the corresponding gain element should be considered essential.

DISCUSSION

The integration of the three major airplane subsystems (airframe, inlet, and engine) primarily affected the airframe stability characteristics. The effects on the inlet and engine modes were negligible. Table 7 shows the open-loop eigenvalues for the coupled and uncoupled systems and each mode is identified with its corresponding subsystem. The aerodynamic model is fifth order; the five states are V, α, q, θ, and h. As shown in table 7, the oscillatory short-period mode, involving primarily α and q, has changed from $\omega_{n,sp} = 4$, $\zeta_{sp} = 0.45$ to $\omega_{n,sp} = 4.5$, $\zeta_{sp} = 0.39$ after coupling the subsystems. The phugoid mode, involving primarily V and θ, becomes oscillatory and remains unstable after integrating the subsystems. The phugoid natural frequency is 0.054 rad/sec and the time to damp to double amplitude t_2 is 23.6 sec. The last mode of the airplane model is associated with the altitude h. It has a stable response both before and after the systems are combined. The times to damp to one-half amplitude $t_{1/2}$ before and after coupling the systems are 10.1 and 7.2 sec, respectively.

After integrating the subsystems with coupling equation (1), the standard LQR problem was solved. The open- and closed-loop eigenvalues along with an indication of what subsystem they correspond to are given in table 8. The results show that the inlet modes were virtually unaffected by the addition of feedback. The engine modes were slightly changed primarily through increases in damping, although the mode associated with fan speed had a more substantial change. This mode changed its time to damp to one-half amplitude from $t_{1/2} = 1.6$ sec (open loop) to $t_{1/2} = 0.45$ sec (closed loop). The aerodynamic model was the most affected by feedback. The short period mode changed its natural frequency from $\omega_{n,sp} = 4.5$ to $\omega_{n,sp} = 5.4$ and the damping ratio changed from $\zeta_{sp} = 0.39$ (open loop) to $\zeta_{sp} = 0.64$ (closed loop). Of course, this was designed into the model by the appropriate choice of weighting matrices Q and R in the performance index. The phugoid mode became stable with feedback and the eigenvalue associated with altitude changed from -0.096 (open loop) to -0.053 (closed loop).

The resulting airplane model has enough interaction between the propulsion subsystems and the airframe subsystem to demand cross feedback in the solution to the LQR problem. This provides a useful tool for testing the gain significance matrix S_n as a measure of feedback gain importance to the optimal solution. The values of S_n for this model are given in table 9. To demonstrate the utility of the gain significance matrix, gain elements were eliminated according to the following rule: if the (k,ℓ) element of all the S_n's is less than β, remove the (k,ℓ) element of the feedback gain matrix. The β parameter was chosen to be 0., 0.01, 0.1, 0.5, 1., and ∞. Choosing $\beta = 0.$ results in no elements of the gain matrix being eliminated. This is the full state feedback case and the optimal solution to the LQR problem. Choosing $\beta = \infty$ eliminates all the gains and so represents the open-loop solution. The closed-loop eigenvalues for each value of β are given in table 10. The location of the short-period pole for varying β is given in figure 3. As mentioned before, when an element of S_n is close to unity, the corresponding gain element is essential for the optimal solution. This is demonstrated in table 10 and figure 3 since negligible change in the eigenvalues occurs for $\beta < 0.01$, even though 25 percent of the gains have been eliminated. In addition, with $\beta < 0.1$, 65 percent of the gains are eliminated while still maintaining acceptable performance. For larger values of β, the change in eigenvalues becomes much larger and the system designer must choose the acceptable trade-off between closeness to the optimal solution and reduction in the number of feedback gains. In addition, for large values of β, the possibility exists for eliminating certain combinations of gains which may cause an amplified adverse response; this is shown in figure 3 for $\beta = 1$.

The S_n elements also provide a relative measure of the penalty for removing a particular gain. It is only a relative measure since eigenvalues and their sensitivities to gains can be complex numbers. In simplifying these numbers with the S_n formula, only the modulus is used; therefore, phase information is lost (relative size comparisons make sense only on the real axis). Therefore, the S_n elements cannot be used directly to compute the actual change in an eigenvalue after modifying a gain. This is a penalty for simplification. The S_n can still be a useful indicator of the cost of changing or removing a gain element. As was seen in table 10(b) for $\beta = 0.01$, very small S_n elements indicate very small or negligible changes in the associated eigenvalue when the corresponding gains are removed. Now, for example, consider S_n element (1,3) for eigenvalues 7 and 8, the short-period mode (table 9); this gives $S_n(1,3) \approx 0.69$. This corresponds to the gain element which feeds back pitch rate q to the elevator control δ_e. The result of removing only this gain should be significant. The resulting closed-loop eigenvalues are given in table 11. The change in natural frequency is from $\omega_{n,sp} = 5.4$ to $\omega_{n,sp} = 5.6$, and the change in damping ratio is from $\zeta = 0.64$ to $\zeta = 0.13$, almost a factor of 5. These changes show that S_n indicates where the sensitive and more important gain elements are for each mode, although it does not indicate whether the sensitivity will be expressed in the magnitude or phase of the eigenvalue.

CONCLUDING REMARKS

This report satisfies two objectives. The first objective was to develop a simple model of a modern fighter with interactive propulsion and airframe subsystems. This provides a working tool for studying integrated control methodology. The second objective was to develop a method for eliminating ineffectual feedback gains. This provides a method for simplifying the control system without major performance penalties.

There are limitations to these two efforts, however. The airplane model has only longitudinal aerodynamics and is linearized about one flight condition. The engine is operating with afterburner on. This flight condition represents only one small area in the fighter's flight envelope. The gain significance matrix S_n provides an easily implemented measure of the relative importance of the feedback gains and an indirect measure of the actual change that can occur in the eigenvalues. The actual change in the eigenvalue can be calculated from the eigenvalue sensitivities if these are retained during the calculation of S_n.

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DERIVATION OF EIGENVALUE SENSITIVITIES TO FEEDBACK GAINS

The derivation of eigenvalue sensitivities to feedback gains closely follows the development for sensitivities to the system matrix. (See ref. 8.)

