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The Performance of VLA as a Telemetry Receiver for Voyager Planetary Encounters

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The Very Large Array (VLA) has been proposed for use as a supplement to the Deep Space Network (DSN) for telemetry reception at Voyager 2 Uranus and Neptune encounters. The main problem with the use of VLA for telemetry is that it is not capable of producing a continuous stream of data. Gaps of one millisecond follow every 51 milliseconds of data. This report investigates the effect of these millisecond gaps on coded telemetry. An ungapped system of the same aperture as the VLA would be capable of handling data rates of 38.4 kbps at Uranus encounter and 19.2 kbps at Neptune encounter. It is shown here that VLA with (7, 1/2) convolutional coding (the baseline coding scheme for Voyager) will support a data rate of 10.8 kbps but not 19.2 kbps at both Uranus and Neptune. It is also shown that by implementing Voyager's concatenated Reed-Solomon/convolutional coding capability, data rates of 38.4 kbps and 19.2 kbps would be achievable at Uranus and Neptune respectively. Concatenation also offers a factor of 2 improvement in overall throughput.

I. Introduction

As Voyager 2 journeys toward Uranus and Neptune, a need for an increased telemetry receiving capability is arising. One way of increasing this capability is to increase the effective aperture of the DSN receiving sites. More antennas could be built or existing ones enlarged for use in antenna arrays. These efforts are expensive and time consuming. Even so, some additions and enlargements are planned before the 1986 encounter at Uranus (Ref. 1). Another way to increase telemetry reception capability is to use non-DSN facilities. These sites could be used independently or in arrays with DSN stations.

One of the prime candidate non-DSN facilities that is being considered for Voyager encounter enhancement is the Very Large Array (VLA) (Ref. 2). The VLA is a phased array consisting of 27 independently pointable parabolic dishes in the New Mexico desert. Each dish measures 25 meters in diameter. The array is configured in three linear arrays of nine antennas each that radiate from a common center point. The antennas may be moved along these lines by means of railroad-type tracks. Communication between the antennas and the central processing site is by means of a connecting network of waveguides. The total aperture of the VLA is about twice that of a DSN array consisting of a 64-m antenna and three 34-m antennas. The VLA was designed for use as a radio astronomy observatory. Some additional front-end hardware will be needed in order for the VLA to receive the X-band Voyager telemetry transmissions.

The main problem with using the VLA as a telemetry receiver is that signals are not transmitted continuously from the antennas to the processing facility. Instead, the VLA has a duty cycle which consists of 51 ms of data gathering followed by a 1-ms calibration and control period. During these



1-ms intervals, which will be called "gaps," no data is gathered.

If uncoded telemetry data were received by the VLA, then, for data rates over a few thousand bits per second, the best possible bit error rate would be $(1/2)(1/52) = 0.95 \times$ 10^{-2} . Coding must be used in order to achieve bit error rates below the 5 \times 10⁻³ level required for good image quality. In this report it is shown that the use of the Viterbi-decoded (7, 1/2) convolutional code would allow the VLA to receive at a data rate of 10.8 kbps at both Uranus and Neptune encounters. It would not, however, support a data rate of 19.2 kbps at either encounter. It is further shown that the data rate can be improved by concatenating the (255, 233) Reed-Solomon code with the (7, 1/2) convolutional code. Such coding would allow the VLA to support a data rate of 38.4 kbps at Uranus and 19.2 kbps at Neptune. The reason for the improvement is that the Reed-Solomon code tends to correct the burst errors caused by the gaps. In terms of throughput (bits per day received at an acceptable average bit error rate), concatenated coding used in conjunction with source data compression provides for about a 100% increase over convolutional coding alone.

II. Mathematical Model of VLA Coded Performance

The performance of convolutional codes has always been difficult to model analytically. This is mainly due to the complex nature of the Viterbi decoding process. It is well known that when a Viterbi decoder makes a decoding error due to excessive channel noise, it has a tendency to produce a burst of decoded bit errors (Ref. 3). This is due to the fact that the Viterbi decoder is a multistate machine with a trellis structure that interconnects the states (Ref. 4). Each state corresponds to a possible sequence of decoded bits. A decoding error occurs when the decoder is in a state that corresponds to a sequence other than the one that was transmitted. Even if the channel bit signal-energy-to-noise-spectral-density ratio (E_b/N_0) is infinite from that time forward, it will still take the decoder several bit times, on average, to wander through the trellis from state to state until it returns to the correct one. During this period of wandering, a burst of errors occurs.

