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## RF MODEL OF THE DISTRIBUTION SYSTEM AS A COMMUNICATION CHANNEL

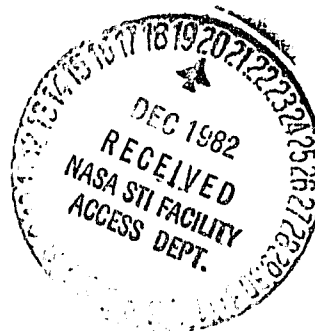
### PHASE II

### VOLUME III — APPENDICES

FINAL REPORT  
Contract No. 955647

July 28, 1982

R.C. Rustay  
J.T. Gajjar  
R.W. Rankin  
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Schenectady, New York 12345

Prepared for

Jet Propulsion Laboratory  
California Institute of Technology  
4800 Oak Grove Drive  
Pasadena, California 91103

SRD-82-055-3

# **RF MODEL OF THE DISTRIBUTION SYSTEM AS A COMMUNICATION CHANNEL**

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## ABSTRACT

This four-volume final report is concerned with Phase II of the DOE/JPL project "RF Model of the Distribution System As a Communication Channel." An earlier Phase I effort was concerned with the design, implementation, and verification of a computerized model for predicting the steady-state sinusoidal response of radial (tree) configured distribution feeders. That work demonstrated the feasibility and validity based on verification measurements made on a limited size portion of an actual live feeder. The Phase II effort is concerned with 1) extending the verification based on a greater variety of situations and network size, 2) extending the model capabilities for reverse direction propagation, 3) investigating parameter sensitivities, 4) improving transformer models, and 5) investigating procedures/fixes for ameliorating propagation "trouble spots."

## PREFACE

This volume contains a collection of miscellaneous appendices, most of which are essentially verbatim copies of selected internal memos on subjects of general interest. The subject/titles are listed on the following pages.

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**APPENDIX 1**  
**TUTORIAL REVIEW OF "PERTURBATION" THEORY AS USED**  
**IN DIFNAP PROGRAM NTWKANSj**

R.C. Rustay

Late in the Phase I work, J.T. Gajjar conceived an exact perturbation theory which enables efficient calculation of the effects of single load variation (say, for example, short circuit fault to ground effect) and propagation due to any single source not located at the original root section source. This work was reported in the following reports by J.T. Gajjar.

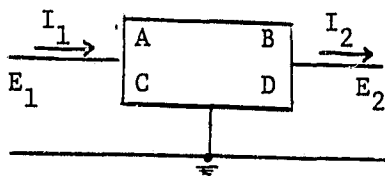
- [1] Matrix Analysis of Effects of Load Variation on Signal Propagation on Multiconductor Lines, 7/14/77.
- [2] Reciprocity in Multiconductor Systems With Application To Effects of Load Perturbations on Multiconductor Lines.

This Appendix, benefitting from the insight gained from these reports, presents an alternative approach to the understanding of this theory.

Further relevant comments are contained in Appendix 2 concerning using reciprocity and precomputed driving point admittance to compute reverse direction voltage transfer ratio matrices.

**Circuit Analysis Background**

The following is a summary of pertinent circuit analysis which will be of use in explaining J. Gajjar's Perturbation Theory. Consider first a two port network with equal number\*\* of terminals in each port and represented by *ABCD* parameters, i.e.,



$E_1, E_2, I_1, I_2$  each  $n$ -vectors  
Sketch 1

\*\* A requirement for the use of *ABCD* representation with invertibility capability. If the number of terminals are not equal, then "one way" representation may be possible if "controllability" conditions are satisfied. Even with equal number of terminals, "controllability" conditions may not be satisfied. In applications of interest bidirectional controllability is assumed.

The circuit equations are\*:

$$E_1 = AE_2 + BI_2 \quad \text{for Sketch 1} \quad \text{A-1}$$

$$I_1 = CE_2 + DI_2 \quad \text{A-2}$$

from which (and using other fundamental circuit properties) can obtain

$$E_2 = D^T E_1 - B^T I_1 \quad \text{for Sketch 1} \quad \text{A-3}$$

$$I_2 = -C^T E_1 + A^T I_1 \quad \text{A-4}$$

First note the reciprocity of transimpedance and transadmittance.

The transadmittance is concerned with the short circuit output current resulting from an input voltage. From A-1 with  $E_2 = 0$

$$E_1 = BI_2 \quad E_2 = 0 \quad \text{A-5}$$

so that  $B^{-1T}$  is the transadmittance from port 1 to port 2. Similarly from A-3 with  $E_1 = 0$

$$E_2 = -B^T I_1 \quad E_1 = 0 \quad \text{A-6}$$

so that (taking into account the positive sign/direction of  $I_1$  defined in the Sketch 1)  $B^{-1T}$  represents the transadmittance from port 2 to port 1, and it is again seen that these two transadmittances are related simply by their transpose, i.e., reciprocity.

The transimpedance is concerned with the open circuit output voltage resulting from an input current. From A-2 with  $I_2 = 0$

$$I_1 = CE_2 \quad I_2 = 0 \quad \text{A-7}$$

so that  $C^{-1}$  is the transimpedance from port 1 to port 2. Similarly from A-4 with  $I_1 = 0$

$$I_2 = -C^T E_1 \quad I_1 = 0 \quad \text{A-8}$$

so that (taking into account the positive sign/direction of  $I_2$  defined in the Sketch 1)  $C^{-1T}$  represents the transimpedance from port 2 to port 1, and it is seen that these two transimpedances are related simply by their transpose, i.e., reciprocity.

It is of interest to note that the transimpedance which can be derived from equations A-1 and A-2 with  $I_1 = 0$ , i.e., from port 2 to port 1, can easily be shown to be

$$E_1 = [B - AC^{-1}D] I_2 \quad \text{A-9}$$

which implies that

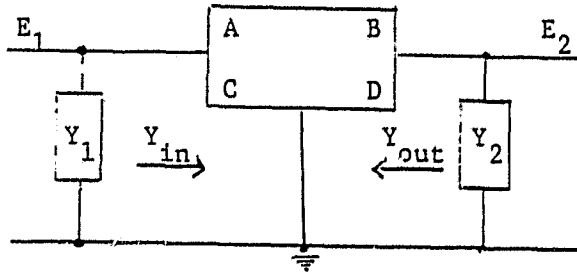
$$C^{-1T} \equiv [AC^{-1}D - B] \quad \text{A-10}$$

This can be proved utilizing, appropriately, various identities relating the  $A$ ,  $B$ ,  $C$ ,  $D$  matrices. This exercise illustrates the "redundancy" of the  $ABCD$  parameters and that results derived using them should always be carefully examined for possible simplifications.

Next consider the following circuit.

\* The  $ABCD$  matrices are related by  $A^T D - C^T B = I$  from which many equivalent relationships can be derived.

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Sketch 2

Using equations A-1 and A-2 the following relationships are easily derived

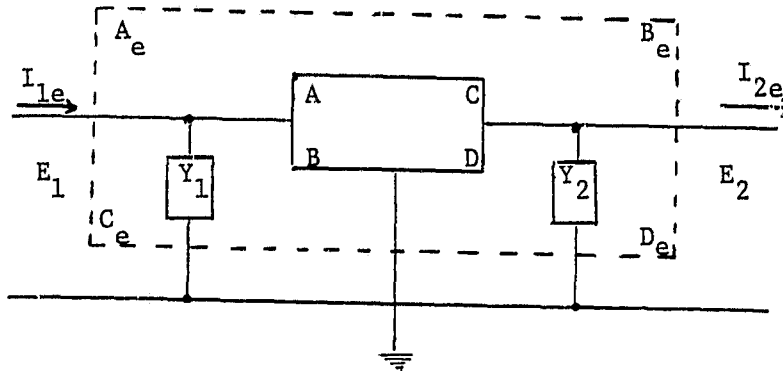
$$Y_{IN} = (C + DY_2)(A + BY_2)^{-1} \quad \text{A-11}$$

for Sketch 2

$$Y_{OUT} = (D + Y_1B)^{-1}(C + Y_1A) \quad \text{A-12}$$

where  $Y_{IN}$  is the driving point admittance looking into port 1 with (only)  $Y_2$  connected to port 2. Similarly  $Y_{OUT}$  is the driving point admittance looking (back) into port 2 with (only)  $Y_1$  connected to port 1.

Next consider the equivalent circuit corresponding to Sketch 3, i.e.,



Sketch 3

It is easy to show using equations A-1 and A-2 that the  $A_e$ ,  $B_e$ ,  $C_e$ ,  $D_e$  matrices for the equivalent circuit (bounded by dashed lines) are

$$A_e = A + BY_2 \quad \text{A-13}$$

$$B_e = B \quad \text{A-14}$$

$$C_e = Y_1A + C + (Y_1B + D) Y_2 \quad \text{for Sketch 3} \quad \text{A-15}$$

$$D_e = Y_1B + D \quad \text{A-16}$$

so that

$$E_1 = A_e E_2 + B_e I_{2e} \quad \text{A-17}$$

for Sketch 3

$$I_{1e} = C_e E_2 + D_e I_{2e} \quad \text{A-18}$$

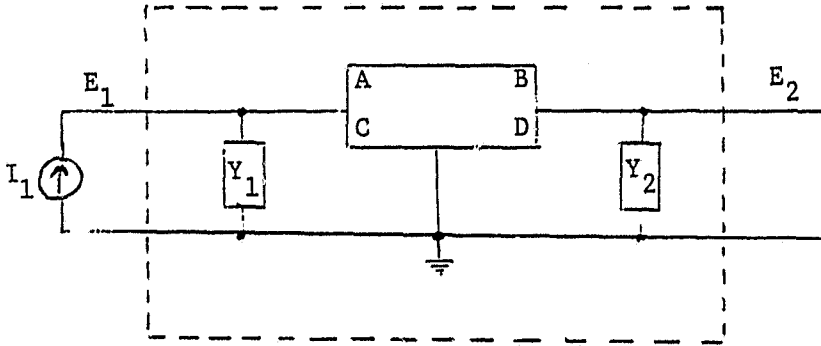
and their inverse from A-3 and A-4

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$$E_{2e} = D_e^T E_1 - B_e^T I_{1e} \quad \text{for Sketch 3} \quad \text{A-19}$$

$$I_{2e} = C_e^T E_1 + A_e^T I_{1e} \quad \text{A-20}$$

Next consider the following circuit with a current drive  $I_1$ . (Note  $I_1$  and  $Y_1$  form a Norton's equivalent source.)



Sketch 4

Using equations A-9 and A-10 the following equation is easily obtained

$$E_2 = \underbrace{[A + BY_2]^{-1}}_{\text{voltage transfer}} \left[ Y_1 + \underbrace{(C + DY_2)(A + BY_2)^{-1}}_{Y_{IN}} \right]^{-1} I_1 \quad \text{A-21}$$

or after some simplification (or directly from A-18)

$$E_2 = [Y_1(A + BY_2) + C + DY_2]^{-1} I_1 \quad \text{A-22}$$

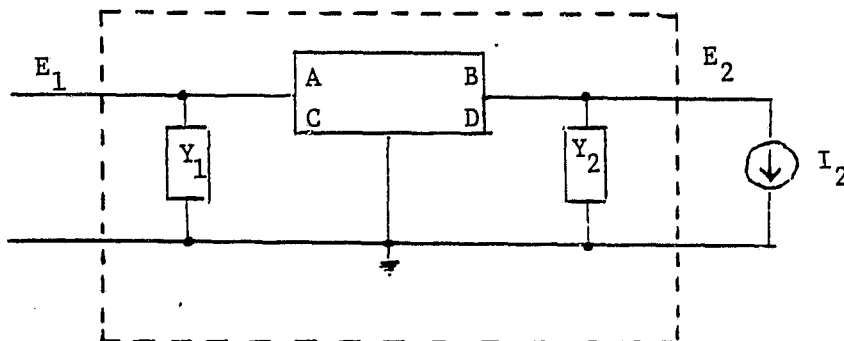
for Sketch 4

What has been obtained is the forward (port 1 to port 2) transimpedance for the total network denoted by the dashed line boundary. Equation A-22 will be used to develop the fundamental perturbation equations. Note that the result A-21 can be directly obtained using A-5 and A-18, i.e.,  $C_e^{-1}$ . For port 2 to port 1 transimpedance use A-20

$$E_1 = -C_e^T I_{2e}$$

as expected.

Finally consider the following circuit with a drive  $I_2$



Sketch 5

Using reciprocity stated earlier

$$E_1 = - \left[ Y_1(A+BY_2) + C+DY_2 \right]^{-1T} I_2 \quad \text{A-23}$$

It is of interest for subsequent analysis to develop a rather obvious expression for  $E_2$ . Using A-12 obtain

$$E_2 = - \left[ \underbrace{Y_2 + (D+Y_1B)^{-1}(C+Y_1A)}_{Y_{out}} \right]^{-1} I_2 \quad \text{A-24}$$

which can be expanded to yield

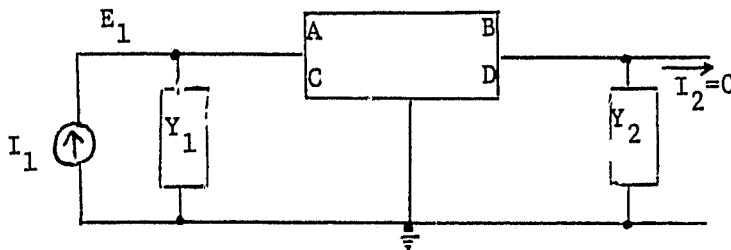
$$E_2 = - \left[ (D+Y_1B)Y_2 + (C+Y_1A) \right]^{-1} (D+Y_1B) I_2 \quad \text{A-25}$$

This same result can be obtained from A-18 with  $I_{1e} = 0$ .

### Development of Exact Analysis for Single Load Perturbation

The procedure for exactly computing changes in voltages due to a change in a (single) load was originally devised by Prof. J. Gajjar and reported in references [1] and [2]. The analytical development to follow arrives at the same procedure via a somewhat different logical route, which however, has depended on the insight obtained from Gajjar's approach.

Frequently we are interested in the following circuit with a current drive  $I_1$  and zero output current (i.e., all loading is represented by  $Y_2$ ).



Sketch 6

The current  $I_1$  and voltage  $E_2$  is given by (see A-22)

$$I_1 = [Y_1(A+BY_2) + C + DY_2] E_2 \quad \text{for } I_2 \equiv 0 \quad \text{D-1}$$

Also note the voltage transfer relationship (see A-17).

$$E_1 = [A+BY_2] E_2 \quad \text{for } I_2 \equiv 0 \quad \text{D-2}$$

Now suppose that the load  $Y_2$  changes from  $Y_2$  to  $Y_2 + \Delta Y_2$  while  $I_1$  and other circuit parameters remain unchanged. Then  $E_2$  will change to  $E_2 + \Delta E_2$  and  $E_1$  will change to  $E_1 + \Delta E_1$ . By substituting

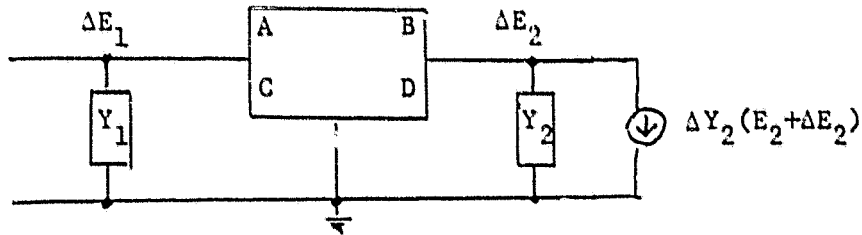
$$E_1 \rightarrow E_1 + \Delta E_1 \quad \text{D-3}$$

$$E_2 \rightarrow E_2 + \Delta E_2 \quad \text{D-4}$$

$$Y_2 \rightarrow Y_2 + \Delta Y_2 \quad \text{D-5}$$

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into equation D-1 and D-2 it will be shown that  $\Delta E_1$  and  $\Delta E_2$  resulting from the change  $\Delta Y_2$  can be evaluated computationally by driving the output side of the circuit in Sketch 6 by a current  $-\Delta Y_2(E_2 + \Delta E_2)$ , i.e.,



Sketch 7

and that  $\Delta E_2$  can be gotten from  $\Delta Y_2$  using various circuit parameters associated with the original unperturbed networks.

As a first step make the substitutions D-3, D-4, D-5 into the transimpedance relationship of D-1 and then subtracting D-1 obtain the following equivalent forms, each of which are exact

$$\Delta E_2 = -[(D + Y_1 B) Y_2 + (C + Y_1 A)]^{-1} (D + Y_1 B) (E_2 + \Delta E_2) \quad \text{D-6}$$

$$\Delta E_2 = -[Y_2 + \underbrace{(D + Y_1 B)^{-1} (C + Y_1 A)}_{\text{out}}]^{-1} \Delta Y_2 (E_2 + \Delta E_2) \quad \text{D-7}$$

$$\Delta E_2 = -[Y_2 + \Delta Y_2 + \underbrace{(D + Y_1 B)^{-1} (C + Y_1 A)}_{\text{out}}]^{-1} \Delta Y_2 E_2 \quad \text{D-8}$$

Next make similar substitutions into the voltage transfer relationship of D-2 and then subtracting D-2 obtain the following exact form

$$\Delta E_1 = B \Delta Y_2 (E_2 + \Delta E_2) + (A + B Y_2) \Delta E_2 \quad \text{D-9}$$

Substituting D-7 into D-9 obtain the exact form

$$\Delta E_1 = \{ B - (A + B Y_2) [Y_2 + \underbrace{(D + Y_1 B)^{-1} (C + Y_1 A)}_{\text{out}}]^{-1} \} \Delta Y_2 (E_2 + \Delta E_2) \quad \text{D-10}$$

Now make some observations. First compare Sketch 7 with Sketch 5 and correspondingly equations D-6, D-7 with A-25, A-24 and conclude that if a current source  $\Delta Y_2(E_2 + \Delta E_2)$  is computationally applied to the unperturbed circuit, in the manner shown in the sketch, then the voltage at port 2 will be  $\Delta E_2$ . Similarly using equations A-17, A-18 (with  $I_1 e \equiv 0$ ) with equations A-13, A-14, A-15, A-16, the result of D-10 can be obtained with  $I_2 e = -\Delta Y_2(E_2 + \Delta E_2)$  and the voltage at port 1 being  $\Delta E_1$ . Hence, if this equivalent current  $\Delta Y_2(E_2 + \Delta E_2)$  can be evaluated, then the voltages  $\Delta E_1$  and  $\Delta E_2$  can be evaluated by computationally driving the unperturbed circuit as shown in Sketch 7.

