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DESIGN OF A HELICOPTER AUTOPILOT BY MEANS OF LINEARIZING TRANSFORMATIONS

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SUMMARY

A method for designing automatic flight control systems for aircraft that have complex characteristics and operational requirements, such as the powered-lift STOL and V/STOL configurations, is presented. The method is effective for a large class of dynamic systems that require multiaxis control and that have highly coupled nonlinearities, redundant controls, and complex multidimensional operational envelopes. The method exploits the possibility of linearizing the system over its operational envelope by transforming the state and control. The linear canonical forms used in the design are described, and necessary and sufficient conditions for linearizability are stated. The control logic has the structure of an exact model follower with linear decoupled model dynamics and possibly nonlinear plant dynamics. The design method is illustrated with an application to a helicopter autopilot design.

1. INTRODUCTION

Consider in general terms the control-system design problem. Let us take the usual hardware/software model of the problem in which the hardware consists of the plant together with all the sensors and actuators which are connected to a digital computer, and in which the software embodies the complete control strategy. The hardware is fixed; we may change only the software, over which, however, we have full control. So one may say that since the underlying physical process is to remain fixed, only its representation may be changed. If this point of view is taken, then much of the control-system design problem may be interpreted in terms of transformations.

This paper outlines a design approach that is being developed from the transformations point of view, and describes an application to the control of a helicopter. This approach, first outlined in Ref. 1, has been applied to several aircraft of increasing complexity, and the completely automatic flight-control system was first tested on a DHC-6. The reference trajectory used in the flight test exercised a substantial part of the operational envelope of the aircraft. Despite disturbances and variations in plant dynamics, the system performed well (see Ref. 2). Next, the technique was applied to the Augmentor Wing Jet STOL Research aircraft, the successful flight tests of which are reported in Ref. 3. Methods for providing pilot inputs to this design were examined in Ref. 4, and application of the scheme to the control of an A-7 aircraft for carrier landing and testing in manned simulation is reported in Refs. 5 and 6. The design method is currently being applied to the UH-1H helicopter, again with the substantial portion of the operational envelope of this aircraft being used.

The key concept of the approach is to simplify the representation of the plant dynamics by means of a change of coordinates of the state and control. The design proceeds in three steps. First, the given nonlinear system – possibly time-varying, multiaxis, and cross-coupled – is transformed into a constant, decoupled linear representation. Second, standard linear and nonlinear design techniques, such as Bode plots, pole placement, LQG, and phase plane, are used to design a control law for this simple representation. And third, the resulting control law is transformed back out into the original coordinates to obtain the control law in terms of the available controls.

The mathematical foundation for the approach is provided by modern differential geometry, the necessary and sufficient conditions for linearizing have been established, and the theory is given in Refs. 7-15.

2. CANONICAL FORM

Suppose that the given natural representation S_1 of the physical process is given by the state x_1, control u_1, and field f_1,

$$\dot{x}_1 = f_1(x_1,u_1) \tag{1}$$

and that we wish to change S_1 into S_2 with state x_2, control u_2, and field f_2,

$$\dot{x}_2 = f_2(x_2,u_2) \tag{2}$$

This change will be accomplished, and hence a large part of the design problem solved, once we construct the appropriate transformations of the state and control,

$$x_2 = T(x_1) \tag{3a}$$

$$u_1 = W(x_1,u_2) \tag{3b}$$

which relate S_1 to S_2 so that for all admissible (x_1,u_2),

$$\frac{\partial T}{\partial x_1}(x_1)f_1[x_1,W(x_1,u_2)] = f_2[T(x_1),u_2] \tag{4}$$

The function W is the control law.

The construction of the transformation (T,W) is often greatly simplified by the introduction of an intermediate canonical representation S_0 as shown in Fig. 1. To obtain (T_{12},W_{12}), which links S_1 to S_2, both S_1 and S_2 are first transformed into the canonical representation S_0. Then

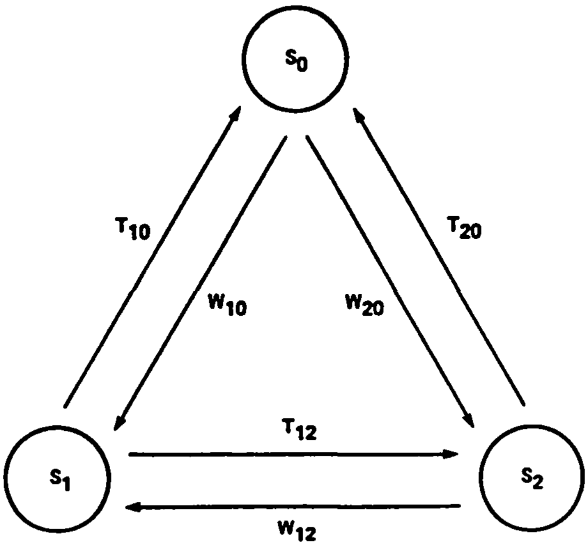

Fig. 1. Manipulation of system representations.