In the case that E_b/N_0 is infinite except during the gaps, an anlysis can be performed. Suppose a convolutional code with constraint length k is used. Prior to encountering a gap, the Viterbi decoder always decodes correctly. During the gap, only noise is input to the decoder. Since received channel symbols are determined uniquely by the current data bit and the k - 1 previous data bits, no information is known about the bits in the gap with the exception of the final k - 1bits. The decoder can correct these bits. This reasoning leads to the following formula for the performance of Viterbi decoding in the presence of a periodically gapped channel and in the limit of high channel SNR.

$$P_{b} = 0.5 \left(R_{b} T_{2} - k + 1 \right) / \left[R_{b} \left(T_{1} + T_{2} \right) \right]$$
(1)

In (1), R_b is data rate in bits per second, T_1 is the time in seconds spent in that portion of the cycle representing a perfect channel, and T_2 is the time in seconds spent in each gap. It is assumed that R_b is large enough so that the gaps are at least k bits long. A graph of P_b as a function of R_b for $T_1 = 51$ ms, $T_2 = 1$ ms, and k = 7 is shown in Fig. 1. This represents the best possible performance of the VLA with Viterbi-decoded (7, 1/2) convolutional coding. It is important to notice that even in the case of infinite channel SNR and with no other system losses, the VLA can support a data rate of at most 13.0 kbps at a bit error rate of $P_b = 5 \times 10^{-3}$.

For now, the only practical way to determine the performance of the VLA at finite channel SNRs and arrayed with other sites is through the use of simulations. The software simulations that were used in this study are described in the next section.

III. Software Simulations of VLA Performance

The starting point for the simulation software for this investigation was the software Viterbi decoder that was used in the studies of Viterbi decoder burst statistics and concatenated code performance (Refs. 3, 5, 6). This decoder is a modified version of a program written by J.W. Layland. The inputs to the program are the parameters of the convolutional code to be simulated, the channel E_b/N_0 , and the length of the simulation in terms of the number of bits to be processed. Any convolutional code of constraint length up to 12 and rate 1/2, 1/3, or 1/4 may be simulated. The software decoder assumes a 4-bit quantized input and has a path memory of 64 bits. It should be noted that the performance advantage of 4-bit quantization over the 3-bit quantization used by the DSN is negligible (Ref. 7). The outputs of the simulation include the Viterbi-decoded bit error probability, the error rate for 8-bit Reed-Solomon symbols (see Section IV), and various statistics concerning burst errors. A 90% confidence interval for the output error rates is also calculated. It should be noted that the software Viterbi decoder always maintains perfect node synchronization (for a discussion of node synchronization, Ref. 8). Also, no system losses other than the Gaussian noise of the channel are simulated.

For the purposes of this study, the software Viterbi decoder was modified so that the E_b/N_0 of the channel could be changed periodically in order to simulate the gaps in VLA data reception. For example, simulation of the VLA system at a data rate of 19.2 kbsp was achieved by alternating E_b/N_0 between one value for 979 bits and -9.99 dB for 19 bits. The output of the software decoder at the latter SNR is random to within the accuracy of the simulation.

Four scenarios were simulated in this fashion. For each of these, simulations were performed at three data rates that are typical of those that might be implemented for Voyager in conjunction with VLA telemetry reception (10.8, 19.2, and 38.4 kbps). These scenarios are as follows:

- Ideal performance. In this scenario, it was assumed that no gaps were present in the data. This represents a best-possible case and it can be used to measure the magnitude of losses that occur in the other scenarios.
- (2) Normal VLA. This scenario represents the VLA as described in the introduction. The gaps are assumed to be periods of time where the channel E_b/N_0 is arbitrarily small. As was remarked above, an SNR of -9.99 dB was actually used for the gaps.
- (3) VLA arrayed with an equal aperture. The gaps in this scenario are 3.0 dB lower in E_b/N_0 than the rest of the data. Since there is no complex anywhere near the VLA of a comparable aperture, this scenario is best interpreted as arraying half of the VLA with the Goldstone DSN complex. The arraying is assumed to be perfect combined carrier referencing (CCR). Two different methods for achieving near ideal CCR performance have been described in Refs. 9 and 10.
- (4) Rotated gaps. In this scenario, the VLA is assumed to be reconfigured so that the three arms receive the 1-ms calibration signal at three different, equally spaced times. Each arm still has the same 51-ms/1-ms duty cycle. This is equivalent to a system with 1-ms periods of 1.76-dB signal attenuation every 17-1/3 ms.