Also by noting that the form of equation D-10 is exactly that of A-9, the reciprocity relationship of equation A-10 can be used to obtain the relationship (see A-23)

$$\Delta E_1 = - \underbrace{[Y_1 (A + B Y_2) + C + D Y_2]^{-1}}_{\text{transimpedance}} \Delta Y_2 (E_2 + \Delta E_2) \quad \text{D-11}$$

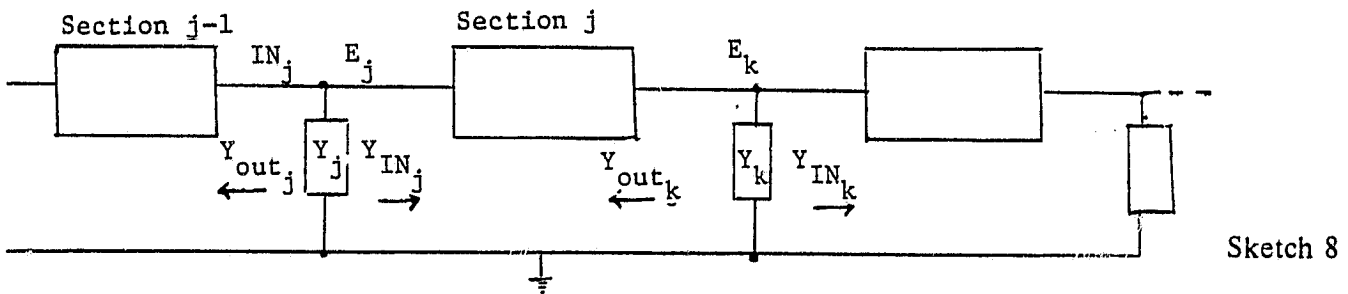
as a replacement for D-10

Finally note that the equivalent drive current  $\Delta Y_2(E_2 + \Delta E_2)$  can be evaluated if  $\Delta E_2$  can be computed, which can be done using equation D-8 which involves known and available parameters.

Thus the computational procedure for evaluating the change  $\Delta E_1$  caused by a change in  $\Delta Y_2$  is as follows:

- Compute  $\Delta E_2$  using D-8, noting that  $Y_{OUT}$ ,  $Y_2$ ,  $Y_1$  are available from the previously computed unperturbed network for any two locations comprising port 1 and port 2
- Compute the "drive" current  $\Delta Y_2(E_2 + \Delta E_2)$  noting again that  $E_2$  is also available at every port of interest in the original unperturbed network analysis.
- Determine the forward transimpedance from port 1 to port 2 using  $Y_{IN}$  and the various voltage transfer ratios also available from the original unperturbed network analysis
- Transpose the transimpedance found in step c) above and use it in equation D-11 to find the desired voltage change  $\Delta E_1$  at the input port.

Specifically then to carry out the exact perturbation analysis for a single load variation all that remains is to establish the forward transimpedance from port 1 to port 2. Consider the following sketch which shows a portion of a total (tree) network



where it is of interest to determine the change in voltage of  $E_j$  (and  $E_k$ ) due to a change in  $Y_k$ , a load connected to port 2 of section  $j$ . As a result of analyzing the unperturbed network, the following quantities (vectors and matrices) are available.

- $Y_{OUT_j}$  - equivalent Norton's source admittance looking back into the output port 2 of section  $j-1$
- $I_{N_j}$  - equivalent Norton's source current out of section  $j-1$
- $Y_{IN_j}$  - admittance looking into port 1 of section  $j$
- $E_k$  - original voltage at port 2 of section  $j$
- $Y_{OUT_k}$  - equivalent Norton source admittance looking back into the output port 2 of section  $j$
- $Y_{IN_k}$  - admittance looking into port 1 of section  $k=j+1$
- $Y_j$  - external load (not including  $Y_{IN_j}$ ) connected to port 2 of section  $j-1$
- $Y_k$  - external load (not including  $Y_{IN_k}$ ) connected to port 2 of section  $j$

$T_{jk}$  - voltage transfer ratio from port 1 to port 2 of section  $j$ . Note this voltage transfer ratio may itself be the product of many sub-sections in cascade which form section  $j$ .

Then to conduct the perturbation analysis associated with a change  $\Delta Y_k$  in  $Y_k$ , form or compute

$$Y_1 = Y_j + Y_{OUT_j} \quad D-12$$

$$Y_2 = Y_k + Y_{IN_k} \quad D-13$$

$$E_2 = E_k \quad D-14$$

$$\Delta Y_2 = \Delta Y_k \quad D-15$$

$$C^{-1} = T_{ij} [Y_{OUT_j} + Y_j]^{-1} \quad \text{forward transimpedance across section } j \quad D-16$$

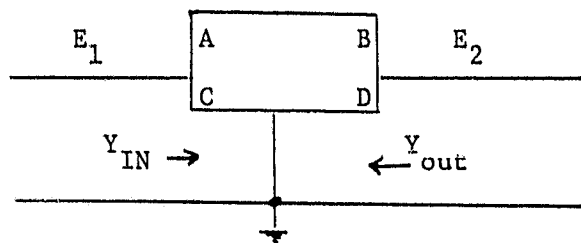
$$\Delta E_2 = - [Y_2 + \Delta Y_2 + Y_{OUT_k}]^{-1} \Delta Y_2 E_2 \quad D-17$$

$$\Delta E_1 = - C^{-1 T} \Delta Y_2 (E_2 + \Delta E_2) = [Y_{OUT_j} + Y_j]^{-1} T_{ij}^T \Delta Y_2 (E_2 + \Delta E_2) \quad D-18$$

Note that either or both  $Y_j$  and  $Y_k$  may be zero (null matrices).

#### Development of Exact Analysis for Reverse Direction Propagation

This procedure is concerned with the computation of voltage propagation in the reverse direction, i.e., considering Sketch 9



Sketch 9

with any relevant load and termination admittances absorbed into the equivalent  $A$ ,  $B$ ,  $C$ ,  $D$  as shown in Sketch 3 and equations A-13 thru A-20, what is  $E_1$  given  $E_2$ ? Letting, as before,  $Y_{out}$  to be the admittance looking backward into port 2, and using the transimpedance reciprocity relationship, then

$$E_1 = \overset{\text{reverse direction transimpedance}}{C^{-1 T} Y_{OUT}} E_2 \quad E-1$$

This is easy to show analytically, i.e., using A-1, A-2 with  $I_1 \triangleq 0$  obtain

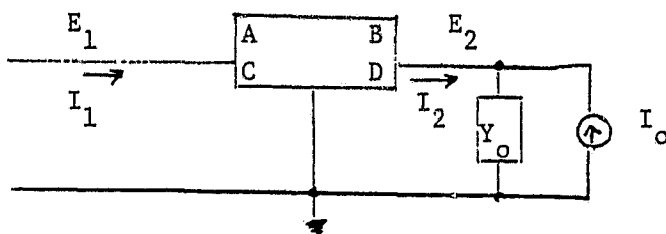
$$E_1 = [A - BD^{-1}C] E_2 = [AC^{-1}D - B] D^{-1} C E_2 \quad E-2$$

From A-10 note  $AC^{-1}D - B = C^{-1 T}$  the reverse direction transimpedance and for the "unloaded" circuit of Sketch 9  $D^{-1}C$  is  $Y_{OUT}$  (see A-12 with  $Y_1=0$ ). Thus all that is required is to develop the transimpedance as explained previously for the load perturbation case. All necessary circuit parameters such as  $Y_{OUT}$ ,  $T_{ij}$  etc are known and available from the original unperturbed circuit analysis.



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The previous form suggests an equivalent Norton source current effect. Consider the following circuit



where  $Y_0$  and  $I_0$  represent a Norton's source. With  $I_1 \equiv 0$  it is easy to show that

$$E_1 = [(A + BY_0)(C + DY_0)^{-1}D - B]I_0 \quad \text{E-3}$$

Now using A-13, A-14, A-15 and A-10 note that the matrix  $[(A + BY_0)(C + DY_0)^{-1}D - B]$  is the reverse direction transimpedance. (This can be observed easily by assuming  $Y_0$  to be imbedded in  $A, B, C, D$  and letting  $Y_0 \rightarrow 0$  yielding  $AC^{-1}D - B$  as before. Hence finding the equivalent Norton's source may be more convenient than specifying the voltage  $E_2$ .)

A final note of interest is to expand equation E-1 into terms involving external behavior, i.e.  $Y_{IN}$ ,  $Y_{OUT}$ , and the voltage transfer matrix  $A^{-1}$ , i.e.

$$E_2 = A^{-1} E_1 \quad \text{E-4}$$

Again emphasizing that all load and termination admittances are absorbed into the  $ABCD$  network of sketch 9 then

$$Y_{IN} = CA^{-1} \quad \text{E-5}$$

so that

$$C^{-1T} = Y_{IN}^{-1} A^{-1T} \quad \text{E-6}$$

Substituting into E-1

$$E_1 = Y_{IN}^{-1} A^{-1T} Y_{OUT} E_2 \quad \text{E-7}$$

where  $A^{-1}$  is recognized to be the forward voltage transfer ratio matrix,  $Y_{IN}$ ,  $Y_{OUT}$  and the "overall forward voltage transfer matrix" are all available (i.e., computed and saved) and provides a compact procedure. Equation E-7 has been derived using  $ABCD$  network representation which implies  $A$  to be square (equal number of terminals at input and output ports). However, if

$$E_2 = RE_1 \quad \text{E-8}$$

then

$$E_1 = Y_{IN}^{-1} R^T Y_{OUT} E_2 \quad \text{E-9}$$

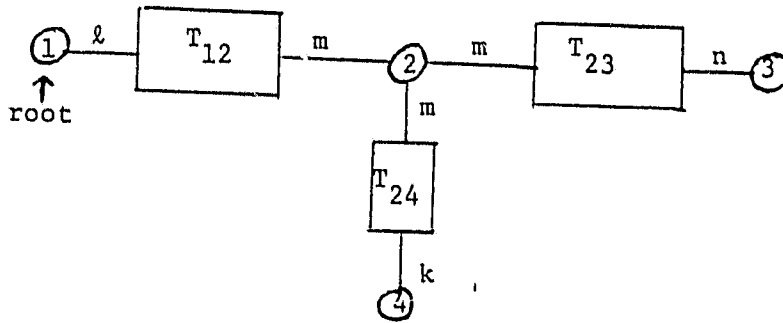
is perfectly general whether or not input and output ports have equal number of terminals. This can be shown using general admittance matrix representation of the two port network or more directly utilizing the reciprocity of transimpedances, i.e., (with suitable sign conventions)

$$E_2 = Z_{12} Y_{IN} E_1 \Rightarrow R = Z_{12} Y_{IN} \Rightarrow Z_{21} = Z_{12}^T = Y_{IN}^{-1} R^T \quad \text{E-10}$$

$$E_1 = Z_{21} Y_{OUT} E_2 = Y_{IN}^{-1} R^T Y_{OUT} E_2 \quad E-11$$

**Note on Cumulative Voltage Transfer Matrices**

The preceding discussions have shown how reciprocity properties makes possible the use of a cumulative forward transfer function matrix (R) to compute reverse path propagation to the original source. Also implied in this approach is the possibility of computing point to point propagation. However, in using reciprocity and cumulative voltage transfer matrices for this purpose, a subtle computational situation arises as pointed out by J.T. Gagjar. This problem can be illustrated by the following sketch.



Sketch 11

The voltage transfer ratio matrix  $T_{ij}$  relates input and output by

$$E_j = T_{ij} E_i \quad F-1$$

Suppose that this network has been analyzed in the conventional outbound direction so that the voltage transfer ratios  $T_{12}$ ,  $T_{13}$ , and  $T_{14}$ \* are available (along with the various  $Y_{IN}$  and  $Y_{OUT}$  admittances), and it is desired to propagate from point 3 to point 4, or from point 3 to point 2. In either case it will be necessary to obtain  $R_{23}$  (among other items).

Now

$$T_{13} = T_{23} T_{12} \quad \begin{array}{l} T_{13} \text{ } n \times l \\ T_{12} \text{ } m \times l \\ T_{14} \text{ } k \times l \\ T_{23} \text{ } n \times m \end{array} \quad F-2$$

\* Note these are "cumulative" voltage transfer rates from a common source point and are available (computed and saved during outbound computations). Using them instead of multiplicative chain multiplies could be a computational advantage during the reverse direction computation, albeit the "cumulative" ratios need to be computed during the outbound processing, but these can be done once and for all.

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Obviously if  $m > n$  then  $T_{12}^{-1}$  does not exist. In this case, the above equation can be partitioned as indicated by the following equation (as pointed out by J.T. Gajjar).

$$\begin{matrix} & T_{13} & & T_{23} & & T_{12} \\ \left[ \begin{array}{cc} nxm & nx(l-m) \\ T_{13P} & T_{13R} \end{array} \right] & = & \left[ \begin{array}{c} nxm \end{array} \right] & \times & \left[ \begin{array}{cc} mxm & mx(l-m) \\ T_{12P} & T_{12R} \end{array} \right] & \text{F-3} \end{matrix}$$

Then

$$T_{13P} = T_{23} T_{12P} \quad \text{F-4}$$

$$T_{13R} = T_{23} T_{12R} \quad \text{F-5}$$

Now  $T_{12P}$  is square and presumably  $T_{12P}^{-1}$  exists so that

$$T_{23R} = T_{13P} T_{12P}^{-1} \quad \text{F-6}$$

Obviously by recording the rows and columns of  $T_{13} = T_{23} T_{12}$  could generate  $\frac{l!}{m!(l-m)!}$  combinations for  $T_{12P}$ , i.e., the number of equations in  $T_{13} = T_{23} = T_{12}$  represent an over-determined case for determining  $T_{23}$ . In practice those columns (of  $T_{12}$ ) associated with the  $m$  terminals of  $E_2$  will be used. Note that the ability to make this identification is possible since a record is kept during the outbound processing of any transitions (3:2, 2:1, 3:1) and transpositions which may have been involved in the  $T_{12}$  path.

**APPENDIX 2**  
**USING RECIPROCITY TO COMPUTE REVERSE DIRECTION**  
**VOLTAGE TRANSFER RATIO MATRICES**

R.C. Rustay

The following attachment is a verbatim copy of an internal memo concerning the relationship of voltage transfer ratio matrices, driving point admittance matrices and the transadmittance matrices. It is included in this report for possible tutorial interest.

DIFNAP SYSTEM MEMO # 21

10/14/80

TO: C.A. Stutt  
J. Gajjar  
W. Hughes  
R. Wooding  
R. Rustay

FROM: R.C. Rustay

SUBJECT: Comments on Voltage Transfer Ratios with Potential Application to the DIFNAP Main Program NTKANS

Recent discussion with W. Hughes regarding the mathematical properties and relationship between outbound and inbound voltage transfer ratios have stimulated the writing of this memo. Also, recent discussions with J. Gajjar regarding my desire to compute the outbound voltage transfer ratio matrix from the source to any section, also has stimulated comments included herein regarding the application of these analytical results to reverse path computations.

### Reciprocity of Transimpedance and Transadmittance

The following is a short tutorial discussion of transimpedance and transadmittance reciprocity. This discussion will begin assuming reader acceptance of the completeness and generality of the "y" parameter representation of linear circuits for those which "y" parameters exist. Consider the following circuit:

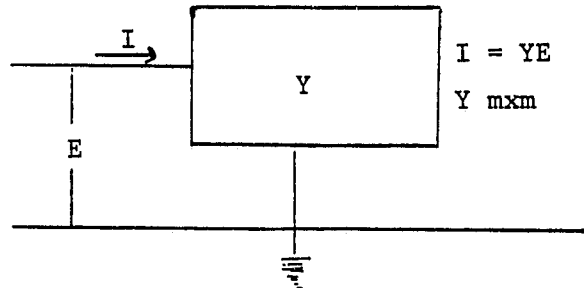


Figure 1

where  $I$  is an  $m$  vector corresponding to the currents entering the  $m$  terminals. Similarly  $E$  is an  $m$  vector of the terminal voltages measured relative to a (zero potential) ground. Next separate the vectors  $I$  and  $E$  each into two partitioned subvectors  $I_1, I_2$  and  $E_1, E_2$ , i.e.

$$I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

where  $I_1$  and  $E_1$  are  $n$  vectors associated with the "first"  $n$  terminals and similarly  $I_2$  and  $E_2$  are  $m-n$  vectors associated with the remaining  $m-n$  terminals. Then Figure 1 can be redrawn to

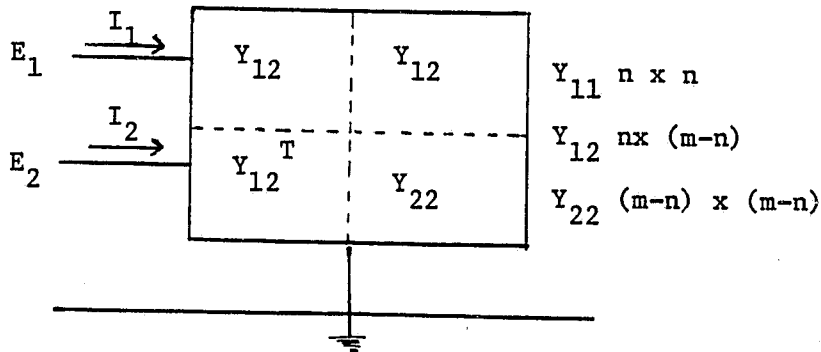


Figure 2

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where the  $Y$   $m \times m$  matrix has been partitioned as shown.  $Y_{12}^T$ , the transpose of  $Y_{12}$  occurs because of the intrinsic symmetry of matrix  $Y_{11}$ ,  $Y^T = Y$ . Figure 2 can be redrawn to frame it into a more conventional two port, i.e. an input and output

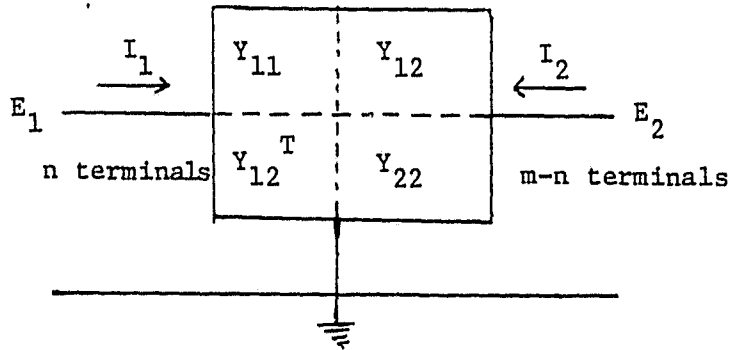


Figure 3

The vector matrix equation associated with Figures 2 and 3 are

$$I_1 = Y_{11}E_1 + Y_{12}E_2 \quad 1)$$

$$I_2 = Y_{12}^T E_1 + Y_{22}E_2 \quad 2)$$

$$\Leftrightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \times \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

from which all subsequent results can be obtained.

### Transadmittance

Suppose all the  $E_2$  terminals are shorted to ground, i.e.,  $E_2 = 0$  then we observe from equation 1 that  $Y_{11}$  is the input driving point admittance\* with the output terminals shorted and that the transadmittance\* is  $Y_{12}^T$  (from equation 2). Similarly if input terminals are shorted to ground, i.e.,  $E_1 = 0$  then the driving point admittance\* seen looking into the output terminals is  $Y_{22}$  and the (output to input) transadmittance\* is seen to be  $Y_{12}$ . Hence the transadmittances between the two parts are said to be reciprocal, i.e. one is equal to the transpose of the other.

