TDA Progress Report 42-71

July – September 1982

A Compact Presentation of DSN Array Telemetry Performance

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The telemetry performance of an arrayed receiver system, including radio losses, is often given by a family of curves giving bit error rate vs bit SNR, with tracking loop SNR at one receiver held constant along each curve. This study shows how to process this information into a more compact, useful format in which the minimal total signal power and optimal carrier suppression, for a given fixed bit error rate, are plotted vs data rate. Examples for baseband-only combining are given. When appropriate dimensionless variables are used for plotting, receiver arrays with different numbers of antennas and different threshold tracking loop bandwidths look much alike, and a universal curve for optimal carrier suppression emerges.

I. Introduction

In an internal memorandum, J. W. Layland wrote:

The task of comparing the performance of the Arrayed Deep Space Network with and without combined carrier references (CCR) seems to be needlessly complex. While the analysis machinery which has been built for us is capable of precisely describing the link performance under a plethora of conditions, interpretation of those results in terms useful for deciding whether to deploy the CCR or not is difficult, and potentially even ambiguous. There are too many "free" variables which seem to be available — even though physical constraints will interrelate many of them. Our task is somewhat analogous to driving a car with a separate steering wheel for each front wheel.

For example, the telemetry performance of a DSN receiver array can be represented by a family of "waterfall" curves giving bit error rate P_E as a function of array total E_b/N_0, the

sum of the bit signal-to-noise ratios at all the array elements, with ρ_{L1}, the tracking loop SNR at the strongest array element, held constant (Refs. 1 and 2 and Fig. 1). We need such a plot (a family of six curves in this instance) for each channel code, each DSN antenna array, each signal combining method, and each choice of threshold tracking loop bandwidth $w_{I0} = 2b_{L0}$. (It is assumed that all receivers in the array use the same w_{L0}.)

This presentation is more convenient for the person generating the plots than for the person who has to interpret them. To reduce the volume of results, one can settle on a fixed telemetry performance, $P_E = 0.005$, for example, and use each waterfall curve set to generate one "radio loss" curve:

$$ E_b/N_0 - (E_b/N_0)_\infty \text{ vs } \rho_{L1} , $$

where E_b/N_0 is the array total bit SNR needed to attain P_E for the given ρ_{L1}, and $(E_b/N_0)_\infty$ is the SNR needed to attain P_E for $\rho_{L1} = +\infty$, i.e., perfect phase tracking (Ref. 1).

In the memorandum quoted above, Layland suggested another presentation format, in which one plots total power threshold as a function of data rate, again for a fixed P_E. Further, for each data rate, one uses the optimal modulation angle θ, i.e, the θ that requires the least total power. This eliminates θ as a free parameter and tells how to design the signal.

In the present paper, this idea is applied to a set of results for baseband-only combining. As Layland suggests, we hold to a bit error rate of 0.005. It turns out that a judicious choice of dimensionless variables allows us to use a "universal" curve for the optimal θ (or carrier suppression $\cos^2\theta$) as a function of data rate. Further, when regarded from this point of view, all the arrays look the same within 0.5 dB. In other words, we have a nearly universal curve for total power-to-noise ratio vs data rate. The differences among the arrays can then be seen by plotting E_b/N_0 vs data rate.

II. A Sample of Results

The source data come from internal memoranda of D. Hansen and D. Divsalar, and from unpublished results of L. Deutsch. Let us list the conditions:

(1) (7, 1/2) convolutional code, maximum likelihood decoding.

(2) One-way radio losses only.

(3) High-rate bit error probability model.

(4) Baseband-only combining with weights optimal for perfect phase tracking.

(5) w_{L0} = 10 Hz or 30 Hz, the same for all receivers in the array.

(6) Antenna array combinations (diameters in meters): 64, 64-34, 64-34-34, 64-34-34-34. We assume that gain-to-noise ratio is proportional to area.

For each array and w_{L0} there is a set of six waterfall curves giving P_E vs E_b/N_0 for ρ_{L1} = 10, 11, 12, 13.5, 15, and 40 dB. Figure 1 is the plot for the four-element array and w_{L0} = 10 Hz.

Setting a telemetry performance threshold of P_E = 0.005, we compute optimal modulation angle and minimal total power-to-noise ratio. Results are plotted in Figs. 2-4.

The dimensionless variables used for plotting are listed below. In the plots, the term X (dB) always means $10 \log_{10} X$.

(1) R_d/w_{L0} = normalized data rate, where R_d = data rate.

(2) $\cos^2\theta$ = carrier suppression.

