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(NASA-CR-169546) AT WHAT WAVELENGTHS SHOULD
WE SEARCH FOR SIGNALS FROM EXTRATERRESTRIAL
INTELLIGENCE? (SETI/INFRARED
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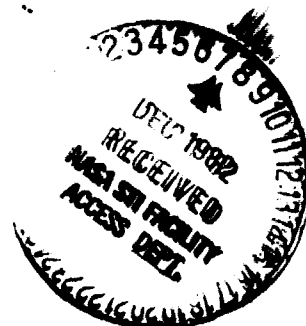
AT WHAT WAVELENGTHS SHOULD WE SEARCH
FOR SIGNALS FROM EXTRATERRESTRIAL INTELLIGENCE?

(SETI/infrared communication/
interstellar communication/extraterrestrial intelligence)

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ABSTRACT

It has often been concluded that searches for extraterrestrial intelligence (SETI) should concentrate on attempts to receive signals in the microwave region, the argument being given that communication can occur there at minimum broadcasted power. Such a conclusion is shown to result only under a restricted set of assumptions. If generalized types of detection are considered, in particular photon detection rather than linear detection alone, and if advantage is taken of the directivity of telescopes at short wavelengths, then somewhat less power is required for communication at infrared wavelengths than in the microwave region. Furthermore, a variety of parameters other than power alone may be chosen for optimization by an extraterrestrial civilization. Hence, while partially satisfying arguments may be given about optimum wavelengths for a search for signals from extraterrestrial intelligence, considerable uncertainty must remain.

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Introduction

The initial proposal (1) of a search for extraterrestrial intelligence (SETI) suggested the search take place in the microwave region, and at the 21 cm wavelength of the hydrogen hyperfine transition in particular. The substantial investment which may in the future be needed for such searches makes pertinent a skeptical review of whether the microwave region is so uniquely advantageous as to clearly be selected by an extraterrestrial civilization. The relative advantages of SETI at various wavelengths is hence examined. This appears to show that, while the microwave region is indeed favored under some sets of conditions, substantially shorter wavelengths can be advantageous under other conditions and hence cannot be ruled out of consideration if a broad search for extraterrestrial intelligence is undertaken.

SETI will be taken as a search for purposeful communication from an intelligent extraterrestrial civilization within a radius from the Earth which is small enough to be practical from a technical point of view but large enough to contain a substantial number of suitable stars where such civilizations might exist. We will thus not discuss the eavesdropping mode--listening to the leakage of local communications--which is both much more limited in range for a given effort and much more difficult to assess because it involves guesses about what stray radiation might exist. A radius of 100 light years provides a volume with approximately 1000 F and G stars; a radius of 1000 light years one with approximately 10^6 such stars.

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100-1000 L.Y. thus appears to be a desired range of radii. Techniques to be used in this enterprise will be assumed to be some reasonable extension of what we on Earth can presently do. Thus various gargantuan possibilities are ruled out, such as modulation of an x-ray star or of an interstellar maser which, if practical, would make such communication otherwise easy.

The first proposal (1) made the important point that our microwave technology is advanced enough to engage in powerful searches for broadcasts by intelligent beings on other planets, and since then some searches have been carried out. However, as additional important resonances were discovered in the microwave region, attention has broadened to include the H_2O resonance at 1.35 cm, and the microwave region more generally⁽²⁾.

General Principles and Strategy

In attempting to examine optimum wavelengths for SETI, we must of course ask with respect to what is the wavelength to be optimized. Unfortunately, there is no very clear-cut answer to such a question. It is attractive and common to single out the broadcast power required for successful communication as a parameter for optimization. Certainly total energy is one possibly important parameter, but it might not be very critical to another civilization and other parameters to be considered might include simplicity, the total amount of unusual materials required such as metals, difficulties of transmission through possible planetary atmospheres, or weather hazards to a broadcast installation such as wind and ice. For lack of any more precise principle, we shall use as a guide the minimizing of costs as measured by those our own civilization might face in a foreseeable future. This will of course include considerations of the total energy requirements as well as manufacturing and materials costs. We must, however, recognize that on another planet any other one parameter, including energy, might be very different in cost from what it is on Earth.