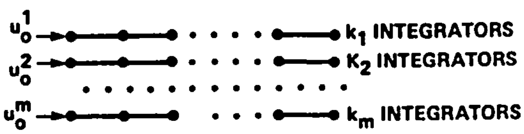

Fig. 2. Diagram of Brunovsky form.

$$T_{12} = T_{20}^{-1} T_{10} \tag{5a}$$

$$W_{12} = W_{10} W_{20}^{-1} \tag{5b}$$

In the design procedure being described, the Brunovsky form (Ref. 14) is taken to be canonical. In linear theory this form is basic. It consists of decoupled strings of integrators which may be diagrammed as shown in Fig. 2 where each dot represents a scalar integrator.

The number of strings, which may be of different lengths, equals the number of controls which itself equals the number of Kronecker indexes $k_1 \geq k_2 \geq . . . \geq k_m$. The lengths of the ith string is given by k_i and the dimension of the state space

$$n = \sum_{i=1}^{m} k_i .$$

Let the canonical state $x_0 \in R^n$ control $u_0 \in R^m$, and denote the canonical field f_0 by

$$\dot{x}_0 = A_0 x_0 + B_0 u_0 \tag{6}$$

According to linear theory any constant, linear, controllable system may be viewed as a nonsingular transformation of an appropriate Brunovsky form. Thus, if S_1 is given by the state $x_1 \in R^n$, control $u_1 \in R^m$, and field

$$\dot{x}_1 = A_1 x_1 + B_1 u_1 \tag{7}$$

with controllable (A_1, B_1), then there is a Brunovsky form that can be transformed into S_1 by means of nonsingular transformations (T,W nonsingular)

$$x_1 = T^{-1} x_0 \tag{8a}$$

$$u_1 = W u_0 + Q x_1 \tag{8b}$$

Of course, S_0 may also be transformed into nonlinear and time-varying systems without any loss of information by allowing the transformations in Eq. (8) to be nonlinear and time-varying but still nonsingular:

$$x_1 = T^{-1}(x_0, t) \tag{9a}$$

$$u_1 = W(x_1, u_0, t) \tag{9b}$$

This fact gives rise to our design procedure. When presented with a nonlinear system S_1, the first step is to try to linearize the system over its whole operational envelope by constructing (T,W) which maps S_1 into S_0. Then a control law is synthesized for the much simpler S_0. Finally, the S_0 control law is transformed into the coordinates of S_1 to obtain the control law for S_1.

3. TRANSFORMABILITY

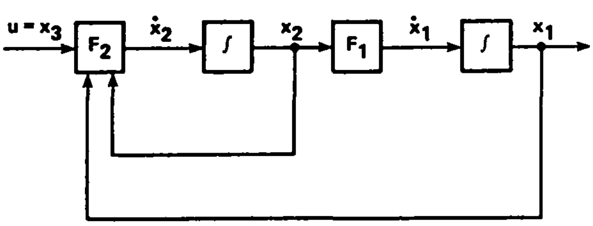

Fig. 3. An example of a block triangular system.

A class of systems particularly amenable to this approach has the following form. The control $u \in R^m$, the state $x \in R^m \times R^m \times . . . \times R^m$ and the field f is without transmission zeroes and invertible. An example is shown in Fig. 3. In this example, the state $x \in R^m \times R^m$, the control $u \in R^m$, and the field f,

$$\left.\begin{aligned} \dot{x}_1 &= F_1(x_1, x_2, t) \\ \dot{x}_2 &= F_1(x_1, x_2, u, t) \end{aligned}\right\} \tag{10}$$

and F_1 and F_2 are invertible relative to $(x_2, \dot{x}_1)$ and $(u, \dot{x}_2)$. That is, functions h_1 and h_2 can be constructed so that if

$$\left.\begin{aligned} x_2 &= h_1(x_1, \dot{x}_1, t) \\ u &= h_2(x_1, x_2, \dot{x}_2, t) \end{aligned}\right\} \tag{11}$$

then

$$\left.\begin{aligned} F_1[x_1, h_1(x_1, \dot{x}_1, t), t] &= \dot{x}_1 \\ F_2[x_1, x_2, h_2(x_1, x_2, \dot{x}_2, t), t] &= x_2 \end{aligned}\right\} \tag{12}$$

on the operational envelope. Because of the form of Eq. (10), such systems will be called block-triangular.

For this example, the canonical form S_0 has m Kronecker indexes, all equal to 2. The state $x^\circ \in R^m \times R^m$, the control $u^\circ \in R^m$, and the field,

$$\left.\begin{aligned} \dot{x}_1^\circ &= x_2^\circ \\ \dot{x}_2^\circ &= u^\circ \end{aligned}\right\} \tag{13}$$

The map linking Eq. (10) with Eq. (13) may be obtained by letting $x_1 = x_1^\circ(t)$ and pushing, as it were, the time-history $x_1^\circ(t)$ upstream through f to obtain u(t). Thus,

$$\left.\begin{aligned} x_1 &= x_1^\circ \\ \dot{x}_1 &= \dot{x}_1^\circ = x_2^\circ \\ x_2 &= h_1(x_1^\circ, x_2^\circ, t) \\ \dot{x}_2 &= \frac{\partial h_1}{\partial x_1} x_2^\circ + \frac{\partial h_1}{\partial x_2} u^\circ + \frac{\partial h_1}{\partial t} \\ u &= h_2(x_1^\circ, x_2^\circ, \dot{x}_2, t) \end{aligned}\right\} \tag{14}$$

In general, the canonical form S_0 of a block triangular S_1 will have m Kronecker indexes, all equal to n/m where n is the dimension of the state space of S_1.

