Department of Geodetic Science and Surveying

# BASIC RESEARCH FOR THE GEODYNAMICS PROGRAM

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# 1. CURRENT TECHNICAL OBJECTIVES

1. Optimal Utilization of Laser and VLBI Observations for Reference Frames for Geodynamics (Grant NSG 5265)

2. Utilization of Range Difference Observations in Geodynamics (Contract NAS 5-25888)

3. Development of Models for Ice Sheet and Crustal Deformations (Grant NSG 5265)

# 2. ACTIVITIES

## 2.1 Effects of Adopting New Precession, Nutation and Equinox Corrections on the Terrestrial Reference Frame

A paper on this topic was presented at the XVII General Assembly of the International Astronomical Union, Patras, Greece, August 17-26, 1982, and appears in its entirety below. It will also appear in Bulletin Geodesique.

PREFACE

These projects are under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science and Surveying, The Ohio State University. The Science Advisor of RF 711055 is Dr. David E. Smith, Code 921, Geodynamics Branch, and the Technical Officer is Mr. Jean Welker, Code 903, Technology Applications Center. The Technical Officer for RF 712407 is Mr. C. Stephanides, Code 942. The latter three are at NASA/GSFC, Greenbelt, Maryland 20771.

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## EFFECTS OF ADOPTING NEW PRECESSION, NUTATION AND EQUINOX CORRECTIONS ON THE TERRESTRIAL REFERENCE FRAME [1]

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ABSTRACT. First, the paper is devoted to the effects of adopting new definitive precession and equinox corrections on the terrestrial reference frame: The effect on polar motion is a diurnal periodic term with an amplitude increasing linearly in time; on UT1 it is a linear term. Second, general principles are given the use of which can determine the effects of small rotations (such as precession, nutation or equinox corrections) of the frame of a Conventional Inertial Reference System (CIS) on the frame of the Conventional Terrestrial Reference System (CTS). Next, seven CTS options are presented, one of which is necessary to accommodate such rotations (corrections). The last of these options requiring no changes in the origin of terrestrial longitudes and in UT1 is advocated; this option would be maintained by eventually referencing the Greenwich Mean Sidereal Time to a fixed point on the equator, instead of to the mean equinox of date, the current practice. Accomodating possible future changes in the astronomical nutation is discussed in the last section. The Appendix deals with the effects of differences which may exist between the various CTS's and CIS's (inherent in the various observational techniques) on earth rotation parameters (ERP) and how these differences can be determined. It is shown that the CTS differences can be determined from observations made at the same site, while the CIS differences by comparing the ERP's determined by the different techniques during the same time period.

## INTRODUCTION

New general precession and equinox corrections are being introduced in the 1984 star catalogues and ephemerides. These corrections in turn will affect the earth rotation parameters (ERP), i.e., polar motion coordinates and UT1, and thus may change the frame of the Conventional Terrestrial Reference System

*On leave from Shanghai Observatory, China.
[1] Presented at XVIII General Assembly of the International Astronomical Union, Patras, Greece, August 17-26, 1982.

(CTS) [Mueller 1981]. Williams and Melbourne [1981] have already given a detailed discussion of these effects on UT1 and on the origin of terrestrial longitudes. In fact, it was this work which gave us motivation to expand the discussion to include the effects on all ERP's and offer additional options on how the necessary changes in the CTS could be accommodated. The approach is strictly geometric, i.e., we try to answer the question how definitive corrections to precession, nutation and the equinox affect the ERP's, and thus the CTS. Williams and Melbourne [1981] emphasize the point of how UT1 and the origin of longitudes will be affected in the future by the uncertainties in the newly adopted corrections or how these corrections can be improved in the future from ensembles of Very Long Baseline Interferometer (VLBI) or Lunar Laser Ranging (LLR) orbservations, with the desire that no or minimum additional changes result in the CTS. They assume that VLBI sources are observed randomly over the sky, while LLR observations are equally distributed only along the ecliptic, and therefore the resulting equations defining the changes of the origin of terrestrial longitudes and UT1 are technique dependent, whereas ours are not. (Putting it differently, they imply that if the analysis of future VLBI or LLR ensemble observations indicate necessary changes in UT1 and in the origin of terrestrial longitudes, such changes are due to the still existing imperfections in the newly adopted corrections to precession, equinox, etc., and when determined they will be biased with respect to each other because of the different sensitivities of the two ensembles of observations.) This difference in the results should not confuse the reader who recognizes the different purposes for which these papers were written.

## 1. EFFECTS OF ADOPTING NEW PRECESSION AND EQUINOX CORRECTIONS ON THE FRAME OF THE CONVENTIONAL TERRESTRIAL REFERENCE SYSTEM

### 1.1 Transformation Between Conventional Inertial (CIS) and Terrestrial Reference Frames (CTS)

The transformation at an epoch T between the CIS at some fundamental epoch (e.g., 1950.0) and the CTS is

$$[\underline{CTS}] = SNP(M)\ [\underline{CIS}] \tag{1}$$

(see [Mueller 1981]). Here

$$S = R_2(-x_p)\ R_1(-y_p)\ R_3(\theta)$$

is the earth rotation matrix, in which $x_p$ and $y_p$ are the polar motion components, and $\theta$ is the Greenwich Apparent Sidereal Time (corresponding to the epoch T) computed from

$$\theta = (GMST)_0 + \omega_e \, UT1 + Eq.\ E.$$

where $(GMST)_0$ is the Greenwich Mean Sidereal Time at $0^h$ UT1, $\omega_e$ is the conversion factor from mean time to mean sidereal time, and Eq. E. is the equation of the equinox (nutation in right ascension). The other matrices N, P, M in equation (1) are the nutation, precession, and proper motion matrices respectively [Mueller 1969, p. 123]. Parentheses around the M matrix indicate that proper motion is applied only in the case of a stellar CIS.

Let prime (') denote the case with the precession, nutation and equinox changes introduced. The transformation equation (1) also holds for the corrected case:

$$\underline{[CTS]}' = S'N'P'(M') \, \underline{[CIS]}' \qquad (1')$$

In this section only the precession and equinox changes are considered so that N' = N. From the definitions (or stipulations), one can determine directly or indirectly the relations between P' and P, M' and M, and [CIS]' and [CIS] at some epoch, leaving S' and [CTS]' to be solved for.

One cannot solve for both S' and [CTS]' simultaneously, hence some additional constraint is needed. There are several options for the constraint, and they will be discussed later in Section 2.2. For the time being we will conform with the IAU adopted constraint, namely: Let the new ERP's be the same as the old ones at some epoch $T_u$ (in this paper T denotes the epoch, and t the time interval between T and some fundamental epoch, e.g., 1950.0); solve for [CTS]' at this time, then keep it time invariant and solve for the resulting time variations in the new ERP's.

## 1.2 The Effect in the Case of a Stellar CIS

The new (1976) corrections for lunisolar precession in longitude and planetary precession in right ascension are [Williams and Melbourne 1981]

$$\Delta p_1 = 1''\!.1/cy$$
$$\Delta \dot{\chi} = -0''\!.029/cy$$

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The correction to the equinox is $E_0 + \dot{E}t$, where $E_0 = 0''.525$ is the offset at 1950.0, $\dot{E} = 1''.275/cy$, and t is the time elapsed from 1950.0 [Fricke 1981].

The new precession matrix P' can be written with sufficient approximation as

$$P' = R_2(\Delta nt)\ R_3(-\Delta mt)\ P \tag{2}$$

with

$$\Delta n = \Delta p_1 \sin\varepsilon$$
$$\Delta m = \Delta p_1 \cos\varepsilon - \Delta\dot{\chi}$$

where $\varepsilon$ is the obliquity of the ecliptic, and $\Delta n$, $\Delta m$ are the general precession changes in declination and in right ascension. Due to the equinox correction, the equation for the Greenwich Mean Sidereal Time is to change(without terms of higher order) to [Aoki et al. 1982]

$$(GMST)_0' = (GMST)_0 + E_0 + \dot{E}t \tag{3}$$

For the stellar (i.e., classical optical) CIS the change caused by the equinox correction at the fundamental epoch 1950.0 is

$$\underline{[CIS]}' = R_3(-E_0)\ \underline{[CIS]} \tag{4}$$

The new proper motion matrix is

$$M' = R_2(-\Delta nt)\ R_3[(\Delta m - \dot{E})t]\ M \tag{5}$$

The proper motion components in right ascension and declination are

$$(\mu_\alpha)' = (\mu_\alpha) + \dot{E} - \Delta m - \Delta n \sin\alpha \tan\delta$$
$$(\mu_\delta)' = (\mu_\delta) - \Delta n \cos\alpha$$

Substituting the above new values of P', M', $\underline{[CIS]}'$ and $(GMST)_0'$(i.e., eqs. (2) - (5)) into eq. (1'), one gets

$$\underline{[CTS]}' = R_2(-x_p')\ R_1(-y_p')\ R_3[(GMST)_0 + \omega_e\ UT1' + Eq.\ E.]\ R_3(E_0 + \dot{E}t)\ N \cdot$$
$$\cdot\ R_2(\Delta nt)\ R_3(-\Delta mt)P\ R_2(-\Delta nt)\ R_3[(\Delta m - \dot{E})t]\ M\ R_3(-E_0)\ \underline{[CIS]}$$

Except for $(GMST)_0$, all rotation angels are small; neglecting the second-order terms, approximately,

$$\underline{[CTS]}' = R_2(-x_p')\ R_1(-y_p')\ R_3[(GMST)_0 + \omega_e\ UT1' + Eq.\ E.]\ NPM\ \underline{[CIS]} \tag{6}$$

(For the above given $\Delta p_1$ and $\dot{E}$ values, neglecting the modulation of NP will cause an error of less than 0".0001 in t = 10 yr.) Combining the above equation with the mentioned constraints at epoch $T_u$: $x_p' = x_p$, $y_p' = y_p$, and UT1' = UT1, one obtains

$$\boxed{[\underline{CTS}]' = [\underline{CTS}]}$$

The well-known conclusion is that in the case of the stellar CIS, the CTS and ERP's are unaffected because changes in the proper motion compensate for the equinox and precession changes. This statement is naturally valid not only at the epoch $T_u$ but at any time before or after.

## 1.3 The Effect in the Case of a Non-Stellar CIS

For any non-stellar (e.g., VLBI or LLR) CIS, the proper motion matrix is no longer taken into consideration; the P' and $(GMST)_0'$ are the same as in the stellar case (eq. (2) - (3)). The relationship between $[\underline{CIS}]'$ and $[\underline{CIS}]$ depends on the particular CIS under consideration. Generally,

$$[\underline{CIS}]' = E_I\ [\underline{CIS}]$$

If the considered CIS is aligned with the dynamic equator and equinox, then $E_I = I$, where I is a unit matrix.

If the non-stellar CIS is aligned with the stellar system equinox at some epoch $T_0$, then $E_I$ will be a little complicated. At this time due to the equinox correction,

$$[\underline{CIS}]'_{T_0} = R_3(-E_0 - \dot{E}t_0)\ [\underline{CIS}]_{T_0} \tag{7}$$

(More exactly, a second-order term could be considered.) The precession effect on the CIS's for the time interval $t_0$ between the fundamental epoch 1950.0 and the alignment epoch $T_0$ is

$$[\underline{CIS}]_{T_0} = P(t_0)\ [\underline{CIS}]$$

$$[\underline{CIS}]'_{T_0} = P'(t_0)\ [\underline{CIS}]'$$

With equations (2) and (7) one gets at the fundamental epoch

$$[\underline{CIS}]' = R_2(-\Delta n t_0)\ R_3[(\Delta m - \dot{E})t_0]\ R_3(-E_0)\ [\underline{CIS}] = E_I\ [\underline{CIS}] \tag{8}$$

i.e.,

$$E_I = R_2(-\Delta n t_0)\, R_3[(\Delta m - \dot{E})t_0 - E_0] \tag{9}$$

The corresponding corrections in right ascension ($\Delta\alpha_{E_I}$) and declination ($\Delta\delta_{E_I}$) are

$$\Delta\alpha_{E_I} = E_0 + (\dot{E} - \Delta m)t_0 - \Delta n t_0 \sin\alpha \tan\delta$$

$$\Delta\delta_{E_I} = -\Delta n t_0 \cos\alpha\ .$$

Now, substituting P', $(GMST)'_0$, and [CIS]' (i.e., eq. (2), (3) and (8)) into eq. (1'),

$$[\underline{CTS}]' = R_2(-x'_p)\, R_1(-y'_p)\, R_3[(GMST)_0 + E_0 + \dot{E}t + \omega_e\, UT1' + Eq.\ E]\, N \cdot$$
$$\cdot\, R_2(\Delta n t)\, R_3(-\Delta m t)\, P\, E_I\, [\underline{CIS}] \tag{1''}$$

As stated before, the ERP's are continuous, that is, at the alignment epoch $T_u$, $x'_p = x_p$, $y'_p = y_p$, UT1' = UT1. Thus

$$[\underline{CTS}]' = R_2(-x_p)\, R_1(-y_p)\, R_3[(GMST)_0 + \omega_e\, UT1 + Eq.\ E]\, R_3(E_0 + \dot{E}t_u)\, N \cdot$$
$$\cdot\, R_2(\Delta n t_u)\, R_3(-\Delta m t_u)\, P\, E_I\, [\underline{CIS}] =$$
$$\doteq SNP\, R_2(\Delta n t_u)\, R_3[E_0 + (\dot{E} - \Delta m)\, t_u]\, E_I\, [\underline{CIS}] \tag{10}$$

If the CIS is linked with the stellar system equinox at epoch $T_0$, i.e., $E_I$ is expressed by eq. (9), then

$$[\underline{CTS}]' \doteq SNP\, R_2[\Delta n(t_u - t_o)]\, R_3[(\dot{E}-\Delta m)(t_u - t_o)][\underline{CIS}]$$
$$\doteq S\, R_2[\Delta n(t_u - t_o)]\, R_3[(\dot{E}-\Delta m)(t_u - t_o)]\, NP\, [\underline{CIS}] \tag{10'}$$

As pointed out previously, the modulation of NP is negligible, but the modulation of $R_3(\theta)$, included in S, must be taken into consideration.

$$R_3(\theta)\, R_2[\Delta n(t_u - t_o)] = \{R_3(\theta)\, R_2[\Delta n(t_u - t_o)]\, R_3(-\theta)\}\, R_3(\theta)$$
$$= R_1[\Delta n(t_u - t_o)\sin\theta]\, R_2[\Delta n(t_u - t_o)\cos\theta]\, R_3(\theta)$$

Substituting this into equation (10'),

$$[\underline{CTS}]' \doteq R_1[\Delta n(t_u - t_o)\sin\theta]\, R_2[\Delta n(t_u - t_o)\cos\theta]\, R_3[(\dot{E}-\Delta m)(t_u - t_o)]\, SNP\, [\underline{CIS}]$$

Thus for the case of <u>CIS alignment with the stellar system</u>

$$[\underline{CTS}]' = R_1[\Delta n(t_u-t_o)\sin\theta]\, R_2[\Delta n(t_u-t_o)\cos\theta]\, R_3[(\dot{E}-\Delta m)(t_u-t_o)][\underline{CTS}] \quad (11)$$

If the CIS is aligned with the dynamic equinox, that is, $E_I = I$, then

$$[\underline{CTS}]' = SNP\, R_2(\Delta n\, t_u)\, R_3[E_0 + (\dot{E}-\Delta m)\, t_u][\underline{CIS}]$$

Thus

$$[\underline{CTS}]' = R_1(\Delta n\, t_u \sin\theta)\, R_2(\Delta n\, t_u \cos\theta)\, R_3[E_0 + (\dot{E}-\Delta m)\, t_u][\underline{CTS}] \quad (12)$$

If the alignment is made over some time period (say, five days or so) $T_u$ is the mean epoch of alignment, and the values sinθ and cosθ are the mean values within this time span and can be averaged to zero. In this case

$$[\underline{CTS}]' = R_3[(\dot{E}-\Delta m)(t_u-t_o)][\underline{CTS}] \quad (11')$$

for the CIS linked with the stellar system equinox, and

$$[\underline{CTS}]' = R_3[E_0 + (\dot{E}-\Delta m)\, t_u][\underline{CTS}] \quad (12')$$

when aligned with the dynamic equinox. Thus the relation between the new and old CTS's is a small rotation around the third axis. Expressed in longitude (positive to the East),

$$\delta\lambda = \lambda' - \lambda = (\Delta m-\dot{E})(t_u-t_o) \quad (11'')$$

for the CIS linked with the stellar system equinox, and

$$\delta\lambda = \lambda' - \lambda = (\Delta m-\dot{E})\, t_u - E_0 \quad (12'')$$

when aligned with the dynamic equinox.

For a CIS linked with the stellar system, if $t_u = t_o$, then $\delta\lambda = 0$; otherwise a shift in longitude is necessary. As for a CIS aligned with the dynamic equinox, the CTS longitude origin shift generally cannot be avoided.

## 1.4 The Effect of the Time-Invariant CTS on the ERP's

The new CTS' at the time of alignment $T_u$ can then be determined as outlined in the previous section, i.e., in the stellar CIS case $[\underline{CTS}]' = [\underline{CTS}]$, and in the non-stellar cases as given by eqs. (11), (11') or (12), (12').

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The next step is to keep the new CTS time invariant and to find the resulting ERP's at any time other than $T_u$. Substituting eq. (11') for the left side of eq. (1"), and eq. (9) in the right-hand side, after some derivation and neglecting second-order terms, one gets

$$[\underline{CTS}] \doteq R_2(-x_p') \, R_1(-y_p') \, R_3[(GMST)_0 + \omega_e \, UT1' + Eq.\ E] \cdot$$
$$\cdot \, R_2[\Delta n(t-t_o)] \, R_3[(\dot{E}-\Delta m)(t-t_u)] \; NP \; [\underline{CIS}]$$

Comparing this equation with eq. (1),

$$R_2(-x_p') \, R_1(-y_p') \, R_3[(GMST)_0 + \omega_e \, UT1' + Eq.\ E] \, R_2[\Delta n(t-t_o)] \cdot$$
$$\cdot \, R_3[(\dot{E}-\Delta m)(t-t_u)] \;=\; S$$

or

$$R_2[-x_p' + \Delta n(t-t_o)\cos\theta] \, R_1[-y_p' + \Delta n(t-t_o)\sin\theta] \, R_3\{(GMST)_0 + Eq.\ E +$$
$$+ [\omega_e \, UT1' + (\dot{E}-\Delta m)(t-t_u)]\} =$$
$$= R_2(-x_p) \, R_1(-y_p) \, R_3[(GMST)_0 + \omega_e \, UT1 + Eq.\ E]$$

From the above it is obvious that over a limited time span (otherwise second-order terms must be added),

$$\Delta x_p = x_p' - x_p = \Delta n(t-t_o) \cos\theta$$
$$\Delta y_p = y_p' - y_p = \Delta n(t-t_o) \sin\theta \qquad (13)$$
$$\Delta UT1 = UT1' - UT1 = (\Delta m-\dot{E})(t-t_u)/\omega_e$$

The above are in the case of a non-stellar CIS linked with the stellar system. For the dynamic equinox alignment, substitute eq. (12') for the left side of eq. (1") and let $E_I$ = I. The results are

$$\Delta x_p = \Delta nt \cos\theta$$
$$\Delta y_p = \Delta nt \sin\theta \qquad (14)$$
$$\Delta UT1 = (\Delta m-\dot{E})(t-t_u)/\omega_e$$

For both cases $\Delta$UT1 is the same; so is the rate of $\Delta$UT1:

$$\frac{d\Delta UT1}{dt} = (\Delta m-\dot{E})/\omega_e \;=\; -0.157 \text{ ms/yr} \qquad (15)$$

In conclusion, in the case of a non-stellar CIS, changes in the precessional constant and the equinox will result in changes in both the CTS and the ERP's. The CTS change is a longitude origin shift. The ERP changes are diurnal terms in the polar motion components with amplitudes linearly increasing with time and a constant rate change in UT1. One point worth stressing is that these are the differences of the same system (technique) between the new and old cases, not the differences between different systems (techniques). Also the diurnal term which is evident in polar motion is not the diurnal true polar motion, but an artifact due to the time invariant CTS constraint applied.

## 2. GENERAL SOLUTION: SMALL CIS ROTATIONS AND THEIR EFFECT ON THE CTS

### 2.1 Changes in the Earth Rotation Parameters

In the general case, eq. (13) or (14) can be written in the following form

$$\Delta x_p = -\alpha_1 \sin\theta + \alpha_2 \cos\theta$$

$$\Delta y_p = \alpha_1 \cos\theta + \alpha_2 \sin\theta \qquad (16)$$

$$\Delta\theta = -\alpha_3$$

where the small angles $\alpha_i$ represent the changes in the sense

$$N'P'M' \underline{[CIS]}' = R_1(\alpha_1)\ R_2(\alpha_2)\ R_3(\alpha_3)\ NPM\ \underline{[CIS]}$$

Since

$$\theta = (GMST)_0 + \omega_e\ UT1 + Eq.\ E$$

there are several possibilities for changing $\theta$. If the nutation is assumed to be unchanged, $\alpha_3$ either may be absorbed into $(GMST)_0$, i.e., it becomes a change in the Greenwich Mean Sidereal Time; or, as before, it may go into UT1 ($\Delta UT1 = \alpha_3/\omega_e$); or it can be incorporated partially in $(GMST)_0$ and partially in UT1. When $\alpha_3$ is placed (fully or partly) into UT1, then if UT1 is still to be continuous at the epoch $T_u$, the longitude $\lambda$ has to absorb the one-time discontinuity as shown before. Finally, if $\alpha_3$ is a nutation correction, than $\alpha_3$ must be combined with Eq. E (see Section 3).

The corrections for precession, for the equinox, and for proper motion may be written in the following forms respectively

$$\Delta p = R_2(\Delta nt)\ R_3(-\Delta mt)$$
$$E_I = R_2(E_{I_2})\ R_3(E_{I_3})$$
$$\Delta M = R_2(\Delta M_2)\ R_3(\Delta M_3)$$

where, from comparisons with earlier results, in the case of the stellar CIS, $\Delta M_2 = -\Delta nt$, $\Delta M_3 = (\Delta m-\dot{E})t$, $E_{I_2} = 0$, $E_{I_3} = -E_0$; for the non-stellar CIS aligned with the dynamic equinox, $E_{I_2} = E_{I_3} = \Delta M_2 = \Delta M_3 = 0$; and in the case of the non-stellar CIS linked with the stellar system, $E_{I_2} = -\Delta nt_0$, $E_{I_3} = -E_0 + (\Delta m-\dot{E})t_0$, $\Delta M_2 = \Delta M_3 = 0$.

In any of the above cases

$$\alpha_1 = 0$$
$$\alpha_2 = \Delta nt + E_{I_2} + \Delta M_2 \qquad (17)$$
$$\alpha_3 = -\Delta mt + E_{I_3} + \Delta M_3$$

Thus, for example, in the case of the stellar CIS

$$\alpha_1 = \alpha_2 = 0$$
$$\alpha_3 = -E_0 - \dot{E}t \qquad (17')$$
$$\Delta\theta = E_0 + \dot{E}t$$

If we let $\Delta\theta$ be the $\Delta(GMST)_0$, as we did before, then eq. (17') is equivalent to eq. (3).

In the case where the non-stellar CIS is linked at $T_0$ with the stellar system,

$$\alpha_1 = 0$$
$$\alpha_2 = \Delta n(t-t_0)$$
$$\alpha_3 = -E_0 + (\Delta m-\dot{E})t_0 - \Delta mt \qquad (17'')$$
$$\Delta\theta = E_0 + (\dot{E}-\Delta m)t_0 + \Delta mt$$

If we let $\Delta(GMST)_0 = E_0 + \dot{E}t$, and let the ERP's be continuous at $T_u$, then eq. (17") is equivalent to eqs. (11") and (13). The analogy can also be established for the case of the non-stellar CIS linked to the dynamic equinox (eq. (14)).

## 2.2 Options to Change the CTS (Due to $\Delta\theta$)

As shown, in the case of equinox and precession constant corrections,

$\Delta\theta = E_0 + \dot{E}t$ for the stellar system, and

$\Delta\theta = -E_{I_3} + \Delta mt$ for the non-stellar systems

$\Delta\theta$ can also be written (assuming no change in nutation and the one-time discontinuity in UT1 absorbed in the longitudes, $\delta\lambda$, mentioned earlier)

$$\Delta\theta = \Delta(GMST)_0 + \omega_e \, \Delta UT1 + \delta\lambda$$

Thus, as stated above, one can absorb $\Delta\theta$ either in $\Delta(GMST)_0$, or $\Delta UT1$, or $\delta\lambda$, or in some combinations of these. To get a definite (unique) solution, some constraint is needed. Mathematically, there are quite a number of possible choices for such a constraint, but practically only a few are meaningful. Below we deal with three sets of options. Which option is the best surely will be the subject of many discussions.

Set A Options. Here the basic requirements are: (i) no discontinuity in ERP's at the epoch $T_u$, (ii) the change in the Greenwich Mean Sidereal Time formula is the same for all CIS's, though different for each option.

| Set A Options | | Stellar CIS $\Delta\theta = E_0 + \dot{E}t$ | Non-stellar CIS $\Delta\theta = -E_{I_3} + \Delta mt$ |
|---|---|---|---|
| Option I | $\Delta(GMST)_0$ | $E_0 + \dot{E}t$ | $E_0 + \dot{E}t$ |
| | $\omega_e \, \Delta UT1$ | 0 | $(\Delta m-\dot{E})(t-t_u)$ |
| | $\delta\lambda$ | 0 | $(\Delta m-\dot{E})t_u - E_{I_3} - E_0$ |
| Option II | $\Delta(GMST)_0$ | 0 | 0 |
| | $\omega_e \, \Delta UT1$ | $\dot{E}(t-t_u)$ | $\Delta m(t-t_u)$ |
| | $\delta\lambda$ | $E_0 + \dot{E}t_u$ | $-E_{I_3} + \Delta mt_u$ |
| Option III | $\Delta(GMST)_0$ | $\Delta mt$ | $\Delta mt$ |
| | $\omega_e \, \Delta UT1$ | $(\dot{E}-\Delta m)(t-t_u)$ | 0 |
| | $\delta\lambda$ | $E_0 + (\dot{E}-\Delta m)t_u$ | $-E_{I_3}$ |

Options I, II, and III above are similar to Tables 1, 2, and 3 respectively in [Williams and Melbourne 1981]. (The main difference appears to be that for the non-stellar cases the general precession in right ascension $\Delta m$ is replaced by what they call "the average value over all observations of the effects of the precession corrections in right ascension" $<\dot{\alpha}_p>$. For VLBI $<\dot{\alpha}_p> = \Delta p_1 \cos\varepsilon - \Delta\dot{\chi} = \Delta m$, but for LLR $<\dot{\alpha}_p> = \Delta p_1 - \Delta\dot{\chi} \neq \Delta m$.) We already elaborated on Option I in Section 1.1. Option 1 is the presently accepted approach for the new FK5 CIS. But, as pointed out by [Williams and Melbourne 1981], for future possible new improvement of the precession constant and equinox corrections, this option might not be the best. They favor Option III because the space techniques are becoming the dominant source of information about the transformation parameters between the CIS and CTS frames and because this option keeps UT1 invariant to improved values of the precession constant and the equinox position for the space techniques. The common geodetic disadvantage of Set A options is the required shift in the longitude origin (except in Option I for the stellar CIS case), the worst thing being that these shifts are different in the cases of stellar and non-stellar CIS's.

Set B Options. Here the basic requirements are: (i) no change in the CTS, i.e., $\delta\lambda = 0$, (ii) as before, the Greenwich Mean Sidereal Time formula change is the same in all CIS cases, but different for each option.

The major inconvenience of Set B options is the change in UT1, not only in the rate, but also in the necessary discontinuity. The value of the discontinuity would need to be added with opposite sign to the UT1 at the epoch when the changes (new constants) are introduced.

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| Set B Options | | Stellar CIS $\Delta\theta = E_0 + \dot{E}t$ | Non-stellar CIS $\Delta\theta = -E_{I_3} + \Delta mt$ |
|---|---|---|---|
| | $\delta\lambda$ | 0 | 0 |
| Option IV | $\Delta(GMST)_0$ | 0 | 0 |
| | $\omega_e\ \Delta UT1$ | $E_0 + \dot{E}t$ | $-E_{I_3} + \Delta mt$ |
| | $\delta\lambda$ | 0 | 0 |
| Option V | $\Delta(GMST)_0$ | $E_0 + \dot{E}t$ | $E_0 + \dot{E}t$ |
| | $\omega_e\ \Delta UT1$ | 0 | $-E_{I_3} - E_0 + (\Delta m-\dot{E})t$ |
| | $\delta\lambda$ | 0 | 0 |
| Option VI | $\Delta(GMST)_0$ | $-E_{I_3} + \Delta mt$ | $-E_{I_3} + \Delta mt$ |
| | $\omega_e\ \Delta UT1$ | $E_0 + E_{I_3} + (\dot{E}-\Delta m)t$ | 0 |

<u>Set C Option</u>. Here the basic requirements are: (i) no change in CTS, (ii) no change in UT1, i.e., $\Delta\theta$ is entirely absorbed in $\Delta(GMST)_0$.

| Set C Option | | Stellar CIS $\Delta\theta = E_0 + \dot{E}t$ | Non-stellar CIS $\Delta\theta = -E_{I_3} + \Delta mt$ |
|---|---|---|---|
| | $\delta\lambda$ | 0 | 0 |
| Option VII | $\omega_e\ \Delta UT1$ | 0 | 0 |
| | $\Delta(GMST)_0$ | $E_0 + \dot{E}t$ | $-E_{I_3} + \Delta mt$ |

Although this option is probably the preference of geodesists, it may seem to be unorthodox from the traditional astronomical point of view. How can the formulae for Greenwich Mean Sidereal Time for different CIS's be different? What will the astronomical meaning of $(GMST)_0$ be? However, one can view the formula for Greenwich Mean Sidereal Time as composed of two parts: The first part, $(GMST)_0$, has its original astronomical meaning, while the second part, $\Delta(GMST)_0$ is only a correction particular for a given CIS. It would make sense that since the changes in precession and the equinox affect different CIS's in different ways, this correction should also be different. From this point of view, Option VII seems plausible and even preferable for geodetic use.

It should also be noted that after the new equinox and precession changes are introduced (once) into $\Delta(\text{GMST})_0$, this option could become the equivalent of referencing the GMST to a fixed point on the equator, instead of to the mean equinox of date, the current practice. As pointed out by a number of authors, the advantage of such a change would be overwhelming and would make the future CTS stable against changes in the precession constant, etc. [Guinot 1979, Murray 1979, Williams and Melbourne 1981, Mueller 1981].