### Transimpedances

The equations 1) and 2) can be inverted to the form

$$E_1 = Z_{11}I_1 + Z_{12}I_2 \quad 3)$$

$$E_2 = Z_{12}^T I_1 + Z_{22}I_2 \quad 4)$$

$$\Leftrightarrow \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where  $Z_{11}$ ,  $Z_{12}$ ,  $Z_{22}$  are the corresponding matrix partitions of  $Z$  and

$$Z = Y^{-1} \quad 5)$$

Since  $Y$  is symmetric, so also is  $Z$ .

Now suppose all the  $E_2$  terminals are open circuited, i.e.,  $I_2 = 0$  and a set of input currents are caused to occur. Then from equation 3)  $Z_{11}$  is seen to be the input driving point im-

\* all admittances are in general matrices which can be  $1 \times 1$

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pedance while from equation 4)  $Z_{12}^T$  is seen to be the transimpedance (input to output). Similarly with the input terminal open circuited, i.e.,  $I_1 = 0$ , then  $Z_{22}$  is the driving point impedance looking into the output port and  $Z_{12}$  is the (output to input) transimpedance. Hence as before the transimpedances are also seen to be reciprocal, i.e., one is equal to the transpose of the other.

Note that the matrix elements in equations 3 and 4 are the equally well known "z" parameters which can obviously be expressed in terms of the "y" parameters.

**Relationship of  $Z_{11}$ ,  $Z_{12}$ ,  $Z_{22}$  to  $Y_{11}$ ,  $Y_{12}$ ,  $Y_{22}$**

For possible future reference the relationship of  $Z_{11}$ ,  $Z_{12}$ ,  $Z_{22}$ , will now be given. These relationships can be obtained by matrix algebra operations on equations 1) and 2) or equivalently making use of well known partitioned matrix inversion procedure, the following results can be obtained

$$Z_{11} = [Y_{11} - Y_{12} Y_{22}^{-1} Y_{12}^T]^{-1} \quad 6)$$

$$Z_{12} = - [Y_{11} - Y_{12} Y_{22}^{-1} Y_{12}^T]^{-1} Y_{12} Y_{22}^{-1} = - Z_{11} Y_{12} Y_{22}^{-1} * \quad 7)$$

$$Z_{22} = [Y_{22} - Y_{12}^T Y_{11}^{-1} Y_{12}]^{-1} \quad 8)$$

**Admittance Matrix Equivalent for a Two Port Circuit With External Loading**

The purpose of discussion is to show that a two port linear circuit, with external loading, i.e.,

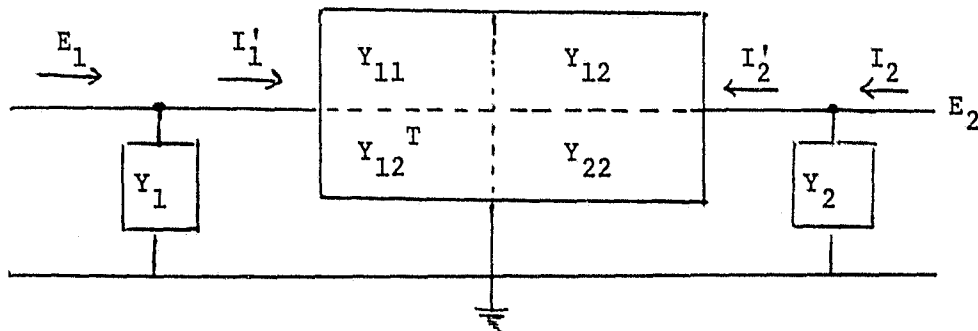


Figure 4

can be easily converted to an equivalent unloaded two port of the type discussed in the preceding section. Writing obvious equations

$$I_1 = Y_1 E_1 + I_1' = (Y_{11} + Y_1) E_1 + Y_{12} E_2 \quad 9)$$

$$I_2 = Y_2 E_2 + I_2' = Y_{12}^T E_1 + (Y_{22} + Y_2) E_2 \quad 10)$$

Hence, the equivalent  $Y$  matrix is seen to be

\* Many other equivalent expressions can be formulated



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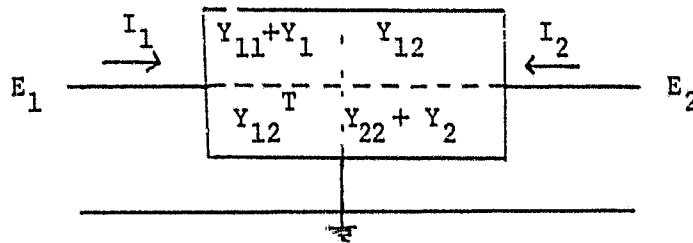


Figure 5

Thus the discussions of the preceding and following sections are perfectly general.

#### Relationship Between Input to Output and Output to Input Voltage Transfer Matrices

This section is concerned with the properties of the voltage transfer ratio matrices from input to output and output to input for a two port linear circuit of the configuration shown in Figure 3 where it is assumed that any external loading has been imbedded into the network  $Y$  matrix as discussed in the preceding section. This discussion also assumes observability and controlability (also implied for the preceding sections but more likely to be overlooked in this context) which imply that  $Y_{11}^{-1}$  and  $Y_{22}^{-1}$  exist.

#### Voltage Transfer Matrix $G_{12}$ from Input to Output Port

Given a voltage  $E_1$  (vector) applied somehow to the input port of Figure 3, the voltage  $E_2$  at the output port with those terminals open circuit, i.e.  $I_2 = 0$  (remember any loading on the output port has been imbedded into the circuit  $Y$  matrix\*) is easily seen to be from equation 2)

$$E_2 = Y_{22}^{-1} Y_{12}^T E_1 = G_{12} E_1 \quad I_2 = 0 \quad (11)$$

i.e.,

$$G_{12} = - Y_{22}^{-1} Y_{12}^T \quad (12)$$

Alternatively using the "Z" representation of equations 3) and 4) it is easy to obtain, for the same condition that  $I_2 = 0$

$$E_2 = Z_{12}^T Z_{11}^{-1} \quad I_2 = 0 \quad (13)$$

i.e.,

$$G_{12} = Z_{12}^T Z_{11}^{-1} \quad (14)$$

\* i.e., from the preceding section this output loading admittance would be linearly added to the original circuit  $Y_{22}$  and thus  $Y_{22}^{-1}$  in equations 11) and 12) will involve this loading

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**Voltage Transfer Matrix  $G_{21}$  from Output to Input Port**

Similarly, given a voltage  $E_2$  (vector) applied somehow to the output port of Figure 3, the voltage  $E_1$  at the input port with those terminals open circuit, i.e.,  $I_1 = 0$  (remember any loading on the input port has to be considered and also imbedded into the circuit  $Y$  matrix) is easily seen to be from equation 1)

$$E_1 = Y_{11}^{-1} Y_{12} E_2 = G_{21} E_2 \quad I_1 = 0 \quad 15)$$

i.e.,

$$G_{21} = - Y_{11}^{-1} Y_{12} \quad 16)$$

Similarly and alternatively using the "Z" representation of equations 3) and 4) it is easy to obtain for the same condition that  $I_1 = 0$

$$E_1 = Z_{12} Z_{22}^{-1} E_2 \quad I_1 = 0 \quad 17)$$

i.e.,

$$G_{21} = Z_{12} Z_{22}^{-1} \quad 18)$$

**Observations Regarding Voltage Transfer Ratio Matrices**

Examining the voltage transfer ratio matrices, i.e.,

$$G_{12} = - Y_{22}^{-1} Y_{12}^T = Z_{12}^T Z_{11}^{-1} \quad 19)$$

$$G_{21} = - Y_{11}^{-1} Y_{12} = Z_{12} Z_{22}^{-1} \quad 20)$$

observe that they involve the "transadmittance"  $Y_{12}$  or the "transimpedance"  $Z_{12}$  commonly between them. Obviously  $G_{12}$  and  $G_{21}$  are not reciprocal. Also if  $G_{12}$ ,  $Y_{22}$ , and  $Y_{11}$  (or  $G_{12}$ ,  $Z_{22}$ ,  $Z_{11}$ ) are known then  $G_{21}$  can be calculated, i.e.,

$$G_{21} = Y_{11}^{-1} G_{12}^T Y_{22} = Z_{11} G_{12}^T Z_{22}^{-1} \quad 21)$$

Similarly for  $G_{12}$  (but not usually of interest)

$$G_{12} = Y_{22}^{-1} G_{21}^T Y_{11} = Z_{22} G_{21}^T Z_{11}^{-1} \quad 22)$$

It is appropriate at this point to reemphasize that as used in these discussions concerning voltage transfer ratio matrices,  $Y_{11}$ ,  $Y_{22}$  and the  $Z_{ij}$  contain the embedded effects of any termination (see Figures 4 and 5) (including any "source" admittance) since obviously the  $G_{12}$  and  $G_{21}$  are dependent, in the context implied here, on both terminations.

The alternate formulations given above, in terms of  $Y$  or  $Z$  parameters represent short circuit (admittance) and open circuit (impedance) approaches respectively. The latter form is used in the DIFNAP programs because open circuit conditions (with terminations imbedded) are generally of interest, i.e. terminal voltages. In the context and terminology of the DIFNAP system  $Z_{11}^{-1}$  is referred to as  $Y_{in}$  and  $Z_{22}^{-1}$  is referred to as  $Y_{out}$  so that equations 21) and 22) become

$$G_{21} = Y_{in}^{-1} G_{12}^T Y_{out} \quad 23)$$

$$G_{12} = Y_{out}^{-1} G_{21}^T Y_{in} \quad 24)$$

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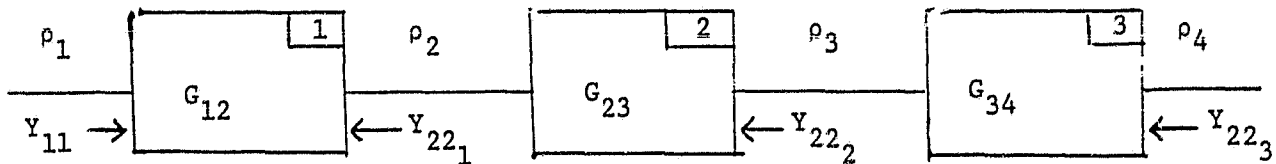
It is interesting to observe the physical interpretation of  $Z$  form of Equations 19) and 20). Consider the voltage transfer  $G_{12}$  from port 1 to port 2. The current  $I_1$  into port 1 is  $I_1 = Y_{in} E_1$  and the port 2 voltage  $E_2$  due to  $I_1$  is  $E_2 = Z_{12}^T I_1$ , so that  $E_2 = Z_{12}^T Y_{in} E_1$  or  $G_{12} = Z_{12}^T Y_{in}$  as before.

The relationships 21 and 22 involving  $Z_{11} = Y_{in}^{-1}$  and  $Z_{22} = Y_{out}^{-1}$  offer a procedure for calculating the reverse path voltage transfer ratio matrices in the DIFNAP program NTWKANSi. Some discussions on such an algorithmic procedure are discussed in the next section.

### Comments Regarding Algorithmic Procedure for Computing Reverse Path Voltage Transfer Matrix

This discussion is concerned with some tentative and embryonic ideas regarding algorithmic procedures which could be implemented into the DIFNAP computer program NTWKANSi for computing the reverse path voltage transfer ratio.

Consider, for the purpose of an example to illustrate the possible algorithm, the following cascaded sections. (It is obvious that branching presents no conceptual difficulties; practically it will depend upon having available appropriate matrix admittances associated with branching.)



As a result of normal outbound response calculations, the voltage transfer ratios  $G_{12}$ ,  $G_{23}$ ,  $G_{34}$ , the network driving point admittance  $Y_{IN}$  and the "equivalent Norton source admittances"  $Y_{OUT_1}$ ,  $Y_{OUT_2}$ ,  $Y_{OUT_3}$  are available. (As mentioned above there may be a lack of complete saved data when branching is involved.) Now consider the three following cases:

$G_{21}$

From equation 23

$$G_{21} = Y_{IN}^{-1} G_{12}^T Y_{OUT_1} \quad (25)$$

$G_{31}$

Similarly

$$G_{31} = Y_{IN_1}^{-1} G_{13}^T Y_{OUT_2} \quad (26)$$

But

$$G_{13} = G_{23} G_{12} \Rightarrow G_{13}^T = G_{12}^T G_{23}^T \quad (27)$$

$\therefore$

$$G_{31} = Y_{IN_1}^{-1} G_{12}^T G_{23}^T Y_{OUT_2} \quad (28)$$

$G_{41}$

Similarly

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$$G_{41} = Y_{IN_1}^{-1} G_{14}^T Y_{OUT_3} \quad 29)$$

But

$$G_{14} = G_{34} G_{23} G_{12} \Rightarrow G_{14}^T = G_{12}^T G_{23}^T G_{34}^T \quad 30)$$

∴

$$G_{41} = Y_{IN_1}^{-1} G_{12}^T G_{23}^T G_{34}^T Y_{OUT_3} \quad 31)$$

Hence, it is easily observed that a recursive computational algorithm can be realized in the same sense that the present Norton's equivalent source admittances (looking in the reverse direction) such as  $Y_{OUT_1}$ ,  $Y_{OUT_2}$ , and  $Y_{OUT_3}$  represent the total affect of the network upstream from that point towards the source. For example during the normal outbound processing, say to get the voltage  $e_3$ , the result of the computation

$$Y_{IN_1}^{-1} G_{12}^T G_{23}^T \quad 32)$$

could be saved with the other data saved with section 2. Then progressing to section 3 the corresponding quantity

$$Y_{IN_1}^{-1} G_{12}^T G_{23}^T G_{34}^T = (Y_{IN_1}^{-1} G_{12}^T G_{23}^T) * G_{34}^T \quad 33)$$

could be computed and saved with that section's data. (Note an appropriate RCR subroutine already exists for computing  $(A^T)^T = AB^T$ .)

Then either on demand, or at the time the above computations are made (and saved for later recall) the reverse path voltage transfer ratio could be computed, i.e., for example

$$G_{41} = (Y_{IN_1}^{-1} G_{12}^T G_{23}^T G_{34}^T) * Y_{OUT_3} \quad 34)$$

Obviously many "bookkeeping" activities need to be considered. For example, during the normal outbound NTWKANSi calculation, the recursively accumulated matrix such as  $Y_{IN_1}^{-1} G_{12}^T G_{23}^T$  should be saved in core and saved with the data associated with the next downstream section (or two with branching) to avoid an extra disc access when later recalling this data for computing on demand the reverse direction voltage transfer ratio matrix. However, if the extra computing time is acceptable, the reverse direction voltage transfer ratio matrix could be computed for every section. However, it is not necessary that all sections will be evaluated tending to "throw out" this procedure. For the "on demand" procedure an extra dimension would be required to the LIFO (push down stack) matrix array to accommodate to the potential level of branching traversed while computing an outbound propagation.

Finally it is noted that the above procedure implies transfers back to the original source.

**APPENDIX 6**  
**MAIN PROGRAM NETGENS1 FOR GENERATING GENERIC NETWORKS**  
**R.C. Rustay**

Included in this Appendix is a copy of an internal memo describing a main program NETGENS1 for generating generic networks.

DIFNAP MEMO #24

11/17/80

Memo To: C. A. Stutt  
J. Gajjar  
R. Wooding  
D. Rodriguez  
W. Hughes  
R. Rankin  
R. Rustay

From: R. C. Rustay

Subject: Main Program NETGENSI for Generating Generic Networks

Please be advised that a main program NETGENSI is now operational. This program provides a convenient procedure for generating a simple class of generic network files, of the form DNWKINij, and which are characterized by the following items:

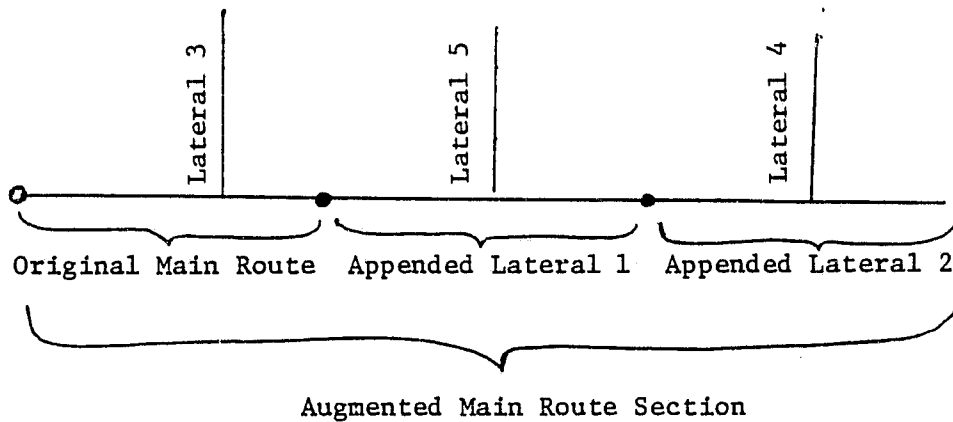
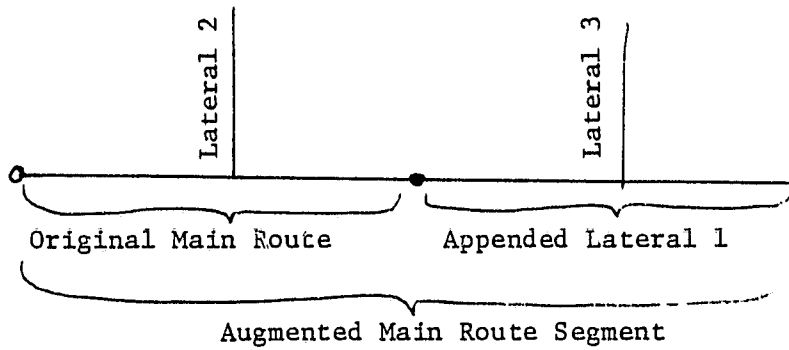
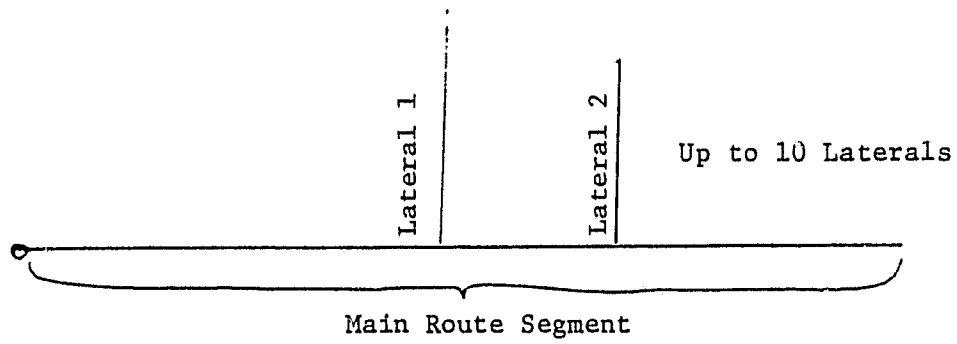
- 1) The generated network is everywhere three phase
- 2) Only one level of branching is available, ie. as illustrated on the attached sketch.
- 3) The networks shown in the sketch are composed of segments, the one originating at the source or root section being designated as the main "route" segment; all others being designated as laterals with up to a maximum of 10 being user definable.
- 4) Each segment will have an invariant line/cable configuration which is user definable.
- 5) Each segment can be composed of many sections with the sections in any segment being equal length (except possibly for the last one). These section lengths are user definable for the main route and all laterals.
- 6) The user can invoke an option to cause the program to automatically place a distribution transformer at the end of each section. The program automatically balances this loading by cycling the assignment of the DT in turn to each phase. The user can elect to defeat this option in which case all lines will be unloaded.