An important strategic question is whether a civilization wanting to communicate would broadcast an isotropic signal or one directed towards likely selected stars. For a directional broadcast, we assume here that a planetary system rather than an individual planet would be singled out, since the latter would require more directivity and precision than we can presently achieve at reasonable costs. The power received in such a communication can be expressed as

$$P_R = \frac{A_R P_B}{\Omega_B R^2} \quad (1a)$$

$$\text{or } P_R = \frac{A_R A_B P_B}{\lambda^2 R^2} \quad (1b)$$

where P_B is the broadcast power, A_R the receiver antenna area, A_B the broadcasting antenna area, Ω_B the broadcast solid angle, R the distance between broadcast and reception, and λ the wavelength. Expression (1b) assumes a diffraction limited broadcast and an approximately uniform excitation of the broadcast area used. Higher directivity may in principle be achieved with specialized excitations, but that seems likely a more expensive route than using simple excitations of larger areas, and hence has not been considered. To obtain an approximate magnitude of power needed for communication in an isotropic broadcast ($\Omega_B = 4\pi$), we assume a receiving antenna of diameter 100 meters and that an adequate signal might correspond to about one photon per second striking such an antenna. If the source were at 1000 L.Y. distance, this implies a broadcast power of $\frac{2 \times 10^{12}}{\lambda}$ Watts, where λ is in centimeters. For 100 L.Y., the power is of course 100 times smaller but it is still very high. By comparison, the small solid angle afforded by the diffraction limit of a 100 meter broadcasting antenna would allow the broadcaster to emit only $2 \times 10^3 \lambda$ Watts

to achieve the same photon density at the receiver. It thus appears reasonable to expect that a broadcaster would choose to send separate beams of energy towards a finite, though perhaps large, number of stars rather than use the enormous amount of power required for an isotropic signal. This choice would be particularly advantageous if multiplex systems are used so that multiple beam directions can be transmitted from a single antenna dish.

A second question of strategy is whether to attempt to eliminate frequency variation due to varying relative motion of the sender and receiver. The sender could easily know the variation of velocity of his own planet along the direction of the antenna beam and correct for it. Likewise, a receiver could easily correct for his own variation in velocity along the direct line of sight of a search. Hence, some correction for Doppler effects might well be adopted. However, there will still be some uncorrected Doppler effects and we will assume here that it is Doppler effects which determine the ultimate frequency bandwidths used.* This assumption implies that a comparison between the efficacy of different wavelengths does not in fact depend on whether some of the Doppler shifts are removed; it ensures that the bandwidth increases linearly with the broadcast frequency.

*This is different from the interesting suggestion of Drake and Helou (3) that bandwidths used should be limited only by scatter due to interstellar material, leading them to an optimum frequency for SETI, based on this and other assumptions, near 75 Gigahertz and a bandwidth of about 0.1 Hz. This implies that varying planetary velocities would be corrected to 0.03 cm/sec, or 5×10^{-9} that of the Earth's orbital velocity.

Comparison of Technology at Different Wavelengths

In comparing different wavelengths one must consider the nature of sources, antennas, detectors, and spectrum analyzers. It is clear that our civilization has had more experience with sources and detectors in the radio and microwave region than at some shorter wavelengths, such as the far infrared, although the difference in experience represents only a few decades and could easily be negligible in a somewhat older civilization. There seems also to be no a priori reason why electronic vacuum tubes and amplifiers were discovered before lasers, which are the intense sources we now know at shorter wavelengths. All of the basic physics for laser or maser oscillators was understandable by about 1917 when Einstein discussed stimulated emission, although certain coherence properties were not easily treatable until the quantum mechanics of the 1920's. This was of course the period of development of the vacuum tube, so that our own inventions of lasers and vacuum tubes could well have been almost simultaneous and their relative timing for another civilization may be somewhat arbitrary. We will hence assume that so far as power sources are concerned there is no necessary choice as a function of wavelength from the radio region down at least into the ultraviolet. Our detection of electromagnetic energy is perhaps best developed in the visible region, where we can come closer to the limit of detecting single photons than in the radio region. While at radio wavelengths we are now fairly close to the limit of single photon detection with maser amplifiers, on the surface of the Earth we miss this physical limit by one or two orders of magnitude. There are good detectors in some parts of the infrared region, but the quality of our detection technology at infrared wavelengths is very spotty and generally not fully developed. Nevertheless, there appears to be no basic reason why, with the use of cryogenics and suitable materials, appro-