## 3. EFFECT OF ASTRONOMICAL NUTATION CHANGES ON EARTH ROTATION PARAMETERS

According to the principle in Section 2.1, it is also easy to deal with any future changes in nutation. The nutation matrix N is [Mueller 1969]

$$N = R_1(-\varepsilon - \Delta\varepsilon)\, R_3(-\Delta\psi)\, R_1(\varepsilon)$$
$$\doteq R_1(-\Delta\varepsilon)\, R_2(\Delta\psi \sin\varepsilon)\, R_3(-\Delta\psi \cos\varepsilon)$$

where $\Delta\psi$ and $\Delta\varepsilon$ are the nutation in longitude and obliquity respectively, and $\varepsilon$ is the obliquity of the ecliptic. If $\delta\Delta\varepsilon$ and $\delta\Delta\psi$ are the respective corrections to $\Delta\varepsilon$ and $\Delta\psi$, then one can easily obtain the nutation correction matrix,

$$\Delta N \doteq R_1(-\delta\Delta\varepsilon)\, R_2(\delta\Delta\psi \sin\varepsilon)\, R_3(-\delta\Delta\psi \cos\varepsilon)$$

Thus in the notation of eq. (16),

$$-\delta\Delta\varepsilon = \alpha_1$$
$$\delta\Delta\psi \sin\varepsilon = \alpha_2$$
$$-\delta\Delta\psi \cos\varepsilon = \alpha_3$$

and, therefore,

$$\Delta x_p = \delta\Delta\varepsilon \sin\theta + \delta\Delta\psi \sin\varepsilon \cos\theta$$
$$\Delta y_p = -\delta\Delta\varepsilon \cos\theta + \delta\Delta\psi \sin\varepsilon \sin\theta$$
$$\Delta\theta = -\alpha_3 = \delta\Delta\psi \cos\varepsilon$$

Thus, as expected, the effects on polar motion components are diurnal terms ($\delta\Delta\psi$ and $\delta\Delta\varepsilon$ are long periodic). Again, this is a diurnal artifact in polar motion due to the introduction of the new nutation and not diurnal true polar motion.

As far as the term $\Delta\theta = \delta\Delta\psi \cos\varepsilon$ is concerned, if it is incorporated into the Eq. E, neither the longitude origin nor the UT1 will be affected.

# APPENDIX

## EFFECTS OF DIFFERENCES BETWEEN VARIOUS CTS'S AND CIS'S ON EARTH ROTATION PARAMETERS AND THE DETERMINATION OF SUCH DIFFERENCES

The two CIS's (and two CTS's) inherent in two different techniques (e.g., SLR and VLBI) are generally not exactly identical [Mueller 1981]. Suppose the relation between the two CIS's at any epoch is (common nutation (N) and precession (P) matrices are assumed to be used in both techniques)

$$[\underline{CIS}]^{II} = R_1(\alpha_1)\ R_2(\alpha_2)\ R_3(\alpha_3)\ [\underline{CIS}]^{I} \tag{A.1}$$

Similarly, the relation between the two CTS's is

$$[\underline{CTS}]^{II} = R_1(\beta_1)\ R_2(\beta_2)\ R_3(\beta_3)\ [\underline{CTS}]^{I} \tag{A.2}$$

where $\alpha_i$ and $\beta_i$ are small rotation angles about the axes "i".

The transformation from CIS to CTS again is

$$[\underline{CTS}]^{I} = S^{I}\ N\ P\ [\underline{CIS}]^{I} \tag{A.3}$$

and

$$[\underline{CTS}]^{II} = S^{II}\ N\ P\ [\underline{CIS}]^{II} \tag{A.4}$$

Substituting eq. (A.1) for the last term of the right-hand side of eq. (A.4), and eq. (A.2) for the left-hand side,

$$R_1(\beta_1)\ R_2(\beta_2)\ R_3(\beta_3)[\underline{CTS}]^{I} = S^{II}\ N\ P\ R_1(\alpha_1)\ R_2(\alpha_2)\ R_3(\alpha_3)[\underline{CIS}]^{I}$$

After some reduction, neglecting second-order terms,

$$[\underline{CTS}]^{I} = R_1(-\beta_1 + \alpha_1 \cos\theta + \alpha_2 \sin\theta)\ R_2(-\beta_2 - \alpha_1 \sin\theta + \alpha_2 \cos\theta) \cdot$$
$$\cdot\ R_3(-\beta_3 + \alpha_3)\ S^{II}\ N\ P\ [\underline{CIS}]^{I}$$

Comparing the above equation with (A.3)

$$S^{I} = R_1(-\beta_1 + \alpha_1 \cos\theta + \alpha_2 \sin\theta)\ R_2(-\beta_2 - \alpha_1 \sin\theta + \alpha_2 \cos\theta) \cdot$$
$$\cdot\ R_3(-\beta_3 + \alpha_3)\ S^{II}$$

Or

$$-\Delta y_p = -(y_p^{I} - y_p^{II}) = -\beta_1 + \alpha_1 \cos\theta + \alpha_2 \sin\theta$$

$$-\Delta x_p = -(x_p^{I} - x_p^{II}) = -\beta_2 - \alpha_1 \sin\theta + \alpha_2 \cos\theta$$

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$$\omega_e \, \Delta UT1 = \omega_e (UT1^{I} - UT1^{II}) = -\beta_3 + \alpha_3 \tag{A.5}$$

Thus the CTS differences ($\beta$ angles) cause biases in all earth rotation parameters. Because of the modulation of the earth's diurnal rotation, the effect of CIS differences ($\alpha_1$, $\alpha_2$) on polar motion components are diurnal terms, while the effect of $\alpha_3$ on UT1 is again a bias.

The direct way to determine all the $\beta$ angles is the method of station collocation, i.e., to position two different types of techniques at the same location.

The "observation" equation is

$$\Delta \underline{x}_i = \underline{x}_i^{I} - \underline{x}_i^{II} = - \begin{vmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{vmatrix} + \begin{vmatrix} 0 & \beta_3 & -\beta_2 \\ -\beta_3 & 0 & \beta_1 \\ \beta_2 & -\beta_1 & 0 \end{vmatrix} \begin{vmatrix} x_i \\ y_i \\ z_i \end{vmatrix} + c \begin{vmatrix} x_i \\ y_i \\ z_i \end{vmatrix} + \underline{v}_i$$

where $\underline{x}_i^{I}$ and $\underline{x}_i^{II}$ are the determined coordinates of the same collocated station i in the two CTS's, $\underline{\delta}_i$ is the translation vector, and c is the scale difference. One must have at least three collocated stations if all seven unknowns are to be solved for.

For connecting the CIS's, there are a few methods such as the use of space astrometry to connect the stellar CIS and the radio source CIS, or using differential VLBI (which, for example, was used when the Viking Mars Orbiters and a quasar were near eclipsing) to connect the planetary and radio source CIS's (see [Kovalevsky and Mueller 1981]). These are direct approaches. One indirect method is via station collocation, i.e., using the earth as an intermediate body (see [Kovalevsky 1980]): First by station collocation one determines the CTS difference ($\beta$ angles) as above, then through earth rotation parameter differences determined over the same time period one finds the CIS difference ($\alpha$ angles). Eq. (A.5) is the basis for connecting the two CIS's. More details on this subject may be found in [Mueller et al. 1982].

When considering the above method one should note that the diurnal polar motion difference terms in eq. (A.5) will show up as long as there are differences between the two CIS's (i.e., $\alpha_1$ and $\alpha_2$ exist). This may even be the case in situations when one (or both) of the techniques solve for rotations of its CIS, resulting in no (individual) diurnal polar motion. This, of course, would mean that the adopted precessional constant is discarded.

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REFERENCES

Aoki, S., B. Guinot, G.H. Kaplan, H. Kinoshita, D.D. McCarthy and P.K. Seidelmann (1982) The new Definition of Universal Time, Astron. Astrophys. 105, 359-361.

Fricke, W. (1981) Definition and Practical Realization of the Reference Frame in the FK5--The Role of Planetary Dynamics and Stellar Kinematics in the Definition, Reference Coordinate Systems for Earth Dynamics, E.M. Gaposchkin and B. Kołaczek, eds., Reidel.

Guinot, B. (1979) Basic Problems in the Kinematics of the Rotation of the Earth, Time and the Earth's Rotation, D.D. McCarthy and J.D.H. Pilkington, eds., Reidel, 6-18.

Kovalevsky, J. (1980) Modern Approaches to Research in Dynamical Astronomy, Astronomische Gessellschaft Mitteilungen 48, Hamburg.

Kovalevsky, J. and I.I. Mueller (1981) Comments on Conventional Terrestrial and Quasi-Inertial Reference Systems, Reference Coordinate Systems for Earth Dynamics, E.M. Gaposchkin and B. Kołaczek, eds., Reidel.

Mueller, I.I. (1969) Spherical and Practical Astronomy As Applied to Geodesy Ungar Publ., New York.

Mueller, I.I. (1981) Reference Coordinate Systems for Earth Dynamics: A Preview, Reference Coordinate Systems for Earth Dynamics, E.M. Gaposchkin and B. Kołaczek, eds., Reidel.

Mueller, I,I. S.-Y. Zhu and Y. Bock (1982) Reference Frame Requirements and the MERIT Campaign, Dep. of Geodetic Science and Surveying Rep. 329, Ohio State Univ., Columbus.

Murray, C.A. (1979) The Ephemeris Reference Frame for Astrometry, Time and the Earth's Rotation, D.D. McCarthy and J.D.H. Pilkington, eds., Reidel, 165-168.

Williams, J.G. and W.G. Melbourne (1981) Comments on the Effect of Adopting New Precession and Equinox Correction, Proc. of IAU Colloquium 63: High Precision Earth Rotation and Earth-Moon Dynamics: Lunar Distances and Related Observations, O. Calame, ed., Reidel.

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## 2.2 Utilization of Range-Difference Observations in Geodynamics (Research Contract NAS5-25888)

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### 2.21 Utilization of Simultaneous Lageos Range-Differences in Geodynamics

Introduction

The following is a summary of the research performed during the past six months under the Lageos project, dealing with the utilization of simultaneous laser range-differences (SRD) for the determination of earth orientation and baseline variations. Reported are some results from the Aug. 1980 Lageos data collected during the short MERIT campaign, and simulations for a possible station arrangement for the main campaign (to begin in 1983).

2.211 Simulations for a proposed MERIT83 laser network.

Based on an optimal global laser station distribution (likely to be realizable by mid-1983) proposed at a recent meeting of the study group (cf. GOTES proposal in last semiannual report), a simulation study for baseline recovery was performed. Except for the fact that different stations (seventeen total) are involved, this simulation was similar to the one previously reported for the MERIT80 network in the last report. The station locations and the data distribution are given in Table 1 [ (a)-(b) ]. Baseline estimates and their statistics were computed for both the range and the SRD adjustments. In order to assess the effect of orbital biases on the baseline recovery, the orbit used in the adjustments (range and SRD) was biased as follows :

| | | |
|---|---|---|
| Radial bias | : | 2.00 m |
| Along track bias | : | 0.60 m |
| Across track bias | : | -1.20 m |

Two different adjustments were performed.In the first case the coordinates of all stations were obtained in a simultaneous adjustment based on the data collected from all baseline pairs. On the basis of this solution the baselines between all possible station combinations were obtained along with their formal accuracies and differences with respect to their "true" values. The results of this solution for the station coordinates are given in Table 2 for the range adjustment and in Table 3 for the SRD adjustment. The baseline results are shown in Table 4.

As it can be seen from the last table, in all cases except for two, the baseline lengths have been overestimated although the errors in the SRD case are about an order of magnitude smaller than the ones for the range adjustment. Since the radial bias results in an "expansion" of the network of satellite positions, this should come as no surprise. The stations have a global distribution and since the observations from all stations are adjusted simultaneously, their positions become interdependent and the aforementioned expansion affects all of them similarly.

The results of this first adjustment prompted us to test the recovery of baselines from individual adjustments. In this second case the data collected from each pair of stations are adjusted independently and the estimated baselines are only the ones defined by coobserving station pairs. The results of this second type solution are shown in Table 5. What is obvious again is that the SRD results are again superior to the range results for baselines and station coordinates alike. The quality of the results with respect to the latter is characterized by the norm $||X||$ of the six coordinate differences between the true and estimated positions of the stations defining each baseline.

The most interesting observation though in this solution is that on the basis of the same data, the range adjustment now underestimates the baselines and the recovery errors are all negative. For the SRD results, there seems to be no bias preference and those errors are rather randomly distributed and in almost all cases at the centimeter level. The three baselines for which the range adjustment has given better results than the SRD, all have lengths in excess of 7000 km and very few observations. As it has been previously reported the SRD mode is much more geometry dependent than the range mode, and as the results of this table show it admits of its limitations very eagerly (note the formal accuracies on those baselines !). Unlike the SRD mode, the formal accuracies for the range mode give no hint whatsoever as to the real accuracy of the results. Even though the recovery errors are of the order of a few decimeters in all cases, the reported $\sigma$'s are hardly ever higher than 2 cm !

On the basis of these simulations one can conclude that the

Table 1 (a)

PARTICIPATING STATION COORDINATE LIST

| STATION | # | LATITUDE | | | LONGITUDE | | | HEIGHT | X (M) | Y (M) | Z (M) | SIGMA (M) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| FTDAVS | 7086 | 30 | 38 | 0.0 | 256 | 3 | 0.0 | 1580.000 | -1324510.442 | -5332139.932 | 3231791.956 | 0.100 |
| WETTZE | 7914 | 49 | 9 | 0.0 | 12 | 53 | 0.0 | 0.0 | 4074613.305 | 931963.678 | 4801492.271 | 0.100 |
| HAWAII | 7120 | 20 | 42 | 0.0 | 203 | 44 | 0.0 | 0.0 | -5464096.683 | -2402363.153 | 2240358.273 | 0.100 |
| STALAS | 7063 | 39 | 1 | 0.0 | 283 | 10 | 0.0 | 0.0 | 1130304.818 | -4831721.449 | 3993759.624 | 0.100 |
| WESTFO | 7091 | 42 | 36 | 46.518 | 288 | 30 | 22.720 | 67.400 | 1492212.742 | -4458121.791 | 4296085.489 | 0.100 |
| ONSALA | 7095 | 57 | 0 | 0.0 | 13 | 0 | 0.0 | 0.0 | 3392750.872 | 783278.257 | 5325986.607 | 0.100 |
| HERSTM | 7911 | 51 | 0 | 0.0 | 0 | 0 | 0.0 | 0.0 | 4022035.768 | 0.0 | 4933550.635 | 0.100 |
| GRAZ | 7999 | 48 | 0 | 0.0 | 15 | 0 | 0.0 | 0.0 | 4130031.490 | 1106638.602 | 4716882.075 | 0.100 |
| JAPAN | 7935 | 35 | 0 | 0.0 | 142 | 0 | 0.0 | 0.0 | -4121637.800 | 3220176.370 | 3637871.320 | 0.100 |
| RICHMO | 7069 | 25 | 36 | 47.130 | 279 | 37 | 4.200 | 100.000 | 961533.601 | -5674186.968 | 2740519.741 | 0.100 |
| QUINCY | 7051 | 39 | 58 | 0.0 | 239 | 4 | 0.0 | 0.0 | -2516274.896 | -4198843.469 | 4075154.589 | 0.100 |
| YARAGA | 7090 | -29 | -3 | 0.0 | 115 | 21 | 0.0 | 0.0 | -2389125.331 | 5042839.038 | -3078750.728 | 0.100 |
| CHILBO | 7901 | 53 | 0 | 0.0 | 358 | 0 | 0.0 | 0.0 | 3844341.319 | -134247.357 | 5070549.690 | 0.100 |
| ORRORA | 7943 | -45 | 0 | 0.0 | 160 | 0 | 0.0 | 1000.000 | -4245816.653 | 1545350.882 | -4488060.975 | 0.100 |
| DIONYS | 7940 | 37 | 0 | 0.0 | 22 | 0 | 0.0 | 0.0 | 4728637.251 | 1910493.462 | 3817397.791 | 0.100 |
| AREQUI | 7907 | -16 | -28 | 0.0 | 288 | 30 | 0.0 | 0.0 | 1941330.115 | -5802924.122 | -1796312.986 | 0.100 |
| GRASSE | 7942 | 44 | 0 | 0.0 | 8 | 0 | 0.0 | 0.0 | 4550759.258 | 639567.505 | 4408696.973 | 0.100 |

(b) Distribution of Observations per Baseline

BASELINE

| PASS# | B0608 | B0815 | B0602 | B0510 | B0416 | B0116 | B1016 | B1001 | B0411 | B1101 | B0311 | B0301 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3739 | 3310 | 3806 | 2248 | 512 | 497 | 858 | 2055 | 1972 | 2648 | 1475 | 925 |
| *TOTAL* | 3739 | 3310 | 3806 | 2248 | 512 | 497 | 858 | 2056 | 1972 | 2648 | 1475 | 925 |

BASELINE

| PASS# | B1302 | B0615 | B1708 | B0617 | B0506 | B0407 | B1017 | B0715 | B1317 | B1702 | B0706 | B1715 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3601 | 2988 | 3730 | 3477 | 1297 | 1221 | 223 | 2942 | 3531 | 3734 | 3725 | 3248 |
| *TOTAL* | 3601 | 2988 | 3730 | 3477 | 1297 | 1221 | 223 | 2942 | 3531 | 3734 | 3725 | 3248 |

BASELINE

| PASS# | B0104 | B0316 | B0501 | B0314 | B0309 | B0914 | B0912 | B1214 | B1203 | B0911 | *TOTAL* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2333 | 0 | 2174 | 48 | 671 | 0 | 183 | 1613 | 0 | 206 | 64991 |
| *TOTAL* | 2333 | 0 | 2174 | 48 | 671 | 0 | 183 | 1613 | 0 | 206 | 64991 |

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Table 2 Recovered Station Coordinates (Range Mode)

| | | | |
|---|---|---|---|
| STATION NO. : 7051 | X | Y | Z |
| APRIORI ESTIMATE | -2516274.896042 | -4198843.469479 | 4075154.588717 |
| ADJUSTMENTS | -1.287283 | -1.351438 | 1.822092 |
| ADJUSTED POSITION | -2516276.183325 | -4198844.820917 | 4075156.410809 |
| STATION NO. : 7063 | X | Y | Z |
| APRIORI ESTIMATE | 1130304.817676 | -4831721.449137 | 3993759.624496 |
| ADJUSTMENTS | -0.267394 | -1.937618 | 1.854899 |
| ADJUSTED POSITION | 1130304.550281 | -4831723.386754 | 3993761.479395 |
| STATION NO. : 7069 | X | Y | Z |
| APRIORI ESTIMATE | 961533.600910 | -5674186.967561 | 2740519.740502 |
| ADJUSTMENTS | -0.264610 | -2.204586 | 1.526215 |
| ADJUSTED POSITION | 961533.336300 | -5674189.172147 | 2740521.266717 |
| STATION NO. : 7086 | X | Y | Z |
| APRIORI ESTIMATE | -1324510.442373 | -5332139.932091 | 3231791.055906 |
| ADJUSTMENTS | -0.902349 | -1.830874 | 1.691947 |
| ADJUSTED POSITION | -1324511.344721 | -5332141.762966 | 3231792.747852 |
| STATION NO. : 7090 | X | Y | Z |
| APRIORI ESTIMATE | -2389125.331291 | 5042839.037557 | -3078750.728221 |
| ADJUSTMENTS | -0.975459 | 1.621840 | -1.359841 |
| ADJUSTED POSITION | -2389126.306750 | 5042840.659397 | -3078752.088062 |
| STATION NO. : 7091 | X | Y | Z |
| APRIORI ESTIMATE | 1492212.741998 | -4458121.790935 | 4296005.488571 |
| ADJUSTMENTS | -0.037315 | -1.836603 | 1.862885 |
| ADJUSTED POSITION | 1492212.704682 | -4458123.627538 | 4296007.351456 |
| STATION NO. : 7095 | X | Y | Z |
| APRIORI ESTIMATE | 3392750.871854 | 783278.256725 | 5325906.606633 |
| ADJUSTMENTS | 1.469007 | -0.126506 | 2.043865 |
| ADJUSTED POSITION | 3392752.340862 | 783278.130219 | 5325908.650498 |
| STATION NO. : 7120 | X | Y | Z |
| APRIORI ESTIMATE | -5464096.682969 | -2402363.153199 | 2240358.272655 |
| ADJUSTMENTS | -2.035242 | -0.395526 | 1.552528 |
| ADJUSTED POSITION | -5464098.718211 | -2402363.540725 | 2240359.825183 |
| STATION NO. : 7901 | X | Y | Z |
| APRIORI ESTIMATE | 3844341.318863 | -134247.357044 | 5070549.689834 |
| ADJUSTMENTS | 1.470786 | -0.208830 | 1.992617 |
| ADJUSTED POSITION | 3844342.789649 | -134247.565874 | 5070551.682451 |

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Table 2 (cont'd)

| | X | Y | Z |
|---|---|---|---|
| STATION NO. : 7907 | | | |
| APRIORI ESTIMATE | 1941330.114913 | -5802024.122161 | -1796312.985770 |
| ADJUSTMENTS | -0.233499 | -2.273246 | 0.774573 |
| ADJUSTED POSITION | 1941329.881414 | -5802026.395407 | -1796312.211197 |
| STATION NO. : 7911 | | | |
| APRIORI ESTIMATE | 4022035.767630 | 0.0 | 4933550.635358 |
| ADJUSTMENTS | 1.390632 | -0.343076 | 2.103931 |
| ADJUSTED POSITION | 4022037.158262 | -0.343076 | 4933552.739290 |
| STATION NO. : 7914 | | | |
| APRIORI ESTIMATE | 4074613.304579 | 931963.678222 | 4801492.271034 |
| ADJUSTMENTS | 1.547344 | -0.189697 | 2.059665 |
| ADJUSTED POSITION | 4074614.851923 | 931963.488525 | 4801494.330699 |
| STATION NO. : 7935 | | | |
| APRIORI ESTIMATE | -4121637.799587 | 3220176.370484 | 3637871.319704 |
| ADJUSTMENTS | -1.710228 | 1.738149 | 1.791909 |
| ADJUSTED POSITION | -4121639.509815 | 3220178.108633 | 3637873.111614 |
| STATION NO. : 7940 | | | |
| APRIORI ESTIMATE | 4728637.250678 | 1910493.461735 | 3817397.791492 |
| ADJUSTMENTS | 1.733165 | -0.080564 | 1.959695 |
| ADJUSTED POSITION | 4728638.983843 | 1910493.381171 | 3817399.751187 |
| STATION NO. : 7942 | | | |
| APRIORI ESTIMATE | 4550759.258444 | 639567.504711 | 4408096.973499 |
| ADJUSTMENTS | 1.616610 | -0.203136 | 2.022975 |
| ADJUSTED POSITION | 4550760.875054 | 639567.301575 | 4408098.996474 |
| STATION NO. : 7943 | | | |
| APRIORI ESTIMATE | -4245816.653287 | 1545350.881948 | -4488060.975056 |
| ADJUSTMENTS | -0.801772 | 2.071760 | -1.331585 |
| ADJUSTED POSITION | -4245817.455059 | 1545352.953708 | -4488062.306640 |
| STATION NO. : 7999 | | | |
| APRIORI ESTIMATE | 4130031.489874 | 1106638.602427 | 4716882.074958 |
| ADJUSTMENTS | 1.644699 | -0.043508 | 1.998229 |
| ADJUSTED POSITION | 4130033.134572 | 1106638.558919 | 4716884.073187 |

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Table 3 Recovered Station Coordinates (SRD Mode)

| | X | Y | Z |
|---|---|---|---|
| STATION NO. : 7051 | | | |
| APRIORI ESTIMATE | -2516274.896042 | -4198843.469479 | 4075154.588717 |
| ADJUSTMENTS | -0.562246 | 0.077679 | 0.552835 |
| ADJUSTED POSITION | -2516275.458287 | -4198843.391800 | 4075155.141552 |
| STATION NO. : 7063 | | | |
| APRIORI ESTIMATE | 1130304.817676 | -4831721.449137 | 3993759.624496 |
| ADJUSTMENTS | -0.412984 | -0.339656 | 0.568841 |
| ADJUSTED POSITION | 1130304.404692 | -4831721.788792 | 3993760.193337 |
| STATION NO. : 7069 | | | |
| APRIORI ESTIMATE | 961533.600910 | -5674186.967561 | 2740519.740502 |
| ADJUSTMENTS | -0.493799 | -0.372649 | 0.504026 |
| ADJUSTED POSITION | 961533.107111 | -5674187.340210 | 2740520.244528 |
| STATION NO. : 7086 | | | |
| APRIORI ESTIMATE | -1324510.442373 | -5332139.932091 | 3231791.055906 |
| ADJUSTMENTS | -0.614059 | -0.110826 | 0.512840 |
| ADJUSTED POSITION | -1324511.056431 | -5332140.042917 | 3231791.568746 |
| STATION NO. : 7090 | | | |
| APRIORI ESTIMATE | -2389125.331291 | 5042839.037557 | -3078750.728221 |
| ADJUSTMENTS | 0.287588 | 0.480930 | 0.222993 |
| ADJUSTED POSITION | -2389125.043703 | 5042839.518487 | -3078750.505229 |
| STATION NO. : 7091 | | | |
| APRIORI ESTIMATE | 1492212.741998 | -4458121.790935 | 4296005.488571 |
| ADJUSTMENTS | -0.348903 | -0.347854 | 0.591585 |
| ADJUSTED POSITION | 1492212.393095 | -4458122.138789 | 4296006.080157 |
| STATION NO. : 7095 | | | |
| APRIORI ESTIMATE | 3392750.871854 | 783278.256725 | 5325906.606633 |
| ADJUSTMENTS | 0.250456 | -0.309695 | 0.704342 |
| ADJUSTED POSITION | 3392751.122310 | 783277.947029 | 5325907.310975 |
| STATION NO. : 7120 | | | |
| APRIORI ESTIMATE | -5464096.682969 | -2402363.153199 | 2240358.272655 |
| ADJUSTMENTS | -0.589684 | 0.486807 | 0.474197 |
| ADJUSTED POSITION | -5464097.272654 | -2402362.666392 | 2240358.746853 |
| STATION NO. : 7901 | | | |
| APRIORI ESTIMATE | 3844341.318863 | -134247.357044 | 5070549.689834 |
| ADJUSTMENTS | 0.175767 | -0.397336 | 0.686721 |
| ADJUSTED POSITION | 3844341.494631 | -134247.754380 | 5070550.376555 |

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Table 3 (cont'd)

| | X | Y | Z |
|---|---|---|---|
| STATION NO. : 7907 | X | Y | Z |
| APRIORI ESTIMATE | 1941330.114913 | -5802024.122161 | -1796312.985770 |
| ADJUSTMENTS | -0.444374 | -0.522985 | 0.256872 |
| ADJUSTED POSITION | 1941329.670539 | -5802024.645146 | -1796312.728899 |
| STATION NO. : 7911 | X | Y | Z |
| APRIORI ESTIMATE | 4022035.767630 | 0.0 | 4933550.635358 |
| ADJUSTMENTS | 0.198181 | -0.407657 | 0.673034 |
| ADJUSTED POSITION | 4022035.965812 | -0.407657 | 4933551.308392 |
| STATION NO. : 7914 | X | Y | Z |
| APRIORI ESTIMATE | 4074613.304579 | 931963.678222 | 4801492.271034 |
| ADJUSTMENTS | 0.301524 | -0.373080 | 0.668115 |
| ADJUSTED POSITION | 4074613.606103 | 931963.305142 | 4801492.939149 |
| STATION NO. : 7935 | X | Y | Z |
| APRIORI ESTIMATE | -4121637.799587 | 3220176.370484 | 3637871.319704 |
| ADJUSTMENTS | 0.044690 | 0.580912 | 0.490394 |
| ADJUSTED POSITION | -4121637.754897 | 3220176.951396 | 3637871.810098 |
| STATION NO. : 7940 | X | Y | Z |
| APRIORI ESTIMATE | 4728637.250678 | 1910493.461735 | 3817397.791492 |
| ADJUSTMENTS | 0.444574 | -0.387945 | 0.619332 |
| ADJUSTED POSITION | 4728637.695252 | 1910493.073790 | 3817398.410824 |
| STATION NO. : 7942 | X | Y | Z |
| APRIORI ESTIMATE | 4550759.258444 | 639567.504711 | 4408096.973499 |
| ADJUSTMENTS | 0.296323 | -0.433382 | 0.648499 |
| ADJUSTED POSITION | 4550759.554768 | 639567.071329 | 4408097.621998 |
| STATION NO. : 7943 | X | Y | Z |
| APRIORI ESTIMATE | -4245816.653287 | 1545350.881948 | -4488060.975056 |
| ADJUSTMENTS | -0.249487 | 0.494039 | 0.067337 |
| ADJUSTED POSITION | -4245816.902774 | 1545351.375987 | -4488060.907719 |
| STATION NO. : 7999 | X | Y | Z |
| APRIORI ESTIMATE | 4130031.489874 | 1106638.602427 | 4716882.074958 |
| ADJUSTMENTS | 0.324886 | -0.372003 | 0.668722 |
| ADJUSTED POSITION | 4130031.814759 | 1106638.230423 | 4716882.743680 |