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- 7) The user defines the total length of all segments.
- 8) The user defines the distance along the main route (from the root/source) at which each lateral will be attached.  
Note: In the present first edition, the logic necessary to introduce "zero length" sections to permit tertiary or higher order branching has not been included. Therefore, at the present time, it will be the user's responsibility to preplan to avoid this situation. If a need occurs and time/resources allow, this additional logic for higher order branching could be included.
- 9) The user can arrange for a lateral to be appended to the end of the original main route segment by specifying connection distance for a lateral to be greater than the total length of the main route. This procedure can be cascaded. This procedure allows the user to construct a cascaded sequence of segments where each segment may have its own and different line types and/or section lengths. Subsequent laterals can be specified to branch off anywhere on the, now, augmented main route.
- 10) The resulting network is placed in a file whose name is of the form DNWKINij where ij is specified by the user. The program includes a check/alert to the user to guard against over-writing an existing file of the same name.
- 11) The output file includes an additional column which gives the accumulated length along any segment (including any augmented main route). This added column will not interfere with the reading of this file by the main network program NTWISERSj; this last column is simply ignored.

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**APPENDIX 7**  
**MAIN PROGRAM SUBNETS1**

**R.C. Rustay**

Included in this Appendix is a copy of an internal memo describing a main program SUBNETS1 which can extract as a subnetwork a portion of a larger network.

DIFNAP SYSTEM MEMO #23

11/13/80

Memo To: C. A. Stutt  
J. Gajjar  
W. Hughes  
R. Rankin  
D. Rodriguez  
R. Rustay  
R. Wooding

From: R. C. Rustay

Subject: Availability of Main Program SUBNETSI to Extract Subnetworks

Please be advised that a main program SUBNETSI is now operational. This program allows the user to extract from a given network file (usually with a file name like DNWKINij) a "subtree" of that network. This subtree is defined to the program by specifying the section number (which, of course, must already exist) which is to become the new root section. The program then extracts the portion of the original network which is outbound from that new root section, and places the resulting subnetwork specification into a file whose name is specified by the user. The program SUBNETSI employs conversational input and is self-explaining during running of the program.

Conforming to RCR policy, the user can and should exit from the program by typing a negative number when prompted.

Any section identification number cannot exceed 2000 with the present dimensioning. This arbitrary limit can easily be increased, if necessary, at the expense of additional core requirement.

RCR:jw

## **APPENDIX 8**

### **NETWORK DESCRIPTION**

This Appendix contains specific information on the parameter variations analyzed during the study. Each variation is identified by its network (or system) number which is also used to identify the output data for that variation. Unless a parameter is specifically mentioned, it can be assumed that the parameter is the same as it was for the associated reference network.

## Reference Networks

Network #8 - 1/4 Wavelength underground cable network.

- . Same as Network #9 except section lengths reduced to 42.5m for a total length of 3400m.

Network #9 - Nominal underground cable network

- . Aluminum phase conductors GMR - .385 inches
- . 21 copper neutral strands, each with a radius of .64 inches, evenly spaced around the phase conductor at a radius of .896 inches.
- . Main insulation has an inner radius of .515 inches and an outer radius of .750 inches.
- . Outer jacket has an inner radius of .780 inches and an outer radius of .880 inches.
- . Center to center spacing of phase cables is 2.0 inches, equilateral spacing assumed.
- . Phase conductor temperature = 75°C
- . Neutral temperature = 45°C
- . Earth resistivity = 100  $\Omega/\text{m}^3$
- . Signal frequency = 8.13 kHz
- . Total of eighty transformers, one loading each section, rated at 25 kva with load admittances of -78.8 dB at -27.3 degrees.
- . Eighty sections, each 200m long for a total length of 16,000m. No branches, no loops, entire line is three phase.
- . Diagonal load at the end of the line has three equal elements,  $4.77 \times 10^{-4}$  mhos -68.22° each.

Network #10 - Nominal overhead reference network.

- . Geometric arrangement as shown in Fig. (6a).  
Phase conductors are 336 Widgeon ACSR with 1/0 ALCOA ACSR neutral wire. Diameters are .721 inches and .398 inches, respectively. Cross-sectional areas are .408 inches<sup>2</sup> and .124 inches<sup>2</sup>, respectively.
- . This line type code is 101.
- . Total of eighty transformers, one loading each section, rated at 25 kva, with load admittances of -78.8 dB at -27.3 degrees.
- . Eighty sections, each 200m long, for a total length of 16,000m. No branches, no loops, entire line is three phase.
- . Temperature of conductors = 25°C
- . Earth resistivity = 100  $\Omega/m^3$
- . Signal frequency = 8.13 kHz
- . Diagonal load at the end of the line has three equal elements,  $4.77 \times 10^{-4}$  -68.22° mhos each.

## 8.2 Overhead Geometry Variation Networks

- System #11 - Referred to network #10; Phase conductor changed to 4/Ø ALCOA ACSR. Diameter reduced from .721 inches to .563 inches, a 21.9% reduction. Cross-sectional area reduced 39.0%.
- System #12 - Referred to network #10; Phase conductor changed to 2/Ø ALCOA ACSR. Diameter reduced from .721 inches to .447 inches - a 38% reduction. Cross-sectional area reduced 61.6%.
- System #13 - Referred to network #10; Phase conductor changed to 447 HEN ACSR. Diameter increased from .721 inches to .883 inches, a 22.5% increase. Cross-sectional area increased 50.0%.

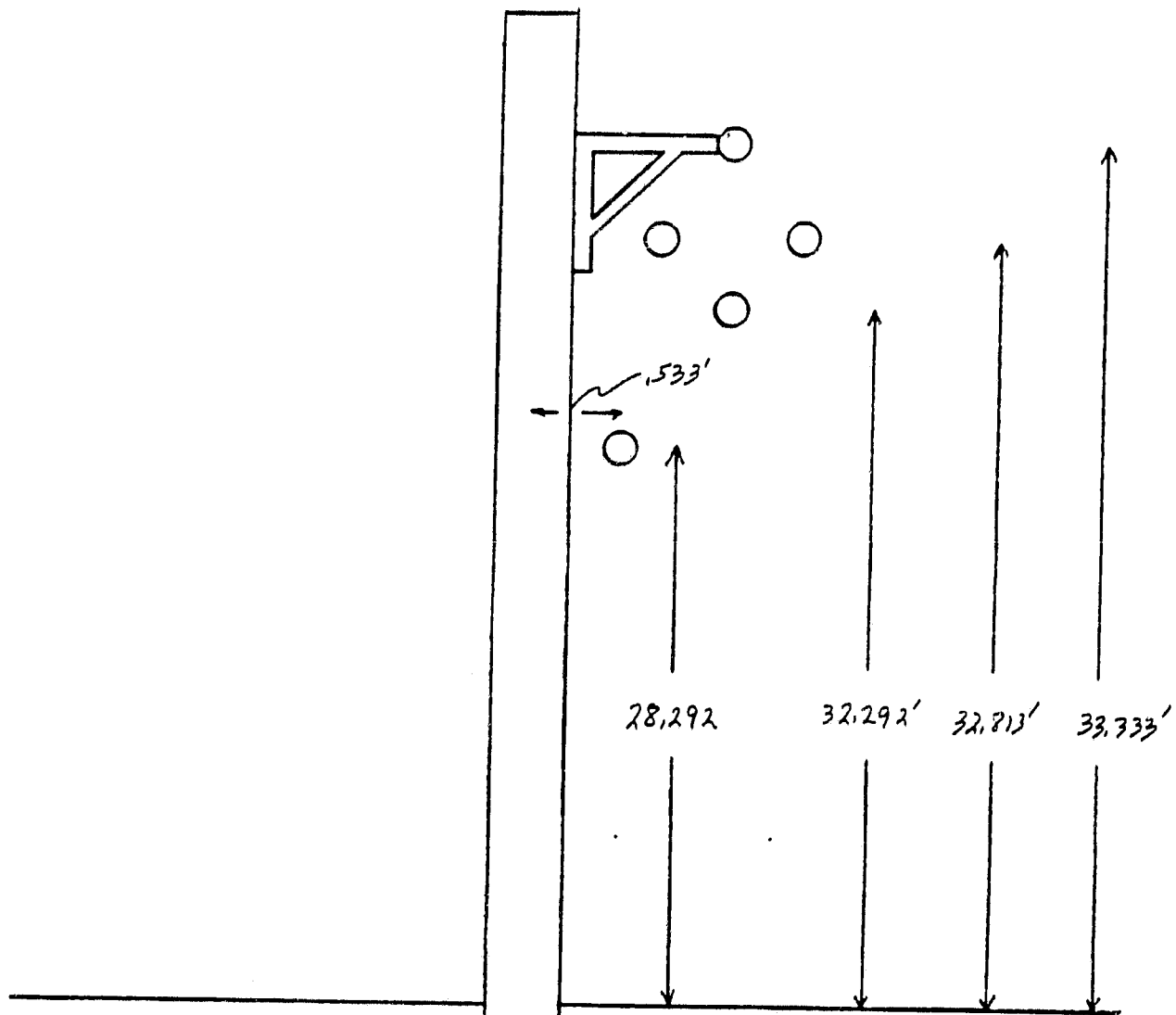
- System #14 - Referred to Network #10; Neutral conductor changed to #4 ALCOA ACSR. Diameter reduced from .398 inches to .250 inches, a 37.2% difference. Cross-sectional area reduced 60.5%.
- System #15 - Referred to network #10; Phase conductor to phase conductor spacing increased from 3.667 feet to 4.033 feet, a 10.0% difference.
- System #16 - Referred to network #10; Phase conductor to earth spacing increased from 35.063 feet (for Phase A) and 33.667 feet (for phase B & C) to 38.569 feet and 37.033 feet, respectively. This is an increase of 10.0%.
- System #17 - Referred to network #10; Neutral conductor to earth spacing increased from 27.333 feet to 30.066 feet, a 10.0% difference.
- System #18 - Referred to network #10; Miscellaneous conductor added, 23.458 feet above the earth. Assumed to be at ground potential.
- System #19 - Referred to network #10; Two miscellaneous conductors added at 23.208 and 23.458 feet above the earth. Both assumed to be at ground potential.
- System #20 - Referred to network #10; Two open circuit secondary conductors added at 26.000 and 26.667 feet above the earth. Both assumed to be at ground potential.
- System #21 - Referred to network #10; Neutral conductor changed to Triplex. Diameter increased from .398 inches to .92 inches, a 131% difference. Cross-sectional area increased 434%.

- System #22 - Referred to network #10; Configuration changed to spacer cable. See Figure (8-1)
- Network #28 - Referred to 1/4 wavelength version of network #10; same variation as system #11.
- Network #29 - Referred to 1/4 wavelength version of network #10; same variation as system #12.
- Network #30 - Referred to 1/4 wavelength version of network #10; same variation as system #13.
- Network #31 - Referred to 1/4 wavelength version of network #10; Same variation as system #14.
- Network #32 - Referred to 1/4 wavelength version of network #10; same variation as system #15.
- Network #33 - Referred to 1/4 wavelength version of network #10; same variation as system #16.
- Network #34 - Referred to 1/4 wavelength version of network #10; same variation as system #17.
- Network #35 - Referred to 1/4 wavelength version of network #10; same variation as system #18.
- Network #36 - Referred to 1/4 wavelength version of network #10; same variation as system #19.
- Network #37 - Referred to 1/4 wavelength version of network #10; same variation as system #20.
- Network #38 - Referred to 1/4 wavelength version of network #10; same variation as system #21.
- Network #39 - Referred to 1/4 wavelength version of network #10; same variation as system #22.

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L TYP = 201

← 1.667 →  
← 1.333 →  
← 1.333' →



Feeder Conductor 336 Widgcon AJCR  
Neutral Conductor  
Carrier Cable

Figure (8-1)



### 8.3 Transformer Loading Variation Networks

Network #13 - Referred to network #10; The diagonal load at the end of the line is removed, leaving it open circuited.

Network #14 - Referred to network #10; The admittance of each distribution transformer load is doubled to -72.8 dB at -27.3 degrees.

Network #17 - Referred to network #10; The diagonal load at the end of the line is removed while at the same time the admittance of each distribution transformer load is doubled to -72.8 dB at -27.3 degrees.

Network #42 - Referred to 1/4 wavelength version of network #10; same variation as network #13.

Network #45 - Referred to 1/4 wavelength version of network #10; same variation as network #17.

Network #56 - Referred to 1/4 wavelength version of network #10; same variation as network #14.

Network #57 - Referred to 1/4 wavelength version of network #10; all distribution transformer loading is neglected.

Network #58 - Referred to network #10; same variation as network #57.

Network #59 - Referred to network #10; admittance of each distribution transformer load is cut in half to -84.8 dB at -27.3 degrees.

Network #60 - Referred to network #10; overhead geometry varied as in system #11, loading varied as in network #59.

Network #61 - Referred to network #10; overhead geometry varied as in system #12, loading varied as in network #59.

- Network #62 - Referred to network #10; overhead geometry varied as in system #13, loading varied as in network #59.
- Network #63 - Referred to network #10; overhead geometry varied as in system #14, loading varied as in network #59.
- Network #64 - Referred to network #10; overhead geometry varied as in system #15, loading varied as in network #59.
- Network #65 - Referred to network #10; overhead geometry varied as in system #16, loading varied as in network #59.
- Network #66 - Referred to network #10p overhead geometry varied as in system #17, loading varied as in network #59.
- Network #67 - Referred to network #10; overhead geometry varied as in system #18, loading varied as in network #59.
- Network #68 - Referred to network #10; overhead geometry varied as in system #19, loading varied as in network #59.
- Network #69 - Referred to network #10; overhead geometry varied as in system #20, loading varied as in network #59.
- Network #70 - Referred to network #10; overhead geometry varied as in system #21, loading varied as in network #59.
- Network #71 - Referred to network #10; overhead geometry varied as in system #22, loading varied as in network #59.

#### 8.4 "Set up" Time Reduction Networks

- Network #11 - Referred to network #10; the length of every other section is doubled to 400m with the in-between sections reduced to zero length. Same overall length and loading are maintained. Number of sections is effectually cut in half, though not actually.

- Network #12 - Referred to network #10; the length of every third section is tripled to 600m with the in-between sections reduced to zero length. Because 80 is not evenly divisible by 3, the next to last section is doubled instead of tripled in length to maintain the same overall length as the reference network. Same loading is maintained. Number of sections is effectively reduced to almost a third of the original number.
- Network #15 - Referred to network 10; same variation as network #11 with the admittance of each transformer load doubled to -72.8 dB at -27.3 degrees.
- Network #16 - Referred to network #10; same variation as network #12 with the admittance of each transformer load doubled to -72.8 dB at -27.3 degrees.
- Network #18 - Referred to network #10; Temperature changed from 35°C to 30°C, a 20.0% increase.
- Network #19 - Referred to network #10; Temperature changed from 25°C to 20°C, a 20.0% decrease.
- Network #20 - Referred to network #10; temperature changed from 25°C to 10°C, a 60.0% decrease.
- Network #21 - Referred to network #10; Temperature changed from 25°C to 0°C, a 100.0% decrease
- Network #22 - Referred to network #10; temperature changed from 25°C to -10°C, a 140.0% decrease.
- Network #23 - Referred to network #10. Earth resistivity changed from  $100 \Omega/m^3$  to  $50 \Omega/m^3$ , a 50.0% reduction.
- Network #24 - Referred to network #10. Earth resistivity changed from  $100 \Omega/m^3$  to  $75 \Omega/m^3$ , a 25.0% reduction.

- Network #25 - Referred to network #10. Earth resistivity changed from  $100 \text{ } \Omega/\text{m}^3$  to  $125 \text{ } \Omega/\text{m}^3$ , a 25.0% increase.
- Network #26 - Referred to network #10. Earth resistivity changed from  $100 \text{ } \Omega/\text{m}^3$  to  $150 \text{ } \Omega/\text{m}^3$ , a 50.0% increase.
- Network #27 - Referred to network #10. It is not assumed that the neutral conductor is at ground potential everywhere along the line.
- Network #40 - Referred to 1/4 wavelength version of network #10. Same variation as network #11 except that section lengths are adjusted for 1/4 wavelength.
- Network #41 - Referred to 1/4 wavelength version of network #10. Same variation as network #12 except that section lengths are adjusted for 1/4 wavelength.
- Network #43 - Referred to 1/4 wavelength version of network #10. Same variation as network #15 except that section lengths are adjusted for 1/4 wavelength.
- Network #44 - Referred to 1/4 wavelength version of network #10. Same variation as network #16 except that section lengths are adjusted for 1/4 wavelength.
- Network #46 - Referred to 1/4 wavelength version of network #10. Same variation as network #18.
- Network #47 - Referred to 1/4 wavelength version of network #10. Same variation as network #19.
- Network #48 - Referred to 1/4 wavelength version of network #10. Same variation as network #20.
- Network #49 - Referred to 1/4 wavelength version of network #10. Same variation as network #21.

- Network #50 - Referred to 1/4 wavelength version of network #10.  
Same variation as network #22.
- Network #51 - Referred to 1/4 wavelength version of network #10.  
Same variation as network #23.
- Network #52 - Referred to 1/4 wavelength version of network #10.  
Same variation as network #24.
- Network #53 - Referred to 1/4 wavelength version of network #10.  
Same variation as network #25.
- Network #54 - Referred to 1/4 wavelength version of network #10.  
Same variation as network #26.
- Network #55 - Referred to 1/4 wavelength version of network #10.  
Same variation as network #27.
- Network #74 - Referred to network #10. Each section is reduced  
in length to 190m to give an overall length reduc-  
tion of 800m, or 5%.
- Network #75 - Referred to network #10. Each section is increased  
in length to 210m to give an overall length increase  
of 800m, or 5%.

## 8.5 Cable Networks

- Network #76 - Referred to network #9. Earth resistivity changed  
from  $100 \Omega/m^3$  to  $50 \Omega/m^3$ , a 50.0% reduction.
- Network #77 - Referred to network #9. Earth resistivity changed  
from  $100 \Omega/m^3$  to  $150 \Omega/m^3$ , a 50.0% increase.
- Network #78 - Referred to network #9. Earth resistivity changed  
from  $100 \Omega/m^3$  to  $200 \Omega/m^3$ , a 100.0% increase.
- Network #79 - Referred to network #9. Temperature of phase  
conductors changed from 75°C to 65°C, a 13.3% decrease.  
Temperature of neutral strands changed from 45°C to  
35°C, a 22.2% reduction.