appropriate quantum counting detectors or linear amplifiers cannot be produced throughout the infrared region. We therefore assume that the broadcasting civilization may have at its disposal detectors of sensitivity close to the ultimate limit dictated by the quantum properties of radiation over the whole range of wavelengths. We already have some considerable experience with antennas and spectrum analyzers throughout this region, and hence a comparison of the relative advantages of different wavelengths can probably be based on presently known technology so far as these two components are concerned. Multiplex use of antennas appears to be as easy at short wavelengths as in the microwave region, perhaps easier because the relative size of sources to the antenna diameter or focal length is smaller as the wavelength is decreased. Spectral analysis by gratings and multiple detectors in the short wavelength region need also not be enormously different in cost from multichannel spectrometers at radio frequencies.

At least one further element is important in any comparison of communication at various wavelengths, and that is transmission of the atmospheres of the two planets involved in any communication. Probably the atmospheric transparency of another distant planet cannot be very completely known. Some absorption by an ionosphere, by water vapor, and reasonably good transparency in the visible region if clouds are not present seem reasonable assumptions. However, it also seems reasonable that where transparency of the atmosphere is poor or uncertain, broadcast and reception from nearby space could be undertaken. We will hence assume that if needed, the use of space is to be expected, though of course the costs for space operations will be at least somewhat greater for most civilizations than for work on the planetary surface.

General Consideration of Signal to Noise Ratios

The limiting noise in a receiver depends on whether linear detection and amplification is used or some kind of photon counter, which is of course a square law detector. Photon counting is much the more sensitive if there is little background radiation. However, in the radio region background radiation is always present so that linear amplification represents no disadvantage. The minimum noise power achievable for the two cases is

$$P_N \text{ (photon counter)} = h\nu \sqrt{n(n+1)} \sqrt{\frac{A_R \Omega_R \Delta\nu}{\lambda^2 t}} \quad (2a)$$

$$P_N \text{ (heterodyne detector)} = h\nu \left[\sqrt{n(n+1)} + 1 \right] \sqrt{\frac{A_R \Omega_R \Delta\nu}{\lambda^2 t}} \quad (2b)$$

Here Ω_R is the solid angle received by the antenna of area A_R , $\Delta\nu$ is the bandwidth received, t the time duration of reception, $h\nu$ the quantal energy, and n the occupation number of the radiation field. This occupation number depends on the nature and sources of background radiation impinging on the receiver, and will be discussed in some detail below. It is assumed here that the photon counter, like the heterodyne detector, receives only a single polarization. Such an assumption makes a difference of only $\sqrt{2}$. If the antenna is diffraction limited, then the quantity $A_R \Omega_R / \lambda^2$ is unity. However, we shall want to consider receiving surfaces which are not necessarily diffraction limited and hence the expression for noise is kept in the more generalized form given by expressions 2. The quantity n represents the number of photons per second flowing through any diffraction-limited channel of bandwidth one Hz. For black body radiation of temperature T , $n = \frac{1}{e^{h\nu/kT} - 1}$. From this and expressions 2, it is easy to see that when the number of photons per second is large and an antenna is diffraction limited, one obtains the form familiar in the radio region,

$kT \sqrt{\frac{\Delta\nu}{c}}$. When n is very small, it takes on the familiar form of photon fluctuations, with noise power proportional to the square root of the photon counting rate. We will be dealing with some intermediate cases where n is neither small nor large, so that the complete expression is needed rather than one of these limiting approximations. Most treatments which optimize wavelengths for SETI assume linear amplification and do not consider photon counting, which is a reason they provide optima in the microwave region [Cf. Kardashev, (3) where there is a rather general treatment but with effective background assumed to be $h\nu/k$ at short wavelengths].