Consider the mth order system matrix $\hat{A}$, the closed-loop system $\tilde{A}$ with distinct eigenvalues, the distribution matrix $\hat{B}$, and the gain matrix F where

$$\tilde{A} = \hat{A} + \hat{B}F \tag{A1}$$

Let V_j and U_i be left and right eigenvectors of $\tilde{A}$ so that

$$\tilde{A}U_i = \lambda_i U_i \tag{A2}$$

and

$$\tilde{A}^T V_j = \lambda_j V_j \tag{A3}$$

and

$$U_i^T V_j = V_j^T U_i = \delta_{ij} \tag{A4}$$

where δ is the Kronecker delta. Differentiating equation (A2) with respect to gain $f_{k\ell}$ gives

$$\frac{\partial \tilde{A}}{\partial f_{k\ell}} U_i + \tilde{A} \frac{\partial U_i}{\partial f_{k\ell}} = \frac{\partial \lambda_i}{\partial f_{k\ell}} U_i + \lambda_i \frac{\partial U_i}{\partial f_{k\ell}} \tag{A5}$$

Multiplying equation (A5) on the left by V_i^T gives

$$V_i^T \frac{\partial \tilde{A}}{\partial f_{k\ell}} U_i + V_i^T \tilde{A} \frac{\partial U_i}{\partial f_{k\ell}} = V_i^T \frac{\partial \lambda_i}{\partial f_{k\ell}} U_i + V_i^T \lambda_i \frac{\partial U_i}{\partial f_{k\ell}} \tag{A6}$$

Substituting equation (A3) into equation (A6) and dropping the i subscript gives

$$V^T \frac{\partial \tilde{A}}{\partial f_{k\ell}} U = \frac{\partial \lambda}{\partial f_{k\ell}} \tag{A7}$$

Note that

$$\frac{\partial \tilde{A}}{\partial f_{k\ell}} = \frac{\partial}{\partial f_{k\ell}}(\hat{A} + \hat{B}F) = \hat{B}\,\frac{\partial F}{\partial f_{k\ell}} \tag{A8}$$

Substitute equation (A8) into equation (A7) to get

$$\frac{\partial \lambda}{\partial f_{k\ell}} = V^T\hat{B}\,\frac{\partial F}{\partial f_{k\ell}}\,U \tag{A9}$$

or

$$\frac{\partial \lambda}{\partial f_{k\ell}} = V^T\hat{B}\delta_{ik}\,\delta_{j\ell}U = (V^T\hat{B})_k U_\ell \tag{A10}$$

Thus the kth element of the vector $V^T\hat{B}$ times the ℓth element of vector U is the eigenvalue sensitivity to the gain $f_{k\ell}$. Equation (A10) can also be written as

$$\frac{\partial \lambda_n}{\partial f_{k\ell}} = \left(\sum_{i=1}^{m} V_{n,i}\hat{b}_{ik}\right)U_{n,\ell} \tag{A11}$$

which emphasizes that the equation is for the nth eigenvalue.

REFERENCES

1. Michael, Gerald J.; and Farrar, Florence A.: Development of Optimal Control Modes for Advanced Technology Propulsion Systems. Rep. No. N911620-2 (Contract N00014-73-C-0281), United Aircraft Res. Lab., Mar. 1974. (Available from DTIC as AD 775 337.)

2. Pennington, Jack E.; and Meintel, Alfred J., Jr.: Simulation Study of Nonaxisymmetric Nozzle for Air-Combat Maneuvering for an F-15 Class Airplane. NASA TM-80230, 1980.

3. Heimbold, Richard L.; Hauge, James A.; and Miller, Ronald J.: Flight Propulsion Control Coupling (FPCC) and Dynamic Interaction Investigation. Volumes I and II. AFFDL-TR-75-153, U.S. Air Force, Sept. 30, 1975. (Available from DTIC as AD B026 174L and AD B025 832L.)

4. Lallman, Frederick J.: Simplified Off-Design Performance Model of a Dry Turbofan Engine Cycle. NASA TM-83204, 1981.

5. Armstrong, Ernest S.: ORACLS - A Design System for Linear Multivariable Control. Marcel Dekker, Inc., c.1980.

6. Bryson, Arthur E., Jr.; and Ho, Yu-Chi: Applied Optimal Control, Revised Printing. Hemisphere Publ. Corp., c.1975.

7. Chalk, C. R.; Neal, T. P.; Harris, T. M.; Pritchard, F. E.; and Woodcock, R. J.: Background Information and User Guide for MIL-F-8785B(ASG), "Military Specification - Flying Qualities of Piloted Airplanes." AFFDL-TR-69-72, U.S. Air Force, Aug. 1969. (Available from DTIC as AD 860 856.)

8. Porter, Brian; and Crossley, Roger: Modal Control: Theory and Applications. Taylor & Francis (London), 1972.

9. Horowitz, Isaac M.: Synthesis of Feedback Systems. Academic Press, Inc., c.1963.

10. Elliott, Jarrell R.; and Teague, William F.: A Method for Reducing the Sensitivity of Optimal Nonlinear Systems to Parameter Uncertainty. NASA TN D-6218, 1971.

11. Cruz, J. B., Jr.; and Perkins, W. R.: A New Approach to the Sensitivity Problem in Multivariable Feedback System Design. IEEE Trans. Autom. Control, vol. AC-9, no. 3, July 1964, pp. 216-223.

12. Faddeev, D. K.; and Faddeeva, V. N. (Robert C. Williams, transl.): Computational Methods of Linear Algebra. W. H. Freeman and Co., c.1963.

TABLE 1.- NOMINAL OPERATING VALUES

Airframe	Inlet	Engine
M = Subsonic	ϕ_R = 5.°	n_f = 10 040 rpm
α = 1.7°	x_R = 0.293	n_c = 12 930 rpm
q = 0. deg/sec	$w_{a,I}$ = 227 lb/sec	w_f = 8182 lb/hr
θ = 1.7°	$p_{t,2}$ = 11.4 psi	$w_{f,A/B}$ = 24 870 lb/hr
δ_e = -0.6°	K_{a2} = 0.56	T = 16 930 lb/engine
	T_f = 520°R	$w_{a,E}$ = 173.2 lb/sec
		PLA = 120°
		A_j = 4.7 ft^2
		m_f = 19.35 percent
		m_c = 20.39 percent
		T_{fT} = 2171°R
		$(w_{a,E})_{tr}$ = 225 lb/sec

TABLE 2.- AIRFRAME MODEL

$$\begin{bmatrix} X_A = [V \quad \alpha \quad q \quad \theta \quad h]^T \\ Y_A = [M \quad \alpha \quad q \quad \gamma \quad h]^T \\ U_A = [\delta_e \quad 2T \quad 2C_{D,I} \quad 2C_{M,I}]^T \\ \dot{X}_A = A_A X_A + B_A U_A \\ Y_A = C_A X_A + D_A U_A \end{bmatrix}$$

A_A matrix

-3.397E-02	-4.037E+01	0.	-3.217E+01	1.328E-04
-3.915E-05	-1.565E+00	1.000E+00	0.	1.493E-06
-1.374E-03	-1.292E+01	-2.142E+00	0.	7.503E-06
0.	0.	1.000E+00	0.	0.
0.	-9.332E+02	0.	9.332E+02	0.