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Table 4 Recovered Baselines and Associated Statistics

| | Baseline for Stations | A Priori Length | Ranges | | | SRD's | | |
|---|---|---|---|---|---|---|---|---|
| | | | Adjusted | Error | $\hat{\sigma}$ | Adjusted | Error | $\hat{\sigma}$ |
| 1 | 7051 ==> 7063 | 3701986.397 | 3701987.501 | 1.104 | 0.014 | 3701986.615 | 0.218 | 0.009 |
| 2 | 7051 ==> 7069 | 4006624.584 | 4006625.884 | 1.300 | 0.014 | 4006624.825 | 0.241 | 0.010 |
| 3 | 7051 ==> 7086 | 1848222.236 | 1848222.838 | 0.602 | 0.012 | 1848222.336 | 0.100 | 0.006 |
| 4 | 7051 ==> 7090 | 11687738.184 | 11687742.486 | 4.302 | 0.025 | 11687738.714 | 0.530 | 0.076 |
| 5 | 7051 ==> 7091 | 4022930.960 | 4022932.239 | 1.279 | 0.014 | 4022931.203 | 0.242 | 0.010 |
| 6 | 7051 ==> 7095 | 7829591.498 | 7829594.393 | 2.895 | 0.010 | 7829591.889 | 0.391 | 0.018 |
| 7 | 7051 ==> 7120 | 3909408.182 | 3909409.312 | 1.130 | 0.020 | 3909408.428 | 0.246 | 0.011 |
| 8 | 7051 ==> 7901 | 7613750.166 | 7613753.103 | 2.936 | 0.013 | 7613750.547 | 0.380 | 0.017 |
| 9 | 7051 ==> 7907 | 7544174.051 | 7544175.685 | 1.634 | 0.037 | 7544174.479 | 0.428 | 0.022 |
| 10 | 7051 ==> 7911 | 7817713.002 | 7817715.815 | 2.812 | 0.012 | 7817713.391 | 0.389 | 0.017 |
| 11 | 7051 ==> 7914 | 8384065.582 | 8384068.542 | 2.960 | 0.011 | 8384065.995 | 0.413 | 0.018 |
| 12 | 7051 ==> 7935 | 7603305.998 | 7603309.103 | 3.106 | 0.037 | 7603306.364 | 0.366 | 0.029 |
| 13 | 7051 ==> 7940 | 9480463.508 | 9480466.631 | 3.123 | 0.012 | 9480463.975 | 0.468 | 0.019 |
| 14 | 7051 ==> 7942 | 8571116.796 | 8571119.846 | 3.050 | 0.011 | 8571117.219 | 0.423 | 0.018 |
| 15 | 7051 ==> 7943 | 10455416.980 | 10455421.363 | 4.383 | 0.014 | 10455417.555 | 0.575 | 0.078 |
| 16 | 7051 ==> 7999 | 8528384.534 | 8528387.646 | 3.112 | 0.011 | 8528384.954 | 0.420 | 0.018 |
| 17 | 7063 ==> 7069 | 1519487.440 | 1519487.859 | 0.419 | 0.012 | 1519487.521 | 0.081 | 0.006 |
| 18 | 7063 ==> 7086 | 2618612.747 | 2618613.369 | 0.622 | 0.013 | 2618612.908 | 0.161 | 0.007 |
| 19 | 7063 ==> 7090 | 12645700.300 | 12645705.074 | 4.774 | 0.023 | 12645700.939 | 0.639 | 0.076 |
| 20 | 7063 ==> 7091 | 601586.746 | 601586.951 | 0.205 | 0.013 | 601586.791 | 0.045 | 0.006 |
| 21 | 7063 ==> 7095 | 6198507.839 | 6198510.154 | 2.315 | 0.010 | 6198508.137 | 0.298 | 0.016 |
| 22 | 7063 ==> 7120 | 7243088.373 | 7243090.573 | 2.200 | 0.022 | 7243088.834 | 0.461 | 0.018 |
| 23 | 7063 ==> 7901 | 5530979.463 | 5530981.811 | 2.348 | 0.013 | 5530979.726 | 0.263 | 0.016 |
| 24 | 7063 ==> 7907 | 5926566.472 | 5926567.587 | 1.115 | 0.036 | 5926566.802 | 0.331 | 0.019 |
| 25 | 7063 ==> 7911 | 5708839.391 | 5708841.621 | 2.230 | 0.012 | 5708839.660 | 0.269 | 0.016 |
| 26 | 7063 ==> 7914 | 6522380.757 | 6522383.146 | 2.389 | 0.011 | 6522381.062 | 0.305 | 0.016 |
| 27 | 7063 ==> 7935 | 9619907.288 | 9619911.155 | 3.867 | 0.034 | 9619907.812 | 0.524 | 0.030 |
| 28 | 7063 ==> 7940 | 7644381.054 | 7644383.631 | 2.577 | 0.012 | 7644381.414 | 0.360 | 0.017 |
| 29 | 7063 ==> 7942 | 6465770.398 | 6465772.873 | 2.475 | 0.011 | 6465770.699 | 0.301 | 0.016 |
| 30 | 7063 ==> 7943 | 11895840.209 | 11895844.871 | 4.663 | 0.017 | 11895840.939 | 0.731 | 0.072 |
| 31 | 7063 ==> 7999 | 6692188.453 | 6692191.006 | 2.553 | 0.011 | 6692188.766 | 0.313 | 0.016 |
| 32 | 7069 ==> 7086 | 2363121.040 | 2363121.745 | 0.705 | 0.013 | 2363121.196 | 0.156 | 0.007 |
| 33 | 7069 ==> 7090 | 12646954.988 | 12646959.747 | 4.759 | 0.024 | 12646955.633 | 0.646 | 0.073 |
| 34 | 7069 ==> 7091 | 2044497.683 | 2044498.217 | 0.534 | 0.013 | 2044497.802 | 0.119 | 0.006 |
| 35 | 7069 ==> 7095 | 7368439.441 | 7368442.015 | 2.575 | 0.011 | 7368439.812 | 0.371 | 0.017 |
| 36 | 7069 ==> 7120 | 7227981.538 | 7227983.929 | 2.391 | 0.020 | 7227982.014 | 0.476 | 0.018 |
| 37 | 7069 ==> 7901 | 6665624.561 | 6665627.134 | 2.572 | 0.013 | 6665624.894 | 0.333 | 0.017 |
| 38 | 7069 ==> 7907 | 4643187.992 | 4643188.735 | 0.743 | 0.035 | 4643188.248 | 0.256 | 0.016 |
| 39 | 7069 ==> 7911 | 6809732.429 | 6809734.911 | 2.481 | 0.012 | 6809732.766 | 0.336 | 0.017 |
| 40 | 7069 ==> 7914 | 7588155.202 | 7588157.844 | 2.642 | 0.012 | 7588155.572 | 0.370 | 0.018 |
| 41 | 7069 ==> 7935 | 10283655.504 | 10283659.652 | 4.148 | 0.037 | 10283656.062 | 0.557 | 0.031 |
| 42 | 7069 ==> 7940 | 8536867.895 | 8536870.719 | 2.823 | 0.012 | 8536868.310 | 0.415 | 0.019 |
| 43 | 7069 ==> 7942 | 7451634.061 | 7451636.775 | 2.713 | 0.011 | 7451634.423 | 0.361 | 0.018 |
| 44 | 7069 ==> 7943 | 11466935.187 | 11466939.925 | 4.738 | 0.015 | 11466935.897 | 0.710 | 0.073 |
| 45 | 7069 ==> 7999 | 7741122.810 | 7741125.605 | 2.795 | 0.012 | 7741123.188 | 0.378 | 0.018 |
| 46 | 7086 ==> 7090 | 12190017.682 | 12190022.207 | 4.525 | 0.025 | 12190018.257 | 0.575 | 0.074 |
| 47 | 7086 ==> 7091 | 3135345.207 | 3135346.041 | 0.834 | 0.013 | 3135345.406 | 0.199 | 0.008 |
| 48 | 7086 ==> 7095 | 8002263.047 | 8002265.839 | 2.792 | 0.009 | 8002263.455 | 0.408 | 0.018 |
| 49 | 7086 ==> 7120 | 5167466.031 | 5167467.779 | 1.748 | 0.019 | 5167466.358 | 0.327 | 0.014 |
| 50 | 7086 ==> 7901 | 7557522.680 | 7557525.492 | 2.812 | 0.011 | 7557523.066 | 0.385 | 0.017 |

Table 4 (cont'd)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 51 | 7086 ==> | 7907 | 6014011.635 | 6014012.800 | 1.165 | 0.035 | 6014011.974 | 0.338 | 0.018 |
| 52 | 7086 ==> | 7911 | 7740365.514 | 7740368.214 | 2.699 | 0.011 | 7740365.906 | 0.392 | 0.017 |
| 53 | 7086 ==> | 7914 | 8417451.703 | 8417454.565 | 2.861 | 0.010 | 8417452.125 | 0.421 | 0.018 |
| 54 | 7086 ==> | 7935 | 9007271.328 | 9007274.972 | 3.644 | 0.038 | 9007271.779 | 0.451 | 0.030 |
| 55 | 7086 ==> | 7940 | 9457233.777 | 9457236.821 | 3.044 | 0.011 | 9457234.249 | 0.472 | 0.020 |
| 56 | 7086 ==> | 7942 | 8459537.776 | 8459540.720 | 2.945 | 0.009 | 8459538.199 | 0.423 | 0.018 |
| 57 | 7086 ==> | 7943 | 10743836.617 | 10743841.260 | 4.643 | 0.013 | 10743837.225 | 0.608 | 0.078 |
| 58 | 7086 ==> | 7999 | 8568278.243 | 8568281.260 | 3.018 | 0.010 | 8568278.671 | 0.428 | 0.019 |
| 59 | 7090 ==> | 7091 | 12638040.638 | 12638045.406 | 4.769 | 0.022 | 12638041.280 | 0.643 | 0.077 |
| 60 | 7090 ==> | 7095 | 11054963.383 | 11054967.922 | 4.540 | 0.013 | 11054964.034 | 0.651 | 0.081 |
| 61 | 7090 ==> | 7120 | 9652947.997 | 9652951.496 | 3.498 | 0.029 | 9652948.411 | 0.413 | 0.069 |
| 62 | 7090 ==> | 7901 | 11492146.332 | 11492150.861 | 4.529 | 0.015 | 11492146.996 | 0.664 | 0.080 |
| 63 | 7090 ==> | 7907 | 11747703.941 | 11747708.043 | 4.102 | 0.031 | 11747704.602 | 0.661 | 0.059 |
| 64 | 7090 ==> | 7911 | 11433730.158 | 11433734.779 | 4.621 | 0.015 | 11433730.816 | 0.657 | 0.080 |
| 65 | 7090 ==> | 7914 | 10989879.121 | 10989883.735 | 4.613 | 0.013 | 10989879.768 | 0.647 | 0.080 |
| 66 | 7090 ==> | 7935 | 7171939.095 | 7171942.195 | 3.100 | 0.031 | 7171939.379 | 0.284 | 0.073 |
| 67 | 7090 ==> | 7940 | 10393796.101 | 10393800.671 | 4.570 | 0.013 | 10393796.733 | 0.632 | 0.077 |
| 68 | 7090 ==> | 7942 | 11117719.497 | 11117724.116 | 4.619 | 0.013 | 11117720.151 | 0.654 | 0.079 |
| 69 | 7090 ==> | 7943 | 4203079.994 | 4203079.533 | -0.461 | 0.042 | 4203080.272 | 0.279 | 0.039 |
| 70 | 7090 ==> | 7999 | 10897934.223 | 10897938.794 | 4.571 | 0.013 | 10897934.872 | 0.649 | 0.080 |
| 71 | 7091 ==> | 7095 | 5669657.481 | 5669659.600 | 2.119 | 0.011 | 5669657.738 | 0.257 | 0.015 |
| 72 | 7091 ==> | 7120 | 7539368.002 | 7539370.323 | 2.321 | 0.022 | 7539368.484 | 0.482 | 0.019 |
| 73 | 7091 ==> | 7901 | 4982802.192 | 4982804.336 | 2.145 | 0.013 | 4982802.411 | 0.220 | 0.015 |
| 74 | 7091 ==> | 7907 | 6254928.000 | 6254929.140 | 1.140 | 0.036 | 6254928.357 | 0.357 | 0.019 |
| 75 | 7091 ==> | 7911 | 5165396.234 | 5165398.253 | 2.018 | 0.012 | 5165396.461 | 0.226 | 0.014 |
| 76 | 7091 ==> | 7914 | 5998110.612 | 5998112.791 | 2.179 | 0.012 | 5998110.876 | 0.264 | 0.015 |
| 77 | 7091 ==> | 7935 | 9534396.737 | 9534400.606 | 3.869 | 0.032 | 9534397.260 | 0.523 | 0.030 |
| 78 | 7091 ==> | 7940 | 7159802.313 | 7159804.669 | 2.356 | 0.012 | 7159802.634 | 0.321 | 0.016 |
| 79 | 7091 ==> | 7942 | 5945898.372 | 5945900.626 | 2.254 | 0.011 | 5945898.632 | 0.260 | 0.015 |
| 80 | 7091 ==> | 7943 | 12088278.997 | 12088283.622 | 4.625 | 0.017 | 12088279.749 | 0.752 | 0.070 |
| 81 | 7091 ==> | 7999 | 6172664.180 | 6172666.525 | 2.345 | 0.012 | 6172664.452 | 0.271 | 0.016 |
| 82 | 7095 ==> | 7120 | 9905183.912 | 9905187.285 | 3.373 | 0.020 | 9905184.478 | 0.567 | 0.026 |
| 83 | 7095 ==> | 7901 | 1054037.162 | 1054037.247 | 0.085 | 0.010 | 1054037.210 | 0.049 | 0.005 |
| 84 | 7095 ==> | 7907 | 9808100.813 | 9808103.428 | 2.615 | 0.034 | 9808101.384 | 0.571 | 0.025 |
| 85 | 7095 ==> | 7911 | 1078641.514 | 1078641.604 | 0.090 | 0.010 | 1078641.566 | 0.052 | 0.004 |
| 86 | 7095 ==> | 7914 | 872957.116 | 872957.157 | 0.041 | 0.010 | 872957.167 | 0.051 | 0.004 |
| 87 | 7095 ==> | 7935 | 8077993.108 | 8077996.681 | 3.573 | 0.022 | 8077993.613 | 0.505 | 0.033 |
| 88 | 7095 ==> | 7940 | 2308853.694 | 2308853.924 | 0.230 | 0.009 | 2308853.824 | 0.130 | 0.006 |
| 89 | 7095 ==> | 7942 | 1484591.097 | 1484591.233 | 0.135 | 0.008 | 1484591.180 | 0.082 | 0.004 |
| 90 | 7095 ==> | 7943 | 12459631.945 | 12459636.131 | 4.185 | 0.025 | 12459632.803 | 0.857 | 0.058 |
| 91 | 7095 ==> | 7999 | 1009482.790 | 1009482.972 | 0.182 | 0.010 | 1009482.846 | 0.056 | 0.004 |
| 92 | 7120 ==> | 7901 | 9990062.600 | 9990066.034 | 3.434 | 0.021 | 9990063.173 | 0.573 | 0.026 |
| 93 | 7120 ==> | 7907 | 9093555.719 | 9093558.233 | 2.515 | 0.035 | 9093556.311 | 0.592 | 0.028 |
| 94 | 7120 ==> | 7911 | 10149450.364 | 10149453.725 | 3.361 | 0.021 | 10149450.941 | 0.577 | 0.026 |
| 95 | 7120 ==> | 7914 | 10424208.891 | 10424212.360 | 3.469 | 0.020 | 10424209.479 | 0.588 | 0.026 |
| 96 | 7120 ==> | 7935 | 5947116.046 | 5947118.193 | 2.147 | 0.045 | 5947116.282 | 0.236 | 0.028 |
| 97 | 7120 ==> | 7940 | 11179428.021 | 11179431.635 | 3.615 | 0.019 | 11179428.647 | 0.626 | 0.027 |
| 98 | 7120 ==> | 7942 | 10688768.532 | 10688772.104 | 3.572 | 0.020 | 10688769.136 | 0.604 | 0.026 |
| 99 | 7120 ==> | 7943 | 7895585.976 | 7895589.858 | 3.882 | 0.016 | 7895586.379 | 0.403 | 0.078 |
| 100 | 7120 ==> | 7999 | 10511591.643 | 10511595.224 | 3.581 | 0.020 | 10511592.237 | 0.594 | 0.027 |

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Table 4 (cont'd)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 101 | 7901 | ==> | 7907 | 9104885.947 | 9104888.507 | 2.560 | 0.034 | 9104886.479 | 0.532 | 0.025 |
| 102 | 7901 | ==> | 7911 | 261469.713 | 261469.531 | -0.182 | 0.013 | 261469.730 | 0.017 | 0.006 |
| 103 | 7901 | ==> | 7914 | 1123487.006 | 1123487.024 | 0.018 | 0.011 | 1123487.060 | 0.053 | 0.004 |
| 104 | 7901 | ==> | 7935 | 8761366.855 | 8761370.526 | 3.670 | 0.023 | 8761367.381 | 0.526 | 0.033 |
| 105 | 7901 | ==> | 7940 | 2556038.730 | 2556038.940 | 0.210 | 0.012 | 2556038.864 | 0.134 | 0.007 |
| 106 | 7901 | ==> | 7942 | 1239620.646 | 1239620.717 | 0.070 | 0.011 | 1239620.713 | 0.067 | 0.005 |
| 107 | 7901 | ==> | 7943 | 12634822.678 | 12634826.951 | 4.273 | 0.025 | 12634823.537 | 0.859 | 0.059 |
| 108 | 7901 | ==> | 7999 | 1321551.217 | 1321551.409 | 0.191 | 0.012 | 1321551.278 | 0.061 | 0.005 |
| 109 | 7907 | ==> | 7911 | 9126000.454 | 9126003.032 | 2.578 | 0.034 | 9126000.981 | 0.527 | 0.025 |
| 110 | 7907 | ==> | 7914 | 9665843.112 | 9665845.834 | 2.722 | 0.033 | 9665843.662 | 0.550 | 0.026 |
| 111 | 7907 | ==> | 7935 | 12152779.136 | 12152783.306 | 4.170 | 0.053 | 12152779.816 | 0.680 | 0.037 |
| 112 | 7907 | ==> | 7940 | 9938096.258 | 9938099.181 | 2.923 | 0.031 | 9938096.817 | 0.559 | 0.027 |
| 113 | 7907 | ==> | 7942 | 9316540.457 | 9316543.238 | 2.781 | 0.032 | 9316540.987 | 0.530 | 0.026 |
| 114 | 7907 | ==> | 7943 | 9975480.531 | 9975484.652 | 4.121 | 0.023 | 9975481.210 | 0.679 | 0.064 |
| 115 | 7907 | ==> | 7999 | 9743805.430 | 9743808.250 | 2.821 | 0.033 | 9743805.985 | 0.555 | 0.026 |
| 116 | 7911 | ==> | 7914 | 942740.742 | 942740.909 | 0.167 | 0.010 | 942740.783 | 0.041 | 0.005 |
| 117 | 7911 | ==> | 7935 | 8852555.558 | 8852559.214 | 3.655 | 0.023 | 8852556.086 | 0.528 | 0.033 |
| 118 | 7911 | ==> | 7940 | 2322728.588 | 2322728.978 | 0.389 | 0.011 | 2322728.705 | 0.117 | 0.006 |
| 119 | 7911 | ==> | 7942 | 982189.734 | 982189.990 | 0.256 | 0.010 | 982189.783 | 0.049 | 0.005 |
| 120 | 7911 | ==> | 7943 | 12629816.243 | 12629820.537 | 4.294 | 0.025 | 12629817.099 | 0.855 | 0.058 |
| 121 | 7911 | ==> | 7999 | 1132809.489 | 1132809.826 | 0.337 | 0.011 | 1132809.536 | 0.048 | 0.005 |
| 122 | 7914 | ==> | 7935 | 8588856.921 | 8588860.579 | 3.659 | 0.022 | 8588857.444 | 0.523 | 0.033 |
| 123 | 7914 | ==> | 7940 | 1534180.499 | 1534180.712 | 0.213 | 0.010 | 1534180.582 | 0.083 | 0.005 |
| 124 | 7914 | ==> | 7942 | 683352.290 | 683352.365 | 0.075 | 0.009 | 683352.323 | 0.033 | 0.003 |
| 125 | 7914 | ==> | 7943 | 12486056.145 | 12486060.345 | 4.200 | 0.026 | 12486057.002 | 0.857 | 0.056 |
| 126 | 7914 | ==> | 7999 | 201844.964 | 201845.143 | 0.179 | 0.010 | 201844.972 | 0.007 | 0.004 |
| 127 | 7935 | ==> | 7940 | 8948456.154 | 8948459.829 | 3.675 | 0.021 | 8948456.694 | 0.540 | 0.033 |
| 128 | 7935 | ==> | 7942 | 9080928.389 | 9080932.138 | 3.748 | 0.022 | 9080928.931 | 0.542 | 0.033 |
| 129 | 7935 | ==> | 7943 | 8297664.519 | 8297667.497 | 2.978 | 0.047 | 8297664.955 | 0.436 | 0.044 |
| 130 | 7935 | ==> | 7999 | 8586113.915 | 8586117.604 | 3.689 | 0.022 | 8586114.442 | 0.526 | 0.034 |
| 131 | 7940 | ==> | 7942 | 1412734.544 | 1412734.695 | 0.151 | 0.009 | 1412734.616 | 0.072 | 0.004 |
| 132 | 7940 | ==> | 7943 | 12233347.755 | 12233351.785 | 4.030 | 0.028 | 12233348.613 | 0.858 | 0.053 |
| 133 | 7940 | ==> | 7999 | 1346693.532 | 1346693.574 | 0.043 | 0.010 | 1346693.608 | 0.077 | 0.004 |
| 134 | 7942 | ==> | 7943 | 12543596.675 | 12543600.914 | 4.239 | 0.026 | 12543597.537 | 0.862 | 0.056 |
| 135 | 7942 | ==> | 7999 | 700368.121 | 700368.199 | 0.079 | 0.009 | 700368.153 | 0.033 | 0.003 |
| 136 | 7943 | ==> | 7999 | 12453042.887 | 12453047.068 | 4.181 | 0.026 | 12453043.748 | 0.861 | 0.056 |

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Table 5 Baseline Recovery from Individual Adjustments

| BASELINE FOR STATIONS | APRIORI LENGTH | RANGE ADJUSTMENT #OBS | RANGE ADJUSTMENT ‖x‖ | RANGE ADJUSTMENT ERROR | RANGE ADJUSTMENT $\hat{\sigma}$ | SRD ADJUSTMENT #OBS | SRD ADJUSTMENT ‖x‖ | SRD ADJUSTMENT ERROR | SRD ADJUSTMENT $\hat{\sigma}$ |
|---|---|---|---|---|---|---|---|---|---|
| 7901 ==> 7914 | 1123487.000 | 7202 | 3.572 | -0.117 | 0.017 | 3601 | 1.959 | -0.005 | 0.018 |
| 7095 ==> 7940 | 2308853.094 | 5976 | 3.489 | -0.362 | 0.019 | 2988 | 1.712 | -0.047 | 0.026 |
| 7942 ==> 7999 | 700300.121 | 7460 | 3.730 | -0.096 | 0.019 | 3730 | 1.196 | 0.010 | 0.016 |
| 7095 ==> 7942 | 1434591.097 | 6954 | 3.582 | -0.228 | 0.020 | 3477 | 1.371 | 0.004 | 0.020 |
| 7091 ==> 7095 | 5009057.481 | 2594 | 2.848 | -0.973 | 0.027 | 1297 | 0.454 | 0.413 | 0.326 |
| 7053 ==> 7911 | 5700039.391 | 2442 | 2.805 | -1.076 | 0.033 | 1221 | 0.445 | 0.323 | 0.279 |
| 7069 ==> 7942 | 7451634.001 | 446 | 2.605 | -1.397 | 0.165 | 223 | 0.591 | 0.311 | 5.1[illegible]3 |
| 7911 ==> 7940 | 2322728.588 | 5884 | 3.518 | -0.382 | 0.020 | 2942 | 1.619 | -0.038 | 0.030 |
| 7901 ==> 7942 | 1239020.046 | 7062 | 3.631 | -0.187 | 0.020 | 3531 | 1.471 | -0.006 | 0.020 |
| 7942 ==> 7914 | 683352.290 | 7468 | 3.728 | -0.091 | 0.019 | 3734 | 1.630 | -0.001 | 0.015 |
| 7911 ==> 7095 | 1078641.514 | 7450 | 3.594 | -0.153 | 0.020 | 3725 | 1.541 | -0.009 | 0.017 |
| 7942 ==> 7940 | 1412734.544 | 6496 | 3.684 | -0.207 | 0.020 | 3248 | 1.578 | -0.008 | 0.021 |
| 7095 ==> 7999 | 1009482.790 | 7478 | 3.618 | -0.158 | 0.020 | 3739 | 1.261 | 0.002 | 0.017 |
| 7999 ==> 7940 | 1346093.532 | 6620 | 3.671 | -0.194 | 0.019 | 3310 | 1.392 | 0.006 | 0.019 |
| 7095 ==> 7914 | 672957.116 | 7612 | 3.629 | -0.122 | 0.020 | 3806 | 1.074 | 0.025 | 0.016 |
| 7091 ==> 7069 | 2044497.683 | 4496 | 3.544 | -0.305 | 0.022 | 2248 | 1.336 | 0.010 | 0.029 |
| 7063 ==> 7907 | 5926566.472 | 1024 | 2.809 | -1.165 | 0.057 | 512 | 0.888 | 0.059 | 0.483 |
| 7086 ==> 7907 | 6014011.635 | 994 | 2.798 | -1.264 | 0.060 | 497 | 1.023 | -0.083 | 0.027 |
| 7009 ==> 7907 | 4643187.992 | 1716 | 3.052 | -0.864 | 0.037 | 858 | 0.766 | 0.168 | 0.175 |
| 7069 ==> 7086 | 2303121.040 | 4112 | 3.352 | -0.375 | 0.025 | 2056 | 1.250 | 0.041 | 0.036 |
| 7063 ==> 7051 | 3701930.397 | 3944 | 3.264 | -0.006 | 0.025 | 1972 | 1.150 | 0.038 | 0.069 |
| 7051 ==> 7086 | 1848222.236 | 5296 | 3.641 | -0.388 | 0.023 | 2648 | 1.145 | 0.020 | 0.027 |
| 7120 ==> 7051 | 3909403.182 | 2950 | 3.220 | -0.724 | 0.030 | 1475 | 1.366 | -0.035 | 0.085 |
| 7120 ==> 7080 | 5167465.031 | 1850 | 2.968 | -0.991 | 0.040 | 925 | 0.687 | 0.249 | 0.220 |
| 7080 ==> 7063 | 2513612.747 | 4666 | 3.500 | -0.452 | 0.025 | 2333 | 0.925 | 0.090 | 0.039 |
| 7091 ==> 7086 | 3135345.207 | 4348 | 3.396 | -0.569 | 0.026 | 2174 | 1.520 | -0.041 | 0.050 |
| 7120 ==> 7935 | 3947116.046 | 1342 | 2.836 | -1.211 | 0.052 | 671 | 1.570 | -0.302 | 0.443 |
| 7935 ==> 7090 | 7171939.095 | 306 | 2.609 | -1.008 | 0.155 | 183 | 7.275 | 4.760 | 3.401 |
| 7090 ==> 7943 | 4203079.994 | 3226 | 3.160 | -0.611 | 0.029 | 1613 | 1.428 | -0.009 | 0.096 |
| 7935 ==> 7051 | 7003305.998 | 412 | 2.582 | -1.458 | 0.202 | 206 | 10.245 | 7.029 | 8.130 |

SRD mode will in all likelihood provide more meaningful results in the presence of unmodeled orbital biases than the range mode, and it will also give more reliable accuracy estimates for those results. Comparing the batch (global) solution to that of individual adjustments, the latter seems to be by far a better approach in the case of SRD observations, although the opposite is true for the range observations. Compare for instance the level of recovery errors between Tables 4 and 5.

2.212 Preliminary Results from Lageos Data Analysis.

Lageos ranging data collected from ten stations over the period August 14-29,1980 (during the short MERIT campaign) were used in GEOSPP81 for baseline recovery. A total of 24240 ranges were selected with effort to balance the distribution among stations whose observability performance shows wild variations (cf. station 7090 with over 60000 ranges during August, and station 7092 with hardly over 3000 in the same period of time). The summary of the data distribution per pass per station is given in Table 6 [ (a)-(j) ]. The ill-conditioning of the normal equations due to the lack of origin of longitudes definition is overcome by applying a small weight in all three coordinates of all stations, corresponding to a $\sigma=\pm 50$ m. This way the origin of longitudes does not depend on a single station but rather the ensemble of them. The separation of the X and Y coordinates is thus not as good as it would be if one longitude were fixed absolutely, but that has no effect on the baselines. This high correlation between X and Y is also reflected in the estimated formal accuracies for these coordinates, Table 7. The orbital model and the constants used in the solution are shown in Table 8. Baseline results of the adjustment are given in Table 9, and an analytical breakdown of the residuals after the adjustment are given in Table 10 [ (a)-(j) ]. Notice that the fact that station 7907 (ARELAS) is the one with the fewest observations (only 489) shows very clearly in the estimation of baselines which emanate from that station (Table 9).

Care should be taken in comparing these results with other solutions for the fact that these baselines are reckoned between the optical centers of the corresponding laser instruments and not the stations' validation points.

This investigation is now being completed, and the final report is in preparation by E. Pavlis, to appear in the report series of the Department of Geodetic Science and Surveying, The Ohio State University.