This procedure of constructing the linearizing transformations will fail if S_1 does have transmission zeroes. In that case, one obtains differential equation constraints on u, thereby destroying its status as an independent control variable. Nevertheless, such systems may still be linearizable. For example, the scalar system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -x_3 \\ 0 \\ 1 \end{pmatrix} u \tag{15}$$

is linearized by the transformation

$$\left.\begin{aligned} x_1^\circ &= x_1 + \frac{1}{2}(x_3)^2 \\ x_2^\circ &= x_2 \\ x_3^\circ &= x_3 \\ u &= u^\circ \end{aligned}\right\} \tag{16}$$

On the other hand it will be shown that the system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -x_2 \\ 0 \\ 1 \end{pmatrix} u \tag{17}$$

is not linearizable; the nonlinearity in this case is intrinsic and cannot be removed by a change of coordinates. The conditions under which linearization is possible are summarized next.

Let $x \in R^n$, $u \in R^m$, and the field

$$\dot{x} = F(x,u) \tag{18}$$

There are four conditions for this system to be linearizable. First, it is necessary to be able to construct a new control variable, ϕ,

$$\begin{aligned} \phi &= h^{-1}(x,u) \\ u &= h(x,\phi) \end{aligned} \tag{19}$$

so that ϕ enters linearly into the field:

$$F[x,h(x,\phi)] = f(x) + \sum_{i=1}^{m} g_i(x)\phi_i \tag{20}$$

The remaining three conditions are technical, and they are best expressed by means of Lie brackets defined as follows. If f and g are C^∞ vector fields on R^n, the Lie bracket of f and g is

$$[f,g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g \tag{21}$$

and we set

$$(ad^0 f,g) = g$$
$$(ad^1 f,g) = [f,g]$$
$$(ad^2 f,g) = [f,[f,g]]$$
$$\vdots$$
$$(ad^k f,g) = [f,(ad^{k-1} f,g)]$$

A collection of C^∞ vector fields $h_1, h_2, \ldots, h_r$ on R^n is involutive if there exist C^∞ functions γ_{ijk} with

$$[h_i, h_j] = \sum_{k=1}^{r} \gamma_{ijk} h_k \,, \quad 1 \le i \,, \quad j \le r \,, \quad i \ne j$$

Now, suppose that we wish to transform Eq. (20) into S_0 with Kronecker indexes $k_1 \ge k_2 \ge \ldots \ge k_m$. Define the sets

$$\begin{aligned} C = \{&g_1, [f_1 g_1], \ldots, (ad^{k_1-1} f_1 g_1), \\ &g_2, [f_1 g_2], \ldots, (ad^{k_2-1} f_1 g_2), \\ &\text{-----------------} \\ &g_m, [f_1 g_m], \ldots, (ad^{k_m-1} f_1 g_m)\} \end{aligned}$$

$$\begin{aligned} C_j = \{&g_1, [f_1 g_1], \ldots, (ad^{k_j-2} f_1 g_1), \\ &g_2, [f_1 g_2], \ldots, (ad^{k_j-2} f_1 g_2), \\ &\text{-----------------} \\ &g_m, [f_1 g_m], \ldots, (ad^{k_j-2} f_1 g_m)\} \quad \text{for} \quad j = 1, 2, \ldots, m \end{aligned}$$

Then it can be shown that the transformation is possible if and only if at each admissible x,

1. The set of C spans an n-dimensional space
2. Each C_j is involutive for $j = 1, 2, \ldots, m$
3. The span of C_j equals the span of $C_j \cap C$ for $j = 1, 2, \ldots, m$

For a linear field, $\dot{x} = Ax + Bu$, the spanning condition (1) or C is equivalent to controllability, rank $(B, AB, \ldots, A^{n-1}B) = n$. The other conditions are automatically satisfied. The new coordinate surface $T_1(x)$ = constant in a plane through the origin in the old state space. For nonlinear field, $T_1(x)$ = constant will be a general surface. The involutivity condition (2) guarantees that this surface is constructible from local conditions (integrability theorem of Frobenius).

Suppose we wish to transform system (17) into S_0 with $k = 3$. Here the set $C = \{g, [f,g], (ad^2 f, g)\}$ spans R^3. But the $C_1 = \{g, [f,g], [g,[f,g]]\}$ also spans R^3; therefore, system (17) is not transformable into S_0. For further details on the transformation theory see Refs. 7-15. Let us turn our attention next to the control structure in which these ideas of transformability may be implemented in practical cases.

4. EXACT MODEL FOLLOWER

If the plant S_1 is equivalent to a system S_2 in the sense that each can be transformed into the other by nonsingular transformations, then one may construct an exact model follower in which the plant S_1 will, except for disturbances, follow exactly the system S_2, which is interpreted as being the model. Let the system representing the tracking error be denoted by S_3. The three systems, are related to each other and to the Brunovsky form S_0 as shown in Fig. 4.