B_A matrix

-9.223E+00	8.958E-04	-3.005E+02	0.
-1.910E-01	-2.812E-08	0.	0.
-2.483E+01	1.261E-08	0.	3.207E+01
0.	0.	0.	0.
0.	0.	0.	0.

C_A matrix

9.646E-04	0.	0.	0.	0.
0.	1.000E+00	0.	0.	0.
0.	0.	1.000E+00	0.	0.
0.	-1.000E+00	0.	1.000E+00	0.
0.	0.	0.	0.	1.000E+00

D_A matrix

$$D_A = \{0\}$$

TABLE 3.- ENGINE MODEL

$$X_E = [n_f \quad n_c \quad w_f \quad w_{f,A/B}]^T$$

$$Y_E = [T \quad w_{a,E} \quad m_f \quad m_c \quad T_{fT}]^T$$

$$U_E = [PLA \quad (w_{a,E})_{tr} \quad p_{t,2} \quad T_f \quad p_{st} \quad K_{a2} \quad A_j]^T$$

$$\dot{X}_E = A_E X_E + B_E U_E$$

$$Y_E = C_E X_E + D_E U_E$$

A_E matrix

-3.385E-01	-1.089E-01	1.269E-01	-2.394E-03
3.318E-01	-1.270E+00	3.654E-01	-3.013E-03
-1.462E+00	-4.795E+00	-4.503E-01	-3.114E-03
-1.362E+01	6.116E+01	1.214E+01	-7.291E+00

B_E matrix

1.148E+01	6.292E+01	7.687E+01	1.330E+00	0.	0.	3.256E+02
2.254E+00	8.389E+00	-2.575E+02	1.462E+01	0.	0.	5.239E+01
-4.486E+00	-3.679E+01	1.067E+03	-4.153E+01	0.	0.	-1.214E+03
1.118E+04	-1.222E+02	3.526E+03	1.043E+02	0.	0.	-1.440E+03

C_E matrix

-1.281E+00	9.396E-01	1.147E+00	1.502E-01
2.029E-02	-1.167E-02	1.045E-03	9.343E-05
2.485E-02	-1.756E-02	-7.241E-03	6.049E-05
-1.595E-03	5.036E-03	-8.618E-04	1.122E-05
9.524E-02	-2.514E-02	7.329E-02	-1.879E-04

D_E matrix

-2.036E+01	-2.690E+01	8.939E+01	-5.816E+01	-8.769E+02	0.	-1.564E+03
-5.862E-02	7.717E-02	1.371E+01	-2.416E-01	0.	0.	8.646E+00
1.643E-02	3.773E-01	7.705E+00	-2.665E-02	0.	-1.030E+01	1.863E+01
-1.779E-02	-2.870E-02	1.091E+00	-6.044E-02	0.	0.	-7.235E-02
8.597E-01	1.371E+00	-7.130E+01	2.149E+00	0.	0.	8.800E+00

TABLE 4.- INLET MODEL

$$X_I = [\phi_R \quad x_R \quad w_{a,I}]^T$$

$$Y_I = [p_{t,2} \quad K_{a2} \quad T_f \quad C_{D,I} \quad C_{M,I}]^T$$

$$U_I = [M \quad \alpha \quad h \quad w_{a,E}]^T$$

$$\dot{X}_I = A_I X_I + B_I U_I$$

$$Y_I = C_I X_I + D_I U_I$$

A_I matrix

-3.125E+01	0.	0.
0.	-3.125E+01	0.
0.	0.	-2.857E+01

B_I matrix

0.	1.791E+03	0.	0.
0.	2.472E+01	0.	0.
0.	0.	0.	2.857E+01

C_I matrix

-6.800E-03	5.475E-01	3.430E-03
2.200E-02	-3.490E+00	8.000E-03
0.	0.	0.
-3.500E-05	-9.100E-04	-1.400E-05
-1.063E-03	0.	-4.660E-05

D_I matrix

1.215E+01	-4.583E-01	-4.050E-04	0.
2.040E-01	1.500E+00	0.	0.
8.320E+01	0.	-1.670E-02	0.
1.150E-03	2.570E-03	0.	0.
7.300E-03	-8.600E-03	0.	0.

TABLE 5.- G AND $\hat{F}$ MATRICES FOR COUPLING EQUATION

[Matrix order is indicated by first row elements]

Coupling equation: $U = G\hat{U} + \hat{F}Y$

G matrix

1.000E+00	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	1.000E+00	0.	0.
0.	0.	1.000E+00	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	1.000E+00

$\hat{F}$ matrix

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(1, 7)
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
1.000E+00	0.	0.	0.	0.	0.	0.
0.	1.000E+00	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.000E+00	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.000E+00	0.
0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-2.830E-04	0.	0.
0.	0.	0.	0.	0.	0.	1.000E+00
0.	0.	0.	0.	0.	0.	0.

(1, 8)	(1, 9)	(1, 10)	(1, 11)	(1, 12)	(1, 13)	(1, 14)	(1, 15)
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	2.000E+00	0.	0.	0.	0.
0.	2.000E+00	0.	0.	0.	0.	0.	0.
0.	0.	2.000E+00	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.000E+00	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
1.000E+00	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.

TABLE 6.- INTEGRATED MODEL

Matrix order is similar to table 5

$$X = [V \quad \alpha \quad q \quad \theta \quad h \quad \phi_R \quad x_R \quad w_{a,I} \quad n_f \quad n_c \quad w_f \quad w_{f,A/B}]^T$$

$$Y = [M \quad \alpha \quad q \quad \gamma \quad h \quad p_{t,2} \quad K_{a2} \quad T_{fT} \quad C_{D,I} \quad C_{M,I} \quad T \quad w_{a,E} \quad m_f \quad m_c \quad T_{fT}]^T$$

$$\hat{U} = [\delta_e \quad PLA \quad (w_{a,E})_{tr} \quad A_j]^T$$

$$\dot{X} = \hat{A}X + \hat{B}U$$

$$Y = \hat{C}X + \hat{D}U$$

$\hat{A}$ matrix

-4.112E-02	-4.199E+01	0.	-3.217E+01	2.253E-03	1.994E-02	6.345E-01
-3.895E-05	-1.505E+00	1.000E+00	0.	1.427E-06	3.418E-08	-2.752E-06
-8.727E-04	-1.347E+01	-2.142E+00	0.	7.533E-06	-6.818E-02	1.234E-06
0.	0.	1.000E+00	0.	0.	0.	0.
0.	-9.332E+02	0.	9.332E+02	0.	0.	0.
3.858E-14	1.791E+03	0.	0.	-1.295E-15	-3.125E+01	2.407E-11
5.326E-16	2.472E+01	0.	0.	-1.788E-17	-2.252E-15	-3.125E+01
4.037E+00	-1.795E+02	0.	0.	-4.336E-02	-2.664E+00	2.145E+02
1.008E+00	-3.523E+01	0.	0.	-5.334E-02	-5.227E-01	4.209E+01
-1.844E+00	1.180E+02	0.	0.	-1.399E-01	1.751E+00	-1.410E+02
9.172E+00	-4.890E+02	0.	0.	2.614E-01	-7.256E+00	5.842E+02
4.969E+01	-1.616E+03	0.	0.	-3.170E+00	-2.398E+01	1.930E+03