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Table 6 (a)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7063   NUMBER OF PASSES TRACKED : 10   NUMBER OF OBSERVATIONS : 4284

| PASS NO. | BEGINNING DATE YYMMDD HHMMSS.S | ENDING DATE YYMMDD HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|
| 1 | 800814 1611 4.0 | 800814 163236.0 | 1292.0 | 4 | 323.00 |
| 2 | 800820 814 6.0 | 800820 84127.0 | 1641.0 | 477 | 3.44 |
| 3 | 800822 90040.0 | 800822 92534.0 | 1494.0 | 202 | 7.40 |
| 4 | 800822 122631.0 | 800822 125440.0 | 1689.0 | 6 | 281.50 |
| 5 | 800825 82148.0 | 800825 901 6.0 | 2358.0 | 859 | 2.75 |
| 6 | 800825 153110.0 | 800825 153110.0 | 0.0 | 1 | 0.0 |
| 7 | 800827 90532.0 | 800827 95222.0 | 2810.0 | 1550 | 1.81 |
| 8 | 800827 160010.0 | 800827 162513.0 | 1503.0 | 4 | 375.75 |
| 9 | 800827 192326.0 | 800827 200556.0 | 2550.0 | 14 | 182.14 |
| 10 | 800828 74317.0 | 800828 82441.0 | 2484.0 | 1167 | 2.13 |

(b)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7090   NUMBER OF PASSES TRACKED : 30   NUMBER OF OBSERVATIONS : 4143

| PASS NO. | BEGINNING DATE YYMMDD HHMMSS.S | ENDING DATE YYMMDD HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|
| 1 | 800814 73524.0 | 800814 81120.0 | 2156.0 | 97 | 22.23 |
| 2 | 800814 110144.0 | 800814 1138 6.0 | 2182.0 | 167 | 13.07 |
| 3 | 800814 1735 8.0 | 800814 1817 7.0 | 2519.0 | 182 | 13.84 |
| 4 | 800814 210657.0 | 800814 2148 8.0 | 2471.0 | 207 | 11.94 |
| 5 | 800815 93831.0 | 800815 102222.0 | 2631.0 | 196 | 13.42 |
| 6 | 800815 162235.0 | 800815 165245.0 | 1810.0 | 141 | 12.84 |
| 7 | 800815 1941 5.0 | 800815 202838.0 | 2853.0 | 263 | 10.85 |
| 8 | 800818 91445.0 | 800818 94842.0 | 2037.0 | 119 | 17.12 |
| 9 | 800818 124141.0 | 800818 1259 1.0 | 1040.0 | 67 | 15.52 |
| 10 | 800818 190634.0 | 800818 1953 1.0 | 2787.0 | 171 | 16.30 |
| 11 | 800819 111532.0 | 800819 114928.0 | 2036.0 | 136 | 14.97 |
| 12 | 800819 175015.0 | 800819 183136.0 | 2481.0 | 203 | 12.22 |
| 13 | 800819 212421.0 | 800819 213355.0 | 574.0 | 50 | 11.48 |
| 14 | 800820 629 6.0 | 800820 65033.0 | 1287.0 | 29 | 44.38 |
| 15 | 800820 200224.0 | 800820 204123.0 | 2339.0 | 136 | 17.20 |
| 16 | 800821 83132.0 | 800821 91537.0 | 2645.0 | 104 | 25.43 |
| 17 | 800822 72745.0 | 800822 74943.0 | 1318.0 | 55 | 23.96 |
| 18 | 800822 104041.0 | 800822 111838.0 | 2277.0 | 162 | 14.06 |
| 19 | 800822 171526.0 | 800822 175657.0 | 2491.0 | 173 | 14.40 |
| 20 | 800822 204756.0 | 800822 2128 5.0 | 2409.0 | 136 | 17.71 |
| 21 | 800826 84746.0 | 800826 928 7.0 | 2421.0 | 155 | 15.62 |
| 22 | 800826 122016.0 | 800826 123826.0 | 1090.0 | 41 | 26.59 |
| 23 | 800826 1534 1.0 | 800826 155759.0 | 1438.0 | 88 | 16.34 |
| 24 | 800826 1847 5.0 | 800826 193427.0 | 2842.0 | 233 | 12.20 |
| 25 | 800827 105652.0 | 800827 112919.0 | 1947.0 | 115 | 16.93 |
| 26 | 800827 143042.0 | 800827 143215.0 | 93.0 | 6 | 15.50 |
| 27 | 800827 172746.0 | 800827 181144.0 | 2638.0 | 169 | 13.96 |
| 28 | 800827 210429.0 | 800827 213938.0 | 2109.0 | 154 | 13.69 |
| 29 | 800828 161247.0 | 800828 164735.0 | 2088.0 | 154 | 13.56 |
| 30 | 800828 1937 5.0 | 800828 2022 5.0 | 2700.0 | 214 | 12.62 |

(c)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7091   NUMBER OF PASSES TRACKED : 4   NUMBER OF OBSERVATIONS : 1168

| PASS NO. | BEGINNING DATE YYMMDD HHMMSS.S | ENDING DATE YYMMDD HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|
| 1 | 800815 181925.0 | 800815 183830.0 | 1145.0 | 137 | 8.36 |
| 2 | 800818 72153.0 | 800818 75031.0 | 1718.0 | 352 | 4.88 |
| 3 | 800826 65525.0 | 800826 72644.0 | 1879.0 | 240 | 7.83 |
| 4 | 800826 103656.0 | 800826 110941.0 | 1965.0 | 439 | 4.48 |

(d)

O B S E R V A T I O N   S U M M A R Y

STATION IDENTIFICATION NO. : 7092   NUMBER OF PASSES TRACKED : 5   NUMBER OF OBSERVATIONS : 2253

| PASS NO. | BEGINNING DATE YYMMDD HHMMSS.S | ENDING DATE YYMMDD HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|
| 1 | 800823 153228.0 | 800823 160149.0 | 1761.0 | 322 | 5.47 |
| 2 | 800824 141353.0 | 800824 1433 6.0 | 1153.0 | 286 | 4.03 |
| 3 | 800824 173125.0 | 800824 1811 5.0 | 2380.0 | 1273 | 1.87 |
| 4 | 800825 1613 4.0 | 800825 165055.0 | 2271.0 | 363 | 6.26 |
| 5 | 800826 150956.0 | 800826 151250.0 | 174.0 | 9 | 19.33 |

Table 6 (cont'd)

(e) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7096 NUMBER OF PASSES TRACKED : 6 NUMBER OF OBSERVATIONS : 2450

| PASS NO. | BEGINNING DATE YYMMDD | BEGINNING HHMMSS.S | ENDING DATE YYMMDD | ENDING HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|---|---|
| 1 | 800817 | 130518.0 | 800817 | 1345 7.0 | 2389.0 | 969 | 2.47 |
| 2 | 800818 | 150931.0 | 800818 | 1543 0.0 | 2009.0 | 461 | 4.36 |
| 3 | 800819 | 141549.0 | 800819 | 143418.0 | 1109.0 | 268 | 4.14 |
| 4 | 800820 | 62538.0 | 800820 | 63030.0 | 652.0 | 91 | 7.16 |
| 5 | 800827 | 35759.0 | 800827 | 41323.0 | 924.0 | 45 | 20.55 |
| 6 | 800827 | 135148.0 | 800827 | 141436.0 | 1368.0 | 616 | 2.22 |

(f) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7114 NUMBER OF PASSES TRACKED : 11 NUMBER OF OBSERVATIONS : 1866

| PASS NO. | BEGINNING DATE YYMMDD | BEGINNING HHMMSS.S | ENDING DATE YYMMDD | ENDING HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|---|---|
| 1 | 800820 | 1148 6.0 | 800820 | 120748.0 | 1182.0 | 182 | 6.49 |
| 2 | 800820 | 2210 3.0 | 800820 | 222859.0 | 1136.0 | 17 | 66.82 |
| 3 | 800821 | 102434.0 | 800821 | 110649.0 | 2535.0 | 855 | 2.96 |
| 4 | 800821 | 205140.0 | 800821 | 211427.0 | 1367.0 | 9 | 151.89 |
| 5 | 800822 | 1234 9.0 | 800822 | 125059.0 | 1010.0 | 161 | 6.27 |
| 6 | 800825 | 115845.0 | 800825 | 123348.0 | 2103.0 | 390 | 5.39 |
| 7 | 800826 | 210644.0 | 800826 | 212131.0 | 887.0 | 6 | 147.83 |
| 8 | 800827 | 92627.0 | 800827 | 94352.0 | 1045.0 | 228 | 4.58 |
| 9 | 800827 | 125149.0 | 800827 | 1303 5.0 | 676.0 | 7 | 96.57 |
| 10 | 800827 | 194956.0 | 800827 | 200756.0 | 1080.0 | 7 | 154.29 |
| 11 | 800827 | 230754.0 | 800827 | 231434.0 | 400.0 | 4 | 100.00 |

(g) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7115 NUMBER OF PASSES TRACKED : 12 NUMBER OF OBSERVATIONS : 2265

| PASS NO. | BEGINNING DATE YYMMDD | BEGINNING HHMMSS.S | ENDING DATE YYMMDD | ENDING HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|---|---|
| 1 | 800814 | 131518.0 | 800814 | 133542.0 | 1224.0 | 264 | 4.64 |
| 2 | 800814 | 195338.0 | 800814 | 200529.0 | 711.0 | 29 | 24.52 |
| 3 | 800815 | 1143 3.0 | 800815 | 121624.0 | 2001.0 | 384 | 5.21 |
| 4 | 800815 | 215545.0 | 800815 | 221246.0 | 1021.0 | 27 | 37.81 |
| 5 | 800818 | 110623.0 | 800818 | 114414.0 | 2271.0 | 500 | 4.54 |
| 6 | 800818 | 143855.0 | 800818 | 150048.0 | 1313.0 | 38 | 34.55 |
| 7 | 800818 | 211928.0 | 800818 | 214154.0 | 1346.0 | 172 | 7.83 |
| 8 | 800819 | 95759.0 | 800819 | 101248.0 | 889.0 | 63 | 14.11 |
| 9 | 800819 | 131239.0 | 800819 | 134432.0 | 1913.0 | 119 | 16.08 |
| 10 | 800820 | 114827.0 | 800820 | 123354.0 | 2727.0 | 588 | 4.64 |
| 11 | 800821 | 140436.0 | 800821 | 143124.0 | 1608.0 | 37 | 43.46 |
| 12 | 800822 | 1244 7.0 | 800822 | 125459.0 | 652.0 | 44 | 14.82 |

(h) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7120 NUMBER OF PASSES TRACKED : 10 NUMBER OF OBSERVATIONS : 1904

| PASS NO. | BEGINNING DATE YYMMDD | BEGINNING HHMMSS.S | ENDING DATE YYMMDD | ENDING HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|---|---|
| 1 | 800814 | 131956.0 | 800814 | 135240.0 | 1964.0 | 225 | 8.73 |
| 2 | 800814 | 165838.0 | 800814 | 171227.0 | 829.0 | 44 | 18.84 |
| 3 | 800815 | 25247.0 | 800815 | 31424.0 | 1297.0 | 160 | 8.11 |
| 4 | 800815 | 121219.0 | 800815 | 122237.0 | 618.0 | 42 | 14.71 |
| 5 | 800815 | 152236.0 | 800815 | 160610.0 | 2614.0 | 187 | 13.98 |
| 6 | 800821 | 141436.0 | 800821 | 150035.0 | 2759.0 | 346 | 7.97 |
| 7 | 800822 | 125343.0 | 800822 | 133636.0 | 2573.0 | 401 | 6.42 |
| 8 | 800822 | 163425.0 | 800822 | 165250.0 | 865.0 | 50 | 17.30 |
| 9 | 800825 | 155656.0 | 800825 | 162431.0 | 1655.0 | 121 | 13.68 |
| 10 | 800828 | 1519 7.0 | 800828 | 155924.0 | 2417.0 | 328 | 7.37 |

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Table 6 (cont'd)

(i) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7907 NUMBER OF PASSES TRACKED : 20 NUMBER OF OBSERVATIONS : 469

| PASS NO. | BEGINNING DATE YYMMDD | HHMMSS.S | ENDING DATE YYMMDD | HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|---|---|
| 1 | 800814 | 62230.1 | 800814 | 62722.5 | 292.4 | 12 | 24.37 |
| 2 | 800814 | 955 7.4 | 800814 | 1014 0.3 | 1133.0 | 41 | 27.63 |
| 3 | 800815 | 82930.0 | 800815 | 84352.5 | 862.5 | 51 | 16.91 |
| 4 | 800816 | 70052.5 | 800816 | 718 0.2 | 1027.8 | 19 | 54.09 |
| 5 | 800816 | 234237.7 | 800816 | 234837.8 | 360.2 | 5 | 72.03 |
| 6 | 800817 | 92152.3 | 800817 | 93645.2 | 892.9 | 52 | 17.17 |
| 7 | 800818 | 75452.3 | 800818 | 80552.6 | 660.3 | 24 | 27.51 |
| 8 | 800819 | 633 7.5 | 800819 | 64315.1 | 607.6 | 19 | 31.96 |
| 9 | 800819 | 101022.6 | 800819 | 102945.2 | 1162.6 | 35 | 33.22 |
| 10 | 800820 | 84452.5 | 800820 | 900 7.4 | 914.9 | 53 | 17.26 |
| 11 | 800821 | 71730.1 | 800821 | 73337.6 | 967.5 | 34 | 28.46 |
| 12 | 800822 | 60230.0 | 800822 | 60830.1 | 360.1 | 5 | 72.02 |
| 13 | 800822 | 93315.1 | 800822 | 95145.1 | 1110.0 | 24 | 46.25 |
| 14 | 800823 | 80922.6 | 800823 | 82052.6 | 690.0 | 17 | 40.59 |
| 15 | 800824 | 65237.6 | 800824 | 65937.6 | 420.0 | 8 | 52.50 |
| 16 | 800825 | 52522.5 | 800825 | 533 7.6 | 465.0 | 9 | 51.67 |
| 17 | 800825 | 90122.3 | 800825 | 91530.3 | 848.0 | 22 | 38.54 |
| 18 | 800827 | 95015.0 | 800827 | 1007 7.5 | 1012.5 | 28 | 36.16 |
| 19 | 800828 | 45537.5 | 800828 | 457 7.5 | 90.0 | 2 | 45.00 |
| 20 | 800828 | 82622.6 | 800828 | 83922.6 | 780.0 | 29 | 26.90 |

(j) OBSERVATION SUMMARY

STATION IDENTIFICATION NO. : 7943 NUMBER OF PASSES TRACKED : 32 NUMBER OF OBSERVATIONS : 3416

| PASS NO. | BEGINNING DATE YYMMDD | HHMMSS.S | ENDING DATE YYMMDD | HHMMSS.S | DURATION SECONDS | OBSERVATIONS | DENSITY LAG X (ONE POINT PER X SECS) |
|---|---|---|---|---|---|---|---|
| 1 | 800814 | 1101 0.2 | 800814 | 112015.2 | 1154.9 | 56 | 20.62 |
| 2 | 800815 | 92745.0 | 800815 | 100630.3 | 2325.3 | 122 | 19.06 |
| 3 | 800815 | 124959.9 | 800815 | 132145.0 | 1905.1 | 82 | 23.23 |
| 4 | 800815 | 1603 7.7 | 800815 | 1641 0.0 | 2272.3 | 98 | 23.19 |
| 5 | 800815 | 1945 7.6 | 800815 | 200952.4 | 1484.7 | 83 | 17.89 |
| 6 | 800816 | 81544.9 | 800816 | 84152.7 | 1567.8 | 65 | 18.44 |
| 7 | 800816 | 114022.8 | 800816 | 1204 7.8 | 1425.0 | 90 | 15.83 |
| 8 | 800816 | 144522.7 | 800816 | 1531 7.7 | 2745.0 | 212 | 12.95 |
| 9 | 800816 | 181515.0 | 800816 | 190345.0 | 2910.0 | 159 | 18.30 |
| 10 | 800817 | 101652.8 | 800817 | 1049 7.7 | 1934.9 | 47 | 41.17 |
| 11 | 800817 | 134030.4 | 800817 | 140545.0 | 1514.7 | 54 | 28.05 |
| 12 | 800817 | 170115.0 | 800817 | 1728 0.0 | 1605.0 | 75 | 21.40 |
| 13 | 800818 | 90422.7 | 800818 | 93445.2 | 1822.5 | 139 | 13.11 |
| 14 | 800818 | 1217 0.2 | 800818 | 124837.7 | 1897.5 | 100 | 18.97 |
| 15 | 800818 | 153015.1 | 800818 | 161930.1 | 2955.0 | 170 | 17.38 |
| 16 | 800818 | 191822.5 | 800818 | 194952.5 | 1890.0 | 106 | 17.83 |
| 17 | 800819 | 110015.2 | 800819 | 113215.1 | 1919.9 | 90 | 21.33 |
| 18 | 800820 | 125830.2 | 800820 | 133437.7 | 2167.5 | 44 | 49.26 |
| 19 | 800821 | 1507 7.7 | 800821 | 154515.2 | 2287.5 | 79 | 28.96 |
| 20 | 800821 | 183030.1 | 800821 | 191159.9 | 2489.8 | 178 | 13.99 |
| 21 | 800822 | 102622.7 | 800822 | 105115.2 | 1492.5 | 38 | 39.28 |
| 22 | 800822 | 134252.8 | 800822 | 141745.3 | 2092.5 | 88 | 23.78 |
| 23 | 800822 | 170915.1 | 800822 | 175622.4 | 2827.3 | 172 | 16.44 |
| 24 | 800823 | 92930.2 | 800823 | 940 0.1 | 629.9 | 73 | 8.63 |
| 25 | 800825 | 131730.2 | 800825 | 134430.1 | 1619.9 | 67 | 24.18 |
| 26 | 800825 | 163237.7 | 800825 | 165937.4 | 1619.7 | 96 | 16.87 |
| 27 | 800826 | 84152.6 | 800826 | 91352.5 | 1919.9 | 163 | 11.78 |
| 28 | 800826 | 115522.7 | 800826 | 122837.7 | 1995.0 | 191 | 10.44 |
| 29 | 800826 | 150922.7 | 800826 | 155922.5 | 2999.8 | 262 | 11.45 |
| 30 | 800828 | 92537.5 | 800828 | 958 0.2 | 1942.7 | 97 | 20.03 |
| 31 | 800828 | 1238 0.0 | 800828 | 131415.1 | 2175.1 | 61 | 35.66 |
| 32 | 800828 | 155722.7 | 800828 | 162230.1 | 1507.4 | 41 | 36.77 |

## Table 7 A Priori Station Information and Final Solution Summary

APRIORI STATION INFORMATION :

| STATION # | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| 7063 | 1130711.7 | -4831371.3 | 3994088.7 |
| 7090 | -2389002.3 | 5043333.5 | -3078528.5 |
| 7091 | 1492450.9 | -4457261.7 | 4296817.0 |
| 7092 | -6143448.5 | 1364706.9 | 1034164.6 |
| 7096 | -6100049.6 | -996197.8 | -1568978.3 |
| 7114 | -2410426.2 | -4477802.2 | 3838606.1 |
| 7115 | -2350667.4 | -4655546.1 | 3660999.2 |
| 7120 | -5466003.7 | -2404404.3 | 2242228.6 |
| 7907 | 1942786.1 | -5804078.9 | -1796938.6 |
| 7943 | -4447545.7 | 2677137.8 | -3694998.0 |

APRIORI WEIGHT MTX. :

STATION# 1
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 2
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 3
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 4
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 5
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 6
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 7
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 8
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 9
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

STATION# 10
```
0.40000-03
0.0         0.40000-03
0.0         0.0         0.40000-03
```

ADJUSTMENT STATISTICS FOR ITERATION : 2

```
DEGREES OF FREEDOM FOR THIS ADJUSTMENT.......................: 24234
PREVIOUS WEIGHTED SUM OF SQUARES OF THE RESIDUALS/D.F.......:    0.1760
CURRENT  WEIGHTED SUM OF SQUARES OF THE RESIDUALS/D.F.......:    0.1760
IMPROVEMENT IN PERCENT (NEG.SIGN INDICATES DETERIORATION)..:    0
CONTRIBUTION FROM STATION PARAMETER CONSTRAINTS.............:  1745.D-04
CONTRIBUTION FROM POLAR MOTION PARAMETER CONSTRAINTS.......:  5508.D-12
CONTRIBUTION FROM ORBITAL PARAMETER CONSTRAINTS.............:    0.
```

PASS BY PASS BREAKDOWN OF ADJUSTMENT STATISTICS FOR ITERATION : 2

| PASS NO. | CONSTRAINTS FROM : STATIONS | P.M.STEPS | ORBIT | TOTAL NO.OF CONSTRAINTS | NUMBER OF OBSERVATIONS | NUMBER OF PARAMETERS | WEIGHTED SS. OF RESIDUALS | DEGREES OF FREEDOM | VARIANCE COMPONENT | VIPVIII #OBS.-6 | RMS OF THE RESIDUALS | MEAN OF RESIDUALS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 30 | 6 | 0 | 36 | 24240 | 42 | 0.42660D+04 | 24234 | 0.1761 | 0.18 | 0.4195 | -0.0009 |

```
TOTAL NUMBER OF OBSERVATIONS..:   24240
ARITHMETIC MEAN OF RESIDUALS..: -0.0009
RMS OF ALL RESIDUALS..........:  0.4195
```

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Table 7 (cont'd)

FINAL RESULTS FOR STATION # : 7063

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | 1130708.42204 | -4831371.20638 | 3994088.93494 |
| CURRENT ADJUSTMENT : | 0.02237 | 0.00509 | -0.00009 |
| TOTAL ADJUSTMENT : | -3.25499 | 0.03870 | 0.23484 |
| ADJUSTED ESTIMATE : | 1130708.44501 | -4831371.26130 | 3994088.93484 |
| STANDARD DEVIATION : | 5.72796 | 1.34052 | 0.03958 |

FINAL RESULTS FOR STATION # : 7092

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -6143446.15561 | 1364709.14351 | 1034165.06444 |
| CURRENT ADJUSTMENT : | -0.00637 | -0.02604 | -0.00007 |
| TOTAL ADJUSTMENT : | 0.33802 | 2.21467 | -1.75563 |
| ADJUSTED ESTIMATE : | -6143443.16196 | 1364709.11457 | 1034165.06437 |
| STANDARD DEVIATION : | 1.01296 | 7.26364 | 0.64074 |

FINAL RESULTS FOR STATION # : 7090

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -2300999.96444 | 5043334.84229 | -3078527.61217 |
| CURRENT ADJUSTMENT : | -0.02355 | -0.01114 | -0.00007 |
| TOTAL ADJUSTMENT : | 2.28901 | 1.34315 | 0.93176 |
| ADJUSTED ESTIMATE : | -2300000.00799 | 5043334.83115 | -3078527.61224 |
| STANDARD DEVIATION : | 5.97927 | 2.63255 | 0.03542 |

FINAL RESULTS FOR STATION # : 7096

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -6100049.96964 | -996195.17401 | -1566977.31336 |
| CURRENT ADJUSTMENT : | 0.00456 | -0.02844 | -0.00006 |
| TOTAL ADJUSTMENT : | -0.38128 | 2.62454 | 1.00356 |
| ADJUSTED ESTIMATE : | -6100049.96528 | -996195.20246 | -1566977.31342 |
| STANDARD DEVIATION : | 1.18123 | 7.23222 | 0.04062 |

FINAL RESULTS FOR STATION # : 7091

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | 1452448.19366 | -4457262.67104 | 4296817.84539 |
| CURRENT ADJUSTMENT : | 0.02069 | 0.00677 | -0.00015 |
| TOTAL ADJUSTMENT : | -2.06505 | -0.96427 | 0.84524 |
| ADJUSTED ESTIMATE : | 1452448.21435 | -4457262.66427 | 4296817.84524 |
| STANDARD DEVIATION : | 5.26441 | 1.76962 | 0.04135 |

FINAL RESULTS FOR STATION # : 7114

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -2410430.13262 | -4477801.27485 | 3638688.97676 |
| CURRENT ADJUSTMENT : | 0.02092 | -0.01121 | -0.00004 |
| TOTAL ADJUSTMENT : | -1.91570 | 0.93494 | 0.90572 |
| ADJUSTED ESTIMATE : | -2410430.11170 | -4477801.28606 | 3638688.97672 |
| STANDARD DEVIATION : | 5.30866 | 2.85773 | 0.03962 |

Table 7 (cont'd)

FINAL RESULTS FOR STATION # : 7115

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -2350869.23763 | -4655545.15656 | 3660999.85808 |
| CURRENT ADJUSTMENT : | 0.02169 | -0.01091 | 0.00002 |
| TOTAL ADJUSTMENT : | -1.85894 | 0.92454 | 0.63010 |
| ADJUSTED ESTIMATE : | -2350869.21594 | -4655545.16746 | 3660999.85810 |
| STANDARD DEVIATION : | 5.51947 | 2.76733 | 0.03718 |

FINAL RESULTS FOR STATION # : 7907

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | 1942783.20050 | -5804080.21407 | -1796919.71300 |
| CURRENT ADJUSTMENT : | 0.02749 | 0.00906 | -0.00004 |
| TOTAL ADJUSTMENT : | -2.87201 | -1.30502 | 1.88696 |
| ADJUSTED ESTIMATE : | 1942783.22799 | -5804080.20502 | -1796919.71304 |
| STANDARD DEVIATION : | 6.68123 | 2.30354 | 0.09142 |

FINAL RESULTS FOR STATION # : 7120

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -5466004.08925 | -2404402.05471 | 2242229.46133 |
| CURRENT ADJUSTMENT : | 0.01128 | -0.02552 | 0.00007 |
| TOTAL ADJUSTMENT : | -0.99198 | 2.22478 | 0.86840 |
| ADJUSTED ESTIMATE : | -5466004.07798 | -2404402.08022 | 2242229.46140 |
| STANDARD DEVIATION : | 2.85082 | 6.48051 | 0.03966 |

FINAL RESULTS FOR STATION # : 7943

| | X (M) | Y (M) | Z (M) |
|---|---|---|---|
| APRIORI ESTIMATE : | -4447544.46005 | 2677140.24779 | -3694997.07404 |
| CURRENT ADJUSTMENT : | -0.01248 | -0.02076 | -0.00009 |
| TOTAL ADJUSTMENT : | 1.20847 | 2.41503 | 0.87687 |
| ADJUSTED ESTIMATE : | -4447544.47253 | 2677140.22703 | -3694997.07413 |
| STANDARD DEVIATION : | 3.17406 | 5.27300 | 0.03368 |

FINAL RESULTS FOR PASS # :7603901.01

| | X (M) | Y (M) | Z (M) | XDOT (M/S) | YDOT (M/S) | ZDOT (M/S) |
|---|---|---|---|---|---|---|
| APRIORI ESTIMATE : | -5097881.44816 | -5961549.88883 | -9377290.43571 | -4720.07387 | -871.25436 | 3111.28230 |
| CURRENT ADJUSTMENT : | 0.02741 | -0.02379 | 0.00014 | 0.00000 | -0.01002 | 0.00003 |
| TOTAL ADJUSTMENT : | -82.04308 | 22.97666 | 29.38775 | 0.00603 | 0.04008 | 0.02003 |
| ADJUSTED ESTIMATE : | -5097881.42075 | -5961549.91262 | -9377290.43557 | -4720.07387 | -871.25438 | 3111.28230 |
| T.O.R.D. ESTIMATE : | -5098243.47143 | -5961677.20100 | -9377012.07520 | -4720.05663 | -870.98060 | 3111.38509 |
| STANDARD DEVIATION : | 7.06781 | 6.04394 | 0.04245 | 0.00103 | 0.00560 | 0.00002 |

RMS POSITION : 5.36920

RMS VELOCITY : 0.00329

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# Table 8 Constants and Force Model for GEOSPP

EARTH CONSTANTS USED BY GEOSPP80 :

| | | | | | |
|---|---|---|---|---|---|
| SEMI-AXIS MAJOR................: | 0.637814411000000D+07 | (M) | SPEED OF LIGHT......: | 0.299792458000000D+09 | (M/S) |
| INVERSE FLATTENING.............: | 0.298255000000000D+03 | --- | ASTRONOMICAL UNIT..: | 0.149597870950000D+12 | (M) |
| GRAVITATIONAL CONSTANT (GM)...: | 0.398600441250000D+15 | (M3/S2) | SOLAR PRES AT 1AU..: | 0.462576125869690D-05 | (N/M2) |
| ROTATIONAL RATE................: | 0.729211585470000D-04 | (RAD/S) | SINGLE RANGE ACC...: | 0.100000000000000D+01 | (M) |

GEOPOTENTIAL MODEL USED IN THIS RUN : PGSL1 ( 12 X 12 )

ZONALS

| INDEX N M | VALUE | INDEX N M | VALUE | INDEX N M | VALUE | INDEX N M | VALUE | INDEX N M | VALUE |
|---|---|---|---|---|---|---|---|---|---|
| 2 0 | -1.08263D-03 | 3 0 | 2.53389D-06 | 4 0 | 1.62421D-06 | 5 0 | 2.29821D-07 | 6 0 | -5.43249D-07 |
| 7 0 | 3.57373D-07 | 8 0 | 2.10548D-07 | 9 0 | 1.21266D-07 | 10 0 | 2.44660D-07 | 11 0 | -2.33408D-07 |
| 12 0 | 1.93042D-07 | | | | | | | | |

SECTORIALS AND TESSERALS

| INDEX N M | VALUE C | VALUE S | INDEX N M | VALUE C | VALUE S | INDEX N M | VALUE C | VALUE S |
|---|---|---|---|---|---|---|---|---|
| 2 1 | -3.40933D-09 | -3.45464D-09 | 2 2 | 1.57362D-06 | -9.03069D-07 | 3 1 | 2.18996D-06 | 2.66524D-07 |
| 3 2 | 3.08402D-07 | -2.09768D-07 | 3 3 | 1.00587D-07 | 1.97364D-07 | 4 1 | -5.05141D-07 | -4.44071D-07 |
| 4 2 | 7.93024D-08 | 1.47716D-07 | 4 3 | 5.93923D-08 | -1.20966D-08 | 4 4 | -4.04056D-09 | 6.45692D-09 |
| 5 1 | -4.44673D-08 | -7.98052D-08 | 5 2 | 1.04731D-07 | -5.34060D-08 | 5 3 | -1.50726D-08 | -7.06860D-09 |
| 5 4 | -2.25580D-09 | 3.88035D-10 | 5 5 | 4.35650D-10 | -1.63033D-09 | 6 1 | -6.15351D-08 | 1.47418D-08 |
| 6 2 | 6.19517D-09 | -4.40707D-08 | 6 3 | 1.24902D-09 | 3.43425D-11 | 6 4 | -3.89362D-10 | -1.75704D-09 |
| 6 5 | -2.13260D-10 | -4.33520D-10 | 6 6 | 1.88956D-12 | -5.52509D-11 | 7 1 | 1.95429D-07 | 6.70658D-08 |
| 7 2 | 3.34179D-08 | 1.07341D-08 | 7 3 | 3.49184D-09 | -3.05535D-09 | 7 4 | -5.99304D-10 | -2.66841D-10 |
| 7 5 | 4.65443D-12 | 1.51414D-11 | 7 6 | -2.50399D-11 | 1.05350D-11 | 7 7 | 1.79248D-13 | 5.63543D-13 |
| 8 1 | 1.84982D-08 | 4.08088D-08 | 8 2 | 6.16066D-09 | 4.16045D-09 | 8 3 | -5.81017D-11 | -8.89240D-10 |
| 8 4 | -3.16583D-10 | 9.78554D-11 | 8 5 | -2.72342D-12 | 1.46759D-11 | 8 6 | -1.90388D-12 | 8.88305D-12 |
| 8 7 | 3.46777D-13 | 3.76174D-13 | 8 8 | -1.56858D-13 | 1.64256D-13 | 9 1 | 1.02546D-07 | 1.25022D-08 |
| 9 2 | 1.54669D-09 | -1.96536D-09 | 9 3 | -1.21291D-09 | -7.32492D-10 | 9 4 | -8.51484D-12 | 7.97852D-12 |
| 9 5 | 2.48948D-13 | -6.90571D-12 | 9 6 | 5.71572D-13 | 2.94202D-12 | 9 7 | -1.91564D-13 | -1.55864D-13 |
| 9 8 | 6.01006D-14 | -4.85784D-15 | 9 9 | -4.32941D-15 | 7.20317D-15 | 10 1 | 4.89494D-08 | -8.25066D-08 |
| 10 2 | -4.68674D-09 | -8.27266D-10 | 10 3 | -1.02258D-10 | -9.86654D-10 | 10 4 | -5.70476D-11 | -3.83382D-11 |
| 10 5 | -3.92963D-12 | -2.45444D-12 | 10 6 | -2.56467D-13 | -6.03466D-13 | 10 7 | 2.16386D-15 | 7.61696D-15 |
| 10 8 | 4.56440D-15 | -8.95563D-15 | 10 9 | 2.28234D-15 | -9.24323D-16 | 10 10 | 4.52539D-16 | -1.03802D-16 |
| 11 1 | -5.62091D-10 | -9.12224D-09 | 11 2 | 2.47802D-09 | -4.87164D-09 | 11 3 | -9.47966D-11 | -6.37254D-10 |
| 11 4 | -1.80082D-11 | -2.39313D-11 | 11 5 | 1.90660D-12 | 3.26441D-12 | 11 6 | 8.33275D-15 | 1.77378D-13 |
| 11 7 | 5.61605D-15 | -3.24726D-14 | 11 8 | 1.72316D-16 | 1.14172D-15 | 11 9 | -1.83983D-16 | 2.00592D-16 |
| 11 10 | -5.91281D-17 | -4.83426D-18 | 11 11 | 6.33816D-18 | -1.00384D-17 | 12 1 | -3.54722D-08 | -3.12321D-08 |
| 12 2 | 3.93978D-10 | -3.28817D-10 | 12 3 | 2.50930D-10 | 1.12897D-10 | 12 4 | -2.16549D-11 | -2.83157D-12 |
| 12 5 | 1.17221D-12 | 3.61694D-13 | 12 6 | 2.15485D-14 | 8.43442D-14 | 12 7 | -2.23636D-15 | 9.64344D-15 |
| 12 8 | -5.56339D-16 | 5.27651D-16 | 12 9 | 1.04973D-16 | 3.20229D-17 | 12 10 | 2.00297D-16 | 1.43332D-17 |
| 12 11 | 7.21746D-19 | -2.91149D-19 | 12 12 | -6.81526D-20 | -1.08451D-19 | | | |

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Table 8 (cont'd)

```
VARIABLE ORDER-VARIABLE STEPSIZE NUMERICAL INTEGRATION :
=========================================================

NOMINAL STEPSIZE........:   140.0 SEC
MINIMUM STEPSIZE........:     0.0 SEC
MAXIMUM STEPSIZE........:   600.0 SEC
RELATIVE ACCURACY
EQUATIONS OF MOTION.....: 1.0D-07
VARIATIONAL EQUATIONS...: 1.0D-04
INT.MESSAGE OUTPUT UNIT :      21


PERTURBATION MODEL :
==================

GEOPOTENTIAL..............................: (12,12)
FOR VARIATIONAL EQUATIONS.................: ( 4, 4)
MOON......................................:   YES
SUN.......................................:   YES
UNMODELED ACCELERATIONS ..................:   YES
     ALONG TRACK    = -0.34800-11 (M/S2)
     CROSS TRACK    =  0.0        (M/S2)
     RADIAL         =  0.0        (M/S2)
SOLAR RADIATION PRESSURE..................:   YES
     SATELLITE AREA =    0.2827 (M2)
     SATELLITE MASS = 406.9650 (KG)
     SATELLITE REFLECTIVITY CK= 1.1729
SOLID EARTH TIDES.........................:   YES
     LOVE NO.       K2=0.2740
     PHASE ANGLE    E2=2.3300


INPUT INFORMATION FOR PASS : 7603901.01
===========================
                            YYMMDD HHMMSS.SSS
EPOCH OF ELEMENTS.......: 80 813 235930.028
OBSERVATIONS START AT...: 80 814  0 0 0.0
OBSERVATIONS END AT.....: 80 829  0 0 0.0
REFERENCE SYSTEM EPOCH..: 80 731


INERTIAL CARTESIAN ELEMENTS AT THE EPOCH :
==========================================
      X (M)            Y (M)            Z (M)
 -5098161.430     -5964700.182     -9377042.063

   XDOT (M/S)       YDOT (M/S)       ZDOT (M/S)
 -4720.0626632    -871.0200757     3111.3650676

APRIORI WEIGHT MATRIX :

0.0
0.0       0.0
0.0       0.0       0.0
0.0       0.0       0.0       0.0
0.0       0.0       0.0       0.0       0.0
0.0       0.0       0.0       0.0       0.0       0.0


LOVE NUMBER FOR RADIAL EXPANSION    H2 =  0.600
================================

SHIDA NUMBER FOR HORIZONTAL SHEAR   L2 =  0.075
=================================


10  STATIONS OBSERVING THIS PASS :
   7063    7090    7091    7092    7096    7114    7115    7120    7907    7943
```