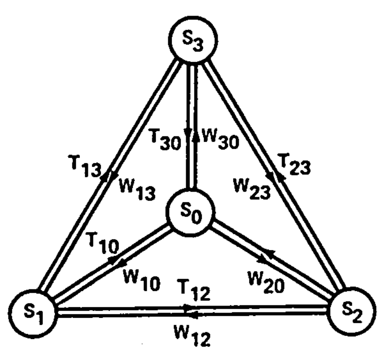

Fig. 4. Plant S_1, model S_2, and regulator S_3.

Now consider the structure of the model follower as shown in Fig. 5. There are five subsystems: model servo, regulator, plant, and two transformations. The desired system behavior is defined in the model servo. The model servo law may be nonlinear, time-varying, and dynamic. The TW-map (T_{23},W_{23}) transforms the S_2 system

$$\dot{x}_2^m = f_2(x_2^m,u_2^m) \qquad (22)$$

into S_3,

$$\dot{x}_3^m = f_3(x_3^m,u_3^m) \qquad (23)$$

The TW-map (T_{13},W_{13}) transforms the plant S_1

$$\dot{x}_1 = f_1(x_1,u_1) \qquad (24)$$

into S_3

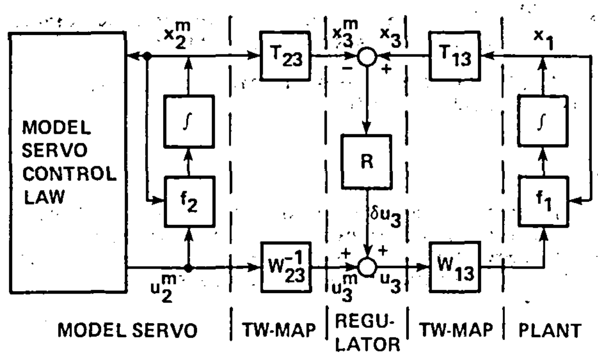

Fig. 5. Structure of the model follower.

$$\dot{x}_3 = f_3(x_3,u_3) \qquad (25)$$

That is, when seen through the TW-map (T_{13},W_{13}) in terms of (x_3,u_3), the plant looks like Eq. (25); which is also how the model looks through the TW-map (T_{23},W_{23}).

In the absence of disturbances and with proper initialization of the model (i.e., $x_2^m(0) = T_{12}[x_1(0)]$), the plant will follow the model exactly in the sense that the error defined by

$$e_3 = x_3 - x_3^m \qquad (26)$$

will be zero for $t \geq 0$. In a realistic situation, disturbances, say, d, are present, and they are controlled by means of the regulator which transforms the tracking error e_3 into corrective control δu_3. The error dynamics are given by

$$\dot{e}_3 = f_3(x_3,u_3) - f_3(x_3^m,u_3^m) \qquad (27)$$

which for small errors simplifies to

$$\dot{e} = \frac{\partial f_3}{\partial x_3} e_3 + \frac{\partial f_3}{\partial u_3} \delta u_3 + d$$

In particular, if S_3 is chosen to coincide with S_0, so that $f_3 = f_0$, then without approximation,

$$\dot{e}_0 = A_0 e_0 + B_0 \delta u_0 + d \qquad (28)$$

It may be noted that the regulator need not be gain-scheduled; that function is accomplished automatically by the TW-map.

This completes the outline of the design approach. Consider next an application to a helicopter.

5. THE PLANT-A HELICOPTER

The helicopter will be represented by a rigid body moving in three-dimensional space in response to gravity, aerodynamics, and propulsion. The state,

$$x = (r,v,C,\omega)^T \in X \subset R^3 \times R^3 \times SO(3) \times R^3 \qquad (29)$$

where r and v are the inertial coordinates of body center-of-mass position and velocity, respectively, and C is the direction cosine matrix of the body-fixed axes relative to the runway-fixed axes (taken to be inertial). The attitude C moves on the sphere SO(3). The body coordinates of angular velocity are represented by ω.

The controls,

$$u = (u^M,u^P)^T \in U \subset R^3 \times R \qquad (30)$$

where u^M is the three-axis moment control, that is, roll cyclic and pitch cyclic, which tilt the main-rotor thrust, and the tail-rotor collective, which controls the yaw moment; and u^P is the main-rotor collective, which controls the main-rotor thrust.

The effectively 12-dimensional state equation consists of the translational and rotational kinematic and dynamic equations:

$$\left.\begin{aligned} \dot{r} &= v \\ \dot{v} &= f^F(x,u) \\ \dot{C} &= S(\omega)C \\ \dot{\omega} &= f^M(x,u) \end{aligned}\right\} \qquad (31)$$

where f^F and f^M are the total force and moment generation processes, and (x,u) are defined by Eqs. (29) and (30). This is the natural representation S_1 of the helicopter.

Consider next the transformation of Eqs. (31) into a Brunovsky form S_0 by means of the transformations $u = W(x,u_0)$ and $x_0 = T(x)$.