8.962E-03	-2.295E-03	1.683E-03	2.055E-03	2.691E-04
-1.724E-08	7.203E-08	-5.284E-08	-6.450E-08	-8.446E-09
-2.989E-03	-3.231E-08	2.370E-08	2.893E-08	3.788E-09
0.	0.	0.	0.	0.
0.	0.	0.	0.	0.
-3.921E-14	0.	0.	0.	0.
-5.413E-16	0.	0.	0.	0.
-2.723E+01	5.797E-01	-3.334E-01	2.986E-02	2.669E-03
2.637E-01	-3.385E-01	-1.089E-01	1.269E-01	-2.394E-03
-8.832E-01	3.318E-01	-1.270E+00	3.654E-01	-3.013E-03
3.660E+00	-1.462E+00	-4.795E+00	-4.503E-01	-3.114E-03
1.209E+01	-1.362E+01	6.116E+01	1.214E+01	-7.291E+00

TABLE 6.- Continued

$\hat{B}$ matrix

-9.223E+00	-3.648E-02	-4.819E-02	-2.802E+00
-1.910E-01	1.145E-06	1.513E-06	8.795E-05
-2.483E+01	-5.135E-07	-6.784E-07	-3.944E-05
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	-1.675E+00	2.205E+00	2.470E+02
0.	1.148E+01	6.292E+01	3.256E+02
0.	2.254E+00	8.389E+00	5.239E+01
0.	-4.486E+00	-3.679E+01	-1.214E+03
0.	1.118E+04	-1.222E+02	-1.440E+03

$\hat{D}$ matrix

0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	0.	0.	0.
0.	-2.036E+01	-2.690E+01	-1.564E+03
0.	-5.862E-02	7.717E-02	8.646E+00
0.	1.643E-02	3.773E-01	1.863E+01
0.	-1.779E-02	-2.870E-02	-7.235E-02
0.	8.597E-01	1.371E+00	8.800E+00

TABLE 6.- Concluded

$\hat{C}$ matrix

9.646E-04	-5.551E-17	0.	0.	1.735E-18	-7.550E-19	6.078E-17
2.155E-17	1.000E+00	0.	0.	-7.233E-19	-9.109E-17	1.344E-14
0.	0.	1.000E+00	0.	0.	0.	0.
0.	-1.000E+00	0.	1.000E+00	0.	0.	0.
0.	0.	0.	0.	1.000E+00	0.	0.
1.172E-02	-4.583E-01	0.	0.	-4.050E-04	-6.800E-03	5.475E-01
1.968E-04	1.500E+00	0.	0.	-2.168E-18	2.200E-02	-3.490E+00
8.025E-02	-3.553E-15	0.	0.	-1.670E-02	-4.832E-17	3.890E-15
1.109E-06	2.570E-03	0.	0.	-5.082E-21	-3.500E-05	-9.100E-04
7.041E-06	-8.600E-03	0.	0.	2.711E-20	-1.063E-03	9.023E-18
-3.620E+00	-4.097E+01	0.	0.	1.183E+00	-6.079E-01	4.894E+01
1.413E-01	-6.283E+00	0.	0.	-1.518E-03	-9.323E-02	7.506E+00
8.613E-02	-1.898E+01	0.	0.	-2.675E-03	-2.790E-01	4.017E+01
7.936E-03	-5.000E-01	0.	0.	5.675E-04	-7.419E-03	5.973E-01
-6.631E-01	3.268E+01	0.	0.	-7.012E-03	4.848E-01	-3.904E+01

3.808E-19	0.	0.	0.	0.
-2.190E-17	0.	0.	0.	0.
0.	0.	0.	0.	0.
0.	0.	0.	0.	0.
0.	0.	0.	0.	0.
3.430E-03	0.	0.	0.	0.
8.000E-03	0.	0.	0.	0.
2.437E-17	0.	0.	0.	0.
-1.400E-05	0.	0.	0.	0.
-4.660E-05	0.	0.	0.	0.
3.066E-01	-1.281E+00	9.396E-01	1.147E+00	1.502E-01
4.703E-02	2.029E-02	-1.167E-02	1.045E-03	9.343E-05
-5.597E-02	2.485E-02	-1.756E-02	-7.241E-03	6.049E-05
3.742E-03	-1.595E-03	5.036E-03	-8.618E-04	1.122E-05
-2.446E-01	9.524E-02	-2.514E-02	7.329E-02	-1.879E-04

TABLE 7.- OPEN-LOOP EIGENVALUES FOR COUPLED AND UNCOUPLED SYSTEMS

Subsystem	Re(λ)	Im(λ)	ω	ζ	$t_{1/2}$ or t_2
Open-loop eigenvalues for uncoupled system					
Airframe	9.350 E-03	0.			7.41 E+01
Airframe	3.109 E-02	0.			2.23 E+01
Airframe	-6.895 E-02	0.			1.01 E+01
Engine	-4.938 E-01	0.			1.40 E+00
Engine	-7.985 E-01	1.313 E+00	1.537 E+00	5.195 E-01	8.68 E-01
Engine	-7.985 E-01	-1.313 E+00	1.537 E+00	5.195 E-01	8.68 E-01
Airframe	-1.826 E+00	3.582 E+00	4.021 E+00	4.542 E-01	3.79 E-01
Airframe	-1.826 E+00	-3.582 E+00	4.021 E+00	4.542 E-01	3.79 E-01
Engine	-7.259 E+00	0.			9.55 E-02
Inlet	-2.857 E+01	0.			2.43 E-02
Inlet	-3.125 E+01	0.			2.22 E-02
Inlet	-3.125 E+01	0.			2.22 E-02
Open-loop eigenvalues for coupled system					
Airframe	2.941 E-02	4.471 E-02	5.352 E-02	5.496 E-01	2.36 E+01
Airframe	2.941 E-02	-4.471 E-02	5.352 E-02	5.496 E-01	2.36 E+01
Airframe	-9.664 E-02	0.			7.17 E+00
Engine	-4.443 E-01	0.			1.56 E+00
Engine	-8.106 E-01	1.314 E+00	1.544 E+00	5.250 E-01	8.55 E-01
Engine	-8.106 E-01	-1.314 E+00	1.544 E+00	5.250 E-01	8.55 E-01
Airframe	-1.757 E+00	4.174 E+00	4.529 E+00	3.880 E-01	3.95 E-01
Airframe	-1.757 E+00	-4.174 E+00	4.529 E+00	3.880 E-01	3.95 E-01
Engine	-7.259 E+00	0.			9.55 E-02
Inlet	-2.725 E+01	0.			2.54 E-02
Inlet	-3.125 E+01	0.			2.22 E-02
Inlet	-3.139 E+01	0.			2.21 E-02