- Network #81 - Referred to network #9. Temperature of phase conductors changed from 75°C to 95°C, a 26.6% increase. Temperature of neutral strands changed from 45°C to 65°C, a 44.4% increase.
- Network #82 - Referred to network #9. The length of every other section is doubled to 400m with the in-between sections reduced to zero length. Same overall length and loading are maintained. Number of sections is effectively cut in half.
- Network #83 - Referred to network #9. The length of every third section is tripled to 600m with the in-between sections reduced to zero length. Because 80 is not evenly divisible by 3, the next to last section is doubled instead of tripled in length to maintain the same overall length as the reference network. Same loading is maintained. Number of sections is effectively reduced to almost a third of the original number.
- Network #84 - Referred to network #9. The admittance of each distribution transformer load is doubled to -72.8 dB at -27.3 degrees.
- Network #85 - Referred to network #9. The admittance of each distribution transformer load is cut in half to -84.8 dB at -27.3 degrees.
- Network #86 - Referred to network #9. All distribution transformer loading is neglected.
- Network #87 - Referred to network #8. Same variation as network #76.
- Network #88 - Referred to network #8. Same variation as network #77.
- Network #89 - Referred to network #8. Same variation as network #78.

- Network #90 - Referred to network #8. Same variation as network #79.
- Network #91 - Referred to network #8. Same variation as network #80.
- Network #92 - Referred to network #8. Same variation as network #81.
- Network #93 - Referred to network #8. Same variation as network #82 except that section lengths are adjusted for 1/4 wavelength network.
- Network #94 - Referred to network #8. Same variation as network #83 except that section lengths are adjusted for 1/4 wavelength network.
- Network #95 - Referred to network #8. Same variation as network #84.
- Network #96 - Referred to network #8. Same variation as network #85.
- Network #97 - Referred to network #8. Same variation as network #86.
- Network #98 - Referred to network #9. The length of each section is reduced to 190m to give an overall length reduction of 800m, or 5.0%.
- Network #99 - Referred to network #9. The length of each section is increased to 210m to give an overall length increase of 800m, or 5.0%.

### APPENDIX 3

## Y PARAMETER ANALYSIS OF SYMMETRIC DISTRIBUTION TRANSFORMER WITH BALANCED LOADING AND BRIEF DISCUSSION OF A RLC LUMPED PARAMETER MODEL

R.C. Rustay

The material contained in this Appendix was originally issued as an internal memo early in the Phase II work as an attempt to summarize, at that time, what in our judgement was apt to be the most useful type of distribution transformer model (and also any single phase two winding transformer). At that time, two candidate models;

Y Parameter Derived Algebraic Model with  
Frequency Dependent Parameters

A RLC Lumped Parameter Model

were selected.

The writing of the original memo followed laboratory measurements and analytical studies performed by J.T. Gajjar which demonstrated the feasibility of the Y parameter derived algebraic model. Also, at that time, R.W. Rankin had performed response measurements (see plot at the end of this Appendix) which strongly suggested the feasibility of a simplified RLC lumped parameter model for frequencies less than, say, 50-100 kHz. R. Wooding subsequently showed that indeed an RLC model, with appropriate selection of the lumped parameter values could account for the measured response. Subsequent "full scale" measurements were made to develop the parameters for both models. Also at that time, measurements were made of critical admittance and transfer characteristics which were used to verify the selected model.



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DIPNAP SYSTEM MEMO #14

MEMO TO: C.A. Stutt  
J. Gajjar  
R. Wouding  
R. Rankin  
W. Hughes

FROM: R.C. Rustay

SUBJECT:  $Y$  Parameter Analysis of Symmetric Distribution Transformer  
with Balanced Loading and Brief Discussion of a RLC  
Lumped Parameter Model

1. INTRODUCTION

This memo presents, for reference, the results of " $Y$ " parameter analysis of a distribution transformer for the special conditions of symmetric construction and balanced secondary loading. These conditions were suggested by J. Gajjar for modeling/simulation because:

1. in most cases the algebra is greatly simplified and leads to inverse linear relationships involving loading
2. these conditions/assumptions are the best that can be assumed for modeling in the sense that there is no apriori basis for assuming other than balanced secondary loading and that for the lower frequencies at least the DT's do appear to be electrically symmetric with respect to the secondary electrical characteristics
3. these conditions and the resulting simple algebraic models reveal and suggest easy to measure sets of parameters (open circuit voltage transfer ratios, open circuit impedances) as well as some of the conventional  $Y$  parameters which avoid the sensitivity to measurement precision/error that appear to occur when all " $y$ " parameters are used.
4. these conditions lead to particularly convenient and computationally stable algebraic formulations for driving point admittances

Also presented are comments on a "zeroeth" order two winding RLC (plus ideal transformer model).

## 2. ALGEBRAIC MODEL DERIVED FROM "Y" PARAMETER ANALYSIS

The material contained in this memo represents a simplification of the more general results contained in Appendix Q of Vol. 3 of the "RF Model ---" final report, plus rearrangements to include other than "Y" parameters.

Figure 1-1 is the generic/general distribution transformer schematic model with connected loads from which the various special subcases can be derived. This schematic serves to define terminology and convection labeling.

- $Y_1$  Admittance load connected across primary terminals
- $Y_2, Y_3, Y_{23}$  Admittance representation of secondary load
- $Y_N$  Neutral to ground admittance

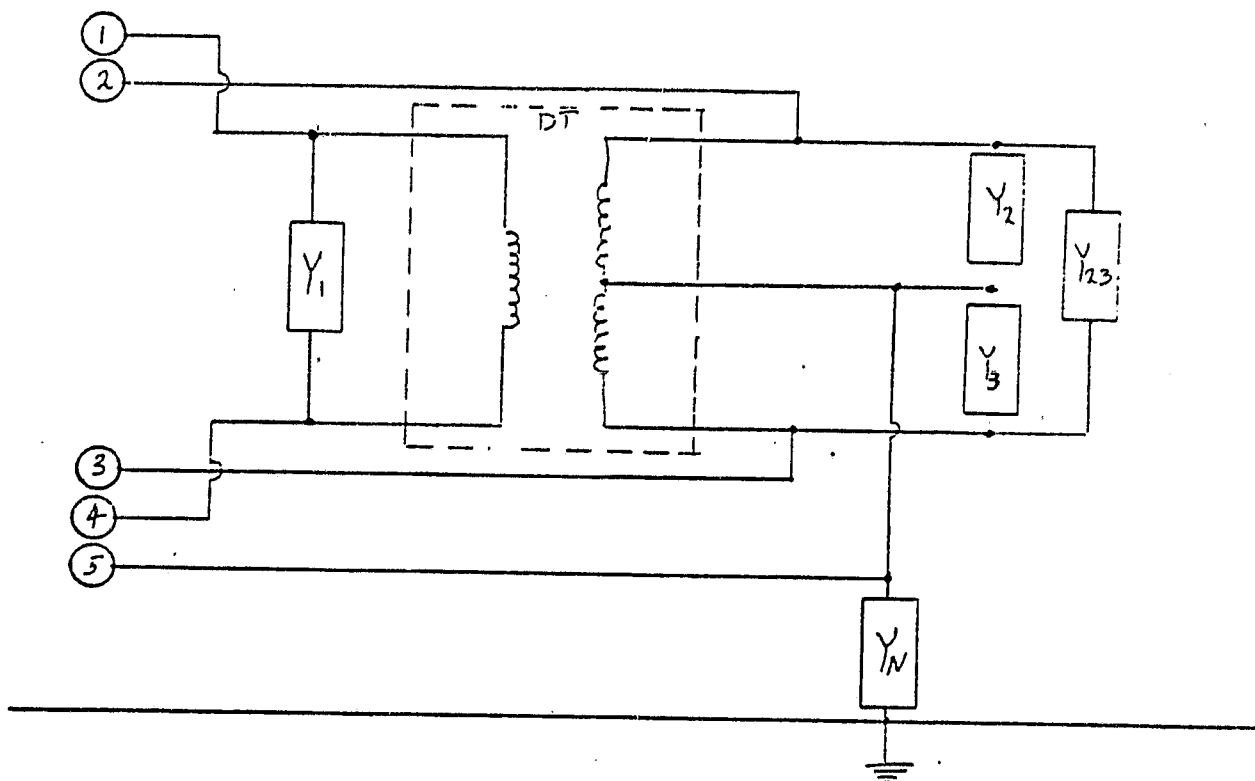


Figure 1-1. General Distribution Transformer Schematic Model with Connected Loads

Figure 1-2 is a general distribution transformer schematic (without loading) and serves to define its associated  $y_{ij}$  parameter representation in terms of the associated  $Y_I$  admittance matrix shown on Figure 1-3. In Figure 1-3 only the independent matrix elements  $y_{ij}$  are shown; the other matrix elements can be derived/deduced from symmetry and the (row-column) zero sum property of an ungrounded network. Figure 1-4 is similar schematic for the terminal loads, including the earth to ground admittance  $Y_N$ . Figure 1-5 is the associated terminal load matrix  $Y_L$ . The matrix admittance representation corresponding to Figure 1-1 is then seen to be the sum  $Y_L + Y_I$ .

Various special cases can be easily obtained using the admittance matrix  $Y_L + Y_I$  and applying the terminal conditions associated with the special cases.

Finally, it should be noted that the convention adhered to in this memo has all terminal voltages  $e_i$  representing terminal to ground potential, and all terminal currents  $i_i$  positive into circuit as shown on various schematics.

As mentioned above, this memo is concerned with results associated with electrically symmetric transformer characteristics and balanced secondary loading. These conditions are represented by the statements:

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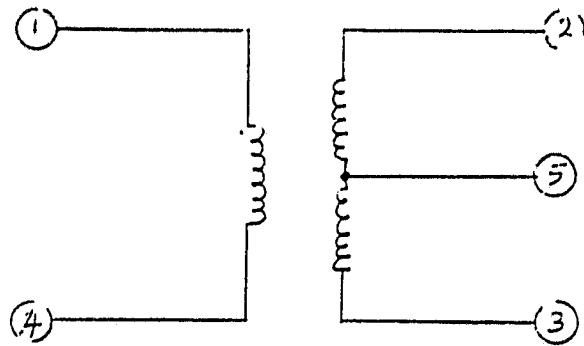


Figure 1-2. General Distribution Transformer Schematic

$$Y_T = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} & -- \\ -- & y_{22} & y_{23} & y_{24} & -- \\ -- & -- & y_{33} & y_{34} & -- \\ -- & -- & -- & y_{44} & -- \\ -- & -- & -- & -- & -- \end{bmatrix}$$

Figure 1-3. Associated Distribution Transformer Admittance Matrix  $Y_T$

$$\begin{aligned} Y_3 &= Y_2 \\ Y_{33} &= Y_{22} \\ Y_{13} &= -Y_{12} \\ Y_{44} &= Y_{11} \\ Y_{24} &= Y_{13} = -Y_{12} \\ Y_{34} &= Y_{12} \end{aligned}$$

leaving only the following "independent" elements

$$y_{11} \quad y_{22} \quad y_{12} \quad y_{23} \quad y_{14}$$

Figure 1-6 shows the associated distribution transformer admittance matrix  $Y_T$  subject to these electrical symmetry conditions.

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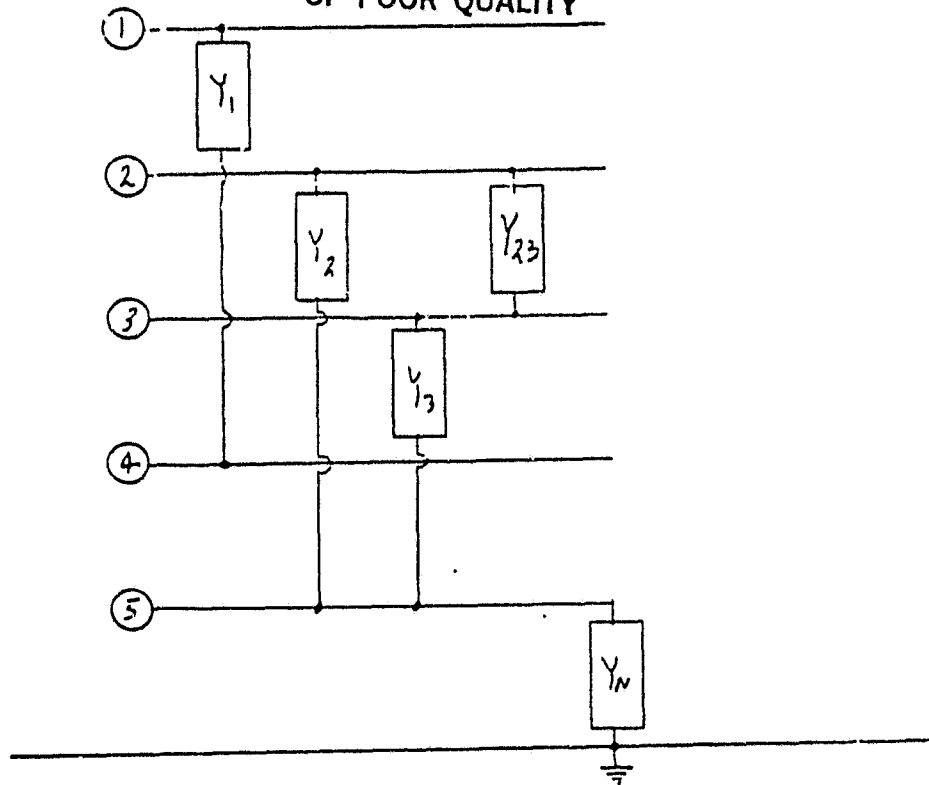


Figure 1-4. Terminal Load Schematic

$$Y_L = \begin{bmatrix} Y_1 & 0 & 0 & -Y_1 & 0 \\ 0 & Y_2 + Y_{23} & -Y_{23} & 0 & -Y_2 \\ 0 & -Y_{23} & Y_3 + Y_{23} & 0 & -Y_3 \\ -Y_1 & 0 & 0 & Y_1 & 0 \\ 0 & -Y_2 & -Y_3 & 0 & Y_1 + Y_2 + Y_3 \end{bmatrix}$$

Figure 1-5. Associated Terminal Load Admittance Matrix  $Y_L$

$$Y_T = \begin{bmatrix} y_{11} & y_{12} & -y_{12} & y_{14} & -- \\ -- & y_{22} & y_{23} & -y_{12} & -- \\ -- & -- & y_{22} & y_{12} & -- \\ -- & -- & -- & y_{11} & -- \\ -- & -- & -- & -- & -- \end{bmatrix}$$

Figure 1-6. Associated Distribution Transformer Admittance Matrix  $Y_T$  Symmetric Electrical Properties

Case A - Phase to Neutral Primary Connection, Neutral at Zero Potential

This case represents the most frequently occurring connection and assumption regarding the neutral being at zero potential. This case can be derived from the general case of Figure 1-1 by applying the terminal conditions.\*

$$e_4 = e_5 = 0 \quad (Y_N = \infty)$$

The resulting schematic is shown on Figure A-1 and the associated total admittance matrix  $Y$  is given in the following equation

$$Y = \begin{bmatrix} y_{11} + Y_1 & y_{12} & -y_{12} \\ y_{12} & y_{22} + Y_2 + Y_{23} & y_{23} - Y_{23} \\ -y_{12} & y_{23} - Y_{23} & y_{22} + Y_3 + Y_{23} \end{bmatrix} \quad \text{A-1}$$

or in equation form

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} y_{11} + Y_1 & y_{12} & -y_{12} \\ y_{12} & y_{22} + Y_2 + Y_{23} & y_{23} - Y_{23} \\ -y_{12} & y_{23} - Y_{23} & y_{22} + Y_3 + Y_{23} \end{bmatrix} \times \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad \text{A-2, A-3, A-4}$$

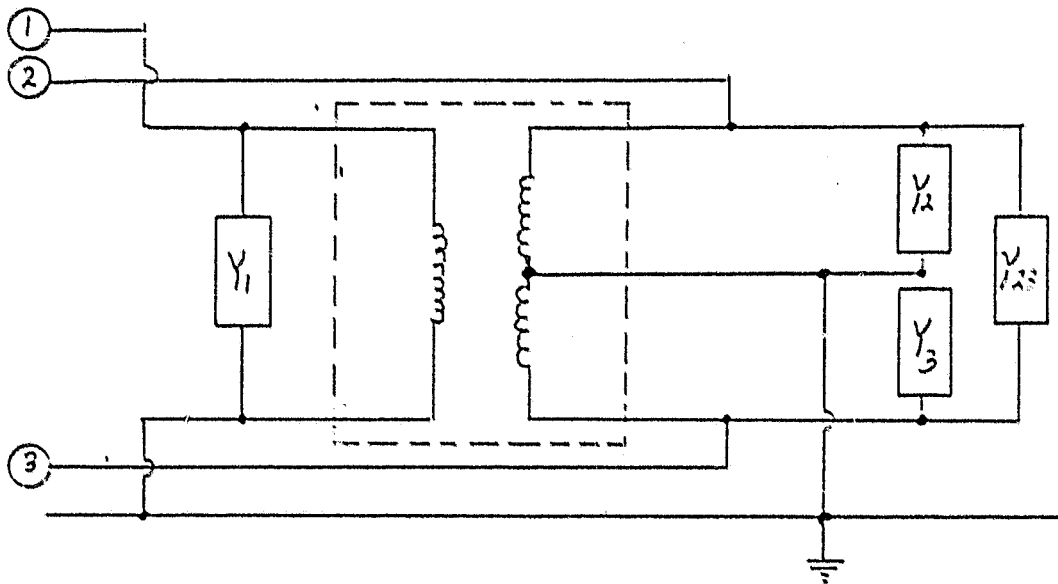


Figure A-1. Distribution Transformer Schematic - Phase to Neutral Primary Connection, Neutral at Zero Potential ( $Y_N = \infty$ )

In accordance with the assumption stated earlier regarding a balanced secondary loading, it will be assumed for the remainder of this note that

$$Y_3 = Y_2 \quad \text{A-5}$$

With this balanced load assumption the equations become

\* Which effectively deletes the 4<sup>th</sup> and 5<sup>th</sup> row and columns.

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$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} y_{11} + Y_1 & y_{12} & -y_{12} \\ y_{12} & y_{22} + Y_2 + Y_{23} & y_{23} - Y_{23} \\ -y_{12} & y_{23} - Y_{23} & y_{22} + Y_2 + Y_{23} \end{bmatrix} \times \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \quad \text{A-6, A-7, A-8}$$

By applying appropriate terminal conditions to these equations, all desired results can be obtained. Finally, it is evident that with symmetric electrical characteristics and balanced secondary loading,  $e_3 \triangleq -e_2$  for primary to secondary considerations.\* Also it is of interest to note that for  $i_1 \triangleq 0$

$$e_1 = - \frac{y_{12}}{y_{11} + Y_1} (e_2 - e_3) \quad i_1 \triangleq 0 \quad \text{A-9}$$

which is valid for any condition where both  $e_2$  and  $e_3$  (or their difference  $e_2 - e_3$ ) is specified by external conditions.