Table 9

BASELINE ESTIMATES AND RELATED STATISTICS :

| BASELINE# | STATION# | | STATION# | APRIORI EST. | ADJUSTED VAL. | DIFF.(A-E) | SIGMA | RELATIVE ACC. |
|---|---|---|---|---|---|---|---|---|
| 1 | 7063 | ==> | 7090 | 12645951.761 | 12645950.847 | -0.914 | 0.018 | 3.340-09 |
| 2 | 7063 | ==> | 7091 | 602032.143 | 602032.169 | 0.026 | 0.036 | 1.440-07 |
| 3 | 7063 | ==> | 7092 | 10003296.515 | 10003295.833 | -0.682 | 0.025 | 6.030-09 |
| 4 | 7063 | ==> | 7096 | 9896473.055 | 9896471.526 | -1.528 | 0.022 | 5.190-09 |
| 5 | 7063 | ==> | 7114 | 3562138.713 | 3562137.442 | -1.272 | 0.041 | 2.740-08 |
| 6 | 7063 | ==> | 7115 | 3501893.178 | 3501891.797 | -1.381 | 0.037 | 2.520-08 |
| 7 | 7063 | ==> | 7120 | 7244020.742 | 7244019.261 | -1.482 | 0.028 | 9.230-09 |
| 8 | 7063 | ==> | 7907 | 5928036.951 | 5928019.003 | -17.948 | 0.085 | 3.410-08 |
| 9 | 7063 | ==> | 7943 | 12108539.054 | 12108538.064 | -0.990 | 0.018 | 3.480-09 |
| 10 | 7090 | ==> | 7091 | 12638160.062 | 12638160.219 | 0.156 | 0.024 | 4.520-09 |
| 11 | 7090 | ==> | 7092 | 6674009.770 | 6674008.743 | -1.027 | 0.024 | 8.610-09 |
| 12 | 7090 | ==> | 7096 | 7247520.432 | 7247520.743 | 0.311 | 0.023 | 7.680-09 |
| 13 | 7090 | ==> | 7114 | 11768618.014 | 11768618.337 | 0.323 | 0.018 | 3.070-09 |
| 14 | 7090 | ==> | 7115 | 11810628.856 | 11810629.014 | 0.158 | 0.016 | 3.200-09 |
| 15 | 7090 | ==> | 7120 | 9656458.579 | 9656458.910 | 0.331 | 0.021 | 5.120-09 |
| 16 | 7090 | ==> | 7907 | 11750456.119 | 11750458.620 | 2.500 | 0.034 | 6.920-09 |
| 17 | 7090 | ==> | 7943 | 3196328.733 | 3196328.646 | -0.087 | 0.021 | 1.550-08 |
| 18 | 7091 | ==> | 7092 | 10141371.223 | 10141371.602 | 0.379 | 0.031 | 7.210-09 |
| 19 | 7091 | ==> | 7096 | 10199643.124 | 10199642.536 | -0.587 | 0.025 | 5.840-09 |
| 20 | 7091 | ==> | 7114 | 3929728.803 | 3929728.019 | -0.782 | 0.039 | 2.340-08 |
| 21 | 7091 | ==> | 7115 | 3900598.445 | 3900597.570 | -0.876 | 0.034 | 2.070-08 |
| 22 | 7091 | ==> | 7120 | 7540273.824 | 7540273.123 | -0.701 | 0.029 | 9.100-09 |
| 23 | 7091 | ==> | 7907 | 6257037.782 | 6257020.271 | -17.511 | 0.088 | 3.350-08 |
| 24 | 7091 | ==> | 7943 | 12249596.212 | 12249596.272 | 0.059 | 0.022 | 4.380-09 |
| 25 | 7092 | ==> | 7096 | 3514556.686 | 3514554.371 | -2.316 | 0.027 | 1.840-08 |
| 26 | 7092 | ==> | 7114 | 7479017.596 | 7479018.461 | 0.865 | 0.027 | 8.500-09 |
| 27 | 7092 | ==> | 7115 | 7584680.410 | 7584681.155 | 0.745 | 0.028 | 8.760-09 |
| 28 | 7092 | ==> | 7120 | 4015538.430 | 4015538.979 | 0.550 | 0.028 | 1.680-08 |
| 29 | 7092 | ==> | 7907 | 11171115.715 | 11171110.424 | -5.291 | 0.041 | 8.740-09 |
| 30 | 7092 | ==> | 7943 | 5192643.026 | 5192640.982 | -2.044 | 0.024 | 1.120-08 |
| 31 | 7096 | ==> | 7114 | 7414696.951 | 7414696.912 | -0.040 | 0.023 | 7.480-09 |
| 32 | 7096 | ==> | 7115 | 7402692.901 | 7402692.731 | -0.170 | 0.025 | 8.030-09 |
| 33 | 7096 | ==> | 7120 | 4112220.542 | 4112220.461 | -0.081 | 0.025 | 1.440-08 |
| 34 | 7096 | ==> | 7907 | 9373094.052 | 9373093.497 | -0.554 | 0.044 | 1.110-08 |
| 35 | 7096 | ==> | 7943 | 4554571.701 | 4554572.165 | 0.464 | 0.024 | 1.240-08 |
| 36 | 7114 | ==> | 7115 | 258289.958 | 258290.167 | 0.210 | 0.036 | 3.330-07 |
| 37 | 7114 | ==> | 7120 | 4022959.527 | 4022959.505 | -0.022 | 0.031 | 1.810-08 |
| 38 | 7114 | ==> | 7907 | 7243602.178 | 7243588.024 | -14.154 | 0.076 | 2.490-08 |
| 39 | 7114 | ==> | 7943 | 10587702.281 | 10587702.701 | 0.420 | 0.018 | 4.050-09 |
| 40 | 7115 | ==> | 7120 | 4096904.174 | 4096904.146 | -0.027 | 0.031 | 1.790-08 |
| 41 | 7115 | ==> | 7907 | 7038726.657 | 7038712.246 | -14.411 | 0.074 | 2.510-08 |
| 42 | 7115 | ==> | 7943 | 10595990.172 | 10595990.425 | 0.253 | 0.018 | 4.070-09 |
| 43 | 7120 | ==> | 7907 | 9097407.601 | 9097399.389 | -8.212 | 0.052 | 1.360-08 |
| 44 | 7120 | ==> | 7943 | 7880988.899 | 7880989.300 | 0.401 | 0.022 | 6.750-09 |
| 45 | 7907 | ==> | 7943 | 10787493.058 | 10787496.735 | 3.676 | 0.041 | 8.960-09 |

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Table 10 Residual Summaries by Station

(a)

CONSOLIDATED STATISTICS FOR STATION : 7063

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 4 | -0.9288 | 5.383 | 6.123 | 1292.00 | -8.493 | 5.007 | -0.93 |
| 2 | 477 | 0.1070 | 0.237 | 0.212 | 1641.00 | -2.866 | 0.398 | 0.11 |
| 3 | 202 | 0.1237 | 0.643 | 0.632 | 1494.00 | -6.840 | 5.480 | 0.12 |
| 4 | 6 | -2.0976 | 3.459 | 3.012 | 1689.00 | -5.903 | 1.408 | -2.10 |
| 5 | 859 | 0.1242 | 0.225 | 0.187 | 2358.00 | -2.436 | 0.473 | 0.12 |
| 6 | 1 | 0.0458 | 0.046 | 0.0 | 0.0 | 0.046 | 0.046 | 0.05 |
| 7 | 1550 | 0.0139 | 0.322 | 0.321 | 2810.00 | -4.383 | 8.987 | 0.01 |
| 8 | 4 | -4.4022 | 5.625 | 4.043 | 1503.00 | -9.652 | -1.045 | -4.40 |
| 9 | 14 | -0.4982 | 2.473 | 2.514 | 2550.00 | -4.545 | 5.946 | -0.50 |
| 10 | 1167 | -0.1706 | 0.464 | 0.432 | 2484.00 | -6.694 | 7.124 | -0.17 |

(b)

CONSOLIDATED STATISTICS FOR STATION : 7090

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 97 | 0.0882 | 0.130 | 0.095 | 2156.00 | -0.221 | 0.344 | 0.09 |
| 2 | 167 | -0.0325 | 0.104 | 0.099 | 2182.00 | -0.337 | 0.177 | -0.03 |
| 3 | 182 | -0.0892 | 0.131 | 0.096 | 2519.00 | -0.513 | 0.131 | -0.09 |
| 4 | 207 | 0.0322 | 0.109 | 0.105 | 2471.00 | -0.282 | 0.264 | 0.03 |
| 5 | 196 | 0.0335 | 0.140 | 0.137 | 2631.00 | -0.433 | 0.448 | 0.03 |
| 6 | 141 | -0.0832 | 0.142 | 0.116 | 1810.00 | -0.386 | 0.192 | -0.08 |
| 7 | 263 | -0.0764 | 0.119 | 0.091 | 2853.00 | -0.427 | 0.177 | -0.08 |
| 8 | 119 | 0.0891 | 0.138 | 0.106 | 2037.03 | -0.153 | 0.384 | 0.09 |
| 9 | 67 | -0.0531 | 0.093 | 0.078 | 1040.00 | -0.235 | 0.145 | -0.05 |
| 10 | 171 | -0.0780 | 0.122 | 0.094 | 2787.00 | -0.471 | 0.181 | -0.08 |
| 11 | 136 | -0.0981 | 0.138 | 0.097 | 2036.00 | -0.451 | 0.101 | -0.10 |
| 12 | 203 | -0.0640 | 0.112 | 0.092 | 2481.00 | -0.526 | 0.145 | -0.06 |
| 13 | 50 | 0.0123 | 0.080 | 0.079 | 574.00 | -0.157 | 0.216 | 0.01 |
| 14 | 29 | 0.0674 | 0.143 | 0.128 | 1287.00 | -0.219 | 0.286 | 0.07 |
| 15 | 136 | -0.0940 | 0.124 | 0.081 | 2339.00 | -0.347 | 0.092 | -0.09 |
| 16 | 104 | 0.1106 | 0.435 | 0.422 | 2645.00 | -3.875 | 0.378 | 0.11 |
| 17 | 55 | 0.1305 | 0.233 | 0.194 | 1318.01 | -0.282 | 0.690 | 0.13 |
| 18 | 162 | -0.1606 | 0.204 | 0.126 | 2277.00 | -0.472 | 0.071 | -0.16 |
| 19 | 173 | 0.0735 | 0.136 | 0.114 | 2491.00 | -0.272 | 0.326 | 0.07 |
| 20 | 136 | -0.1598 | 0.212 | 0.140 | 2409.00 | -0.567 | 0.111 | -0.16 |
| 21 | 155 | 0.0943 | 0.185 | 0.159 | 2421.00 | -0.323 | 0.378 | 0.09 |
| 22 | 41 | 0.0313 | 0.069 | 0.062 | 1090.00 | -0.113 | 0.143 | 0.03 |
| 23 | 88 | 0.1992 | 0.218 | 0.088 | 1438.00 | -0.102 | 0.348 | 0.20 |
| 24 | 233 | -0.0433 | 0.112 | 0.104 | 2842.00 | -0.597 | 0.195 | -0.04 |
| 25 | 115 | 0.1264 | 0.156 | 0.092 | 1947.00 | -0.168 | 0.345 | 0.13 |
| 26 | 6 | 0.1585 | 0.172 | 0.073 | 93.00 | 0.071 | 0.280 | 0.16 |
| 27 | 189 | 0.0197 | 0.128 | 0.126 | 2638.00 | -0.289 | 0.294 | 0.02 |
| 28 | 154 | 0.0887 | 0.123 | 0.085 | 2109.00 | -0.164 | 0.328 | 0.09 |
| 29 | 154 | -0.0249 | 0.223 | 0.223 | 2088.00 | -0.567 | 0.398 | -0.02 |
| 30 | 214 | 0.0598 | 0.128 | 0.114 | 2700.00 | -0.379 | 0.292 | 0.06 |

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Table 10 (cont'd)

(c) CONSOLIDATED STATISTICS FOR STATION : 7091

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 137 | 0.0824 | 0.271 | 0.259 | 1145.00 | -0.750 | 0.683 | 0.08 |
| 2 | 352 | -0.0646 | 0.182 | 0.171 | 1718.00 | -1.040 | 0.346 | -0.06 |
| 3 | 240 | 0.0505 | 0.169 | 0.162 | 1879.00 | -0.545 | 0.450 | 0.05 |
| 4 | 439 | -0.0202 | 0.314 | 0.313 | 1965.01 | -1.142 | 4.742 | -0.02 |

(d) CONSOLIDATED STATISTICS FOR STATION : 7092

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 322 | 0.2185 | 0.268 | 0.156 | 1761.01 | -0.258 | 0.527 | 0.22 |
| 2 | 286 | -0.1712 | 0.268 | 0.206 | 1153.00 | -0.845 | 0.666 | -0.17 |
| 3 | 1273 | -0.0004 | 0.239 | 0.239 | 2380.00 | -1.234 | 0.986 | -0.00 |
| 4 | 363 | -0.0324 | 0.304 | 0.303 | 2271.00 | -1.293 | 0.734 | -0.03 |
| 5 | 9 | 0.0926 | 0.331 | 0.337 | 174.00 | -0.367 | 0.[illegible]26 | 0.09 |

(e) CONSOLIDATED STATISTICS FOR STATION : 7096

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 969 | 0.0078 | 0.189 | 0.189 | 2389.00 | -0.583 | 0.546 | 0.01 |
| 2 | 461 | 0.0359 | 0.150 | 0.146 | 2008.99 | -0.731 | 0.331 | 0.04 |
| 3 | 268 | -0.1355 | 0.257 | 0.219 | 1109.01 | -0.931 | 0.313 | -0.14 |
| 4 | 91 | -0.3075 | 0.391 | 0.244 | 652.00 | -0.953 | 0.122 | -0.31 |
| 5 | 45 | 0.0547 | 0.166 | 0.158 | 924.00 | -0.451 | 0.356 | 0.06 |
| 6 | 616 | 0.0351 | 0.213 | 0.210 | 1368.01 | -1.019 | 0.531 | 0.04 |

(f) CONSOLIDATED STATISTICS FOR STATION : 7114

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 182 | -0.0405 | 0.176 | 0.172 | 1181.99 | -0.458 | 1.003 | -0.04 |
| 2 | 17 | -0.1346 | 1.475 | 1.514 | 1136.00 | -4.979 | 2.632 | -0.13 |
| 3 | 855 | 0.0155 | 0.263 | 0.262 | 2535.00 | -3.965 | 1.966 | 0.02 |
| 4 | 9 | 1.1201 | 2.490 | 2.358 | 1367.00 | -1.729 | 5.392 | 1.12 |
| 5 | 161 | 0.0939 | 0.155 | 0.124 | 1009.99 | -0.310 | 0.465 | 0.09 |
| 6 | 390 | -0.0036 | 0.129 | 0.129 | 2102.99 | -0.387 | 0.968 | -0.00 |
| 7 | 6 | -0.7358 | 3.655 | 3.922 | 887.00 | -4.838 | 6.072 | -0.74 |
| 8 | 228 | -0.0292 | 0.350 | 0.349 | 1045.00 | -0.384 | 4.296 | -0.03 |
| 9 | 7 | 0.0771 | 0.111 | 0.086 | 676.00 | 0.001 | 0.236 | 0.08 |
| 10 | 7 | 0.4243 | 0.703 | 0.606 | 1080.00 | 0.099 | 1.779 | 0.42 |
| 11 | 4 | -4.5052 | 5.795 | 4.208 | 400.00 | -8.096 | 0.651 | -4.51 |

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Table 10 (cont'd)

(g) CONSOLIDATED STATISTICS FOR STATION : 7115

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 264 | 0.0850 | 0.131 | 0.099 | 1224.01 | -0.178 | 0.379 | 0.08 |
| 2 | 29 | 0.2660 | 0.280 | 0.088 | 711.00 | 0.077 | 0.410 | 0.27 |
| 3 | 384 | -0.0730 | 1.057 | 1.056 | 2001.00 | -6.960 | 0.595 | -0.07 |
| 4 | 27 | -6.7929 | 6.793 | 0.090 | 1021.00 | -6.969 | -6.617 | -6.79 |
| 5 | 500 | 0.0934 | 0.151 | 0.119 | 2271.00 | -0.422 | 1.167 | 0.09 |
| 6 | 38 | 0.3169 | 0.328 | 0.088 | 1313.00 | 0.120 | 0.488 | 0.32 |
| 7 | 172 | 0.3589 | 0.511 | 0.364 | 1346.00 | -0.525 | 4.824 | 0.36 |
| 8 | 63 | -0.0468 | 0.145 | 0.138 | 889.00 | -0.346 | 0.257 | -0.05 |
| 9 | 119 | -0.2538 | 0.312 | 0.181 | 1913.00 | -0.739 | 0.504 | -0.25 |
| 10 | 588 | 0.1708 | 0.213 | 0.128 | 2727.00 | -0.255 | 1.811 | 0.17 |
| 11 | 37 | 0.1305 | 0.165 | 0.102 | 1608.00 | -0.075 | 0.363 | 0.13 |
| 12 | 44 | 0.2436 | 0.286 | 0.151 | 652.00 | -0.040 | 0.949 | 0.24 |

(h) CONSOLIDATED STATISTICS FOR STATION : 7120

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 225 | -0.1213 | 0.142 | 0.073 | 1964.00 | -0.380 | 0.120 | -0.12 |
| 2 | 44 | -0.0037 | 0.157 | 0.159 | 829.00 | -0.772 | 0.180 | -0.00 |
| 3 | 160 | 0.0996 | 0.140 | 0.098 | 1297.00 | -0.225 | 0.313 | 0.10 |
| 4 | 42 | -0.0689 | 0.108 | 0.084 | 618.00 | -0.218 | 0.098 | -0.07 |
| 5 | 187 | 0.0268 | 0.133 | 0.131 | 2614.00 | -0.337 | 0.857 | 0.03 |
| 6 | 346 | -0.0766 | 0.114 | 0.085 | 2759.00 | -0.348 | 0.247 | -0.08 |
| 7 | 401 | 0.0931 | 0.138 | 0.102 | 2573.00 | -0.259 | 0.298 | 0.09 |
| 8 | 50 | -0.2583 | 0.294 | 0.141 | 865.00 | -0.511 | 0.034 | -0.26 |
| 9 | 121 | -0.1879 | 0.216 | 0.107 | 1655.00 | -0.380 | 0.104 | -0.19 |
| 10 | 328 | 0.1102 | 0.163 | 0.120 | 2417.00 | -0.221 | 0.409 | 0.11 |

(i) CONSOLIDATED STATISTICS FOR STATION : 7907

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLOS |
|---|---|---|---|---|---|---|---|---|
| 1 | 12 | 0.1231 | 0.366 | 0.360 | 292.40 | -0.408 | 0.939 | 0.12 |
| 2 | 41 | -0.0135 | 0.604 | 0.611 | 1132.95 | -1.520 | 0.761 | -0.01 |
| 3 | 51 | 0.0917 | 0.509 | 0.505 | 862.51 | -1.291 | 0.803 | 0.09 |
| 4 | 19 | -0.0486 | 0.335 | 0.341 | 1027.76 | -0.767 | 0.561 | -0.05 |
| 5 | 5 | -0.0084 | 0.596 | 0.666 | 360.15 | -1.164 | 0.446 | -0.01 |
| 6 | 52 | 0.1667 | 0.521 | 0.498 | 892.90 | -0.951 | 2.032 | 0.17 |
| 7 | 24 | 0.0601 | 0.401 | 0.405 | 660.27 | -0.925 | 0.724 | 0.06 |
| 8 | 19 | -0.2028 | 0.393 | 0.346 | 607.64 | -0.868 | 0.360 | -0.20 |
| 9 | 35 | 0.0493 | 0.288 | 0.287 | 1162.58 | -0.648 | 0.640 | 0.05 |
| 10 | 53 | -0.0616 | 0.408 | 0.408 | 914.95 | -1.008 | 0.881 | -0.06 |
| 11 | 34 | -0.0226 | 0.398 | 0.403 | 967.53 | -1.199 | 0.697 | -0.02 |
| 12 | 5 | 0.2501 | 0.257 | 0.065 | 360.10 | 0.169 | 0.303 | 0.25 |
| 13 | 24 | -0.5918 | 1.005 | 0.830 | 1110.02 | -2.043 | 0.962 | -0.59 |
| 14 | 17 | 0.0004 | 0.813 | 0.838 | 689.98 | -1.536 | 1.569 | 0.00 |
| 15 | 8 | 0.2717 | 0.528 | 0.484 | 420.00 | -0.434 | 1.126 | 0.27 |
| 16 | 9 | -0.1124 | 0.568 | 0.590 | 465.05 | -1.312 | 0.452 | -0.11 |
| 17 | 22 | -0.1859 | 0.744 | 0.737 | 847.96 | -1.970 | 1.863 | -0.19 |
| 18 | 28 | 0.2387 | 0.405 | 0.333 | 1012.50 | -0.646 | 0.796 | 0.24 |
| 19 | 2 | -0.3987 | 0.425 | 0.207 | 90.00 | -0.545 | -0.252 | -0.40 |
| 20 | 29 | 0.0272 | 0.411 | 0.417 | 779.97 | -0.855 | 0.698 | 0.03 |

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Table 10 (cont'd)

(J) CONSOLIDATED STATISTICS FOR STATION : 7943

| PASS | OBSERV | RESID MEAN | RMS | DEVIATION | LENGTH | MIN RESD | MAX RESD | MEAN CLUS |
|---|---|---|---|---|---|---|---|---|
| 1 | 56 | 0.0012 | 0.433 | 0.437 | 1154.92 | -1.039 | 0.959 | 0.00 |
| 2 | 122 | 0.0426 | 0.338 | 0.336 | 2325.29 | -1.432 | 0.815 | 0.04 |
| 3 | 82 | -0.0668 | 0.439 | 0.437 | 1905.10 | -1.252 | 1.074 | -0.07 |
| 4 | 98 | -0.0301 | 0.408 | 0.409 | 2272.34 | -0.970 | 0.715 | -0.03 |
| 5 | 83 | 0.0633 | 0.426 | 0.424 | 1484.75 | -1.229 | 1.222 | 0.06 |
| 6 | 85 | 0.0516 | 0.337 | 0.335 | 1567.79 | -0.917 | 0.872 | 0.05 |
| 7 | 90 | 0.1375 | 0.292 | 0.259 | 1424.99 | -0.489 | 0.991 | 0.14 |
| 8 | 212 | -0.0796 | 0.318 | 0.309 | 2744.95 | -0.864 | 0.679 | -0.08 |
| 9 | 159 | 0.0983 | 0.344 | 0.330 | 2910.00 | -0.720 | 0.861 | 0.10 |
| 10 | 47 | -0.0243 | 0.511 | 0.516 | 1934.86 | -1.052 | 1.198 | -0.02 |
| 11 | 54 | 0.0337 | 0.429 | 0.432 | 1514.65 | -1.193 | 1.353 | 0.03 |
| 12 | 75 | -0.1024 | 0.552 | 0.546 | 1605.00 | -1.306 | 0.974 | -0.10 |
| 13 | 139 | 0.0414 | 0.239 | 0.236 | 1822.48 | -0.681 | 0.594 | 0.04 |
| 14 | 100 | -0.0034 | 0.375 | 0.377 | 1897.49 | -0.853 | 1.041 | -0.00 |
| 15 | 170 | -0.0420 | 0.414 | 0.413 | 2955.00 | -1.097 | 1.295 | -0.04 |
| 16 | 106 | -0.0837 | 0.347 | 0.338 | 1890.00 | -1.137 | 0.715 | -0.08 |
| 17 | 90 | -0.0132 | 0.280 | 0.281 | 1919.92 | -0.878 | 0.702 | -0.01 |
| 18 | 44 | -0.0889 | 0.470 | 0.467 | 2167.47 | -1.129 | 0.643 | -0.09 |
| 19 | 79 | 0.1667 | 0.424 | 0.392 | 2287.55 | -0.943 | 0.899 | 0.17 |
| 20 | 178 | 0.0212 | 0.282 | 0.282 | 2489.80 | -0.607 | 1.253 | 0.02 |
| 21 | 38 | -0.1376 | 0.625 | 0.617 | 1492.55 | -1.638 | 1.460 | -0.14 |
| 22 | 88 | -0.0665 | 0.485 | 0.483 | 2092.54 | -1.147 | 1.249 | -0.07 |
| 23 | 172 | -0.0355 | 0.263 | 0.261 | 2827.32 | -0.671 | 0.611 | -0.04 |
| 24 | 73 | -0.3053 | 0.399 | 0.259 | 629.94 | -0.896 | 0.119 | -0.31 |
| 25 | 67 | 0.0351 | 0.448 | 0.450 | 1619.87 | -1.124 | 0.798 | 0.04 |
| 26 | 96 | -0.0945 | 0.596 | 0.592 | 1619.69 | -1.605 | 1.349 | -0.09 |
| 27 | 163 | -0.1419 | 0.279 | 0.241 | 1919.90 | -0.771 | 0.511 | -0.14 |
| 28 | 191 | 0.1105 | 0.330 | 0.312 | 1994.97 | -0.727 | 0.898 | 0.11 |
| 29 | 262 | 0.1799 | 0.365 | 0.318 | 2999.78 | -0.819 | 0.837 | 0.18 |
| 30 | 97 | 0.1417 | 0.282 | 0.245 | 1942.65 | -0.673 | 0.703 | 0.14 |
| 31 | 61 | 0.1398 | 0.466 | 0.448 | 2175.10 | -0.953 | 0.875 | 0.14 |
| 32 | 41 | -0.0845 | 0.485 | 0.483 | 1507.44 | -1.351 | 0.607 | -0.09 |

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## 2.22 Doppler Experiments

### 2.221 Geometric Adjustment of Simultaneous Doppler-Derived Range Differences

The results of work on this topic are described in a paper presented at the Third Internationl Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Ermioni, Greece, September 20-25, 1982. It appears on the following pages and will be published in the proceedings of the symposium obtainable from the National Technical University, Athens.

*Third International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Ermioni, Greece, September 20-25, 1982*

# GEOMETRIC ADJUSTMENT OF SIMULTANEOUS DOPPLER-DERIVED RANGE DIFFERENCES

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ABSTRACT. A mathematical model for the use of simultaneous Doppler-derived correlated ranges in the geometric mode is presented. The model is tested with data taken during the EDOC-2 campaign with different integration intervals. The results of this adjustment are compared with the EDOC-2 adopted solution and those from an uncorrelated model [Schneeberger et al. 1982] used earlier to provide more economical calculations.

The analysis of the comparison shows that the correlated mode is superior to the uncorrelated one when the optimum integration interval of 23 seconds is used.

## 1. INTRODUCTION

The geometric purpose of satellite geodesy is to tie remote stations together in the same geometric system. Its ultimate aim is to determine the coordinates of unknown ground stations [Mueller 1984].

Satellite geodesy with Doppler techniques is based on the principle that a frequency transmitted from a satellite-borne transmitter moving relative to a ground receiver is observed shifted by the Doppler effect. The observations are Doppler counts which are measures of the range change between the satellite and the receiver during the integration interval [Wells 1974].

In the geometric mode for Doppler observations, the satellite is regarded as a benchmark in space and its coordinates at the observation instants are unknowns which are solved in an adjustment with the unknown coordinates of ground stations. Such solutions are based on geometric rather than dynamic principles; therefore the calculations are relatively simple and do not require extensive computer programs.

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In a previous study [Schneeberger et al. 1982], the Doppler-derived ranges were regarded as uncorrelated pseudo-observations as a further simplification (to save computer time). In fact, since the Doppler-derived ranges are calculated from Doppler counts, it is obvious that there exist correlations between them in a given pass. The purpose of this study is to investigate the use of Doppler-derived correlated ranges in the geometric mode.

This method is then tested against a data set from which a dynamic solution is available. The results are compared with both the dynamic solutions and the uncorrelated geometric one.

## 2. SUMMARY OF THE PREVIOUS STUDY BASED ON UNCORRELATED OBSERVATIONS [SCHNEEBERGER ET AL. 1982]

### 2.1 Definitions

The *coordinate system* in which the computations are performed is an earth-fixed Cartesian system. It is defined by the assigned six coordinates distributed among at least three ground stations. A *satellite point* is the position of a satellite at a certain epoch. An *event* is the set of all observations to a satellite point. A *pass* is a set of satellite points between two epochs which are observed without interruption from at least six ground stations. A *Doppler-derived range* is a pseudo-observation derived by adding the range differences computed from Doppler counts to an estimated initial range.

### 2.2 Doppler-Derived Ranges

The basic equation which related the ratio between the received frequency f and the transmitted frequency $f_0$ to the range rate between transmitter and receiver ($\dot{r}$) is accredited to Doppler (1803-1853):

$$\frac{f}{f_0} = \left(\frac{c}{c + \dot{r}}\right) \approx \left(1 - \frac{\dot{r}}{c}\right)$$

where c is the velocity of propagation for electromagnetic waves in a vacuum. This equation has to be integrated to find a relation between the shifted frequency and the range difference during a time interval t. A detailed derivation can be found in [Brown and Trotter 1969] resulting in

$$r_j - r_{j-1} = \lambda_0(N_j - \Delta f_{00}\, t_j) + S \qquad (1)$$

where

$r_j$ = range between receiver and transmitter at epoch $T_j$

$r_{j-1}$ = range at epoch $T_{j-1}$

$N_j$ = the integrated Doppler shift over time interval $t_j = T_j - T_{j-1}$ (referred to as the Doppler count)

$\Delta f_{00}$ = the difference between the transmitted frequency and the reference frequency generated in the receiver

$\lambda_0 = \frac{f_0}{c}$ = wavelength corresponding to the frequency of transmission $f_0$

S = correction term representing all systematic errors such as bias in the difference between the adopted transmitted and reference frequencies, and/or the drift rates of transmitter and receiver frequencies.