TABLE 8.- OPEN- AND CLOSED-LOOP EIGENVALUES FOR COUPLED SYSTEM

Subsystem	Re(λ)	Im(λ)	ω	ζ	$t_{1/2}$ or t_2
Open-loop eigenvalues for coupled system					
Airframe	2.941 E-02	4.471 E-02	5.352 E-02	5.496 E-01	2.36 E+01
Airframe	2.941 E-02	-4.471 E-02	5.352 E-02	5.496 E-01	2.36 E+01
Airframe	-9.664 E-02	0.			7.17 E+00
Engine	-4.443 E-01	0.			1.56 E+00
Engine	-8.106 E-01	1.314 E+00	1.544 E+00	5.250 E-01	8.55 E-01
Engine	-8.106 E-01	-1.314 E+00	1.544 E+00	5.250 E-01	8.55 E-01
Airframe	-1.757 E+00	4.174 E+00	4.529 E+00	3.880 E-01	3.95 E-01
Airframe	-1.757 E+00	-4.174 E+00	4.529 E+00	3.880 E-01	3.95 E-01
Engine	-7.259 E+00	0.			9.55 E-02
Inlet	-2.725 E+01	0.			2.54 E-02
Inlet	-3.125 E+01	0.			2.22 E-02
Inlet	-3.139 E+01	0.			2.21 E-02
Closed-loop eigenvalues for coupled system					
Airframe	-5.329 E-02	0.			1.30 E+01
Airframe	-7.954 E-01	7.665 E-01	1.105 E+00	7.201 E-01	8.71 E-01
Airframe	-7.954 E-01	-7.665 E-01	1.105 E+00	7.201 E-01	8.71 E-01
Engine	-1.540 E+00	0.			4.50 E-01
Engine	-1.213 E+00	1.135 E+00	1.661 E+00	7.302 E-01	5.71 E-01
Engine	-1.213 E+00	-1.135 E+00	1.661 E+00	7.302 E-01	5.71 E-01
Airframe	-3.430 E+00	4.169 E+00	5.399 E+00	6.353 E-01	2.02 E-01
Airframe	-3.430 E+00	-4.169 E+00	5.399 E+00	6.353 E-01	2.02 E-01
Engine	-7.212 E+00	0.			9.61 E-02
Inlet	-2.693 E+01	0.			2.57 E-02
Inlet	-3.125 E+01	0.			2.22 E-02
Inlet	-3.135 E+01	0.			2.21 E-02

TABLE 9.- SIGNIFICANCE MATRICES FOR COUPLED, CLOSED-LOOP SYSTEM

[Matrix order is similar to table 5]

Eigenvalue 1

1.615E-01	1.687E-04	7.695E-05	1.021E-02	1.121E-01	8.990E-06
7.233E-01	2.706E-04	4.237E-06	9.720E-03	1.195E-01	5.302E-05
9.168E-02	2.744E-05	4.301E-08	1.020E-04	5.858E-03	2.092E-05
3.053E-01	3.408E-04	1.685E-07	6.972E-04	3.493E-02	3.032E-04
2.800E-05	1.049E-03	9.418E-04	2.712E-02	2.379E-02	6.933E-03
5.782E-05	9.823E-03	6.775E-03	1.711E-02	1.496E-01	9.478E-04
1.691E-05	5.826E-03	6.198E-03	1.183E-02	1.294E-01	5.924E-03
5.585E-04	9.320E-03	1.759E-02	2.855E-02	3.358E-01	4.589E-02

Eigenvalue 2

1.991E-01	6.471E-01	1.659E-01	1.062E+00	8.389E-01	3.532E-02
8.830E-03	1.028E-02	9.045E-05	1.001E-02	8.853E-03	2.062E-03
4.256E-02	3.964E-02	3.492E-05	3.995E-03	1.651E-02	3.095E-02
6.470E-02	2.248E-01	6.243E-05	1.246E-02	4.493E-02	2.047E-01
1.100E-01	6.489E-04	1.542E-02	6.418E-02	5.196E-02	2.284E-03
2.249E-03	6.015E-05	1.098E-03	4.009E-04	3.236E-03	3.091E-06
2.502E-02	1.357E-03	3.821E-02	1.055E-02	1.064E-01	7.347E-04
3.771E-01	9.907E-04	4.951E-02	1.162E-02	1.261E-01	2.598E-03

Eigenvalue 3

1.991E-01	6.471E-01	1.659E-01	1.062E+00	8.389E-01	3.532E-02
8.830E-03	1.028E-02	9.045E-05	1.001E-02	8.853E-03	2.062E-03
4.256E-02	3.964E-02	3.492E-05	3.995E-03	1.651E-02	3.095E-02
6.470E-02	2.248E-01	6.243E-05	1.246E-02	4.493E-02	2.047E-01
1.100E-01	6.489E-04	1.542E-02	6.418E-02	5.196E-02	2.284E-03
2.249E-03	6.015E-05	1.098E-03	4.009E-04	3.236E-03	3.091E-06
2.502E-02	1.357E-03	3.821E-02	1.055E-02	1.064E-01	7.347E-04
3.771E-01	9.907E-04	4.951E-02	1.162E-02	1.261E-01	2.598E-03

Eigenvalue 4

1.500E-02	8.242E-02	3.774E-03	1.733E-02	5.497E-02	4.612E-03
1.956E-03	3.850E-03	6.052E-06	4.804E-04	1.706E-03	7.920E-04
3.951E-02	6.221E-02	9.788E-06	8.034E-04	1.333E-02	4.980E-02
7.146E-03	4.198E-02	2.083E-06	2.983E-04	4.317E-03	3.921E-02
1.436E-02	4.500E-04	2.455E-02	2.401E-03	9.670E-03	2.222E-03
8.637E-04	1.227E-04	5.142E-03	4.410E-05	1.771E-03	8.846E-06
4.026E-02	1.159E-02	7.495E-01	4.860E-03	2.439E-01	8.809E-03
7.222E-02	1.007E-03	1.156E-01	6.370E-04	3.441E-02	3.707E-03