Since the ratio of signal to noise obtainable depends on the occupation number of the radiation field, one must examine carefully the sources of background radiation. The two most notable sources are the 3° black body radiation and stellar radiation. The first has an easily expressible form, with $n = \frac{1}{e^{h\nu/kT} - 1}$ where T is approximately 3 K. At the surface of a star, T is typically about 10^4 K. Average stellar radiation density in space corresponds to that at a stellar surface diluted by a factor of approximately 10^{14} . However, since a search for signals would be in the vicinity of a star, the background is not the average stellar intensity but is instead given by an occupation number $\frac{1}{e^{h\nu/kT} - 1}$ times the fraction of the beam filled by the stellar disk. To estimate this fraction, stars of solar diameter will be assumed in subsequent calculations. There are also a number of other significant sources of radiation which cannot be so simply described. These include the radio radiation from synchrotron-type sources and H II regions, infrared radiation from warm dust in interstellar clouds, the background radiation from other galaxies, and zodiacal radiation. Two

general summaries giving estimates of these miscellaneous sources have already been published (5,6) and, while some aspects of them are rather uncertain, we shall use these sources for an approximate evaluation of the background radiation.

From the above expressions, the ratio of signal-to-noise for a given wavelength can be written

$$\frac{S}{N} = \frac{\lambda^2 \Omega_B}{h\nu R^2 \Omega_B} \sqrt{\frac{A_R t}{\Omega_R \Delta\nu}} \frac{1}{\sqrt{n(n+1)} + (1 \text{ or } 0)} \quad (3)$$

Here the numbers 1 or zero apply when linear detection or photon counting is used respectively. So far as the frequency of wavelength variation is concerned, this expression for signal-to-noise is overtly proportional to $\nu^{-5/2}$, since we have assumed above that Doppler effects dominate in the bandwidth $\Delta\nu$. The $\nu^{-5/2}$ dependence can give the immediate impression that the lowest frequencies are the most favored. This is of course not true in the radio region because the noise background, represented by n , increases rapidly as the frequency decreases; the fact that wavelengths shorter than about 30 cm are therefore disadvantageous is already well-recognized. The apparent rapid decrease of S/N with increasing frequency comes from several sources: the quantum noise is proportional to ν for linear amplification, the Doppler bandwidth is proportional to $\sqrt{\nu}$, and the number of modes received by an antenna of fixed $A_R \Omega_R$ increases as $\frac{1}{\lambda^2}$. On the other hand, an antenna of reasonable size can give a higher directivity at shorter wavelengths and hence Ω_R or Ω_B can be smaller at short wavelengths. In addition, the occupation number n decreases as the frequency increases, dropping substantially after ν is well past the peak of the black body radiation. These last factors can in some cases more than compensate for the $\nu^{-5/2}$ dependence which is more overtly evident in expression (3).

Possible Design Choices which Determine S/N

To proceed to a more quantitative comparison of different wavelengths it is necessary to consider some of the necessary technical choices. We will try to avoid being limited to specific and arbitrary choices, and to simply lay out what the various alternatives would give. The reasonable possibilities for various parameters seem to be the following:

I The area of the receiving antenna might be chosen to be either constant (choice IA) on the basis that the total structure size is a likely limitation, or it might be decreased in size as the wavelength is decreased (choice IB) on the basis that a given fractional accuracy is what must be held constant for a given cost. Our own technology shows that such a size decrease should not continue indefinitely. The largest fully steerable antennas which we have been willing to build are about 100 meters in diameter, whereas the largest optical telescopes are about 5 meters and optical telescopes of 7-25 meter diameter are being designed. Hence, we take the choice IB to be a constant diameter of 100 meters throughout the microwave region down to a 1 cm wavelength, and then a diameter decreasing linearly with decreasing wavelength to 10 meters in the optical region. This implies diameters of 19 meters at a wavelength of 1 mm, 10.9 at 1/10th mm, and 10.1 m at 10 micron wavelengths, which are reasonably consistent with present plans on Earth. While this choice IB is somewhat more complex than a simple constant antenna size, it is also probably more realistic.

II The receiving solid angle may be taken to be either diffraction limited and hence $\approx \lambda^2/A_R$ (choice IIA) or alternatively assumed to be diffraction limited only for wavelengths greater than 1 cm, and remaining constant for shorter wavelengths (choice IIB). Choice IIB would represent, then, a multimode telescope for wavelengths shorter than 1 cm. This may be

realistic if the total telescope area is taken to be constant, as in choice IA above. For choice IB, involving a decreasing size of telescope area with decreasing wavelength, the diffraction limited assumption, choice IIA, seems the more appropriate one.