6. THE TW-MAP

In general, the moment generation process f^M is invertible with respect to the pair $(\dot{\omega},u^M)$, and for the restricted class of maneuvers being considered in this experiment (i.e., no 360° rolls), f^F is invertible with respect to the pair $(\dot{v}_3,u^P)$. Thus, in the present case for the set of angular and vertical acceleration commands restricted to the set

$$U_{\dot{\omega}} = \{(\dot{\omega},\dot{v}_3) : |\dot{\omega}_i| \le 1.0 \text{ rad/sec}^2, \quad i = 1,2,3, \ |\dot{v}_3| \le 0.5 \text{ g}\} \tag{32}$$

a function $h^M : X \times U_{\dot{\omega}} \to U$ can be constructed so that if

$$u = h^M[x,(\dot{\omega}_0,\dot{v}_{30})] \tag{33}$$

then

$$\left.\begin{aligned} \dot{\omega} &= \dot{\omega}_0 \\ \dot{v}_3 &= \dot{v}_{30} \end{aligned}\right\} \tag{34}$$

If $(\dot{\omega}_0,\dot{v}_{30})$ are chosen to be the new independent control variables to replace the natural controls (u^M,u^P), then the state equation (31) becomes the following:

$$\left.\begin{aligned} \dot{r} &= v \\ \begin{pmatrix}\dot{v}_1\\ \dot{v}_2\end{pmatrix} &= f^\circ(r,v,C) + \epsilon\, f^1[r,v,C,\omega,(\dot{\omega}_0,\dot{v}_{30})] \\ \dot{v}_3 &= \dot{v}_{30} \\ \dot{C} &= S(\omega)C \\ \dot{\omega} &= \dot{\omega}_0 \end{aligned}\right\} \tag{35}$$

where $\epsilon = 1$ and f^1 is such that $f^1[r,v,C,0,(0,0)] = 0$.

The function f° is invertible with respect to the pair, $\{[\dot{v}_1,\dot{v}_2,E_3(\psi)],C\}$ in which $E_3(\psi)$ is an elementary rotation about the runway z-axis, representing the heading of the helicopter.

$$E_3(\psi) = \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{36}$$

If the horizontal acceleration commands are restricted to the set

$$U_{\dot{v}} = \{(\dot{v}_1,\dot{v}_2) : |\dot{v}_i| \le 0.5 \text{ g}, \quad i = 1,2\} \tag{37}$$

then a function $h^F : R^3 \times R^3 \times U_{\dot{v}} \times SO(2) \to SO(3)$ can be constructed so that the helicopter attitude given by

$$C_0 = h^F[r,v,\dot{v}_0,E_3(\psi_0)] \tag{38}$$

results in the commanded acceleration,

$$\dot{v} = \dot{v}_0 \tag{39}$$

Equations (33) and (38) are the trim equations of the helicopter (31) without the parasitic effects ($\epsilon = 0$ in Eq. (35)). That is, for a given motion $\{r(t),E_3[\psi(t)]\}, t \ge 0$, the corresponding trim state and control may be computed as follows:

$$\left.\begin{aligned} r_0 &= r(t) \\ v_0 &= \dot{r}(t) \\ C_0 &= h^F\{r_0,v_0,\dot{v}_0(t),E_3[\psi_0(t)]\} \\ \omega_0 &= q[\dot{C}_0(t)C_0^T] \\ \dot{\omega}_0 &= \dot{\omega}_0(t) \\ u_0 &= h^M[r_0,v_0,C_0,\omega_0,(\dot{\omega}_0,\dot{v}_{30})] \end{aligned}\right\} \tag{40}$$

where the function q extracts ω from $S(\omega) = \dot{C}C^T$. The required time derivatives in Eq. (40) are computable provided that the motion (r,E_3) is generated by the strings of integrators shown in Fig. 6 where a dot represents a scalar integrator, and $y^5 \in R^4$ is an independent control variable. The system shown in Fig. 6 (Kronecker indexes {4,4,2,2}) will be taken as the canonical model of the helicopter. The canonical variables will be denoted by y rather than x_0 to reduce the number of subscripts. The transformation and feedback that change the natural representation (Eq. (31)) to the canonical representation are approximately the following.

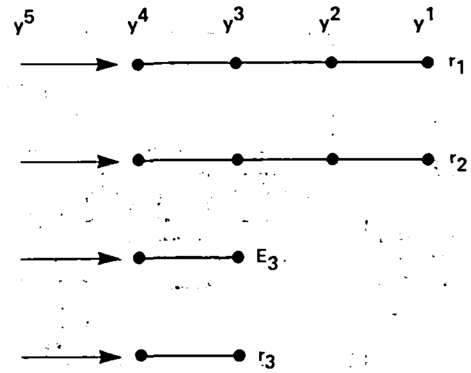

Fig. 6. Canonical model of helicopter.