TABLE 9.- Continued

Eigenvalue 5

6.190E-02	2.764E-01	8.506E-02	3.619E-01	1.594E-01	1.529E-02
1.161E-02	1.857E-02	1.961E-04	1.443E-02	7.114E-03	3.776E-03
2.636E-02	3.373E-02	3.566E-05	2.712E-03	6.249E-03	2.669E-02
4.593E-02	2.193E-01	7.310E-05	9.701E-03	1.950E-02	2.024E-01

4.762E-02	4.182E-04	1.276E-02	8.107E-02	3.535E-02	2.430E-03
4.118E-03	1.639E-04	3.843E-03	2.141E-03	9.310E-03	1.391E-05
2.158E-02	1.742E-03	6.298E-02	2.654E-02	1.442E-01	1.557E-03
3.729E-01	1.458E-03	9.355E-02	3.351E-02	1.959E-01	6.313E-03

Eigenvalue 6

6.190E-02	2.764E-01	8.506E-02	3.619E-01	1.594E-01	1.529E-02
1.161E-02	1.857E-02	1.961E-04	1.443E-02	7.114E-03	3.776E-03
2.636E-02	3.373E-02	3.566E-05	2.712E-03	6.249E-03	2.669E-02
4.593E-02	2.193E-01	7.310E-05	9.701E-03	1.950E-02	2.024E-01

4.762E-02	4.182E-04	1.276E-02	8.107E-02	3.535E-02	2.430E-03
4.118E-03	1.639E-04	3.843E-03	2.141E-03	9.310E-03	1.391E-05
2.158E-02	1.742E-03	6.298E-02	2.654E-02	1.442E-01	1.557E-03
3.729E-01	1.458E-03	9.355E-02	3.351E-02	1.959E-01	6.313E-03

Eigenvalue 7

4.740E-02	5.418E-01	6.878E-01	9.007E-01	3.199E-02	3.202E-02
1.829E-03	7.489E-03	3.264E-04	7.390E-03	2.937E-04	1.627E-03
1.587E-04	5.197E-04	2.267E-06	5.307E-05	9.856E-06	4.394E-04
9.763E-04	1.193E-02	1.641E-05	6.704E-04	1.086E-04	1.177E-02

9.973E-02	6.965E-04	3.068E-03	1.200E-02	4.589E-03	1.543E-03
1.774E-03	5.617E-05	1.902E-04	6.523E-05	2.487E-04	1.817E-06
3.552E-04	2.280E-05	1.190E-04	3.088E-05	1.471E-04	7.771E-06
2.168E-02	6.741E-05	6.244E-04	1.377E-04	7.059E-04	1.113E-04

Eigenvalue 8

4.740E-02	5.418E-01	6.878E-01	9.007E-01	3.199E-02	3.202E-02
1.829E-03	7.489E-03	3.264E-04	7.390E-03	2.937E-04	1.627E-03
1.587E-04	5.197E-04	2.267E-06	5.307E-05	9.856E-06	4.394E-04
9.763E-04	1.193E-02	1.641E-05	6.704E-04	1.086E-04	1.177E-02

9.973E-02	6.965E-04	3.068E-03	1.200E-02	4.589E-03	1.543E-03
1.774E-03	5.617E-05	1.902E-04	6.523E-05	2.487E-04	1.817E-06
3.552E-04	2.280E-05	1.190E-04	3.088E-05	1.471E-04	7.771E-06
2.168E-02	6.741E-05	6.244E-04	1.377E-04	7.059E-04	1.113E-04

TABLE 9.- Concluded

Eigenvalue 9

3.706E-04	9.386E-04	1.541E-03	1.510E-03	2.364E-05	6.492E-05
5.942E-03	5.392E-03	3.038E-04	5.150E-03	9.020E-05	1.371E-03
7.091E-05	5.147E-05	2.903E-07	5.088E-06	4.164E-07	5.093E-05
3.945E-05	1.068E-04	1.900E-07	5.810E-06	4.148E-07	1.233E-04
2.022E-04	3.349E-05	6.733E-05	7.237E-04	4.598E-05	2.363E-03
1.495E-03	1.123E-03	1.734E-03	1.635E-03	1.035E-03	1.157E-03
4.117E-05	6.268E-05	1.494E-04	1.064E-04	8.425E-05	6.806E-04
2.271E-04	1.675E-05	7.082E-05	4.291E-05	3.655E-05	8.809E-04

Eigenvalue 10

1.467E-07	1.743E-05	1.410E-04	3.700E-05	8.349E-08	6.712E-06
1.952E-07	8.305E-06	2.306E-06	1.047E-05	2.643E-08	1.176E-05
9.735E-08	3.314E-06	9.211E-08	4.322E-07	5.100E-09	1.826E-05
9.729E-07	1.236E-04	1.083E-06	8.867E-06	9.126E-08	7.943E-04
2.090E-05	3.321E-04	1.611E-05	2.406E-05	3.678E-06	1.083E-05
1.282E-05	9.235E-04	3.443E-05	4.510E-06	6.873E-06	4.397E-07
1.476E-05	2.155E-03	1.239E-04	1.227E-05	2.338E-05	1.081E-05
1.463E-03	1.035E-02	1.056E-03	8.889E-05	1.822E-04	2.514E-04

Eigenvalue 11

2.468E-15	7.312E-28	1.235E-14	2.795E-15	5.404E-17	5.949E-14
5.531E-17	5.871E-30	3.405E-18	1.332E-17	2.883E-19	1.756E-15
2.943E-17	2.499E-30	1.451E-19	5.868E-19	5.934E-20	2.909E-15
2.219E-16	7.030E-29	1.287E-18	9.082E-18	8.011E-19	9.546E-14
3.781E-13	2.609E-14	1.214E-15	1.103E-14	1.171E-14	2.216E-15
3.908E-15	1.222E-15	4.370E-17	3.483E-17	3.687E-16	1.516E-18
4.799E-15	3.043E-15	1.678E-16	1.011E-16	1.338E-15	3.978E-17
3.589E-13	1.102E-14	1.079E-15	5.526E-16	7.867E-15	6.979E-16

Eigenvalue 12

1.996E-05	6.313E-05	4.391E-04	9.903E-05	4.152E-07	1.089E-03
2.879E-07	3.262E-07	7.789E-08	3.037E-07	1.425E-09	2.069E-05
3.062E-08	2.776E-08	6.635E-10	2.675E-09	5.865E-11	6.854E-06
4.315E-07	1.459E-06	1.100E-08	7.737E-08	1.480E-09	4.203E-04
3.393E-03	2.001E-04	1.103E-05	5.163E-05	6.107E-05	8.750E-06
2.257E-05	6.034E-06	2.557E-07	1.049E-07	1.237E-06	3.853E-09
5.540E-06	3.003E-06	1.963E-07	6.090E-08	8.975E-07	2.021E-08
7.742E-04	2.033E-05	2.358E-06	6.219E-07	9.862E-06	6.626E-07