(3) $P_T/(N_0 w_{L0})$ = normalized array total power-to-noise ratio, where P_T/N_0 is the sum of the P_T/N_0 values of the receivers in the array (P_T = carrier power + data power, N_0 = 1-sided noise spectral density).

(4) E_b/N_0 = array total bit SNR, the sum of the E_b/N_0 values of the receivers in the array (E_b = data energy per bit).

Figure 2 is a "universal" curve giving optimal carrier suppression vs *normalized* data rate. The modulation angle is truncated at 80 deg. Actually, the several arrays yield distinct curves, which all agree within about 1 dB. Since the optimization problem is smooth, a deviation from optimal carrier suppression causes only a second-order increase in the total power needed to maintain the required performance. This is why one curve will do for all the array-w_{L0} combinations.

Figure 3 shows the array total P_T/N_0 needed to achieve P_E = 0.005. Total power has been minimized by the proper choice of carrier suppression (Fig. 2). Although this curve is again billed as universal, it is actually about 0.5 dB thick in the vertical direction. Thus it is intended only as a long-range snapshot. We shall soon tell how to compute values with greater accuracy.

For each array, Fig. 4 shows the array total E_b/N_0 needed to achieve P_E = 0.005. Since E_b/N_0 varies much less than P_T/N_0, we can now easily distinguish the different arrays. Still, the curves all fit within a 0.5-dB band. Having these E_b/N_0 curves, one can compute P_T/N_0 more accurately than before by the formula

$$\frac{P_T}{N_0 w_{L0}} = \frac{1}{1 - \cos^2\theta} \frac{E_b}{N_0} \frac{R_d}{w_{L0}} \tag{1}$$

where $\cos^2\theta$ is read from Fig. 2 and E_b/N_0 from Fig. 4.

III. Remarks and Cautions

These results, though incomplete and inaccurate, are adequate for a pilot study. The E_b/N_0 curves are probably good within ±0.1 dB. The author's source data consist of meager sets of numbers read from photocopied computer-generated graphs. Although both the Hansen-Divsalar and the Deutsch graphs were processed, we show only the Hansen-Divsalar results because Deutsch's basic Viterbi decoding error curve (used in the high-rate model) is slightly different, and his graphs are harder to read accurately. Thus, for each array there

is only one w_{L0}. The Deutsch results indicate that the E_b/N_0 curves for w_{L0} = 10 Hz lie perhaps 0.1 dB *above* the corresponding curves for w_{L0} = 30 Hz. This, of course, does not mean that narrowing the loop bandwidth makes telemetry worse; recall that data rate is normalized by w_{L0}. Thus for a given array, if we were to plot the 10 Hz and 30 Hz E_b/N_0 values against unnormalized data rate, the 10-Hz curve would lie to the left of the 30-Hz curve, and therefore below it.

Notice, too, that the E_b/N_0 curve for any multiple-antenna array lies *above* the curve for one antenna. This is because we are, in effect, comparing the array to a single large fictitious antenna with the same total area. The array is worse because the tracking loop of the fictitious receiver gets more carrier power than any of the loops of the array receivers. To compare a 64-m station to the 4-element array on the basis of E_b/N_0 at the 64-m station, one can simply move the 4-element curve down by $1 + 3(34)^2/64^2 = 2.66$ dB.

A particular point on any of these curves is valid only if ρ_{L1}, the loop SNR at the strongest array element, is at least 10 dB. The left-hand endpoint of each E_b/N_0 curve shows the R_d/w_{L0} value beyond which this condition fails. For the array

in question, the "universal" carrier suppression and total power curves must not be used to the left of this value.

IV. Conclusions

Layland's suggestion for presenting array telemetry performance appears to work well when applied to a set of results for baseband-only combining. Each set of waterfall curves is distilled into one curve of E_b/N_0 vs normalized data rate R_d/w_{L0}, with P_E kept constant and modulation angle optimized. By plotting array total E_b/N_0 and using w_{L0} as a normalizing factor, we absorb the gross differences among the different antenna arrays and receiver bandwidths, thus leaving a set of curves that differ by at most 0.5 dB. Further, there emerge approximate universal curves for optimal carrier suppression and minimal array total $P_T/(N_0 w_{L0})$ vs R_d/w_{L0}.

Although this work is merely a small pilot study, the method may be useful for comparing a variety of situations, including various carrier combining methods and channel codes. If this is the case, then the method could also be used for handbook specification of the arrayed DSN. The carrier suppression curve (Fig. 2) may be convenient for mission design.