For the purposes of displaying the results of the simulations, E_b/N_0 , refers to the bit SNR of the channel during the 51-ms periods of good reception whenever it appears in the figures.

Each scenario was simulated over a range of SNRs from 0.0 to 4.0 dB in 0.25-dB increments. At each SNR, one million bits of data were simulated for each data rate. The results of the simulations are shown graphically in Figs. 2 through 5. The asymptotes that appear in Fig. 3 agree with the performance that was predicted by Eq. (1) for high SNRs. Also, the burst statistics produced by these simulations corroborates the theory outlined in Section II. Notice that scenarios 3 and 4 both allow the VLA to support data rates higher than 10.8 kbps. In fact, arbitrarily high data rates can be supported provided the channel SNR is high enough. This is in contrast to scenario 2.

IV. Concatenated Coding

It was shown in section III that even at bit SNRs as high as 4 dB, the VLA as presently configured will not support a data rate of 19.2 kbps with only Viterbi-decoded (7, 1/2) convolutional coding. One way to improve this performance is to use Voyager's capability for a concatenated coded downlink. The Voyager project has the option of adding a (255, 223) Reed-Solomon code to the telemetry downlink coding system. This code would be the outer code, with the (7, 1/2) convolutional code being the inner code. The advantage to this concatenated scheme comes from the fact that the Reed-Solomon code has the ability to correct many errors provided that they occur in bursts. It was already remarked in section II that the errors produced by Viterbi decoding tend to be burst errors. Also, a gap may be considered to be a burst error.

A Reed-Solomon code is, in general, a nonbinary code. The role of the bit is played by the Reed-Solomon symbol. The (255, 223) code can correct up to 16 symbol errors in each 255-symbol codeword. Since each symbol is 8 bits long, a symbol error could represent a burst of errors from the inner convolutional code. If one assumes that symbol errors occur independently, then the bit error probability of the concatenated channel, without gaps, is given by (Refs. 5, 11),

$$P_{cb} = \frac{p}{\pi} \sum_{i=17}^{255} \left(\frac{i}{255}\right) {\binom{255}{i}} \pi^i (1-\pi)^{255\cdot i}$$
(2)

where p is the bit error rate of the Viterbi decoder, and π is the average error rate for a Reed-Solomon symbol. Since both p and π are calculated in the software simulations described in section III, P_{cb} may be calculated in this case of independent symbol errors.

The Reed-Solomon symbols produced by Voyager will be interleaved to a depth of four before being convolutionally encoded, modulated, and transmitted. This means that since Viterbi decoder burst errors of 32 bits or more are unlikely (Ref. 5), the symbol errors produced by the Viterbi decoder will be very nearly independent. The effect of the gaps is to introduce a certain number of symbol errors in known positions in the data stream. For all the data rates that were investigated, these errors are uniformly distributed over the four levels of interleaving. This means that no Reed-Solomon codeword will receive an inordinate number of symbol errors caused by gaps. Each Reed-Solomon codeword would be corrupted by no more than 6 symbol errors from the gaps. Since the Reed-Solomon code can correct this number of symbol errors, the asymptotic behavior of scenario 2 would be corrected by using concatenated coding. It should also be noted that a further, as yet undetermined, performance gain can be achieved by flagging the symbols that occur during gaps as erasures. The (255, 223) Reed-Solomon code can correct *E* errors and *e* erasures as long as $2E + e \leq 32$. Such an erasure handling decoder would have to be built to take advantage of this gain.

Graphs of P_{cb} for the four scenarios of section III are shown in Figs. 6 through 9. These were graphed using Eq. (2) and so, by the above remarks, should be viewed as an upper bound on the actual expected error rate, with the exception of the ideal reception case (scenario 1).

Some additional facts concerning concatenated coding should be mentioned at this point. The motivation for using concatenated coding in a mission like Voyager is to be able to achieve very low overall bit error rates at a lower SNR than convolutional coding alone. This is important if one is to implement source data compression. In source data compression, the number of bits needed to represent a Voyager image at Uranus and Neptune encounters will be reduced, through source coding techniques, by a factor of 2.5 (Ref. 12). Since the bits to be transmitted are less redundant, a compressed image is more vulnerable to channel bit errors than an uncompressed image. Concatenated coding allows source data compression to work by providing the low bit error rates required. A bit error rate of 10^{-5} is considered necessary for data compression. In comparing the performance of the VLA with and without Reed-Solomon coding, one must take into account the 255/223 overhead of the code, the factor of 2.5 in data throughput, and the lower bit error rate required for concatenated coding. This will be explained further in the following section.