The coordinate change, $y = T(x)$, is given by

$$\left.\begin{aligned} y^1 &= (y_1^1,y_2^1)^T = (r_1,r_2)^T \\ y^2 &= (y_1^2,y_2^2)^T = (v_1,v_2)^T \\ y^3 &= (y_1^3,y_2^3,y_3^3,y_4^3)^T = (f_1^\circ,f_2^\circ,E_3(\psi),r_3)^T \\ y^4 &= (y_1^4,y_2^4,y_3^4,y_4^4)^T = \left(\frac{\partial f^\circ}{\partial C}\,\omega,\omega_3,r_3\right)^T \end{aligned}\right\} \quad (41)$$

where

$$\cos\psi = c_{11}/c,\ \sin\psi = c_{12}/c,\ c = (c_{11}^2 + c_{12}^2)^{1/2},\ \text{and}\ (c_{ij}) = C\,.$$

The control variable change, $u = w(x,y^5)$ is defined in two steps:

$$\dot{\omega} = \begin{pmatrix} \dfrac{\partial f^\circ}{\partial C} \\ 0 \quad 0 \quad 1 \end{pmatrix}^{-1} \begin{pmatrix} y_1^5 \\ y_2^5 \\ y_3^5 \end{pmatrix} \qquad (42)$$

$$\dot{v}_3 = y_4^5$$

and

$$u = h^M[r,v,C,\omega,(\dot{\omega},\dot{v}_3)] \qquad (43)$$

The effects of the various approximations made in the construction of these transformations are relegated to the regulator.

7. MODEL SERVO AND REGULATOR

The field of the model S_2 is chosen to be canonical as defined in Fig. 6. The design of the three trajectory channels of the model servo is shown in Fig. 7. There are four 3-axis integrators. Two axes correspond to the horizontal strings (r_1,r_2) in Fig. 6. The third axis represents the vertical (r_3) string, but with two additional smoothing integrators. The exceedingly simple structure of the field (i.e., linear, decoupled) greatly simplifies the servo design process. The inner loop $(a_0,\dot{a}_0)$ is designed to be an acceleration servo whose input a_I is the sum of coarse acceleration command a^* from the coarse command generator, and δa_0 which is generated by the outer loop to smoothly reduce any discontinuities in the commanded position and velocity vectors, r^* and v^*, respectively. The acceleration servo bandwidth is 0.63 rad/sec. The bandwidth of the outer loop is 0.1 rad/sec. Large position errors are reduced at the rate of 6 m/sec.

The heading model servo is shown in Fig. 8. It is designed to have two scalar integrators corresponding to the E_3 string in Fig. 6. The heading error,

$$e_{\psi_0} = \cos\psi_0 \sin\psi^* - \sin\psi_0 \cos\psi^* \qquad (44)$$

is computed in the q-block. Block S represents, together with the integrator, the kinematic equation of E_3.

The model reference state y_0 and control y_0^5 in Fig. 5 are defined (see Figs. 6-8) as follows:

$$\left.\begin{aligned} y_0^1 &= (r_{10},r_{20})^T \\ y_0^2 &= (v_{10},v_{20})^T \\ y_0^3 &= [a_{10},a_{20},E_3(\psi_0),r_{30}]^T \\ y_0^4 &= (\dot{a}_{10},\dot{a}_{20},\omega_{30},v_{30})^T \\ y_0^5 &= (\ddot{a}_{10},\ddot{a}_{20},\dot{\omega}_{30},a_{30})^T \end{aligned}\right\} \qquad (45)$$

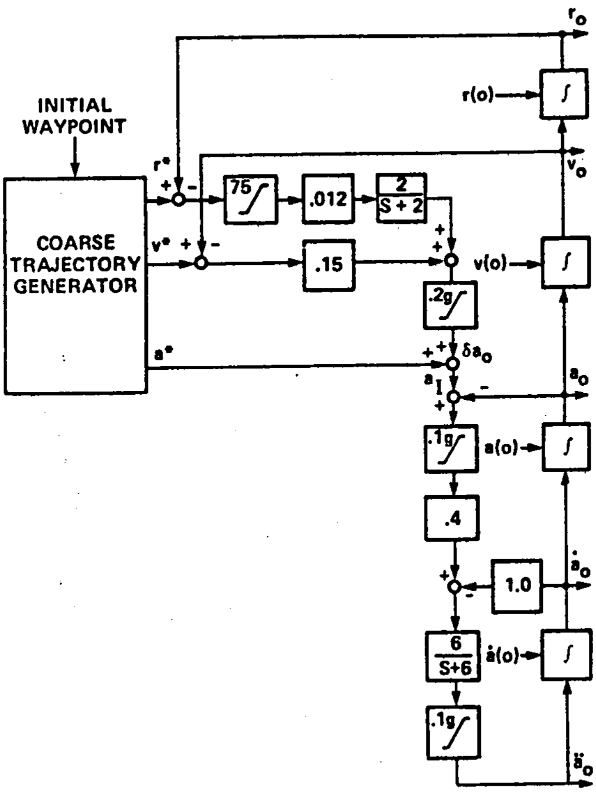

Fig. 7. Model servo-trajectory, $r = (r_1, r_2, r_3)$.

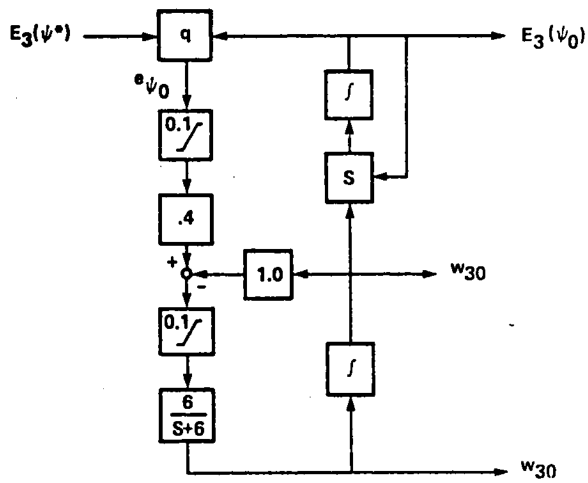

Fig. 8. Model servo-heading $E_3(\psi_0)$.