References

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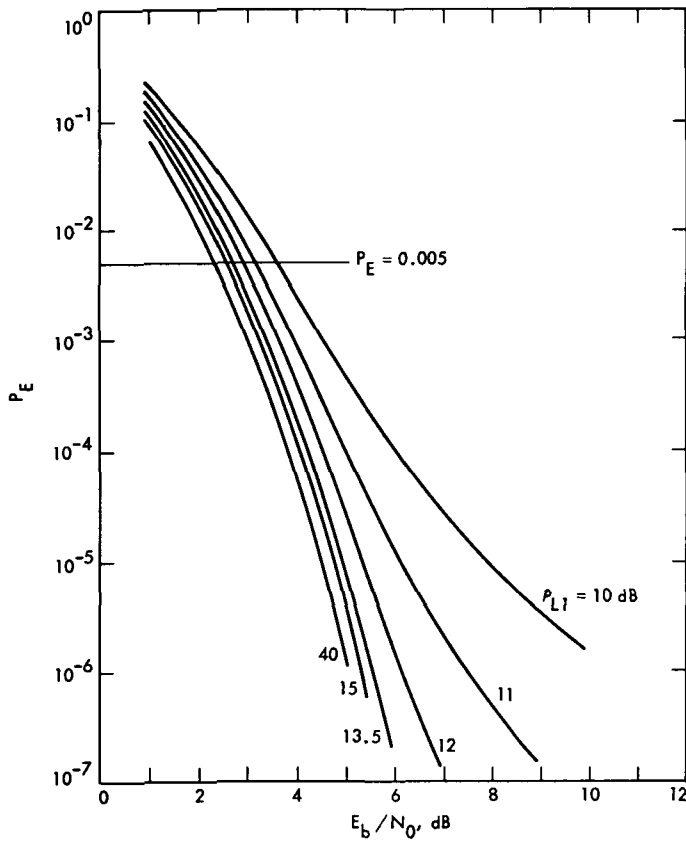
Fig. 1. A family of waterfall curves giving bit error rate P_E vs array total E_b/N_0, with ρ_{L1} = tracking loop SNR at the largest antenna. Conditions: (7, 1/2) code, one-way radio losses, baseband-only combining, 10-Hz tracking loops, four-element array: 64–34–34–34 m (after Hansen)
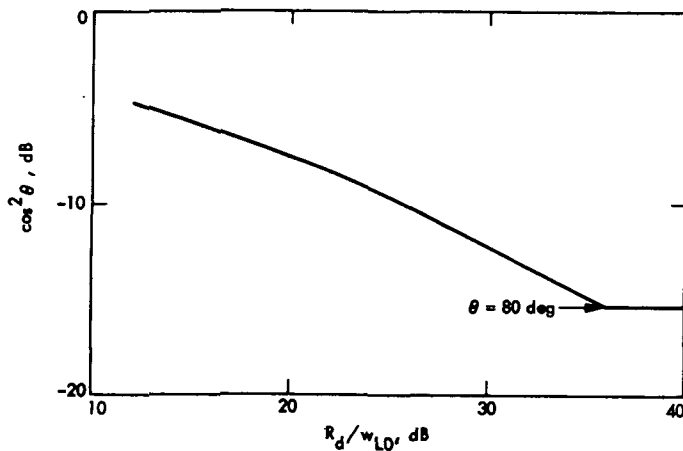

Fig. 2. An approximate universal curve for optimal carrier suppression vs data rate normalized by threshold loop bandwidth. Conditions: (7, 1/2) code, one-way radio losses, one to four DSN stations, baseband-only combining, P_E = 0.005. The modulation angle θ is truncated at 80 deg
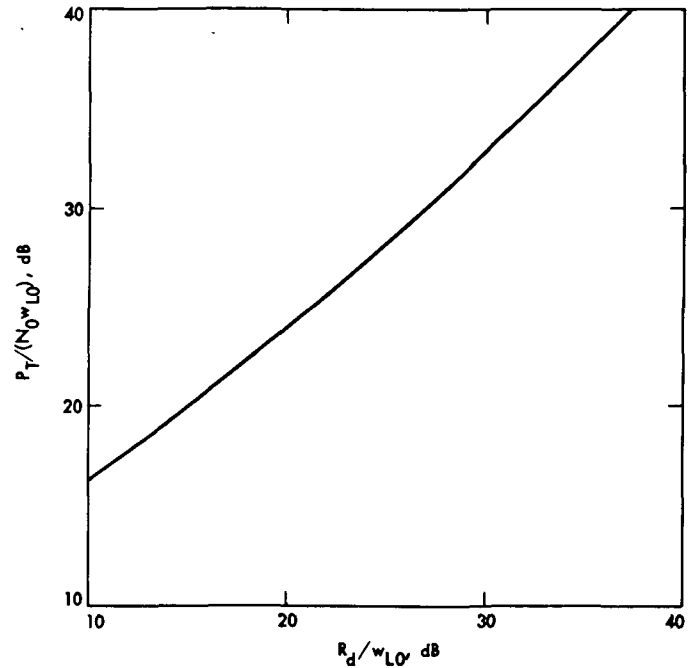