V. VLA Throughput at Voyager Encounters

The most reasonable measure of performance for a telemetry receiving system such as the VLA is the amount of data throughput that it is capable of handling. The particular measure of throughput used in this study is the number of "good bits per day." During each day of an encounter, Voyager will be "visible" from the VLA for some number of hours. Depending on the data rate and coding that are implemented, only a fraction of this viewing time may support reception at the required bit error rate or better. This is due to the fact that received power is a function of the elevation angle of the antennas. The raw data used in this section comes from design control tables for Voyager 2 Uranus encounter. The curve shown in Fig. 10 represents the total-power-to-noise-spectral-density ratio (P_T/N_0) that is expected to be incident on a threeelement array at the Goldstone complex on day 34 of 1986 (Uranus encounter). This curve includes a 90% weather confidence level and the effects of antenna elevation angle. Since the VLA is at approximately the same latitude as Goldstone, the values of P_T/N_0 for the VLA at Uranus encounter were derived from these simply by scaling them up by the ratio of the apertures involved. For Neptune, 3.5 dB was subtracted to account for the additional space loss. It was assumed that the maximum viewing time is 8.3 hours.

For each data rate and scenario described in sections III and IV, the P_T/N_0 required for a bit error rate of 5×10^{-3} (for convolutional coding only) or 1×10^{-5} (for concatenated coding) was calculated from the corresponding E_b/N_0 given by Figs. 2 through 9. For convolutional coding,

$$P_T^{-}/N_0 = (E_b/N_0) \times R_b/\sin^2 \theta$$
 (3)

and for concatenated coding,

$$P_T / N_0 = (223/255) (E_b / N_0) \times R_b / \sin^2 \theta.$$
 (4)

The data rate R_b is the number of bits per second processed by the Viterbi decoder – hence the need for the factor 223/ 255 in Eq. (4). Modulation indices of $\theta = 76^\circ$ for Uranus and $\theta = 70^\circ$ for Neptune were assumed for this study. These insure that the carrier loop SNRs are at least 15 dB (see section VI). The length of time that the VLA can receive at the required bit error rate for each scenario was then calculated by observing how long the P_T/N_0 incident on the VLA is at least as high as that given by (3) or (4). Call this time T_0 . In the case of convolutional coding alone, the throughput of the system in terms of the number of good bits per day is given by

$$B = T_0 R_b.$$
(5)

In the case of concatenated coding, the throughput of the system is given by

$$B = 2.5 \times (223/255) \times T_0 R_b \tag{6}$$

where the factor of 223/255 is due to the Reed-Solomon coding overhead, the factor of 2.5 is due to the source data compression, and R_b is, again, the data rate of the inner convolutional channel.

Figures 11 and 12 show the expected number of good bits per day for the VLA at Uranus and Neptune encounters respectively. The four scenarios of section III are represented with scenario 2 (normal VLA) shown for 1/2 VLA aperture as well as full aperture. These histograms show clearly that the VLA, as presently configured, can support a convolutionalonly channel at a rate of 10.8 kbps but not 19.2 kbps. Concatenated coding allows the VLA to be used at rates of 38.4 kbps at Uranus encounter and 19.2 kbps at Neptune encounter. Also, concatenated throughput is about twice that of the convolutional-only channel. Performance can also be improved by arraying the VLA with Goldstone or rotating the gaps.

VI. Other Degradations

Up to this point, the only degradations that were modeled or simulated were the space loss, the thermal noise in the receivers, and the gaps. Other sources of degradation may include radio loss (noisy carrier reference), imperfect subcarrier tracking, imperfect symbol tracking, Viterbi decoder node synchronization losses, and Reed-Solomon frame synchronization losses.