III The simplest assumption in evaluating n would be that only the black body background radiation and stellar radiation are present. However, even though the other sources are rather uncertain, they can be important and it would appear that the only realistic choice is to take the sum of all known and estimated radiations. This will be what is used in further discussion.

IV We may assume our receiver is either a linear amplifier (choice IVA), or a quantum counting detector (choice IVB). Both choices seem logical enough in principle, though in fact almost surely a linear amplifier would be used in the radio region and a quantum detector at very short wavelengths. At intermediate wavelengths the natural choice is less obvious, and hence both assumptions will be explored at all wavelengths.

V The broadcast solid angle, as in the case of the receiving solid angle, may be taken to be either diffraction limited (choice VA), or diffraction limited only for wavelengths longer than 1 cm, and constant at shorter wavelengths (choice VB).

Numerical Evaluations of S/N

Since we are primarily interested in relative signal-to-noise values, expression (3) may be simplified to

$$\frac{S}{N} = \frac{P_B}{\nu^{5/2} \Omega_B} \sqrt{\frac{A_R}{\Omega_R}} \frac{1}{\sqrt{n(n+1)} + (1 \text{ or } 0)} \quad (4)$$

It appears reasonable to assume a fixed broadcast power P_B independent of frequency, as argued above. The relative effect on S/N of each of the

remaining factors in expression (4) are given in Table 1 for the various choices outlined above. Each factor, corresponding to a column in the Table, is normalized to unity at 1 cm wavelength. Values for the quantity n are based on background fluxes listed in Table 2 as a function of wavelength. It can be seen from this Table that in the long wavelength range the isotropic black body radiation is dominant, whereas at shorter wavelengths radiation directly from a star in the field of view is dominant, except that near 1 mm wavelength some of the miscellaneous background sources are important. Obviously, there are more intense localized sources which have been omitted, such as ionized regions which produce additional noise in the microwave region or dust clouds radiating in the infrared. An evaluation of the magnitude of each of these and the solid angles effectively obscured by them requires a detailed examination which is not attempted here.

From Table 1 we can now compare the efficacy of different wavelength ranges with various combinations of choices of the parameters involved. Table 3 shows the result of two such sets of choices. One set clearly favors the longer wavelengths; the other favors the shorter wavelengths. The first set of choices, favoring longer wavelengths, involves linear detection of all wavelengths and a constant antenna area, but solid angles corresponding to the diffraction limit only for wavelengths longer than 1 cm. In the infrared this would mean a large multimode antenna having an angular precision no better than at 1 cm. Such an assumption clearly destroys much of the advantage of the shorter wavelengths, and does not seem especially reasonable in view of our own experience with the technical possibilities. However, it is a choice the reader may wish to consider as an example. The other set of assumptions, which indicates that the

shorter wavelengths are more favored, involves a quantum counting detector and an antenna of fixed diameter for long wavelengths down to 1 cm and then decreasing linearly in size, as indicated above, to 10 meters in the infrared. It also assumes solid angles limited only by diffraction. The assumption of a quantum counter for detection makes a large difference at the shorter wavelengths, but gives essentially the same sensitivity as linear detection in the microwave region.

It should be emphasized that the precise sizes of antennas one might wish to assume do not in themselves change the relative efficacy of different wavelengths. Rather, it is the functional form of variation with wavelength which is important here, so that if all sizes are scaled up as might be the case for a civilization more technically capable or interested than our own, the results in Table 3 would be identical although the power requirement for a given signal-to-noise would be substantially decreased.

There are various other sets of assumptions that can be made with some logic. The two represented in Table 3 are two fairly extreme ones. While both may be defensible, the first set, with fixed solid angles Ω_R and Ω_B and linear detection at the shorter wavelengths gives rather arbitrary handicaps to the shorter wavelengths. The second set, showing an advantage for short wavelengths, is perhaps more logical. A counter argument against the shorter wavelengths may be that the necessary operations from space are too awkward.