\* i.e.,  $i_2 = i_3 = 0$

Case A - Primary to Secondary:  $i_2=i_3=0$ ,  $Y_3=Y_2$ ,  $Y_1=0$ ,  $Y_4 \rightarrow \infty$

This case represents the situation looking into the primary terminal 1 with  $Y_1=0$  (since we are interested in the input driving point admittance to the transformer). Applying the terminal conditions  $i_2=i_3=0$  and the fact that  $e_3=-e_2$  obtain easily

$$-e_3=e_2 = \frac{-y_{12}}{y_{22}-y_{23}+Y_2+2Y_{23}} e_1 \quad \text{A-10}$$

$$i_1 = \left\{ y_{11} - \frac{2y_{12}^2}{y_{22}-y_{23}+Y_2+2Y_{23}} \right\} e_1 \quad \text{A-11}$$

If the open (secondary) circuit primary driving point impedance  $z_{11}$

$$\frac{1}{z_{11}} = y_{11} - \frac{2y_{12}^2}{y_{22}-y_{23}} \quad Y_2=0=Y_{23} \quad \text{A-12}$$

is available, J.T. Gajjar has suggested the following equivalent expression

$$i_1 = \left\{ \frac{1}{z_{11}} + \left( y_{11} - \frac{1}{z_{11}} \right) \frac{Y_2+2Y_{23}}{y_{22}-y_{23}+Y_2+2Y_{23}} \right\} e_1 \quad \text{A-13}$$

Noting that the open (secondary) primary to secondary line to line voltage transfer ratio  $g_{12}$  is given by

$$g_{12} = \frac{e_2-e_3}{e_1} = \frac{-2y_{12}}{y_{22}-y_{23}} \quad Y_2=0=Y_{23} \quad \text{A-14}$$

so that

$$e_2 = -e_3 = \left[ \frac{2}{g_{12}} - \frac{Y_2+2Y_{23}}{y_{12}} \right]^{-1} e_1 \quad \text{A-15}$$

$$i_1 = \left\{ \frac{1}{z_{11}} + \left( y_{11} - \frac{1}{z_{11}} \right) \frac{Y_2+2Y_{23}}{Y_2+2Y_{23} - \frac{2y_{12}}{g_{12}}} \right\} e_1 \quad \text{A-16}$$

If the parameters  $z_{11}, y_{11}, y_{12}, g_{12}$  are measured, the following identity-redundancy can be used to validate the measurements.

$$\frac{1}{z_{11}} = y_{11} + y_{12}g_{12} \quad \text{A-17}$$

It is seen that the dependency on the secondary load condition  $Y_2+2Y_{23}$  (remember  $Y_3=Y_2$ ) has a simple inverse linear fashion. The forms A-15 and A-16 involve easily measured quantities  $g_{12}, \frac{1}{z_{11}}, y_{11}, y_{12}$ . The form A-16 is intuitively and computationally attractive because  $\frac{1}{z_{11}}$  and  $y_{11}$  represent the asymptotic limits. Also further analysis will show that  $Y_2+2Y_{23} - \frac{y_{12}}{g_{12}}$  always has a positive real part and therefore will not ever be zero.

\* An equivalent and alternate expression is

$$i_1 = \left\{ y_{11} + \left( \frac{1}{z_{11}} - y_{11} \right) \left( \frac{1}{1 - \frac{g_{12}}{y_{12}} \left( \frac{Y_2+2Y_{23}}{2} \right)} \right) \right\} e_1 \quad \text{A-17a}$$

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Case A - Secondary to Primary:  $i_1=0$ ,  $Y_3=Y_2$ ,  $Y_N \rightarrow \infty$

The purpose of this condition is to reemphasize a conclusion reached earlier [see (A-9)] that for any condition where  $e_2$  and  $e_3$  or their difference  $e_2-e_3$  is specified, then

$$e_1 = - \frac{y_{12}}{y_{11} + Y_1} (e_2 - e_3) \quad \text{A-18}$$

Because of various manners in which the secondary can be driven various special subcases will be considered.



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Case A - Secondary to Primary  $i_1=0$ ,  $i_2+i_3=0$ ,  $Y_3=Y_2$ ,  $Y_1 \rightarrow \infty$

This case corresponds to a "floating" drive with a voltage  $e_2-e_1$  or where a "balanced" drive with  $e_3 \triangleq -e_2$ . In the former case obviously  $i_2+i_3=0$ . In the latter case  $i_2+i_3=0$  because of the assumptions that  $Y_3=Y_2$  and the transformer is electrically symmetric. Then (as before)

$$e_1 = -\frac{y_{12}}{y_{11}+Y_1} (e_2-e_3) \quad A-19$$

$$\frac{i_2}{e_2-e_3} = \frac{i_2}{2e_2} = \underbrace{Y_{23} + \frac{Y_2}{2}}_{\text{secondary load}} + \underbrace{\frac{y_{22}-y_{23}}{2}}_{\text{leakage}} - \underbrace{\frac{y_{12}^2}{y_{11}+Y_1}}_{\text{reflected primary}} \quad A-20$$

Note that  $Y_{23} + \frac{Y_2}{2}$  just represents (with  $Y_3=Y_2$ ) the line to line admittance as expected. As before, the measured primary to secondary open circuit voltage transfer ratio

$$s_{12} = \frac{-2y_{12}}{y_{22}-y_{23}} \quad A-22$$

could be used to replace the difficult to measure  $y_{22}-y_{23}$ . Similarly if the line to line secondary to open circuit primary voltage transfer ratio is  $s_{21}$ , where analytically

$$s_{21} = \frac{-y_{12}}{y_{11}} \quad A-23$$

is measured then

$$e_1 = \frac{s_{21}}{1 + \frac{Y_1}{y_{11}}} (e_2-e_3) \quad A-24$$

and

$$\frac{i_2}{e_2-e_3} = Y_{23} + \frac{Y_2}{2} - \frac{y_{12}}{s_{12}} + \frac{y_{12}s_{21}}{1 + \frac{Y_1}{y_{11}}} \quad A-25$$

Other variations and measurements are possible. For example, if the following measurements were available

$$y_{sc} = \left. \frac{i_2}{e_2-e_3} \right|_{\substack{Y_1 \rightarrow \infty \\ Y_{23}=Y_2=0}} = \frac{y_{22}-y_{23}}{2} \quad (\text{leakage effect}) \quad A-26$$

$$y_{oc} = \left. \frac{i_2}{e_2-e_3} \right|_{\substack{Y_1=0 \\ Y_{23}=Y_2=0}} = \frac{y_{22}-y_{23}}{2} - \frac{y_{12}^2}{y_{11}} \quad A-27$$

they could be used to obtain the desirable form

$$\frac{i_2}{e_2-e_3} = Y_{23} + \frac{Y_2}{2} + y_{sc} + \frac{y_{oc}-y_{sc}}{1 + \frac{Y_1}{y_{11}}} \quad A-28$$

\* Nongrounded

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Obviously for this case,  $Y_{23} + \frac{Y_2}{2}$  is the line-to-line secondary load and has been included to show that it does not effect the secondary to primary voltage transfer and enters into  $\frac{I_2}{e_2 - e_3}$  only as a parallel load, all as expected. The set of measurements  $y_{11}, y_{12}, y_{21}, g_{23}$  seem appropriate for secondary to primary in this case of floating line to line drive. Note that these measurements are redundant (if  $y_{12}$  is measured) and are related by

$$y_{0c} = y_{3c} + g_{21}y_{12}$$

A-28a

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Case A - Secondary to Primary:  $i_1=0$ ,  $i_3=0$ ,  $Y_3 \neq Y_2$ ,  $Y_3 \rightarrow \infty$

This case corresponds to a "line to neutral" secondary drive with the other "line" free to take on a voltage determined by circuit considerations. It will be observed that for this case that the secondary to primary voltage transfer will be theoretically dependent on the connected secondary load ( $Y_3$ ,  $Y_1 \neq Y_2$ ,  $Y_{23}$ ). Note in order to show the dependance on the secondary loading  $Y_3$  has not been assumed equal to  $Y_2$ . Using equations A-2, A-4 ( $i_1=i_3=0$ ) obtain

$$e_1 = \frac{y_{12}(y_{23}+y_{22}+Y_3)}{y_{12}^2 - (y_{11}+Y_1)(y_{22}+Y_3+Y_{23})} e_2$$

$$= - \frac{y_{12}}{y_{11}+Y_1} \left( 1 - \frac{y_{12}^2 + (y_{11}+Y_1)(y_{23}-Y_{23})}{y_{12}^2 - (y_{11}+Y_1)(y_{22}+Y_3+Y_{23})} \right) e_2 \quad A-28$$

where the last form on the right is an algebraic rearrangement to show the difference from the line to line drive situation (see A-9 or A-18). For reference

$$e_3 = \frac{y_{12}^2 + (y_{11}+Y_1)(y_{23}-Y_{23})}{y_{12}^2 - (y_{11}+Y_1)(y_{22}+Y_3+Y_{23})} e_2$$

$$= \left( -1 + \frac{2y_{12}^2 - (y_{11}+Y_1)(y_{22}-y_{23}+Y_3+2Y_{23})}{-y_{12}^2 - (y_{11}+Y_1)(y_{22}+Y_3+Y_{23})} \right) e_2 \quad A-29$$

where the last form on the right shows a difference factor from the "ideal" transformer situation.\*

The driving point admittance  $\frac{i_2}{e_2}$  can be obtained using the above and equation A-3, i.e.

$$\frac{i_2}{e_2} = \frac{Y_2 + \frac{y_{12}^2 [2(y_{22}+y_{23})+Y_3] + [y_{11}+Y_1] [y_{23}^2 - y_{12}^2 - Y_{23} (2(y_{22}+y_{23})+Y_3) - Y_{23}y_{22}]}{y_{12}^2 - (y_{11}+Y_1)(y_{22}+Y_3+Y_{23})}}{\downarrow}}{\quad} \quad A-30$$

secondary load shunting the  $e_2$  voltage drive

It is of interest to observe:

- The voltage transfer  $\frac{e_1}{e_2}$  (see A-28) depends on the secondary loading  $Y_3$  and  $Y_{23}$
- If  $Y_3 \rightarrow \infty$ , i.e., shorting terminal 3 to ground yields (see A-28) the same voltage transfer ratio as A-18 or A-19. No attempt so far has been made to examine the right hand form of A-28 to examine how  $\frac{e_1}{e_2}$  varies with  $Y_3$  and  $Y_{23}$ .

It is of interest also to note the special cases for  $Y_2 = Y_3 = Y_{23} = 0$ , i.e. open secondary, i.e.,

$$e_1 = \frac{y_{12}(y_{23}+y_{22})}{y_{12}^2 - (y_{11}+Y_1)y_{22}} e_2 = - \frac{y_{12}}{y_{11}+Y_1} \left( 1 - \frac{y_{12}^2 + (y_{11}+Y_1)y_{23}}{y_{12}^2 - (y_{11}+Y_1)y_{22}} \right) e_2 \quad A-31$$

$$e_3 = \frac{y_{12}^2 + (y_{11}+Y_1)y_{23}}{y_{12}^2 - (y_{11}+Y_1)y_{22}} e_2 \quad Y_2 = Y_3 = Y_{23} = 0 \quad A-32$$

$$\frac{i_2}{e_2} = \frac{(y_{23}+y_{22}) [2y_{12}^2 + (y_{11}+Y_1)(y_{23}-y_{22})]}{y_{12}^2 - (y_{11}+Y_1)y_{22}} \quad A-33$$

Because of the dependency on the secondary load which, of course, is highly variable, no attempt is made here to suggest a parameter model.

\* Note  $y$  parameters cannot be used to represent an ideal transformer

**Case B - Phase to Neutral Primary Connection, Neutral Not at Zero Potential**

When the neutral is not at zero potential, i.e.,  $Y_N$  finite (see Figure 4), then certain modifications are required. In order to appreciate these modification requirements a short review of certain aspects of modeling is in order. First note that with a pole mounted distribution transformer it is obvious that the only current path to ground is via the intentional ground wire and associated ground rod. For vault mounted distribution transformers which obviously are in closer proximity of ground it is still reasonable to assume that any tank or conductor to ground stray capacity reactance is much greater than the intentional grounding impedance. The significance of these observations is that the whole transformer has a single known current path to ground (through a footing admittance  $Y_N$ ) and the effects of  $Y_N$  are easily accounted for using the procedures discussed in Appendix I of Volume 3 "Technical Appendices" Final Report. Next note that for the same reasons the operation of the transformer (voltage transfers, driving point admittances, etc.) is unaffected essentially\* by imperfect grounding.

For outbound feeder to feeder model computed propagation, the only effect of the distribution transformer is to load the feeder from phase to neutral (for grounded wye connection) or phase to phase (delta connected system and for some cases of grounded wye systems). Letting  $Y_T$  be the complex-scalar driving point admittance seen looking into the primary terminals of the distribution transformer the three phase admittance loadings are of the form

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_T & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_T & -Y_T \\ 0 & -Y_T & Y_T \end{bmatrix}$$

$Y_T$  phase 2 connected  
 $Y_N \rightarrow \infty$

$\Delta$  connected phase 2 + 3  
 $Y_N \rightarrow \infty$

for perfect grounding, i.e.,  $Y_N \rightarrow \infty$ , with imperfect grounding through a finite  $Y_N$  these admittances become

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & Y_T & 0 & -Y_T \\ 0 & 0 & 0 & 0 \\ 0 & -Y_T & 0 & Y_T + Y_N \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & Y_T & -Y_T & 0 \\ 0 & -Y_T & Y_T & 0 \\ 0 & 0 & 0 & Y_N \end{bmatrix}$$

$Y_T$  phase 2 connected  
 $Y_N \neq \infty$

$\Delta$  connected phase 2 + 3  
 $Y_N \neq \infty$

The above are single examples of many combinations that can occur and serve to illustrate that given  $Y_T$  which is unaffected by grounding considerations, the feeder loading admittance can be easily synthesized algorithmically for perfect or imperfect grounding.

\* Conceivably a ground loop could exist via the residential grounding. It is implicitly assumed that no voltage drop exists along the secondary neutral.

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**3. Analysis of Simple RLC (plus ideal transformers) Two Winding Transformer**

This section examines a simple RLC (plus ideal transformers) two winding model which takes into account stray capacity effects, as a single lumped capacitor across the primary terminals (a somewhat arbitrary choice). This is considered to be a "zeroth" order or very simplistic approach and is offered only to the extent that it may result in a model which can be used reasonably well to predict the response characteristics of two winding transformers at lower frequencies, say up to 50 kHz or possibly somewhat greater. A next higher order model might assume lumped capacitors between all external terminals and from all external terminals to the case and/or core. Obviously as frequency range is further extended the "finite element" representation would require further and further fineness to represent all inter and intra-distributed capacity effects.

The "zeroth" order approximation is being suggested here for consideration because based on driving point admittance measurement made by R. Rankin 5/8/80 on a 25 kVA distribution transformer, it appeared that such a model could be used to fit the measurements quite well. However, no attempt was made to see how well it would predict voltage transfer ratios and secondary to primary characteristics.

Also it should be noted that this model is only applicable to:

- a. electrically symmetric distribution transformers
- b. balanced secondary loading
- c. L-L secondary drive
- d. phase to neutral primary connection

The "zeroth" order lumped parameter model is shown on Figure 2-1 and is observed to be the well-known 60 Hertz model with the lumped capacitor across the primary terminals. The above conditions (a, b, c) effectively allow the actual three-winding distribution transformer to be considered a two-winding transformer (since for these conditions no current flows out/into the secondary mid-tap terminal). See "Magnetic Circuits and Transformers" (The Technology Press) for a good discussion of this model and especially leakage reactance.

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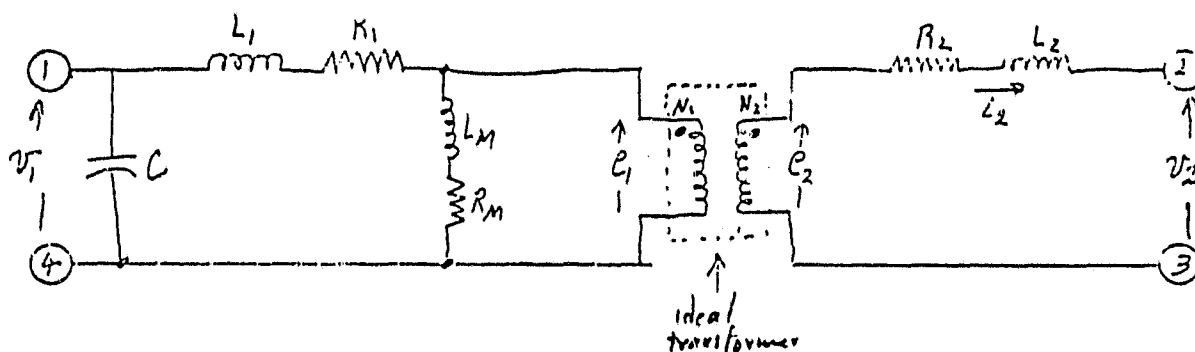


Figure 2-1. "Zeroeth" Order Approximation for Two-Winding Transformer, with Stray Capacity Iron Core

- $C$  = "zeroeth" approximation for stray capacity effects
- $L_1$  = primary leakage reactance effect
- $R_1$  = primary winding effective series resistance
- $L_2$  = secondary leakage reactance effect
- $R_2$  = secondary winding effective series resistance
- $L_m^*$  = equivalent series magnetizing inductance (can be frequency dependent)
- $R_m^*$  = equivalent series magnetizing loss (can be frequency dependent)
- $N_1, N_2$  = primary and secondary turns — ideal transformer
- $a = \frac{N_1}{N_2}$

\*The magnetizing shunt branch is usually, at lower frequencies, basically inductive with some dissipation. Its representation could be either a series  $L_m$  and  $R_m$  as shown or a parallel shunt arrangement. In either representation the lumped parameters  $L$  and  $R$  will not be exactly frequency independent. For the series representation  $R$  is generally small and for the shunt representation  $R$  is usually large.