Substituting the range difference computed from the Doppler count

$$\Delta r_j = \lambda_0 (N_j + \Delta f_{00}\ t_j)$$

into equation (1), the range at epoch $T_j$ is

$$r_j = r_{j-1} + \Delta r_j + S_j$$

If the range $r_0$ at an initial epoch $T_0$ is known, the range for an epoch $T_k$ can then be calculated from (taking into account that most instruments reset the Doppler count for each interval)

$$r_k = r_0 + \sum_{j=1}^{k} \Delta r_j + S_k \qquad (2)$$

This equation is correct only in a vacuum. Since the signal is passing through the ionosphere and the troposphere, the range has to be corrected for refractive effects. The ionospheric refraction is automatically compensated (to first order) by measuring the Doppler shift of the two different frequencies (400 and 150 MHz) [Krakiwsky and Wells 1971]. Each range has to be corrected therefore only for the tropospheric refraction $\Delta T_r$. The tropospheric refraction model used in this study is the one outlined in [Brown and Trotter, 1973], using the Smith-Weintraub model for the index of refraction [Jordan et al. 1966].

Since the initial range in equation (2) is not known, we must use an approximate initial range $r_0'$ and add a correction term $a_0$ to be estimated from the adjustment,

$$r_0 = r_0' + a_0$$

$a_0$ is considered part of the systematic error term $S_k$ in equation (2). The modelling of the other systematic effects in $S_k$ is given in great detail in [Brown and Trotter 1969; Kouba and Boal 1976]. In this study only two major terms are used: a + bt. The main cause of the constant term a is the possible bias in the adopted frequency $f_0$, and the initial range error $a_0$ above. The time dependent term bt is caused mainly by the difference in the adopted values for the transmitter and receiver frequencies (frequency offset) $\Delta f_{00}$.

Other terms in the systematic error model mentioned by Brown and Trotter [1969] but not considered in this study are range dependency, a function of the second power of time, and a function of the elevation angle (for residual refraction errors). An explanation of why only the above two terms are used here may be found in [Schneeberger 1982].

Substituting all terms for S and the correction for tropospheric refraction equation (2) can be written as

$$r_{ik} = r_0 + \sum_{j=1}^{k} \Delta r_j + \Delta T_r + a_i + b_i t_k$$

where the subscript i refers to ground station i. Defining the *Doppler-derived range* as

$$r_{D_k} = r_0 + \sum_{j=1}^{k} \Delta r_j + \Delta T_r \ , \qquad (2')$$

and recalling that

$$r_{ik} = \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2}$$

and changing the signs of a and b, we arrive at the mathematical model

$$r_{D_{ik}} = \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2} + a_{i\ell} + b_{i\ell} t_k \qquad (3)$$

where $r_{D_{ik}}$ is the Doppler-derived pseudo-range (derived from the measured Doppler counts and corrected for tropospheric refraction), and the unknown parameters to be solved for in a least squares adjustment are

| | |
|---|---|
| $X_i, Y_i, Z_i$ | the unknown station (i) coordinates |
| $X_k, Y_k, Z_k$ | the unknown satellite (k) coordinates |
| $a_{i\ell}, b_{i\ell}$ | the unknown coefficients used to model systematic errors for each station (i) and pass ($\ell$) |

$t_k$ is the time elapsed from the epoch of the initial range $r_0$.

## 2.3 Least Squares Adjustment

The mathematical model developed above has the form of an observation equation:

$$L_a = F(X_a) \qquad (4)$$

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where $L_a$ is the adjusted Doppler-derived range, and $X_a$ is the vector of the unknown parameters which can be divided into three subvectors:

$XG_a = XG_0 + XG$ containing the coordinates of the ground stations
$XC_a = XC_0 + XC$ containing the error coefficients a, b
$XS_a = XS_0 + XS$ containing the satellite coordinates

Equation (3) can be written in linearized form

$$r_{Dik\ell} + v_{ik\ell} = F^0_{ik\ell} + \left.\frac{\partial F}{\partial XG}\right|_{X_0} \cdot XG_i + \left.\frac{\partial F}{\partial XC}\right|_{X_0} \cdot XC_{i\ell} + \left.\frac{\partial F}{\partial XS}\right|_{X_0} \cdot XS_k + \ldots \qquad (5)$$

or, after neglecting higher-order terms

$$v_{ik\ell} = A_{ik\ell} \cdot XG_i + C_{ik\ell} \cdot XC_{i\ell} + S_{ik\ell} \cdot XS_k - W_{ik\ell}$$

where

$$A_{ik\ell} = \left.\frac{\partial F}{\partial XG}\right|_{XG^0_i \; XC^0_{i\ell} \; XS^0_k} = \left( - \frac{X^0_k - X^0_i}{r_{0_{ik\ell}}}, \; - \frac{Y^0_k - Y^0_i}{r_{0_{ik\ell}}}, \; - \frac{Z^0_k - Z^0_i}{r_{0_{ik\ell}}} \right)$$

$$C_{ik\ell} = \left.\frac{\partial F}{\partial XC}\right|_{XG^0_i \; XC^0_{i\ell} \; XS^0_k} = (1, \; t_k)$$

$$S_{ik\ell} = \left.\frac{\partial F}{\partial XS}\right|_{XG^0_i \; XC^0_{i\ell} \; XS^0_k} = -A_{ik\ell}$$

$$W_{ik\ell} = r_{Dik\ell} - (r_{0_{ik\ell}} + a_{0_{i\ell}} + b_{0_{i\ell}} t_k)$$

$$r_{0_{ik}} = \sqrt{(X^0_k - X^0_i)^2 + (Y^0_k - Y^0_i)^2 + (Z^0_k - Z^0_i)^2}$$

In this study all pseudo-range observations are assumed to have equal weight. For reason of convenience in programming, the a priori variance of unit weight is chosen to be equal to the variance of a range observation

$$\sigma_0^2 = \sigma_{DR}^2$$

Therefore all observations have the weight one. Further details of this least squares adjustment as used in the Geometric Doppler (GEODOR) computer program may be found in [Schneeberger 1982].

# 3. ADJUSTMENT WITH CORRELATIONS CONSIDERED

## 3.1 Mathematical Model

The correlation existing in the Doppler-derived ranges are considered in this study by assuming that the range differences (computed from the Doppler counts) are independent observations.

Under this consideration, substituting eq. (2') into (3) and moving all the terms to the left side, we obtain

$$\sqrt{(X_k-X_i)^2 + (Y_k-Y_i)^2 + (Z_k-Z_i)^2} + a_{i\ell} + b_{i\ell}t_k - r_0 - \sum_{j=1}^{k} \Delta r_j - \Delta T_r = 0 \tag{6}$$

Thus the model becomes the form of a condition equation with parameters:

$$F(L_a, X_a) = 0 \tag{7}$$

Eq. (6) can be written in a linearized form, using the same notation as before,

$$A_{ik\ell}X_i + C_{ik}\, XC_{ik\ell} + S_{ik\ell}\, XS_k + \sum_{j=1}^{k} B_{ik\ell}\, v_j - W_{ik\ell} = 0 \tag{8}$$

where $B_{ik\ell}$ stands for the derivatives of F with respect to $\Delta r_j$, i.e.,

$$B_{ik\ell} = \frac{\partial F}{\partial \Delta r_j} = \begin{cases} -1 & \text{if } j \leq k \\ 0 & \text{if } j > k \end{cases} \tag{9}$$

All the observations are assumed to have equal weight. For convenience in programming, the a priori variance of unit weight is chosen to be equal to the variance of a range difference observation

$$\sigma_0^2 = \sigma_{\Delta r}^2$$

Therefore, all the observations have unit weight. For the detail of the derivations of the mathematical model and the method of solving this problem, see [Zhang 1982].

## 3.2 Construction of Normal Equations

The solution of the normal equation system for the least squares model of condition equations with parameters has the following form [Uotila 1976]:

$$X = -(A^T M^{-1} A)^{-1} A^T M^{-1} W \tag{10}$$

where

$$M^{-1} = (B\, P^{-1}\, B^T)^{-1}$$

Therefore, before constructing the normal equation system, $M^{-1}$ has to be found first. Fortunately, the matrix B has a regular configuration, and so does the matrix $M^{-1}$ [Ashkenazi et al. 1980]. For the sake of simplicity, we investigate a matrix B for one station and one pass. From eqs. (8) and (9) it is evident that the matrix B has the form

$$B = \begin{vmatrix} -1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & -1 & 0 & 0 & \cdots & 0 \\ -1 & -1 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -1 & -1 & -1 & -1 & \cdots & -1 \end{vmatrix} \tag{12}$$

If one assumes uniform weight and no correlations between the range differences, and chooses the variance of unit weight equal to the variance of range difference observation, the matrix P will become an identity matrix. Then the matrix M can be written as

$$M = B\, P^{-1} B^T = \begin{vmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 2 & 3 & 4 & \cdots & n \end{vmatrix} \tag{13}$$

where n is the number of observations in this pass. $M^{-1}$ is found by inverting M:

$$M^{-1} = \begin{vmatrix} 2 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 & 1 \end{vmatrix} \tag{14}$$

Since $M^{-1}$ is a regular diagonal matrix, it will not invite much difficulty when constructing normal equations. For the case of more than one station and more than one pass, matrices B and $M^{-1}$ can easily be found by using the same method [Zhang 1982].

After the matrix $M^{-1}$ is found, all the coefficients of the normal equation system can be calculated. Since this normal equation system is still of the sparsity pattern, a method called second-order partitioned regression can be used to eliminate the unknowns to save storage and computing time [Brown and Trotter 1969].

# 4. NUMERICAL TEST

## 4.1 Solutions and Their Comparisons

The data taken during EDOC-2 was used for testing the uncorrelated and correlated modes. Fig. 1 shows the network used which is chosen from EDOC-2. There were many solutions for each mode, but only the best

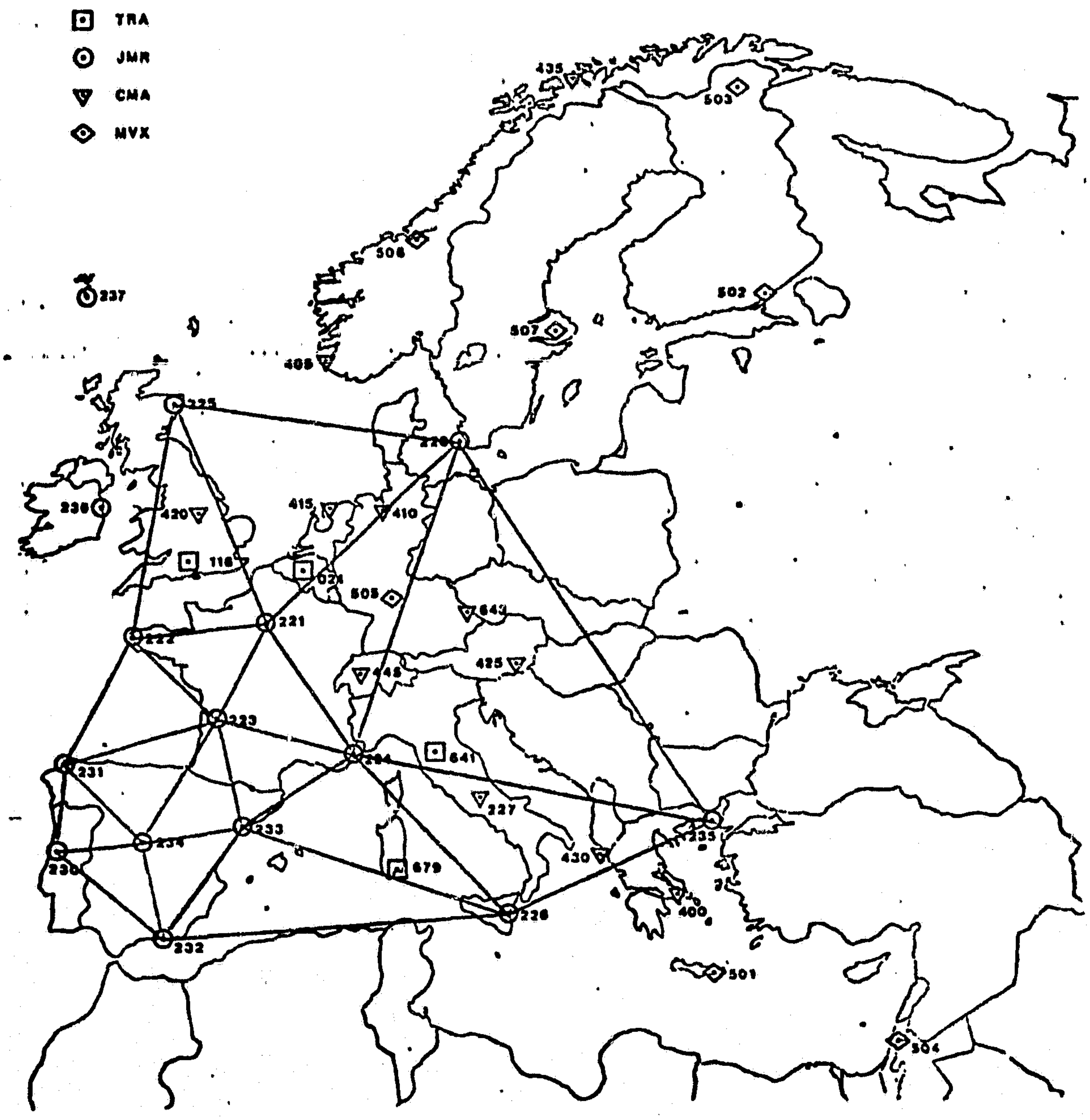


Fig. 1 EDOC-2 network [Boucher et al. 1981]

one of each mode can be presented here. Table 1 is a summary of these two solutions. Solution F4-5 is in the uncorrelated mode; solution C-5 in the correlated mode. The integration intervals of both solutions are 5 x 4.6 = 23 seconds.

The information of solutions using different integration intervals from the correlated mode is collected in Table 2. In the designation C-i, i indicates the integration intervals used, e.g., in case of i = 2, the range change is over 2 x 4.6 = 9.2 s. Fig. 2 gives a visual comparison of these solutions. It is obvious that the solution with i = 5 is the best.

4.2 Test of the Systematic Error Model

In this study as in the earlier one only the two major terms are used for modeling the systematic effects: a + bt. The residuals of the observations of randomly selected passes from the total of 193 passes were plotted for each station. Fig. 3 is one example. Investigating the distributions of the residuals of the observations at each station, no significant remaining systematic effect is found, which indicates that the two major terms used for modeling the systematic effects are reasonable.

4.3 Test of the Residuals

From Table 1 we can find that the correlated mode is superior to the uncorrelated one. In spite of that, there are still significant differences between solutions C-5 and EDOC-2. In order to find the reason, the residuals of all observations were investigated. Table 3 lists the statistics of the residuals of the observations over the ten worst passes.

Checking this table, one can see that the maximum residual is as large as 160 m, and the ratio of the number of the observations whose absolute residuals are larger than three times the standard deviation, to the total number of the observations for each one of the worst passes is high. The worst one is as high as 11.2%. This indicates that there may be blunders in the data set.

4.4 Problem of Weights

As stated earlier, all observations are assumed to have equal weight and the a priori variance of unit weight is chosen to be equal to the variance of a range difference observation

$$\sigma_0^2 = \sigma_{\Delta r}^2 = 1.0$$

In Table 2 one can see that the a posteriori standard deviations of unit weight for all the solutions are much larger than the chosen a priori one. For instance the a posteriori standard deviation of unit weight of the best solution, C-5, is as large as 3.4.

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Table 1 Summary of Solutions F4-5 and C-5

| Solution No.: | | F4-5 | | | C-5 | | |
|---|---|---|---|---|---|---|---|
| Total No. of Passes Processed | | 193 | | | 193 | | |
| Total No. of Events | | 3,430 | | | 3,430 | | |
| No. of Unknowns | | | | | | | |
| Station Coordinates | | 30 | | | 30 | | |
| Error Coefficients | | 3,312 | | | 3,312 | | |
| Satellite Coordinates | | 10,290 | | | 10,290 | | |
| Total No. of Unknowns | | 13,632 | | | 13,632 | | |
| Total No. of Observations | | 27,531 | | | 27,531 | | |
| Degrees of Freedom | | 13,899 | | | 13,899 | | |
| A Priori Weight Information: | | | | | | | |
| Range (or Range Difference) | | 1 m | | | 1 m | | |
| Error Coefficient $\sigma_a$ | | 50 m | | | 50 m | | |
| Error Coefficient $\sigma_b$ | | 38 m / 2 min | | | 38 m / 2 min | | |
| Fixed Station Coordinates $\sigma_X$, $\sigma_Y$, $\sigma_Z$ | | 1 mm | | | 1 mm | | |
| Other Station Coordinates $\sigma_X$, $\sigma_Y$, $\sigma_Z$ | | 100 m | | | 100 m | | |
| 3 Satellite Events/Pass $\sigma_X$, $\sigma_Y$, $\sigma_Z$ | | 10 m | | | 10 m | | |
| A Posteriori Standard Deviation of Unit Weight | | 3.5 m | | | 3.4 m | | |
| Coordinate Differences with Respect to EDOC-2 Solution (all units in m) | | $\Delta\phi$ | $\Delta\lambda$ | $\Delta H$ | $\Delta\phi$ | $\Delta\lambda$ | $\Delta H$ |
| Station No. (* indicates fixed station) | 220* | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | 221 | 3.5 | 16.2 | -4.4 | 3.0 | 7.0 | -4.6 |
| | 222 | 3.4 | -4.3 | -1.2 | 1.9 | -3.0 | -1.4 |
| | 223 | 1.5 | 9.8 | -7.1 | 1.7 | 5.2 | -4.2 |
| | 224 | 5.6 | 27.1 | -12.6 | 5.3 | 13.2 | 13.1 |
| | 225 | -4.8 | -10.9 | 6.7 | -0.2 | 18.4 | 3.0 |
| | 226* | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | 230 | 1.0 | -4.5 | 13.6 | 4.6 | 3.2 | 9.9 |
| | 231* | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| | 232 | 4.5 | -3.1 | 15.7 | 2.1 | -4.6 | 20.5 |
| | 233 | 0.8 | 8.7 | -2.0 | 1.6 | 2.0 | -0.9 |
| | 234 | 0.1 | -1.7 | 5.3 | -1.0 | 1.1 | 7.5 |
| | 235 | 1.0 | 44.2 | 70.0 | 8.2 | -0.4 | 1.9 |
| Average absolute difference (m) (10 stations) | | 2.6 ±2.8 | 13.0 ±5.2 | 14.0 ±4.6 | 2.2 ±4.7 | 5.8 ±8.9 | 6.7 ±7.9 |
| Average absolute difference in position (m) | | 20.8 ±21.7 | | | 10.8 ±6.6 | | |
| Average absolute station-to-station chord distance difference (m) | | 10.2 ±10.5 | | | 5.5 ±4.5 | | |

Table 2 Comparison of the Different Integration Intervals Used in Adjustment

| Name of Solution: | C-2 | C-5 | C-10 | C-15 |
|---|---|---|---|---|
| Integration interval (seconds) | 9.2 | 23 | 46 | 69 |
| Computing time (minutes)* | 25.20 | 8.83 | 5.96 | 2.79 |
| A posteriori standard deviation of unit weight | 2.4 | 3.4 | 4.5 | 5.9 |
| Average total absolute difference in position (m) (10 stations) | 10.9 ±7.3 | 10.9 ±6.6 | 22.0 ±19.3 | 33.8 ±37.1 |
| Average absolute station-to-station chord distance difference (m) | 6.4 ±5.6 | 5.5 ±4.4 | 8.9 ±8.2 | 17.1 ±19.2 |

*using an Amdahl 470

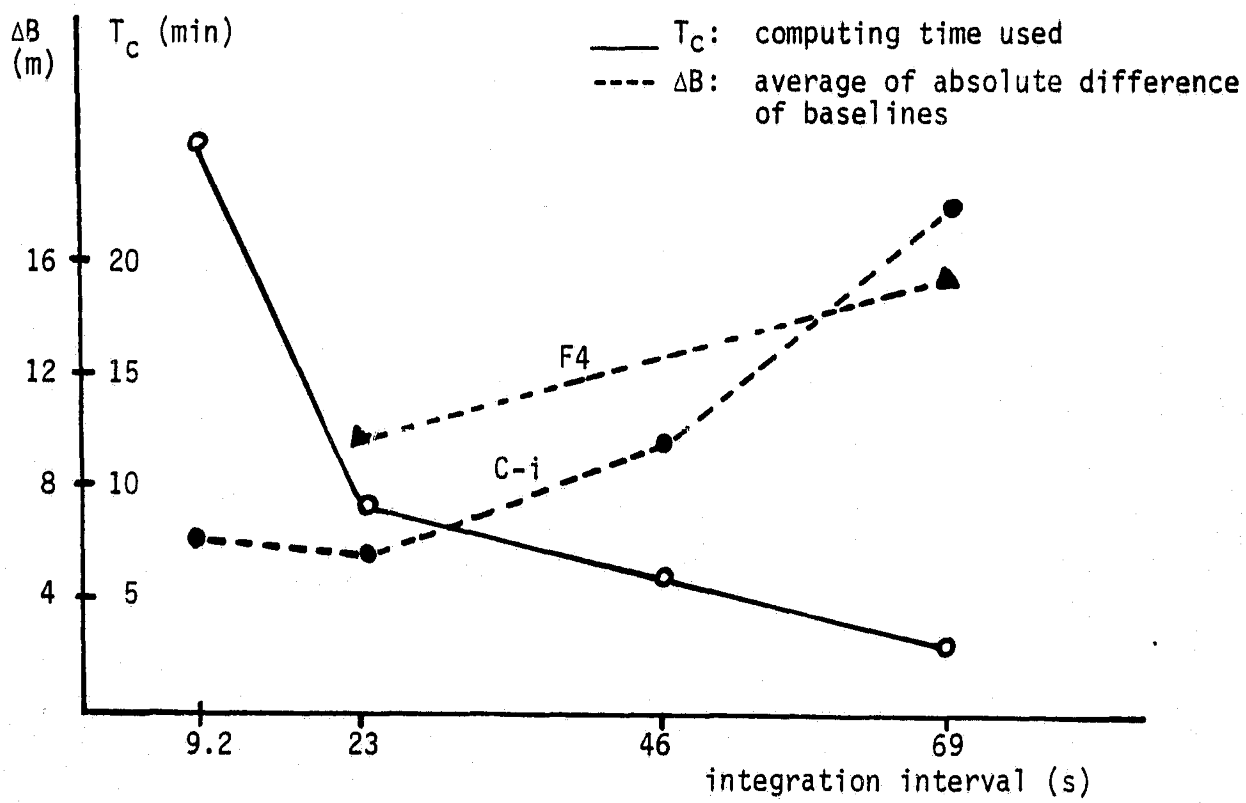


Fig. 2 Computing time used and average of absolute differences of baselines plotted against the length of integration interval

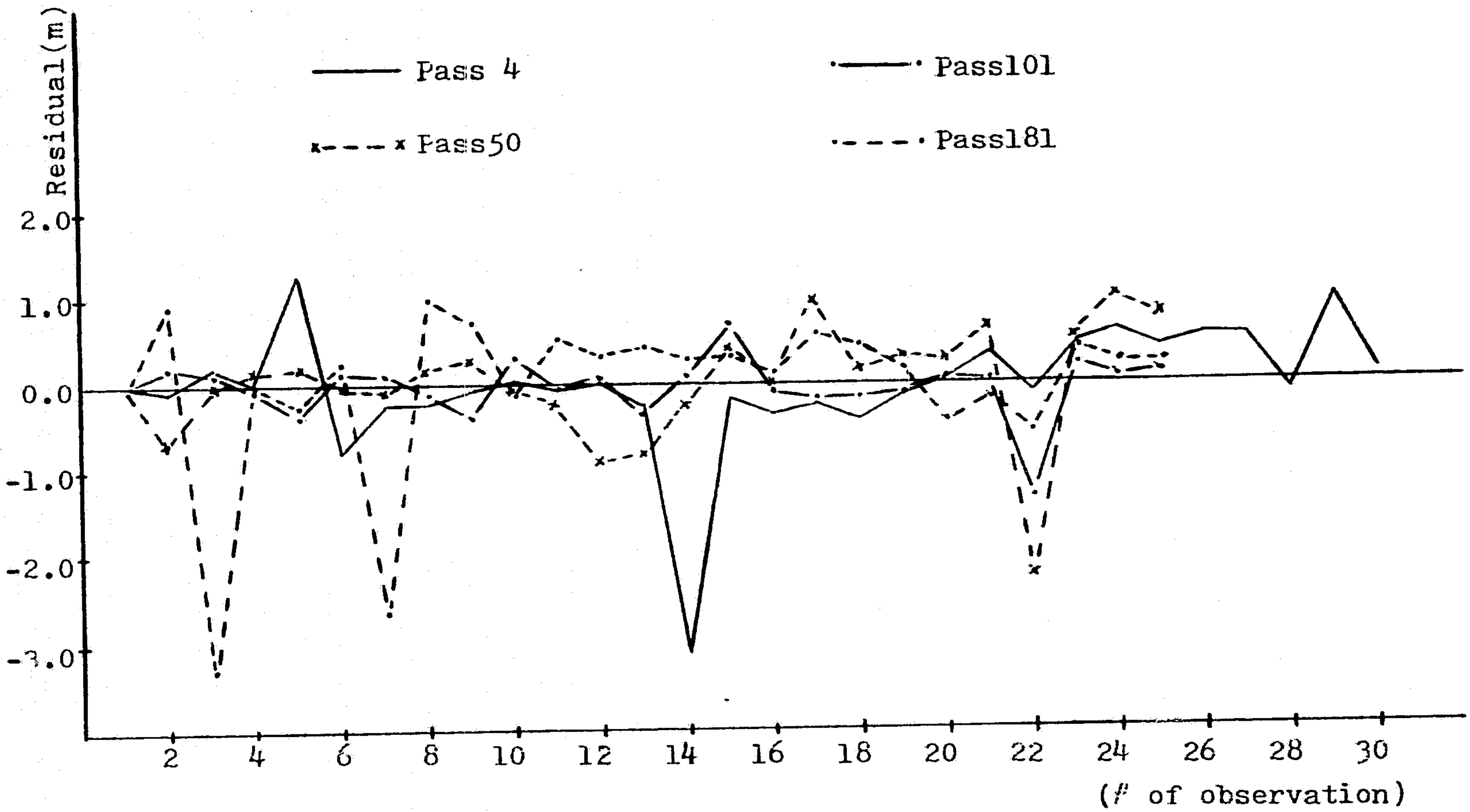


Fig. 3 Distribution of residuals at station 223 in passes 4, 50, 101 and 181.

Table 3 Statistics of the Residuals of the Observations of the Ten Worst Passes

| No. | Pass No. | Number of Observations | | | | | | | $\|v\|_{max}$ (m) |
|---|---|---|---|---|---|---|---|---|---|
| | | Total | $\|v\| > 3\sigma$ | | $\|v\| > 2\sigma$ | | $\|v\| > 10.0$ m | | |
| | | | Number | % | Number | % | Number | % | |
| 1 | 49 | 207 | 20 | 9.7 | 27 | 13.0 | 17 | 8.2 | 160.8 |
| 2 | 46 | 187 | 18 | 9.6 | 38 | 20.3 | 13 | 7.0 | 126.1 |
| 3 | 187 | 143 | 16 | 11.2 | 27 | 18.9 | 12 | 8.4 | 41.4 |
| 4 | 21 | 262 | 19 | 7.3 | 32 | 11.8 | 10 | 3.8 | 53.0 |
| 5 | 43 | 181 | 9 | 5.0 | 17 | 9.4 | 8 | 4.4 | 39.5 |
| 6 | 180 | 142 | 9 | 6.3 | 15 | 10.6 | 8 | 5.6 | 25.6 |
| 7 | 51 | 221 | 10 | 4.5 | 15 | 6.8 | 7 | 3.2 | 17.2 |
| 8 | 26 | 186 | 10 | 5.4 | 12 | 6.5 | 5 | 2.7 | 26.0 |
| 9 | 25 | 105 | 6 | 5.7 | 7 | 6.7 | 5 | 4.8 | 31.6 |
| 10 | 16 | 195 | 7 | 3.6 | 10 | 5.1 | 4 | 2.1 | 37.4 |

Table 4 presents the comparison of the weights of each station calculated from the residuals over all passes. The weights of the stations differ from each other for solutions C-5; the largest one is ninefold as large as the smallest one. When the ten worst passes are taken out, the weights are close to each other, and the a posteriori standard deviation of unit weight is decreased from 3.4 to 2.0. It is seen that the existence of blunders is probably the most important detrimental factor in the solution.

Unfortunately, neither taking out the ten worst passes nor repeating the computation with the different weights for each station improved the result. It is likely that although taking out the ten worst passes removed the major blunders, it also resulted in losing many useful observations.

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Table 4 Comparisons of the Weights of Each Station

| Station No. | All Passes Used | | | W/o 10 Worst Passes | | |
|---|---|---|---|---|---|---|
| | No. of Obs. | $\hat{\sigma}$ | p | No. of Obs. | $\hat{\sigma}$ | p |
| 220 | 1579 | 2.08 | 2.7 | 1473 | 1.73 | 1.3 |
| 221 | 1668 | 1.72 | 4.0 | 1606 | 1.56 | 1.6 |
| 222 | 2912 | 3.30 | 1.1 | 2711 | 1.33 | 2.2 |
| 223 | 1862 | 1.30 | 6.9 | 1777 | 1.27 | 2.4 |
| 224 | 2821 | 1.77 | 3.7 | 2631 | 1.49 | 1.7 |
| 225 | 1893 | 2.19 | 2.4 | 1757 | 1.33 | 2.2 |
| 226 | 845 | 2.56 | 1.8 | 763 | 1.26 | 2.4 |
| 230 | 2505 | 2.26 | 2.3 | 2435 | 1.21 | 2.7 |
| 231 | 2789 | 3.85 | 0.8 | 2609 | 1.87 | 1.1 |
| 232 | 2391 | 2.42 | 2.0 | 2205 | 1.30 | 2.3 |
| 233 | 2760 | 1.67 | 4.2 | 2575 | 1.30 | 2.3 |
| 234 | 2641 | 2.36 | 2.1 | 2444 | 0.99 | 4.0 |
| 235 | 865 | 1.25 | 7.5 | 809 | 1.01 | 3.8 |
| Degree of Freedom | 13,899 | | | 12,910 | | |
| $\hat{\sigma}_0$ | 3.4 | | | 2.0 | | |

## 5. CONCLUSIONS

On the basis of the comparisons, the folloiwng conclusions can be drawn:

(1) The geometric mode of solving the problem of simultaneous Doppler-derived ranges without considering the correlation is a weak one.

(2) The correlated geometric mode leads to better results. Comparing with the uncorrelated solution, the correlated mode reduced the average total absolute differences (with respect to EDOC-2) in position from 20.8 ±21.7 m to 10.9 ±6.6 m; and the average absolute station-to-station chord distance differences from 10.2 ±10.5 m to 5.5 ±4.5 m.

(3) The choice of the optimum integration interval is very important for the use of simultaneous Doppler-derived ranges in the geometric mode. The examples of this study demonstrate that the optimum integration interval is 23 s, which agrees with that suggested by [Ashkenazi et al. 1980].

ACKNOWLEDGMENTS. The EDOC-2 data set was obtained through the efforts of Peter Wilson, Inst. f. Angewandte Geodäsie, Frankfurt, FRG, and Claude Boucher, Inst. Geographique National, France. Mr. R. Schneeberger developed the uncorrelated geometric mode and wrote the program GEODOR. The Instruction and Research Computer Center of The Ohio State University provided computer support.