The field of the regulator S_3 is also chosen to be canonical S_0. The regulator design is outlined in Figs. 9-11.

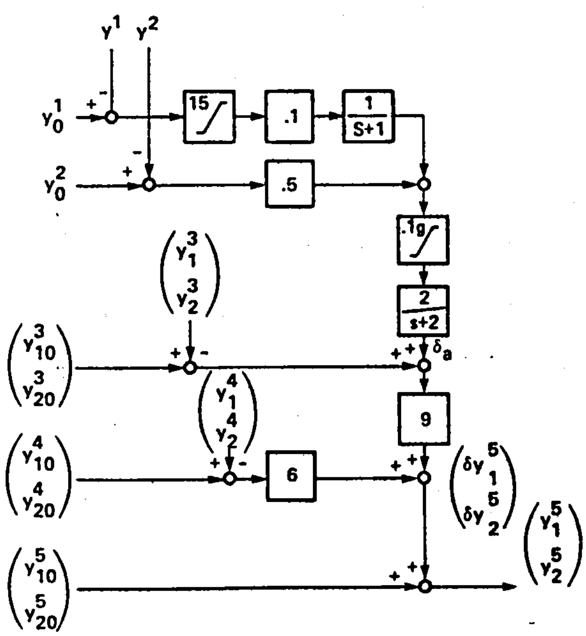

Fig. 9. Regulator-horizontal axes.

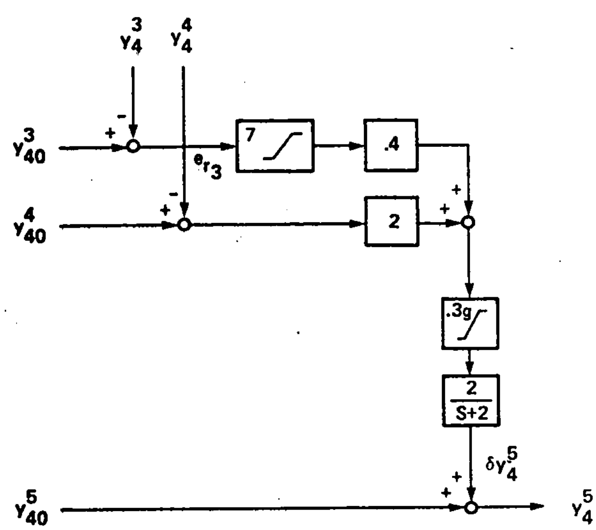

Fig. 10. Regulator — vertical channel.

The estimated canonical state y is computed from the estimated natural state by means of the transformation T defined by Eq. (41). The outer loop of the horizontal channels (Fig. 9) has a bandwidth of 0.3 rad/sec, with a 3 m/sec large-error reduction rate and 0.1-g authority limit. The inner loop bandwidth is 3 rad/sec. The vertical channel, shown in Fig. 10, has a bandwidth of 0.63 rad/sec, a 1.5-m/sec large-error reduction speed, and 0.3-g authority. Finally, the heading channel is regulated as shown in Fig. 11, where the heading error is given by

$$e_{\psi} = \cos \psi \sin \psi_0 - \sin \psi \cos \psi_0 \qquad (46)$$

The bandwidth of the heading regulator is 3 rad/sec.

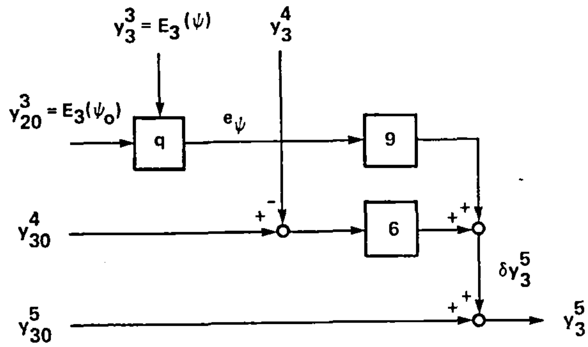

Fig. 11. Regulator — the heading channel.

The total regulator output, δy^5, is added to the open-loop command, y_0^5, resulting in the total canonical control, y^5. It is then transformed by means of the W-map given by Eqs. (42) and (43) into the natural control u which, in turn, drives the actual plant.

8. SYSTEM PERFORMANCE

The results of a manned simulation are summarized in this section. The code was implemented on the flight computer to be used in the flight test, and the mathematical model of the helicopter (UH-1H) was driven through the actual hydraulics.

The experimental flightpath is defined by a set of way points, segments of lines and helixes, and a speed profile, as shown in Figs. 12 and 13. As can be seen from Fig. 12, the flightpath is a closed curve. The time dependence is shown in Fig. 13.

The experiment, which consists of automatically flying this trajectory, exercises the system over a wide range of flight conditions. The helicopter is taken from hover (way-point 1 in Fig. 12) to high-speed (50 m/sec) accelerating, turning, and ascending flight. This input to the system is coarse, with a variety of discontinuities. The required smoothing is provided by the model servo discussed in the preceding section.