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The two winding transformer will now be analyzed to derive various parameters identified in Part I. For convenience the analysis will be made in terms of notation of Figure 2-2 which has obvious relationship to Figure 2-1. For Primary to Secondary we have:

Open Circuit (Secondary) Primary to Secondary Voltage Transfer Ratio  $g_{12}$

$$\left. \frac{v_2}{v_1} \right|_{Z_L=0} = g_{12} = \frac{1}{a(1+Y_0Z_1)} \quad 3-1$$

Open Circuit (Secondary) Primary Driving Point Admittance  $\frac{1}{z_{11}}$

$$\frac{1}{z_{11}} = Y_c + \frac{Y_0}{1+Y_0Z_1} \quad 3-2$$

Short Circuit (Secondary) Primary Driving Point Admittance  $y_{11}$

$$y_{11} = Y_c + \frac{1+a^2Y_0Z_2}{Z_1+a^2Z_2(1+Y_0Z_1)} \quad 3-3$$

Primary to Secondary Voltage Transfer with Load

$$\begin{aligned} \frac{v_2}{v_1} &= \frac{a}{a^2(1+Y_0Z_1)+[Z_1+a^2Z_2(1+Y_0Z_1)]Y_L} \\ &= \frac{1}{a(1+Y_0Z_1)} \left[ \frac{1}{1+\left[\frac{Z_1+a^2(1+Y_0Z_1)Z_2}{a^2(1+Y_0Z_1)}\right]Y_L} \right] \end{aligned} \quad 3-4$$

Short Circuit (Secondary) Transadmittance  $y_{12}$

$$\left. -\frac{i_2}{v_1} \right|_{Z_L=\infty} = y_{12} = \frac{-a}{Z_1+a^2Z_2(1+Y_0Z_1)} \quad 3-5$$

Primary Driving Point Admittance  $\frac{i_1}{v_1}$  with Load

$$\frac{i_1}{v_1} = Y_c + \frac{a^2Y_0 + (1+a^2Y_0Z_2)Y_L}{a^2(1+Y_0Z_1)+[Z_1+a^2Z_2(1+Y_0Z_1)]Y_L} \quad 3-6$$

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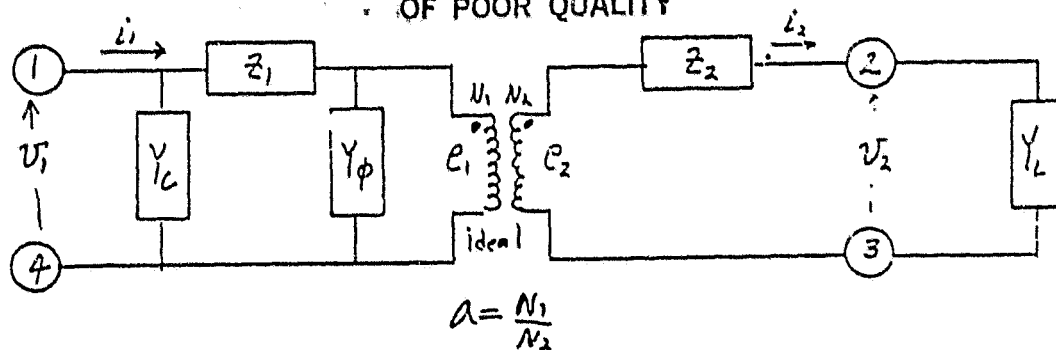


Figure 2-2. Two Winding Transformer Analysis Model

Now this expression for the primary driving point admittance  $\frac{i_1}{v_1}$  with load can be manipulated and expressed in terms of  $\frac{1}{z_{11}}$ ,  $y_{11}$ ,  $g_{12}$ ,  $y_{12}$  as given above to yield

$$\frac{i_1}{v_1} = y_{11} + \left( \frac{1}{z_{11}} - y_{11} \right) \left( \frac{1}{1 - \frac{g_{12}}{y_{12}} Y_L} \right) \quad 3-7$$

which should be compared to equation A-17a and where  $Y_L$  is obviously equivalent to  $Y_{23} + \frac{Y_2}{2}$  which is the line to line loading as assumed in Part A.

Next we will look at the primary to secondary voltage transfer ratio. Equation 3-4 can be manipulated to yield

$$\frac{v_2}{v_1} = \left[ \frac{1}{g_{12}} - \frac{Y_L}{y_{12}} \right]^{-1} \quad 3-8$$

which is exactly equivalent to equation A-15 (realizing that the result in equation A-15 needs to be multiplied by two to correspond to the line to line condition shown in 3-8).

The purpose of the above exercise in deriving equations 3-7 and 3-8 has been to demonstrate that indeed the equations derived in Section 1 under the assumption of balanced secondary loading and symmetric distribution transformer electrical characteristics do represent a two winding transformer. We will not make any attempt now to derive the secondary (line to line) to primary characteristics.

Reviewing frequency response data taken by J.J. LaForest it appears that the simple RLC+ideal transformer being considered here, could account for most of the observed behavior out to 80-100 kHz, i.e., all the "y" parameters seem to follow similar patterns. After 80-100 kHz a variety of "squirrely" behavior is noticed due to probably the distributed-higher order stray capacity effects and possibly instrumentation vagaries.

C. Conley and others have noted some additional resonance effects, usually small, in the 35-50 kHz region. Before adopting the simplified model suggested above, our transformer measurements should be designed and examined to see if other resonances occur.

Assuming the simplified RLC+ideal transformer model of Figure 2-1 is acceptable, then measurements can be designed to determine the appropriate lumped parameters. Generally it is not possible by measurements to determine separately  $L_1$  and  $L_2$ . However, for most transformer loading situations, either on the secondary for primary to secondary considerations or on the primary for secondary to primary considerations, the current through the magnetizing admittance  $Y_0$  (Figure 2-2) is small compared to the current through  $Z_1$ , and therefore the  $Z_1$  voltage drop is little affected by the current through  $Y_0$ . At very light loadings, say open circuit, the series impedance effect  $Z_1 + \frac{1}{Y_0}$  is dominated by  $\frac{1}{Y_0}$ .



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i.e.,  $Z_1 + \frac{1}{Y_0} \approx \frac{1}{Y_0}$  or  $1 + Y_0 Z_1 \approx 1$ . Net result of these considerations is that the following model is usually a reasonable approximation.

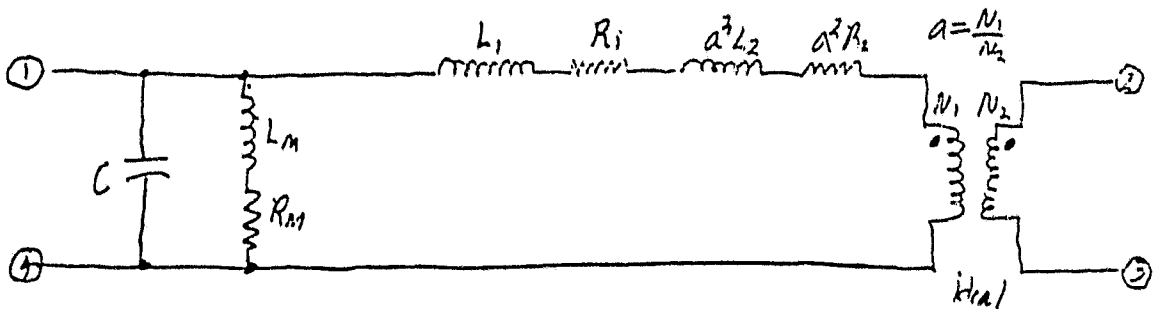


Figure 2-3

Based on the preceding discussion concerning the observation that  $|\frac{1}{Y_0}| \gg |Z_0|$ , i.e.,  $1 + Y_0 Z_1 \approx 1$ ,\* and examining equations 3-1 to 3-6 where the factor  $1 + Y_0 Z_1$  is seen to occur frequently, procedures for determining the lumped parameters  $C$ ,  $L_m$ ,  $R_i$ ,  $L_1 + a^2 L_2$ ,  $R_1$ ,  $R_2$ ,  $a$  can probably be developed with ingenuity. For example equation 3-1 indicates that the open circuit voltage transfer ratio should be reasonably constant with frequency and equal to  $a^{-1}$ . Equation 3-2 indicates, along with the model, that  $\frac{1}{z_{11}}$  should asymptotically increase 6 db/octave at high frequencies and could be used to determine  $C$  (see attached graph prepared by R.W. Rankin 5/8/80). Equation 3-5 or the model of figure indicates that  $y_{12}$  could be used to determine  $R_1 + a^2 R_2$  and  $L_1 + a^2 L_2$ .  $R_1$  and  $R_2$  can be frequently estimated from design standards or approximated by dc measurements. Similarly % leakage impedance (primarily reactance) data is available for all distribution transformers and can be correlated with measurements as appropriate. Other measurements such as  $y_{11}$  (equation 3-3) could be used to help deduce the magnetizing branch lumped parameters, either by a shunt or series representation whichever tends to lead to the least frequency sensitive components. Since  $L_m$  and  $C$  tend to resonate at very low frequencies, approximately 300 Hz, such a resonant frequency test could be used to deduce  $L_m$ .  $L_m$  is usually more difficult to determine and various schemes (involving connecting the primary and secondary windings in series bucking and series riding are used and any book of transformer measurements could be consulted). However, at PLC frequencies, i.e., greater than say 3 or 4 kHz, we could ignore  $Y_0$  since it will be dominated by  $Y_c$ .

If the simplified model of Figure 2-3 is shown to be satisfactory, then an extremely simple model determination may be possible which will be good enough (since we never know exactly what the secondary loading is in practice, we probably cannot justify a high degree of perfection in the model anyway). The procedure would be to:

1. Determine the effective capacity  $C$  using the asymptotic behavior of  $\frac{1}{z_{11}}$
2. Use nameplate available leakage impedance (reactance) specification to determine  $L_1 + a^2 L_2$
3. Use known design standards, to estimate  $R_1$  and  $R_2$
4. Ignore  $Y_0$

The adequacy of this approach should be given serious consideration providing our contractual upper frequency limit is reduced to 50 to 100 kHz.

\* For PLC frequencies, i.e.,

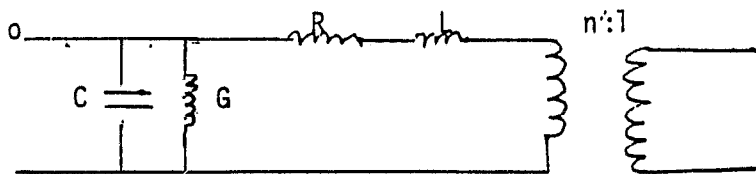
$$Y_0 Z_1 = \frac{R_1 + j\omega L_1}{R_m + j\omega L_m}$$

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**APPENDIX 4**  
**PROGRAM FOR COMPUTING AND PLOTTING**  
**RLC TRANSFORMER MODEL PREDICTED RESPONSES**

**R.C. Rustay and R.C. Wentz**

This Appendix describes a software program called ZPTRANS1 which given the RLC parameters, computes, and plots the predicted bidirectional driving point admittance and voltage transfer ratio associated with the following transformer model.



This model is used to predict distribution transformer (DT) responses with the assumption that the DT is electrically symmetric and has a balanced secondary load, (see Appendix 3) and where

- C = Lumped Stray Capacitance (Including Bushing Capacity)
  - G = Core Conductance
  - R = Winding Resistance
  - L = Winding (Leakage Inductance
  - n = Voltage Ratio (essentially turns ratio for iron core)
- } Referred to primary side

The model includes a frequency effect for the actual value of R.

The attachment included with this Appendix is a verbatim copy of an internal memo written by R. Wentz further describing this program, its operation, and output.



DIFNAP SYSTEMS MEMO #52

3/3/81

Memo To: C. A. Stutt  
R. Wooding  
J. Gajjar  
R. Rankin  
R. Rustay  
R. Wentz  
A. Dunham

From: R. C. Wentz

Subject: New Program for Analysis of Two Winding Distribution Transformers

Please be advised that there is a new program in the catalog that is available for use in the analysis of two winding distribution transformers. Output of the program includes a plot file which may be plotted on the Zeta plotter in Building #37. The source code of the program is contained in a file named ZPTRANS1.

The program utilizes a lumped parameter model to calculate four characteristics of the transformer, each over a range of frequencies from 2 kHz to 100 kHz. The four characteristics are:

- 1) Voltage transfer ratio (in DB)
- 2) Phase angle of the voltage transfer ratio (in degrees)
- 3) Input admittance (in DB)
- 4) Phase angle of the input admittance (in degrees)

The plot file produced is composed of two separate graphs. The first is a plot of the voltage transfer ratio and its phase angle. The second is a plot of the input admittance and its phase angle. Also included in the output of ZPTRANS1 is an output file which contains the numerical values of the four plotted characteristics, the frequencies at which calculations were made, and the input parameters that produced those results.

During operation of ZPTRANS1, the user is requested to specify the following input parameters:

- 1) Mode of Operation
  - 1 = Primary to Secondary
  - 2 = Secondary to Primary
- 2) Load Characteristics - Used only for Mode 1 calculations. The user specifies the values of load resistance and load inductance. For Mode 2 calculations there is no consideration of loads "downstream" of the transformer.
- 3) Feeder Characteristics - Used only for Mode 2 calculations. The user specifies the values of a shunt feeder capacitance and a shunt feeder conductance. For Mode 1 calculations, there is no consideration of feeders "upstream" of the transformer.
- 4) Base Frequency - Since the winding resistance has been determined to be dependent on frequency, a base value must be known so that internal logic will produce the proper equation for frequency dependency. See me for details concerning this feature.
- 5) Winding Characteristics - The user specifies values for the winding resistance and winding inductance, both referred to secondary.
- 6) Turns ratio of the ideal transformer.
- 7) Stray Capacitance - A shunt capacitance on the primary side of the model to account for stray capacitive effects.
- 8) Core Characteristics - The user specifies values for the core inductance and core conductance.
- 9) Output File - The user can specify the name for the output file or request output at the terminal, in which case no output file is written.
- 10) Ordinate Range - The user can specify separate ranges for the ordinates of each of the four characteristics being plotted.

Plots produced by ZPTRANS1 are intended to be on the exact same scale as those produced in the lab. This is to facilitate comparison of model predictions with results of laboratory measurements.

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Attached is an example of a typical terminal execution of ZPTRANS1, a sample page of the output file, and reduced copies of the two graphs produced by the Zeta plotter.

If you have any questions concerning the operation of ZPTRANS1, please do not hesitate to see me.

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03/03/81 10.611

THIS PROGRAM USES THE LUMPED PARAMETER MODEL  
TO ANALYZE DISTRIBUTION TRANSFORMERS

MODE 1 CALCULATES PRIMARY ---> SECONDARY  
MODE 2 CALCULATES SECONDARY --> PRIMARY

ENTER 1 FOR MODE 1 OR 2 FOR MODE 2

ENTER A VALUE OF 0. FOR ELEMENTS YOU WISH TO IGNORE

FEEDER CAPACITANCE = 0. MICROFARADS  
FEEDER CONDUCTANCE = 0.100000 MHOS  
ENTER CR IF OKAY, ELSE ENTER 1  
=

BASE FREQUENCY FOR FREQUENCY DEPENDANT  
WINDING RESISTANCE = 20000.000000 HERTZ  
ENTER CR IF OKAY, ELSE ENTER 1  
=

WINDING RESISTANCE = 0.140000 OHMS REFERRED TO  
WINDING INDUCTANCE = 0.036000 MILLIHENRYS SECONDARY  
ENTER CR IF OKAY, ELSE ENTER 1  
=

URNS RATIO OF IDEAL TRANSFORMER = 31.300000  
STRAY CAPACITANCE = 0.001000 MICROFARADS  
ENTER CR IF OKAY, ELSE ENTER 1

ENTER TURNS RATIO

ENTER STRAY CAPACITANCE (IN MICROFARADS)

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TURNS RATIO OF IDEAL TRANSFORMER = 31.750000  
STRAY CAPACITANCE = 0. MICROFARADS  
ENTER CR IF OKAY, ELSE ENTER 1  
=

CORE INDUCTANCE = 0. MILLIHENRYS  
CORE CONDUCTANCE = 0. MHOS  
ENTER CR IF OKAY, ELSE ENTER 1  
=

OUTPUT FILE = DTOUT1 ;  
ENTER CR IF OKAY, ELSE ENTER 1  
=

DO YOU WANT OUTPUT AT THE TERMINAL ?  
ENTER CR IF OKAY, ELSE ENTER 1  
=

ENTER NAME OF OUTPUT FILE  
=

OUTPUT FILE = DTOUT2 ;  
ENTER CR IF OKAY, ELSE ENTER 1  
=

OPENED FILE = DTOUT2 ;

RANGE OF VTR MAGNITUDE = -50.0 TO 20.0 DB  
RANGE OF VTR PHASE ANGLES = -180.0 TO 180.0 DEGREES  
RANGE OF YINPUT MAGNITUDE = -50.0 TO 20.0 DB  
RANGE OF YINPUT PHASE = -180.0 TO 180.0 DEGREES  
ENTER CR IF OKAY, ELSE ENTER 1  
=

WROTE OUTPUT FILE = DTOUT2 ;  
PLOTTING VTR VS. FREQUENCY  
PLOTTING VTR PHASE VS. FREQUENCY  
PLOTTING YINPUT VS. FREQUENCY  
PLOTTING YINPUT PHASE VS. FREQUENCY  
WROTE PLOT FILE = RPLOTOO1 ;

DO YOU WISH TO ANALYZE ANOTHER TRANSFORMER ?  
ENTER CR IF OKAY, ELSE ENTER 1  
=



\*\*\*\*\*

DTOUT1

03/03/81 8:42 AM

\*\*\*\*\*

MODE OF OPERATION = 2

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BASE FREQUENCY FOR FREQUENCY DEPENDANT  
WINDING RESISTANCE = 20000.000000 HERTZ

FEEDER CAPACITANCE = 0. MICROFARADS  
FEEDER CONDUCTANCE = 0.100000 MHOS

WINDING RESISTANCE = 0.140000 OHMS REFERRED TO  
WINDING INDUCTANCE = 0.086000 MILLIHENRYS SECONDARY

TURNS RATIO OF IDEAL TRANSFORMER = 31.750000  
STRAY CAPACITANCE = 0.001000 MICROFARADS

CORE INDUCTANCE = 0. MILLIHENRYS  
CORE CONDUCTANCE = 0. MHOS

FREQ. (HZ)	VTR (IN DB)	VTR PHASE	YINPUT (DB)	YINPUT PHASE
2000.0	-1.071821E 01	-8.737249E 01	-6.833358E-01	-8.736528E 01
2500.0	-1.265706E 01	-8.728262E 01	-2.622187E 00	-8.727362E 01
3000.0	-1.424145E 01	-8.718020E 01	-4.206578E 00	-8.716940E 01
3500.0	-1.558121E 01	-8.707449E 01	-5.546335E 00	-8.706189E 01
4000.0	-1.674190E 01	-8.696929E 01	-6.707022E 00	-8.695489E 01
4500.0	-1.776581E 01	-8.686627E 01	-7.730932E 00	-8.685007E 01
5000.0	-1.868182E 01	-8.676610E 01	-8.646947E 00	-8.674810E 01
5500.0	-1.951054E 01	-8.666902E 01	-9.475665E 00	-8.664922E 01
6000.0	-2.026717E 01	-8.657506E 01	-1.023230E 01	-8.655346E 01
6500.0	-2.096327E 01	-8.648412E 01	-1.092840E 01	-8.646072E 01
7000.0	-2.160782E 01	-8.639608E 01	-1.157295E 01	-8.637088E 01
7500.0	-2.220793E 01	-8.631078E 01	-1.217306E 01	-8.628378E 01
8000.0	-2.276935E 01	-8.622806E 01	-1.273447E 01	-8.619926E 01
8500.0	-2.329676E 01	-8.614776E 01	-1.326189E 01	-8.611716E 01
9000.0	-2.379407E 01	-8.606974E 01	-1.375919E 01	-8.603733E 01
9500.0	-2.426451E 01	-8.599385E 01	-1.422964E 01	-8.595965E 01
10000.0	-2.471086E 01	-8.591997E 01	-1.467598E 01	-8.588397E 01
10500.0	-2.513546E 01	-8.584799E 01	-1.510058E 01	-8.581019E 01
11000.0	-2.554033E 01	-8.577778E 01	-1.550545E 01	-8.573818E 01
11500.0	-2.592723E 01	-8.570925E 01	-1.589236E 01	-8.566785E 01
12000.0	-2.629770E 01	-8.564231E 01	-1.626282E 01	-8.559911E 01
12500.0	-2.665306E 01	-8.557687E 01	-1.661819E 01	-8.553187E 01
13000.0	-2.699452E 01	-8.551285E 01	-1.695964E 01	-8.546605E 01