REFERENCES

Ashkenazi, V., D.N. McLintock and R.M. Sykes (1980) "Doppler Integration Intervals and Correlation," Phil. Trans. Roy. Soc., A 294, 337-384.

Boucher, C., P. Paquet and P. Wilson (1981) "Final Report on the Observations Carried Out in the Second European Doppler Observation Campaign (EDOC-2) for Position Determinations at 37 Satellite Tracking Stations," D.G.K., B 225, Frankfurt.

Brown, D.C. and J.E. Trotter (1969) SAGA, A computer Program for Short Arc Geodetic Adjustment of Satellite Observations, DBA Systems Inc., Melbourne, Florida.

Jordan/Eggert/Kneissl (1966) Handbuch der Vermessungskunde, Band IV. J.B. Metzlersche Verlagsbuchhandlung, Stuttgart, FRG.

Kouba, J. and J.D. Boal (1976) "Program GEODOP," Dept. of Engineering, Mines and Resources Canada, Ottawa.

Krakiwsky, E.J. and D.E. Wells (1971) "Mathematical Models for Positioning by Satellite," Lecture Notes No. 17, Dept. of Surveying Engineering, Univ. of New Brunswick, Fredericton, Canada.

Mueller, I.I. (1964) Introduction to Satellite Geodesy, Ungar Publ., New York.

Schneeberger, R. (1982) Use of Simultaneous Doppler-Derived Ranges from the EDOC-2 Campaign in the Geometric Mode, MS thesis, Dept. of Geodetic Science and Surveying, Ohio State Univ., Columbus.

Schneeberger, R., D. Pavlis and I.I. Mueller (1982) "Use of Simultaneous Doppler-Derived Ranges in the Geometric Mode," Proc. of 3rd International Symp. on Satellite Doppler Positioning, Physical Science Laboratory, New Mexico State University, Las Cruces.

Uotila, U. (1976) "Adjustment Computation with Matrices," Lecture Notes, Department of Geodetic Science, Ohio State Univ., Columbus.

Wells, D.E. (1974) "Doppler Satellite Control," Tech. Rep. 29, Univ. of New Brunswick, Fredericton, N.B., Canada.

Zhang, C. (1982) Use of Simultaneous Doppler-Derived Correlated Ranges in the Geometric Mode, MS thesis, Dept. of Geodetic Science and Surveying, Ohio State Univ., Columbus.

## 2.222 Doppler Intercomparison Experiment

In the previous semi-annual report, preliminary results of the 1979 CSU comparison test of Doppler receivers are given. Since that time a final report on the comparison has been completed [Archinal, 1982] as a Master's thesis (and soon as a report of the Department of Geodetic Science and Surveying).

In this report, some of the results presented in the previous report are revised, and some additional final results are presented as well. For a more detailed discussion of the following, refer to [Archinal, 1982], and [Archinal and Mueller, 1982].

### FINAL RESULTS OF DATA REDUCTION

As mentioned above, some of the results presented here are slightly different than those given in the last report. This is primarily due to:

a) The determination and use of receiver time delays in the GEODOP processing.

b) The modification of GEODOP to allow the input of a "common station noise" estimate, and use of this option, along with the use of a variance estimation process in GEODOP as well.

Therefore revised versions of the tracking statistics and chord difference results are given here, along with new information concerning the estimation of the receivers' observational (range rate) error and oscillator stability.

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## Tracking Statistics

The statistics on the number of passes tracked, used in PREDOP, and two types of GEODOP runs are presented in table 1. Although several numbers have changed substantially from those given in the last report, most of the results given there are still valid. In addition to these results, it should be noted that if these statistics are broken down by antenna setup (as in [Archinal, 1982, pp. 61-70]), it becomes clear that:

a) The CMA-751 and the JMR-1As generally tracked about the same number of passes, and slightly more than the MX1502 (when operating correctly and tracking continuously).

b) There is no bias due to antenna location, at least when PREDOP rejections are considered. The relative percentages of rejections stayed fairly constant for all setups for the JMR-1A #2 and MX1502. No conclusions can be drawn for the CMA-751 due to its faulty antenna cable (on all but one site), or for the JMR-1A #1, since it only occupied one site.

The GEODOP statistics (for a multi-station broadcast ephemeris solution and a single station precise ephemeris solution) show a fairly consistant observations/pass value for all instruments, except for the MX1502, which has a higher value in both solutions. This higher value is due to the fact that the MX1502 was recording only (the better) passes which reached over 15 degrees altitude on the first setup, which strongly affects the grand totals shown here. The observations/pass for the CMA-751 are not representative here either, since it was operating properly only during the first and last setup.

## Chord Difference Results

Table 2 shows absolute differences obtained in the chord distance between all pairs of instruments for each antenna setup for multi-station and precise ephemeris solutions. Many of these values are different from those given in the previous report, with generally smaller standard deviations and chord differences than previously reported. This is probably due to the changes in weighting and the better determined delays respectively, and points out the value of the

TABLE 1 SUMMARY OF TRACKING AND PASS/DOPPLER COUNT ACCEPTANCE
1979 $274^D$ $14^H$ - $317^D$ $16^H$ ($43^D$ $02^H$ TOTAL)

| INSTRUMENT | NO. PASSES TRACKED | NO. PASSES TRACKED PER DAY | NO. PASSES AFTER PREDOP | | NO. PASSES (DOPPLER COUNT/PASS) AFTER GEODOP SOLUTION | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | MULTI-STA. - B.E. | | | POINT PS. - P.E. | |
| CMA-751 [1] | 827 | 19.2 | 680 | 82% [4] | 594 | (17.6) | 72% [4] | 290 | (17.5) |
| JMR-1A #1 [2] | 231 | 20.6 | 197 | 85% | 185 | (16.5) | 80% | 77 | (17.6) |
| JMR-1A #2 | 919 | 21.3 | 770 | 84% | 642 | (16.5) | 70% | 317 | (16.5) |
| MX-1502 [3] | 805 | 18.7 | 483 | 60% | 429 | (17.9) | 53% | 190 | (18.4) |

1 CMA-751 LOST PASSES DUE TO FAULTY ANTENNA CABLE, INTERMITTENTLY BETWEEN $286^D$ AND $302^D$.

2 JMR-1A #1 OBSERVED ONLY DURING $274^D$ $14^H$ - $285^D$ $19^H$.

3 MX-1502 WAS NOT TRACKING CONTINUOUSLY UNTIL $291^D$ $17^H$ AND HAD BREAKDOWNS INTERMITTENTLY BETWEEN $289^D$ $18^H$ AND $310^D$ $04^H$. THE LARGE DIFFERENCE BETWEEN NUMBER OF PASSES TRACKED AND AVAILABLE AFTER PREDOP IS DUE MAINLY TO 206 PASSES (26% OF THOSE TRACKED) WHICH THE MX-1502 WAS UNABLE TO MAJORITY VOTE.

4 %'s ARE WITH RESPECT TO NUMBER OF PASSES TRACKED.

TABLE 2 ABSOLUTE DIFFERENCES IN CM OVER CHORD DISTANCES RANGING FROM 7.2 M TO 22.2 M IN LENGTH

| ANTENNA SETUP # | MULTI-STATION SOLUTIONS BROADCAST EPHEMERIS - 5 SATELLITES | | | | | | MULTI-STATION SOLUTIONS PRECISE EPHEMERIS - 2 SATELLITES | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | CMA-751 TO JMR-1A #2 | | CMA-751 TO MX-1502 | | JMR-1A #2 TO MX-1502 | | CMA-751 TO JMR-1A #2 | | CMA-751 TO MX-1502 | | JMR-1A #2 TO MX-1502 | |
| | DIFF | $\sigma$ | DIFF | $\sigma$ | DIFF | $\sigma$ | DIFF | $\sigma$ | DIFF | $\sigma$ | DIFF | $\sigma$ |
| 1 | -17 | 6 | -13 | 7 | -20 | 13 | 1 | 10 | -15 | 10 | 18 | 22 |
| 2 | -60 | 11 | 3 | 5 | -34 | 26 | -30 | 16 | 63 | 27 | -47 | 54 |
| 3 | -25 | 25 | - 5 | 22 | -55 | 29 | - 7 | 36 | 58 | 36 | 49 | 56 |
| 4 | 11 | 10 | 6 | 12 | 1 | 20 | 18 | 21 | -12 | 20 | 21 | 36 |
| 5 | 6 | 10 | -33 | 12 | - 9 | 21 | - 7 | 16 | -83 | 17 | 73 | 35 |
| WEIGHTED MEAN | 21 | 4 | 14 | 5 | 20 | 8 | 17 | 7 | 33 | 7 | 33 | 15 |
| | CMA-751 TO JMR-1A #1 | | JMR-1A #1 TO JMR-1A #2 | | JMR-1A #1 TO MX-1502 | | CMA-751 TO JMR-1A #1 | | JMR-1A #1 TO JMR-1A #2 | | JMR-1A #1 TO MX-1502 | |
| 1 | 3 | 12 | 13 | 7 | 11 | 10 | 28 | 20 | 13 | 10 | 16 | 15 |

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more rigorus weighting and better determination of delay for these solutions.

Even with the differences, the previous result still holds, that most of the differences (all execept two) lie within their three sigma value. A new result is that the single baseline determined between instruments of the same type (between the two JMR-1As on setup #1) did not show significantly better results than pairings between any other instrument combination. In conclusion, it appears that there is no evidence that any of these instruments are biased against one another for chord determinations (over short distances).

Another additional result shown by this table is that the precise ephemeris two satellite solutions for chord distances do not appear to have necessarily higher accuracy or precision than the corresponding broadcast ephemeris five satellite solutions, and in fact the precision of the broadcast ephemeris solution is better in <u>all</u> cases. This simply indicates that the greater number of observations in the broadcast ephemeris solution improves the results more than the corresponding increase in ephemeris accuracy of the precise ephemeris solution. This would imply that if only chord distances were needed from Doppler observations, then generally broadcast ephemeris solutions would be preferable to precise ephemeris solutions, since the former usually have more data available.

## Range Rate Measurement Errors

Using procedures described in detail in [Archinal, 1982, pp. 70-79], estimates of the common station noise and each instrument's range rate standard deviation were made for each setup and precise ephemeris satellite. The results are shown in table 3 and discussed here.

First of all, the common station noise was estimated by processing only simultaneous observatins and precise ephemeris orbits. The common station (or "interstation" or "satellite" noise) estimates were made using the common station estimated variance-covariance matrix output by GEODOP to obtain the values shown in column three of table 3. The results vary with satellite and time during the entire test, with an amount between 3.4 and 7.5 cm/30 seconds. The overall average value (weighted mean of all observation pairs) is 4.9 cm/30 seconds. Since the range was not too great,

TABLE 3 ESTIMATION OF COMMON STATION NOISE AND INSTRUMENT RANGE RATE STANDARD DEVIATION

| SETUP | SATELLITE NO. | NO. OF OBSERVATIONS (1) | COMMON STATION NOISE (2) | CMA-751 (3) | JMR-1A #1 (3) | JMR-1A #2 (3) | MX-1502 (3) |
|---|---|---|---|---|---|---|---|
| 1 | 14 | 263 | 4.8 | 8.8 | 12.0 | 9.2 | 12.3 |
| | 19 | 277 | 3.4 | 7.9 | 11.8 | 9.5 | 11.4 |
| 2 | 14 | 75 | 3.4 | -- | | -- | -- |
| | 19 | 123 | 6.5 | -- | | -- | -- |
| 3 | 14 | 26 | -- | -- | | -- | -- |
| | 19 | 52 | 4.0 | -- | | -- | -- |
| 4 | 14 | 161 | 7.5 | 12.8 | | 12.4 | 14.5 |
| | 19 | 157 | 3.5 | -- | | -- | -- |
| 5 | 14 | 160 | 5.5 | 6.4 | | 6.5 | 13.5 |
| | 19 | 276 | 5.6 | 11.3 | | 11.4 | 12.8 |
| ALL | 14 | | 5.4 | 9.6 | 12.0 | 9.9 | 13.0 |
| | 19 | | 4.4 | 9.7 | 11.8 | 10.6 | 12.1 |
| | BOTH | | 4.9 | 9.7 | 11.9 | 10.4 | 12.5 |

(1) NUMBER OF "30" SECOND COUNTS OBTAINED SIMULTANEOUSLY BY ALL INSTRUMENTS AND USED IN ESTIMATE. NO ESTIMATE DONE IF LESS THAN 10 OBS. PER INTERVAL.

(2) WEIGHTED MEAN OF RANGE RATE INTERSTATION NOISE FOR EACH 30-SECOND INTERVAL.(CM).

(3) WEIGHTED MEAN OF RESIDUALS (NOISE) FOR EACH 30-SECOND INTERVAL (CM).

and rather than change the value for each setup/satellite (perhaps based on too few observations), the GEODOP default of 5.0 cm/30 seconds was then used for all subsequent processing.

Secondly, the estimated receiver range rate standard deviations were computed for each setup and satellite, using the weighted mean of the diagonal elements of the estimated variance-covariance matrix of the residuals. The results for all three instrument types are shown in the last four columns of table 3. These values were obtained from GEODOP single station precise ephemeris solutions, in which the observations were approximately (the rejections due to statistical testing cause some exceptions) simultaneous. Although some of the individual solutions did not have enough data to be considered significant, using at least 600 observations in each case (but only 350 for the JMR-1A #1), the estimated range rate standard deviations were found to be 9.7, 11.9, 10.4, and 12.5 cm/30 seconds for the CMA-751, JMR #1, JMR-#2, and the MX1502 respectively, over the entire period of the test. The variations shown during the test may be partially instrument related, but they are probably due mostly to the satellite noise just discussed. The relative precision of the three instruments continuously observing also stays approximately the same for all time periods and satellites, which also indicates that the variations are non-receiver related. It is also significant that the variation between instruments is usually less than 3 cm/30 seconds, showing that these instruments are generally very similar, and that the variation in the common station noise is generally greater than this. The conclusion can therefore be drawn that the variation of the measurement precision between these instruments is not significant. The even more important conclusion which can be drawn is that the range rate accuracy obtainable depends in many cases more on the time and the satellite than it does on the receiver itself.

Lastly, to obtain the best possible estimates of the final variance-covariance matrices in GEODOP, the GEODOP option was used to allow an internal estimate of the range rate standard deviation and adjacent observation correlation for each pass to be made, with the previously estimated range rate standard deviation value (given in the last paragraph) used as an input approximate value. Although increasing the computational time by over 50% (all of the passes are processed twice), this method takes into account the variation of the satellite noise and possible variations in the receiver noise during the period under consideration. It is felt that this procedure, in conjunction with the first two above would result in the most rigorus processing of the data, to provide the best solutions.

## Frequency Drift Results

The frequency drift of an instrument's oscillator is an important quantity which can be determined to fairly high accuracy during data reduction. In general, the more stable an oscillator (over the period of a satellite pass) the better the timing and Doppler count measurements can be made. If a drift is occurring, and remains fairly constant in time, it can be taken into account in the adjustment of the data (as in GEODOP), although it must still be assumed to be linear over a pass, and should not be very large in magnitude. If the drift is erratic, either changing during a pass or over just a few passes, the data will be very noisy due to these unmodeled changes in the oscillator. It is therefore important to check the frequency drift variations of these instruments. Ideally, one would like to check the short term drift which corresponds to the length of a satellite pass (over 100 seconds to about 15 minutes), but this is generally not possible unless an atomic standard is available for comparison. Instead, the long term drift of these instruments can be checked for variations (which may provide an indication of the short term stability), or at least checked against the manufacturer's specifications.

In the case of the data collected here, the frequency drift for each instrument for each setup and precise ephemeris satellite has been determined. The values have been obtained from the difference between the first and last (reasonable) frequency offsets computed for each instrument during a setup. The frequency offsets were determined from two satellite (one satellite at a time) precise ephemeris, single station solutions, and the antenna setup periods which ranged from about five to fifteen days in length. Note that to obtain the per day values given here, the assumption has been made that the frequency drift is constant during each setup. Examination of the GEODOP frequency plots supports this assumption.

The results for frequency drift are shown in table 4, and have been graphed in figure 1. They can be summarized as follows:

a) The CMA-751 had a fairly uniform value for frequency drift, using either satellite, and easily met its spe-

TABLE 4 LONG-TERM OSCILLATOR FREQUENCY DRIFT [1]

| SETUP NO. | SATELLITE NO. | CMA-751 | JMR-1A #1 | JMR-1A #2 | MX-1502 |
|---|---|---|---|---|---|
| 1 | 14 | 0.45 | 1.65 | 3.15 | 0.41 [2] |
| | 19 | 0.23 | 1.78 | 3.22 | 0.11 [2] |
| 2 | 14 | 0.14 | | 2.50 [2] | -- [3] |
| | 19 | -0.32 | | 2.66 [2] | -- [3] |
| 3 | 14 | 0.50 | | 2.57 [2] | -2.88 |
| | 19 | 0.33 | | 2.87 [2] | -2.72 [2] |
| 4 | 14 | 0.39 | | 2.27 [2] | 0.47 |
| | 19 | 0.10 | | 2.68 [2] | 1.11 |
| 5 | 14 | 0.27 | | 2.06 | 0.78 [2] |
| | 19 | 0.50 | | 2.47 | 0.74 [2] |
| SPECIFICATION: | | | | | |
| | /DAY | ±1.00 | ±0.50 | ±0.50 | ? |
| | /100 s | ±0.01 | ±0.05 | ±0.05 | ±0.08 |

[1] $10^{-10}$ PARTS PER DAY. DETERMINED FROM FREQUENCY OFFSET OF FIRST AND LAST PASS OF SINGLE STATION, PRECISE EPHEMERIS SOLUTION.

[2] SOLUTION SHOWS FREQUENCY JUMP AFTER FIRST OR SECOND PASS.

[3] TOO FEW PASSES IN SOLUTION, WITH TWO FREQUENCY JUMPS (OSCILLATOR DISTURBED DUE TO MAINTENANCE)

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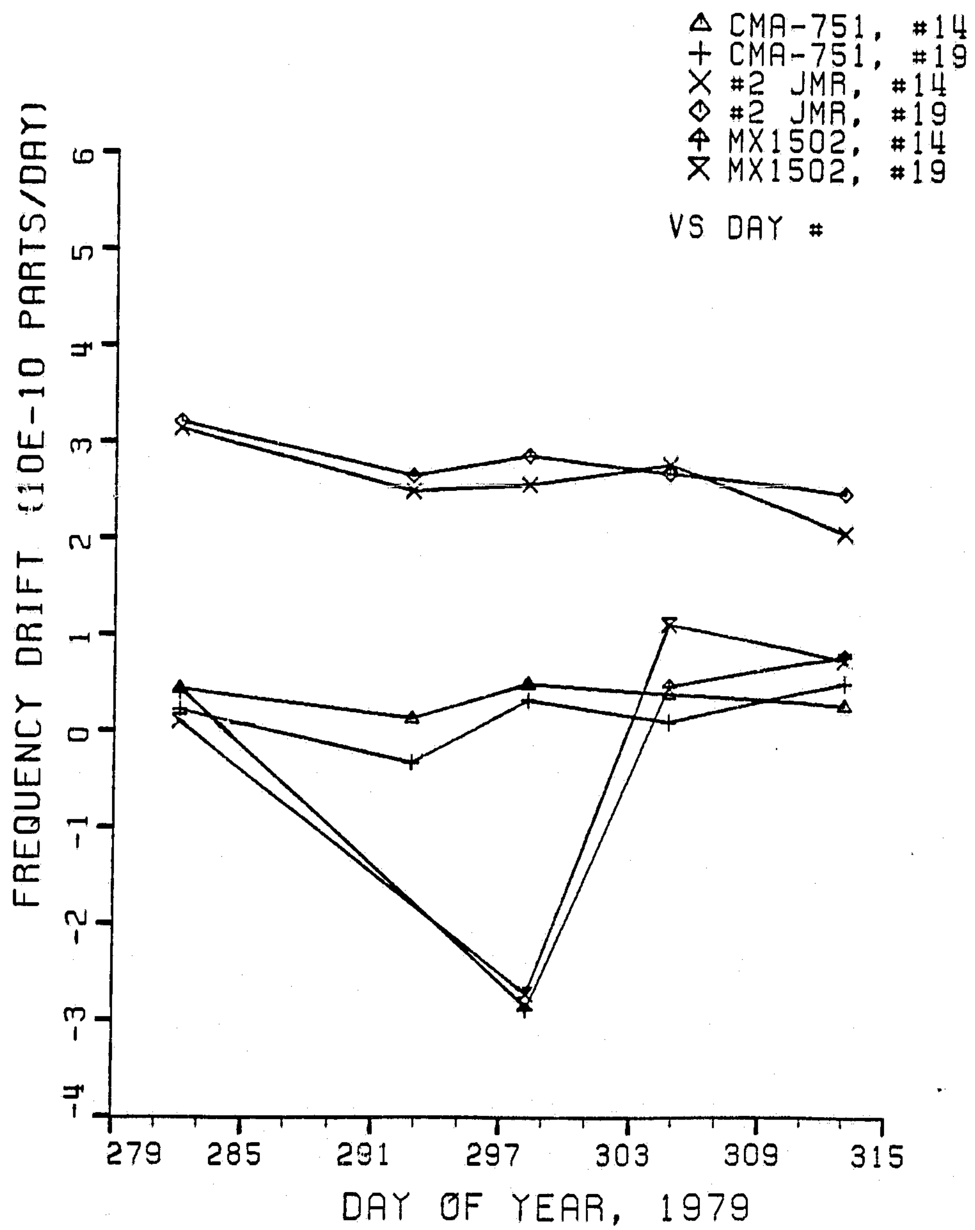


Fig. 4 Oscillator frequency drift versus time.

cified $10^{-10}$ parts/day precision. The frequency drift was usually from one half to one tenth of that value, and even approached its 100 second specification.

b) The JMR-1A #2 also had a fairly uniform value for frequency drift, using either satellite. However, both it and the JMR-1A #1 failed to meet their $0.5 \times 10^{-10}$ parts/day specified precision. (Note that this specification is actually for the JMR-1. It is assumed that the JMR-1A would have the same or a better specification.)

c) The MX1502 did not have a consistent value for its frequency drift, which shows oscillations during the second through fourth setups. Since the values for the first, fourth and last setups are at least similar, one would suspect that the frequency drift changes are mostly due to the various times that the instrument was opened (and its oscillator turned off) for repairs. No specification for the MX1502 drift per day is available for comparison purposes.

## FINAL COMMENTS

The results presented here should be considered as the final ones of this comparison, although if time permits, some additional material will possibly be added to the report version of [Archinal, 1982] and the final version of [Archinal and Mueller, 1982]. Work is also continuing on the documentation and further testing of the IBM version of the GEODOP Program System.

As to the further use of the data obtained, the recommendation is made here that the data from both this comparison and the Ottawa comparison be finally processed together in multi-station solutions, to provide a comparison of how well the various possible instrument pairs can measure the long Columbus-Ottawa baselines involved. Further, it is also suggested that a similar reduction be made (if the data can be obtained) using the "Quebec" data described in [Moreau, 1981], which was also obtained during the operational phase of this comparison.

Other investigations are also possible, including extending the results given above by making further comparisons of the chords, comparing the vertical and horizontal positions

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of the stations through their coordinates, and comparing the computed coordinates with the available control coordinates. These items were not done in this study mainly because they are considered to be of lesser importance than the other results presented, and due to a general lack of time for these lengthy investigations. Other work concerning program options or comparisons of programs could also be done with this data.

## REFERENCES


Archinal, Brent A. (1982) A Comparison of Geodetic Doppler Satellite Receivers, MS thesis, Ohio State University, Dept. of Geodetic Science and Surveying (report of the Dept. of Geodetic Science and Surveying in preparation).

Archinal, Brent A. and Ivan I. Mueller (1982) "Results of a Comparison of Geodetic Doppler Satellite Receivers," Third International Symp. on the Use of Artificial Satellites for Geodesy and Geodynamics, Ermioni, Greece, Sept. 20-25 (National Technical University of Athens).

Moreau, Richard (1981) "Integrated Densification by Doppler," ACSM Fall Technical Meeting, American Congress on Surveying and Mapping, Falls Church, VA 22046, pp. 272-281.

## 2.3 Earth Deformation Considerations for the Maintenance of a Conventional Terrestrial Reference System

The role of deformation analysis in the maintenance of a new Conventional Terrestrial Reference Frame has been outlined in previous semiannual reports and in [Bock and Zhu, 1982]. Basically, a set of fundamental coordinates $x_0$ of a global network of stations adopted at an initial epoch define the reference frame. The initial size and shape of this network is defined by the corresponding baseline lengths, $D_0$. By comparing the estimated baseline lengths at a later epoch to $D_0$, the deformations of the network can be estimated. This information is then used to improve the global estimates of variations in polar motion and earth rotation, with respect to the conventional axes defined by $X_0$.

Mathematical Model and Preliminary Estimation Model

The mathematical model for the deformation analysis is simply the chord length of baseline i-j

$$D_{ij} = [(x_j-x_i)^2 + (y_j-y_i)^2 + (z_j-z_i)^2]^{\frac{1}{2}}$$

This model is linearized about $X_0$ to yield

$$L = AX + V$$

where the observation vector L for the $k^{th}$ baseline is

$$L_k = (D_{ij} - D_{ij_0})_k$$

and the parameter vector X represents the deformations, i.e., the change in coordinates between the initial epoch and a later one. V denotes the noise vector.

Since the design matrix A is rank deficient by 6, we are restricted to a Generalized Gauss-Markoff (GGM) model $(L, AX, \sigma_0{}^2P^{-1})$ where

$$E(L) = AX$$
$$D(L) = \sigma_0{}^2P^{-1}$$

If there is no a priori information for the deformations, the minimum bias P-least squares estimate for X is given by

$$\hat{X} = N^{+}U = A^{+}_{PI}\ L ; \qquad N = A^{T}PA, \quad U = A^{T}PL$$

using the notation of [Rao and Mitra, 1971], where P is the weight matrix of the observations. This estimate can be shown to be equivalent to that obtained from augmenting the normal matrix N by a set of constraints C such that [Blaha, 1971]

$$AC^{T} = 0$$
$$CX = 0$$

and

$$\hat{X} = (N + C^{T}C)^{-1} - C^{T}(CC^{T}CC^{T})^{-1}\ C$$

This means that we constrain the origin and orientation defined by the coordinates at some later epoch t to be equivalent to that defined by $X_0$.

Extended Models for A Priori Deformation Information

In the case of the availability of a priori information on the deformations of the network, e.g., as provided by absolute plate motion models, four possible estimators have been outlined and analyzed in [Bock, in preparation]. We briefly outline here the corresponding estimates and their respective properties.

Consider an expanded GGM model $(L, AX, Q_V, Q_{\overline{X}})$ where

$$E(L) = AX$$
$$D(L) = Q_V = E\{VV^{T}\} = \sigma_0{}^2P^{-1}$$
$$E(\overline{X}\ \overline{X}^{T}) = Q_{\overline{X}}$$
$$= \Sigma_{\overline{X}} + \mu_{\overline{X}}\ \mu_{\overline{X}}{}^{T} \qquad (\mu_{\overline{X}} = E\{\overline{X}\} = X)$$

where $\overline{X}$ is an independent estimate of the parameter vector. The resulting minimum M-norm P-least squares minimum variance estimate for X

$$\hat{X}_1 = Q_X N(NQ_X N)^{+}\ U$$
$$= M^{-1}\ N(NM^{-1}\ N)^{+}\ U$$

where $M = Q_X^{-1}$ (positive definite). $\hat{X}_1$ has the property of minimum bias. Therefore, this estimate is termed the BLIMBE (Best Linear Minimum Bias Estimate). In this case, it can be shown that this estimate is equivalent to that obtained from augmenting the normal equations by CM such that

$$AC^T = 0$$
$$CM\hat{X}_1 = 0$$

and

$$\hat{X}_1 = [(N + MC^TCM)^{-1} - C^T(CMC^T\ CMC^T)^{-1}\ C]^{-1}\ U$$

Therefore, we can say that the reference frame is maintained in a minimum M-norm P-least squares sense by a specified number of CTS stations.

For positive semidefinite $Q_X$, which would be the case for any plate model

$$\hat{X}_1 = (N + M)^{-1}\ N[N(N + M)^{-1}\ N]^+\ U$$

with $M = Q_X^+$. In this case, the estimate is minimum M-seminorm P-least squares but is no longer minimum bias.

For the BLIMBE we assume that the parameter vector X is deterministic and define a weighted norm in the parameter space on the basis of a priori information on X. Another possible biased estimator can be obtained by considering X as a random variable. Our estimation model is $(L, A\overline{X}, Q_V, Q_X)$ where

$$E\{X\} = \overline{X}$$
$$D[X] = E\{(X-\overline{X})(X-\overline{X})^T\} = \Sigma_X$$

which gives

$$Q_X = E\{XX^T\} = \Sigma_X + \overline{X}\ \overline{X}^T$$

The vector X includes the deformations computed from, say, a plate motion model and $\Sigma_X$ is its covariance matrix. The distribution of L is given by

$$E\{L\} = A\overline{X}$$
$$D[L] = A\ \Sigma_X\ A^T + \sigma_0{}^2P^{-1}$$

from which

$$Q_L = E\{LL^T\} = A\ Q_X\ A^T + \sigma_0{}^2P^{-1}$$

In addition,

$$Q_{XL} = E\{XL^T\} = Q_X A^T$$

and we assume

$$Q_{XV} = 0$$

By the Gauss-Markoff theorem [Liebelt, 1967]

$$\hat{X}_2 = Q_{XL} Q_L^{-1} L$$
$$= Q_X A^T (A Q_X A^T + \sigma_0^2 P^{-1})^{-1} L$$

Which, for positive definite $Q_X$,

$$\hat{X}_2 = (N + M)^{-1} U : \qquad M = Q_X^{-1}$$

This estimate has been referred to as the Best (or Bayes) Linear Estimate or BLE for short [Rao, 1973, 1976]. While the BLIMBE has the minimum bias property, the BLE has minimum mean square error, i.e., it minimizes the sum of covariance and biased squares

$$MSE(\hat{X}) = \Sigma_{\hat{X}} + [X - E(\hat{X})][X - E(\hat{X})]^T$$

in the class of biased estimators. Note that the BLE requires some knowledge of the deformations in order to compute $Q_X$. Furthermore, while the BLIMBE reference system is maintained through the constraints $CM\hat{X}_1 = 0$, the deformations estimated by the BLE are with respect to an underlying reference frame of the deformation model from which $Q_X$ is computed.