Fig. 3. An approximate universal curve giving normalized array total power-to-noise ratio vs normalized data rate, for an error rate P_E = 0.005. Carrier suppressions from Fig. 2 are used. This curve actually consists of several curves differing by at most 0.5 dB. Conditions: same as Fig. 2
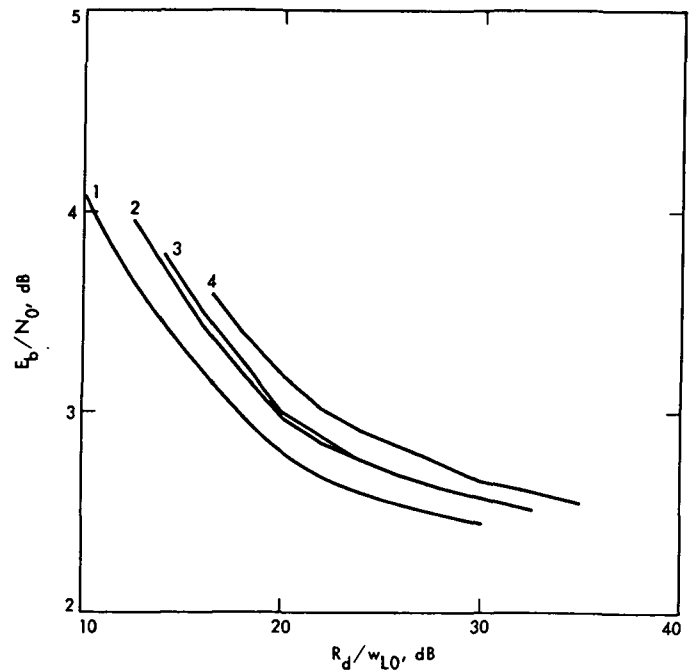

Fig. 4. Array total E_b/N_0 needed to achieve an error rate P_E = 0.005, with carrier suppression from Fig. 2. Arrays: (1) One antenna, w_{L0} = 30 Hz; (2) 64 m – 34 m, 10 Hz; (3) 64–34–34 m, 10 Hz; (4) 64–34–34–34 m, 10 Hz. Conditions: same as Fig. 2

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Appendix

Processing Details

I. Tracking Loop Formulas

We are given a set of waterfall curves, giving P_E vs array total E_b/N_0 with ρ_{L1} fixed at 10, 11, 12, 13.5, 15, and 40 dB. Since loop SNR ρ_L is a nonlinear function of carrier margin m, a more well-behaved quantity, we first convert the ρ_L values to the equivalent m values. Recall that the DSN definition of m is

$$m = \frac{P_c}{N_0 w_{L0}}$$

where P_c = carrier power.

For the reader's convenience, here is a set of formulas relating ρ_L to m for the DSN carrier tracking loops.

w_H = 2-sided IF predetection bandwidth

$$\rho_{H0} = \frac{2w_{L0}}{w_H} \tag{A-1}$$

= threshold predetection SNR

Define

$$\phi(x) = e^{-x}[I_0(x) + I_1(x)] \tag{A-2}$$

Then

$$\alpha_0 = \frac{1}{2}\sqrt{\pi\rho_{H0}}\,\phi\left(\frac{1}{2}\rho_{H0}\right) \tag{A-3}$$

= threshold limiter suppression factor

$$\rho_H = m\rho_{H0} \tag{A-4}$$

= predetection SNR

$$\alpha = \frac{1}{2}\sqrt{\pi\rho_H}\,\phi\left(\frac{1}{2}\rho_H\right) \tag{A-5}$$

= limiter suppression factor

$$r = 2\alpha/\alpha_0 \tag{A-6}$$

$$w_L = \frac{r+1}{3}w_{L0} \tag{A-7}$$

= two-sided loop noise bandwidth

$$\Gamma = \frac{1 + \rho_H}{0.862 + \rho_H} \tag{A-8}$$

= limiter performance factor

This is a rough approximation due to Tausworthe (Ref. 3). For more accuracy, one should use a formula of Springett and Simon (Ref. 4):

$$\Gamma = \frac{1 - \exp(-\rho_H)}{\alpha^2(1 + 0.0975 \exp(-0.2146\,\rho_H))} \tag{A-9}$$

Then we have

$$\rho_L = \frac{2m\,w_{L0}}{w_L\Gamma} = m\,\frac{6}{(r+1)\,\Gamma} \tag{A-10}$$

This gives loop SNR in the form $\rho_L = m\psi(m;\rho_{H0})$. To obtain m from ρ_L, one can iterate the formula $m = \rho_L/\psi(m;\rho_{H0})$ from some initial guess for m. The iteration, which is convergent because ψ is a slowly varying function, can be accelerated by Steffensen's method (Ref. 5).