TABLE 10.- CLOSED-LOOP EIGENVALUES FOR DIFFERENT β

Re(λ)	Im(λ)	Re(λ)	Im(λ)	Re(λ)	Im(λ)
(a) β = 0.		(b) β = 0.01		(c) β = 0.1	
-5.329 E-02	0.	-5.302 E-02	0.	-4.385 E-02	0.
-7.954 E-01	7.665 E-01	-7.987 E-01	7.623 E-01	-9.374 E-01	6.938 E-01
-7.954 E-01	-7.665 E-01	-7.987 E-01	-7.623 E-01	-9.374 E-01	-6.938 E-01
-1.540 E+00	0.	-1.532 E+00	0.	-1.507 E+00	0.
-1.213 E+00	1.135 E+00	-1.208 E+00	1.139 E+00	-1.114 E+00	1.207 E+00
-1.213 E+00	-1.135 E+00	-1.208 E+00	-1.139 E+00	-1.114 E+00	-1.207 E+00
-3.430 E+00	4.169 E+00	-3.431 E+00	4.166 E+00	-3.460 E+00	3.993 E+00
-3.430 E+00	-4.169 E+00	-3.431 E+00	-4.166 E+00	-3.460 E+00	-3.993 E+00
-7.212 E+00	0.	-7.197 E+00	0.	-7.230 E+00	0.
-2.693 E+01	0.	-2.692 E+01	0.	-2.726 E+01	0.
-3.125 E+01	0.	-3.125 E+01	0.	-3.125 E+01	0.
-3.135 E+01	0.	-3.134 E+01	0.	-3.131 E+01	0.
(d) β = 0.5		(e) β = 1.0		(f) β = ∞	
-5.123 E-02	0.	-2.191 E-03	0.	2.941 E-02	4.471 E-02
-6.418 E-01	6.968 E-01	-3.708 E-02	0.	2.941 E-02	-4.471 E-02
-6.418 E-01	-6.968 E-01	-4.465 E-01	0.	-9.664 E-02	0.
-1.394 E+00	0.	-9.264 E-01	0.	-4.443 E-01	0.
-8.750 E-01	1.308 E+00	-8.106 E-01	1.314 E+00	-8.106 E-01	1.314 E+00
-8.750 E-01	-1.308 E+00	-8.106 E-01	-1.314 E+00	-8.106 E-01	-1.314 E+00
-3.719 E+00	4.599 E+00	-7.259 E+00	0.	-1.757 E+00	4.174 E+00
-3.719 E+00	-4.599 E+00	-1.294 E+00	7.374 E+00	-1.757 E+00	-4.174 E+00
-7.212 E+00	0.	-1.294 E+00	-7.374 E+00	-7.259 E+00	0.
-2.725 E+01	0.	-2.725 E+01	0.	-2.725 E+01	0.
-3.125 E+01	0.	-3.125 E+01	0.	-3.125 E+01	0.
-3.141 E+01	0.	-3.138 E+01	0.	-3.139 E+01	0.

TABLE 11.- CLOSED-LOOP EIGENVALUES WITH GAIN ELEMENT (1,3) REMOVED

Subsystem	Re(λ)	Im(λ)	ω	ζ	$t_{1/2}$ or t_2
Airframe	-5.328 E-02	0.			1.301 E+01
Airframe	-9.024 E-01	6.250 E-01	1.098 E+00	8.221 E-01	7.681 E-01
Airframe	-9.024 E-01	-6.250 E-01	1.098 E+00	8.221 E-01	7.681 E-01
Engine	-1.535 E+00	0.			4.516 E-01
Engine	-1.113 E+00	1.149 E+00	1.600 E+00	6.958 E-01	6.228 E-01
Engine	-1.113 E+00	-1.149 E+00	1.600 E+00	6.958 E-01	6.228 E-01
Airframe	-7.404 E-01	5.607 E+00	5.656 E+00	1.309 E-01	9.362 E-01
Airframe	-7.404 E-01	-5.607 E+00	5.656 E+00	1.309 E-01	9.362 E-01
Engine	-7.207 E+00	0.			9.618 E-02
Inlet	-2.694 E+01	0.			2.573 E-02
Inlet	-3.125 E+01	0.			2.218 E-02
Inlet	-3.134 E+01	0.			2.212 E-02