A natural question is how far into the short wave region one should press in order to capitalize on the advantage of small solid angles Ω_R and Ω_B and the relative ease of quantum counting at short wavelengths. One natural stopping point is where the solid angle is so small that the guiding problems become difficult or that an antenna beam might not cover all planets of a given solar system at the same time. Thus a beam of

about 1 arcsec size may be a reasonable minimum value for Ω_R and Ω_B , which is why 10 μ m is the shortest wavelength listed here for a 10 meter antenna. This would give a beam $\frac{1}{2}$ " in size and may hence be a somewhat shorter wavelength than is desired.

Summary

While the above discussion indicates that the infrared is as good as, and may be a more favorable region for SETI than the microwave region on the basis of reasonable assumptions, it does not indicate that we should either search only in the infrared or even search at all in this wavelength region with present technology. There is considerable uncertainty as to what design parameters would be considered most critical for interstellar communication by an extraterrestrial civilization. Furthermore, the microwave region does have one unique property--that we are prepared now, during the coming decade, to search the microwave spectrum rather efficiently. Hence such searches are probably quite justified. But I believe the above discussion does show that we have no assurance the microwave region is the one of choice for a civilization trying to communicate with us. This may affect the scale and style with which SETI is carried out on Earth even in the immediate future.

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2. The Search for Extraterrestrial Intelligence (eds. P. Morrison J. Billingham, and J. Wolf) NASA-SP-419 (1977) Cf in particular B.M. Oliver, pg. 64.
3. Drake, F.D. and Helou, G., National Astronomy and Ionospheric Center, NAIC 76 (1977).
4. Kardashev, N.S., Nature 278, 28 (1979).
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Table 2

$\nu(\text{cm}^{-1})$	"Big bang" radiation	Background flux in quanta per second per mode $\left(\frac{\text{quanta/sec}}{\Delta\Omega\Delta\nu} \right)$	Star at 100 L.Y. in field of view Option VB for Ω_B	Dust emission	Other sources
$\frac{1}{30}$	64	6.0×10^{-8}	1.7×10^{-5}		
$\frac{1}{10}$	21	1.8×10^{-7}	1.7×10^{-5}		
$\frac{1}{3}$	6	6.0×10^{-7}	1.7×10^{-5}		
1	1.7	1.8×10^{-6}	1.6×10^{-5}		
3	3.3×10^{-1}	7.4×10^{-7}	1.4×10^{-5}		<Big Bang source
10	9.5×10^{-3}	6.5×10^{-7}	9.0×10^{-6}		8.4×10^{-4}
30	8.3×10^{-7}	8.7×10^{-7}	1.9×10^{-6}		1.0×10^{-5}
100	5.4×10^{-21}	2.1×10^{-6}	1.7×10^{-9}		8.4×10^{-8}
300	1.6×10^{-61}	5.7×10^{-6}	3.8×10^{-13}		1.0×10^{-9}
1000	~ 0	1.8×10^{-5}	4.3×10^{-48}		8.4×10^{-12}

Table 3

Relative $\frac{S}{N}$ as a function of wavelength for two different sets of assumptions:

- (1) Linear (heterodyne) detection, constant solid angle of broadcast radiation, a solid angle of reception antenna proportional to λ^2 from long wavelengths down to $\lambda=1$ and then constant, and a constant antenna area (multimode for $\lambda < 1$ cm). This combination tends to favor longer wavelengths.
- (2) Photon counting detection, antenna area constant (e.g. 100 meters) from long wavelength to $\lambda=1$ cm, then decreasing linearly with wavelength by a factor of 10 (e.g. 10 meters) at short wavelengths, both broadcast and receiving solid angles diffraction limited for the antenna area assumed. This combination tends to favor shorter wavelengths.

$\nu(\text{cm}^{-1})$ [$\lambda=1/\nu$]	$\frac{S}{N}$ under assumptions (1) favoring long wavelength	$\frac{S}{N}$ under assumptions (2) favoring short wavelength
$\frac{1}{30}$	7.8	5.9×10^{-3}
$\frac{1}{10}$	4.5	3.2×10^{-2}
$\frac{1}{3}$	2.2	1.9×10^{-1}
1	1	1
3	1.5×10^{-1}	1.4×10^{-1}
10	1.7×10^{-2}	8.7×10^{-2}
30	8.1×10^{-4}	2.7×10^{-1}
100	3.1×10^{-5}	2.6
300	2.0×10^{-6}	5.5
1000	9.9×10^{-8}	18

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