For the data presented, the helicopter was flown manually to the point ★ marked in Fig. 12. There the automatic system was engaged. It takes about 500 sec for the helicopter to go once around the flightpath. Unlike the coarse accelerations in Fig. 13, the model accelerations are smooth, as is the vertical velocity, v_{3O}. The second panel in Fig. 14 (labeled "acceleration error") shows the effects of the neglected parasitic terms on acceleration. The acceleration errors are quite small — less than 0.05 g. The regulator controls these effects by means of position errors. The resulting horizontal error is less than 2 m, and the vertical error is below 0.5 m. The speed error, $e_V = \|v\| - \|v_0\|$, is below 0.5 m/sec.

Thus, the performance of the design in the simulation tests was good, and, at this writing, flight tests are in progress.

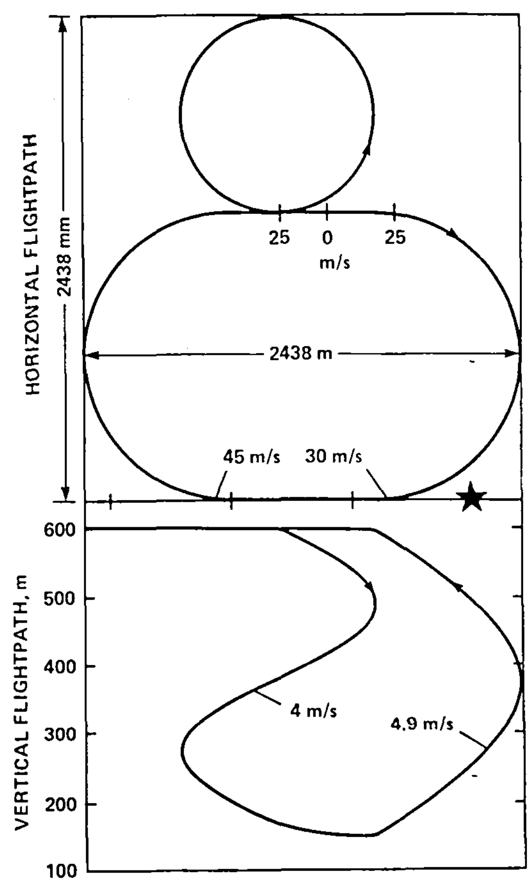

Fig. 12. Experimental flightpath shown in horizontal and vertical planes.

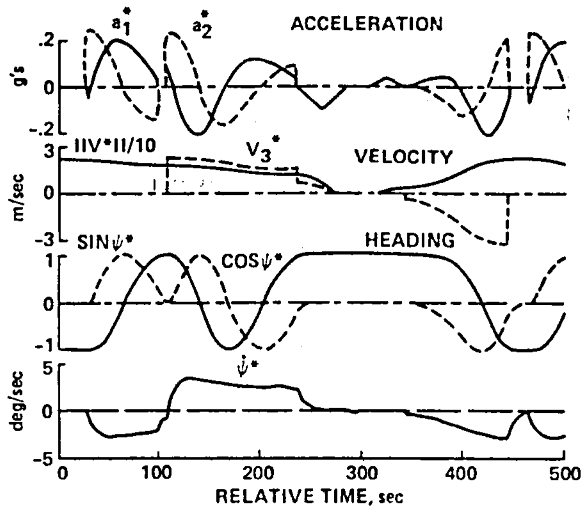

Fig. 13. Time-dependence of the coarse command.

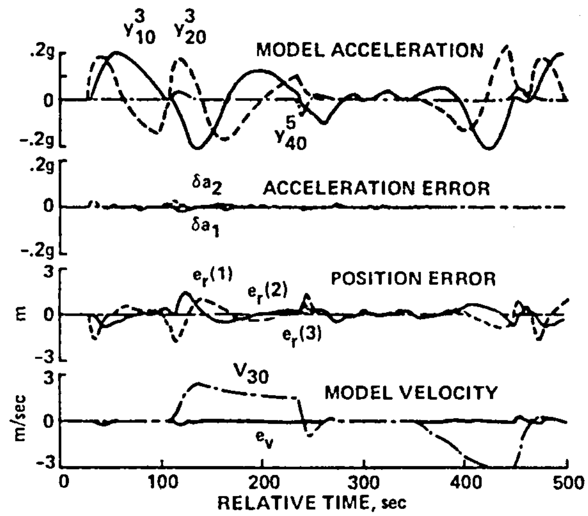

Fig. 14. System response — canonical variables.

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16. Abstract

A method for designing automatic flight control systems for aircraft that have complex characteristics and operational requirements, such as the powered-lift STOL and V/STOL configurations, is presented. The method is effective for a large class of dynamic systems that require multiaxis control and that have highly coupled nonlinearities, redundant controls, and complex multidimensional operational envelopes. The method exploits the possibility of linearizing the system over its operational envelope by transforming the state and control. The linear canonical forms used in the design are described, and necessary and sufficient conditions for linearizability are stated. The control logic has the structure of an exact model follower with linear decoupled model dynamics and possibly nonlinear plant dynamics. The design method is illustrated with an application to a helicopter autopilot design.

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