II. Optimal Carrier Suppression

Having converted each ρ_{L1} to m_1 = carrier margin at the strongest array element, we multiply m_1 by array total antenna area/area of largest antenna to obtain total carrier margin m. We draw a horizontal line across the set of waterfall curves at the desired P_E (5×10^{-3} in this study). The intersections of this line with the waterfall curves give several points $[m_{dB}, (E_b/N_0)_{dB}]$, through which we interpolate a smooth function

$$(E_b/N_0)_{dB} = g(m_{dB}) \tag{A-11}$$

by means of cubic splines or a similar method. (The author used a local cubic method of Butland and Brodlie, Ref. 6.) Then we can also write

$$\frac{E_b}{N_0} = f(m) = 10^{g(m_{dB}/10)} \tag{A-12}$$

Define the dimensionless variables

$$p = \frac{P_T}{N_0 w_{LO}}$$

$$r = R_d / w_{LO}$$

where P_T / N_0 is the array total. Then we have the fundamental equation

$$p = m + rf(m) \qquad \text{(A-13)}$$

This just says total power = carrier power + data power, and data power is set for the desired P_E. Now, given r (i.e., data rate), we ask for the minimal p (i.e., total power) that gives the desired P_E. To get it we just set $dp/dm = 0$:

$$0 = 1 + rf'(m)$$

$$f'(m) = -1/r \qquad \text{(A-14)}$$

If we solve Eq. (A-14) for m, then we can get p from Eq. (A-13). The optimal carrier suppression is m/p.

There are two tricks for simplifying this procedure. First, there is no need to solve Eq. (A-14) for m in terms of r. Indeed, if we use m as independent variable instead of r, then Eqs. (A-14) and (A-13) give r, p, and m/p as functions of m; we then plot p, E_b/N_0, and m/p vs r. If the modulation angle θ ($m/p = \cos^2 \theta$) comes out greater than 80 deg, then set

$$p = m \sec^2 (80 \text{ deg})$$

and solve for r from Eq. (A-13).

The second trick comes from using Eq. (A-11) directly. By Eq. (A-12),

$$g'(m_{dB}) = mf'(m)/f(m) \qquad \text{(A-15)}$$

which, combined with Eqs. (A-13) and (A-14), gives

$$\frac{rf(m)}{m} = \frac{1}{g'(m_{dB})}$$

$$\frac{p}{m} = 1 - \frac{1}{g'(m_{dB})} \qquad \text{(A-16)}$$

Since $p/m = \sec^2 \theta$, we have

$$\tan^2 \theta = -1/g'(m_{dB}) \qquad \text{(A-17)}$$

We use Eq. (A-17) to determine θ, reduce it to 80 deg if necessary, and compute p and r as before. It is curious that optimal modulation angle can be so easily extracted from the slope of g.

III. Adjusting for the Universal Carrier Suppression Curve

Carrying out all of the above for several array configurations, we may find that the curves of optimal $\cos^2\theta$ vs r almost fall on top of each other, within 1 dB, say. We can then replace these curves by one universal curve. If, however, we combine a universal θ with an E_b/N_0 from above, then Eq. (1), which in our present notation reads

$$p = \frac{r E_b/N_0}{\sin^2 \theta} \qquad \text{(A-18)}$$

gives a wrong answer for p.

For the user's sake, it is necessary to apply a small fudge factor to E_b/N_0. As we mentioned, a small deviation of θ from the optimum hardly changes p, since p has been minimized as a function of θ. Given r, we have exact and universal values of $\cos^2\theta$, and an exact value of E_b/N_0. An adjusted value of E_b/N_0 is obtained from Eq. (A-18) by pretending that adjusted p = exact p:

$$\frac{(E_b/N_0)_{adj}}{\sin^2 \theta_{univ}} = \frac{(E_b/N_0)_{exact}}{\sin^2 \theta_{exact}}$$

These adjusted values were used for Fig. 4.

As a check, we can derive a new value of p from

$$(E_b/N_0)_{adj} = f(p_{new} \cos^2 \theta_{univ})$$

The new p might be about 0.01 dB greater than the old p derived from inserting either $(E_b/N_0, \theta)$ pair into Eq. (A-18).