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INPUT ADMITTANCE VS. FREQUENCY

CASENO= 002

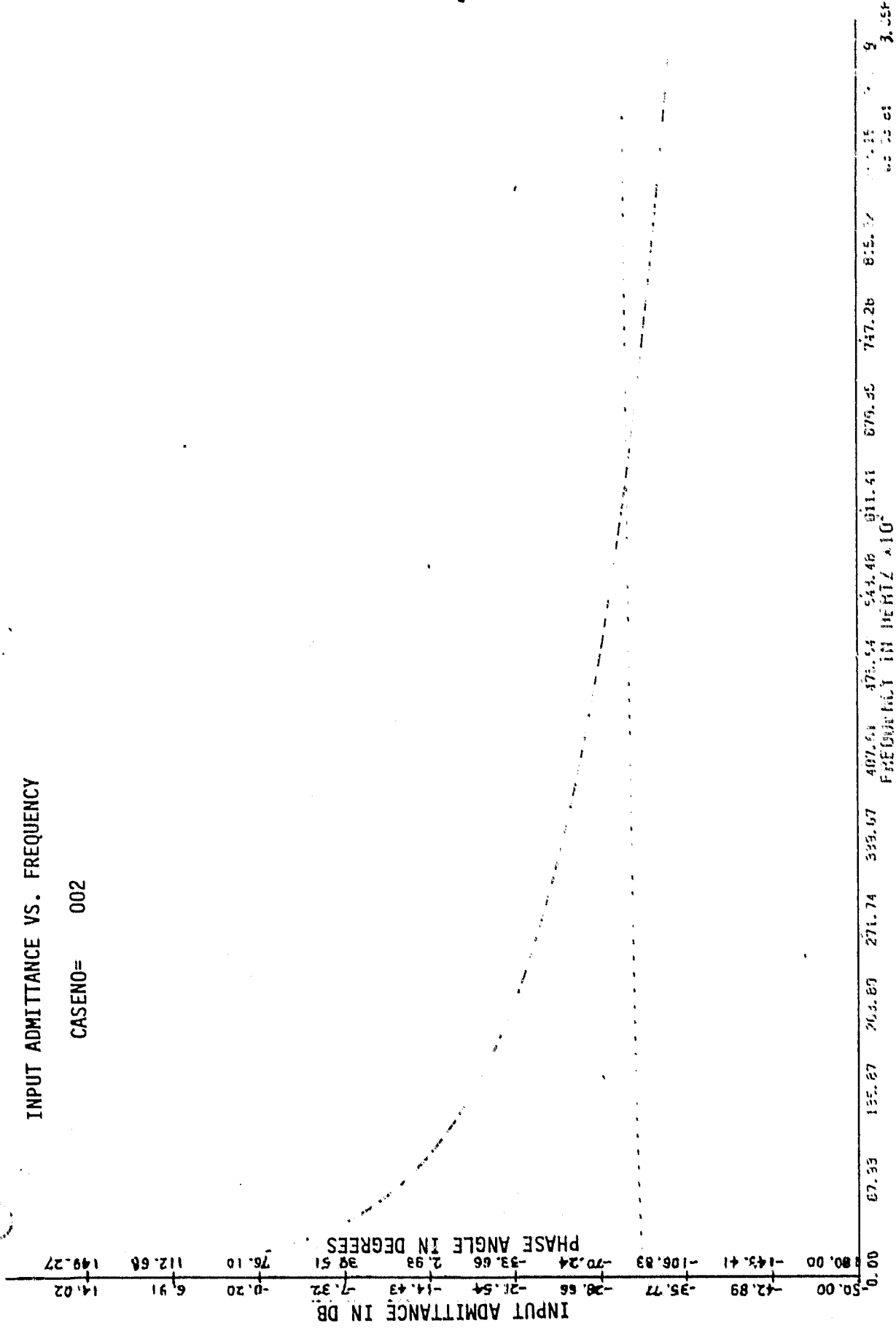
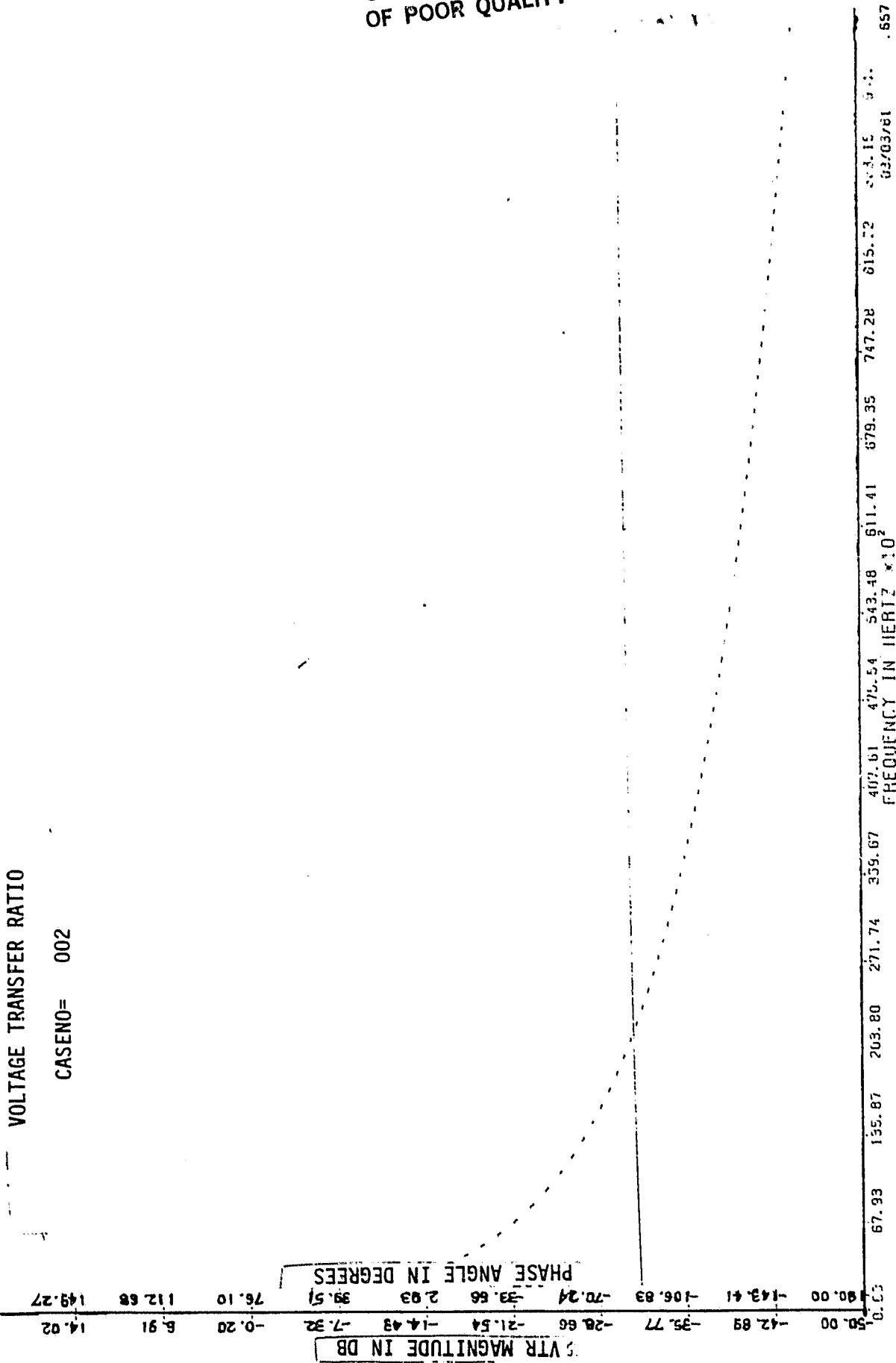


Figure (4-2)

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VOLTAGE TRANSFER RATIO

CASENO= 002



9

**APPENDIX 5**  
**MATRIX BASED GENERALIZED NEUTRAL REDUCTION PROGRAM**

R.C. Rustay

Attached, as part of this Appendix, is a verbatim copy of an internal memo concerned with a generalized procedure to coalesce several electrically paralleled neutrals into one equivalent neutral so as to be compatible with a program FEEDPUSj. The requirement for this procedure arises usually from the analysis of multiphase cables as explained in the attached memo.

## Appendix U

### Matrix Based Generalized Neutral Reduction Program

The purpose of this Appendix is derive a general (matrix) based algorithm for the neutral reduction procedure required for the  $m \times m$ , the  $Z$  and  $Y$  matrices encountered with underground cables. The  $Z$  and  $Y$  matrices are defined by the steady state coupled ordinary differential equations

$$\frac{d\bar{V}}{dX} = - Z \bar{I} \quad \text{U-1}$$

$$\frac{d\bar{I}}{dX} = - Y \bar{V} \quad \text{U-2}$$

The neutral reduction procedure occurs because usually  $Z$  and  $Y$  are of a dimension,\* say  $m$ , which is at least twice as large as the number of phases,  $n$ . This occurs because the cable parameter programs consider also, as part of the dimension  $m$ , the cable sheaths and any miscellaneous additional conductors which may be geometrically paralleling the cables, i.e., such as bare copper neutral wires. The purpose of the neutral reduction program is to obtain "reduced" matrices  $Z_R$  and  $Y_R$ , each of dimension  $n+1$ , by invoking the assumption that at every point position  $X$ , all of the  $n+1$  to  $m$  conductors (the conductor indexing is ordered so that the first  $n$  are associated with the phase conductors) are at equal potential  $V_c$ , i.e.,

$$\dot{V}_{n+1} = \dot{V}_{n+2} = \dots = \dot{V}_m = \dot{V}_e \quad \text{U-3}$$

and that  $i_e$  the equivalent current flowing through the equivalent (parallel) conductor is given by

$$i_e = i_{n+1} + i_{n+2} + \dots + i_m \quad \text{U-4}$$

Thus given the representation of U-1 and U-2, i.e.,

$$\begin{bmatrix} \dot{V}_u \\ \dot{V}_L \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_U \\ I_L \end{bmatrix} \quad \text{U-5}$$

$$\begin{bmatrix} \dot{I}_U \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_U \\ V_L \end{bmatrix} \quad \text{U-6}$$

we seek reduced equations of the form

$$\begin{bmatrix} \dot{V}_U \\ \dot{V}_e \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12e} \\ Z_e & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_U \\ i_e \end{bmatrix} \quad \text{U-7}$$

$$\begin{bmatrix} \dot{I}_U \\ \dot{i}_e \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12e} \\ Y_e & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_U \\ v_e \end{bmatrix} \quad \text{U-8}$$

where  $Z_{12e}$  and  $Y_{12e}$  are  $n \times 1$ ,  $Z_e$  and  $Y_e$  are  $1 \times n$ , and  $Z_{22}$  and  $Y_{22}$  are (complex) scalars.

\* Implied in this Appendix is the fact that  $Z$ ,  $Y$ ,  $Z_R$ ,  $Y_R$  are square

### Reduction of $Y$

Equation U-2 can be partitioned to

$$\begin{bmatrix} i_U \\ i_L \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_U \\ V_L \end{bmatrix} \quad \text{U-9}$$

or

$$i_U = Y_{11} V_U + Y_{12} V_L \quad \text{U-10}$$

$$i_L = Y_{12}^T V_U + Y_{22} V_L \quad \text{U-11}$$

Introducing the  $m-n \times 1$  "summing" vector-matrix  $S$

$$S^T = [1, 1, \dots, 1] \quad \text{U-12}$$

then from U-4

$$i_e = S^T i_L = S^T Y_{12}^T V_U + S^T Y_{22} V_L \quad \text{U-13}$$

Now since all the components in  $V_L$  are equal to  $v_e$ , see U-3,

$$V_L = S v_e \quad \text{U-14}$$

Thus U-10 and U-13 become

$$i_U = Y_{11} V_U + Y_{12} S v_e \quad \text{U-15}$$

$$i_e = S^T Y_{12}^T V_U + S^T Y_{22} S v_e \quad \text{U-16}$$

Equations U-15 and U-16 can be combined to form the desired reduced  $Y$  matrix, i.e.,

$$\begin{bmatrix} i_U \\ i_e \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} S \\ S^T Y_{12}^T & S^T Y_{22} S \end{bmatrix} \times \begin{bmatrix} V_U \\ v_e \end{bmatrix} \quad \text{U-17}$$

reduced  $Y$  matrix

It is clear that

$$S^T Y_{12}^T \text{ is } 1 \times m-n$$

$$Y_{12} S \text{ is } m-n \times 1 \text{ (transpose of above)}$$

$$S^T Y_{22} S \text{ is } 1 \times 1 \text{ scalar and simply sum of } Y_{22} \text{ elements}$$

Computationally the elements of  $Y_{12} S$  could be formed by the individual row sums of  $Y_{12}$ , and  $S^T Y_{22} S$  is simply the sum of all the  $Y_{22}$  elements, a scalar. Note that this algorithm holds for the case where  $n=1$ ,  $m=2$  (corresponding to a single phase cable).

### Reduction of $Z$

Equation U-1 can be partitioned to

$$\begin{bmatrix} \dot{V}_U \\ \dot{V}_L \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix} \times \begin{bmatrix} I_U \\ I_L \end{bmatrix} \quad \text{U-18}$$

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or

$$\dot{V}_L = Z_{11} I_L + Z_{12} I_u \quad \text{U-19}$$

$$\dot{V}_L = Z_{12}^T I_u + Z_{22} I_L \quad \text{U-20}$$

Inverting U-20 obtain

$$I_L = Z_{22}^{-1} \dot{V}_L - Z_{22}^{-1} Z_{12}^T I_u \quad \text{U-21}$$

Utilizing the "summing" vector-matrix of equation U-12 and proceeding as before (see U-13 and U-14) obtain

$$i_e = S^T Z_{22}^{-1} S \dot{v}_e - S^T Z_{22}^{-1} Z_{12}^T I_u \quad \text{U-22}$$

Solving for  $\dot{v}_e$ , noting  $S^T Z_{22}^{-1} S$  is a scalar, obtain

$$\dot{v}_e = \frac{S^T Z_{22}^{-1} Z_{12}^T}{S^T Z_{22}^{-1} S} I_u + \frac{1}{S^T Z_{22}^{-1} S} i_e \quad \text{U-23}$$

Next substitute U-21 into U-19 and obtain (using U-14)

$$\dot{V}_L = (Z_{11} - Z_{12} Z_{22}^{-1} Z_{12}^T) I_u + Z_{12} Z_{22}^{-1} S \dot{v}_e \quad \text{U-24}$$

Substituting U-23 into U-24 obtain

$$\dot{V}_L = \left( Z_{11} - Z_{12} Z_{22}^{-1} Z_{12}^T + \frac{Z_{12} Z_{22}^{-1} S S^T Z_{22}^{-1} Z_{12}^T}{S^T Z_{22}^{-1} S} \right) I_u + \frac{Z_{12} Z_{22}^{-1} S}{S^T Z_{22}^{-1} S} i_e \quad \text{U-25}$$

Now note that  $Z_{12} Z_{22}^{-1} S$  is a single column vector-matrix and that  $S^T Z_{22}^{-1} Z_{12}^T = (Z_{12} Z_{22}^{-1} S)^T$ , since  $Z_{22}^{-1}$  is symmetric, and suggests a computational approach. Also it is observed that digital word precision could be a significant concern since an exact algebraic approach would reveal many cancelling terms in the matrix coefficient of  $I_u$  in U-25. Equation U-23 and U-25 can be combined to form the desired reduced Z matrix, i.e.,

$$\begin{bmatrix} \dot{V}_u \\ \dot{V}_e \end{bmatrix} = \underbrace{\begin{bmatrix} \left( Z_{11} - Z_{12} Z_{22}^{-1} Z_{12}^T + \frac{Z_{12} Z_{22}^{-1} S S^T Z_{22}^{-1} Z_{12}^T}{S^T Z_{22}^{-1} S} \right) & \frac{Z_{12} Z_{22}^{-1} S}{S^T Z_{22}^{-1} S} \\ \frac{S^T Z_{22}^{-1} Z_{12}^T}{S^T Z_{22}^{-1} S} & \frac{1}{S^T Z_{22}^{-1} S} \end{bmatrix}}_{\text{reduced Z matrix}} \times \begin{bmatrix} I_u \\ i_e \end{bmatrix} \quad \text{U-26}$$

Note that for the special case where  $n=1$ ,  $m=2$  (single cable situation), U-26 reduces to U-18 as it should. Also as before  $S^T Z_{22}^{-1} S$  is simply the sum of the elements of  $Z_{22}^{-1}$ .

A rearrangement of equation U-25 using the identity

$$Z_{12} Z_{22}^{-1} Z_{12} \triangleq \frac{Z_{12} Z_{22}^{-1} S^T Z_{22}^{-1} S Z_{22} Z_{22}^{-1} Z_{12}}{S^T Z_{22}^{-1} S} \quad \text{U-27}$$

yields the alternate form

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$$\begin{bmatrix} \dot{V}_u \\ \dot{V}_e \end{bmatrix} = \begin{bmatrix} \left( Z_{11} - \frac{Z_{12}Z_{22}^{-1} [S^T Z_{22}^{-1} S Z_{22} - S S^T] Z_{22}^{-1} Z_{12}}{S^T Z_{22}^{-1} S} \right) & \frac{Z_{12}Z_{22}^{-1} S}{S^T Z_{22}^{-1} S} \\ \frac{S^T Z_{22}^{-1} Z_{12}^T}{S^T Z_{22}^{-1} S} & \frac{1}{S^T Z_{22}^{-1} S} \end{bmatrix} \times \begin{bmatrix} I_u \\ i_e \end{bmatrix} \quad \text{U-28}$$

where it is noted that  $SS^T$  is square matrix of dimension  $m-n$  with unity for every element. This upper left  $n \times n$  submatrix in U-28 may be computationally more accurate than in U-26. (Remember  $S^T Z_{22}^{-1} S$  is a scalar.)

Finally the comment is offered that the utilization of the  $m-n \times 1$  vector-matrix in the above analytical derivation does not imply that it needs to be used in the computational algorithm since by itself operating on a matrix represents, as appropriate summing the elements in each row or column.