The previous two estimates are drawn from the class of biased estimators. If $Q_X$ can be constructed, that is, if there exists a priori deformation information, then the origin and orientation singularities are essentially eliminated. We then are led to investigate whether an unbiased estimate exists and we find the Bayesian estimate. Consider the estimation model $(L, AX, Q_V, \overline{X}, \Sigma_{\overline{X}})$ where X is deterministic, $\overline{X}$ random and the set of observation equations

$$\begin{bmatrix} L \\ L_X \end{bmatrix} = \begin{bmatrix} A \\ I \end{bmatrix} X + \begin{bmatrix} V \\ V_X \end{bmatrix} ; \qquad L_X = \overline{X}$$

such that

$$E\{\overline{X}\} = X$$
$$D[\overline{X}] = E\{(\overline{X} - X)(X - X)^T\} = \Sigma_{\overline{X}}$$
$$E\{L\} = AX$$
$$D[L] = A\ \Sigma_{\overline{X}}\ A^T + \sigma_0{}^2 P^{-1} = \Sigma_L$$

The least squares solution for this model yields

$$\hat{X}_3 = \Sigma_{\overline{X}}\ A^T(A\ \Sigma_{\overline{X}}\ A^T + \Sigma_L)^{-1}\ L + [I - \Sigma_{\overline{X}}\ A^T(A\ \Sigma_{\overline{X}}\ A^T + \Sigma_L)^{-1}\ A]\ \overline{X}$$

for $\Sigma_{\overline{X}}$ positive semidefinite. For positive definite $\Sigma_{\overline{X}}$, this reduces to

$$\hat{X}_3 = (N+M)^{-1}\ U + [I - (N+M)^{-1}\ N]\overline{X}\ ; \qquad M = \Sigma_{\overline{X}}{}^{-1}$$
$$= \overline{X} + (N+M)^{-1}\ A^T P(L - A\overline{X})$$
$$= (N+M)^{-1}\ (U + M\overline{X})$$

It is easiliy seen that given this estimation model, particularly $E\{\overline{X}\} = X$, $E\{\hat{X}_3\} = X$, so that X is unbiased. This estimate has the minimum mean square error property which implies minimum variance since the bias is equal to zero. Note that in the BLE, the a priori information is incorporated into the moment matrix $Q_X$, while for $\hat{X}_3$, $\overline{X}$ is applied directly, and a residual deformation is estimated. Thus, we can consider the BLE ($\hat{X}_2$) as a "weak" Bayesian estimate and $\hat{X}_3$ a "strong" Bayesian estimate.

Assume again that some a priori deformations are available. In this case, the model may indicate that $CX=L_X$ where $L_X \neq 0$ which leads to an alternative approach to the constraints $CM\hat{X}_1 = 0$ of BLIMBE. Consider the following set of observation equations

$$\begin{bmatrix} L \\ L_X \end{bmatrix} = \begin{bmatrix} A \\ C \end{bmatrix} X + \begin{bmatrix} V \\ V_X \end{bmatrix}, \qquad L_X = C\overline{X}$$

We assume the estimation model $(L, AX \mid CX = C\overline{X},\ \Sigma_{\overline{X}},\ Q_V)$ where

$$E\{C\overline{X}\} = CX$$
$$D[C\overline{X}] = C\ \Sigma_{\overline{X}}\ C^T$$
$$E\{L\} = AX$$
$$D[L] = A\ \Sigma_{\overline{X}}\ A^T + \sigma_0{}^2 P^{-1}$$

For this model, the least squares estimate is

$$\hat{X}_4 = [N + C^T P_X C]^{-1} U + C^T P_X C \overline{X}$$

where

$$P_X = (C \Sigma_{\overline{X}} C^T)^{-1}$$

From [Chipman, 1964]

$$A^+_{PI} = [N + C^T P_X C]^{-1} A^T P$$

$$C^+_{P_X I} = [N + C^T P_X C]^{-1} C^T P_X$$

so that

$$\hat{X}_4 = A_{PI}{}^+ L + C^+_{P_X I} \overline{X}$$

$$= N^+ U + C^+_{P_X I} \overline{X}$$

Therefore, $\hat{X}_4$ can be viewed as a correction term to the minimum I-norm P-least squares estimate $\hat{X}_1$, or a combination of the BLIMBE and Bayesian approaches.

The properties of the four estimators are summarized in Table 1.

Addition and Temporary Deletion of CTS Stations

The reference frame is defined by a particular number of CTS stations. It is quite possible that from time to time one or more of the stations will not be able to participate in a particular deformation analysis observing session which should involve all stations. Furthermore, it must be anticipated that new stations will be added to the frame periodically. Both of these occurrences must be dealt with in order to maintain continuity and avoid ambiguity in the reference frame definition. For the addition of CTS stations we use the filtering and estimation capabilities of least squares collocation. The model becomes

$$L = AX + BS + V$$

Where X is deterministic and represents the coordinates of the new stations to be estimated. The vector S, the signal, is random and includes the

Table 1 Properties of Deformation Estimators

| Estimate / Property | BLIMBE | BLE | Bayesian | BLICUE |
|---|---|---|---|---|
| Uniqueness | Yes | Yes | Yes | Yes |
| P-least squares | $\hat{V}^T P \hat{V} = \min$ | $\hat{V}^T P \hat{V} + \hat{X}^T Q_X^{-1} \hat{X} = \min$ | $\hat{V}^T P \hat{V} + \hat{V}_X^T \Sigma_{\overline{X}}^{-1} \hat{V}_X = \min$ | $\hat{V}^T P \hat{V} = \min$ |
| Minimum M-norm | In the class of P-least squares | Yes | No | No |
| Biasedness | Minimum bias* | Biased | Unbiased assuming $E(\overline{X}) = X$ | Unbiased conditional on $E(\overline{CX}) = CX$ |
| Minimum Variance | In the class of minimum bias estimators | In the class of biased estimators | Yes | Conditional |
| Minimum Mean Square Error | No | In the class of biased estimators | Yes | No |
| Estimation Model | $(L, AX, Q_V, Q_{\overline{X}})$ | $(L, A\overline{X}, Q_V, Q_X)$ | $(L, AX, Q_V, \overline{X}, \Sigma_{\overline{X}})$ | $(L, AX \mid CX = C\overline{X}, \Sigma_{\overline{X}}, Q_V)$ |

*Only for positive definite $Q_{\overline{X}}$

filtered deformations. The L and V vectors are as before. From [Moritz, 1980),

$$\hat{X} = [A^T(BQ_SB^T + Q_V)^{-1}A]^+ A(BQ_SB^T + Q_V)^{-1} L$$

$$\hat{S} = Q_SB^T(BQ_SB^T + Q_V)^{-1} (L - A\hat{X})$$

where $Q_S$ is the same as the previous $Q_X$.

If a station cannot observe, we can use the prediction capabilities of least squares collocation to predict the deformation via

$$\hat{S} = Q_SB^T(BQ_tB^T + Q_V)^{-1} L$$

where

$$S = \begin{bmatrix} t \\ u \end{bmatrix},$$

t includes the deformation of the observing station, and u the predicted deformations of the missing stations.

## Conclusions

In order to test the properties of the four estimators and their suitability in estimating deformations, a series of simulations were run as described in [Bock, in preparation]. A 20-station, 8-plate network was chosen for the simulations as depicted in Fig. 1 and Table 2. The AM1-2 absolute plate motion model of [Minster and Jordan, 1978] was "adopted" (see Table 3).

The following conclusions were arrived at based on the simulations. Assuming that the absolute motion models available today are good to within their stated noise levels (this is reasonable considering that [Bender, 1981] indicates that their predicted deformations differ at the centimeter level), it is found that it is advisable to adopt a deformation model than not at all. This was seen from comparing the deformation estimates obtained with a deformation model and those obtained when M = I (no model) is assumed. If a model is adopted, then the BLE appears to be the best candidate for deformation analysis. This conclusion follows from several considerations.

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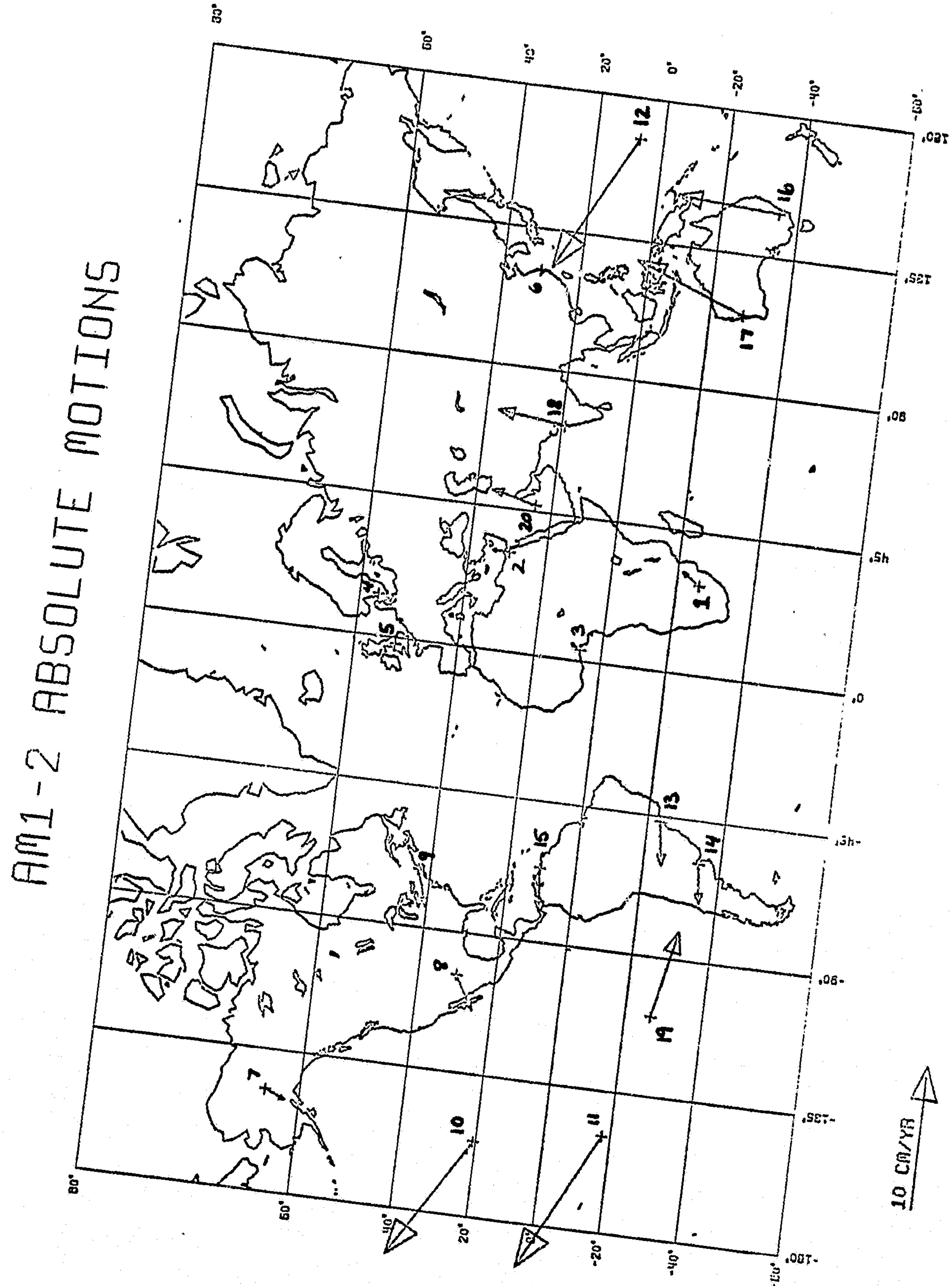


Fig. 1

Table 2 20-Station, 8-Plate Simulation Network and AM1-2 Velocities

| No. | Station | Latitude D | Latitude M | Longitude D | Longitude M | Plate | Velocity (cm/yr) X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Johannesberg* | -29 | 10 | 28 | 02 | AFRC | 0.03 | 0.99 | 1.00 |
| 2 | Cairo | 30 | 3 | 31 | 15 | AFRC | -0.49 | -0.31 | 1.01 |
| 3 | Lagos | 6 | 27 | 3 | 28 | AFRC | -0.09 | 0.34 | 0.62 |
| 4 | Onsala* | 57 | 0 | 13 | 0 | EURA | -0.14 | -0.32 | 0.14 |
| 5 | Jodrell Bank | 53 | 0 | 358 | 0 | EURA | -0.13 | -0.31 | 0.09 |
| 6 | Shanghai | 30 | 45 | 121 | 45 | EURA | -0.09 | -0.20 | 0.21 |
| 7 | Fairbanks | 64 | 50 | 212 | 10 | NOAM | -1.38 | -0.15 | -0.59 |
| 8 | Ft. Davis* | 30 | 38 | 256 | 3 | NOAM | -2.43 | -0.07 | -1.11 |
| 9 | Westford | 42 | 36 | 288 | 30 | NOAM | -2.27 | -1.29 | -0.54 |
| 10 | Maui* | 20 | 30 | 203 | 45 | PCFC | -1.80 | 8.35 | 4.59 |
| 11 | Tahiti | -17 | 30 | 210 | 30 | PCFC | -6.11 | 7.59 | 4.48 |
| 12 | Marshall Isles | 7 | 0 | 167 | 0 | PCFC | 2.73 | 9.24 | 4.76 |
| 13 | Sao Paulo* | -23 | 33 | 313 | 21 | SOAM | -2.26 | -1.94 | -0.33 |
| 14 | Buenos Aires | -34 | 37 | 301 | 36 | SOAM | -2.44 | -1.30 | -0.25 |
| 15 | Caracas | 10 | 35 | 293 | 40 | SOAM | -2.75 | -1.26 | -0.26 |
| 16 | Orroral* | -35 | 18 | 149 | 8 | INDI | -3.63 | 1.70 | 5.63 |
| 17 | Yaragadee | -29 | 3 | 115 | 21 | INDI | -4.20 | 1.99 | 6.47 |
| 18 | Bombay | 18 | 56 | 72 | 51 | INDI | -0.95 | -1.25 | 4.30 |
| 19 | Easter Isle* | -27 | 5 | 250 | 39 | NAZC | 6.04 | -1.56 | -1.04 |
| 20 | Arabia* | 24 | 39 | 46 | 46 | ARAB | -1.42 | -0.36 | 2.70 |

* 8-Station 8-Plate Network

Table 3 AM1-2 Absolute Motion Plate Model (Adapted from [Minster and Jordan, 1981], Table 7)

| | Absolute Rotation Vector | | | | | |
|---|---|---|---|---|---|---|
| Plate | Deg (N) | | Deg (E) | | Deg/M.Y.* | |
| 1. African | 18.76 | 33.93 | 338.24 | 42.20 | 0.139 | 0.055 |
| 2. Eurasian | 0.70 | 124.35 | 336.81 | 146.67 | 0.038 | 0.057 |
| 3. North American | -58.31 | 16.21 | 319.33 | 39.62 | 0.247 | 0.080 |
| 4. Pacific | -61.66 | 5.11 | 97.19 | 7.71 | 0.967 | 0.085 |
| 5. South American | -82.28 | 19.27 | 75.67 | 85.88 | 0.285 | 0.084 |
| 6. Indian-Australian | 19.23 | 6.96 | 35.64 | 6.57 | 0.716 | 0.076 |
| 7. Nazca | 47.99 | 9.36 | 266.19 | 8.14 | 0.585 | 0.097 |
| 8. Arabian | 27.29 | 12.40 | 356.06 | 18.22 | 0.388 | 0.067 |
| 9. Antarctic** | 21.85 | 91.81 | 75.55 | 63.20 | 0.054 | 0.091 |
| 10. Carribean** | -42.80 | 39.20 | 66.75 | 40.98 | 0.129 | 0.[illegible]04 |
| 11. Cocos** | 21.89 | 3.08 | 244.29 | 2.81 | 1.422 | 0.119 |

* Million Years
** Not used in the Simulations

First, the BLE provides the best estimates (in the sense of minimizing the root mean square error between true and estimated deformations) at the same level as the Bayesian estimate, in the case when the deformation model is correct (and then the deformation is just being filtered from the baseline noise). Second, and most important, it is markedly less sensitive to errors in the adopted deformation model. This is particularly apparent in the case that in reality there is no deformation but we assume some deformation model. These results are due to the minimum mean square error and minimum norm properties of the BLE and its "weak" Bayesian interpretation.

Finally, we should stress that the reference system is dependent on the choice of estimation models including the choice of M (as well as P, but to a lesser extent). This leads to the need for investigations concerning how sensitive the reference system is to changes in M and P. For example, what measures should be taken as M and P improve with time.

The algorithms presented here are general enough to incorporate geophysical as well as geodetic evidence of deformations. In [Bock, in preparation] only models for deformations of interplate type have been considered, to be monitored by periodic re-observations of the baseline lengths. Other aspects to be considered include intraplate and local motions (the site stability problem). Local effects can possibly be modeled on the basis of on-site observations such as by tidal gravimeters and local geodetic nets. It is necessary to investigate how to incorporate these and other types of observations (and their corresponding reference frames) into CTS operations.

This investigation is now being completed, and the final report is in preparation by Y. Bock, to appear in the report series of the Department of Geodetic Science and Surveying, The Ohio State University.

## References


Bender, P.L. (1981) "Establishment of Terrestrial Reference Frames by New Observation Techniques (review)," Reference Coordinate Systems for Earth Dynamics, E.M. Gaposchkin and B. Kołaczek, eds., Reidel, 23-36.

Blaha, G. (1971) "Inner Adjustment Constraints with Emphasis on Range Observations," Dept. of Geodetic Science Rep. 148, Ohio State Univ., Columbus.

Bock, Y. (in preparation) "Estimation of Earth Deformations for the Maintenance of a New Conventional Terrestrial Reference System," PhD dissertation, Dep. of Geodetic Science and Surveying Rep., Ohio State Univ., Columbus.

Bock, Y. and S.Y. Zhu (1982) "On the Establishment and Maintenance of a Modern Conventional Terrestrial Reference System," pres. at General Meeting of IAG, Tokyo, Japan, to be publ. as NOAA Technical Memorandum.

Chipman, J.S. (1964) "On Least Squares with Insufficient Observations," J. Amer. Statist. Assoc., 59, 1078-1111.

Liebelt, P.O. (1967) An Introduction to Optimal Estimation, Addison-Wesley, Reading, Massachusetts.

Minster, J.B. and T.H. Jordan (1978) "Present Day Plate Motions," J. Geophys. Res., 83(B11), 5331-5354.

Moritz, H. (1980) Advanced Physical Geodesy, H. Wichmann, Karlsruhe, and Abacus Press, Turnbridge Wells, Kent, U.K.

Rao, C.R. (1973) Linear Statistical Inference and Its Applications, Wiley, New York.

Rao, C.R. and S.K. Mitra (1971) Generalized Inverse of Matrices and Its Applications, Wiley, New York.

Rao, C.R. (1976) "Estimation of Parameters in Linear Models," Annals of Statistics, 4, 6, 1023-1037.

## 2.4 Development of Models for Studying Ice Sheet and Crustal Deformations

The observed locations of survey markers change with time. When random and systematic errors are accounted for, what remains is actual movement. The movements of a network of stations can be described as the translation and rotation of the stations as a group and the deformation occurring within the network. Thus when a network of stations is resurveyed, it should be possible to obtain the geophysical parameters of velocity, rotation rate and strain rate [Dermanis, 1981; Livieratos, 1980; Reilly, 1979]. If the same network is resurveyed more than once, either the derivatives of these quantities or averaged values may be calculated.

As most stations are on the surface of the earth, it is natural to assume that all movements and deformations are two-dimensional. This may be adequate in many cases. However, vertical movement and deformation may occur because of irregularities in the surface, faulting, or from being buried under new material. Also, for networks covering relatively large areas, the surface of the earth cannot be well approximated by a plane. In this case, it may be better to determine the movements in an arbitrary (earth-centered) coordinate system and then transform these results to a latitude, longitude and elevation coordinate system.

A model is being developed to determine these geophysical parameters from the coordinates of a network that has been resurveyed at least once. Several methods have been proposed for obtaining sufficiently accurate coordinates [Brunner et al., 1981, Niemeier, 1979]. One technique that has been proposed for studying tectonic deformation is to use positions determined by Doppler satellite receivers [Malyevac and Anderle, 1979]. The precision of the receivers used individually (point positioning) is meters to tens of meters. But by using translocation between two or more receivers, the relative positions can be determined to within decimeters [Brown, 1976]. However, the movements and deformations of the crust are slow even in tectonically active areas [Savage, 1978; Minster and Jordan, 1978]; thus the time span between resurveying must be of the order of decades. Because the time period between reobservations is so long, it may be difficult to guarantee that the coordinate systems are identical.

For example, the coordinate system defining the broadcast ephemeris of the Navy Navigational Satellite System slowly varies with time. This problem could be overcome by using relative rather than absolute coordinates. Thus the velocities and rotation rates would be relative to some "fixed" stations. However, the deformation within the network can still be obtained by calculating the strains from the changes in the chord lengths between the stations. The only assumption needed for this is that the scale of the coordinate system has not changed. Because the strains obtained this way are theoretically identical to the strains obtained from coordinate differences, any differences can be attributed to rotations and/or translations of the coordinate system.

For the purposes of testing the model, the data set being used is from survey stations placed on the Greenland ice sheet. Seven Magnavox 1502 satellite receivers were used during the summers of 1980 and 1981 to obtain the movement of 22 stations on the ice sheet of Greenland. Using the data reduction program GEODOP [Kouba and Boal, 1976], the coordinates of the stations have been obtained relative to the positions of two stationary stations (which were located on the west coast of Greenland). The formal accuracy of the coordinates is under 20 cm. These stations are moving at velocities of up to 45 m per year, and the magnitude of the maximum strain rates are over 100 ppm.

## References

Archinal, B. and I.I. Mueller (1981) "A Comparison of Geodetic Doppler Satellite Receivers," Proc. 3rd International Geodetic Symp. on Satellite Doppler Positioning, Physical Sciences Lab., New Mexico State Univ., Las Cruces, N.M. 88003.

Brown, Duance C. (1976) "Doppler Positioning by the Short Arc Method," Proc. International Symp. on Satellite Doppler Positioning, Physical Sciences Lab., New Mexico State Univ., Las Cruces, N.M. 88003, pp. 97-140.

Brunner, F.K., R. Coleman and B. Hirsch (1981) "A Comparison of Computation Methods for Crustal Strains from Geodetic Measurements," Tectonophysics, 71, 281-298.

Dermanis, A. (1981) "Geodetic Estimability of Crustal Deformation Parameters," Quaterniones Geodaesiae, 2, 2, 159-169.

Kouba, J. and J.D. Boal (1976) Program GEODOP, Dept. of Energy, Mines and Resources, Ottawa, Ontario, Canada.

Livieratos, E. (1980) "Crustal Strains Using Geodetic Methods," Quaterniones Geodaesiae, 1, 3, 191-211.

Malyevac, C.A. and R.J. Anderle (1979) "Determination of Plate Tectonic Motion from Doppler Observations of Navy Navigation Satellites," Proc. 2nd International Geodetic Symp. on Satellite Doppler Positioning, Univ. of Texas, Austin.

Minster, J.B. and J.H. Jordan (1978) "Present Day Plate Motions," J. Geophysical Res., 83(B11), 5331-5354.

Niemeier, W. (1979) "Statistical Tests for Detecting Movements in Repeated Measured Geodetic Networks," XVII General Assembly of the IUGG, Canberra, Australia.

Reilly, W.I. (1979) "Complete Determination of Local Crustal Deformation from Geodetic Observations," XVII General Assembly of the IUGG, Canberra, Australia.

Savage, J.C. (1978) "Strain Patterns and Strain Accumulations Along Plate Margins," Applications of Geodesy to Geodynamics, I.I. Mueller, ed., Dept. of Geodetic Science Rep. 280, Ohio State Univ., Columbus.

## 3. PERSONNEL

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## 4. TRAVEL

Ivan I. Mueller

Patras, Greece — August 17-20, 1982
Attended XVIII General Assembly of the International Astronomical Union. Presented the paper which appears on pp. 2-18 and a report on progress in planning for the new Conventional Terrestrial Reference System to Commissions 4, 19 and 31. Chaired meetings of the IAG/IAU Working Group COTES.

Budapest, Hungary — August 20-26, 1982
Attended 3rd Symposium on the Study of Movements in Engineering Surveys. Presented a paper on the Greenland Ice Movement Study (see p. 87).

## 5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published under Grant No. NSG 5265:

262 The Observability of the Celestial Pole and Its Nutations
by Alfred Leick
June, 1978

263 Earth Orientation from Lunar Laser Range-Differencing
by Alfred Leick
June, 1978

284 Estimability and Simple Dynamical Analyses of Range (Range-Rate and Range-Difference) Observations to Artificial Satellites
by Boudewijn H.W. van Gelder
December, 1978

289 Investigations on the Hierarchy of Reference Frames in Geodesy and Geodynamics
by Erik W. Grafarend, Ivan I. Mueller, Haim B. Papo, Burghard Richter
August, 1979

290 Error Analysis for a Spaceborne Laser Ranging System
by Erricos C. Pavlis
September, 1979

298 A VLBI Variance-Covariance Analysis Interactive Computer Program
by Yehuda Bock
May, 1980

299 Geodetic Positioning Using a Global Positioning System of Satellites
by Patrick J. Fell
June, 1980

302 Reference Coordinate Systems for Earth Dynamics: A Preview
by Ivan I. Mueller
August, 1980

320 Prediction of Earth Rotation and Polar Motion
by Sheng-Yuan Zhu
September, 1981

329 Reference Frame Requirements and the MERIT Campaign
by Ivan I. Mueller, Sheng-Yuan Zhu and Yehuda Bock
June, 1982

Estimation of Earth Deformations for the Maintenance of a New Conventional Terrestrial Reference System
by Yehuda Bock
November, 1982 (in preparation)

On the Geodetic Applications of Simultaneous Range-Differencing to LAGEOS
by Erricos C. Pavlis
December, 1982 (in preparation)

The following papers were presented at various professional meetings and/or published:

"Concept for Reference Frames in Geodesy and Geodynamics"
AGU Spring Meeting, Miami Beach, Florida, April 17-21, 1978
IAU Symposium No. 82, Cadiz, Spain, May 8-12, 1978
7th Symposium on Mathematical Geodesy, Assisi, Italy, June 8-10, 1978
"Concepts for Reference Frames in Geodesy and Geodynamics: The Reference Directions," Bulletin Geodesique, 53 (1979), No. 3, pp. 195-213.

"What Have We Learned from Satellite Geodesy?
2nd International Symposium on Use of Artificial Satellites for Geodesy and Geodynamics, Lagonissi, Greece, May 30 - June 3, 1978

"Parameter Estimation from VLBI and Laser Ranging"
IAG Special Study Group 4.45 Meeting on Structure of the Gravity Field
Lagonissi, Greece, June 5-6, 1978

"Estimable Parameters from Spaceborne Laser Ranging"
SGRS Workshop, Austin, Texas, July 18-23, 1978

"Defining the Celestial Pole," manuscripta geodaetica, 4 (1979), No. 2 pp. 149-183.

"Three-Dimensional Geodetic Techniques"
Technology Exchange Week, Inter-American Geodetic Survey
Fort Clayton, Canal Zone, May 14-19, 1979
published in Spanish as "Tecnicas Geodesicas Tridimensionales," Memoria de la Semana de Intercambio Tecnologico, Panama, Rep. de Panama, pp. 400-426.

"On the VLBI-Satellite Laser Ranging 'Iron Triangle' Intercomparison Experiment," Meeting on Radio Interferometry Techniques for Geodesy, Massachusetts Institute of Technology, Cambridge, June 19-21, 1979

"Space Geodesy for Geodynamics, A Research Plan for the Next Decade"
Sonderforschungsbereich - Satellitengeodäsie - SFB 78
Colloquium in Viechtach, FRG, October 23-24, 1979

"Concept of Reference Frames for Geodesy and Geophysics"
seminar given at University of Stuttgart, West Germany, June 19, 1980

"Space Geodesy and Geodynamics,"
seminar given at University of Stuttgart, West Germany, June 26, 1980

"Geodetic Applications of the Global Positioning System of Satellites and Radio Interferometry," seminar given at University of Stuttgart, West Germany, July 3, 1980

"Reference Coordinate Systems for Earth Dynamics: A Preview," Proc. of IAU Colloquium 56 on Reference Coordinate Systems for Earth Dynamics," September 8-12, 1980, Warsaw, Poland, E.M. Gaposchkin and B. Kołaczek, eds., D. Reidel. Presented at AGU, San Francisco, December, 1980.

C-2

"Comments on Conventional Terrestrial and Quasi-inertial Reference Systems," with J. Kovalevsky, Proc. of IAU Colloquium 56 on Reference Coordinate Systems for Earth Dynamics, September 8-12, 1980, Warsaw, Poland, E.M. Gaposchkin and B. Kołaczek, eds., D. Reidel.

"Precise Positioning with GPS"
seminar given at Deutsche Geodätische Forschungsinstitut, Munich, West Germany, September 18, 1980

"Tecnicas Geodesicas Tridimensionales"
(translated from English by IAGS), ASIA Journal (Asociacion Salvadorena de Ingenieros y Arquitectos) San Salvador, No. 61, Oct. 80, pp. 40-51; cont'd in No. 62, Dec. 80, pp. 31-39.

"Space Geodetic Techniques and Geodynamics"
Proc. 1st International Symposium on Crustal Movements in Africa
Addis Ababa, Ethiopia, May 5-16, 1981

"Comparison of Polar Motion Data from the 1980 Project MERIT Short Campaign"
Proc. of IAU Colloquium 63 on "High Precision Earth Rotation and Earth-Moon Dynamics: Lunar Distances and Related Observations," May 22-27, 1981, Grasse France, D. Reidel (I.I. Mueller, B.S. Rajal and S.Y. Zhu)

"Geodesy and the Global Positioning System"
Proc. of IAG Symposium on Geodetic Networks and Computations, Aug. 31-Sept. 5, 1981, Munich (I.I. Mueller and B. Archinal)

"The African Doppler Project," Commission for Geodesy in African and Commission on the International Coordination of Space Techniques for Geodesy and Geodynamics, presented at 2nd Symp. on Geodesy in Africa, Nairobi, Kenya, November 9-20, 1981. Also at 3rd International Geodetic Symp. on Satellite Doppler Positioning, Las Cruces, New Mexico, Feb. 8-12, 1982.

"The Future of Space Techniques in Geodesy and Geodynamics," 2nd Symp. on Geodesy in Africa, Nairobi, November 9-20, 1982

"Use of Simultaneous Doppler-Derived Ranges in the Geometric Mode," Proc. of 3rd International Geodetic Symp. on Satellite Doppler Positioning, Las Cruces, New Mexico, Feb. 8-12, 1982 (R. Schneeberger, D. Pavlis and I.I. Mueller.

"A Comparison of Geodetic Doppler Satellite Receivers," Proc. of 3rd International Geodetic Symp. on Satellite Doppler Positioning, Las Cruces, New Mexico, Feb. 8-12, 1982 (B. Archinal and I.I. Mueller)

"Effects of Adopting New Precession, Nutation and Equinox Corrections on the Terrestrial Reference Frame," XVII General Assembly of the International Astronomical Union, Patras, Greece, August 17-26, 1982. Also Bulletin Geodesique, in press (S.Y. Zhu and I.I. Mueller)

"Glacial Movement Measurements in Greenland by Satellite Doppler Geodetic Receivers," Third Symp. on the Study of Movements in Engineering Surveys, August 25-27, 1982, Budapest, Hungary (Alice Drew, Ivan I. Mueller and Ian Whillans)

"On the Establishment and Maintenance of a Modern Conventional Terrestrial Reference System," Proc. IAG General Meeting, Tokyo, May, 1982, Symp. 5 (Y. Bock and S.Y. Zhu)

"A Comparison of Geodetic Doppler Satellite Receiver, II," Third International Symp. on the Use of Artificial Satellites for Geodesy and Geodynamics, Porto Hydra, Greece, Sept. 20-25, 1982 (B. Archinal and I.I. Mueller)

"Geometric Adjustment of Simultaneous Doppler-Derived Range Differences," Third International Symp. on the Use of Artificial Satellites for Geodesy and Geodynamics, Porto Hydra, Greece, Seot. 20-25, 1982 (C. Zhang and I.I. Mueller)