The carrier margin, or the SNR in the design point bandwidth of the carrier tracking loop, is given by

$$m = (P_T/N_0) \cos^2 \theta/2 B_{L0}$$

where B_{L0} is the two-sided threshold carrier loop bandwidth. For Block IV receivers, a typical value would be $B_{L0} = 15$ Hz. Average values of P_T/N_0 for the VLA at Uranus and Neptune encounters will be, according to section V, 47.5 and 44.0 dB respectively. For the assumed modulation indices of 76° and 70°, these lead to carrier margins of 20.4 and 19.9 dB. The loop SNR in the receiver, which is a function of the carrier margin (Ref. 13), will hence be 15.9 dB at Uranus and 15.5 dB at Neptune. These are high enough so that the carrier tracking loops will acquire on their own and cycle slip losses will be negligible. However, some radio loss will be evident.

The effects of a noisy carrier reference on the performance of the VLA were computed using the modified high rate model in Ref. 8. Figure 13 is an example of the results. It represents the normal VLA scenario at 10.8 kbps and various loop SNRs. It can be seen that the radio loss to be expected at the Voyager encounters is about 0.1 dB. Preliminary results on modeling the concatenated link with noisy carrier referencing show that the radio loss should be less than for convolutional coding alone.

The radio losses above were computed assuming a Viterbi decoder node synchronization threshold of 0.0 dB. They will

be about twice as large for a threshold of 2.0 dB. It is expected, however, that the threshold can be reduced in time for the Voyager encounters.

It was assumed, for this study, that carrier synchronization is maintained through the gaps. It was also assumed that the Subcarrier Demodulation Assembly (SDA) and the Symbol Synchronizer Assembly (SSA) perform perfectly. In general, losses occurring in the SDA and SSA are minimal compared to the radio loss. In the case of gapped reception, however, there could be additional losses due to timing inaccuracies.

One other "loss" that should be mentioned is that the DSN Viterbi decoder (MCD) performs about 0.2 dB worse than the software decoder used in this study, according to the DSN/ Flight Project Interface Design Book (JPL Document 810-5). The origin of this discrepancy is not understood by the author. As remarked in section III, there is no significant difference in performance between the 3-bit quantization used in the hardware and the 4-bit in the software. A mission planner may choose to include this 0.2 dB loss in making predictions.

VII. Conclusions

The results of this study show that the VLA can be considered a viable telemetry receiving station for Voyager 2 Uranus and Neptune encounters. The problem of the 1-ms gaps can be solved in one of three ways. First, the VLA can be arrayed with a second site (such as the Goldstone complex) that does not exhibit this gapped behavior. Second, the VLA could be reconfigured so that the gaps occur at different times in each of the three arms. Third, concatenated coding may be implemented. These improvements each offer the capability of supporting data rates of 38.4 kbps at Uranus encounter and 19.2 kbps at Neptune encounter. The VLA, as presently configured and with convolutional coding alone, can support 10.8 kbps at each encounter but even with infinite SNR it can support only 13.0 kbps.

Concatenated coding is particularly attractive since, in addition to enabling the VLA to support higher data rates, it also offers a factor of 2 improvement in data throughput over arraying or reconfiguring. This is due to the fact that concatenated coding provides bit-error rates low enough to support the use of source data compression.

It is suggested that, before the VLA is actually used for this purpose, hardware simulations be performed in a facility such as the Telecommunications Development Laboratory (TDL) of JPL. The TDL channel simulator can be modified to mimic the gapped behavior of the VLA. In this way, the effect of gapped reception on the various DSN subsystems can be directly measured.

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Fig. 1. Theoretical performance of VLA with Viterbi-decoded (7, 1/2) convolutional code at infinitive SNR



Fig. 2. Simulated performance of an ungapped receiving system, convolutional (7, 1/2) coding only



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Fig. 3. Simulated performance of VLA with 1-ms gaps with no signal, convolutional (7, 1/2) coding only



Fig. 4. Simulated performance of VLA with 1-ms gaps attenuated by 3-dB, convolutional (7, 1/2) coding only

Fig. 5. Simulated performance of VLA with rotated gaps, convolutional (7, 1/2) coding only

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Fig. 6. Simulated performance of an ungapped receiving system, concatenated coding

Fig. 7. Simulated performance of VLA with 1-ms gaps with no signal, concatenated coding



Uranus Encounter; array = 64 m + 34 m + 34 m; day 24, 1986, 90% weather

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Fig. 11. Throughput of VLA at Voyager 2 Uranus Encounter



Fig. 12. Throughput of VLA at Voyager 2 Neptune Encounter

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Fig. 13. The effects of noisy carrier referencing on the VLA, carrier loop SNR = ρ

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