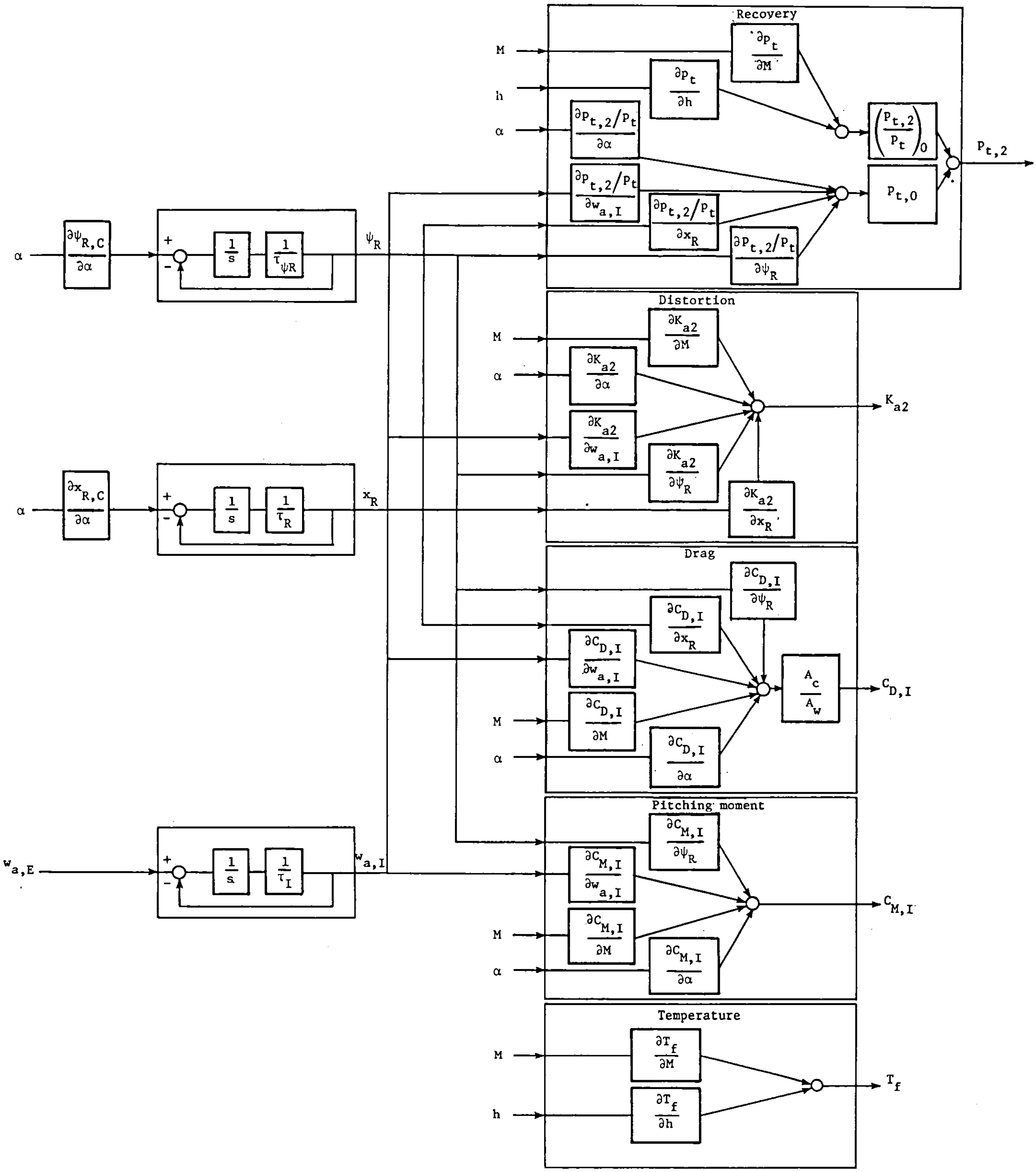

Figure 1.- Inlet model.

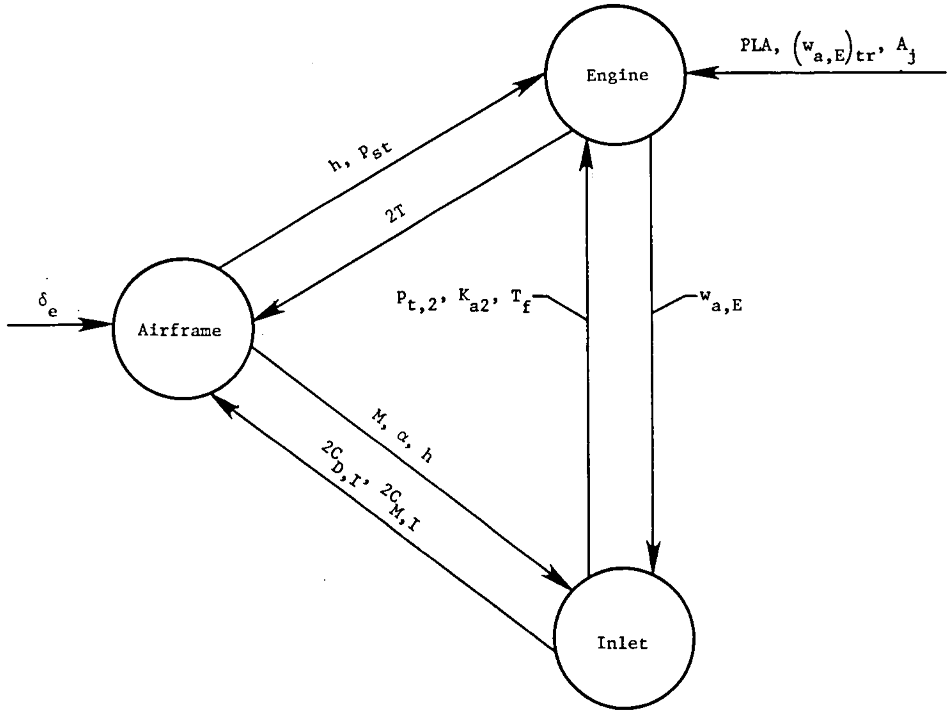

Figure 2.- Input/output relations.

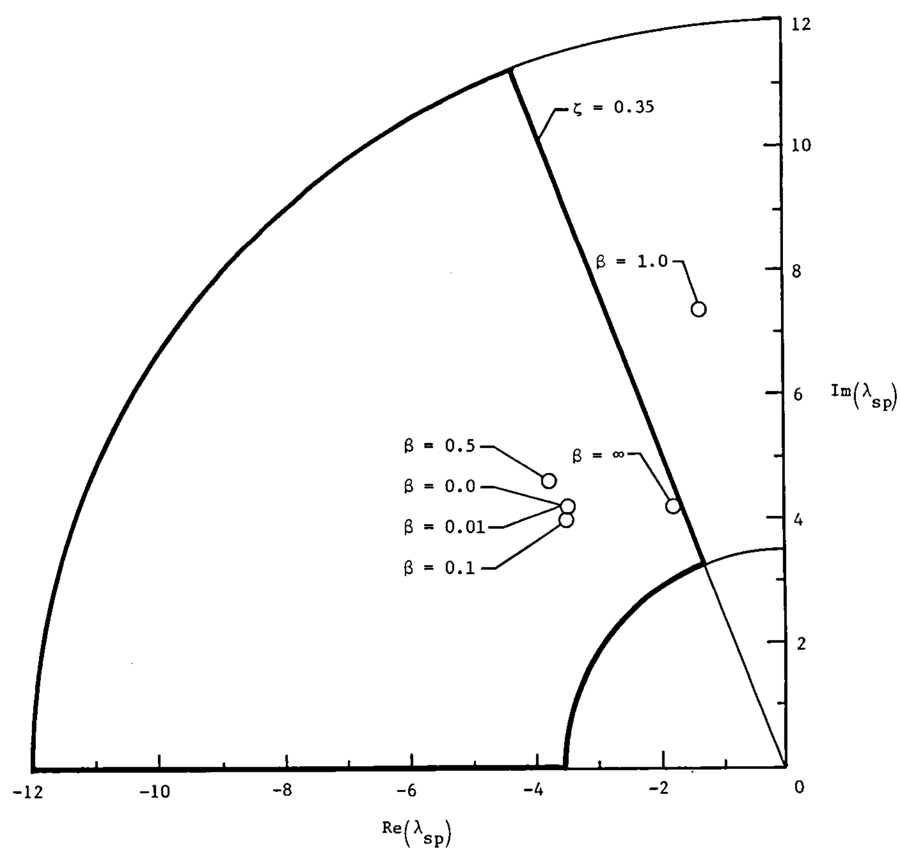

Figure 3.- Locus of short-period mode for varying β. Area inside dark boundary meets requirements of reference 6.

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15. Supplementary Notes

16. Abstract

A longitudinal model is formulated for an advanced fighter from three subsystem models: the inlet, the engine, and the airframe. Notable interaction is found in the coupled system. A procedure, based on eigenvalue sensitivities, is presented which indicates the importance of the feedback gains to the optimal solution. This allows ineffectual gains to be eliminated; thus, hardware and expense may be saved in the realization of the physical controller.

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